Introducing the Economic Decision Framework into Socio-hydrology

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Introducing the Economic Decision Framework into Socio-hydrology

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Kurzfassung

Menschliche Entscheidungen können Wassersysteme verändern, während Wassersysteme menschliche Entscheidungen beeinflussen. Diese komplexen wechselseitigen Feedbackmechanismen sind das Kernstück des jungen aufstrebenden Forschungsbereiches der Soziohydrologie. In dieser Dissertation verwenden wir einen interdisziplinären Zugang um Forschungsfragen der Soziohydrologie zu beantworten. Dabei werden Modelle und Gesetze aus der Hydrologie und dem Ressourcenmanagement in ökonomische Modelle eingebettet. Im Zentrum stehen optimale Entscheidungen von Haushalten, Firmen oder Gesellschaften. Zur Berechnung der optimalen Handlungsstrategien werden verschiedene Methoden der dynamischen Optimierung weiterentwickelt und die entsprechenden Lösungen analytisch und numerisch aufbereitet.

Die in dieser Dissertation entwickelten vielfältigen Modelle stellen formale Modellierungsmöglichkeiten anhand konkreter Beispiele dar. Unseres Wissens nach sind das die ersten mathematisch fundierten Modelle in der Soziohydrologie, die außerdem menschliche Entscheidungen und die verknüpften Feedbacks mit Wassersystemen endogen beschreiben. Drei konkrete Modelle wurden für zwei Anwendungsbereiche entwickelt: Einerseits Hochwassermanagement, welches Wasserdynamiken berücksichtigt, andererseits Phosphormanagement, welches Wasserqualität beeinflusst.

Die ersten beiden Modelle beschreiben eine Gesellschaft, beziehungsweise eine Firma, die in einer Hochwasser-gefährdeten Region angesiedelt ist. Wir identifizieren die optimalen Investitionsstrategien in Hochwasserschutzmaßnahmen. Die gesellschaftliche Perspektive wird in einem ökonomischen Wachstumsmodell beleuchtet, wo sich zwei Investitionsstrategien langfristig als optimal herausstellen. Wir nennen sie reiche und arme Ökonomien, wobei es sich die Reichen leisten können in Hochwasserschutz zu investieren und Kapitalstöcke aufzubauen, während die Armen bevorzugt ihre kleinere Wirtschaftsleistung direkt konsumieren anstatt Hochwasserschutz aufzubauen. Somit müssen sie beim kleinsten Hochwasser Schäden in Kauf nehmen, wie das zum Beispiel im Mekong-Delta in Vietnam der Fall ist. Zusätzlich vergleichen wir Simulationen mit häufigerem, intensiverem oder stochastisch auftretendem überhöhten Wasserpegel.

Für die Firmenperspektive verwenden wir ein partielles Gleichgewichtsmodell, das neben den optimalen Investitionsstrategien auch die optimale Lage der Produktionsstätte berechnet. Wie oft, wann und wieviel ist eine repräsentative Firma bereit in Hochwasser-

schutzmaßnahmen zu investieren? Um diese Frage zu beantworten, verwenden und erweitern wir die Impulskontrolltheorie und entwickeln einen Fortsetzungsalgorithmus für die numerische Lösung. Wir finden heraus, dass höheres Überflutungsrisiko und langfristigere Planung die Investitionsbereitschaft in Hochwasserschutz erhöhen und danach auch immer die Investitionen in Produktionskapital aufstocken und somit mehr produzieren. Wir können daraus schließen, dass langfristige und nachhaltige Investitionsplanung zu einem gesunden Wirtschaftswachstum führt.
Das Ausmaß der wechselseitigen Einflüsse von Hochwasserschutzmaßnahmen und dem Wassersystem sind ausschlaggebend für die optimale Lage der Produktionsstätte, während ökonomische Rahmenbedingungen die optimale Investitionsstrategie prägen. Wenn bereits z.B. staatliche Hochwasserschutzmaßnahmen vorhanden sind, siedeln sich Firmen näher am Wasser an und die geringeren Ausgaben für Hochwasserschutz erlauben höhere erwartete Profite. Wenn Firmen durch Förderungen bereits große Produktionsanlagen haben, reduzieren sie mögliche Hochwasserschäden eher durch Abbau von Produktionskapital als durch eigenständigen Aufbau von Hochwasserschutz.

Das dritte Modell untersucht die Wechselwirkung von Haushaltskonsumentscheidungen, Düngeentscheidungen in der landwirtschaftlichen Produktion, und Umwelt-, speziell der Wasserqualität, um den Einfluss dieser Wechselwirkungen auf Phosphornutzung und -recycling zu verstehen. In einem ökonomischen Gleichgewichtsmodell wird unter anderem Angebot und Nachfrage von Phosphordünger modelliert und eine Materialflussanalyse integriert, um die Dynamiken der Phosphorbestände in Gewässern, Boden und Lebensmittelproduktion einzubinden. In diesem Rahmen wird die Einführung unterschiedlicher Abwasserbehandlungsverfahren zur Rückgewinnung von Phosphor untersucht. Aus dem Modell folgt, dass die Verwendung von recyceltem Phosphor die Umweltqualität und sogar die Profite für die Landwirtschaft erhöht. Außerdem ist die Ökonomie durch die Wiederverwertung von Phosphor weniger von Mineraldüngerimporten abhängig und somit resilienter gegen starke Preisschwankungen am globalen Phosphormarkt.
Insgesamt gibt es einen Bedarf Wiederaufbereitungstechnologien auszubauen, weil Landwirte bereit sind, mehr rückgewonnenen Phosphor als Düngemittel zu verwenden als üblicherweise verfügbar ist.
Insgesamt ist die Reduktion von Phosphor in den Böden und Gewässern nur möglich, wenn Phosphor rückgewonnen wird und der globale Düngerpreis steigt. Der Staat kann diesen technologischen Wandel unterstützen, in dem sie Rückgewinnungstechnologien für Phosphor fördert oder zusätzliche Steuern oder Abgaben für Phosphorimporte einführt. Alternativ müssten sich gesellschaftliche Werte verändern, so dass die Umweltqualität im Vergleich zum Konsum einen höheren Stellenwert einnimmt. Das erhöht die Bereitschaft mehr für Lebensmittel zu zahlen und ermöglicht den Landwirten somit einen finanziellen Beitrag zur Errichtung oder Erhaltung von Phosphorrückgewinnungsanlagen zu leisten.

Aus allen drei Modellen lernen wir, dass die Antizipation von Umweltveränderung, speziell Wasserdynamik und Wasserqualität, wirtschaftliche Entscheidungen und daraus entstehendes menschliches Verhalten ändert. Es gibt immer einen Kompromiss zwischen Investition in Konsum, Produktionskapital und Umwelt. Die optimale Investitionsstrategie hängt jedoch stark von den vorherrschenden ökonomischen Rahmenbedingungen und der Wertehaltung der Gesellschaft ab.

Abstract

How to describe two-way coupled feedbacks of water systems and humans? We use an interdisciplinary approach to study complex scientific questions in the emerging field of socio-hydrology. Economic frameworks are developed to integrate hydrological and resource management approaches, and advanced mathematical optimization methods are developed and improved.

Existing research on socio-hydrology mainly summarizes general qualitative relationships and describes case studies. In this thesis, we develop a variety of quantitative model frameworks to identify and capture selected socio-hydrological phenomena. In particular, we move from a positive to a normative approach. To our knowledge, these are the first socio-hydrological models, which integrate endogenous economic decisions and their feedbacks with water systems. Three models are developed for two socio-hydrological applications: Firstly, water dynamics affected by flood risk management and, secondly, water quality affected by phosphorus management.

The first and second model in this thesis describe a society or a firm, respectively, in a flood risk area and their optimal strategies to invest into flood protection measures.

We frame the societies perspective in an economic growth model, where we identify two optimal long term strategies depending on the initial endowment and denote them rich and poor economies. Whereas rich economies can afford to invest in flood defense and therefore avoid flood damage and develop high living standards, poor economies prefer consumption instead of investing in flood defense capital and end up facing flood damages every time the water level rises like e.g. observed in the Mekong delta.
Moreover, we compare simulations with more frequent, more intense and stochastic high water level events.

The firm's perspective is captured by an economic partial equilibrium model, which helps to understand the firm's optimal location choice and its investment strategies. How often, when and how much are firms willing to invest in flood risk protection measures? We apply Impulse Control Theory and develop a continuation algorithm to solve the model numerically. We find that, the higher the flood risk and the more the firm values the future, i.e. the more sustainable the firm plans, the more the firm invests in flood protection measures. Investments in productive capital follow a similar path. Hence,

planning in a sustainable way leads to economic growth.
Sociohydrological feedbacks are crucial for the location choice of the firm, whereas different economic settings have an impact on investment strategies. If flood defense is already present, e.g. built up by the government, firms move closer to the water and invest less in flood defense, which allows firms to generate higher expected profits. Firms with a large initial productive capital surprisingly try not to keep their market advantage, but rather reduce flood risk by reducing exposed productive capital.

The third model of this thesis provides a general equilibrium framework to study households' consumption and farmers' fertilizer decisions to understand the coupled human-resource-environment feedbacks associated with phosphorus use and recycling. We model the demand and supply of phosphorus and changes in resource stocks by combining a multi-sector economic model with a material flow model.
Within this framework we study the effects of implementing phosphorus recovering technologies from waste water. We show that using recycled phosphorus fertilizer increases environmental quality and profits in the agricultural sector. Furthermore, the economy does not depend as much on mineral fertilizer imports and is therefore more resilient to a price increase on the global phosphorus market. However, there is a need to improve the quantity and quality of recycled phosphorus products, because farmers would be willing to use 100% of the available recovered phosphorus fertilizer.
Overall, reduction of phosphorus in soil and water bodies as result of economic decisions is only possible if phosphorus is recovered from waste water and the prices of imported mineral fertilizer rise. Policy makers can support this technological change by subsidizing recycled phosphorus or introducing taxes or tolls for imported mineral fertilizer.
Alternatively, societal values would have to change. Such a change may be induced by putting a higher value on a healthy environment and hence being willing to pay more for food and consequently production inputs like recycled phosphorus fertilizer.

To sum up, each model demonstrates that economic decisions and consequently human behavior adjust if decision makers anticipate environmental changes, in e.g. water systems. There is always a trade-off between consumption, investments in economic production and investments in prevention and abatement measures for a safe environment. The optimal strategy crucially depends on the economic conditions and the societal values.

Declaration

I hereby certify that no other than the sources and aids referred to were used in this thesis. All parts which have been adopted either literally or in a general manner from other sources have been indicated accordingly.

I certify that the main contribution of this thesis, *Introducing the economic decision framework into socio-hydrology*, is my own work.

Vienna, January 2018
Johanna Grames

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I hope that this thesis is motivation to work across disciplines and an inspiration for future research - not only at TU Wien.

Contents

1 Introduction

Human and water are closely related in a dualistic manner: on one hand water has great influence on human's welfare and social development, on the other hand human's activities greatly affect water and water systems (Falkenmark, 1979). Falkenmark (1997) introduces a conceptual framework to visualize fundamental linkages between the water cycle and human activities. Sivapalan *et al.* (2012) define the field of socio-hydrology - the science of people and water - aiming to understand the dynamics and co-evolution of coupled human-water systems. Many review papers summarize the efforts in the emerging field of socio-hydrology (Kelly *et al.*, 2013b; Wescoat, 2013; Kandasamy *et al.*, 2014; Sivapalan *et al.*, 2014; Di Baldassarre *et al.*, 2015; Sivapalan & Blöschl, 2015; Troy *et al.*, 2015; Blair & Buytaert, 2016; Levy *et al.*, 2016; Lu *et al.*, 2016; McMillan *et al.*, 2016; Bekchanov *et al.*, 2017; Pande & Sivapalan, 2017; Srinivasan *et al.*, 2017; Wada *et al.*, 2017). Quantitative socio-hydrological models oftens consist of coupled differential equations that capture the dynamics of a specific system (Wada *et al.*, 2017). It is time for socio-hydrology to move beyond individual case studies, and find generalized, but locally relevant descriptions of changes in the (large-scale) human-water system (McMillan *et al.*, 2016; Wada *et al.*, 2017). To enable a holistic view, it is important to integrate other disciplines. We introduce the economic decision framework in the field of socio-hydrology. This is important for two reasons: firstly, human decisions cannot be described like natural laws e.g. in hydrology. Secondly, human decisions in a holistic framework go beyond the choice of irrigation, reservoir buildings, water allocation, and dam building. Individuals prioritize consumption decisions of other products and the choice of intangible values like environmental quality, which in turn effect water systems indirectly.

Our conceptual models aim to understand specific two-way coupled human-water systems. We complement socio-hydrology frameworks like M. Carey (2014) and Elshafei *et al.* (2014) and integrate components from economic growth models, agricultural, environmental and ecological economics.

Various disciplines explicitly study human-water systems. However, socio-hydrology

considers the two-way coupled feedbacks between human and water systems to describe and understand interlinks and their consequences. Pande & Sivapalan (2017) and Blair & Buytaert (2016) distinguish socio-hydrology from associated disciplines. In the following we give a short overview of related research fields.
Coupled human and natural systems (CHANS) are integrated systems where humans and nature interact, e.g. social-ecological systems (SES) or human-environment systems (Crook, 1970; Liu *et al.*, 2007; Schlueter *et al.*, 2012; Levin *et al.*, 2013) and aim to understand the complexity of human-nature interactions at the heart of many contemporary problems (Kramer *et al.*, 2017). The hydro-social cycle (Swyngedouw, 2009) centers the water cycle in the interdisciplinary analysis based on Falkenmark (1979), who introduces hydrosociology to describe water and human interaction within a river basin as a basic territorial unit. Large-scale hydrological models (LHM) consider the interaction between terrestrial water fluxes and human activities, including water use and reservoir regulation (Bierkens, 2015; Wada *et al.*, 2017). Integrated water resource management (IWRM), global water resource management (e.g. Wanders *et al.*, 2015) and adaptive water management (e.g. Savenije *et al.*, 2014) are management tools for the development of water resources considering social and economic needs, and the protection of ecosystems. A holistic water resource-economic model (HWEM) embeds water resources and economic components into a consistent mathematical programming model, with the objective of maximizing economic profits from water uses in various sectors (Cai, 2008). Rosegrant (2000), Draper (2003), Ringler (2004), Jenkins (2004), Ward (2006) and Marques (2006) used this model class for water market analysis. (Integrated/Coupled) hydro-economic models (HEM), which are also called water economy models (WEM) (Bekchanov *et al.*, 2017), are driven by the economic value of water or economically evaluated to provide policy insights and reveal opportunities for better management (R.Brouwer, 2008; Harou, 2009; Booker *et al.*, 2012). Bio-economic models integrate biophysical models and economic mathematical programming models (Dellink *et al.*, 2011), but there is a lack of literature regarding the implicit or explicit assumptions of these models and economic theory, their main advantages compared to conventional economic approaches, and their specific contributions in strengthening collaboration and improving integration between different disciplines (Flichman & Allen, 2015). Finally, econometric models in resource management estimate e.g. water demand functions and valuation of ecosystem services.

Different approaches try to answer questions raised in the above research fields, especially socio-hydrology. We use coupled-component modelling (CCM) approaches to integrate (socio-)hydrology and resource managament into economic model frameworks. Additionally, we study different scenarios to investigate the outcomes of specific policy implementations or different initial conditions. The proposed work is also classified into a heuristic or knowledge based modelling approach, since we assume relationships between important model variables. Kelly *et al.* (2013a) and Blair & Buytaert (2016) also list other approaches to model human-water systems: agent-based modelling (ABM), system dynamics, pattern-oriented modelling (POM) and Baysian Networks. We supplement the variety of methods by introducing two more modelling approaches in socio-hydrology. Firstly, optimal decisions and the underlying optimization methods help to deeply understand complex human and water behavior and its feedbacks. Secondly, a general equilibrium framework provides a holistic view and a better understanding of feedbacks between agents, and between agents and natural systems.

D.C. McKinney (1999) classifies three different model types to capture economic criteria in i.e. integrated hydro-economic models like Booker *et al.* (2012): Holistic, compartment (modular) and computable general equilibrium (CGE) models. The compartment approach loosely connects the different economic and hydrologic components and usually only output data is transferred. The holistic approach provides an integrated analytical framework to tightly connect both components within one single unit. Cai (2008) describes holistic models that simulate coupled human and natural systems within a consistent system as effective tools that have been applied to analyze (1) the sustainability of irrigated crop production (Cai (2002), Jenkins (2004), Rodgers (2004), Jenkins (2004), Booker (2005)), and (2) the hydrological ecological sustainability concerning the limit of water quantity supply, water quality, and ecological functions for (a) groundwater sustainability (McCarl, 1999; Marques, 2006; Pulido-Velazquez & Sahuquillo, 2006), (b) destination lake ecosystem (Cai, 2003), and (c) instream ecological water requirement (Ringler, 2006).
Contrary to most holistic and compartment model types, CGE models start the integration procedure from the economic system and attempt to link economic relationships to the hydrological system (R.Brouwer, 2008). Harou (2009) lists the different model approaches in Table 1.1. Models can differ within different perspectives: Is it a simulation or an optimization? Is it a deterministic or stochastic time series? Is it a modular or holistic approach? The proposed thesis captures all approaches, but focuses on dynamic optimization within holistic models.

In the proposed socio-hydrology models we develop, improve and apply dynamic optimization techniques. The aim of dynamic optimization is to control a dynamic system such as mechanical motions, physical processes, and economic systems in an optimal way. Optimization is not only a goal, it could be a tool for understanding the mechanisms of a system. [Veliov, V.M.]
We can use both analytical and numerical methods to solve optimization problems. We focus on analytical solutions, but also consider numerical methods once analytical solutions are no longer feasible. An optimization problem consists of an objective function that is maximized or minimized depending on a control variable, and one or more constraints. The Lagrangian method is used to derive optimal static solutions. Adding a time perspective and state variables requires different methods. We can use optimal control or dynamic programming to solve dynamic optimization problems. For analytical solutions it is more popular to use optimal control theory. The basic concepts are the Pontryagin's maximum principle (Pontryagin, 1962) and the Hamilton-Jacobi-Bellman equation (Bellman, 1954). Additionally we develop the Impulse Control theory further. The Impulse Control Problem is solved using the impulse maximum principle (Blaquiere, 1985; Rempala & Zabczyk, 1988; Chahim, 2012), which we prefer over the more general theory of viscosity analysis and quasivariational inequalities (e.g. Barles, 1985; El Farouq

Options	Summary	Advantages	Limitations
Simulation/optimization			
Simulation	Time-marching, rule-based algorithms; Answers question: "what if?"	Conceptually simple; existing simulation models can be used, reproduces complexity and rules of real systems	Model only investigates simulated scenarios, requires trial and error to search for the best solution over wide feasibility region
Optimization	Maximizes/minimizes an objective subject to constraints*; answers question: "what is best?"	Optimal solutions can recommend system improvements; reveals what areas of decision space promising for detailed simulation	Economic objectives require economic valuation of water uses; ideal solutions often assume perfect knowledge, central planning or complete institutional flexibility
Representing time			
Deterministic time series	Model inputs and decision variables are time series, historical or synthetically generated	Conceptually simple: easy to compare with time series of historical data or simulated results	Inputs may not represent future conditions; limited representation of hydrologic uncertainty (system performance obtained just for a single sequence of events)
Stochastic and multi-stage stochastic	Probability distributions of model parameters or inputs; use of multiple input sequences ('Monte-Carlo' when equiprobable sequences, or 'ensemble approach' if weighted	Accounts for stochasticity inherent in real systems	Probability distributions must be estimated, synthetic time series generated; presentation of results more difficult; difficulties reproducing persistence (Hurst phenomenon) and non-stationarity of time series
Dynamic optimization	Inter-temporal substitution represented	Considers the time varying aspect of value; helps address sustainability issues	Requires optimal control or dynamic programming
Submodel integration			
Modular	Components of final model developed and run separately	Easier to develop, calibrate and solve individual models	Each model must be updated and run separately; difficult to connect models with different scales
Holistic	All components housed in a single model	Easier to represent causal relationships and interdependencies and perform scenario analyses	Must solve all models at once; increased complexity of holistic model requires simpler model components

* If optimized time-horizon is a single time period, the model can be considered a simulation model that uses an optimization computational engine.

Table 1.1: Some design choices, options, and implications for building a hydro-economic model. Reprinted from Harou (2009).

et al., 2010)).
To derive numerical solutions we apply e.g. the (multipoint) boundary value approach (Grass & Chahim, 2012). The idea is to solve a boundary value problem (BVP) based on the system dynamics given by the canonical system. For the impulse control problem the boundary value information is updated at every discontinuity caused by an impulse. We use the specific MATLAB® -Toolbox *OCMat* from Grass & Seidl (2013) for the numerical solutions of the dynamic optimization problems.
Additionally, our third chapter is based on a general equilibrium framework that underpins analytical solutions, and we provide a MATLAB®-Algorithm to run the dynamic model numerically.

We study the socio-hydrological feedbacks between economic decisions and both physical and chemical impacts on water systems. The fields of application are flood and phosphorus management.
Flooding affects agriculture, industry, residences, infrastructure and environment. Consequently, an interdisciplinary approach is necessary to draw conclusions and develop policies. Flood management models have both a technical and a behavioral component. Early models of flood management emphasized the technical component, but more recent models such as Galloway (2009) and R. Suddeth & Lund (2010) have demonstrated the value of the integration of the behavioral aspects with the technical engineering aspects of the model (Booker *et al.*, 2012). Different policies such as levee protection, expanded insurance markets, and flood plain zoning are possible to avoid flooding. Models simu-

late and analyze flood damages under alternative policies or without policy interventions (Z. Sheng & Lacewell, 2005). E.g. economic (growth) models (Estrada *et al.*, 2015) or agent-based models (e.g. Li *et al.*, 2015) aim to understand the impact of floods on individual behavior, Bubeck *et al.* (2017) discusses the impact of flood events on societies. We also consider the impact that societies have on the occurence of flood events based on Di Baldassarre *et al.* (2013) and Viglione *et al.* (2014), who developed a socio-hydrology model to explain the feedbacks between settlements close to rivers and flooding events. Phosphorus (P) is an essential element for plant and animal growth and is necessary to maintain profitable crop and livestock production (Sharpley & Beegle, 2001). The need for enhancing phosphorus management is to protect surface waters from eutrophication, ensure future food security under uncertain supply, and shift to a circular economy (Withers *et al.*, 2015; Zoboli *et al.*, 2016b). Considering environmental impacts of phosphorus recovery from municipal waste water (Zoboli *et al.*, 2016a; Amann *et al.*, 2018) we develop an economic decision framework to understand the two-way coupled feedbacks of phosphorus management and water quality.

The aim of the thesis is to understand socio-hydrological systems by providing economic decision frameworks. Chapter 2 describes a closed economy living close to a river or a coast. Depending on the water level, which is modeled with a continuous periodic function, floods can occur. Investments in defense capital can avoid floods, but still, people maximize their utility depending on consumption. What are the optimal investment decisions and how do the capital stocks aggregate? The described optimization problem consists of one objective function with two control variables and two state variables including a nonautonomous periodic term. We use a MATLAB® -Toolbox to solve that complex periodic problem. Chapter 3 provides a firm's perspective. The firm has to deal with damage costs in case of flooding. How does it optimally invest in production capital and flood protection given a stochastic water function? We enhance impulse control theory to maximize the expected profit of the firm. Chapter 4 models a two sector-economy including a waste-water treatment plant, where phosphorus flows between households, agriculture and industry. Households maximize their utility choosing their consumption and considering environmental quality. Firms in both sectors maximize their profits. Market clearing at four different good markets leads to long-term equilibrium prices and quantities. We can identify policies to protect the environment and understand consumption decisions and fertilizer choices. Finally, Chapter 5 presents an overview of the results, together with the conclusions, and gives an outlook for future research.

2 Modeling the interaction between flooding events and economic growth

The present chapter corresponds to the following scientific publication in its original form:

Grames J., Prskawetz A., Grass D., Viglione A., Blöschl G. (2016): Modeling the interaction between flooding events and economic growth. Ecological Economics, Volume 129, p.193-209, ISSN 0921-8009,
http://dx.doi.org/10.1016/j.ecolecon.2016.06.014.

Abstract

Recently socio-hydrology models have been proposed to analyse the interplay of community risk-coping culture, flooding damage and economic growth.
These models descriptively explain the feedbacks between socio-economic development and natural disasters such as floods. Complementary to these descriptive models, we develop a dynamic optimization model, where the inter-temporal decision of an economic agent interacts with the hydrological system. We assume a standard macro-economic growth model where agents derive utility from consumption and output depends on physical capital that can be accumulated through investment. To this framework we add the occurrence of flooding events which will destroy part of the capital.
We identify two specific periodic long term solutions and denote them rich and poor economies. Whereas rich economies can afford to invest in flood defense and therefore avoid flood damage and develop high living standards, poor economies prefer consumption instead of investing in flood defense capital and end up facing flood damages every time the water level rises like e.g. the Mekong delta. Nevertheless, they manage to sustain at least a low level of physical capital. We identify optimal investment strategies and compare simulations with more frequent, more intense and stochastic high water level events.

2.1 Introduction

Since the beginning of time, people have settled close to rivers and this is still the case nowadays. Rivers enable ways of transport, supply water for industry and agriculture and enhance the quality of living due to lively nature and beautiful scenery. However, living close to rivers also involves the risk of flooding, one of the most devastating natural threats on Earth (Ohl & Tapsell, 2000), whose impact has increased over the past decades in many regions of the world (Dankers *et al.*, 2014; Hall *et al.*, 2014). In order to avoid flood damage, societies have developed projects involving structural defenses (e.g. dams, levees, retention basins) and non-structural measures (e.g. land-planning, insurance, forecasting, see e.g. Kundzewicz (2002)). These investments are costly, but may avoid damage in the future. This is an interesting dynamic trade-off structure which we aim to analyse in a stylized socio-hydrological model that is embedded in a macroeconomic set up. To account for the dynamic nature of optimal investment strategies, we apply dynamic optimization methods.

Floods and their consequences have been studied with different model approaches: Recent Integrated Assessment Models (IAM) aim to understand the interaction of society and floods (Merz *et al.*, 2014) in a broad context. Climate change leads to more and bigger floods in certain regions (Milly *et al.*, 2002). Such models typically do not account for the impact of changes in the environment on economic growth (Estrada *et al.*, 2015). The aim of Agent Based Models (ABMs) such as Dawson *et al.* (2011), Safarzyńska *et al.* (2013) and Li *et al.* (2015) is to understand the impact of floods on individual behaviour. ABMs can provide a qualitative analysis of the consequences of floods on different levels: the individual/micro-level, the aggregated economy/macro-level and the firm level/meso-level. Complementary Input-Output-Models (Koks *et al.*, 2014; Hasegawa Ryoji, 2009) provide a quantitative cost-benefit-analysis of case studies. Okuyama analysed these model frameworks as well as computational equilibrium models for disasters. A dynamic spatial computable general equilibrium model based on the dynamic structure of a Ramsey growth model was developed by Nakajima *et al.* (2014) to numerically measure flood damage costs. It displays the dynamic tradeoff between the costs today and future savings, investments and consumption. Besides simulation modeling approaches, optimization models have been developed to calculate optimal dike heights (Eijgenraam, 2006; Brekelmans *et al.*, 2012; Chahim *et al.*, 2012). Larger stochastic programming models in water resource management and flood management (Li *et al.*, 2007; Liu *et al.*, 2014; Kleywegt *et al.*, 2002; Needham *et al.*, 2000) only allow optimal solutions for discrete variables and finite time horizon. Moreover, most of these models are linear, have only one control variable, either none or linear constraints and are therefore quite different to the proposed economic growth model in our chapter.
While existing models on flood management have focused on the analysis at the firm level (e.g. Chahim *et al.* (2013) and Eijgenraam *et al.* (2014), who apply impulse control models for optimal dike heightening within an economic cost-benefit decision problem

to minimize the sum of the investment and expected damage cost), our model aims to include flood dynamics into a macroeconomic growth model.
So far, floods have been rarely analysed in a macroeconomic model of economic growth considering not only direct and indirect damage costs, but also loss of future potential economic growth through dynamic consumption and investment decisions.
In environmental economics this approach is quite common. Economic growth models have been applied to study, e.g., the effect of climate change on long run economic growth (Xepapadeas *et al.*, 2005). More formally, these models commonly postulate that pollution causes economic losses via a damage function that is positively related to an increasing temperature caused by pollution. (Rezai *et al.*, 2014; Millner & Dietz, 2015; Morisugi & Mutoh, 2012; Zemel, 2015). Pollution itself is commonly modeled via the flow or stock of emissions. Indeed, emissions and investment in emission abatement have strong analogies to extreme water events (floods, droughts) and investment in abatement (flood defense capital, reservoirs), respectively. It therefore seems an obvious choice to apply this modeling framework also in the context of flood modeling. Similar to the increase in the temperature that underlies the economic damage in climate change models, the water level underlies the occurrence of floodings and hence the economic damage.
There is a new research line, socio-hydrology, that deals with such coupled systems. The main thrust of socio-hydrology is to add a new perspective to former models and studies in hydrology by coupling dynamics of human populations, economic growth and general resource availability (Sivapalan *et al.*, 2012; Levy *et al.*, 2016). Socio-hydrology aims at understanding emergent patterns and paradoxes that result from long-term co-evolution of non-linearly coupled human-water systems. Elshafei *et al.* (2014) and Sivapalan & Blöschl (2015) developed prototype frameworks for socio-hydrology models. Di Baldassarre *et al.* (2013) and Viglione *et al.* (2014) developed a socio-hydrology model to explain the feedbacks between settlements close to rivers and flooding events. Di Baldassarre *et al.* (2015) use the model to capture processes such as the "levee effect" (e.g. Montz & Tobin, 2008) and the "adaptation effect" (Penning-Rowsell, 1996; IPCC, 2012; Mechler & Bouwer, 2014), which traditional flood risk models do not include. Pande *et al.* (2014) were one of the first who added a water related problem to a standard economic model of finitely lived agents, the so called overlapping-generations model (OLG).
In this chapter, we build a macro-economic model in the context of floods and use a dynamic optimization model which is a different perspective from the more common descriptive models, simulations and scenario analyses. This is where we regard our model to add to the literature. More specifically, while there exist economic growth models that include the feedback between the environment and economic output, our novel contribution is to add an exogenous time varying water level function and study the resulting optimal path of consumption and investment. Mathematically this poses the challenge that we have to solve a non-autonomous optimization model.

Our model uses the model of Di Baldassarre *et al.* (2013) and Viglione *et al.* (2014) as a starting point. Their simulations show that building high levees leads to fewer flooding events with higher impacts which may slow down economic growth. Protecting a settlement by levees can, however, increase the damage to downstream settlement due to the loss of flood retention volume. Furthermore, building levees or any other defense capital will lower flooding probability and may therefore increase the willingness of citizens to build close to the river. If water levels rise higher than the crest of the levees, the physical capital next to the river is destroyed. Since there is a higher physical capital stock next to the river, the flood hits even harder on the economy.

Based on their model set up we build an economic model to analyse the tradeoffs and feedbacks associated with settlements close to rivers. In the original model, decisions depend on social memory that is accumulated after the experience of flooding events and then decays over time. In our economic model framework memory is captured in the dynamics of the state variables which reflect investment and consumption decisions in the past that are related to flooding events. But also future choices are taken into account. We assume a social planner who decides optimally on investment and consumption to maximize not only current but also long term utility. The concept of utility constitutes a mathematical representation of preferences. Preferences in our model are formed over consumption but may also be influenced by social status (e.g. Fisher & Hof, 2005). We abstract from social status or other forms of social norms and values in our model and our utility function does not change over time to ensure an unambiguous assignment of feedbacks.

Moreover we assume that our decision maker represents a social planner whose aim is to maximize the discounted stream of current and future utility of consumption by choosing the time path of investment and consumption and taking into account the dynamics of physical and defense capital. The trade-off for the decision maker is between consumption and investment where the former reduces and the latter augments the capital stock. As typical for economic problems, this trade-off is constrained by the total output, i.e. consumption and investment cannot exceed the output generated. Hence we are facing a standard economic decision problem of optimization under scarce resources.

We assume two types of capital: physical capital and defense capital. Decision makers can invest in physical capital, such as machines, buildings and infrastructure. On the other hand, investments in defense capital can avoid the actual damage of floods and have thereby a positive influence on output. Total output of the economy consequently depends on both capital stocks.

We apply a periodic non-autonomous exogenous function to represent the water level. The periodic water function is introduced in Grames *et al.* (2015). Even though the assumption of non-stochastic flood occurrence is a strong one, we believe that useful insights on the system can still be obtained. Alternatively, we can interpret our water function as approximation of past flood events. Assuming the periodic non-autonomous exogenous function for flood occurrence allows us to solve the dynamic optimization problem, for which we further develop the solution method of Moser *et al.* (2014) where a similar mathematical problem in the context of renewable energy has been solved.

Including a non-autonomous exogenous deterministic function into a dynamic decision framework over an infinite time horizon requires already quite sophisticated methodologies of optimization and a highly challenging numerical approach. If we would model the water level function stochastically, the long run optimization problem could neither be solved analytically nor numerically. Recent research in that field of stochastic optimization is using much simpler objective and state functions (Nisio, 2015) without such strong nonlinearities as they exist in our model. Climate models include uncertainty in the timing of events (Tsur & Withagen, 2013), where the hazard rate of the event can depend on e.g. a stock of pollution of greenhouse gases (Zemel, 2015). Our exogenous water level function does not depend on any state variable, so the solution method applied in e.g. Zemel (2015) cannot be transferred to our model. Moreover, climate change models with an exogenous hazard rate capture only one random event (Zeeuw & Zemel, 2012), whereas floodings in our model are recurrent random events over an infinite time horizon. Hence the model structure of stochastic climate models and our flood model is fundamentally different. However, in order to investigate the sensitivity of our results to the stochasticity of floods, we also present simulations of our model assuming a stochastic water level function like e.g. Viglione *et al.* (2014).

The aim of this chapter is to understand the mechanisms behind investment decisions in the context of flood risk prevention. For this purpose we choose a stylized macroeconomic model to investigate the optimal investment strategy between flood protection measures and physical capital to enable economic growth.
The remainder of the chapter is organized as follows. The following section provides an introduction to the feedbacks between society and floods and outlines the model framework and its equations. In a first step we present various simulations of our model and show the sensitivity of the resulting dynamics on the investment strategy chosen. To determine the optimal investment strategy between physical and defense capital taking into account the dynamic feedback between the economic and hydrological system we next apply the tools of dynamic optimization. We also show the sensitivity of the model dynamics on the initial endowment of the economy. In particular, the optimal investment strategies will be determined by the state of the economy. Furthermore, we investigate how the optimal investment strategy will change depending on the frequency and amplitude of the high level water events and whether a more efficient flood defense capital may foster economic growth. Last, we embed the optimal solutions in a stochastic simulation run. The chapter concludes by discussing our scenarios in the context of flooding in various regions of the world.

2.2 Modeling the interaction between flooding events and economic growth

2.2.1 Feedbacks between society and floods

Floods affect settlements close to rivers by destroying existing capital. Societies have developed different approaches to prevent or mitigate the damage. Building dikes, levees or flood control basins may prevent flood waters entering the settlements. Warning

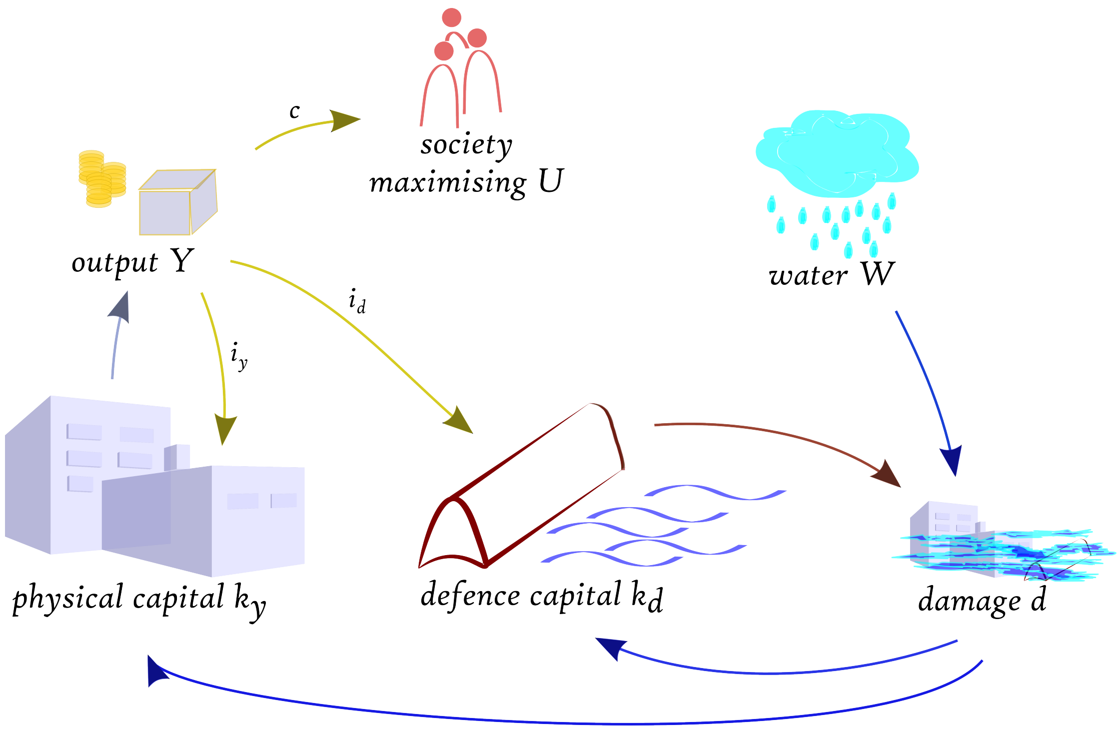


Figure 2.1: Overview of the dynamics within the presented model. The society chooses to consume ($c(t)$) or invest ($i_y(t)$ and $i_d(t)$) the economic output $Y(t)$ into the capital stocks.

systems to assist in evacuations and settling further away from the river (Viglione *et al.*, 2014) may also be regarded as mitigation measures.
In our model we represent all flood prevention technologies by one variable and name it *defense capital*. Similarly we model the physical capital stock — which represents machines, buildings, infrastructure — by one variable named *physical capital*. We assume that a flood causes damage of physical capital if the water level exceeds a specific threshold of the defense capital. The society chooses how much it invests into defense capital and therefore influences the occurrence of floodings.
The physical capital stock is used to produce economic output. Aggregate output in an economy can be used for consumption and investments in either physical or defense capital stock. We assume that the decision of the optimal share of output used for consumption and investment is taken by a social planner. This means we abstract from a market framework where factor renumerations such as interest rates on capital or wages for labour input would determine the optimal allocation of output between consumption and the two types of investment.
We assume a closed economy, which implies that all of the produced output will be used, and no further trade with other communities is possible.

Fig. 2.1 displays the dynamics of the model. Economic output $Y(t)$ depends on the amount of physical capital $k_y(t)$. The output can be either consumed $c(t)$ or invested in physical $i_y(t)$ or defense capital $i_d(t)$. The society chooses the level of consumption and the amount of investment into physical and defense capital in order to maximize utility. The defense capital can prevent the damage $d(W(t), k_d(t))$ caused by flooding events. The occurrence of flooding events depends on the water level $W(t)$. In case of flooding, both capital stocks are damaged.

2.2.2 Model equations

To model the aforementioned interaction between society and flood events we first define the utility function of the social planner and its choice variables. Next, we determine how output is produced in the economy and explain the dynamics of physical and defense capital which constitute the dynamic constraints for the optimization problem of the social planner. To model the water level we introduce an exogenous periodic function over time. Together with the level of defense capital, the water level will then determine the extent of the damage.

Utility function

The objective of the social planner is to maximize the discounted stream of aggregate utility $U(c(t)) = \ln(c(t))$ which depends positively on the consumption level $c(t)$:

$$\max_{\{c(t), i_d(t)\}} \int_0^\infty e^{-\rho t} U(c(t)) dt \tag{2.1}$$

where ρ denotes the time preference and indicates to which extent the social planner prefers utility of consumption today compared to utility of consumption tomorrow. Consumption $c(t)$ and investment in defense capital $i_d(t)$ are control variables [1] to be chosen optimally to maximize equation (2.1), given the level of output and dynamic constraints of physical and defense capital as stated below. More specifically, the dynamic optimization of the social planner guarantees that any decision taken today also incorporates the feedback on the future evolution of the system.

Since at every time period consumption together with investment in physical and defense capital is bounded by the available output, the choice of two variables implies the optimal choice of the third variable (investment in physical capital in our case).

[1] In a less technical setting we refer to the control variables as decision variables.

Economic output

Output $Y(t)$ is given by a Cobb Douglas-production function

$$Y(t) = Ak_y(t)^\alpha \tag{2.2}$$

that depends on the physical capital stock $k_y(t)$ and an exogenous level of technology A. The production input factor labor is normalized to one. $\alpha \in [0,1]$ denotes the elasticity of the production input factor capital.
Output can be used for consumption $c(t)$ as well as for investment in physical capital $i_y(t)$ and investment in defense capital $i_d(t)$. Since output is given in [\$] and the unit of the defense capital is [m] we need to transform investment in defense capital $i_d(t)$ given in [m] into costs $Q(i_d(t)) = \theta_0(\theta_1 i_d(t) + \theta_2 i_d(t)^2)$ given in [\$]. The parameters θ_i weight the linear and quadratic parts of the costs and are calculated according to Slijkhuis *et al.* (1997) and Bedford *et al.* (2008).
The overall budget constraint for the social planner is therefore given as:

$$Y(t) = c(t) + i_y(t) + Q(i_d(t)) \tag{2.3}$$

State dynamics

Following the standard Ramsey model we write the dynamic constraints by the following two state equations for physical and defense capital:

$$\dot{k}_y(t) = i_y(t) - d(k_d(t), W(t))k_y(t) - \delta_y k_y(t) \tag{2.4}$$
$$\dot{k}_d(t) = i_d(t) - \kappa_d d(k_d(t), W(t))k_d(t) - \delta_d k_d(t) \tag{2.5}$$

Each capital stock can be augmented by investments $i_y(t)$ and respectively $i_d(t)$ and depreciates by a constant rate δ_y, respectively δ_d. Moreover, flood damage $d(k_d(t), W(t))$ decreases both capital stocks.[2] The flood damage rate $d(k_d(t), W(t))$ is in the interval [0,1]. We allow for the fact that the damage may be different for physical and defense capital by introducing the parameter κ_d in equation (2.5).

Damage function

Flood damage and flood recovery are complex and discussed in various papers (Di Baldassarre *et al.*, 2015; Merz *et al.*, 2014). Our model constitutes a stylized model with the focus to analytically study and understand the basic feedbacks and mechanisms between society and hydrology. Therefore we assume a damage function $d(k_d(t), W(t))$ analogous to Viglione *et al.* (2014) and a recovery rate based on the economic capital, the technology and the optimal consumption behavior. Since the recovery is endogenous in

[2] Rezai *et al.* (2014) model similar dynamics for pollution.

our optimization framework, we can describe the optimal consumption and investment behavior given an exogenous forcing of the water level $W(t)$.
The amount of damage is related to the flood intensity $W_{eff}(W(t), k_d(t)) = W(t) + \xi_d k_d(t)$ which is a function of the water level $W(t)$ and the additional amount of water $\xi_d k_d(t)$. This additional amount of water occurs due to existing defense capital $k_d(t)$ such as levees: Levees at one place protect this area from flooding, but increase water levels further down the river due to loss of flood plain retention (Di Baldassarre *et al.*, 2013).
If the flood intensity $W_{eff}(W(t), k_d(t)) = W(t) + \xi_d k_d(t)$ exceeds the flood defense capital $k_d(t)$ and the levees spill over, a damage of the overall capital stock occurs. The higher the effective water level $W_{eff}(W(t), k_d(t))$, the higher the direct damage of the flooding (Jonkman *et al.*, 2008). The damage rate $d(k_d(t), W(t)) \in [0, 1]$ gives the relative damage of the capital stocks. Beyond $k_d(t)$, the damage of the flood is proportional to the effective water level of the flood $W_{eff}(t)$ and, also, to the flood duration, which is the time interval when $W_{eff}(W(t), k_d(t)) > k_d(t)$ holds. This assumption reflects the common situation that structural damage is related to the water level, while damage to industry production and stocks is related to the duration of the inundation. The damage rate is then represented as follows.

$$d(k_d(t), W(t)) = \begin{cases} 1 - \exp\left(-W_{eff}(t)\right) & \text{if } W_{eff}(W(t), k_d(t)) > k_d(t) \\ 0 & \text{else} \end{cases} \tag{2.6}$$

For ease of obtaining a numerical solution of the optimization model, we approximate the damage function (2.6) with a continuous function. Still, damage ($d(k_d(t), W(t)) > \epsilon$ with a positive ϵ close to zero) only occurs if $W_{eff}(W(t), k_d(t)) > k_d(t)$. We choose the signum-approximation function and base it on the following four assumptions: First, the minimum value is 0 for the water level $W(t) \leq 0$. Second, if $W_{eff}(W(t), k_d(t)) = W(t) + \xi_d k_d(t) > k_d(t)$ and $W(t) \to \infty$ for $\to \infty$, we reach the maximum value 1. Third, the inflection point is at $W(t) + \xi_d k_d(t) = k_d(t)$. Fourth, the gradient at the inflection point is chosen such as to approach infinity to approximate the jump between 0 and the relative damage $d(t) > 0$ in equation (2.6). Furthermore, we add a multiplicative term $(1 - \frac{1}{1+W(t)^\eta})$ that is increasing in the water level $W(t)$ and bounded by the interval [0,1]. This term ensures that the damage is higher for a more intense flooding.

$$d(k_d(t), W(t)) = \frac{1}{2}\left(\tau_3 + \frac{\tau_2 + W(t) - (1 - \xi_d)k_d(t)}{\sqrt{(W(t) - (1 - \xi_d)k_d(t))^2 + \tau_1}}\right)\left(1 - \frac{1}{1 + W(t)^\eta}\right) \tag{2.7}$$

The coefficients τ_i adjust the accuracy of the approximation of (2.6) with (2.7). For the calculations we used $\tau_1 = 0.001$, $\tau_2 = 0$ and $\tau_3 = 1$.
Fig. 2.2 shows the damage rate with respect to the water level $W(t)$ for different values of defense capital stock $k_d(t)$. If the defense capital is higher than the water level, the damage is closer to zero (no damage) until the inflection point $W(t) = (1 - \xi_d)k_d(t)$ given in equation (2.6) and then close to one (total damage).

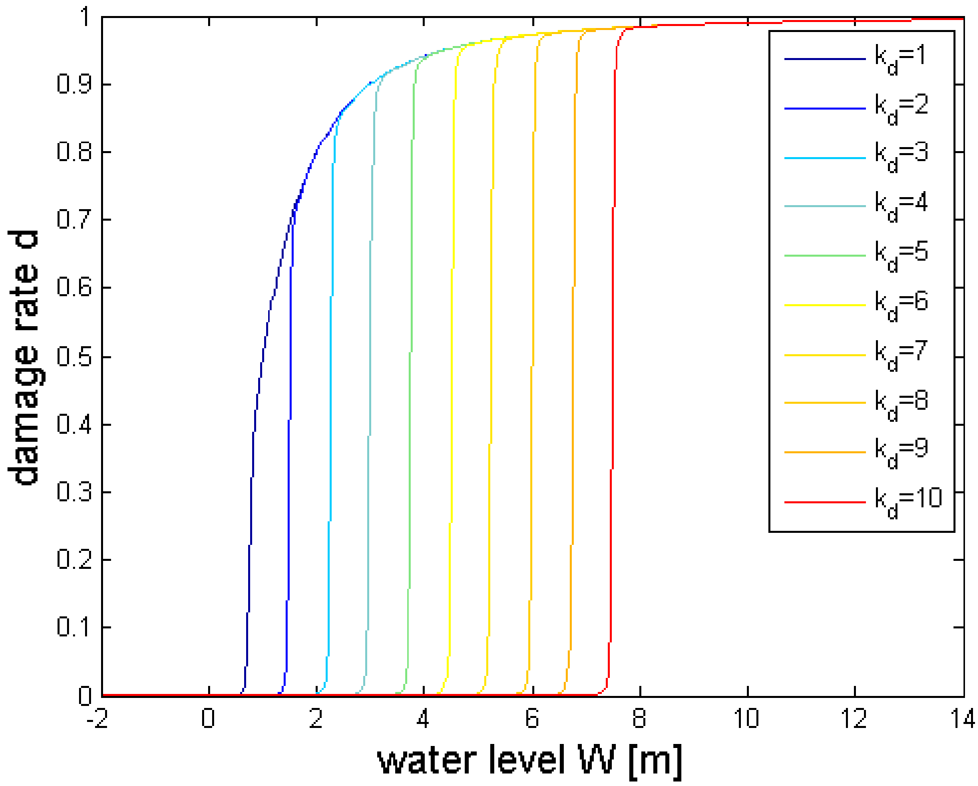


Figure 2.2: Form of the damage rate as a function of the water level $W(t)$ for various levels of the defense capital $k_d(t)$ and for $\xi_d = 0.5$. Both the water level and the defense capital are given in meters.

Water function

The water level $W(t)$ [m] is approximated with a continuous function (Viglione *et al.* (2014) uses a discrete time series for flood events) to allow an analytical solution of the model. A similar function was developed by Langer (2014) and explained in Grames *et al.* (2015). The parameter κ_s determines the maximum level of water to be reached during a flood and κ_m controls the frequency of flood events.

$$W(t) = \frac{1}{2} \sum_{\kappa=1}^{\kappa_s} \cos(\kappa_m \kappa t) \tag{2.8}$$

The water function is shown in Fig. 2.3. The water level equals zero when the river is bankfull and, therefore, the function (2.8) can be negative. Negative water levels, $W(t) < 0$, are simply treated like $W(t) = 0$, since the water level only affects $d(k_d(t), W(t))$, and $d(k_d(t), 0) = d(k_d(t), w_-)$ holds for any w_- below zero.

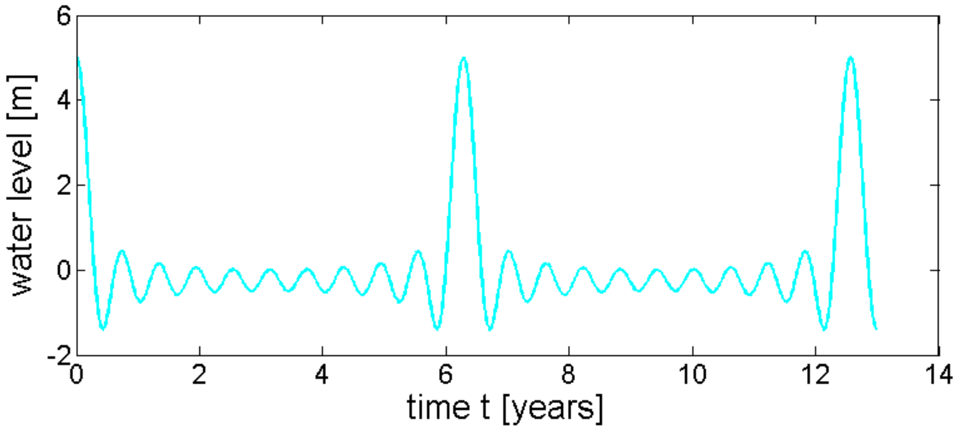


Figure 2.3: The periodic water level function gives quite frequent flood events. In brackets we display the units for the time and the water level itself.

Model summary

In summary, our model is represented by the following set of equations, where we have substituted $i_y(t)$ from equation (2.3) into equation (2.4):

$$\max_{\{c(t)\in[0,Y(t)],i_d(t)\in[0,Y(t)-c(t)]\}} \int_0^{\infty} e^{-\rho t} U(c(t))dt \tag{2.9a}$$

s.t.

$$\dot{k}_y(t) = Ak_y(t)^{\alpha} - c(t) - Q(i_d(t)) - d(k_d(t), W(t))k_y(t) - \delta_y k_y(t) \tag{2.9b}$$

$$\dot{k}_d(t) = i_d(t) - \kappa_d d(k_d(t), W(t))k_d(t) - \delta_d k_d(t) \tag{2.9c}$$

$$U(c(t)) = \ln(c(t)) \tag{2.9d}$$

$$Q(i_d(t)) = \theta_0(\theta_1 i_d(t) + \theta_2 i_d(t)^2) \tag{2.9e}$$

$$W(t) = \frac{1}{2}\sum_{\kappa=1}^{\kappa_s} \cos(\kappa_m \kappa t) \tag{2.9f}$$

$$d(k_d(t), W(t)) = \frac{1}{2}\left(\tau_3 + \frac{\tau_2 + W(t) - (1-\xi_d)k_d(t)}{\sqrt{(W(t) - (1-\xi_d)k_d(t))^2 + \tau_1}}\right)\left(1 - \frac{1}{1+W(t)^{\eta}}\right) \tag{2.9g}$$

The variables and parameters are shown in Table 2.1 and in Table 2.2[3]. We chose them based on existing literature and to replicate the stylized facts discussed in the introduction.

[3] θ_0 is calculated due to Slijkhuis *et al.* (1997) and Bedford *et al.* (2008)

Table 2.1: Variables of the model and their units of measurement

Decision variable	Interpretation	Unit	init.value, function
c	Consumption	10^9 \$	
i_y	Investment in k_y	10^9 \$	
i_d	Increase in k_d after investment of $Q(i_d)$	m	
Endogenous variable	**Interpretation**	**Unit**	
Y	Output	10^9 \$	
k_y	Physical capital	10^9 \$	6.5, 5
k_d	Defense capital	m	2
d	Damage rate	1/year	
W_{eff}	Effective water level	m	$W(t) + \xi_d k_d(t)$
Q	Costs for defense capital	\$	$Q(\theta_0, \theta_1, \theta_2, i_d(t))$
Exogenous variable	**Interpretation**	**Unit**	
W	Water level	m	periodic

2.3 Results

2.3.1 Simulation

To gain a better understanding of the model dynamics we start with numerical simulations of the uncontrolled system where the dynamics of the control variables are exogenously given. Assuming perfect consumption smoothing, we postulate $c(t)$ to be constant over time. Investment into physical and defense capital, $i_y(t)$ and $i_d(t)$ are, therefore, functions of the exogenous consumption level and the aggregate economic output $Y(t)$. To determine the specific investment in either one of the capital stocks we propose two alternative settings. We may keep the defense capital constant and, therefore, choose the investment $i_d(t)$ equal to the sum of the depreciation rate of the flood defense capital $\delta k_d(t)$ and the damage $d(W(t), k_d(t))k_d(t)$. The investment in physical capital $i_y(t)$ is then determined by the budget constraint (2.3). Alternatively, we assume that the total amount available for investments $Y(t) - c(t) = i(t) = i_y(t) + Q(i_d(t))$ is proportionally split between both investment options, i.e. for our simulations we assume $Q(i_d(t)) = 0.3i(t)$ and $i_y(t) = i(t) - Q(i_d(t)) = 0.7i(t)$.
Both cases are shown in the following Figs. 2.4-2.6 where we plot the water level $W(t)$ as well as the effective water level $W_{eff}(W(t), k_d(t)) = W(t) + \xi_d k_d(t)$ and the dynamics of the state variables $k_y(t)$ and $k_d(t)$. The dynamics are qualitatively similar for both cases: Whenever a flooding hits (the effective water level $W_{eff}(t)$ is above the defense capital $k_d(t)$) damage occurs and reduces the total capital stock $k(t) = k_y(t) + k_d(t)$ and

Table 2.2: Parameters of the model and their units of measurement

Parameter	Interpretation	Unit	Base case	Case study
A	Technology	[ ]	2.3	
α	Output elasticity of physical capital	[ ]	0.3	
ρ	Time preference rate	1/year	0.07	
δ_y	Depreciation rate of econ. capital	1/year	0.1	
δ_d	Depreciation rate of defense capital	1/year	0.1	
κ_m	Frequency of floods	$1/(2\pi)$ /year	1	2
κ_s	Water level of floods	1/2 m	5	10
κ_d	Damage of defense capital relative to physical capital	[ ]	1	0.1
η	Increase in damage due to a higher water level	[ ]	2	
τ_1	Approximation parameter in the damage function	[ ]	0.001	
τ_2	Water peak approximation parameter	[ ]	0	
τ_3	Approximation parameter in the damage function	[ ]	1	
θ_0	Scaling parameter for dike heightening costs	10^9\$/m	0.5	
θ_1	Weight for linear dike heightening costs	[ ]	0.5	
θ_2	Weight for quadratic dike heightening costs	[ ]	0.5	
ξ_d	Additional rise of the water level due to existing defense capital	[ ]	0.5	

hence the growth rate of the economy.

We present results of our simulations for two different sets of initial values. Higher initial capital stocks, $k_y(t_0) = 6.5$ and $k_d(t_0) = 2$, enable the economy to grow, see Fig.2.4). Moreover, keeping the amount of defense capital constant (Fig.2.4 a)) allows even faster growth compared to ever increasing amounts of investment in defense capital (Fig.2.4 b)) .

A small change in the initial capital stocks can make a significant difference in the long term behaviour of the capital stocks and hence on economic growth. If the economy does not have enough physical capital in terms of infrastructure, machines and buildings to produce economic output, it cannot withstand floods and economic growth will decline in the long run. If the society still tries to keep the level of the defense capital constant (see Fig.2.5 a)) they even have to invest such a large part of their output in defense capital that their physical capital depreciates and the economy crashes. The situation

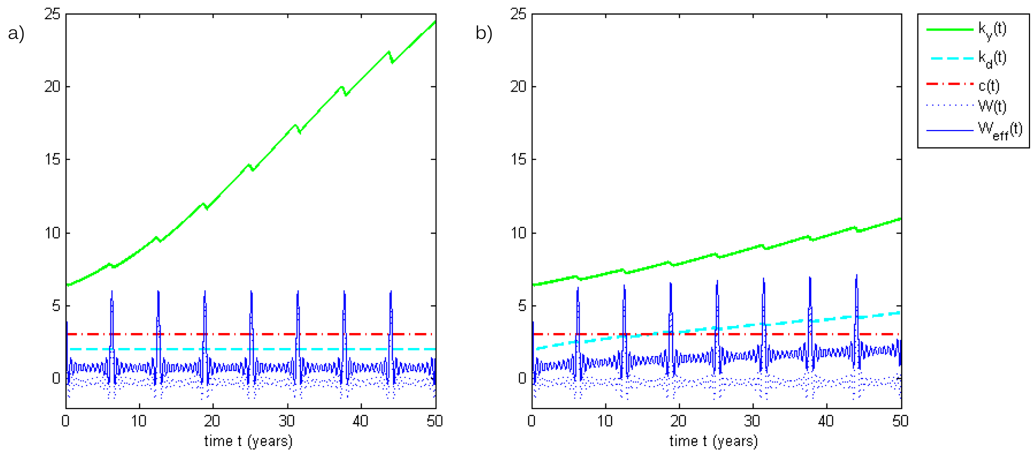


Figure 2.4: Simulation run of the physical capital $k_y(t)$, the defense capital $k_d(t)$, the consumption $c(t)$, the exogenous water level $W(t)$ and the endogenous effective water level $W_{eff}(t)$. a) Constant $k_d = 2$ with $k_y(t_0) = 6.5$ and b) proportional investments with $k_d(t_0) = 2$ and $k_y(t_0) = 6.5$ lead to economic growth. The unit of $k_y(t)$ and $c(t)$ is [$], all the other variables are given in [m].

is not as severe in case two (see Fig.2.5 b)) where an economy invests in defense capital proportional to the existing capital stock. However, also in this case, the economy will shrink in the long run. In order to avoid such a doomsday scenario when initial capital stocks are too low, an alternative is to reduce the amount of investment. For instance, if $Q(i_d(t))$ is only 25% instead of 30% of the total investments, economic growth is sustainable even for low levels of initial capital stocks (see Fig.2.6).

Overall, our simulations indicate that constant levels of decision variables that do not adapt to the state of the economy, may in the long run lead to a collapse of the economy. Therefore, we need to consider dynamic decision rules that react to the state of the model. Dynamic optimization methods are the tools to implement these dynamic decision rules.

2.3.2 Dynamic Optimization

Given the dynamics of the capital stocks, the exogenous water function, and the functional forms of the damage function and aggregate economic output, the social planner maximizes the discounted flow of utility by choosing the optimal consumption and the optimal amount of investments into defense capital. Since the exogenous function of the water level is periodic, the optimal decisions on consumption and investment will also follow a periodic time path.

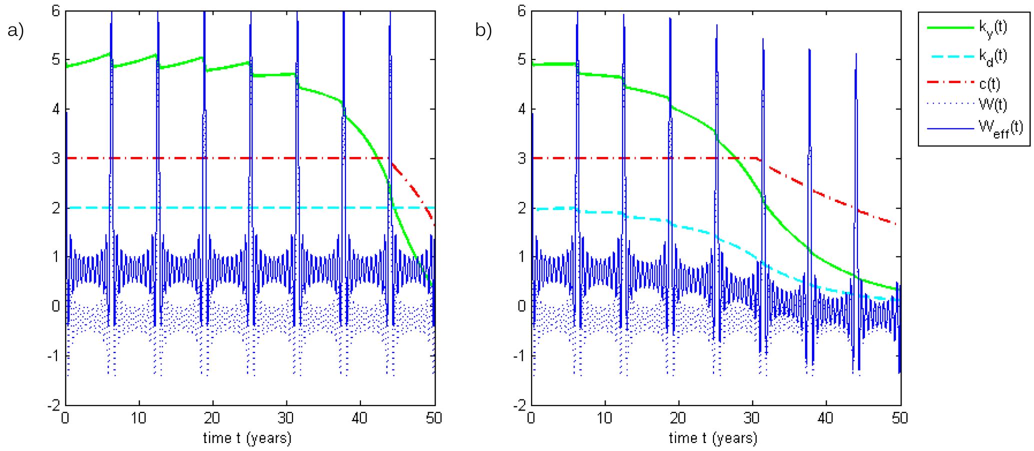


Figure 2.5: Simulation run of the physical capital $k_y(t)$, the defense capital $k_d(t)$, the consumption $c(t)$, the exogenous water level $W(t)$ and the endogenous effective water level $W_{eff}(t)$. a) Constant $k_d = 2$ with $k_y(t_0) = 5$ and b) proportional investments with $k_d(t_0) = 2$ and $k_y(t_0) = 5$ run into economic desaster.

Optimal consumption and investment decisions

Before we present detailed analytical and numerical results of the model we give an intuitive explanation of the dynamics of the model. Total aggregate output of the economy is consumed or reinvested into either one of the capital stocks (see equation (2.3)). Applying optimal control theory (2.5.1), we derive the optimal dynamics of consumption and investment decisions:

$$\begin{aligned}
\dot{c}(t) &= c(t)[A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho] && (2.10)\\
\dot{i_d}(t) &= \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2}[A\alpha k_y(t)^{\alpha-1} + (\kappa_d - 1)d(k_d(t), W(t)) \\
&\quad + \kappa_d d'(k_d(t), W(t))k_d(t) + \delta_d - \delta_y] \\
&\quad + \frac{1}{2\theta_0\theta_2}[d'(k_d(t), W(t))k_y(t)] && (2.11)
\end{aligned}$$

Both, the consumption path and the investment path, depend on the exogenous periodic function $W(t)$ and consequently, they will be periodic as well. Note that $W(t)$ also indirectly influences the dynamics because both capital stocks are a function of $W(t)$. The consumption dynamics are the same as in the standard Ramsey model with a social planner (Ramsey, 1928). A higher marginal product of physical capital (as given by the first derivative of the production function with respect to physical capital) as well as a lower rate of capital depreciation and time preference will positively affect the consumption growth rate. Damage acts like an additional depreciation on the marginal product of physical capital.

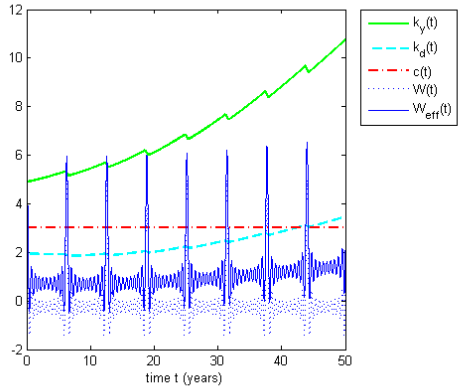


Figure 2.6: Simulation run where the initial values $k_d(t_0) = 2$ and $k_y(t_0) = 5$ are enough to enable economic growth if the investment in defense capital is only 25% of the total investment.

The dynamics of the investment in flood defense capital are more complex. The marginal product of physical capital and a lower rate of depreciation of physical capital positively influence the investment rate $\dot{i}_d(t)$, whereas a low rate of depreciation of the defense capital will reduce the optimal investment rate in flood defense capital because less investment is necessary to sustain the defense capital. Moreover, since the factor $(\kappa_d - 1)$ is nonpositive, when damage occurs, investments in defense capital decreases. The latter effect can be explained by the assumption that, in case of $\kappa_d > 1$, the damage to defense capital is more severe than the damage to physical capital. Consequently, investment in defense capital will be reduced. In case the damage rate for both types of capital is the same, $\kappa_d = 1$, damage does not directly influence the investment behaviour. However, the first derivative of damage with respect to the defense capital is zero or close to zero, so neither of the terms affect the investment dynamics. In general, all investment decisions are scaled by the cost parameters θ_0, θ_1 and θ_2. Lower costs enable higher investments.

Optimal long term capital stocks

Our results indicate that any optimal path of consumption and investment that the social planner decides on will end up in one of two possible long run solutions/limit sets (see 2.5.2) depending on the initial conditions. Note, that mathematical limit sets are different from an economic equilibrium which denotes a situation where all markets clear. We name the inner equilibrium which has high capital stocks and, therefore, high economic output the *rich economy* and the boundary equilibrium which only sustains a comparatively small physical capital stock and no defense capital *poor economy*. This notation will become apparent when we consider the long run economic state of the economy in each case.

To identify both equilibria we solved the optimization problem first analytically using the Pontryagin maximum principle (Pontryagin, 1962) and then numerically using the specific MATLAB® -Toolbox *OCMat* from Grass & Seidl (2013) and the parameter values given in Table 3.2.
The *rich economy* (Fig. 2.7 a)) invests just enough into flood defense capital to avoid

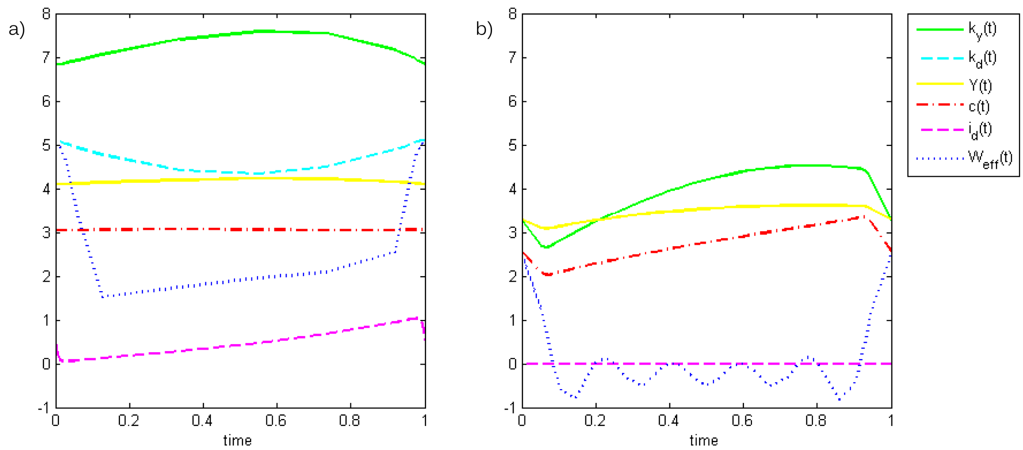


Figure 2.7: One limit cycle (in normalized time) of the long-term behaviour of a a) rich economy, b) poor economy showing the time series of the physical capital $k_y(t)$, the defense capital $k_d(t)$, the economic output $Y(t)$, the consumption $c(t)$, the investment in defense capital $i_d(t)$ and the exogenous effective water level $W_{eff}(t) = W(t) + \zeta_d k_d(t)$.

floodings and consequently flood damage. Thus, the effective water level $W_{eff}(t)$ increases due to the levee effect. Even though the social planner never stops investing into flood risk prevention measures ($i_d(t) > 0$) in the long term, they lower the investments when they are not urgent and rather invest in physical capital $k_y(t)$ to increase the economic output $Y(t)$. In such an economy, the aggregate output is quite high and, therefore, a constant consumption path is sustainable. These so called smooth consumption paths are characteristic of developed economies (Friedman, 1956) and are also commonly shown to be consistent with economic growth (Acemoglu, 2009).
In contrast, *poor economies* (Fig. 2.7 b)) do not invest at all in defense capital. Mathematically they move to a boundary periodic solution with $i_d(t) = 0$. Without any investments $i_d(t)$ the defense capital $k_d(t)$ remains zero (and so the effective water W_{eff} level equals the exogenous water level W). Consequently, the society is vulnerable and every time a high water level occurs, flooding hits the economy. The physical capital stock $k_y(t)$ decreases and less economic output $Y(t)$ is produced. Interestingly, the social planner already anticipates the damage shortly before a flood hits and prefers to distribute the output to consumption rather than investment in physical capital. There-

fore, consumption $c(t)$ strictly increases until a flood hits and less consumption is possible during a flooding event. It takes time to recover and to reach the old consumption level again.
It is useful to highlight the optimal investment strategy for the rich economy: The investments in flood defense capital are always positive and increase before a flood hits. In reality, societies tend to invest in flood defense infrastructure only after big flooding events have occurred. An example is the Danube flood of 1954 which resulted in construction of a flood relief channel in Vienna. Decision processes to invest in flood defense management are mostly based on political decisions and financial considerations and only effective if stakeholders have an immediate memory of past flooding. However, the optimization model shows that investing in flood defense capital before floods would be economically more advisable. Of course we cannot forecast floods, but investing also in times of no flood instead of reacting after flood occurrence is shown to be optimal.

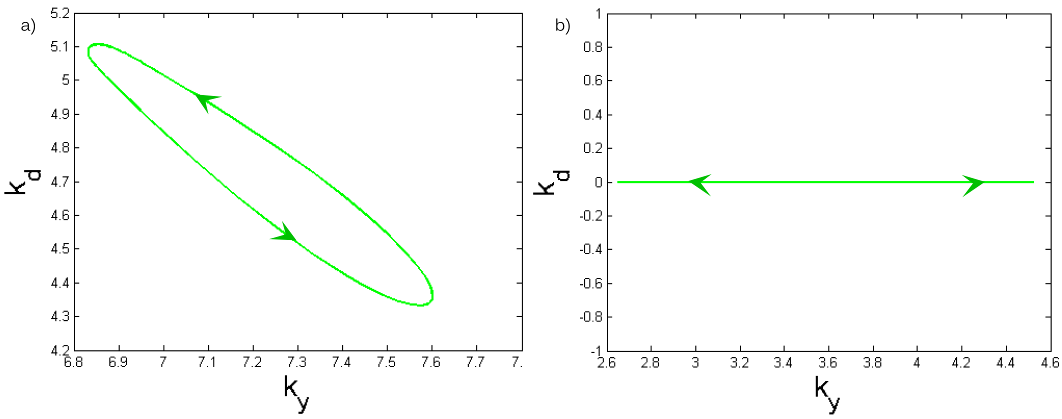


Figure 2.8: The state dynamics of the a) rich economy, b) poor economy.

The long-term state dynamics of the capital stocks $k_y(t)$ and $k_d(t)$ clearly identify the limit cycle. Note that the cycling is counterclockwise. For the rich economy (Fig. 2.8 a)) we see a negative correlation of the capital stocks: Since the social planner wants to keep consumption smooth, increasing investments in one capital stock lowers the investments in the other capital stock. Moreover, a lower physical capital stock yields less output. This allows less investments and, therefore, a lower total capital stock. This is always the case after high water levels, when the priority is to build up defense capital. Hence, floods do not only affect the economy directly via damage, but also indirectly through a lower level of output and, thus, lower capital stocks.
The limit cycle for the poor economy (Fig. 2.8 b)) is trivial. Since there is no defense capital, the physical capital basically increases after a flooding, reaches its maximum slightly before a flooding due to the anticipation effect and decreases quickly when a flood hits the economy.

So far we have studied the long-term behaviour along the limit cycles. It is also

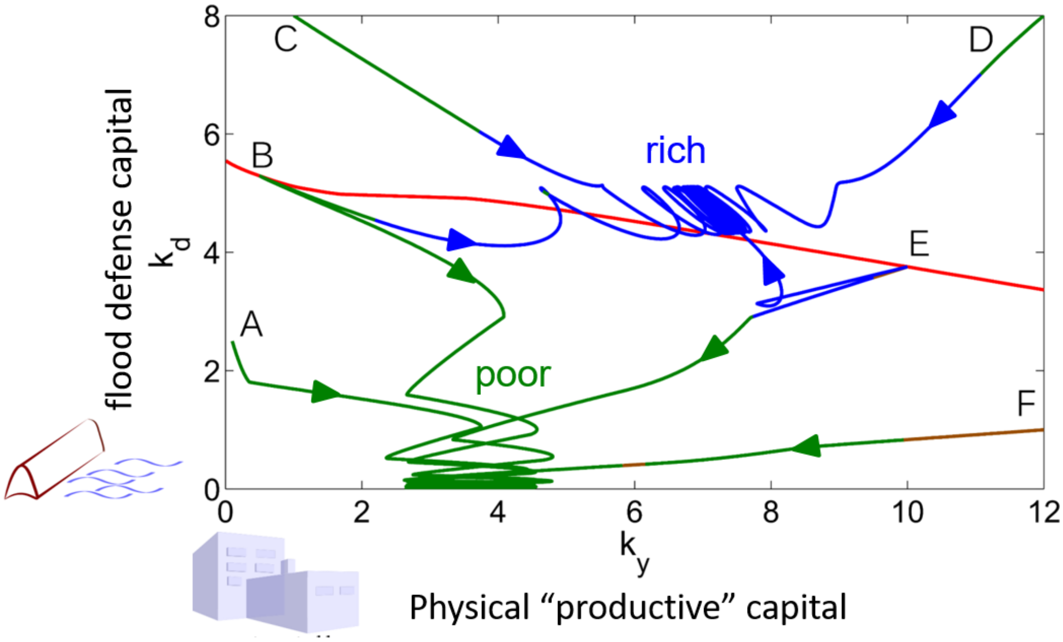


Figure 2.9: Different initial conditions (points A – F) in several economies induce to different long-term behaviour. The red line (Skiba curve) separates the initial values leading to a rich or to a poor economy. $k_y(t)$ and $k_d(t)$ are the physical capital and the defense capital, respectively.

important to understand the path towards one of the two limit cycles. Depending on the initial values of the capital stocks $k_y(t)$ and $k_d(t)$ the economy follows a path to one of the limit cycles, separated by the so called Skiba curve (red line in Fig. 2.9). Starting (slightly) above or below the Skiba curve will lead to a rich economy or poor economy, respectively.

Interestingly, due to the non-autonomous water function, the Skiba curve shifts depending on the starting time relativ to the next flooding event. An economy that e.g. starts slightly below point B but at the same starting time implying that the time it takes to the next flooding has not changed, would converge to a poor economy. However, if in such a situation (i.e. when we start at a point below B) the time to the next flooding would increase as well, the economy would converge to a rich economy.

So it is not only important where the economy starts, but also when the next flood is happening. This allows the paths towards the long term limit cycle to be temporary below the Skiba curve after the starting time.

For the base case where we set the parameters according to Table 2.2 we choose the set of the starting points A-F and show the different paths in Fig. 2.9. Different colours represent different investment combinations. I.e. along the blue line the economy invests

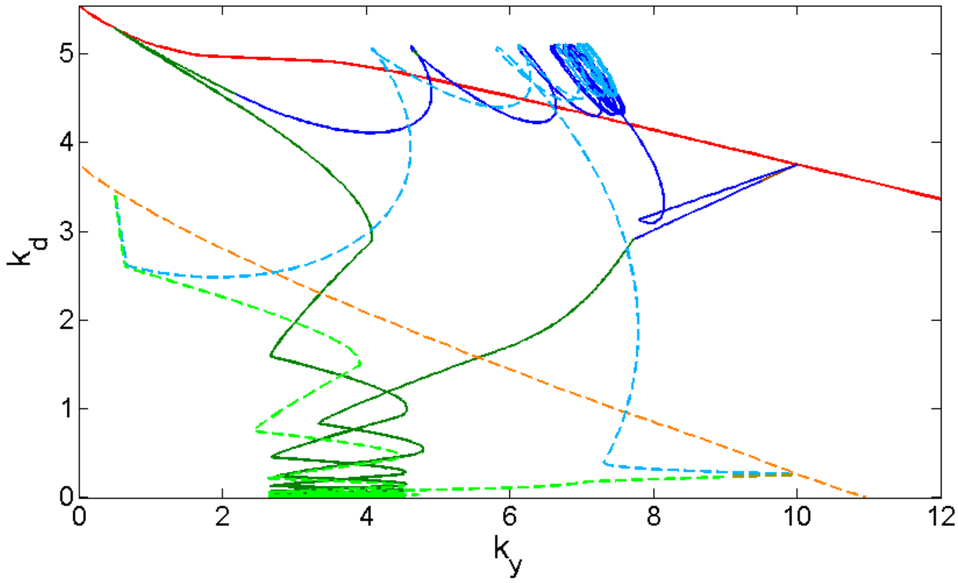


Figure 2.10: Lower costs of investment in defense capital ($\theta_1 = 0.25$, dashed lines) shift the Skiba curve (orange dashed line instead of red solid line) and enlarge the region where economies become a rich economy compared to the base case ($\theta_1 = 0.5$, solid lines). $k_y(t)$ and $k_d(t)$ are the physical capital and the defense capital, respectively.

in both capital stocks and also consumes, the green line indicates that the economy is not investing in flood defense, but still in physical capital, and the brown lines at starting points E and F display that the economy consumes all the produced output without investing in any of the capital stocks.

Economies A, B and C with less physical capital first try to build up physical capital. Economies starting at A or slightly below B do not afford to invest in flood defense and it is optimal to prefer consumption over flood defense. Economies starting at C or slightly above B already have enough defense capital and so it is optimal for them to sustain it. In contrast, if we start with a much higher defense capital at point D, which does not bring any extra benefit compared to the long-term level, investments in defense capital are stopped immediately and the defense capital stock depreciates, while investments in economic capital are slightly positive. The main part of the output is consumed directly, unless the defense capital stock has reached the level where it may be too small to prevent damage from floods. So, even if the community could afford more capital, they prefer to only invest as much as necessary to avoid floodings and rather consume the output right away.

Economies starting close to point E, with a lot of physical capital, but slightly too less flood defense capital, are living on the edge. If they always invest at least a small amount in flood defense they manage to turn into a rich economy, whereas choosing to only consume their economic output in the beginning leads to a poor economy. However, it is

still optimal to invest at some time into flood defense capital to lower the flood damage, but below a certain level of defense capital it is optimal to not invest in it anymore and consume more. Even if the economy is very rich in terms of physical capital but does not have enough knowledge and flood defense to build on (point F), it will not invest in flood defense and rather consume all the economic output. Because it knows that the next flood will destroy a major part of their capital anyways. It starts investing in physical capital when the additional amount of output pays off the damage.
The costs of investment in defense capital are crucial. Fig. 2.10 shows, that decreasing the costs shifts the Skiba curve and significantly enlarges the region where economies develop into a rich economy. I.e. an economy starting with initial values between the red and the orange line would choose the optimal investment given low costs ($\theta_1 = 0.25$) or high costs ($\theta_1 = 0.5$) to end up as a rich or poor economy, respectively.

Higher frequency and higher intensity of floods changes the investment behaviour

So far we have studied the dynamics of the model under one specific set of parameters. We next investigate how the optimal decisions of the social planner will change when she faces a different environment, e.g., a different occurrence of high level water events. We study two cases: First, we assume a higher frequency of floods, and secondly we assume higher water levels which can lead to more pronounced floodings.

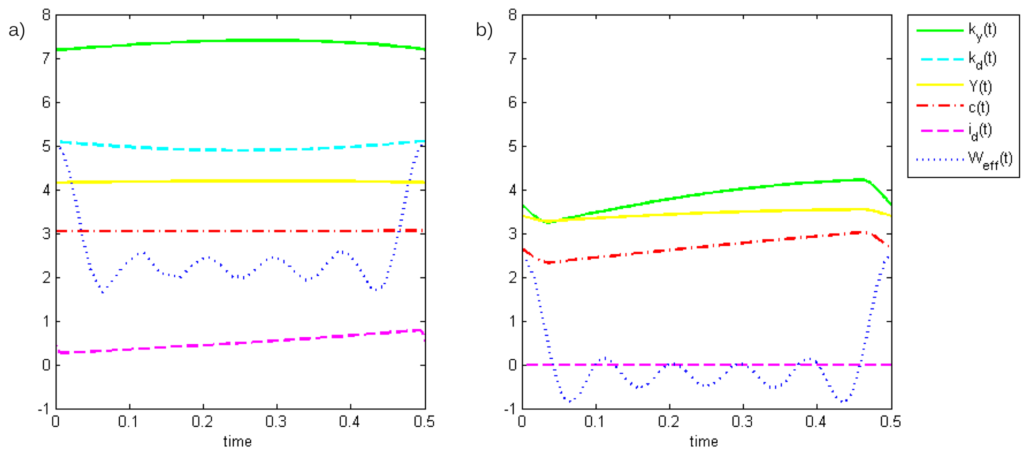


Figure 2.11: One limit cycle in case of a higher frequency of floods and, therefore, time period [0,0.5] for a) a rich economy, b) a poor economy. Parameters as in base case of Table 2.2, but $\kappa_m = 2$. Note, since we plot only one period and the frequency of the periods changed, the time interval is now only [0, 0.5].

Doubling the frequency of high level water events ($\kappa_m = 2$) naturally leads to a smaller time period of the limit cycle. Fig. 2.11 displays less variation in the dynamics of the state and control variables than in the base case. Intuitively, we would expect that a doubling of the flood frequency would translate into a 50%-reduction in the variations of the levels of the state and control variables since the time to accumulate capital without being hit by a flood is only half. However, this is only true for poor economies. For rich economies, the difference between the highest and lowest level of the capital stock along the limit cycle is not even a third in case of double flood frequency. Even more counterintuitive is the finding that a rich economy facing a higher frequency of high water levels manages to have the same consumption rate and even higher capital stocks on average as compared to the case with lower frequencies of high water levels. Both the defense and the physical capital stock are higher on average than in the base case. So only very rich economies manage to stay rich when they are facing higher flood frequencies.
Poor economies suffer from higher flood frequencies. Since more floods lead to shorter flood durations, the damage is not as high, but occurs more often. Not only is the range of the values of the capital stocks smaller than in the base case, also the range of the consumption level is halved. Moreover, on average poor economies facing more floodings consume less and have less economic output.

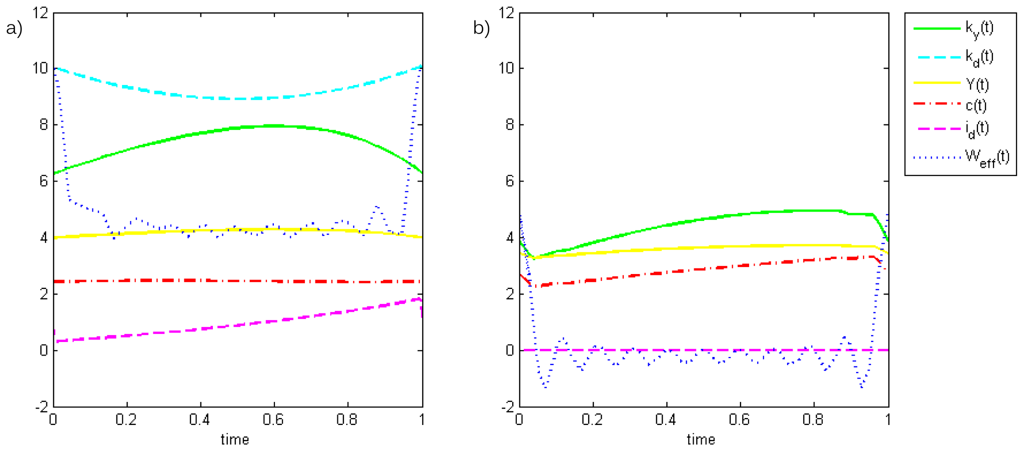


Figure 2.12: One limit cycle in case of bigger floods for a) a rich economy, b) a poor economy. Parameters as in base case of Table 2, but $\kappa_s = 10$.

For the second case we vary the amplitude of the floods ($\kappa_s = 10$) and show the results in Fig. 2.12. In order to protect against higher water levels, rich economies will start to invest in defense capital earlier and to a larger extent. Consequently, less economic output is left to invest in physical capital or for consumption. Rich economies can consume 20% less than rich economies in the base case scenario. This is the only chance they can keep the physical capital almost at the same level and, therefore, produce

a critical amount of economic output.
Surprisingly, poor economies converge in the case of more pronounced floods to an economic state with higher capital stocks and higher consumption levels compared to the base case scenario. Although floodings hit harder, each flood is shorter which results in a wealthier economy.
When we compare rich and poor economies in case of more pronounced floods, the capital stocks are much higher for rich economies, so they seem to be wealthier. However, consumption and, thus, the average utility along one limit cycle of the society is 17% higher for poor economies. This means that poor communities in heavily flooded areas should actually not invest in defense capital but rather invest in physical capital, thereby increasing output and allowing for higher consumption levels, even though they have to give up a smooth consumption path.
The results depend on the parameters and the characteristics of the damage function.

Less damage in the defense capital stock influences the dynamics of the capital stocks

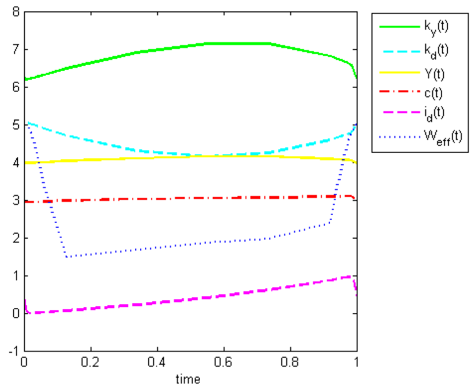


Figure 2.13: One limit cycle in case of a more robust flood defense capital. Parameters as in base case of Table 2.2, but $\kappa_d = 0.1$. Note, due to numerical discretization of the solution W_{eff} is displayed different, but has the same oscillating behaviour as in the other figures.

Fig. 2.13 shows a case where the defense capital is not as vulnerable as the physical capital ($\kappa_d = 0.1$). For this case we only need to analyze rich economies, since poor economies do not even have defense capital and, therefore, defense capital cannot be damaged. Fig. 2.13 shows very similar patterns to the base case. It appears that the floods do not destroy defense capital as heavily as physical capital. Assuming an equilibrium without any damage would simply look like the base case scenario. Since the social planner knows that damage does not affect the defense capital very much, one chooses a lower investment in defense capital than in the base case and, therefore, allows small flooding events for a very short time, where both capital stocks are damaged. As a consequence, the economic output is slightly lower, but the consumption increases in the time in-between the flooding events. This suggests that, in this model, people do care more about the defense capital, if it is more vulnerable.

2.3.3 Simulation with stochastic flooding events

The analysis performed so far does not account for the stochasticity of floods. This has been done to obtain analytically results on the long term optimal behavior of the systems. In this section we investigate how these results change if floods occur stochastically, i.e., when the social planner has no complete knowledge of the future flood occurrences and magnitudes. We present simulations of our model assuming a stochastic water level function like in Viglione *et al.* (2014). The timing of the high water level events is exponentially distributed, as a result of a Poisson process with mean t and arrival rate 0.2 per year, and the height of the water levels is modeled with a generalized Pareto distribution with mean 1 (see Viglione *et al.* (2014), Section 2.1, for details).
Within such a simulation exercise we compare the two policies we derived in Section 2.3.2. We assume that an economy consumes 80% of its economic output in both scenarios. A rich economy invests in flood defense capital, $i_d(t) > 0$, proportional to the output after consumption and possible damage. If the defense capital is high enough to prevent flood damage they only maintain it and do not invest further. A poor economy splits the output after damage proportional into consumption and investment in physical capital, but does not invest in flood defense capital, $i_d(t) = 0$. The obtained scenarios are listed in Table 2.3 together with the different initial capital stocks. To compare the various simulation runs we record the mean and the variance of the present value (discount rate $\delta = 0.07$) of future utility streams $U_0(T)$ for each simulation scenario choosing the simulation run time $T = 750$ years.

$k_y(t_0)$	$k_d(t_0)$	$i_d(t)$	mean($U_0(T)$)	var($U_0(T)$)	Fig.
100	0	> 0	28.4	0.06	2.14 a
100	0	0	27.2	0.14	2.14 b
100	6	> 0	28.6	0.01	
100	6	0	28.5	0.05	
5	0	> 0	18.7	0.03	2.14 c
5	0	0	19.5	0.04	2.14 d
5	6	> 0	18.9	0.01	
5	6	0	19.7	0.04	

Table 2.3: Stochastic simulation runs for different initial levels of physical capital $k_y(t_0)$ and defense capital $k_d(t_0)$ and two different policies (investing in defense capital ($i_d(t) > 0$) versus not investing in defense capital ($i_d(t) = 0$)) tracking the mean and the variance of the present value of future utility streams $U_0(T)$.

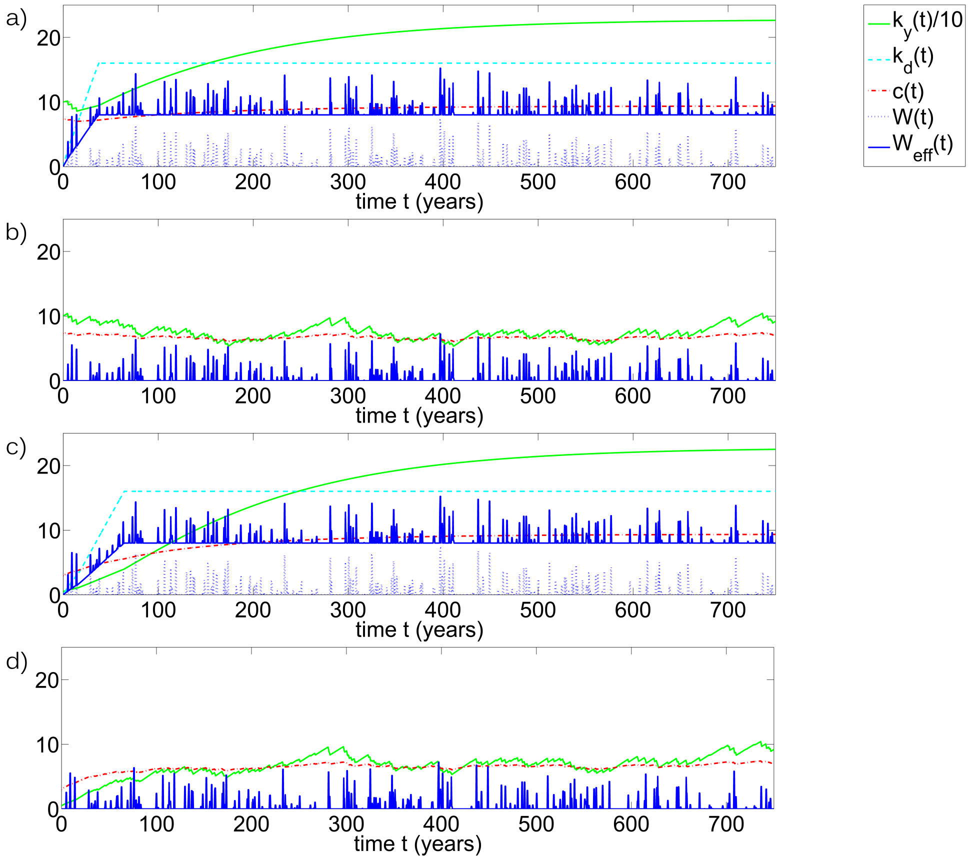


Figure 2.14: Simulation runs of the physical capital $k_y(t)$, the defense capital $k_d(t)$, the consumption $c(t)$, the exogenous water level $W(t)$ and the endogenous effective water level $W_{eff}(t)$. The unit of $k_y(t)$ and $c(t)$ is [\$], all the other variables are given in [m]. a) and c) show the szenario of the rich economy and b) and d) the poor economy. The initial conditions for a) and b) are $k_y(t_0) = 100$ and $k_d(t_0) = 0$, for c) and d) $k_y(t_0) = 5$ and $k_d(t_0) = 0$

In all stochastic simulation runs displayed in Fig. 2.14 we used the same high water level event series. In the long term the stochastic simulations are comparable to the optimal limit cycles. Economies investing in $k_d(t)$, we again refer to them as rich economies, end up in an almost constant state whereas economies without defense capital (poor economies) fluctuate depending on floods.
The initial conditions do not change the long term behavior in the simulation.[4] However, the present value of future utility streams $U_0(T)$ increases for larger initial capital stocks. Nevertheless, floods will cause more damage if the physical capital stock is high as indicated by the dip of the capital stock in the beginning of the simulations displayed in Fig. 2.14 a.
The simulations with existing initial defense capital are qualitatively similar to the simulations in Fig. 2.14 a-d, however these eocnomies reach a higher utility since floodings are avoided in the early years which are discounted less.

In the simulation runs summarized in Table 2.3 economies do not optimally decide on their investment and consumption. Nevertheless, we can compare the discounted stream of utility across the different scenarios. If the initial capital is high, the net present utility value is higher for rich economies, whereas with a low initial capital poor economies are better of in terms of consumption. This coincides with the optimal solution of Section 2.3.2.
Moreover, the variance indicates the different values in the simulation runs based on different flood time series. Poor economies are more sensitive to the flood time series compared to rich economies.

2.4 Discussion

In this chapter we studied a socio-hydrological model of high water level events potentially causing floodings in an economic decision framework. In the model, a social planner, representing the society, decides how to optimally distribute the economic output between consumption, investment in flood defense capital and investment into physical capital. We apply our model to understand the mechanisms between floods and economic growth if the water level follows a specific exogenous fixed water level function that is time varying. Investments in flood defense capital do not only avoid direct damage in the future, but also safe opportunity costs for reconstruction. This allows investments in physical capital and consequently more economic growth in the future (Hochrainer-Stigler *et al.*, 2013).
We applied dynamic optimization methods to determine the long run optimal solution of our system. Depending on the initial capital stocks of the economy, our system either converges to a rich or a poor economy in the long term. This dynamic behaviour is consistent with an extensive literature on economic growth models that have the poten-

[4] The dynamics with higher initial capital stocks would look similar to those in Fig. 2.14.

tial to generate multiple equilibria and poverty traps (Bloom *et al.*, 2003). Graham & Temple (2006) have empirically shown that such multiple equilibria offer a convincing explanation for the income gap between poor and rich countries. Azariadis (2005) provides an excellent survey of plausible economic mechanisms that may induce multiple equilibria (including e.g. increasing returns to scale in production, market failures, etc.) In our model multiple equilibria result from the fact that the social planner might be constrained in choosing enough defense capital to significantly lower flood damage. If the economy does not have enough economic resources to build up defense capital the economy ends up in a low level equilibrium trap because recurrent floodings hit the economy and cause damage.
Besides the initial capital stocks which acts as history dependence in the dynamic evolution of the economy, we have identified the costs of investment in defense capital and the timing until the next flood occurs as crucial parameters for the selection of the low versus high level equilibrium in our model set up.

In order to compare the model results to real world data we use macro-economic data for countries, whereas we are aware that usually only parts of a country are under flood risk. So, whenever we discuss rich or poor economies, we refer to broader regions or countries that are (partly) affected by floods.
The rich economy manages to build up defense capital to avoid damage and, therefore, follows a smooth consumption path. The consumption rate of 70% (Fig. 2.7 a)) equals e.g. the rate in the US[5]. Poor economies, characterized by low levels of initial economic output or initial defense capital, optimally decide not to invest into defense capital and end up with lower capital stocks and lower consumption rates. Every time a flooding hits, physical capital is damaged and consumption decreases strongly. The average consumption rate of poor economies is higher than 80% of their total output, which is around the rate of third world countries such as Cambodia and Kenya [6]
If defense capital such as levees is built, the water level may increase due to the loss of retention volume (Di Baldassarre *et al.*, 2009; Remo *et al.*, 2012; Heine & Pinter, 2012). Also vulnerability may increase because of the levee effect (Montz & Tobin, 2008; Ludy & Kondolf, 2012). However, economic output and consequently consumption and capital stocks are higher since flood damage can be prevented. If the severity of floods is very high we showed that a rich economy investing in defense capital may end up with consuming less out of the total output compared to a poor economy which does not invest in defense capital. Our results are in line with actual observations. For example, the Netherlands are facing severe floods and invest a lot in their flood management systems (Silva *et al.*, 2004; Eijgenraam *et al.*, 2014). The consumption rate of around 50% in this scenario in our model fits the low consumption rate of the Netherlands.[7] The Netherlands have a higher output and the total per capita consumption is higher than in the mentioned third world countries.

[5]http://data.worldbank.org/indicator/NE.CON.PETC.ZS assessed on June 3rd, 2015
[6]http://data.worldbank.org/indicator/NE.CON.PETC.ZS assessed on June 3rd, 2015
[7]http://data.worldbank.org/indicator/NE.CON.PETC.ZS assessed on June 3rd, 2015

Whether an economy is rich or poor depends very much on its economic capabilities including physical capital of firms and governments, infrastructure and technology, but also on existing flood defense capital. If any one of these components is too small, the economy will never have the strength to become a rich economy. It will stop investing in defense capital because it is not worth the opportunity costs of missed consumption. In reality there is always some investment in flood defense since people want to avoid death or very strong flood impacts to human life. Since this is hard to be displayed in economic values we did not explicitly include it in the model. But assuming a minimum investment in defense capital would not change the results qualitatively[8].
We see this scenario in many poor countries: Without any external help, regions such as the Mekong floodplains are flooded regularly and the locals are used to the damage (http://www.mrcmekong.org/). Kahn (2005) also found that rich nations suffer less from natural disasters than poor countries. Higher developed economies invest more in prevention of natural disasters and the total losses after a disaster are smaller (Schumacher & Strobl, 2011).
How is it possible to escape the trap into a poor economy? Since environmental conditions cannot be changed easily, only different economic environments can induce a difference. It is essential to invest into physical capital to bring the economy on a path to the equilibrium of the rich economy. If the country cannot afford this by itself, external help is necessary. This help does not only include capital investment but also ensuring strong institutions to accordingly distribute the investments.
As soon as the economy is on the path towards the long term state of a rich economy, our model predicts that it will never revert to a poor economy given the same environmental and economic conditions. Staying rich when the economy is already there does not require any help from outside anymore. This is the case if no surprise will occur (see e.g., Merz *et al.* (2015)).
In fact, the timing of the expected flooding event plays a crucial role. If a flood is not expected in the near future the optimal behaviour is to invest less in flood defense capital and therefor taking the risk of ending up as a poor economy. This effect is stronger if the costs for flood defense capital are higher.

Fig. 2.15 summarizes the scenarios of this chapter. Each scenario is represented in a different colour and we plot the case of a rich and a poor economy for each scenario. The amount of physical capital of the rich economies is quite similar in every scenario. Naturally, the range differs from scenario to scenario: In case of more floods we observe a lower variation of physical capital while the level of both capital stocks is higher compared to the base case.
In the scenario where we increase the severity of floods, the defense capital has to be very high in order for the economy to remain rich. So it is very hard to obtain such a rich economy and the willingness to invest in flood defense capital has to be very high, too.

[8] Introducing a minimum investment in defense capital would only be a small linear transformation in investment and consequently in consumption and aggregate utility.

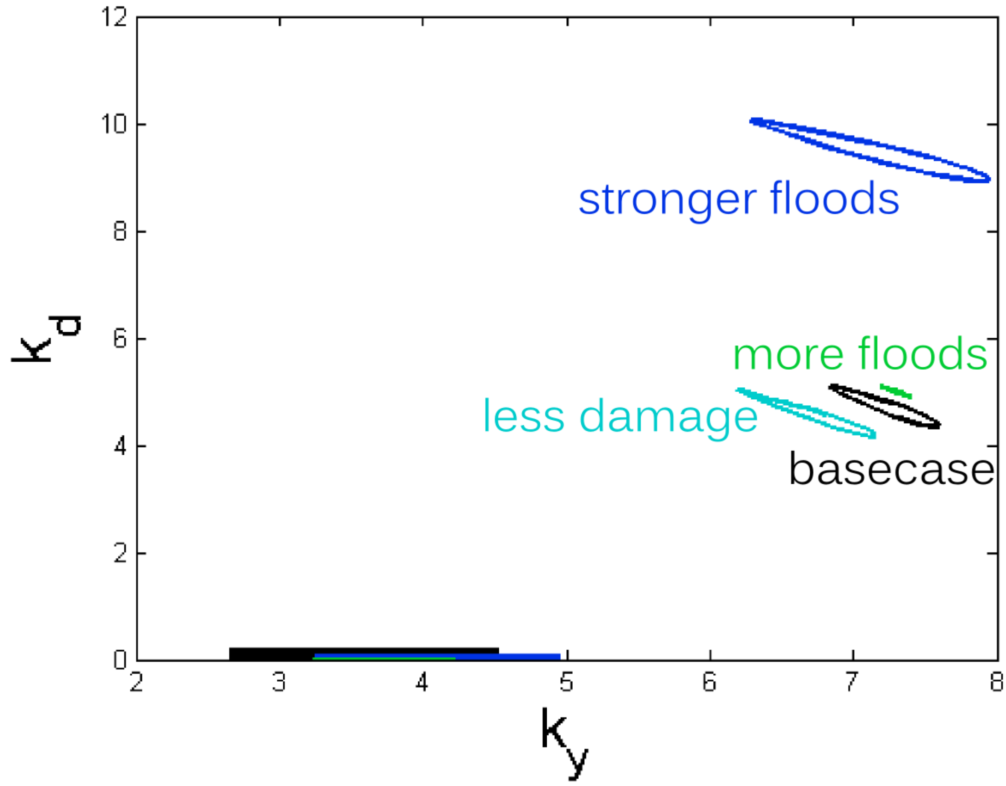


Figure 2.15: Long-term state dynamics for the cases of Figs. 2.7-2.13.

We only encounter this case in first world countries that are highly affected by floods such as the Netherlands. This is very much confronted with floods, can afford defense capital, and is willing to invest in it (Vis *et al.*, 2003).
In the scenario of less damage people are minimalists and only invest in their capital stocks as much as necessary to overcome floods. As a consequence, their capital stocks are lower than in any other scenario. Their consumption is just as high as in the base case, but not as smooth since it decreases during flooding events. The consumption cycle in this scenario has similar dynamics as the poor equilibria of the other cases.

In case of poor economies, flood intensity and frequency directly impact the wealth of the economy. More floods more often cause damage of existing physical capital, but the economies have experience with floodings and rebuild the infrastructure quickly. In contrast, if bigger floods happen less frequently, the damage is much higher and the poor economies need longer and also have to invest more into physical capital to regenerate. In total, the consumption is higher than in the scenario with fewer floods. So even if floods hit harder, as long as they do not appear too often, the living standard can be relatively high in between floods.

Overall, the economic output is almost equal for all rich economies independently of the frequency and intensity of floods. Only the amount of defense capital and the variations of physical capital along the long run economic state differs. Furthermore, the economic output in poor economies is much lower than for rich economies, but it is about the same level for any poor economy in various cases.
Besides the higher economic output and the mostly higher consumption for rich economies, they do have the capacities and resources to anticipate damages before a flood hits. On the other hand, poor economies do not have the economic potential and are therefore not flexible to adjust to floods beforehand. The only anticipation is to stop investing into physical capital shortly before a flooding[9], but basically poor economies are affected by floods every time they occur and have to start over again rebuilding capital stocks and increasing consumption.

Optimization is important to use the resources efficiently. The simulation in Section 2.3.1 shows the dynamics of the model. Even in case of positive economic growth, damage occurs during every high water level event, whereas in the optimization model rich economies can avoid damage in the long run, even though they are investing less, but at the right time. Moreover, in the scenarios with declining economic growth the economies even converge to zero capital stocks. In the optimization case it will never happen that people invest in flood defense capital if they cannot even afford their basic needs for living. Therefore, they always manage to sustain some physical capital and to have enough resources to consume and invest again in production after a flooding event.

Also when we look at simulation runs with stochastic high water level events the present value of utility is larger for rich economies if the initial capital stocks are sufficiently high. This reflects the optimal path towards the limit cycle for rich economies in our dynamic optimization set up. Contrary, the present value of the future utility streams is smaller for economies investing in defense capital than for economies which do not build up a defense capital stock if the initial capital stock is low. This scenario reflects all the paths going towards the limit cycle of the poor economy in our dynamic optimization set up, where the strategy to not invest in defense capital is optimal. Apparently, if the economy starts with low initial capital stocks it will not pay off to invest in flood defense capital and this is the incentive to remain poor and vulnerable. As we have also seen in the optimization, economies do not manage to escape that poor scenario with their own strength, but need external help to do so.

To sum up, if a social planner would base his decision on the present value of future

[9]This is true for our model assumptions. In reality the timing of floods is not known in advance and only last minute protections can be build.

utility given uncertainty of flood events, he would still choose the same policy as in the long term optimization based on a deterministic water level function. For instance, flood frequency analysis is used in hydrology to estimate the expected frequency of exceedence of flood levels for a given time horizon (see, among many, Gumbel (1941, 1958); Chow *et al.* (1988)). In principle our optimization model with the deterministic exogenous water level function based on this expected parameter values can help identifying optimal investment strategies for the long run. Of course, because of the stochasticity of flooding, sensitivity analysis need to be performed for these optimal scenarios, in order to assess their robustness (Blöschl *et al.*, 2013).

Comparing the results in this chapter with the simulation model of Di Baldassarre *et al.* (2013) and Viglione *et al.* (2014), on which our model set up is based, we may highlight further important differences: First, they found that, in certain circumstances, investing in flood defense capital may lead to less economic growth than facing frequent small floodings. This is because rare floodings may be catastrophic since societies erroneously consider floodplains more secure after building levees and invest in building and living there. In our optimization model, in which the social planner has the knowledge of flood occurrence and magnitude, rich economies can manage floods and, therefore, avoid catastrophic floodings.
Second, a lower decay of levees leads to higher growth rates in Viglione *et al.* (2014). In contrast, in our model the social planner decides to invest just a minimum into flood management and physical capital to consume more than in the scenario with a higher depreciation rate.

Our approach is to conceptualize the interaction of human decision making and flood risk management within a macro-economic framework. Our aim is to understand the mechanisms rather than matching specific cases or predicting the future development of societies. As models cannot and should not capture all details of the reality, we do not claim that this is the only true representation of communities in flood risk areas. However, it enables us to discuss certain dynamics and policies in the field of socio-hydrology.

Starting from the results in this chapter, future work will focus on the sensitivity of the model results to the assumptions made, and on the assumption of perfect knowledge of future water levels by the social planner. We expect that, even though uncertainty/stochasticity of natural events will result in more complex dynamics, the results of this work will provide the fundamental baseline over which other mechanism will show up.

2.5 Appendix

2.5.1 Dynamics of the optimal controls

We are analyzing the model analogous to Barro & Sala-i Martin (2004) and Millner & Dietz (2015).

The Hamiltonian

To analytically optimize the model given in equations (2.9) we formulate the Hamiltonian function.

$$\begin{aligned}
&\mathcal{H}(c(t), i_d(t), \mu_y(t), \mu_d(t)) \\
&= U(c(t)) + \mu_y(t)[Ak_y(t)^\alpha - c(t) - Q(i_d(t)) - d(k_d(t), W(t))k_y(t) - \delta_y k_y(t)] \\
&+\mu_d(t)[i_d(t) - \kappa_d d(k_d(t), W(t))k_d(t) - \delta_d k_d(t)]
\end{aligned} \tag{2.12}$$

The Pontryagin conditions are

$$\frac{\partial \mathcal{H}}{\partial c(t)} = U'(c(t)) + \mu_y(t)[-1] = 0 \tag{2.13a}$$

$$\frac{\partial \mathcal{H}}{\partial i_d(t)} = \mu_y(t)[-Q'(i_d(t))] + \mu_d(t) = 0 \tag{2.13b}$$

$$\frac{\partial \mathcal{H}}{\partial k_y(t)} = \mu_y(t)[A(t)\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y] = \rho\mu_y(t) - \dot{\mu}_y(t) \tag{2.13c}$$

$$\begin{aligned}
\frac{\partial \mathcal{H}}{\partial k_d(t)} &= \mu_y(t)[-d'(k_d(t), W(t))k_y(t)] + \mu_d(t)[-\kappa_d d'(k_d(t), W(t))k_d(t) - \kappa_d d(k_d(t), W(t)) - \delta_d] \\
&= \rho\mu_d(t) - \dot{\mu}_d(t)
\end{aligned} \tag{2.13d}$$

$$\frac{\partial \mathcal{H}}{\partial \mu_y(t)} = Ak_y(t)^\alpha - c(t) - Q(i_d(t)) - d(k_d(t), W(t))k_y(t) - \delta_y k_y(t) = \dot{k}_y(t) \tag{2.13e}$$

$$\frac{\partial \mathcal{H}}{\partial \mu_d(t)} = i_d(t) - \kappa_d d(k_d(t), W(t))k_d(t) - \delta_d k_d(t) = \dot{k}_d(t). \tag{2.13f}$$

The canonical system

We rewrite the first order condition (2.13a), use the natural logarithm and take the total time derivative.

$$\mu_y(t) = U'(c(t)) = \frac{1}{c(t)} \tag{2.14}$$

$$\ln(\mu_y(t)) = \ln(\frac{1}{c(t)}) \tag{2.15}$$

$$\frac{\dot{\mu}_y(t)}{\mu_y(t)} = -\frac{\dot{c}(t)}{c(t)} \tag{2.16}$$

Analogous we can use the first order condition (2.13b).

$$\mu_d(t) = \mu_y(t)[Q'(i_d(t))] = \mu_y(t)\theta_0[\theta_1 + 2\theta_2 i_d(t)] \tag{2.17}$$

$$\begin{aligned} \ln(\mu_d(t)) &= \ln(\mu_y(t)\theta_0[\theta_1 + 2\theta_2 i_d(t)]) \\ &= \ln(\mu_y(t)) + \ln(\theta_0) + \ln(\theta_1 + 2\theta_2 i_d(t)) \end{aligned} \tag{2.18}$$

$$\frac{\dot{\mu}_d(t)}{\mu_d(t)} = \frac{\dot{\mu}_y(t)}{\mu_y(t)} + \frac{2\theta_2 \dot{i}_d(t)}{\theta_1 + 2\theta_2 i_d(t)} \tag{2.19}$$

So we use (2.16), (2.19), (2.13c), (2.13d), (2.13e), and (2.13f) to write the canonical system.

$$\dot{c}(t) = -c(t)\frac{\dot{\mu}_y(t)}{\mu_y(t)} \tag{2.20a}$$

$$\dot{i}_d(t) = \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2}[\frac{\dot{\mu}_d(t)}{\mu_d(t)} - \frac{\dot{\mu}_y(t)}{\mu_y(t)}] \tag{2.20b}$$

$$\dot{\mu}_y(t) = -\mu_y(t)[A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho] \tag{2.20c}$$

$$\begin{aligned} \dot{\mu}_d(t) &= \mu_y(t)[d'(k_d(t), W(t))k_y(t)] \\ &\quad + \mu_d(t)[\kappa_d d'(k_d(t), W(t))k_d(t) + \kappa_d d(k_d(t), W(t)) + \delta_d + \rho] \end{aligned} \tag{2.20d}$$

$$\dot{k}_y(t) = Ak_y(t)^\alpha - c(t) - Q(i_d(t)) - d(k_d(t), W(t))k_y(t) - \delta_y k_y(t) \tag{2.20e}$$

$$\dot{k}_d(t) = i_d(t) - \kappa_d d(k_d(t), W(t))k_d(t) - \delta_d k_d(t) \tag{2.20f}$$

Euler equations for optimal controls

The dynamics of the optimal controls are given by the Euler equations. Applying the Pontryagin conditions to this control problem we yield the Euler equations for the optimal controls. We substitute (2.20c) into (2.20a) to describe the optimal consumption and additional (2.20d) and (2.17) into (2.20b) to see the optimal investments in defense capital.

$$
\begin{aligned}
\dot{c}(t) &= -c(t)\frac{-\mu_y(t)[A\alpha k_y(t)^{\alpha-1} - d(k_d(t),W(t)) - \delta_y - \rho]}{\mu_y(t)} \\
&= c(t)[A\alpha k_y(t)^{\alpha-1} - d(k_d(t),W(t)) - \delta_y - \rho] \qquad (2.21) \\
\dot{i_d}(t) &= \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2}\Bigg[\frac{\frac{\mu_d(t)}{\theta_0[\theta_1+2\theta_2 i_d(t)]}[d'(k_d(t),W(t))k_y(t)]}{\mu_d(t)} \\
&\quad + \frac{\mu_d(t)[\kappa_d d'(k_d(t),W(t))k_d(t) + \kappa_d d(k_d(t),W(t)) + \delta_d + \rho]}{\mu_d(t)} \\
&\quad - \frac{-\mu_y(t)[A\alpha k_y(t)^{\alpha-1} - d(k_d(t),W(t)) - \delta_y - \rho]}{\mu_y(t)}\Bigg] \\
&= \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2}\Bigg[\frac{1}{\theta_0[\theta_1 + 2\theta_2 i_d(t)]}[d'(k_d(t),W(t))k_y(t)] \\
&\quad + [\kappa_d d'(k_d(t),W(t))k_d(t) + \kappa_d d(k_d(t),W(t)) + \delta_d + \rho] \\
&\quad + [A\alpha k_y(t)^{\alpha-1} - d(k_d(t),W(t)) - \delta_y - \rho]\Bigg] \\
&= \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2}[A\alpha k_y(t)^{\alpha-1} + (\kappa_d - 1)d(k_d(t),W(t)) + \kappa_d d'(k_d(t),W(t))k_d(t) + \delta_d - \delta_y] \\
&\quad + \frac{1}{2\theta_0\theta_2}[d'(k_d(t),W(t))k_y] \qquad (2.22)
\end{aligned}
$$

2.5.2 Two solutions of the model

To solve the model given in Eqs. 2.9 we proceed as follows. First, to find an initial solution, we redefine the periodic water function $W(\gamma, \bar{W}, t) := \bar{W} + \gamma\Omega(t)$, where $\Omega(t)$ refers to the water function given in Eq. 2.9f.

For the continuation of the function with a periodic solution we consider the more general boundary value problem (BVP)

$$\dot{x}(t) = f(x(t), W(\gamma, \bar{W}, t)), \quad x(t) \in \mathbb{R}^n, \quad t \in [0, 1] \qquad (2.23a)$$

$$x(0) = x(1) \qquad (2.23b)$$

with

$$W(\gamma, \bar{W}, t) = \bar{W} + \gamma\Omega(t), \quad \Omega(0) = \Omega(1). \qquad (2.23c)$$

For $\gamma = 0$ and $\bar{W} = 1$ we found two feasible and optimal solutions $\hat{x}_1$ and $\hat{x}_2$, each corresponding to a different constraint constellation (i.e. $i_d(t) > 0$ and $i_d(t) = 0$). For these two cases the following continuation steps were used: Since $x(\cdot) \equiv \hat{x}$ is an isolated solution and $f_x(\hat{x}, \bar{W})$ is non-singular, $f_\gamma(\hat{x}, \bar{W}) \neq 0$ and the minimal period of $\Omega(t)$ is one. For an isolated solution there exists $\varepsilon > 0$ such that for every $\gamma \in B_\varepsilon(0)$ a unique solution $x(\cdot, \gamma)$ for (2.23) exists. Numerically these solutions can be found e.g. by the pseudo-arclength or Moore-Penrose continuation. As long as $x(\cdot,\gamma)$ itself is an isolated solution and the linearization of Eq. 2.23 is non-singular the continuation proceeds.

For the actual computation the Moore-Penrose continuation in the implementation of the specific MATLAB® -Toolbox *OCMat* from Grass & Seidl (2013) was used, whereas it was shown that in the cases of $\hat{x}_1$ and $\hat{x}_2$ the linearization was always non-singular. This was done in two steps:

1. Continuation along γ from 0 to 1.
2. Continuation along $\bar{W}$ from 1 to 0.

So, we derived the two solutions for the model given in Eqs. 2.9, whereas the periodic water function is $W(\gamma,\bar{W},t)=\bar{W}+\gamma\Omega(t)=\Omega(t)$ and therefore equals Eq. 2.9f.

3 Optimal investment and location decisions of a firm in a flood risk area using Impulse Control Theory

The present chapter corresponds to the following scientific publication in its original form:

Grames J., Grass D., Kort P., and Prskawetz A. (2018): Optimal investment and location decisions of a firm in a flood risk area using Impulse Control Theory. Central European Journal of Operations Research. https://doi.org/10.1007/s10100-018-0532-0

Abstract

Flooding events can affect businesses close to rivers, lakes or coasts. This chapter provides an economic partial equilibrium model, which helps to understand the optimal location choice for a firm in flood risk areas and its investment strategies. How often, when and how much are firms willing to invest in flood risk protection measures? We apply Impulse Control Theory and develop a continuation algorithm to solve the model numerically.
We find that, the higher the flood risk and the more the firm values the future, i.e. the more sustainable the firm plans, the more the firm will invest in flood defense. Investments in productive capital follow a similar path. Hence, planning in a sustainable way leads to economic growth. Sociohydrological feedbacks are crucial for the location choice of the firm, whereas different economic settings have an impact on investment strategies. If flood defense is already present, e.g. built up by the government, firms move closer to the water and invest less in flood defense, which allows firms to generate higher expected profits. Firms with a large initial productive capital surprisingly try not to keep their market advantage, but rather reduce flood risk by reducing exposed productive capital.

3.1 Introduction

Climate change puts increasing environmental pressure on coastal zones (Turner *et al.*, 1996) and on areas around lakes and rivers (Vrösmarty *et al.*, 2000). On top of the list of potential impacts of climate change are effects of sea level rise on coastal cities and effects of extreme events on built infrastructure like floods from heavy precipitation events (Hunt & Watkiss, 2011). Floods and other extreme weather events increase economic losses (Easterling *et al.*, 2000). Large-scale flood disasters from recent years have gained attention among decision makers (e.g. businesses). Implementing actions to reduce disaster risks and build flood resilience facing limited resource needs decision support tools (Mechler & Bouwer, 2014).
River and Coastal engineers develop risk-analysis techniques for high-level planning and detailed designs using simulation models (Sayers *et al.*, 2002). Economic approaches are cost-effectiveness analyses, multi-criteria analyses, robust-decision-making approaches and dynamic programming (Zwaneveld & Verweij, 2014; Eijgenraam *et al.*, 2012, 2014). The most popular tool is cost-benefit analysis applied to cities (Lichfield, 1960; Hunt & Watkiss, 2011), regions, and countries (e.g. Jonkman *et al.* (2004)).

There is a number of methods to control floods. Coastal defenses can be e.g. sea walls, beach nourishment, barrier islands or tide gates in conjunction with dykes and culverts. Next to rivers one can construct levees, lakes, dams, reservoirs, bunds, weirs or retention ponds to hold extra water during floods. Moreover, floodways, water gates, diversion channels, temporary barriers or a property level protection can be built. Often flood control measures significantly change the environment and also influence the water system. E.g. levees increase downstream flow and diversion channels redirect water to another area. Both effects increase flood risk nearby. In addition, flood risk increases due to the levee effect (Collenteur *et al.*, 2015), i.e. people and businesses feel save and move closer to the river, and exposed capital accumulates. Other flood control systems like temporary perimeter barriers are not fool proof and can cause unexpected flood damage Wald (2011). Last, but not least, constructions can restrain the function of a natural flood plain and therefore increase flood risk.
To sum up, installing flood control measures decreases flood risk, but the effect can be significantly reduced when the flood control measure induces feedbacks on the flood hazard or the exposed capital. We include this socio-hydrological feedback mechanismn in our model and study its implications for the system dynamics.

Often investments in flood risk protection measures are done by the government. In this chapter we aim to identify the firm's willingness to pay for flood protection. Furthermore, also actions to reduce flood risk can be taken at the firm-level (Johnson & Priest, 2008). Businesses can install their own prewarning systems, choose a more expensive but safer technology for building the production plant, adjust the production process by using a safer construction technology or another type of machines. Last, but not least, more expensive labour agreements attract better human resources.
While our focus is on the firm's investment decisions we also investigate whether and to

which extend the firm level decisions are influenced by investments of the government.

The aim of this chapter is to understand investment decisions of firms and their implications on businesses in flood risk areas. Viglione *et al.* (2014) and Di Baldassarre *et al.* (2013) developed a conceptual descriptive model to understand the feedbacks of flood risk reduction (i.e. investments in flood defense and moving away from the river) and flood damage from a societal perspective. Grames *et al.* (2016) introduced an optimal decision framework to investigate the interaction of a society's investment in flood defense and productive capital. In this chapter we consider a partial equilibrium model and try to understand the firm's investment decisions in its interrelations with the hydrological system. The focus on a firm instead of the whole society allows to specifically analyze a location choice together with the firm's willingness to pay for flood protection. In contrast to the decisions from a societal point of view, the focus on the firm level also allows us to study the role of firm specific characteristics for the decision process.
A representative firm can have multiple choices: First, it can choose the optimal location for its production plant, second the optimal investment in capital used for production and third, the optimal investment in flood risk reduction measures.

To implement this diverse decision framework this chapter rests on three building stones. One building stone consists of so-called capital accumulation models where optimal control theory is applied to determine the firm's optimal investment behavior over time. This literature starts out with Eisner *et al.* (1963) and later contributions include Davidson & Harris (1981), Barucci (1998) and Grass *et al.* (2012).

Another building stone are the impulse control models that consider e.g. dike height optimization, see Chahim *et al.* (2013). Subject to a water level that increases over time, the decision maker has to decide about the optimal timing and size of the increase of the dike height in order to find an optimal balance between the costs associated with dike height increases and the improved flood protection that results from a dike height increase. This strand of literature abstracts from (firm) investments so that the economic value of the protected land develops exogenously.
The Impulse Control Problem is solved using the impulse maximum principle (Blaquiere, 1985; Rempala & Zabczyk, 1988; Chahim, 2012). The general theory of viscosity analysis and quasivariational inequalities (e.g. Barles (1985), El Farouq *et al.* (2010)) is more consistent, in the sense that it allows more general statements under less restrictive assumptions, covering as many specific cases as possible. However, for the model in this chapter the Impulse Maximum Principle seems quite appropriate for an economic interpretation and its numerical calculation.

The underlying chapter combines these two approaches, i.e. (impulse) investments have to be undertaken to protect the firm from floods while at the same time the firm establishes an optimal investment pattern that directly influences its economic value.

The third building stone is the optimal location choice for the firm's production plant

(Fig.3.1) additional to the investment decisions. This location choice is like the choice of technology explained in Brito (2004). A location closer to the water is more profitable in the sense that the water's infrastructure (transportation, cooling) is easier available and the site is more attractive for the labour force and consumers. But on the other hand being closer to the river implies that the firm faces a larger risk of being flooded.
In location theory Glatte (2015) defines three categories for site selection framework

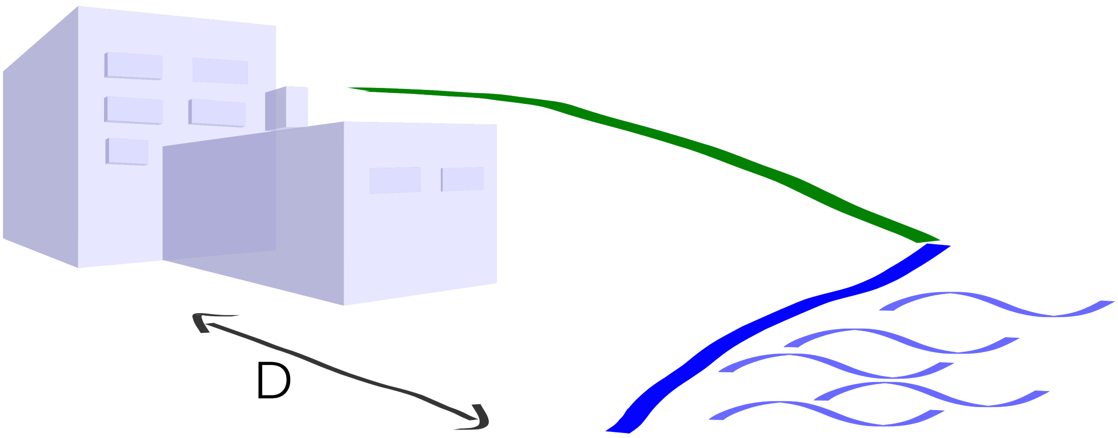


Figure 3.1: The firm chooses where to build its production plant by choosing the distance to the water.

conditions: technical and architectural, economic, and legal, whereas Goette (1994) distinguishes between economic site conditions (sales potential, competitive conditions, infrastructure and transportation costs, labor, monetary conditions), political site conditions (tax legislation, environmental protection, institutional market entry barriers, support of business, political risks), cultural site conditions (differences in language, mentality, religion, and the lack of acceptancy of foreign companies), and geographical site conditions (climate, topography).
Natural hazards are often missing in the site selection literature and the only quantitative methods are cost-benefit analysis or cost-effectivness analysis (Glatte, 2015). We introduce a conceptual framework where firms take economic and environmental conditions into account which in turn are affected by the firm's decisions.

The firm's decisions are based on an optimization problem, where in a first step the firm is controlling its investments. In a second step, the firm aims to find the optimal location knowing that revenues and costs will depend on the location choice. The firm can only choose its optimal location after having determined the set of optimal investment strategies. The planning horizon is finite, but the firm also considers its salvage value at the end of the planning period. Entrepreneurs do consider only a finite life cycle of a firm. Family businesses may plan in longer terms.

For the firm's profit maximization only costs are relevant, which can be transferred to monetary values. Consequently, flood damage is measured by so-called direct tangible costs (Merz *et al.*, 2010). We assume the firm to be a production plant with a lot of tangible capital. Direct flood damage reflects the costs of replacing damaged capital (Veen & Logtmeijer, 2005).

The aim of the chapter is to understand the investment decisions of a representative firm in a flood risk area. We want to identify how much, how often, and when a firm is willing to invest in flood risk protection measures and what the optimal location choice is. Our qualitative model helps to understand feedback mechanisms between the firm's decisions and the hazard of flooding. In Section 3.2 we explain the general model and its analytical solutions. After discussing the numerical solution of the benchmark model in Section 3.3 we investigate the impact of sustainable planning, the economic situation and the sociohydrological feedbacks in Section 3.4. Section 3.5 concludes the chapter and some detailed derivations are given in the Appendix.

3.2 General model

In this section we set out the general model of investment planning in a flood risk area, setting up the Hamiltonian and Impulse Hamiltonian of the representative firm and deriving necessary conditions for optimally determining flood protection and productive investments. The location choice is done in a second step based on the set of optimal decisions.

3.2.1 Flood impact

We model a firm located in a flood risk area. The expected flood water level above bankfull

$$W(t) = W_0 + \eta t, \tag{3.1}$$

is some initial water level $W(0) = W_0$ and increases with η [cm/year] due to climate change (Eijgenraam & Hertog, 2016). Anthropogenic flood risk protection $H(t)$, despite decreasing flooding occurrences, may increase flood water levels and consequently flood risk, because e.g. higher dikes make it more difficult for water to stream back in the sea/river after land has been flooded. We model this like Grames *et al.* (2016) and Viglione *et al.* (2014) by adding an additional amount of water due to man-made flood risk protection measures $\xi_H H(t)$. ξ_H is the sociohydrological parameter describing the feedback of flood risk protection measures $H(t)$ to flood risk. The resulting flood intensity $W(t) + \xi_H H(t)$ can be alleviated by increasing the minimum distance to water D_0 by the amount D for the location of the firm's production plant in the floodplain with slope α_D. Consequently, the flood impact in times of flooding $(W(t) + \xi_H H(t) > H(t))$ is

$$F_I(W(t), D, H(t)) = \frac{W(t) + \xi_H H(t)}{\alpha_D(D_0 + D)}. \tag{3.2}$$

If the flood impact $F_I(W(t), D, H(t))$ exceeds the current height of flood protection (e.g. dikes, levees) $H(t)$, damage occurs. According to Chahim *et al.* (2013) the flood probability $P_F(t)$ [1/year] is given by an initial probability P_0 and increases for a higher water level, but decreases with larger flood risk protection measures (i.e. dikes). This leads to the following flooding probability given a scaling parameter α_F [1/cm].

$$P_F(D, H(t)) \quad = \quad P_0 \exp[\alpha_F(F_I(W(t), D, H(t)) - H(t))] \tag{3.3}$$

Substituting equation (3.1) and (3.2) into equation (3.3) yields equation (3.4), where the socio-hydrological feedback is clearly visible: flooding will reduce the effectiveness of the flood protection by a factor $(1 - \frac{\xi_H}{\alpha_D(D_0+D)})$.

$$P_F(D, H(t)) = P_0 \exp[\alpha_F(\frac{W(0) + \eta t}{\alpha_D(D_0 + D)} - (1 - \frac{\xi_H}{\alpha_D(D_0 + D)})H(t))] \tag{3.4}$$

The relative flood damage in case of floods increases with higher flood impact $F_I(t)$ and is expressed as the proportion $F(W(t), D, H(t)) \in [0, 1]$ of destroyed capital following Grames *et al.* (2016); Viglione *et al.* (2014).

$$F(W(t), D, H(t)) = 1 - \exp[-F_I(W(t), D, H(t))] \tag{3.5}$$

3.2.2 Firm's expected profit

The firm faces a competitive market and produces output $Y(t)$ choosing the production factors capital $K(t)$ and the distance D to a river or coast in the sense that living closer to the water yields advantages for transport, lowers costs of transporting water to households and industry and is attractive for employees (see Viglione *et al.* (2014)). The effect of D on output is similar to a technological parameter as it scales the firm's output level for a given set of the other production factors (e.g.Brito (2004)). We assume a minimum necessary distance to the water body D_0. The production function has a Cobb-Douglas form and reads

$$Y(K(t), D) = \frac{1}{D_0 + D} K(t)^\alpha \tag{3.6}$$

with $\alpha \in [0, 1]$. We assume that the firm can sell all its output $Y(t)$ for a price p normalized to 1.
The firm invests $I_K(t)$ in its physical capital which depreciates with rate $\delta_K \in [0, 1]$.

$$\dot{K}(t) = I_K(t) - \delta_K K(t) \tag{3.7}$$

The costs for capital investment are $\alpha_K I_K(t)^2$ with α_K as a constant scaling parameter.

The value of flood damage is the sum of costs for repairs and cleanup, and costs for lost revenue due to business interruption. First, we assume that repair costs $C_F(K(t), F(t))$

are just as high as the damaged physical capital stock, depending on the impact of flooding $F(t) \in [0, 1]$.

$$C_F(K(t), F(t)) = F(t)K(t) \tag{3.8}$$

Second, the lost revenue due to business interruption is equal to $P_F(D, H(t))Y(K(t), D)$. Hence, revenue times probability that no flood occurs reads

$$[1 - P_F(D, H(t))]Y(K(t), D). \tag{3.9}$$

We assume that everything is repaired immediately after the flooding and production continues with the same capital stock $K(t)$ and level of flood protection $H(t)$ after any flooding. Veen & Logtmeijer (2005); Leiter *et al.* (2009) and Parkatti (2013) are using a similar approach.

To sum up, we can express the expected profit as the difference between expected revenue and expected costs, i.e. investment and damage costs.

$$\begin{aligned}\pi_e(K(t), D, H(t), I_K(t)) = (1 - P_F(D, H(t)))\left[Y(K(t), D) - \alpha_K I_K(t)^2\right] \\ -P_F(D, H(t))C_F(K(t), F(t))\end{aligned} \tag{3.10}$$

3.2.3 Impulse investments in flood defense

Additionally to investments in capital stock K, the firm can invest in flood risk protection at the expense of costs $I_H(u_i, H(t))$ to add an amount $u_i > 0$ to their flood protection $H(t)$ at specific points in time $t = \tau_i$. Therefore the firm chooses the optimal number $N \geq 0$ of investments, the optimal timing τ_i ($i \in \{1, .., N\}$) and the optimal amount $u_i > 0$ ($i \in \{1, .., N\}$).

$$H(\tau_i^+) = H(\tau_i^-) + u_i \tag{3.11}$$

holds for $i \in \{1, .., N\}$. Here $H(\tau_i^-)$ is the level of flood risk protection before and $H(\tau_i^+)$ the level of flood risk protection after the ith investment.
We model exponential investment costs in flood defense capital following Eijgenraam & Hertog (2016) with positive constants θ_1, θ_2 and θ_3.

$$I_H(u, H(\tau^-)) = \begin{cases}(\theta_1 + \theta_2 u)\exp(\theta_3(H(\tau^-) + u)) & \text{if } u > 0 \\ 0 & \text{if } u = 0\end{cases} \tag{3.12}$$

For time $t \notin \{\tau_1, ..., \tau_N\}$ the flood risk protection capital does not change.

$$\dot{H}(t) = 0 \tag{3.13}$$

The firm can invest in flood defense capital during a finite planning period $[0, T]$. The total expected profit considering all types of costs can be displayed as follows using the interest rate r to discount future values.

$$\int_0^T \pi_e(K(\tau), D, H(\tau), I_K(\tau))e^{-r\tau}d\tau - \sum_{i=1}^{N} I_H(u_i, H(\tau_i))e^{-r\tau_i} \tag{3.14}$$

The value of the firm at the end of the planning horizon T is the difference between expected remaining capital $[1 - P_F(D, H(T))]K(T)$ and expected damage $P_F(D, H(T))F(D, H(T))K(T)$.

$$V(K(T), D, H(T)) = [1 - P_F(D, H(T))]K(T) - P_F(D, H(T))F(D, H(T))K(T) \tag{3.15}$$

To model not only the expected profit during the planning period we additionally consider the expected value $V(K(T), D, H(T))$ of the firm after the planning period. Therefore we use the so-called salvage value (Chahim *et al.*, 2013). Note, that the firm does not make any new decisions after the planning period.

$$\int_T^\infty V(K(T), D, H(T))e^{-rt}dt = \frac{1}{r}e^{-rT}V(K(T), D, H(T)) \tag{3.16}$$

3.2.4 The firm's optimal decisions

The firm maximizes accumulated discounted profit given an interest rate r within a finite planning time horizon expecting floods at unknown times. As a first step it solves the problem for a given value of D. It can choose the number N of flood defense investments to increase flood risk protection measures by $u_i > 0$ and its timings τ_i during the finite planning period $[0, T]$. It also controls the investment in physical capital $I_K(t) > 0$ during the planning period and takes into account the salvage value $V(K(T), D, H(T))$ weighted with a time preference δ_S.

$$\max_{\{u_i, \tau_i, N, I_K(t)\}} \int_0^T \pi_e(K(\tau), D, H(\tau), I_K(\tau))e^{-r\tau}d\tau \tag{3.17a}$$
$$- \sum_{i=1}^{N} I_H(u_i, H(\tau_i))e^{-r\tau_i} + \delta_S \frac{1}{r}e^{-rT}V(K(T), D, H(T))$$

To summarize, the dynamics of the state variables $K(t)$ and $H(t)$ are

$$\dot{K}(t) = I_K(t) - \delta_K K(t) \tag{3.17b}$$
$$\dot{H}(t) = 0 \quad \text{for } t \notin \{\tau_1, ..., \tau_N\} \tag{3.17c}$$
$$H(\tau_i^+) = H(\tau_i^-) + u_i \quad \text{for } i \in \{1, ..., N\} \tag{3.17d}$$

and their initial values are $K(0) = K_0$ and $H(0^-) = 0$.
As a second step, the firm chooses the optimal location (D) for its production plant given the solutions of problem (3.17).

We follow the work from Chahim (2012) to derive the necessary optimality conditions for our maximization problem by applying the Impulse Control Maximum Principle (see Appendix 3.6.2). This way we obtain the optimal paths of the decision variables and the costates. We will describe the intuition and for the detailed results refer to the Appendix 3.6.1.

The optimal capital investment is as such that the expected revenue stream, including the increase in the salvage value, equals the expected marginal costs. If the production is capital intense, the firm invests intensively in its production capital but slows the investments down with an increasing stock of physical capital, i.e. an extra unit of physical capital is more valuable if the capital stock is (still) small. However, when the expected damage rate is high, the firm will invest less in physical capital. The investment behaviour does not change much if the expected damage rate (possibly amplified by a high water level) is high, but is very sensitive to small changes of low expected damage rates.
A high current and long term value of the physical capital due to a high shadow price and high interest rates motivates the firm to invest in its physical capital, whereas higher investment costs decelerate the accumulation of physical capital. Nonetheless, the firm wants to sustain its capital stock and invests more if the depreciation rate is higher.
The shadow price for physical capital at the end of the planning period equals the difference of the discounted marginal expected output and the expected damage rate.

For investment in flood defense we derive the optimal timing and the amount of investments. For these decisions the shadow price of flood defense capital is crucial. Whenever the benefits of investing in flood defense (i.e. increase in shadow price and expected profit) exceed the costs, the firm will invest in flood risk measures. The amount of investment will be higher if the previous level of flood defense is low and the shadow price of the increased flood defense capital is high. Moreover investment increases if investment costs are low. Still, the cost structure is important: Significantly lower fixed costs could increase the number of investments and therefore decrease the investment amount.
The shadow price for flood defense capital at the end of the planning period is the expected loss from flooding, i.e. the sum of the revenue due to business interruption, the avoided costs at the time of the flood, and the direct damage described by value of repair and cleanup costs. The net present value of the shadow price for flood defense capital increases with expected future loss (i.e. lost profit and damaged capital) augmented with stronger sociohydrological feedbacks and a closer distance to the water. Contrary, if the expected sustained capital at the end of the planning period is high the value decreases.
The number of impulse investments into flood defense capital is rather small (i.e. less than four investments in a feasible planning period) due to fixed costs. Furthermore, the first investment in flood defense is usually early to ensure a low flood hazard for the location of the production plant.
Last, but not least, we find that the optimal level of flood defense can never exceed an

upper bound $\bar{H}$. Still, this level depends on the propertis of the firm like production capacities and existing capital stock. The upper bound will be lower if the firm locates further away from the river, the sociohydrological feedbacks are small and the initial flooding probability is low.

3.3 Benchmark model

In this section we show the numerical solution of the model and discuss its economic intuition. To derive numerical solutions for our impulse control problem we apply the (multipoint) boundary value approach (Grass, 2017). The idea is to solve a boundary value problem (BVP) based on the system dynamics given by the canonical system and update the according boundary conditions at impulse times. A continuation technique is used to continue and find solutions with different number of impulses. The objective values of such solutions are compared and the optimal solution is chosen. Moreover, the continuation alogrithm allows to continue a solution for every model data. Details about the numerical method, which was developed to solve such types of problems, are described in Grass (2017). Details about the application of the numerical method to our proposed model are found in Appendix 3.6.3. First, we derive the optimal solution for investments depending on the distance D. Second, we plot the objective function evaluated at the optimal investment as a function of D and locate the maximum with respect to D.

We use the following initial conditions. The mean water level above bankfull as well as the flood protection are normalized to zero at the beginning of the planning period. The productive capital initially available for the firm is 10^8 \$. The initial flooding probability is 0.001 per year according to Chahim *et al.* (2013). D is referred to a length measure, but scale free. Still, we can exemplify the minimum distance to the water with 5m. All the variables and their initial conditions are listed in Table 3.1. The parameters are displayed in Table 3.2. Many parameters (r, A, α, α_K, δ_K) are chosen according to standard economic literature, and other parameters (τ_k, α_P) are scaling factors. Most hydrology parameters ξ_H, α_D, α_F are defined in Viglione *et al.* (2014). Investment costs in flood protection θ_1, θ_2, θ_2 and natural water level rise η are introduced in e.g. Chahim *et al.* (2013); Eijgenraam & Hertog (2016). We choose a shorter planning horizon T than Chahim *et al.* (2013) to reflect a feasible life cycle time of a firm (Lumpkin & Dess, 2001). The time discount of the salvage value δ_S is given by $(1+\delta_L)^T = 1+\delta_S$, where δ_L denotes a standard yearly time preference rate. Note, r represents the interest rate of the capital market and is not necessarily equal to the individual time preference rate δ_L.
In addition to the the benchmark values we have also listed the values for sensitivity analysis described in the next sections. Note, that our numerical calculations are aimed to provide a qualitative analysis to understand feedbacks and mechanisms within a sociohydrological model of floodings.

The optimal solution is to locate the firm's production plant rather close to the

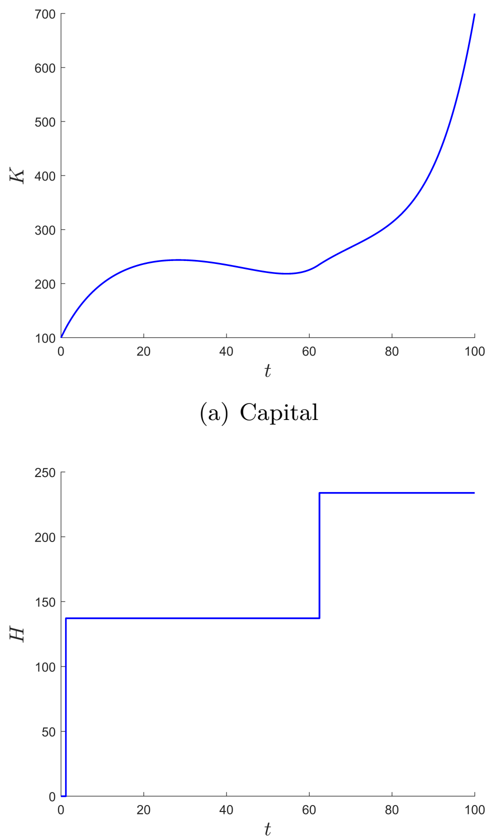
Table 3.1: Variables of the model

Variable	Interpretation	Unit		
t	time	[year]		
$K(t)$	productive capital	10^6 \$		
D	firm's distance to water	10^2[m]		
u_i	height of i^{th} increase in flood protection measures	[cm]		
$H(t)$	level of flood protection	[cm]		
τ_i	timing of i^{th} investment in flood defense	[year]		
N	number of investments i	[ ]		
$I_H(t)$	costs for investment in $H(t)$	10^6 \$		
$\pi_e(t)$	firm's expected profit	10^6 \$		
$Y(t)$	firm's output	10^6 \$		
$I_K(t)$	investment in physical capital	10^6 \$		
$C_F(t)$	total costs of flooding	10^6 \$		
$W(t)$	water level	[cm]		
$F(T)$	proportion of flooding damage	[0,1]		
$F_I(T)$	flood impact	[]		
$P_F(t)$	flooding probability	[1/year]		
$\lambda_K(t)$	shadow price of physical capital	10^6 \$		
$\lambda_H(t)$	shadow price of flood protection	10^6 \$		
Initial values	Interpretation	Unit	Value	case
W_0	initial water level	[cm]	0	
K_0	initial productive capital	10^6 \$	100	500
H_0	initial flood protection	[cm]	0	200
P_0	initial flooding probability	[1/year]	0.001	

Table 3.2: Parameters of the model and their units of measurement

Parameter	Interpretation	Unit	Base case	Case study
D_0	minimal distance to water	10^2[m]	0.05	
T	end time of planning period	[year]	100	50,150
r	interest rate	[1/year]	0.03	
A	technology	[ ]	1	
α	output elasticity of physical capital	[0,1]	0.3	
α_K	scale for expected investment in $K(t)$	[ ]	0.01	
τ_K	scale for deterministic investment in $K(t)$	[ ]	0.01	
δ_K	depreciation rate of $K(t)$	[1/year]	0.05	
θ_1	fixed costs for investing in $H(t)$	10^6 \$	100	
θ_2	linear costs for investing in $H(t)$	10^6 \$/cm	0.5	
θ_3	exponential costs for investing in $H(t)$	[ln(10^6 \$)/cm]	0.005	
$\tilde{\theta}_1$	transformed fixed costs for investing in $H(t)$	10^6 \$	$\theta_2 + \theta_1\theta_3$	
$\tilde{\theta}_2$	transformed linear costs for investing in $H(t)$	10^6 \$	$\theta_2\theta_3$	
η	increase of water level per year	[cm/year]	0.5	
ξ_H	additional rise of the water level due to existing defense capital	[ ]	0.3	0, 0.5
α_D	scale of the slope of the floodplain	[ ]	10	
α_F	scaling of flooding probability	[1/cm]	0.05	
δ_S	time discount of salvage value	[0,1]	0.1	0, 0.25
α_P	approximation parameter for flooding probability	[]	100	

water (Fig.3.4) and to make two impulse investments in flood risk protection measures (Fig.3.2(b)). The dynamics of the capital K and the flood defense H are displayed in Fig.3.2. The first jump occurs very early so that the risk of flooding is very small and the firm can invest in its capital to gain high expected revenues. Since flood risk is increasing with time (Eq.(3.1)) the firm's investments (Fig.3.3) decrease as well. We observe an anticipation effect of the firm, since capital investment increases shortly before the second impulse investment. At the time of an impulse investment I_H the continuous investment I_K jumps too.
The second impulse investment is in the last third of the planning period and just as high as the upper bound $\bar{H}$ derived in Eq.(3.27).



(a) Capital

(b) Flood defense

Figure 3.2: State dynamics in the planning period $[0, T]$. Firm's capital K (a) increases only when the flood risk is low as a result of a high flood risk protection standard (b).

Investments in flood risk protection measures increase economic activity. We can identify that whenever the firm feels saver, it invests more. This is a positive feedback loop and leads to sustainable economic growth, because the firm's capital is high and flood risk is low.

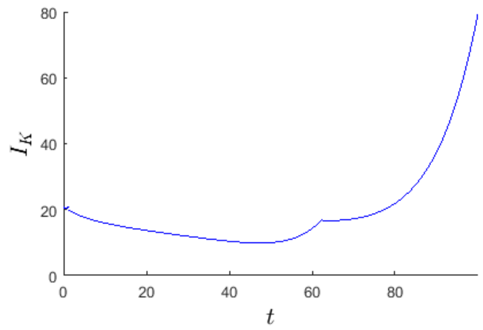


Figure 3.3: Capital investment I_K

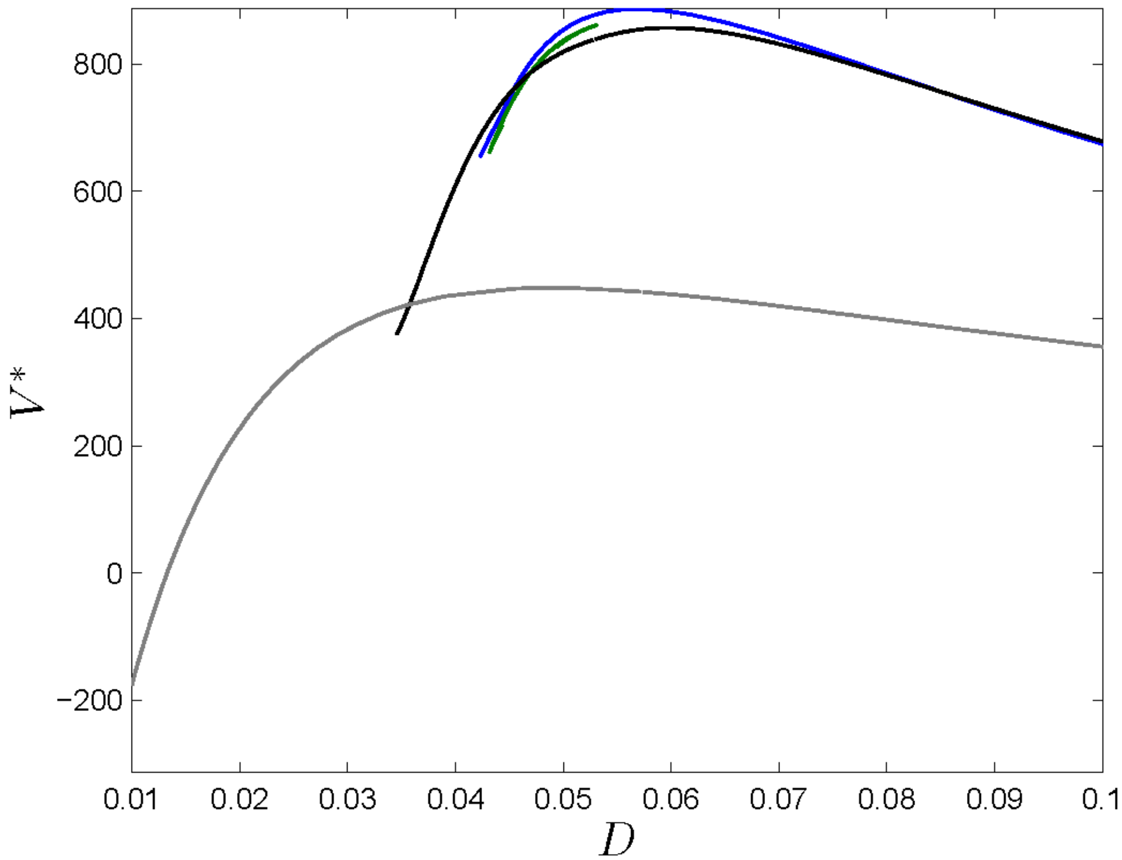


Figure 3.4: Solution structure given by the objective value V^* depending on D for no impulse investments (grey), one impulse investment (black), two impulse investments (blue) and three impulse investments (green). Note, that the objective values do not exist for every value of D for each case.

Moving closer to the water increases production output, but also increases flood risk.

Depending on which effect dominates, the expected profit either increases to the left of the peak or decreases to the right of the peak. One can identify this interesting trade-off in Fig.3.4, where we find the optimal location of the firm's production plant (D) at the peak of the value function V^* of problem set Eqs.(3.17).
It is optimal to make two impulse investments (Fig.3.4). Investing more often is always slightly worse, because fixed costs occur more often. Investing in flood defense only once or even never would only be better if the firm was located closer to the river, but the objective value would decrease. This would imply that the production output is higher in the beginning and the expected profit much less at the end of the planning horizon because flood risk is increasing dramatically. This also leads to a lower salvage value at the end of the planning horizon.

3.4 Alternative scenarios

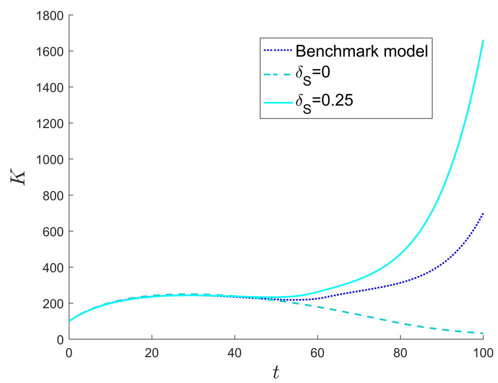
In this section we discuss the optimal investment decisions from the perspective of sustainability, the economic setting and the socio-hydrological feedbacks. The firm has three options to adapt to different situations. Firstly, it can choose the number and amount of investment in flood risk protection measures. Secondly, it can choose the investment strategy in its capital within the planning period. Thirdly, it can choose the location for its production plant.
We will compare the different scenarios to the benchmark model to understand which option is most suitable to adapt optimal investment decisions for a different hydrological and economic setting.

3.4.1 The role of sustainability

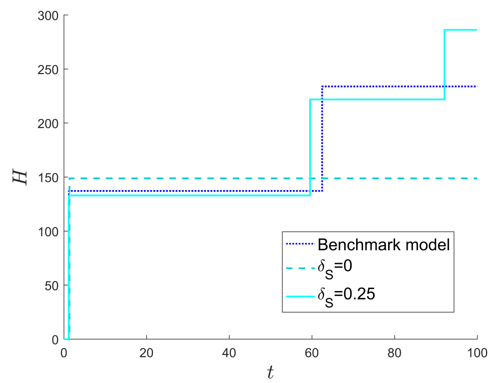
Two parameters reflect how important sustainability is for the decision making firm. On the one hand, the salvage value at the end of the planning period is weighted with a certain time preference rate δ_S. On the other hand, the planning horizon T is important for the investment decisions. We will discuss both options in detail.

If the value of the firm at the end of the planning period is important ($\delta_S > 0$), firms care about flood risk protection measures in the long run and its net present value is significantly higher. The optimal location of the firm's production plant is at an increased distance to the water body. But the more crucial impact is the investment behavior towards the end of the planning horizon. Fig.3.5 shows the time paths of the firm's capital K and the flood defense H for different time preference parameters δ_S. The investment behavior in the beginning is rather similar, but for a higher δ_S the firm invests much more in its productive capital at the end of the planning period. Furthermore, the firm is willing to invest in flood defense more often.
Decision makers in firms with a high time preference rate δ_S can be e.g. families, en-

trepreneurs who are confident about a long life time of their product(s) or entrepreneurs who are able to adapt to a changing environment and market demand.
If the firm cannot be sold at the end ($\delta_S = 0$) because its product will be outdated and the firm cannot survive on the market anymore, it still invests once to protect itself from floods but tries to make a lot of profits only in the short term. After some time it will neither invest in its own capital ($I_K = 0$) nor in flood defense. So the risk of being flooded is much higher.
Even if a firm "only" cares about its own value it is willing to invest in flood risk protection measures and increases economic activity. This is important for the whole region.



(a) Capital



(b) Flood defense

Figure 3.5: State dynamics for $\delta_S = 0$ (dashed blue line), $\delta_S = 0.1$ (dotted dark blue line) and $\delta_S = 0.25$ (solid light blue line) in the planning period $[0, T]$. If the salvage value of the firm is important (a) firm's capital K increases towards the end of the planning horizon and (b) firms invest higher amounts and more often in flood risk protection measures.

Firms that do not expect to be on the market for a long time do not care (much) about flood protection. Fig.3.6 shows the number of impulse investments and the net

present value of the firm for various planning horizons T.
If the planning period is only a few years firms do not invest in flood defense, and the net present value V^* of the firm is also relatively low.
Firms with a planning horizon around thirty years are most valid. They optimally invest once in flood protection after some years and not in the very beginning.
When a firm plans for more than seventy years it optimally invests (at least) twice in flood defense, and the first investment is already very early. Moreover, Fig.3.7 b) shows that the early impulse investment with a planning horizon of 150 years doubles the amount of impulse investment of a firm with a planning horizon of 50 years. Additionally, firms with a longer planning horizon invest more in their capital already at the beginning (see Fig.3.7 a)).
The only disadvantage of investing a lot in flood defense is the necessity to save for these investments and consequently invest less in the firm's productive capital. This could lead to a regression. Firms would be able to keep investing in their physical capital if e.g. the state government built the flood defense.
To sum up, a sustainable planning process (longer planning horizon) of the firms increases GDP already at the beginning and guarantees a safe environment.

3.4.2 The economic situation

Depending on the economic situation firms choose different investment strategies. We first analyze a firm in a region where flood protection already exists like e.g. in the Netherlands. Secondly, we investigate the investment decisions of a firm with a high initial capital stock. This can be a company building a production plant in a country with lower prices or a firm with e.g. state subsidy for its company foundation.

Firms located in a flood risk area where flood protection measures are already installed build their production plant closer to the water and invest in flood defense much later (Fig.3.8). Even though the investment behavior in productive capital is similar to the benchmark model, the expected net present value of the firm is higher because the firm plant is safer and closer to the water.

A firm with high initial productive capital (Fig.3.8) does not invest more often in flood protection, but it will invest earlier, i.e. already at $t = 0$ and even to a higher extent because the firm has more to loose in case of a flood. Still, its location is only slightly closer to the water. Surprisingly, instead of building extra flood defense, the firm is reducing flood risk by decreasing productive capital K_0 to the level in the benchmark scenario. Consequently, the higher value of the firm (Fig.3.9 (b)) is only due to higher expected profits at the beginning of the planning period caused by a higher level of initial capital stock K_0.

We compare the net present value of a firm depending on initial flood defense H_0 and alternatively initial productive capital K_0 (Fig.3.9). In the first case, investment behavior changes (i.e. for higher H_0 less impulse investments are optimal). In the second

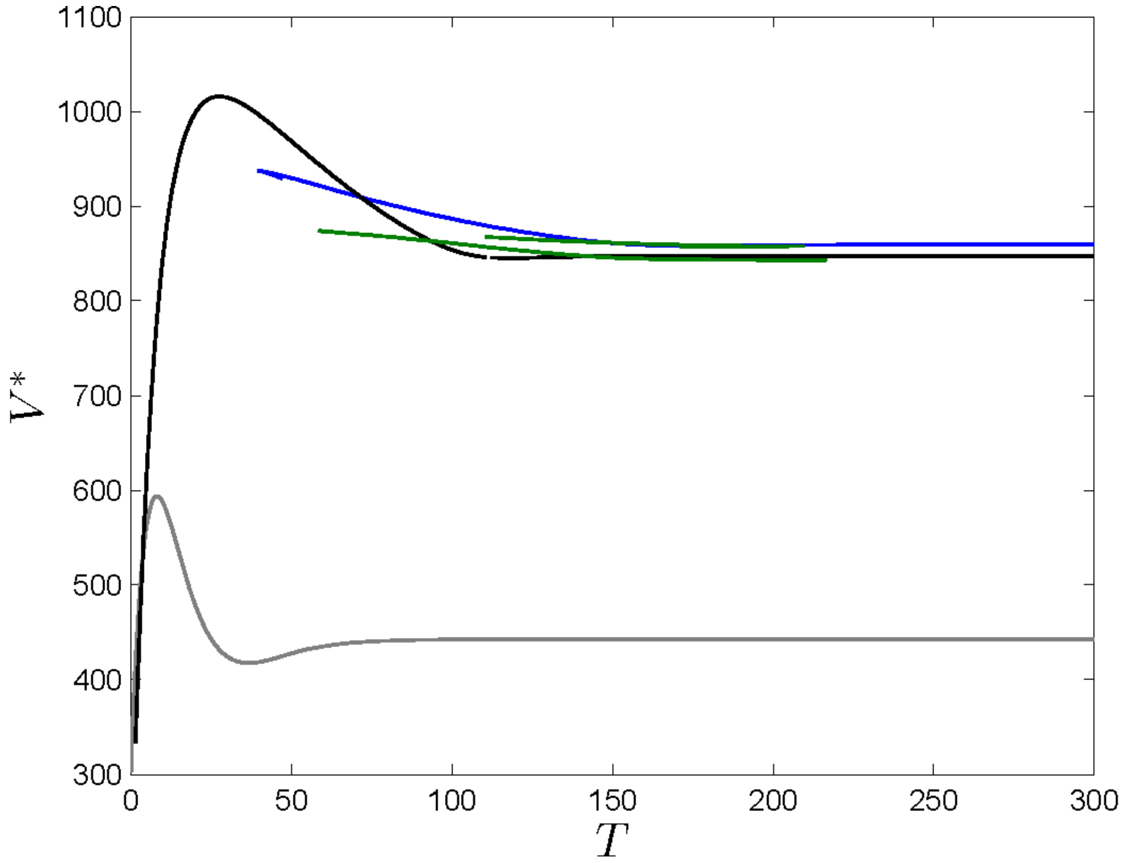


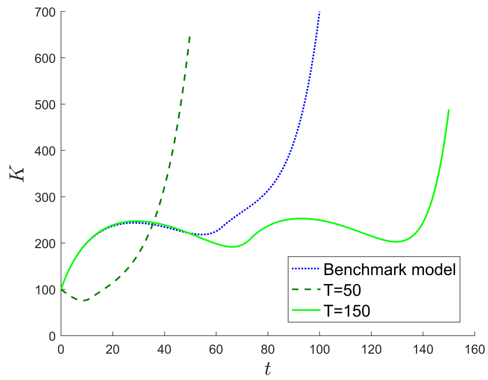
Figure 3.6: Solution structure given by the objective value V^* depending on the planning horizon T for no impulse investments (grey), one impulse investment (black), two impulse investments (blue) and three impulse investments (green).

case, investment behavior does not change (i.e. it is always optimal to invest twice in flood risk protection measures even if the firm could make a large one-time investment at the beginning).

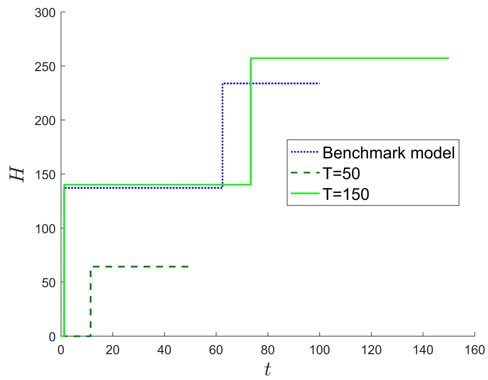
Not investing in flood risk protection measures (grey line in Fig.3.9) becomes more attractive for higher H_0 because the firm is safer anyways, whereas for a higher productive capital K_0 it is less profitable because the exposed capital is larger and therefore possible flood damage is larger. Consequently, flood risk is decreasing for higher (initial) flood defense and increasing for higher (initial) productive capital, since flood risk is defined as the product of flood hazard and exposed capital.

3.4.3 Sociohydrological feedbacks

Building flood risk protection measures often changes the environment and more specifically the water system. This can cause negative feedbacks for investing in flood defense



(a) Capital

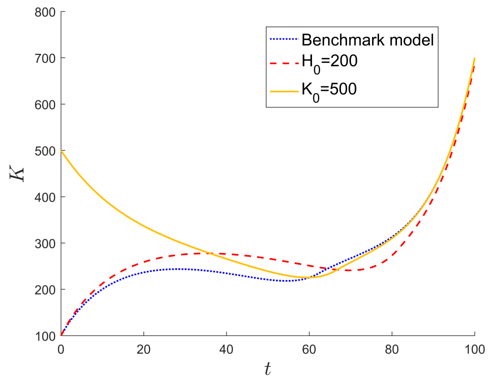


(b) Flood defense

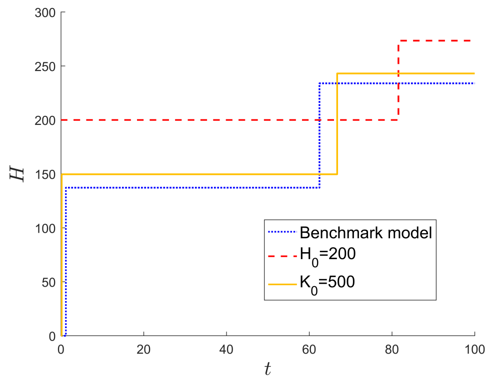
Figure 3.7: State dynamics for planning horizon $T = 50$ (dashed dark green line), $T = 100$ (dotted blue line) and $T = 150$ (solid light green line). If the planning horizon T is longer (a) firms invest more in capital K even at the beginning and (b) firms invest higher amounts and more often in flood risk protection measures.

like e.g. the levee effect or because after a flood it is more difficult for the water to stream back into the river, thereby increasing flood damage. We investigate the effect of these feedbacks on the investment decision of the firm for a scenario with no feedback effects and a scenario with strong feedbacks.

If investment in flood protection affects the water system and increases flood risk, the expected value of the firm decreases dramatically for three reasons: Firstly, firms choose a location much farther away from the water to avoid these negative feedbacks. Secondly, a firm invests less and less often in flood defense, because it increases damage if a flood happens. Thirdly, since the firm is less safe and less profitable because it is located farther away from the water, it will invest less in productive capital, which again leads to a lower production output.



(a) Capital

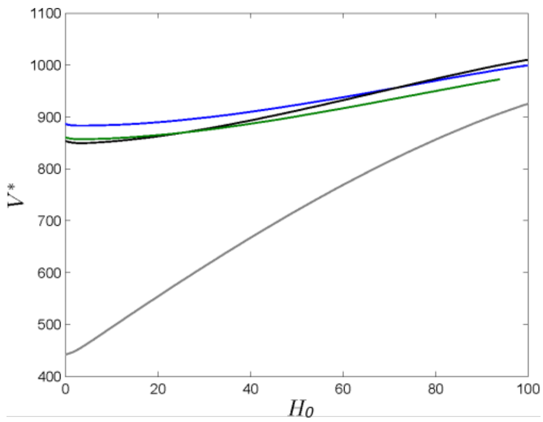


(b) Flood defense

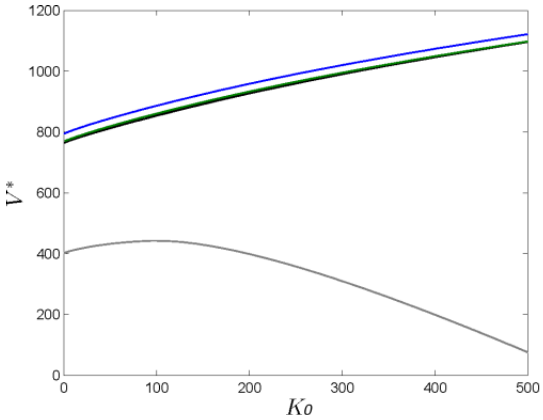
Figure 3.8: State dynamics for different economic situations: benchmark model with $H_0 = 0$ and $K_0 = 10$ (dotted blue line), protected flood plain $H_0 = 200$ (dashed red line), and capital intense firm $K_0 = 500$ (solid orange line).

Fig.3.10 shows the value of the firm in case of no feedbacks ($\xi_H = 0$) and strong feedbacks ($\xi_H = 0.5$). In case of no feedbacks the firm chooses a location much closer to the river and invests three times in flood risk protection measures at almost equal time intervals.
If the hydrological feedbacks are strong the firm builds its premises far away from the water and the value of the firm would not change much if it invests more or less often in flood defense. Still, it is optimal to invest twice. The first investment takes place already after a few years and the second investment is rather at the end of the planning horizon. Interestingly, the total amount of flood defense is almost as high as in the benchmark model, even though the location is much farther away.

We notice that it plays a crucial role if the flood risk protection affects the environment and consequently the water system which is the flood hazard for the firm.



(a) The value of the firm depending on H_0



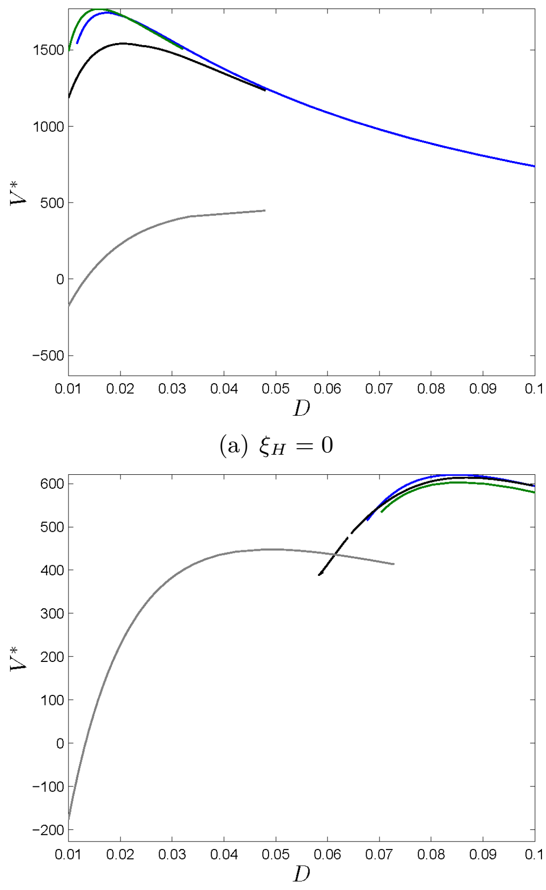
(b) The value of the firm depending on K_0

Figure 3.9: The net present value V^* of the firm increases for higher initial capital stocks. The colors indicate no impulse investments (grey), one impulse investment (black), two impulse investments (blue) and three impulse investments (green).

We conclude that the damage effect of the flood protection level plays a crucial role in affecting optimal firm behavior.

3.5 Conclusions

This chapter provides the investment behaviour and location choice of a firm in a flood risk area within an optimal decision framework. In a first step, the firm chooses timing, number and amount of investments for impulse investments in flood risk protection measures, together with investment in its productive capital within a finite planning period. In a second step, the firm chooses the optimal location for its production plant in the flood risk area.



(a) $\xi_H = 0$

(b) $\xi_H = 0.5$

Figure 3.10: The net present value V^*depending on D for no impulse investments (grey), one impulse investment (black), two impulse investments (blue) and three impulse investments (green). For high feedbacks (ξ_H) it does not pay off to invest in flood defense and will be more profitable to build the production plant further away from the water.

We present analytical and numerical solutions and analyse variations of these solutions under different parameterizations of the model. Sustainable investment planning of the firm doees not lead only to a safer environment with less flood risk, but also to economic growth both in the short and the long run. If the area is already protected against floods, firms still invest in flood defense, but less. And if the firm is more capital intensive potential damage is larger, but the timing and amount of impulse investments do not change.

Anthropogenic flood risk reduction can affect the environment resulting in changes of the water system and consequently again increase flood risk due to negative feedbacks. In this case, production output is much less and the firm decides to build its production far away from the water.

So far we have presented a qualitative numerical analysis of our model set up. It would be interesting in further work to numerically calibrate our model with empirical data from case studies.
Other topics of future research could be to introduce depreciation and maintenance of flood risk protection measures or to simulate random flooding events (e.g. Poisson distribution) like Grames *et al.* (2016) or Viglione *et al.* (2014) and imply them like shocks in the model of Kuhn *et al.* (2017).
Furthermore, our partial equilibrium setup reflecting the firm's decisions could be extended to a general equilibrium framework that also models both the household's behavior and government policies endogenously, in addition to the firm's optimal decisions. This allows for an analysis of the society as a whole given all the economic interactions. Last but not least one could apply the method of impulse control to the decision framework of a social planner who represents the whole society and can include e.g. environmental quality in their objective function.

3.6 Appendix

3.6.1 Details of the firm's optimal decisions

In addition to Section 3.2.4 we provide more detailed insights about the optimal decisions of the firm. To still enable smooth reading we present the derivations of the optimal decisions in Appendix 3.6.2.

Optimal capital investment

The optimal dynamics of the investment in physical capital between the impulse investments is given by Eq.(3.18). The firm increases investments in physical capital if the interest rate is high and the depreciation rate is high. If the expected output per physical capital is already high, the firm slows down investment, whereas investment is increased for a higher capital stock or a higher shadow price of the capital stock. Moreover, the elasticity of physical capital in the production function has a negative impact on the investment decision. Marginal investment increases if the expected damage rate increases. Investment decreases if the water level rises.

$$\begin{aligned}\dot{I}_K(t) &= I_K(t)[r+\delta_K-\frac{\alpha}{\lambda_K}\frac{(1-P_F(D,H(t)))Y(K(t),D)}{K(t)} \\ &\quad +\frac{1}{\lambda_K}P_F(D,H(t))F(D,H(t))+\frac{\alpha_F}{\lambda_K}\frac{P_F(D,H(t))}{1-P_F(D,H(t))}\frac{\eta}{\alpha_D D}] \qquad (3.18)\end{aligned}$$

Solving the differential equation from the first order conditions and the transversality condition yields the net present value for the expected optimal investment in physical capital $I_K(t)$ at time t.

$$\begin{aligned}&(1-P_F(D,H(t)))I_K(t)= \\ &\frac{1}{2\alpha_K}\int_t^T\frac{\alpha[1-P_F(D,H(s))]Y(K(s),D)-P_F(D,H(s))F(D,H(s))K(s)}{K(s)}e^{-(r+\delta_k)(s-t)}ds \\ &+e^{-(r+\delta_K)(T-t)}\frac{1}{r}\frac{\alpha}{2\alpha_K}[1-P_F(D,H(T))]\frac{Y(K(T),D)}{K(T)}-P_F(D,H(T))F(D,H(T)) \qquad (3.19)\end{aligned}$$

Given lower investment costs ($2\alpha_K$) the firm invests more in physical capital. Additionally, more expected output per capital in the future in a more capital intense production (α) increases the investment. On the other hand, a higher expected damage rate decreases the optimal investment. To conclude, expression (3.19) shows that the productive investment rate is determined as such that the resulting expected revenue stream, including the increase in the salvage value, due to a marginal investment, equals the expected marginal investment costs.
At the end of the planning period T the optimal investment rate will be equal to the

difference between expected outcome per capital and expected damage per capital.

$$\begin{aligned}&(1-P_F(D,H(T)))I_K(T) = \\ &\frac{1}{r}\frac{\alpha}{2\alpha_K}[1-P_F(D,H(T))]\frac{Y(K(T),D)}{K(T)} - P_F(D,H(T))F(D,H(T))\end{aligned} \tag{3.20}$$

Shadow prices

Analogous to the derivation of Eq.(3.19) we obtain the net present value of the shadow price for investment in flood defense.

$$\begin{aligned}\lambda_H \;=\; & \int_t^T (P_F(D,H(s))\alpha_F(1-\frac{\xi_H}{\alpha_D(D_0+D)})[Y(K(s),D)-\alpha_K I_K(s)^2 - F(D,H(s))] \\ & -P_F(D,H(s))(1-F(D,H(s)))(\frac{\xi_H}{\alpha_D(D_0+D)}))e^{r(t-s)}ds \\ & -e^{r(t-T)}\frac{1}{r}P_F(D,H(T))\alpha_F(1-\frac{\xi_H}{\alpha_D(D_0+D)})[Y(K(T),D)-\alpha_K I_K(T)^2 \\ & -F(D,H(T))] - P_F(D,H(T))(1-F(D,H(T)))(\frac{\xi_H}{\alpha_D(D_0+D)})\end{aligned} \tag{3.21}$$

The shadow price of flood protection increases with expected future losses (i.e. lost profit and damaged capital) and decreases with expected sustained capital. The shadow price λ_H increases if sociohydrological feedbacks are more intense and if the firm decided to build the production plant closer to the water.

The transversality conditions Eq.(3.35) yield expressions for the shadow prices at time T^+.

$$\lambda_K(T^+) \;=\; \frac{1}{r}\alpha\frac{[1-P_F(H(T^+),D)]Y(K(T^+),D)}{K} - P_F(H(T^+),D)F(H(T^+)) \tag{3.22a}$$

$$\begin{aligned}\lambda_H(T^+) \;=\; & \frac{1}{r}P_F(H(T^+),D)[\alpha_F(1-\frac{\xi_H}{\alpha_D(D_0+D)})(Y(K(T^+),D)-\alpha_K I_K(T^+)^2 \\ & +F(H(T^+),D)K(T^+))]\end{aligned} \tag{3.22b}$$

The shadow price for physical capital at the end of the planning period (3.22a) equals the discounted difference of expected output per capital and the expected damage rate. The shadow price for flood defense capital at T^+ is the expected loss from flooding, i.e. the sum of the revenue due to business interruption $P_F(H,D)Y(K,D)$, the avoided costs at the time of the flooding $-P_F(H,D)(\alpha_K I_K^2)$ and the direct damage as the value of repair and cleanup costs $P_F(H,D)F(H)K$.

Optimal flood defense

Firms invest in flood defense when marginal costs equal marginal gain at the jump point τ_i $(u_i > 0)$. This is shown by the first order impulse conditions

$$\lambda_H(\tau_i^+) = \frac{\partial I_H}{\partial u}(u_i, H(\tau_i^-)) = (\tilde{\theta}_1 + \tilde{\theta}_2 u_i)\exp(\theta_3(H(\tau^-) + u_i)), \tag{3.23}$$

where $\tilde{\theta}_1 := \theta_2 + \theta_1\theta_3$ and $\tilde{\theta}_2 := \theta_2\theta_3$, and the jumping condition,

$$\begin{aligned} \lambda_H(\tau_i^+) - \lambda_H(\tau_i^-) &= \frac{\partial I_H}{\partial H}(H(\tau_i^-), u_i, \lambda_H(\tau_i^+), \tau_i) \\ &= (\tilde{\theta}_3 + \tilde{\theta}_2 u_i)\exp(\theta_3(H(\tau^-) + u_i)), \end{aligned} \tag{3.24}$$

where $\tilde{\theta}_3 := \theta_1\theta_3$.

At the jump points, i.e. when the firm invests in flood protection, the increase in expected profit should equal the investment costs or be higher at the initial point in time provided the firm invests in flood protection at this time.

$$\begin{aligned} &\pi_e(K(\tau_i), D, H(\tau_i^+), I_K(\tau_i^+)) - \pi_e(K(\tau_i), D, H(\tau_i^-), I_K(\tau_i^-)) \\ &+\lambda_K(\tau_i^+)[I_K(\tau_i^+) - \delta_K K(\tau_i^+)] - \lambda_K(\tau_i^-)[I_K(\tau_i^-) - \delta_K K(\tau_i^-)] \\ &-rI_H(u_i, H(\tau_i^-)) = \begin{cases} > 0 & \text{if } \tau_i = 0 \\ = 0 & \text{if } \tau_i \in (0, T) \\ < 0 & \text{if } \tau_i = T \end{cases} \end{aligned} \tag{3.25}$$

If we assume that for every planning horizon T there exists a unique optimal solution for our problem (3.17) with a finite number of jumps, we can derive the optimal impulse control value u_i (Grass & Chahim, 2012).
With the necessary condition Eq.(3.23) we obtain an implicit function of the optimal value u_i at time τ_i.

$$H(\tau_i^-) + u_i = -\ln((\tilde{\theta}_1 + \tilde{\theta}_2 u_i)^{\frac{1}{\theta_3}}) + \ln(\lambda_H(\tau_i^+)^{\frac{1}{\theta_3}}) \tag{3.26}$$

$\frac{\partial^2 IHam}{\partial u^2}(u_i, H(\tau_i^-))$ is always negative for $u \geq 0$ and ensures that u_i is optimal.

Furthermore, we are able to identify an upper bound $\bar{H}$ for the level of flood defense capital H given an optimal solution. The detailed derivation is found in Appendix 3.6.2.

$$\bar{H} = \frac{\ln\left(\frac{P_0\alpha_F(Y+K)}{r\theta_1\theta_3}\right) + \alpha_F\frac{W_0+\eta T^+}{\alpha_D(D_0+D)}}{\theta_3 + \alpha_F(1 - \frac{\xi_H}{\alpha_D(D_0+D)})} \tag{3.27}$$

So we know $\bar{H} > H(T^+)$. Since the water level (3.1) is increasing it holds that $H(T^+) > H(t)$ for all $t \in [0, T]$. $\bar{H}$ still depends on $Y(K, D)$ and K, and can therefore vary for

different properties of the firm.
$\bar{H}$ increases for a higher flood risk. A higher flood risk can be caused by a higher flood hazard ($P_F(t)$), i.e. the initial flooding probability P_0 or the water level increases or the firm is located closer to the water in a flatter flood plain. Defense capital will also be higher if the damage resulting from a flood is higher, which is the case if exposed capital ($K(t)$) is higher. Flood risk also increases if exposed capital increases. The only parameter to lower $\bar{H}$ are the costs of investments I_H in flood defense.

3.6.2 Additional derivations and explanations

Necessary Optimality Conditions

We follow the work from Chahim (2012) to derive the necessary optimality conditions for our maximization problem. We use the current value Hamiltonian form to incorporate the discounting.
To apply the Impulse Control Maximum Principle the functions $\pi_e(t)$ and $I_H(u_i, H(t))$ should be continuously differentiable in H and u_i on $\mathbb{R}_+$, and $\frac{1}{r}\pi_e(T)$ should be continuously differentiable in $K(T)$ and $H(T)$ on $\mathbb{R}_+$. Furthermore, $I_H(u_i, H(\tau^-))$ should be continuous in τ.

The maximization problem displayed in Eq.(3.17) yields the following current value Hamiltonian

$$Ham(K, I_K, \lambda_K, t) = \pi_e(K(t), D, H(t), I_K(t)) + \lambda_K[I_K(t) - \delta_K K(t)] \tag{3.28}$$

and the following current value Impulse Hamiltonian.

$$IHam(H, u, \lambda_H, t) = -I_H(u, H(t)) + \lambda_H u \tag{3.29}$$

The necessary optimality conditions in our model are as follows. For all $t \notin \{\tau_1, ..., \tau_N\}$ it holds that

$$\frac{\partial Ham}{\partial I_K}(K, I_K, \lambda_K, t) = 0 \tag{3.30}$$

$$\frac{\partial Ham}{\partial K}(K, I_K, \lambda_K, t) = r\lambda_K - \dot{\lambda}_K \tag{3.31a}$$

$$\frac{\partial Ham}{\partial H}(K, I_K, \lambda_K, t) = r\lambda_H - \dot{\lambda}_H \tag{3.31b}$$

and for any $u \geq 0$

$$\frac{\partial IHam}{\partial u}(H, 0, \lambda_H, t)u \leq 0. \tag{3.32}$$

For the impulses $t \in \{\tau_1, ..., \tau_N\}$ and non-negative heightenings $u \geq 0$ the following holds.

$$\frac{\partial IHam}{\partial u}(H(\tau_i^-), u_i, \lambda_H(\tau_i^+), \tau_i) = 0 \tag{3.33}$$

$$\lambda_H(\tau_i^+) - \lambda_H(\tau_i^-) = -\frac{\partial IHam}{\partial H}(H(\tau_i^-), u_i, \lambda_H(\tau_i^+), \tau_i) \tag{3.34a}$$

$$\lambda_K(\tau_i^+) - \lambda_K(\tau_i^-) = -\frac{\partial IHam}{\partial K}(H(\tau_i^-), u_i, \lambda_H(\tau_i^+), \tau_i) = 0 \tag{3.34b}$$

At the end of the time interval the transversality conditions

$$\lambda_K(T^+) = \frac{1}{r}\frac{\partial \pi_e}{\partial K}(K(T^+), H(T^+)) \tag{3.35a}$$

$$\lambda_H(T^+) = \frac{1}{r}\frac{\partial \pi_e}{\partial H}(K(T^+), H(T^+)) \tag{3.35b}$$

hold with $K(T^+) = K(T)$ and $H(T^+) = H(T)$ if there is no jump at time T and $\tau_1 < \tau_2 < ... < \tau_N \leq T$.

Derivation of equations for Section 3.2.4

From the condition (3.30) we obtain

$$\lambda_K = (1 - P_F(D, H))2\alpha_K I_K. \tag{3.36}$$

Taking the logarithm and time derivative and combining them with the result from (3.31a) leads to the following optimal dynamics of the investment in physical capital between the impulse investments shown in (3.18).

Solving the differential equation from condition (3.31a) for λ_K, using the transversality condition (3.22a) and Eq.(3.36) yields the net present value for the expected optimal investment in physical capital $I_K(t)$ expressed in Eq.(3.19).

The necessary condition (3.31b) yields the dynamics of the shadow price for investment in flood defense capital.

$$\begin{aligned}\dot{\lambda}_H &= r\lambda_H - P_F(D, H)\alpha_F(1 - \frac{\xi_H}{\alpha_D(D_0 + D)})[Y(K, D) - \alpha_K I_K^2 - F(D, H)] \\ &\quad + P_F(D, H)(1 - F(D, H))(\frac{\xi_H}{\alpha_D(D_0 + D)})\end{aligned} \tag{3.37}$$

We can solve that differential equation (3.37) using the transversality condition (3.22b) to obtain Eq.(3.21).

Derivation of $\bar{H}$

We know investment is only optimal if marginal gain (3.22b) is at least equal to marginal costs (3.23) at time T^+. The resulting equation

$$\frac{1}{r}P_F(D,H)\Big[\alpha_F(1-\frac{\xi_H}{\alpha_D(D_0+D)})[Y-\alpha_K I_K^2+F(D,H)K]-\frac{\xi_H}{\alpha_D(D_0+D)}[1-F(D,H)]K\Big]$$
$$=(\tilde{\theta}_1+\tilde{\theta}_2 u_i)\exp(\theta_3(H(\tau^-)+u_i)) \tag{3.38}$$

ensures that an upper bound $\bar{H}$ exists, because the left hand / the right hand side of the equation converges to 0 / ∞ for $H\to\infty$, respectively. We define $A := \alpha_F(1-\xi_H)Y+\alpha_F(1-\xi_H)F(D,H)K-\frac{\xi_H}{\alpha_D(D_0+D)}[1-F(D,H)]K$ and find $\bar{A}\geq A$ at T^+ with $\bar{A}=\alpha_F(1-\xi_H)(K+Y)$. $\bar{A}$ is constant at T^+. Since we know that $\bar{H}$ still holds for increased marginal gain or decreased marginal costs, we can reduce Eq.(3.38) to

$$\frac{1}{r}P_0\exp(\alpha_F(\frac{W_0+\eta T^+}{\alpha_D(D_0+D)}-(1-\frac{\xi_H}{\alpha_D(D_0+D)})\bar{H})\alpha_F(1-\frac{\xi_H}{\alpha_D(D_0+D)})(Y+K)$$
$$=\theta_1\theta_3\exp(\theta_3\bar{H}) \tag{3.39}$$

and derive $\bar{H}$.

3.6.3 Numerical solution

To apply the continuation algorithm introduced in Section 3.3 we have to derive the model dynamics explicitely. For convenience we do not write the time argument t to the dynamic variables K, H, λ_K, λ_H, I_K.

To avoid a positive product caused by two negative factors $(1-P_F)$ and $(Y-\alpha_K I_K^2)$ and to ensure that $(1-P_F)\in[0,1]$ we approximate the term $(1-P_F)$ with $\frac{1}{1+\alpha_P P_F}$.

We use the following short notations.

$$\begin{aligned}
Y(K,L,D) &= K^\alpha\frac{1}{(D_0+D)} && (3.40a)\\
\pi_e(K,D,H,I_K) &= \frac{1}{1+\alpha_P P_F}[Y(K,D)-\alpha_K I_K^2] &&\\
&\quad -P_F(H,D)F(H,D)K && (3.40b)\\
P_F(H,D) &= P_0\exp[\alpha_F(\frac{W_0+\eta t}{\alpha_D(D_0+D)}-(1-\frac{\xi_H}{\alpha_D(D_0+D)})H)] && (3.40c)\\
F(H,D) &= 1-\exp(-\frac{W+\xi_H H}{\alpha_D(D_0+D)}) && (3.40d)\\
I_H(u,H(\tau^-)) &= (\theta_1+\theta_2 u)\exp(\theta_3(H(\tau^-)+u)) && (3.40e)
\end{aligned}$$

Note, Eq.(3.40e) is only used for u strictly positive.

We can summarize the canonical system dynamics for $t \in (\tau_{i-1}, \tau_i)$ with $i \in \{1, ..., N+1\}$.

$$\dot{K} = I_K - \delta_K K \tag{3.41a}$$
$$\dot{H} = 0 \tag{3.41b}$$
$$\dot{\lambda}_K = (r+\delta_K)\lambda_K - \alpha \frac{\frac{1}{1+\alpha_P P_F} Y(K,L,D)}{K} + P_F(H,D)F(H,D) \tag{3.41c}$$
$$\begin{aligned}\dot{\lambda}_H &= r\lambda_H - P_F(D,H)\alpha_F(1 - \frac{\xi_H}{\alpha_D(D_0+D)})[Y(K,D) - \alpha_K I_K^2 - F(D,H)] \\ &\quad + P_F(D,H)(1-F(D,H))(\frac{\xi_H}{\alpha_D(D_0+D)})\end{aligned} \tag{3.41d}$$
$$\begin{aligned}\dot{I}_K &= I_K[r + \delta_K - \frac{\alpha}{\lambda_K} \frac{\frac{1}{1+\alpha_P P_F})Y(K,L,D)}{K} + \frac{1}{\lambda_K} P_F(H,D)F(H,D) \\ &\quad + \frac{\alpha_F}{\lambda_K} \frac{P_F(H,D)}{1-P_F(H,D)} \frac{\eta}{\alpha_D(D_0+D)}]\end{aligned} \tag{3.41e}$$

Moreover, we rewrite the conditions for the jump points τ_i with $i \in \{1, ..., N\}$.

$$H(\tau_i^+) - H(\tau_i^-) - u_i = 0 \tag{3.42a}$$
$$\lambda_H(\tau_i^+) - (\tilde{\theta}_1 + \tilde{\theta}_2 u_i)\exp(\theta_3(H(\tau^-) + u_i)) = 0 \tag{3.42b}$$
$$\lambda_H(\tau_i^+) - \lambda_H(\tau_i^-) - (\tilde{\theta}_3 + \tilde{\theta}_2 u_i)\exp(\theta_3(H(\tau^-) + u_i)) = 0 \tag{3.42c}$$
$$\begin{aligned}&\pi_e(K(\tau_i), H(\tau_i^+), I_K(\tau_i^+)) - \pi_e(K(\tau_i), H(\tau_i^+), I_K(\tau_i^-)) \\ &+\lambda_K(\tau_i^+)[I_K(\tau_i^+) - \delta_K K(\tau_i^+)] - \lambda_K(\tau_i^-)[I_K(\tau_i^-) - \delta_K K(\tau_i^-)] \\ &-rI_H(u_i, H(\tau_i^-)) = 0\end{aligned} \tag{3.42d}$$

We solve the conditions for every interval assuming $0 < \tau_1 < \tau_2 < ... < \tau_N < \tau_{N+1} = T$. The starting values are

$$K(0) = K_0 \tag{3.43a}$$
$$H(0^-) = 0 \tag{3.43b}$$

and at the end T the transversality conditions have to hold. Note, that here the time argument for all the dynamic variables is time T.

$$\frac{1}{r}[\alpha \frac{[1-P_F(H,D)]Y(K,D)}{K} - P_F(H,D)F(H)] - \lambda_K = 0 \tag{3.43c}$$
$$\frac{1}{r} P_F(H,D)[\alpha_F(1 - \frac{\xi_H}{\alpha_D(D_0+D)})(Y(K,D) - \alpha_K I_K^2 + F(H,D)K)] - \lambda_H = 0 \tag{3.43d}$$

4 Optimizing the phosphorus cycle in a two-sector economy

The present chapter corresponds to the following scientific publication, under submission:

Grames J., Zoboli O., Laner D., Sanchez-Romero M., Zessner M., Rechberger H., Prskawetz A.(2018): Optimizing the phosphorus cycle in a two-sector economy. Submitted to Journal of Environmental Economics and Management

Abstract

Phosphorus is a crucial element for food production. Many economies depend on the import of nonrenewable mineral phosphorus for fertilization. Furthermore, phosphorus losses from fertilized agricultural land impact the environment by causing eutrophication. We present a conceptual economic model describing the agricultural sector that helps to understand the coupled human-resource-environment feedbacks associated with phosphorus use and recycling. We model the demand and supply of phosphorus and changes in resource stocks by combining a multi-sector economic model with a material flow model.

Within this framework we study the effects of implementing phosphorus recovering technologies from waste water. We show that using recycled phosphorus fertilizer increases environmental quality and profits in the agricultural sector. Furthermore, the economy does not depend as much on mineral fertilizer imports and is therefore more resilient to a price increase on the global phosphate market. However, there is a need to improve the quantity and quality of recycled phosphorus products, because farmers would use 100% of the available recovered phosphorus fertilizer.
Overall, reduction of phosphorus in soil and water bodies as result of economic decisions is only possible if phosphorus is recovered from waste water and the prices of imported

mineral fertilizer rise. Policy makers can support this technological change by subsidizing recycled phosphorus or introducing taxes or tolls for imported mineral fertilizer. Alternatively, societal values would have to change. Such a change may be induced by putting a higher value on a healthy environment and hence being willing to pay more for food and consequently production inputs like recycled phosphorus fertilizer.

4.1 Introduction

Phosphorus (P) is an essential element for plant and animal growth and is necessary to maintain profitable crop and livestock production (Sharpley & Beegle (2001)). Food production depends on nonrenewable mineral phosphorus supplies from a finite P stock (Childers *et al.* (2011), Dawson *et al.* (2011)). There is a general consensus that the quality and accessibility of remaining reserves are decreasing and costs will increase in the medium and long term, additional to possible phosphate fertilizer price shocks like in 2008 (Cordell & White (2011)).
Efficient fertilizer use not only increases profits Venezian (1962), also sustainable P use is crucial to preserve food security and this affects households consumption decisions and consequently the whole economy.
Furthermore, P fertilizer use can negatively influence the environment. Mineral P fertilizer contains heavy metals (Zoboli *et al.* (2016a)) and P is the critical element for eutrophication in most fresh waters besides nitrogen (N). Overfertilization leads to increased P stocks in soil and consequently emissions to surrounding water bodies increase. Generally, anthropogenic activities are the main causes of pollution and environmental problems (Ghazi *et al.* (2014)). Pesticide and fertilizer consumption may reach problematic levels (Saysel *et al.* (2002)) and rural living, livestock, paddy field, and precipitation alternately become the leading source of non-point source (NPS) pollution (Carpenter *et al.* (1998),Yuan *et al.* (2017)).

Yet, there are not many policy strategies implemented to ensure P availability, even though the European Commission included phosphate rock into the revised list of Critical Raw Materials (Commission *et al.* (2014)) in 2014. Moreover, economic incentives like subsidies for P recycling or increased prices for mineral fertilizer, their consequences and environmental feedbacks still need to be investigated.

Previous literature suggests different strategies to secure P access for farmers. One strategy is to prospect and explore new sources of mineral P (Childers *et al.* (2011)). Another one is to enhance recycling to close the loop of P resource use (Childers *et al.* (2011), Koppelaar & Weikard (2013), Zoboli *et al.* (2016b)). An alternative strategy is to reduce P losses from soils by erosion abatement (Zoboli *et al.* (2016b)). And the last strategy is to reduce P consumption (Koppelaar & Weikard (2013), Zoboli *et al.* (2016b)).

Different methods are used to study environmental pollution, fertilizer use or phosphorus recycling. Methods from resource management capture detailed material flows and linear cost relations. Economic models allow for consumption and production decisions, but miss profound impacts on resource stocks. We combine material flow analysis (MFA) methodology and terminology according to Baccini & Brunner (1991), Baccini & Brunner (2012) and Brunner & Rechberger (2004) with an economic model framework. It is important to analyze coupled human-environmental feedbacks to investigate household consumption decisions and environmental impact via fertilizer decisions. This gives

additional value to each model type: On the one hand we include farmers and household decisions and resulting optimized profits and utility in a resource management framework. To our knowledge it is the first time to endogenously describe material flows based on economic decisions. On the other hand we introduce the P cycle into a two-sector general equilibrium model. So far only partial equilibrium models have looked at fertilizer decisions and their environmental impacts (Boyle (1982), Larson & Vroomen (1991), Liverpool-Tasie (2017)), but none of these models has integrated material flows. Bouman *et al.* (2000) applied MFA and a partial equilibrium model to the same environmental problem to compare the methods and advocates to integrate them.
The aim of the presented work is to establish a conceptual model with an economy-wide perspective on sustainable P use and understand coupled human-resource-environment feedbacks embedded in an economic framework. Based on that understanding we can investigate implications of introducing P recovery technologies, of price changes in mineral P fertilizer, or of different societal preferences. This contributes to better understand the socio-economic metabolism (Ayres (1989), Fischer-Kowalski & Haberl (1998)) and adds to the fields of bio-economic models (Dellink *et al.* (2011); Flichman & Allen (2015)), coupled hydro-economic models (Bekchanov *et al.* (2017)) and socio-hydrology (Pande & Sivapalan (2017)).

We develop an analytical general equilibrium model for a two sector economy. We face monetary and non-monetary inter-industry linkages between crop production and animal husbandry. Households supply labour and demand food. We add P flows as a second layer to the economic model and introduce a waste water treatment plant to recover P. Crop farmers can choose a combination of recycled P fertilizer and imported mineral fertilizer to produce high-quality grain and fodder. Farmers in animal husbandry decide on the amount of fodder. The model is calibrated to fit an Austrian P time series described in Zoboli *et al.* (2016a) and respective economic data from Österreich & Austria (2010). The parameter choice is location specific, but relevant to many watersheds, regions or countries with a similar soil structure and economy.
The dynamic economic general equilibrium model is solved analytically and numerically to point out the most important mechanisms that influence the decision makers.

We develop the basic model in the first section. In the results section we first present analytical solutions before we continue with numerical results. Within our framework we discuss different scenarios: A price shock of imported fertilizer, using different recycling technologies, and model outcomes considering an environmental friendly society. A sensitivity analysis captures uncertainty. The last sections conclude and we discuss model limitations and future directions. Derivations, proofs, and the idea of the numerical algorithm are found in the Appendixes.

4.2 Conceptual model

The model introduced in this chapter is based on a reduced form of the phosphorus cycle Egle *et al.* (2014) and adds an economic dimension to explain the demand for phosphorus by various stakeholders. The aim of the model is to capture the mechanisms and interlinks between major stakeholders and decision makers (as represented by households and various economic sectors). An overview of the model including the inflows and outflows between the different sectors of the economy and households is given in Fig.4.1.

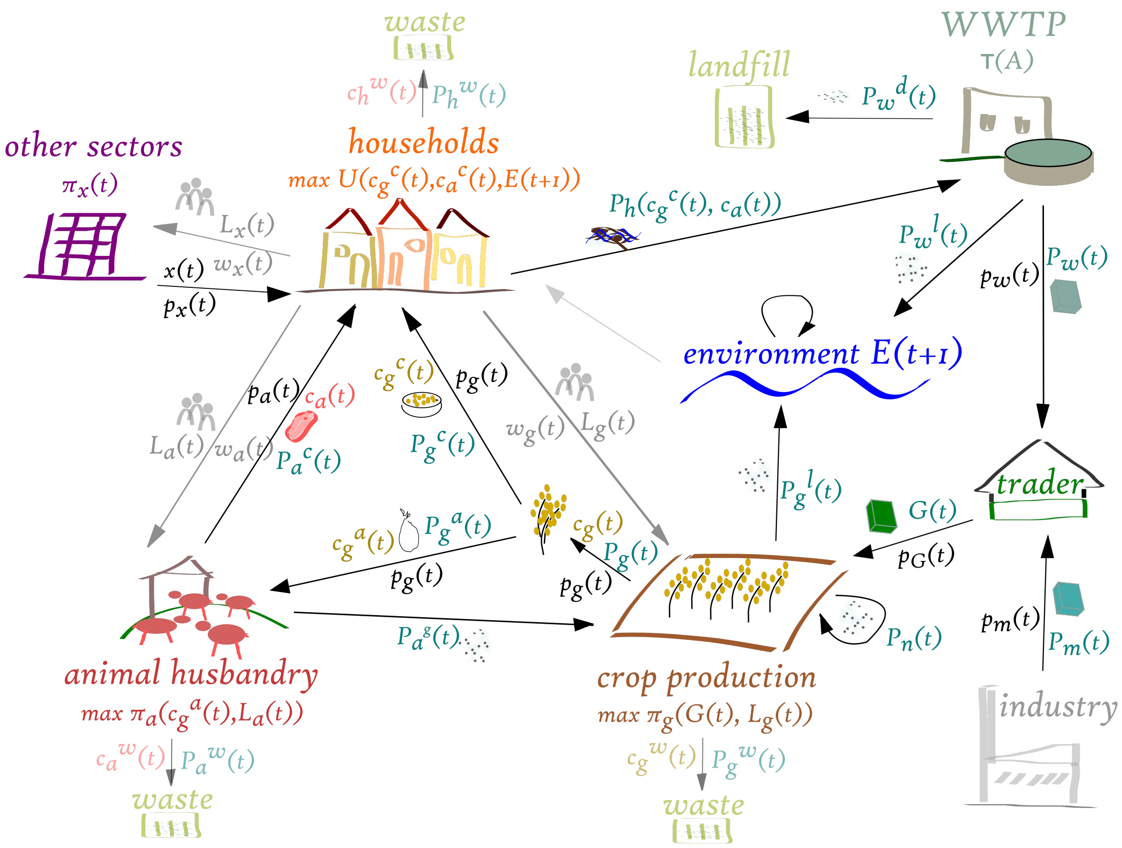


Figure 4.1: Outline of the model framework. We denote with the letter g the grain field, a the animal husbandry, h the households, w the waste water treatment plant (WWTP), m the mineral fertilizer from industry and x the other sectors. The subscripts of the consumption c, prices p and phosphorus P variables describe the source of the flow and the superscript describes the destination.

4.2.1 Economic model

We model a closed economy representing one million inhabitants L living in urban and rural areas. Only a small proportion of the population works in the agricultural sector, everyone else works in other sectors (L_x). Farmers can choose to work in crop production (L_g) or animal husbandry (L_a).

Households We assume that all agents are employed ($L(t) = L_g(t) + L_a(t) + L_x(t)$) and that the labor supply for agriculture is constant. Within the agricultural sector we allow workers to decide to either work for crop production or animal husbandry.

$$L_g(t) + L_a(t) = L - L_x = const. \tag{4.1}$$

Households demand vegetarian products $c_g^c(t)$ (represented by grain) and meat products $c_a^c(t)$ and derive utility from environmental quality $E(t+1)$ in the next period. The environmental quality reflects the water quality with respect to eutrophication as well as the accumulation of heavy metals and organic micropollutants in soils. Household's consumption choice impacts environmental quality indirectly. Consequently feedbacks between households consumption decisions and environmental quality occur.
Households maximize their utility function $u(t)$ in every period. Other goods $x(t)$ than grain $c_g^c(t)$ and animal products $c_a^c(t)$ are assumed constant and therefore not subject to optimal choice. We choose the form of a log-utility function that yields the optimal consumption decision as a constant budget share. γ_c, α_c and ϵ are positive constants reflecting the importance of the respective consumption goods.

$$\begin{aligned}&\max_{\{c_g^c(t),c_a^c(t)\}} u(c_g^c(t), c_a^c(t), E(t+1)) = \\ &\max_{\{c_g^c(t),c_a^c(t)\}} \gamma_c \log(c_g^c(t)) + \alpha_c \log(c_a^c(t)) + \epsilon \log(E(t+1))\end{aligned} \tag{4.2}$$

Household's consumption cannot exceed the income earned, described in Eq.4.3, where $p_g^c(t)$, $p_a^c(t)$, $p_x(t)$ are the prices for grain products, the animal products, and composite other goods, respectively. Households supply labor to the firms in exchange for a wage $w(t)$, and receive profits from grain production $\pi_g(t)$, animal husbandry $\pi_a(t)$, fertilizer trader $\pi_G(t)$, waste water treatment plant $\pi_w(t)$ and other sectors $\pi_x(t)$. Moreover the value of fertilizer imports ($p_m(t)P_m(t)$) is assigned to the households. So the household's budget constraint (Eq.4.3) is aligned with the national budget constraint.

$$\begin{aligned}&p_g^c(t)c_g^c(t) + p_a^c(t)c_a^c(t) + p_x x \\ &\leq w(t)L(t) + \pi_g(t) + \pi_a(t) + \pi_G(t) + \pi_w(t) + \pi_x + p_m(t)P_m(t)\end{aligned} \tag{4.3}$$

Crop production We assume that phosphorus (P) fertilizer denoted by G and labor inputs L_g are chosen to maximize profits of the crop production c_g. All other production inputs like agricultural land, seeds, alternative fertilizer and irrigation are assumed constant and summarized within the crop production technology ϕ_g. The production elasticities α_g and β_g describe the importance of the production inputs P fertilizer and labor, respectively. In addition to labor $L_g(t)$ and P fertilizer $G(t)$, the production of grain also depends on P in manure $P_a(t-1)$ and P in the soil $(P_n(t-1))$. The production function of grain $c_g(t)$ is therefore given as follows:

$$c_g(t) = \phi_g((\chi_a - \ell)P_a(t-1) + (\chi_n - \ell)P_n(t-1) + G(t))^{\alpha_g}(\phi_L L_g(t))^{\beta_g}, \tag{4.4}$$

with χ_a and χ_n denoting the respective efficiency of P in manure and in the soil, ℓ is the P loss via runoff from the field and ϕ_L denotes the labor efficiency. After every period farmers adapt the fixed proportion ϕ_g^c of their yield that is supplied to the households as high-quality products $c_g^c = \phi_g^c c_g(t)$ and the share $\phi_g^a = 1 - \phi_g^c$ that is supplied as fodder $c_g^a = \phi_g^a c_g(t)$ to the animal husbandry sector. The different quality of the products implies different prices on the market. Grain farmers maximize the profit function

$$\max_{\{G(t),L_g(t)\}} \pi_g(G(t), L_g(t)) = \max_{\{G(t),L_g(t)\}} p_g(t)c_g(G(t), L_g(t)) - p_G(t)G(t) - w(t)L_g(t). \tag{4.5}$$

with the composite price $p_g(t) = \phi_g^c p_g^c(t) + \phi_g^a p_g^a(t)$.

Animal husbandry To generate meat products, we assume fodder and labor as the variable production inputs. Other production factors like land are assumed constant with the total factor productivity ϕ_a. The production elasticities α_a and β_a reflect the impact of the production inputs fodder and labor on animal husbandry, respectively. Farmers in animal husbandry demand labor $L_a(t)$ and fodder $c_g^a(t)$ to produce animal products $c_a^c(t)$ with the following production technology:

$$c_a^c(t) = \phi_a\big(\psi c_g^a(t)\big)^{\alpha_a}(\phi_L L_a(t))^{\beta_a} \tag{4.6}$$

The relation of fodder to final meat products is given by the inverse feed conversion ratio (FCR) ψ and labor efficiency ϕ_L scales the work force. Farmers maximize the profit function

$$\max_{\{c_g^a(t),L_a(t)\}} \pi_a(c_g^a(t), L_a(t)) = \max_{\{c_g^a(t),L_a(t)\}} p_a(t)c_a^c(c_g^a(t), L_a(t)) - p_g(t)c_g^a(t) - w(t)L_a(t) \tag{4.7}$$

Market equilibrium Every period farms in crop production and animal husbandry decide on the production inputs to maximize their profit $\pi_g(t)$ and $\pi_a(t)$, respectively, and households maximize their utility function $u(t)$ by choosing the consumption goods. The optimal decisions of the agents result in supply and demand functions for every market. The detailed derivation of the market equilibria for vegetarian products $c_g^c(t)$, grain fodder $c_g^a(t)$, animal products $c_a^c(t)$, labor $L(t)$ and P fertilizer $G(t)$, is given in 4.6.1. The corresponding prices that result in equilibrium are $p_G(t)$, $p_g^c(t)$, $p_g^a(t)$, $p_a(t)$, and $w(t)$.

4.2.2 P dynamics

In addition to the economic model framework we construct a mass balance model to capture the P cycle. Again, we use Fig.4.1 to outline the model dynamics, but focus on the corresponding P flows replicated in Fig.4.2. P can be recovered from the household's

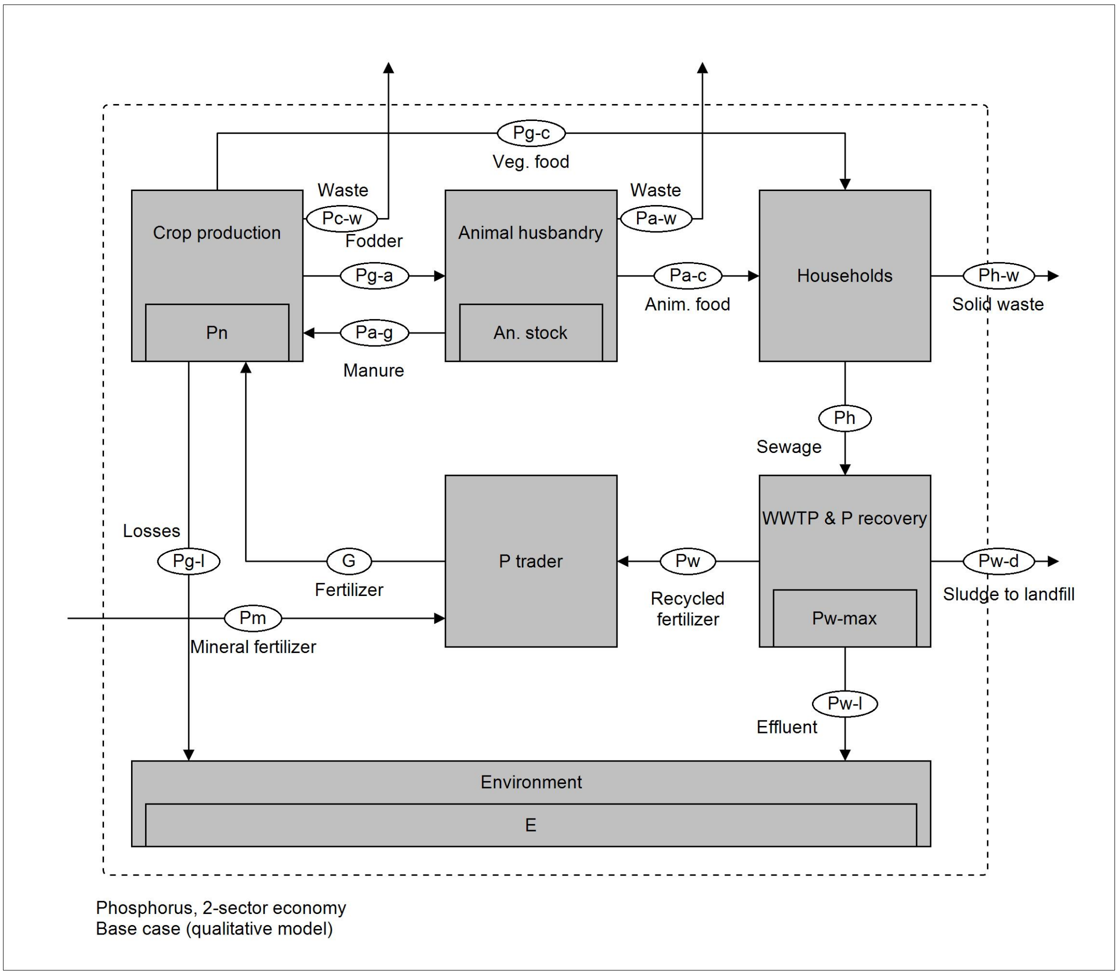


Figure 4.2: Schematic illustration of the phosphorus flow model of the 2-sector economy (Fig.4.1) generated with the MFA software STAN (http://www.stan2web.net/). Processes (= balance volumes) with or without stocks are represented as boxes, and flows are shown as arrows including variable names.

sewage sludge $P_h(t-1)$ and is sold as recycled fertilizer product $P_w(t)$ or goes to landfill $P_w^d(t)$. Additional to the P in the soil $P_n(t)$ grain farmers apply manure $P_a^g(t)$, mineral fertilizer $P_m(t)$ and recycled fertilizer $P_w(t)$ to the field, where mineral and recycled fertilizer are supplied as a composite fertilizer product G by a trader. While plants are growing (Fig.4.3) a part ℓ goes as runoff $P_g^\ell(t) = \ell[P_w(t) + P_m(t) + P_a^g(t) + P_n(t)]$ to the

receiving water bodies.
Plant harvest contains $P_g(t)$ tons of P. The rest remains in the soil as $P_n(t+1)$ for the next period $t+1$. Plant harvest is processed into a share $\frac{c_g^c(t)}{c_g(t)}$ of qualitative vegetarian

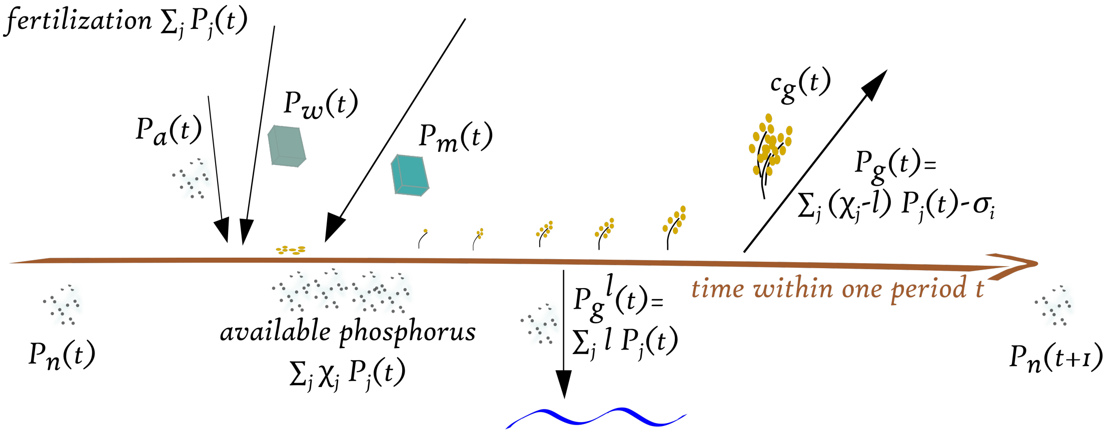


Figure 4.3: Phosphorus dynamics within one period.

food for households containing $P_g^c(t)$ tons of P and a share $\frac{c_g^a(t)}{c_g(t)}$ of fodder for animal husbandry containing $P_g^a(t)$ tons of P, whereas a fraction ω_g^ℓ ends up as food waste ($\omega_g^\ell P_g = P_g^w$).

$$P_g^c(t) + P_g^a(t) = (1 - \omega_g^\ell) P_g(t) \tag{4.8}$$

A share ζ_a^g of $P_g^a(t)$ in the fodder returns as manure $P_a^g(t+1)$ to the field and the remaining share $\zeta_a^c = 1 - \zeta_a^g$ is processed into animal products $c_a^c(t)$ containing $P_a^c(t)$ tons of P. Again, during processing a fraction ω_a^ℓ ends up as food waste ($\omega_a^\ell P_g^a = P_a^w$).

$$P_a^g(t) + P_a^c(t) = (1 - \omega_a^\ell) P_g^a(t) \tag{4.9}$$

The P in sewage sludge is a fraction of P in the household's consumption goods $P_h(t) = \zeta_h^g P_g^c(t) + \zeta_h^a P_a^c(t)$. The rest goes into solid wastes, which are not considered further.

P recovery technology from wastewater The P recovery technology A recovers P from sewage sludge of the households $P_h(t)$ to offer the recycled product $P_w(t+1)$ in the next period to the trader. We choose four different P recovery technologies to represent the various possible recovery routes mentioned in Egle (2014): direct application of sewage sludge without treatment (A1), phosphorus recovery from sludge liquor, i.e. Ostara Pearl Reactor®(A2), from sewage sludge, i.e. Stuttgart process (A3) and from sewage sludge ash, i.e. EcoPhos®(A4).

These technologies are characterized by the following criteria: The fertilizer efficiency $\chi_w(A) \in [0, 1]$ determines the plant availability of the fertilizer product. The accumulation of heavy metals and micropollutants negatively influences the environment with a price $p_w^f(A)$ for one ton of applied fertilizer product. The recovery potential of phosphorus treated with technology A is $\tau(A) \in [0, 1]$. Waste water treatment plants (WWTP) with design capacities ≤100,000 PE treat more than 55% of municipal wastewater in Austria Egle *et al.* (2014). Therefore investment costs are calculated for ten representative 100.000-population-equivalent (PE) plants to serve 1 Mio. inhabitants. The price $p_w(A)$ to sell one ton of recycled fertilizer product is an estimate based on the annualized investment costs $I(A)$ to build technology A, the operating costs $V(A)$ using the technology A and the logistic costs to bring the recycled product to the fields. A technology operates roughly 15 years without large additional investments. We summarize the technologies and their characteristics in Table 4.1.

A	technology	$\chi_w(A)$ [0,1]	$p_w^f(A)$ []	$\tau(A)$ [0,1]	$p_w(A)$ EUR/t	$I(A)$ EUR/a	$V(A)$ EUR/a
1	direct application	0.60	3	1.00	10	0	0
2	Ostara Pearl Reactor®	0.85	1	0.20	100	95,411	35,446
3	Stuttgart process	0.85	1	0.45	2,000	38,164	543,566
4	EcoPhos®	1.00	1	0.85	3,300	38,238	392,686
	mineral fertilizer*	1.00	2	1.00	2,040	-	-

Table 4.1: Characteristics of the P recovery technologies. *Note that mineral fertilizer is not recovered from waste water but imported from abroad.

Before P recovery a proportion ω_ℓ from the waste water is going as effluent $P_w^\ell(t)$ to the water bodies and after applying technology A a certain part $P_w^d(t+1)$ has to go to landfill (Fig.4.1 and Fig.4.2). The rest can be the maximum supply of $P_w(t+1)$ on the P market. Everything that cannot be sold as $P_w(t+1)$ and is not discharged into water bodies as $P_w^\ell(t)$ is landfilled. We summarize the relations in the following equations.

$$\begin{aligned}
P_w^\ell(t) &= \omega_\ell P_h(t) && (4.10)\\
P_w^{maxsupply}(t) &= \tau(A)(P_h(t-1) - P_w^\ell(t-1)) && \\
&= \tau(A)(1-\omega_\ell(A))P_h(t-1) && (4.11)\\
P_w^d(t) &= P_h(t-1) - P_w(t) - P_w^\ell(t-1) && (4.12)
\end{aligned}$$

The profit of the WWTP for selling $P_w(t)$ to the trader is

$$\pi_w(t) = p_w(A)P_w(t). \tag{4.13}$$

P in plants Different P products are applied to the crop fields. The ability of a plant to absorb P depends on the type of fertilizer (composition, chemical species, etc.), the time of application, and the soil conditions at the site. We define these properties as efficiency χ_i, $i \in \{a, n, m, w\}$ of the product type i. With respect to soil type, we assume a generic average representing major soil type in Austria (acid or alkaline). P in the soil ($P_n(t)$) is always available but generally difficult to absorb for the plant (χ_n). Manure $P_a(t)$ from animal husbandry is not always applied when the plants are ready to take it and so its efficiency χ_a is affected. Contrary, the timing of application of mineral fertilizer $P_m(t)$ can be well chosen by the farmers and furthermore, the chemical structure allows for high take up rates of the plant (χ_m). The efficiency χ_w of the recycled P product $P_w(t)$ depends on the recycling technology A. The ideal plant demand for P is $\bar{P}$, whereas the actual for the plant available P might differ (Eq.4.14).

$$P = (\chi_w - \ell)P_w(t) + (\chi_m - \ell)P_m(t) + (\chi_a - \ell)P_a(t-1) + (\chi_n - \ell)P_n(t-1). \tag{4.14}$$

Trader The trader supplies the crop farmers with fertilizer $G(t)$ and produces fertilizer by combining recycled phosphorus $P_w(t)$ and imported mineral fertilizer $P_m(t)$. He aims to meet the fertilizer quantity $G(t)$ (Eq.4.15) demanded by the grain farmers considering fertilizer products' efficiency χ and runoff ℓ after application of the fertilizer $G(t)$.

$$G(t) = (\chi_w - \ell)P_w(t) + (\chi_m - \ell)P_m(t) \tag{4.15}$$

The trader also aims to meet the plant's maximum fertilizer demand $\bar{G}$ according to $\bar{P}$, hence, avoiding overfertilization by fulfilling the quantity constraint $\bar{G} = (\chi_w - \ell)P_w(t) + (\chi_m - \ell)P_m(t)$ in Fig.4.4. However, the trader is only a functional intermediary and cannot earn profits. Therefore he is obliged to fulfill his budget constraint (Eq.4.16) in Fig.4.4.

$$p_G(t)G(t) = p_m(t)P_m(t) + p_w(A)P_w(t) \tag{4.16}$$

The trader cannot sell more recycled P products $P_w(t)$ than the WWTP is supplying ($P_w(t)^{maxsupply}$ in Fig.4.4).

To sum up, the trader aims for an appropriate fertilizer quantity and is restricted to the zero-profit-condition. This yields the amount of $P_m(t)$ and $P_w(t)$ used for agricultural production. If the plants quantity demand $\bar{G}(t)$ cannot be met with the farmers willingness to pay for fertilizer $p_G(t)$ the trader sells $G(t) = \bar{G}(t) + \sigma(t)$ and the budget constraint reads $p_G(t)[\bar{G}(t) + \sigma] = p_m(t)P_m(t) + p_w(A)P_w(t)$. All cases for under- and overfertilization $\sigma(t) \neq 0$ are explained in 4.6.2.

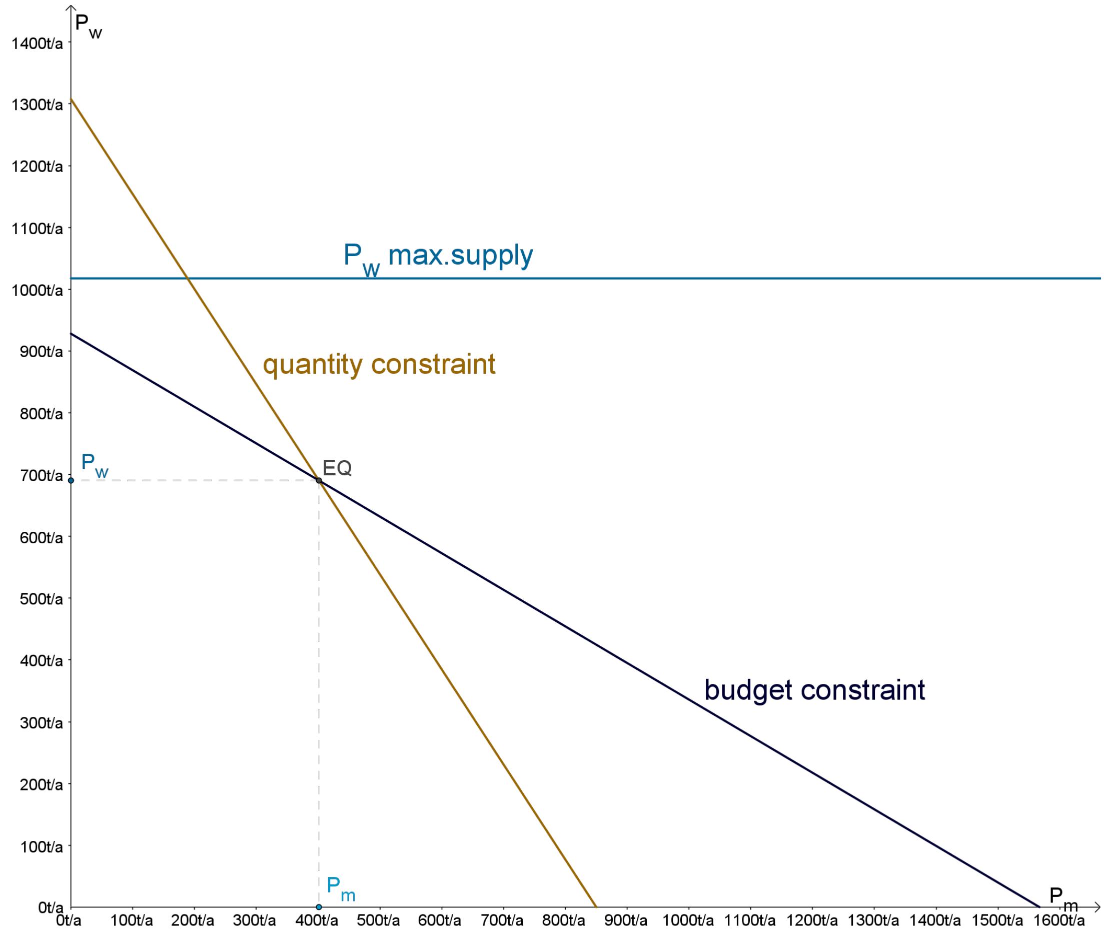


Figure 4.4: The constraints for the traders P_m and P_w supply

Environmental quality The environmental quality decreases by P emissions from agricultural fields $P_g^\ell(t)$ and from waste water treatment effluents $P_w^\ell(t)$, and by accumulation of heavy metals and organic micropollutants via fertilizers $P_w(t)$ or $P_m(t)$ in the fields. Environment can also regenerate with a rate $\delta > 1$. This results in the following dynamics for environmental quality.

$$E(t+1) = \delta(E(t)) - P_g^{\ell}(t) - P_w^{\ell}(t) - p_w^f(A)P_w(t) - p_m^f P_m(t) \tag{4.17}$$

4.3 Results

We first present selected analytical results and continue with numerical results of the model based on a time frame of 15 years reflecting the operating time of a P recovering technology. The parameter values are calibrated according to Austrian data (Österreich & Austria (2010), Zoboli *et al.* (2016a)) and listed in Table 4.4. An overview of all variables and parameters included in the model is given in 4.6.4. For each numerical simulation we assume that the available recycling technology is fixed.

4.3.1 Analytical results

Since a central aim of this chapter is to understand the phosphorus cycle and its relation to the specific fertilizers applied we first consider some analytical results on the market for fertilizers. A further important consideration of our model is to understand the environmental dimension of phosphorus use, because households also demand environmental quality. We therefore also present some analytical feedback mechanisms between household demand of grain and animal products and the environment.

Market for fertilizer In the following we will illustrate the market mechanisms that determine the specific level of the composite fertilizer G that is used by the farmers. First, we need to recall that the grain farmers and the traders are faced by various constraints as illustrated in Fig.4.5 and explained in the following Corollar.

Corollar *The price $p_G(t)$ of the composite fertilizer good $G(t)$ is constrained by the effective price levels $\frac{p_m(t)}{\chi_m}$ (in case only mineral fertilizer is used) and $\frac{p_w(t)}{\chi_w}$ (in case only recycled P is applied). Any combination of mineral and recycled fertilizers results in a price combination of these two prices and cannot exceed its interval.*

Proof See 4.6.3.

Depending on whether the price level of mineral fertilizer or recycled phosphorus is cheaper, the supply curve of the trader will be either upward sloping (Fig.4.5) or downward sloping (Fig.4.6) as we show in Proposition 1. The demand curve of farmers is always downward sloping in the price level of the composite fertilizer good $p_G(t)$.

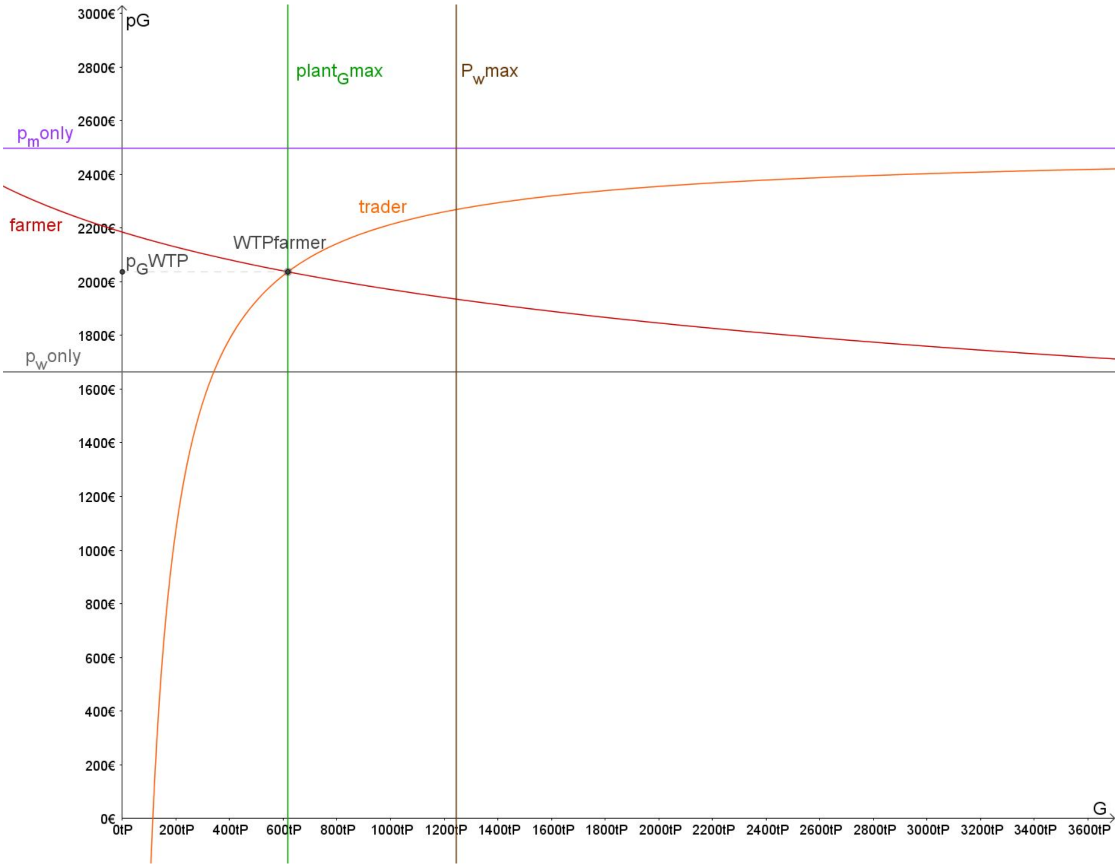


Figure 4.5: The fertilizer market for the composite fertilizer G (x-axis) if the effective price level (y-axis) of the mineral fertilizer $\frac{p_m}{\chi_m}$ is above the effective price level of the recycled P product $\frac{p_w}{\chi_w}$. The ascending traders supply curve intersects the decreasing farmers' demand function. The intersection yields a market equilibrium quantity G^{WTP} below the maximum G-supply and the corresponding market equilibrium price $p_G^{WTP} \in [\frac{p_w}{\chi_w}, \frac{p_m}{\chi_m}]$.

Proposition 1 *The traders' supply function changes its qualitative behavior if the farmers are willing to pay more for the composite fertilizer than the world market price for P, e.g. Fig.4.6.*

Proof See 4.6.3.
If farmers are willing to pay (WTP) more for the composite fertilizer ($p_G^{WTP} > \max(\frac{p_w}{\chi_w}, \frac{p_m}{\chi_m})$) traders will sell more of the expensive product, i.e. recycled fertilizer P_w in Fig.4.7.

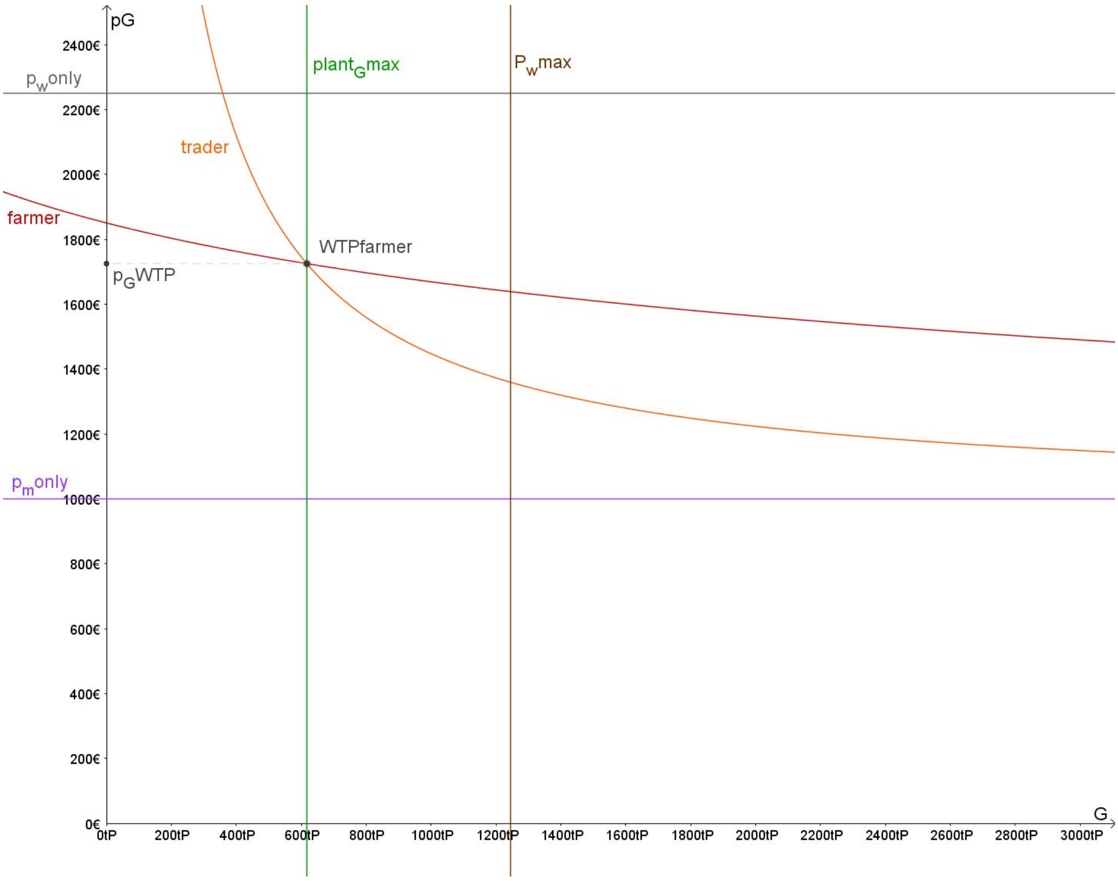


Figure 4.6: The fertilizer market for the composite fertilizer G if the effective price level of the mineral fertilizer $\frac{p_m}{\chi_m}$ is below the effective price level of the recycled P product $\frac{p_w}{\chi_w}$.

Farmer's profit maximization yields their fertilizer demand curve (Figs.4.6-4.8). Their willingness to pay the price p_G^{WTP} for fertilizer G results from meeting the optimal plant supply $G = \bar{G}$ (curve $plant_G max$ in Figs.4.6-4.8) at point $WTPfarmer$.
If the farmer's demand for the composite fertilizer good $G(t)$ is above or below both effective price levels they will over- or underfertilize, respectively. If e.g. the demand of farmers is above the more expensive fertilizer (Fig.4.7-4.8) farmers spend all their

available budget $p_G^{WTP}\bar{G}$ (curve $pGcosts$) on recycled P, i.e. $G(t) = (\chi_w - \ell)P_w(t)$ for its price $p_G(t) = \frac{p_w}{\chi_w} = p_w only$. If the trader would sell a mixed fertilizer product $G(t) = (\chi_w - \ell)P_w(t) + (\chi_m - \ell)P_m(t)$, its price would be lower ($p_G(t) < p_w only$) and allow to buy more fertilizer $G(t)$. Consequently, the trader's supply curve would be above the budget constraint curve $pGcosts$ (Fig.4.7-4.8).
In case the WWTP supply $P_w max$ cannot meet the demand for recovered phosphorus $(\chi_w - \ell)P_w$ (Fig.4.8) the trader would add mineral fertilizer $P_m(t)$ to receive a mixed fertilizer product $Gmix(t) = P_w max(t) + (\chi_m - \ell)P_m(t)$. Hence, farmers spend all available budget $pGcosts$ and pay the composite fertilizer price $p_G(t) = p_G sold$.

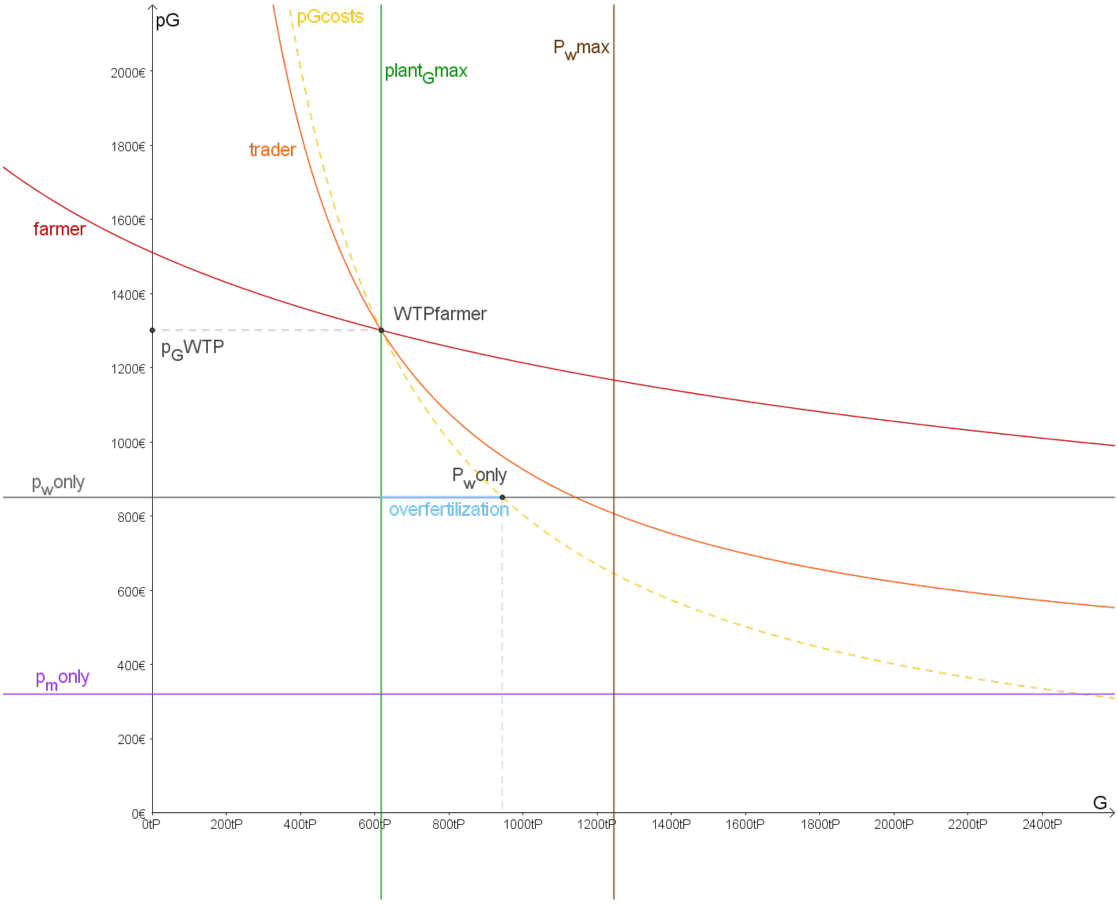


Figure 4.7: The fertilizer market for the composite fertilizer G if only recycled P, P_w is used and farmers budget allows to overfertilize.

Environmental quality Household consumption decisions (c_g^c, c_a^c) influence P flows in water bodies which can cause eutrophication. Since we assume that households care for environmental quality, the consumption decisions will be influenced by their impact on the environment. These interesting feedbacks within the coupled human-water system

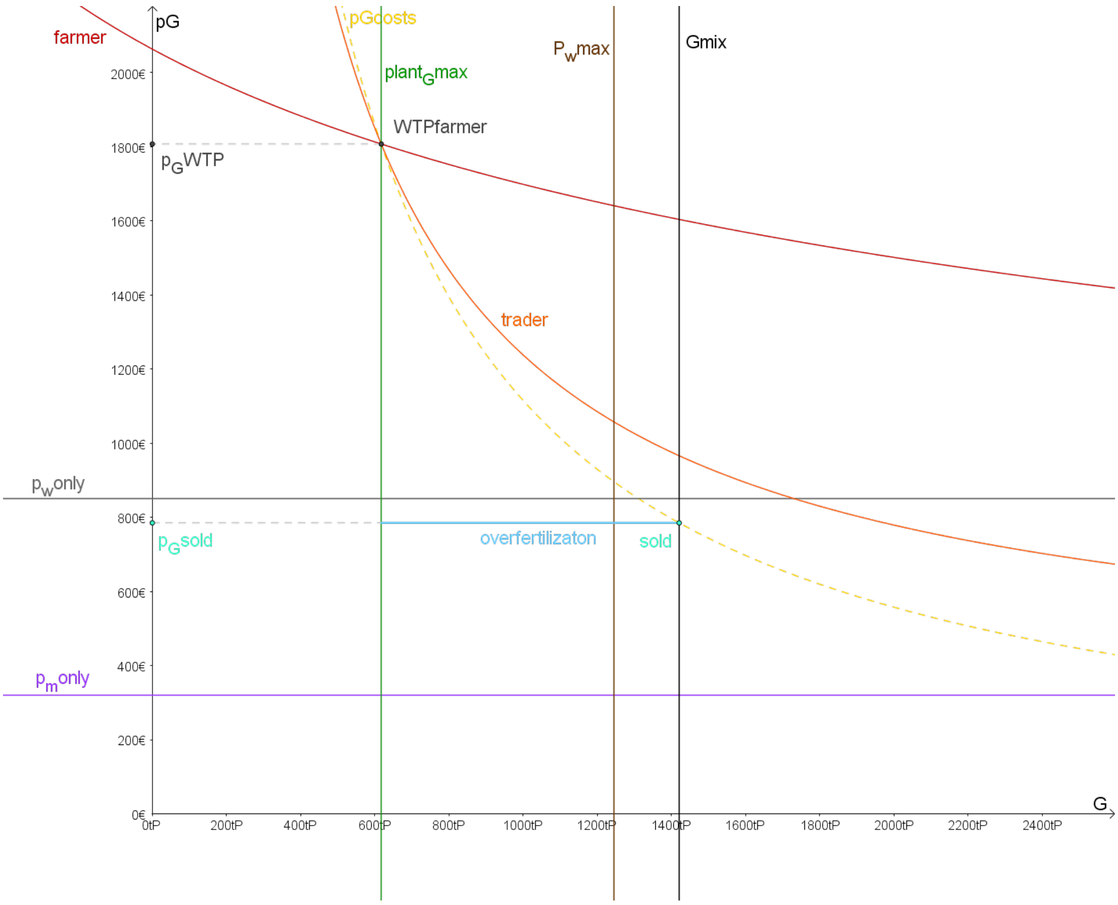


Figure 4.8: The fertilizer market for the composite fertilizer G in case of overfertilization. Farmers take the full potential of recycled P and add some mineral fertilizer to buy the maximum amount of fertilizer $G(t) = Gmix$ for a resulting composite price $p_G(t) = p_G sold$ considering their budget constraint $p_G costs$.

are described by the dynamics of the environment (Eq.4.17) and by the optimal household demand for grain c_g^c (Eq.4.18) and the analogous demand for the meat consumption good c_a^c Considering Eq.4.19 and Eq.4.20 (see 4.6.1) and the fact that $[\frac{p_g^c}{p_a^c}\frac{\partial E}{\partial c_a^c} - \frac{\partial E}{\partial c_g^c}]$ is negative yield the specific coupled feedbacks described below.

$$(c_g^c)^D = \gamma_c \Big[\frac{p_g^c}{p_a^c}\frac{\alpha_c}{c_a^c} + \frac{\epsilon}{E(c_g^c, c_g^a)}[\frac{p_g^c}{p_a^c}\frac{\partial E}{\partial c_a^c} - \frac{\partial E}{\partial c_g^c}]\Big]^{(-1)} \tag{4.18}$$

$$\frac{\partial E}{\partial c_g^c} = -\omega_\ell \frac{(1-\omega_g^\ell)P_g}{c_g(c_g^c, c_a^c)^2}[\zeta_{hg} - \zeta_{ha}\zeta_a^c]c_g^a(c_a^c) \tag{4.19}$$

$$\frac{\partial E}{\partial c_a^c} = -\omega_\ell \frac{(1-\omega_g^\ell)P_g}{c_g(c_g^c, c_a^c)^2}[\zeta_{ha}\zeta_a^c c_g(c_g^c, c_a^c) - \zeta_{hg}c_g^c + \zeta_{ha}\zeta_a^c\frac{c_g^a(c_a^c)}{\alpha_a c_a^c}] \tag{4.20}$$

Generally, more consumption decreases environmental quality. A decrease in environmental quality would again increase consumption, since individuals derive utility from consumption and environmental quality and these are substitutes. A society acting like that would in the long term deteriorate environment.
Otherwise, if e.g. environmental quality is really bad, the impact of environment is the dominant decision driver, and an increase in environmental quality leads to less consumption and this in turn increases environmental quality further. The utility would increase over time until the environment has reached a maximum. Then the negative loop would start again. The only exception is if the society prefers animal products, i.e. α_c is significantly larger than γ_c. In that case grain consumption would actually increase if environmental quality increases.

4.3.2 Numerical Results

In Austria P in sewage sludge is partly recycled via direct application of sewage sludge on the agricultural fields additional to spreading animal manure. The additional P demand is met by importing mineral fertilizer. In the base case for our analysis we therefore allow the choice of a combination of mineral fertilizer and direct application of sewage sludge. The parameters are summarized in Tables 4.4 and 4.5 and the initial values in Table 4.3. Fig.4.9 summarizes the P flows in our model framework and displays the flow values for the first and nineth model period. They represent the most important flows of P in an economy. Such a P flow diagram helps to identify strategies to improve environmental quality with respect to P, to choose a certain P recovery technology and to monitor dynamics of P stock in soil and water bodies. Stocks of P in crop production are P accumulation in soil, whereas stocks of P in animal husbandry and in the P recovery process represent only the transfer of P to the next period. The largest and therefore most important P flows are the fertilizer applications via manure or bought fertilizer on the crop fields. Households consume most P via vegetarian food. Figure

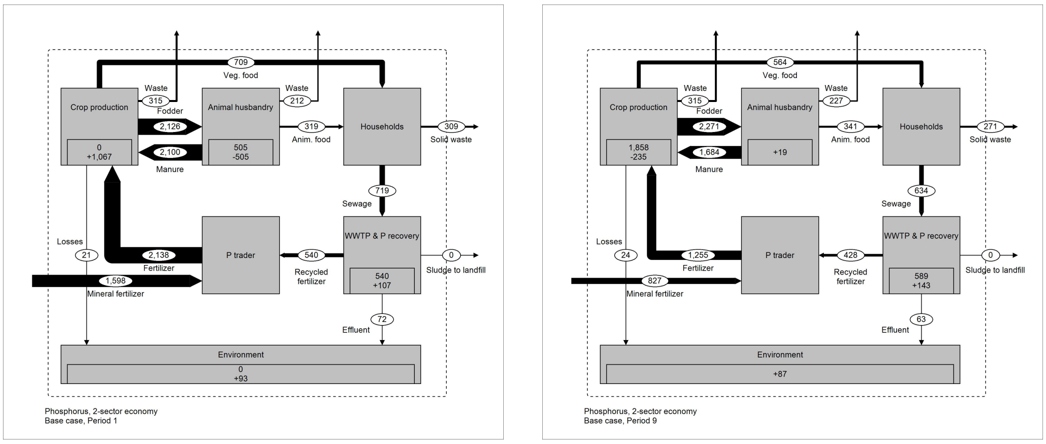


Figure 4.9: Sankey-style diagram (i.e. arrow widths are directly proportional to flow quantities) of P flows in period 1 (left) and period 9 (right) of the base case model

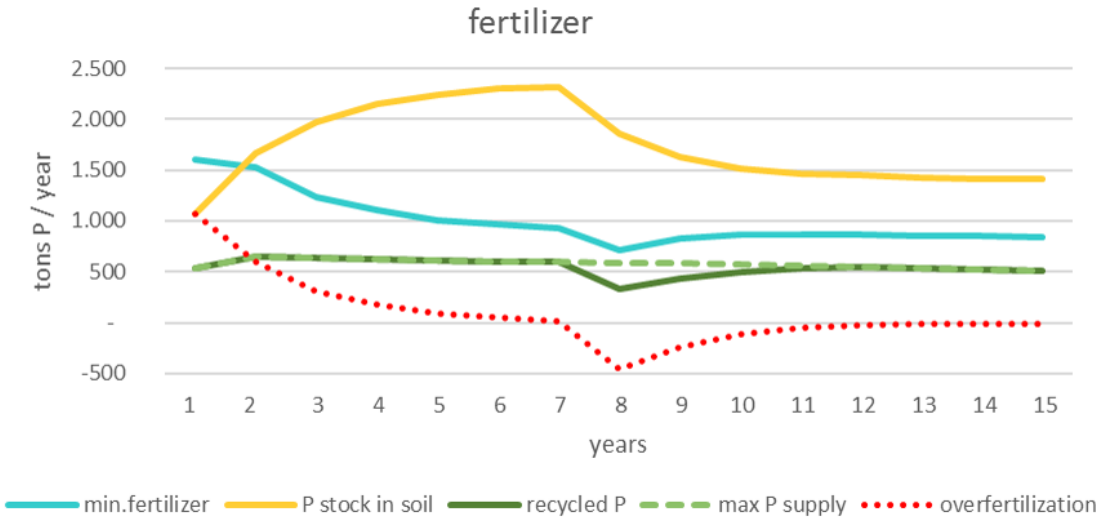


Figure 4.10: Fertilizer use over time given the option of direct application of sewage sludge, i.e. recycled P fertilizer. The price increase of mineral fertilizer in year 7 decreases the use of P fertilizer in the short term and the level of mineral fertilizer in the long term, and consequently lowers the P stock in soil. Max P supply is the maximum available amount of recycled P that the WWTP offers to the farmers and overfertilization show if farmers apply more or less P than the plants can absorb.

4.10 presents a base case simulation of the evolution of the most important P flows and stocks of Figure 4.9: P in soil together with the evolution of the mineral fertilizer and recycled phosphorus. In addition we also record the maximum P supply over time and the quantity of overfertilization, which is the change of stock in ”Crop production” for each period.
In the presented base case scenario, mineral fertilizer is cheap and direct application of sewage sludge is even cheaper. Farmer’s willingness to pay for fertilizer is therefore above the actual market price (cf. Fig. 4.7 and 4.8) and consequently they overfertilize (Fig.4.10 years 1-7). So the P stock in soil increases. This was the case in Austria in the 1980s and 1990s.
A larger P stock implies that farmers do not have to acquire as much additional fertilizer and can afford to spend more on employing labor and hence wages increase since the demand for labor rises. Higher wages in turn increase the income of households, who can then afford more expensive food and buy more meat products. Agricultural farmers adapt and plant more fodder crops. So the supply of vegetarian products decreases and its price increases stronger than the price of fodder products. The fact that inflation for food was higher than for other consumption goods in Austria in the past years fits to that picture.

The consumption behavior and employment structure resulting from the model replicates the current situation in Austria. Households eat slightly more vegetarian products than animal products. And more people are employed in the grain sector (62%) than in animal husbandry.

4.3.3 Price level increase of imported mineral fertilizer

In 2008 the price of imported mineral fertilizer increased from 2040EUR/tP to 3800EUR/tP. Such a price rise is introduced for the years 7-15 (Fig.4.10) to allow us to study how such a price increase in our model will affect the fertilizer market and consequently all the other markets. We can identify decreasing wages and increasing prices of alternative fertilizer and consumption goods, decreasing mineral fertilizer imports and a decreasing P stock.

As soon as the price of the mineral fertilizer rises, the grain farmer’s willingness to pay increases and equals the even higher market price of P fertilizer. So farmers cannot afford overfertilization any longer (Fig.4.10 years 8-15) and apply less fertilizer. In the base case farmers can choose between mineral fertilizer and direct application of sewage sludge. The quality of sewage sludge, i.e. the P availability for the plants, is too low to guarantee that the plant receives enough P to grow to its full extent. Farmers are not willing to buy 100% of the available recycled P (max P supply in Fig.4.10) and have to also decrease the amount of direct application of sewage sludge to spend the money rather on the more efficient mineral fertilizer (Fig.4.10 years 8-11).
Applying less fertilizer lowers the P stock in soil over time. Consequently, demand of P

fertilizer increases to compensate P in soil. Farmers cannot afford the increased demand of P fertilizer and underfertilize. This quantity effect is intensified by a price effect: Lower grain prices decrease revenues and fertilizer costs increase. Sensitivity analysis has shown that even lowering the number of employees and their wages cannot compensate these price changes.
Most important, farmers do not build up P stock after a fertilizer price increase. The degradation of environmental quality is reduced.

The severe overall fertilizer price change affects all other prices. In the short term, the fertilizer price increase in the grain production sector increases production input costs. Consequently, fodder supply decreases (Fig.4.11(a)). Since fertilizer requires more monetary resources for production, farmers can pay less for the complementary production input labor. Wages decrease because less labor is demanded in the grain sector. Consequently the second agricultural production sector, animal husbandry, can spend more on its other production input, fodder, and fodder demand increases in the

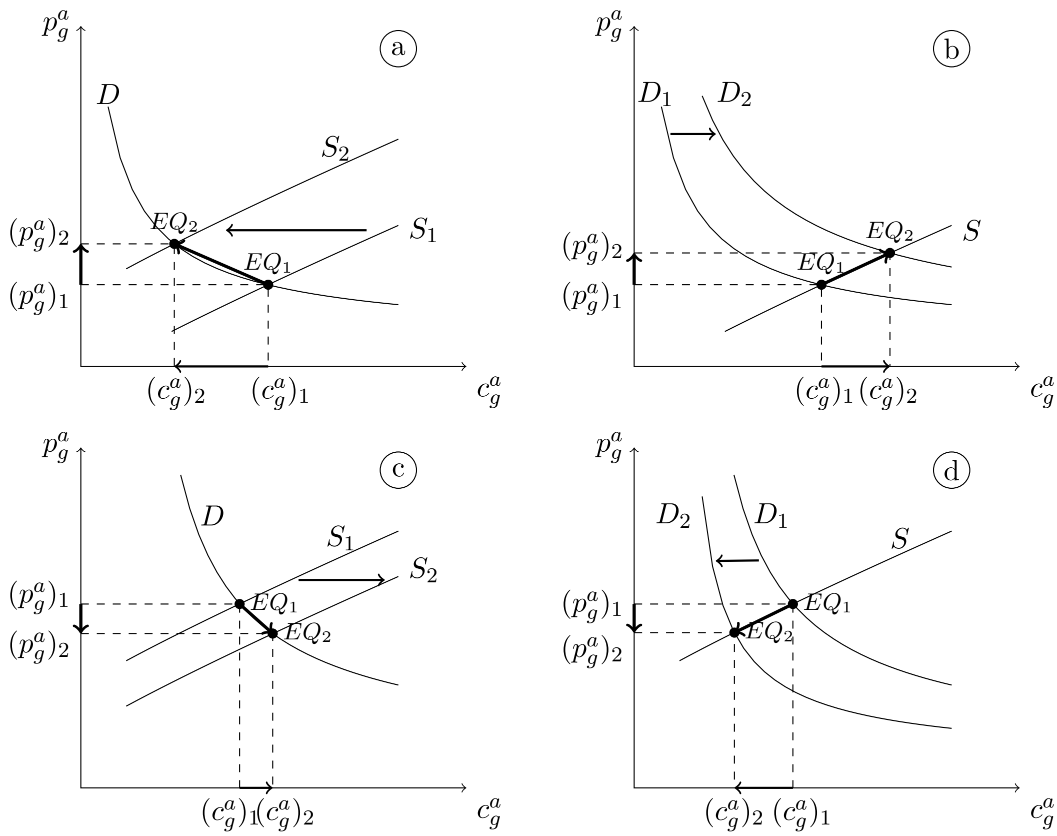


Figure 4.11: Direct (a,b) and indirect (c,d) effects of mineral fertilizer price p_m changes for fodder c_g^a supply S and demand D.

short term (Fig.4.11(b)).
In the long term workers move from the more labor intensive grain sector to the animal husbandry sector. Still, 58% of agricultural labor remain in the grain sector in the base case and, overall, wages decline. Hence, lower income leads to reduced food demand. This results in two opposite effects. Reduced household demand for high quality grain products combined with the short term fodder price increase forces agricultural farmers to shift to a crop variety that produces more lower quality fodder products. Consequently fodder supply increases (Fig.4.11(c)) and supply for high quality grain products decrease. Therefore prices for vegetarian products increase. This shifts the household's demand to a more dairy or meat based diet. To serve the resulting increased demand of animal products, fodder demand increases (Fig.4.11(b)). Contrary, overall reduced food demand also reduces animal husbandry's demand of the intermediate good fodder (Fig.4.11(d)). The resulting fodder price p_g^a dynamics of the first long term effect characterize the final price change, but they are twofold: The increased fodder demand leads to a fodder price increase (Fig.4.11(b)), whereas the increased fodder supply yields a fodder price decrease (Fig.4.11(c)). In the base case, where farmers use mineral fertilizer and direct application of sewage sludge, the fodder price increases 3% (Fig.4.12 (A1)). Whenever a P recovering technology is used to treat sewage sludge, the second effect is dominant, because fertilizer efficiency is higher and therefore it is easier to increase supply. In case of Ostara Pearl Reactor®the fodder price decreases 5%, for applying the Stuttgart process fodder price decreases 8% and for the most expensive technology EcoPhos®the price decreases even 13% (Fig.4.12 (A4)).The lower production input prices in the animal husbandry allow for lower prices for animal products.
The increased share of fodder products in the grain farmer's product portfolio leads to a lower average price per produced ton of grain, even though the price for household's grain consumption goods increased. In the end households pay higher prices with lower

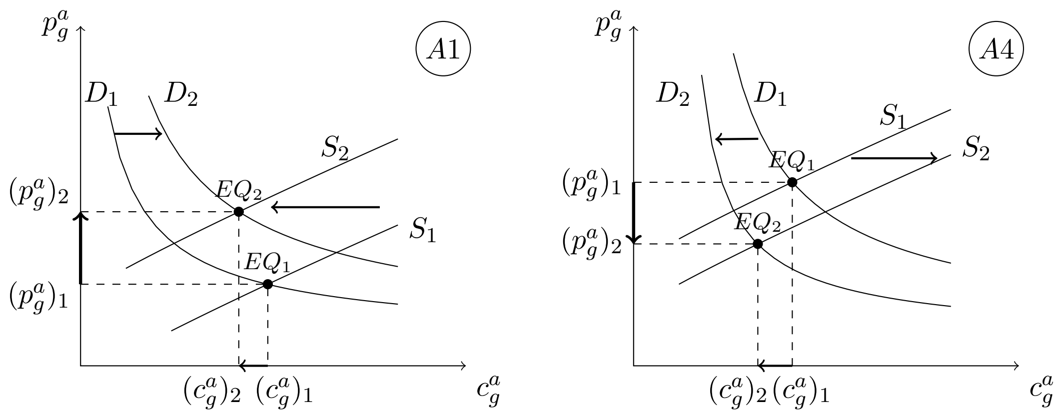


Figure 4.12: Long term effects on the fodder c_g^a market after a mineral fertilizer price p_m change.The fodder price p_g^a increases given the possibility of direct application $A1$ or decreases by using P recycling with EcoPhos®technology $A4$.

income.

If the price of recycled P is low, the effects of the price increase for mineral fertilizer are much stronger. If e.g. recovered P from EcoPhos®treatment can be sold for less than 2000EUR/tP, the reduction of overfertilization due to increased prices for mineral fertilizer would more than double.

4.3.4 Different P recycling technologies

We next study the market changes for the four different recovery methods (A1-A4) introduced in Section 4.2.2.

Offering P recovery technologies (A2, A3, A4) keeps the P stock in the soil significantly lower compared to no recycling or allowing only direct application of sewage sludge (A1) additional to the application of mineral fertilizer (Fig.4.13).
However, for grain farmers efficient grain production is more important than environment. Given any P recycling method (A1-A4) and stable fertilizer prices grain farmers overfertilize and the P stock in the soil increases every year (Fig.4.13, year 1-7). This reflects the current situation in Austria. Even after the fertilizer price increase in year 7, the combination of direct application of sewage sludge (A1) and mineral fertilizer is cheap enough to ensure plants will receive enough P fertilizer. As indicated in Fig.4.13, for year 8-15, overfertilization is close to zero, because enough P fertilizer is applied, and plants absorb less P from the P stock in the soil. This changes when we introduce P recovery technologies (A2, A3, A4). After a price increase of mineral fertilizer the price of the fertilizer composition increases sufficiently and therefore marginal costs of fertilization become larger than the marginal revenue for applying as much fertilizer as necessary for the plant to grow to its full potential. As a result, farmers underfertilize slightly and force the plants to absorb P from the soil. Consequently, environment recovers faster.
After the mineral fertilizer price increase the prices farmers have to pay for different recycled P fertilizer products also change significantly. If recycled P is already expensive the actual price change is not as strong as for cheap recycled fertilizer, but still increases at least 50%.
However, Fig.4.14 displays, that a price increase of imported mineral fertilizer has only short term effects on the use of imported mineral fertilizer if a P recovery technology (A2, A3, A4) is installed. A few years after the price increase firms on the market adjust all other quantities and prices to the higher price of the mineral fertilizer and apply as much or even more mineral fertilizer as before the price increase. Only in case of direct application of sewage sludge (A1) its high P recovery potential and its low price allow a reduction of imported mineral fertilizer.
Generally, the total level of applied fertilizer is decreasing more if, additional to a more expensive mineral fertilizer, the recovered P fertilizer product is more expensive. This leads to a stronger underfertilization (Fig.4.14) and therefore a more significant reduction

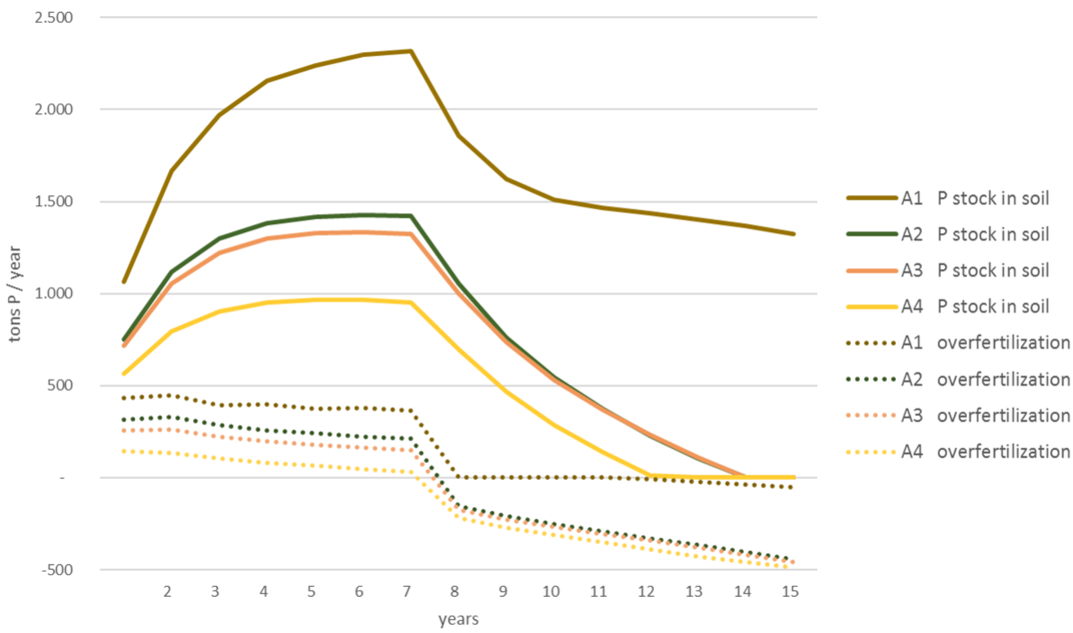


Figure 4.13: P accumulation in soil and corresponding overfertilization (stock and stock changes in "crop production" in Fig.4.2) over time, when grain farmers choose, additional to mineral fertilizer, also fertilizer from different recycling technologies: Direct application of sewage sludge (A1), Ostara Pearl Reactor®(A2), Stuttgart process (A3), EcoPhos®(A4). Note that negative overfertilization is underfertilization and forces the plant to take up P from the soil.

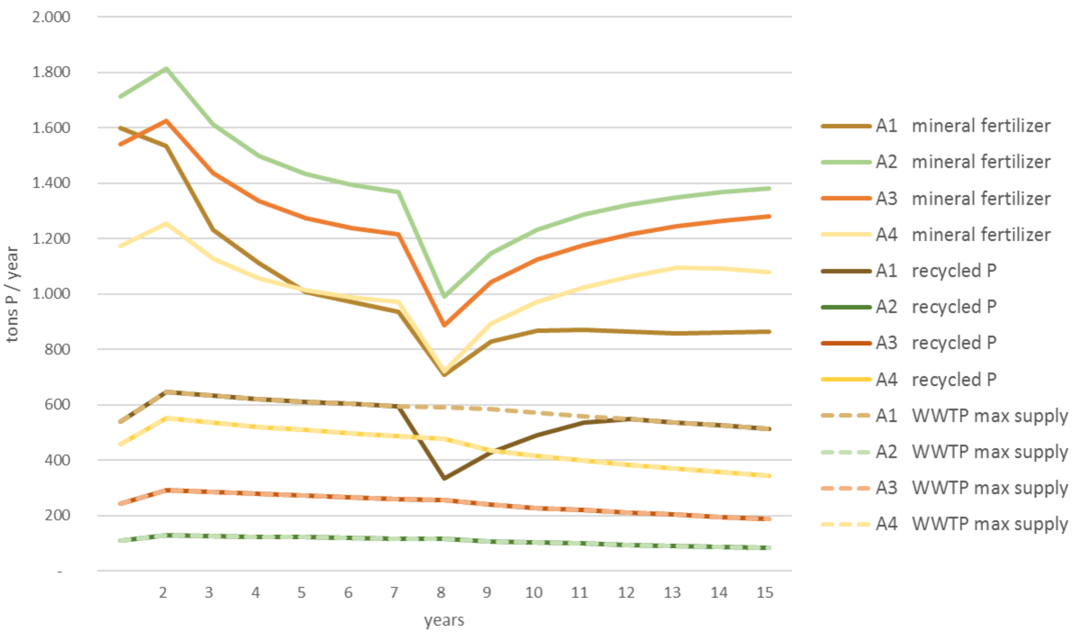


Figure 4.14: Farmer's choice of optimal fertilizer composition given the four different P recovery technologies. Time series of mineral fertilizer and recycled P in Fig.4.2 and the respective maximum amount of available P for the plant.

of P stock in soil.

Implementing any P recovery technology (A2, A3, A4) and offering the recycled product as P fertilizer would lead to reusing 100% of the recovered P (Fig.4.14) independent of the price level of mineral fertilizer. Since P in treated recycled products is more easily available for the plants, less overfertilization happens. An important observation is that farmers would always optimally choose the maximum available amount of recovered P. Nevertheless, this is not yet enough to meet the plant's P demand and farmers have to add imported mineral fertilizer or exploit the P stock in the soil.

Treated recycling products cause a higher fertilizer price. This affects the grain production more than animal husbandry and we can observe a slight increase in meat consumption. This fertilizer price effect due to a more costly recovering technology implies the same mechanisms as in the base case after the price shock. Generally, implementing costly P recovery technologies does not significantly change consumption levels compared to the base case with the possibility of cheap direct application of sewage sludge, but prices change.
Households spend significantly more on food consumption if no P recovering technology is implemented. However, quality has its price. If we only compare the scenarios with implemented P recovering technologies, the most expensive recovering technology leads to the highest prices for vegetarian products. Nevertheless, prices are still lower than in the scenario (A1) with no P recovering technology.

Economies benefit from investing in P recycling. Better recovering technologies, i.e. a greater recovering potential and better plant availability of the recycled material, enable farmers to buy less imported mineral fertilizer. This is important for two reasons: First, the economy depends less on the P world market and its prices. Second, environmental quality is higher since P recovered by a better technology includes less heavy metals and organic micropollutants, and less P ends up in water bodies where P can cause eutrophication.

Any P recovery technology (A2, A3, A4) leads to better environmental quality and profits of grain farmers are generally higher than in the base case scenario. In any case, profits of grain farmers are higher than profits for animal husbandry. However, there are also some economic trade-offs. Wages and consumption levels decrease.
To sum up, the most important benefit of implementing a recovery technology is the increase of environmental quality. Even though direct application of sewage sludge has advantages for plant growth and farmers profits, it yields the worst environmental quality in comparison to recovering technologies A2-A4.

4.3.5 Environmental friendly society

A different mind set causes a different optimal choice of P recovering technologies. In the base case household utility decreases by implementing a better and therefore more costly P recovering technology. This is why there are still no P-recycling technologies implemented e.g. in Austria. If environmental quality plays a crucial role in the societies objective, it is optimal to invest in better technologies (i.e. EcoPhos®) and adjust consumption accordingly.

If people value environment, they also care about it. By slightly adapting the consumption behavior, less fertilizer is needed to meet the demand and consequently less P can run off into surrounding water bodies.
A positive mind set towards environment even leads to higher wages and higher profits in agriculture and animal husbandry. The reason is a higher willingness to pay for food.

4.3.6 Sensitivity analysis

The choice of the parameter values describing the different recovery technologies (Table 4.1) is crucial for the model output. The main results of a sensitivity analysis for fertilizer efficiency $\chi_w(A)$ and recovery potential $\tau(A)$ are discussed below, followed by an outline of model limitations.

A better P availability $\chi_w(A)$ in the recycling product for the plant would obviously lead to a more efficient use of the recycled P product and consequently less application of the product. Still, the qualitative model outcome would only change for expensive P products. If we assume a plant availability of 80% for recovered P from the expensive EcoPhos process instead of 100%, 20% less recycled P from the WWTP would be consumed. Still, P stock in the soil would increase since plants absorb less.
For any other recovering technology reduction in $\chi_w(A)$ would only lead to an increasing use of mineral fertilizer, if imported mineral fertilizer is expensive. The resulting increase of fertilizer prices yields that the farmers are only willing to pay more for more efficient fertilizer, i.e. mineral fertilizer. If e.g. P recovered with the Stuttgart process performs with an efficiency of only 50% P plant availability instead of 85%, farmers would not use any of the recycled P after a price increase for mineral fertilizer. The same would apply to P recovered by the EcoPhos process if efficiency decreases from 100% to 80%.

The recovery potential $\tau(A)$ directly impacts the amount of available P after the recycling process. If more recycled P is available farmers buy adequately less mineral fertilizer. If the plant availability of recycled P is lower than of mineral fertilizer ($\chi_w < \chi_m$), the P stock in soil increases. This is the case for any recycling technology besides EcoPhos, because it would be too expensive to overfertilize.

4.4 Conclusion

In this chapter we have developed a two sector general equilibrium model and studied how household's consumption decision and farmer's profit maximization affects phosphorus (P) flows and resulting P stocks in soil and water bodies. Different P recovery technologies treat waste water and provide recycled P fertilizer products. Farmers can choose recycled P products and alternatively imported P mineral fertilizer.

Environmental quality is best if P is recycled from waste water and then used as fertilizer. This has even positive consequences on the economy. Profits in crop production increase and the economy becomes more independent from imported mineral fertilizer and its world market prices. Withers *et al.* (2015) and Zoboli *et al.* (2016b) also profess that P recovery is the most efficient strategy to reduce P in water bodies and increase independence from imported P fertilizer.

A price increase of imported mineral fertilizer would increase food prices and reduce household income for employees in agriculture. A positive consequence of such a price increase is that the P stock in soil and water bodies would not increase further.

A reduction of P stock in the soil can be achieved by implementing a recovering technology, whereas at the same time prices of imported mineral fertilizer have to be high. Cheap imported mineral fertilizer provides no incentive to lower fertilizer application. If policy makers help to lower prices for recovering P or even to increase mineral P prices by taxes or tolls, a technological change would occur and environmental quality would increase.
A more advanced recovery technology (i.e. recovering P from ashes) allows a stronger decrease of P in soil and water bodies. Whenever recycled P is offered, farmers use 100% of the available recovery products independent of the price of mineral fertilizer. Apparently there is a need, first, to increase the collection of sewage sludge and waste products containing P and, second, to increase the recovery potential of P recovery technologies.

Without any incentives to change the current consumption and fertilization behavior, overfertilization and high P stocks in soil remain a threat for the environment. Furthermore, food prices would further increase as this is the current case in Austria (Österreich & Austria (2010)).

A different mindset can crucially change economic decisions and consequently the environment. A green society - i.e. having higher preferences for environmental quality - would choose to implement the most advanced and expensive P recovery technology. This would not only improve environmental quality, but also increase household's utility, wages and profits. Withers *et al.* (2014) also relates greater public awareness of the environmental consequences to significant economic, environmental, and resource-protection gains. To sum up, agents with a green mindset are willing to pay higher food and consequently production input prices to ensure high environmental quality.

An important outcome of this work is the combination of an economic general equilibrium model with material flow analysis. This helps experts in resource management as well as economists to relate their work to a broader context. We propose a first framework to describe and explain material flows endogenous depending on economic decisions. So we can understand how economic decisions influence the environment based on mapping pollutant flows in a particular case and vice versa.

4.5 Discussion and Outlook

Like any conceptual model it cannot perfectly reflect reality. It has not been econometrically estimated and therefore it does not serve as a forecasting model. However, considering plausible assumptions the results will shed light on what might actually happen if conditions resembling the scenario settings occur. Generally, the interdisciplinary modeling approach includes limitations from a perspective of every single discipline. Bouman *et al.* (2000) also mentions challenges of combining MFA modeling techniques and a partial equilibrium model. In this chapter the nonlinear production functions slightly loosen the animal physiological and biological relationships of fodder and respective meat products. On the other side, linear production functions, as typically applied in material flow models by using transfer coefficients, would not be coherent with standard economic literature where labor plays a crucial role.
To combine the two worlds one has to invent new concepts to show stylized facts observed in data. The standard economic assumption of well informed and rational farmers would lead to no overfertilization, hence, abandon the core of this study. So we introduced the concept of a trader, serving as intermediary and regulatory agency. Aiming for appropriate fertilizer quantity and the zero-profit-condition distinguishes the modeled trader from a commercial fertilizer seller, who would only try to maximize profits. Finding an alternative model approach to capture the phenomenon of overfertilization can be interesting future research.

Further future work can include a more detailed modeling of the agricultural sector by including more cost types or economic regulators.
Moreover, capturing the feedbacks of households decisions and the environment over many periods by aggregating the utility and deriving optimal long term strategies is interesting work for future research.

4.6 Appendix

4.6.1 Market Equilibrium

Demand and Supply functions

We derive the demand and supply function of the households and the firms to obtain the market equilibria.

Households maximize

$$
\begin{aligned}
&\mathcal{L}_h(c_g^c, c_a^c, \mu_h) = \log(c_g^c(t)) + \alpha_c \log(c_a^c(t)) + \epsilon \log(E(t+1)) \qquad (4.21)\\
&+\mu_h[p_g(t)c_g^c(t) + p_a^c(t)c_a^c(t) + p_x x - (w(t)L(t) + \pi_g(t) + \pi_a(t) + \pi_w + \pi_x + p_m(t)P_m(t))]
\end{aligned}
$$

considering

$$
\begin{aligned}
&E(t+1) = E(c_g^c(t), c_g^a(t)) \qquad (4.22)\\
&= \delta E(t) - \ell[P_w(t) + P_m(t) + P_a^g(c_g^c(t-1), c_a^c(t-1)) + P_n(t)]\\
&-P_w^\ell(c_g^c(t), c_a^c(t)) - p_w^f P_w(t) - p_m^f P_m(t).
\end{aligned}
$$

The first order conditions are the following. For easier reading we suppress the time argument t.

$$
\frac{\partial \mathcal{L}_h}{\partial c_g^c} = \frac{\gamma_c}{c_g^c} + \frac{\epsilon}{E(c_g^c, c_a^c)} \frac{\partial E(c_g^c, c_a^c)}{\partial c_g^c} + \mu_h p_g^c = 0 \qquad (4.23)
$$

$$
\frac{\partial \mathcal{L}_h}{\partial c_a^c} = \frac{\alpha_c}{c_a^c} + \frac{\epsilon}{E(c_g^c, c_a^c)} \frac{\partial E(c_g^c, c_a^c)}{\partial c_a^c} + \mu_h p_a^c = 0 \qquad (4.24)
$$

$$
\frac{\partial \mathcal{L}_h}{\partial \mu_h} = p_g^c c_g^c + p_a^c c_a^c + p_x x - (wL + \pi_g + \pi_a + \pi_w + \pi_x + p_m P_m) = 0 \qquad (4.25)
$$

We define

$$
\begin{aligned}
E'(c_g^c, c_a^c) &:= \frac{p_g^c}{p_a^c}\frac{\partial E}{\partial c_a^c} - \frac{\partial E}{\partial c_g^c} \qquad (4.26)\\
&= -w_\ell \frac{(1-w_g^\ell)P_g}{c_g(c_g^c, c_a^c)^2}[(\frac{p_g^c}{p_a^c}\zeta_{ha}\zeta_a^c - \zeta_{hg})c_g(c_g^c, c_a^c)\\
&\quad +(1 - \frac{p_g^c c_g^a(c_a^c)}{\alpha_a p_a^c c_a^c})(\zeta_{hg}c_g^c + \zeta_h a\zeta_a^c c_g^a(c_a^c))].
\end{aligned}
$$

Expressing μ_h from Eq.4.23 and inserting in Eq.4.24 yields an implicite function for the grain demand

$$
c_g^c = \gamma_c[\frac{p_g^c}{p_a^c}\frac{\alpha_c}{c_g^c} + \frac{\epsilon}{E(c_g^c, c_g^a)}E'(c_g^c, c_g^a)]^{(-1)} \qquad (4.27)
$$

and we can reformulate the budget constraint to obtain the animal product demand as function of the grain demand

$$(c_a^c)^D = \frac{(wL + \pi_g + \pi_a + \pi_w + \pi_x + p_m P_m) - (p_g^c c_g^c + p_x x)}{p_g^c}. \quad (4.28)$$

Using $c_g^a(c_a^c) = \frac{1}{\psi}(\frac{c_a^c}{\phi_a} L_a^{1-\beta_a})^{\frac{1}{\alpha_a}}$ inserting Eq.4.28 in Eq.4.27 we obtain an implicite function for $(c_g^c)^D$ that only depends on prices and L_a, which will be a result of the labor market equilibrium.

The supply for labor is

$$L^S = 1000000. \quad (4.29)$$

Grain farmers demand labor and fertilizers and maximize

$$\mathcal{L}_g(G, L_g, \mu_g) = p_g(t)\phi_g[\chi_a P_a + \chi_n P_n + G]^{\alpha_g}[\phi_L L_g]^{\beta_g} - p_G G - wL_g. \quad (4.30)$$

Note, that P_a and P_n are taken from the previous period $t-1$. From the first order conditions

$$\begin{aligned} \frac{\partial \mathcal{L}_g}{\partial G} &= \alpha_g p_g \phi_g[(\chi_a - \ell)P_a + (\chi_n - \ell)P_n + G]^{\alpha_g - 1}[\phi_L L_g]^{\beta_g} - p_G = 0 \quad (4.31)\\ \frac{\partial \mathcal{L}_g}{\partial L_g} &= \beta_g p_g \phi_g[(\chi_a - \ell)P_a + (\chi_n - \ell)P_n + G]^{\alpha_g} \phi_L^{\beta_g} L_g^{\beta_g - 1} - w = 0 \quad (4.32)\end{aligned}$$

we can derive the demands for fertilizer and labor.

$$\begin{aligned} G^D(t) &= \left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{1-\beta_g} \beta_g^{\beta_g} p_g(t)}{p_G(t)^{1-\beta_g} w(t)^{\beta_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \\ &\quad -((\chi_a - \ell)P_a(t-1) + (\chi_n - \ell)P_n(t-1)) \quad (4.33)\\ L_g^D(t) &= \left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{\alpha_g} \beta_g^{1-\alpha_g} p_g(t)}{p_G(t)^{\alpha_g} w(t)^{1-\alpha_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \quad (4.34)\end{aligned}$$

Note, the fertilizer price is constraint with the world market price p_m and the price for recycled P p_w (proof see 4.6.3).

The crop supplies (limited with the crop intake of P to $\bar{P}$) are

$$\begin{aligned} c_g^S(t) &= \left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{\alpha_g} \beta_g^{\beta_g} p_g(t)^{\alpha_g+\beta_g}}{p_G(t)^{\alpha_g} w(t)^{\beta_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \quad (4.35)\\ (c_g^c)^S(t) &= \phi_g^c c_g^S(t) \quad (4.36)\\ (c_g^a)^S(t) &= \phi_g^a c_g^S(t) \quad (4.37)\end{aligned}$$

The average price for grain is $p_g(t) = \phi_g^c p_g^c(t) + \phi_g^a p_g^a(t)$.

Animal husbandry maximizes by choosing the amount of fodder and labor.

$$\mathcal{L}_a(c_g^a, L_a) = p_a^c \phi_a (\psi c_g^a)^{\alpha_a} [\phi_L L_a]^{\beta_a} - p_g^a c_g^a - w L_a \tag{4.38}$$

yields

$$\begin{aligned}
\frac{\partial \mathcal{L}_a}{\partial c_g^a} &= \alpha_a p_a^c \phi_a \psi^{\alpha_a} (c_g^a)^{\alpha_a - 1} [\phi_L L_a]^{\beta_a} - p_g^a = 0 && (4.39)\\
\frac{\partial \mathcal{L}_a}{\partial L_a} &= \beta_a p_a^c \phi_a (\psi c_g^a)^{\alpha_a} \phi_L^{\beta_a} L_a^{\beta_a - 1} - w = 0 && (4.40)
\end{aligned}$$

the optimal demands and supply are

$$\begin{aligned}
(c_g^a)^D(t) &= \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{1-\beta_a} \beta_a^{\beta_a} p_a^c(t)}{(p_g^a)^{1-\beta_a} w^{\beta_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} && (4.41)\\
L_a^D(t) &= \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{\alpha_a} \beta_a^{1-\alpha_a} p_a^c(t)}{(p_g^a)^{\alpha_a} w^{1-\alpha_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} && (4.42)\\
(c_a^c)^S(t) &= \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{\alpha_a} \beta_a^{\beta_a} (p_a^c)^{\alpha_a+\beta_a}}{(p_g^a)^{\alpha_a} w^{\beta_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} && (4.43)
\end{aligned}$$

Trader The zero-profit condition is the budget constraint of the trader.

$$\pi_G = p_G(\bar{G} + \sigma) - p_w(A) P_w - p_m P_m = 0 \tag{4.44}$$

The production function is the combination of the fertilizer types.

$$\bar{G} + \sigma = (\chi_w(A) - \ell) P_w + (\chi_m - \ell) P_m \tag{4.45}$$

For no overfertilization $\sigma = 0$ the quantity constraint for an ideal fertilizer supply $\bar{P}$ of the plant is fulfilled.

$$G(t) = \bar{G} = \bar{P} - (\chi_a - \ell) p_a^c(t-1) - (\chi_n - \ell) P_n(t-1) \tag{4.46}$$

Furthermore the trader is constraint to a maximum supply from the WWTP.

$$P_w \leq P_w^{maxsupply} \tag{4.47}$$

In case budget, quantity and WWTP constraint can be fulfilled we derive the following demand for fertilizer on the world markets (see also Fig.4.4).

$$\begin{aligned}
P_w^D &= \frac{(1 - (\chi_m - \ell)\frac{p_G}{p_m}) G}{(\chi_w - \ell) - (\chi_m - \ell)\frac{p_w}{p_m}} && (4.48)\\
P_m^D &= \frac{p_G G - p_w P_w^D}{p_m} && (4.49)
\end{aligned}$$

Market clearing

We obtain the prices p_g^c, p_g^a, p_a^c, w, p_G from the competitive markets for c_g^c, c_g^a, c_a^c, L, G, respectively. The open markets for P_m and P_w face infinitely elastic supply and exogenous prices p_m and p_w, respectively.

The market clearing conditions are as follows.

$$c_g^c: \quad (c_g^c)^D(p_g^c; p_a^c, E, E') = \phi_g^c\left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{\alpha_g} \beta_g^{\beta_g} p_g(t)^{\alpha_g+\beta_g}}{p_G(t)^{\alpha_g} w(t)^{\beta_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \tag{4.50}$$

$$c_g^a: \quad \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{1-\beta_a} \beta_a^{\beta_a} p_a^c(t)}{(p_g^) a^{1-\beta_a} w^{\beta_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} = \phi_g^a\left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{\alpha_g} \beta_g^{\beta_g} p_g(t)^{\alpha_g+\beta_g}}{p_G(t)^{\alpha_g} w(t)^{\beta_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \tag{4.51}$$

$$c_a^c: \quad \frac{(wL + \pi_g + \pi_a + \pi_w + \pi_x + p_m P_m) - (p_g^c c_g^c + p_x x)}{p_g^c} = \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{\alpha_a} \beta_a^{\beta_a} (p_a^c)^{\alpha_a+\beta_a}}{(p_g^a)^{\alpha_a} w^{\beta_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} \tag{4.52}$$

$$L: \quad \left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{\alpha_g} \beta_g^{1-\alpha_g} p_g(t)}{p_G(t)^{\alpha_g} w(t)^{1-\alpha_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} + \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{\alpha_a} \beta_a^{1-\alpha_a} p_a^c(t)}{(p_g^a)^{\alpha_a} w^{1-\alpha_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} + L_x = L \tag{4.53}$$

$$G: \quad \left(\frac{\phi_g \phi_L^{\beta_g} \alpha_g^{1-\beta_g} \beta_g^{\beta_g} p_g(t)}{p_G(t)^{1-\beta_g} w(t)^{\beta_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} = \bar{P} \tag{4.54}$$

Note, $(c_a^c)^D$ cannot be explicitly expressed with prices and is still a function of $(c_g^c)^D$. So we cannot derive the prices explicitly.

Tatonnement algorithm

We eliminate some equations and apply a tatonnement algorithm for the c_g^c and c_a^c markets for every period:

1. $p_m(t)$, $p_w(A)$, L given; $E(t)$, $P_n(t)$, $P_h(t-1)$ from previous period
2. identify cheaper and more efficient fertilizer by testing $\frac{pm/chi_m}{pw/chi_w}$ smaller or larger than one, derive $\bar{G} = G$ from plan constraint (4.46)
3. initialize p_g^c, p_g^a, p_a, p_G
4. from L market derive w using G from plant constraint (4.46) in the first iteration of the algorithm

$$\left(\phi_g \phi_l^{\beta_g} \beta_g p_g \bar{P}^{\alpha_g}\right)^{\frac{1}{1-\beta_g}} w(t)^{\frac{-1}{1-\beta_g}} + \left(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{\alpha_a} \beta_a^{1-\alpha_a} p_a(t)}{p_g^a(t)^{\alpha_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} w(t)^{\frac{1-\alpha_a}{1-\alpha_a-\beta_a}}$$
$$= L - L_x \tag{4.55}$$

5. from fertilizer G market derive p_G^{WTP} using farmers demand and the plant constraint (4.46)

$$p_G^{WTP} = \Big(\frac{\phi_g \phi_L^{\beta_g} {\alpha_g}^{1-\beta_g} \beta_g^{\beta_g} (\phi_g^c p_g^c + \phi_g^a p_g^a)}{\bar{P}^{1-\alpha_g-\beta_g} w^{\beta_g}} \Big)^{\frac{1}{1-\beta_g}} \tag{4.56}$$

6. derive P_m and P_w, p_G^{sold} and G^{sold}

   (a) check if p_G^{WTP} in $[p_m/\chi_m, p_w/\chi_w]$ without loss of generality or if we have to use a corner solution

   (b) buy only $P_m = p_G^{WTP} \bar{G}/p_m$ if farmer's willingness to pay (WTP) is below the market prices for fertilizers and P_m is more efficient than P_w

   (c) buy only $P_w = p_G^{WTP} \bar{G}/p_w$ or mixed if $P_w > P_w^{maxsupply}$ if farmer's WTP is outside the fertilizer price interval

   (d) if the WTP is within the market fertilizer price range choose fertilizer according to Eq.4.48 and Eq.4.49 , except $P_w > P_w^{maxsupply}$

   (e) derive overfertilization $\sigma = G - \bar{G}$ and the new P stock in the soil $P_n = (1-\ell)(P_m + P_w + P_a^g + P_n) - \bar{P}$

   (f) derive the P in plants $P_g = \min(G, \bar{G}) + (\chi_a - \ell)P_a + (\chi_n - \ell)P_n$

7. derive p_g^a from c_g^a market using $\gamma_g^a = \frac{1}{(\alpha_g+\beta_g)(1-\alpha_a-\beta_a)+(1-\beta_a)(1-\alpha_g-\beta_g)}$

$$p_g^a = \frac{1}{\phi_g^a} \Big(\big(\frac{\phi_a \phi_L^{\beta_a} \psi^{\alpha_a} \alpha_a^{1-\beta_a} \beta_a p_a}{w^{\beta_a}} \big)^{1-\alpha_g-\beta_g} \big(\frac{w^{\beta_g}}{\phi_g \phi_L^{\beta_g} \alpha_g^{\alpha_g} \beta_g^{\beta_g} (p_g^c)^{\alpha_g+\beta_g}} \big)^{1-\alpha_a-\beta_a} \Big)^{\gamma_g^a} \tag{4.57}$$

8. prepare tatonnement

$$\mathbf{(c_g^c)^S} = \phi_g^c\left(\frac{\phi_g\phi_L^{\beta_g}\alpha_g^{\alpha_g}\beta_g^{\beta_g}p_g(t)^{\alpha_g+\beta_g}}{p_G(t)^{\alpha_g}w(t)^{\beta_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \tag{4.58}$$

$$\mathbf{(c_a^c)^S} = \left(\frac{\phi_a\phi_L^{\beta_a}\psi^{\alpha_a}\alpha_a^{\alpha_a}\beta_a^{\beta_a}p_a^{\alpha_a+\beta_a}}{(p_g^a)^{\alpha_a}w^{\beta_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} \tag{4.59}$$

$$L_g^D = \left(\frac{\phi_g\phi_L^{\beta_g}\alpha_g^{\alpha_g}\beta_g^{1-\alpha_g}p_g(t)}{p_G(t)^{\alpha_g}w(t)^{1-\alpha_g}}\right)^{\frac{1}{1-\alpha_g-\beta_g}} \tag{4.60}$$

$$L_a^D = \left(\frac{\phi_a\phi_L^{\beta_a}\psi^{\alpha_a}\alpha_a^{\alpha_a}\beta_a^{1-\alpha_a}p_a(t)}{p_g^a(t)^{\alpha_a}w(t)^{1-\alpha_a}}\right)^{\frac{1}{1-\alpha_a-\beta_a}} \tag{4.61}$$

$$\pi_g = (p_g^c\phi_g^c + p_g^a\phi_g^a)(c_g^c)^S - p_G G - wL_g^D \tag{4.62}$$

$$\pi_a = p_a(c_a^c)^S - p_g(c_g^a)^D - wL_a^D \tag{4.63}$$

$$\pi_w = p_w P_w \tag{4.64}$$

$$P_g^c = \frac{c_g^c}{c_g^c + c_g^a}\zeta_g P_g \tag{4.65}$$

$$P_g^a = \frac{c_g^a}{c_g^c + c_g^a}\zeta_g P_g \tag{4.66}$$

$$P_h = (c_g^c + \zeta_a^c c_g^a)\frac{\zeta_g P_g}{c_g^c + c_g^a} \tag{4.67}$$

$$P_w^\ell = \omega_\ell(A)P_h \tag{4.68}$$

$$P_g^\ell = \ell(P_w + P_m + P_a^g + P_n) \tag{4.69}$$

$$E(t+1) = \delta E - P_g^\ell - P_w^\ell \tag{4.70}$$

$$\mathbf{(c_g^c)^D} = (c_g^c)^D(p_g^c; p_a, E, E') \tag{4.71}$$

$$\mathbf{(c_a^c)^D} = \frac{(wL + \pi_g + \pi_a + \pi_w + \pi_x + p_m P_m) - (p_g^c c_g^c + p_x x)}{p_g^c} \tag{4.72}$$

9. tatonnement with dumping factor d:

$$p_a = p_a^{old} - d((c_a^c)^D - (c_a^c)^S) \tag{4.73}$$

$$p_g^c = (p_g^c)^{old} - d((c_g^c)^D - (c_g^c)^S) \tag{4.74}$$

10. repeat from step 4 and stop if $|((c_a^c)^D - (c_a^c)^S)| + |((c_g^c)^D - (c_g^c)^S)| < tol$

4.6.2 Overfertilization

The fertilizer market is one of the core elements of the proposed model framework. Farmers demand the composite fertilizer good G for a market price p_G (Eq.4.33). A trader combines mineral fertilizer P_m and recycled phosphorus P_w to the fertilizer composition G (Eq.4.15) and sells it for the resulting price p_G (Eq.4.16) under his zero-profit-condition and the price level constraints (see 4.6.3). We can picture the traders fertilizer supply (Eq.4.75), the farmers fertilizer demand, the plant demand $\bar{G}$ and the maximum WWTP supply $P_w^{maxsupply}$ in one graph (e.g. Fig.4.15). If the quantity demand $G(t)$ from the farmers (Eq.4.33) cannot be met with the farmers willingness to pay for fertilizer $p_G(t)$ the trader sells $G(t) + \sigma(t) = (\chi_w - \ell)P_w(t) + (\chi_m - \ell)P_m(t)$. All cases for under- and overfertilization $\sigma(t)$ are explained below.

Farmers are willing to pay more than the necessary fertilizer price and the demand of recycled P can be met. The trader sells only recycled P_w and considers the budget constraint. The sold amount G is displayed in Fig.4.15 and the price p_G is exactly the price p_w for $\chi_w P_w$.

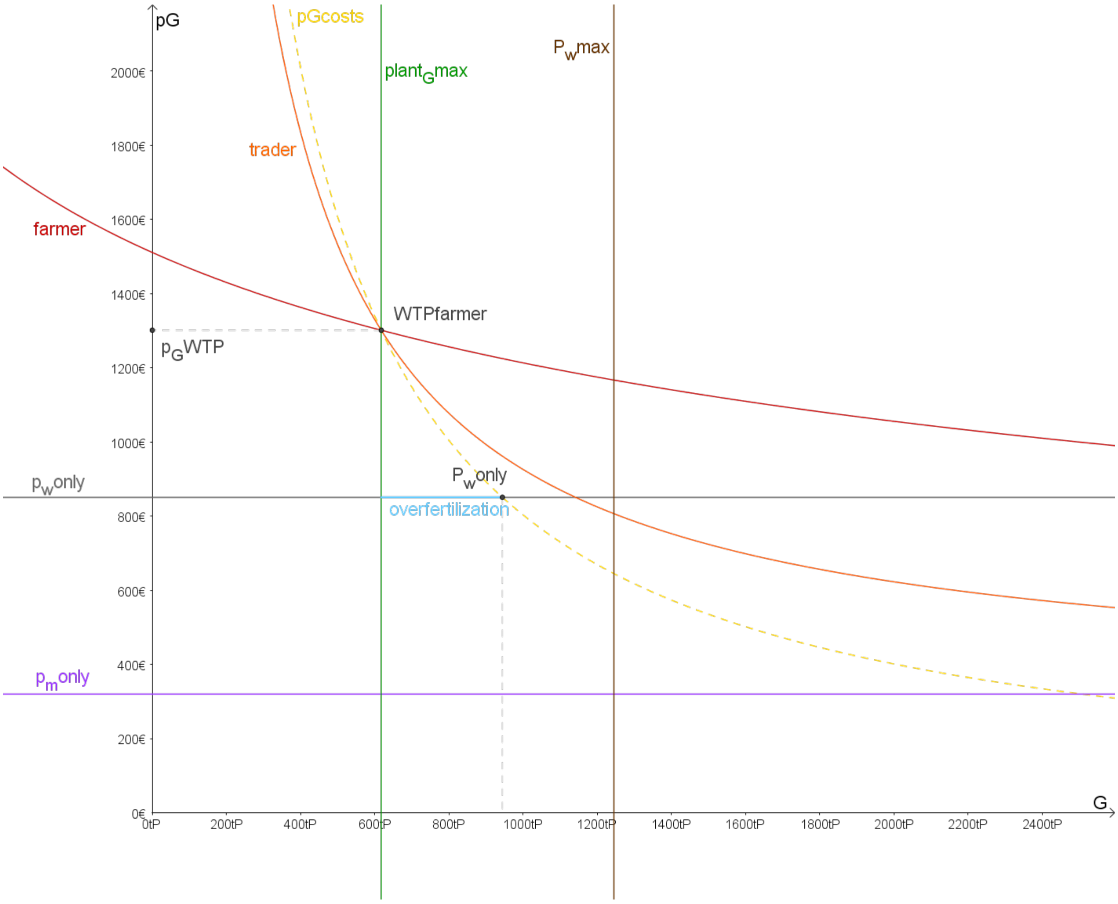


Figure 4.15: Even if farmers pay the highest possible price $p_G = \frac{p_w}{\chi_w}$ the are willing to buy more fertilizer G than the plants actually demand ($plant_G max$) and overfertilize.

Farmers fertilizer demand exceeds the amount of available recycled P fertilizer. Even if the trader sells all the available P_w for the price $p_G = \frac{p_w}{\chi_w}$ farmers are still willing to pay more for fertilizer. So the trader also adds imported mineral fertilizer to the composite fertilizer G_{mix} (Fig.4.16). Consequently, the price for amount $G = \bar{G} + \sigma$ changes to $p_G < \frac{p_w}{\chi_w}$ (Eq.4.16).

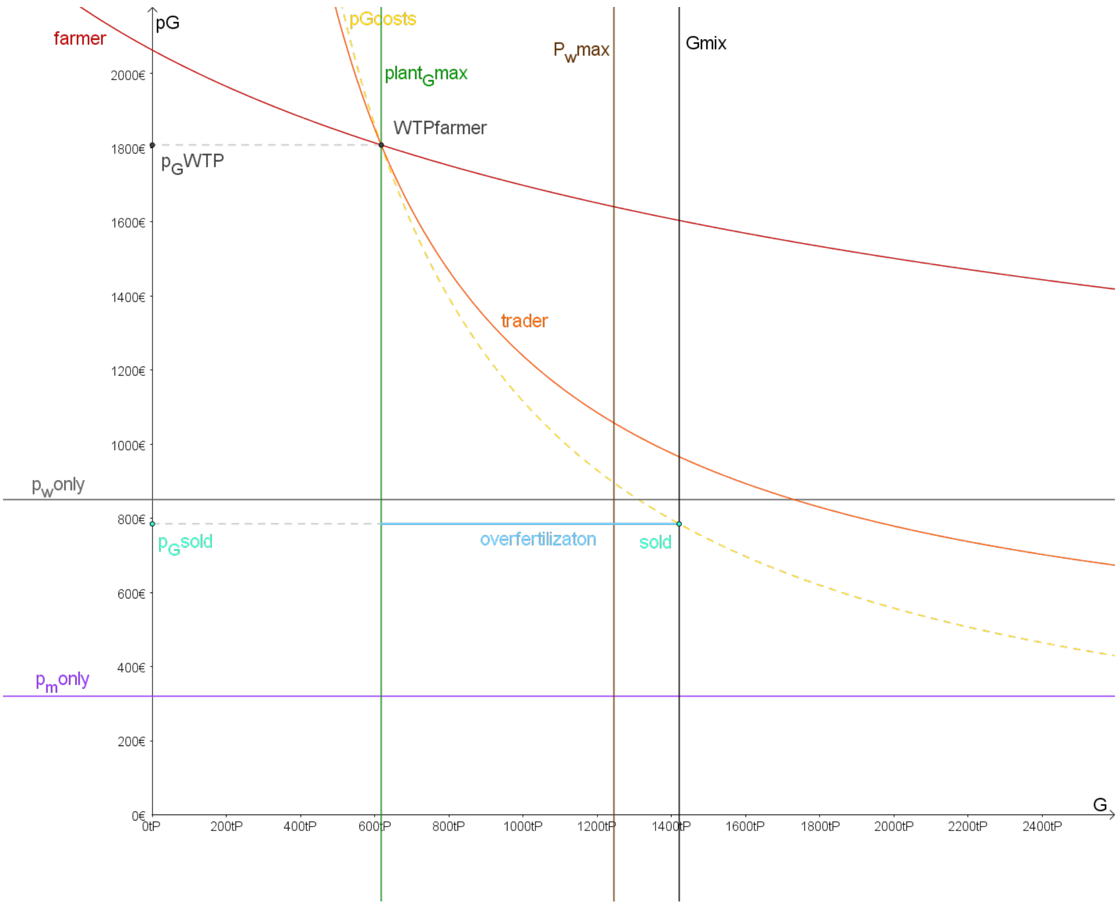


Figure 4.16: The willingness to pay for farmers (G^{WTP}, p_G^{WTP}) allows to sell the fertilizer mix (G^{sold}, p_G^{sold}).

After a price shock of mineral fertilizer price relations change to $p_m > p_w$. If farmers WTP is above p_w traders compose the fertilizer mix G analogous to the cases in the previous paragraphs: The trader would only add P_m if the demand for P_w exceeds $P_w^{maxsupply}$. Contrary to the above case the new price p_G would then increase $(p_G > p_w)$.

Farmers are willing to pay less than the cheapest available fertilizer. If $p_G^{WTP} < \frac{p_m}{\chi_m} < \frac{p_w}{\chi_w}$ or $p_G^{WTP} < \frac{p_w}{\chi_w} < \frac{p_m}{\chi_m}$, the trader sells as much of the cheapest fertilizer as the budget constraint (Eq.4.16) allows. The fertilizer supply is less than the plants demand ($G = \bar{G} - \sigma$) and we are in a case of underfertilization.

4.6.3 Proofs for the fertilizer market

Proof: price p_G is constraint

The price $p_G(t)$ of the composite fertilizer good $G(t)$ has to be in the interval $[\frac{p_m(t)}{\chi_m}, \frac{p_w(t)}{\chi_w}]$ if $\frac{p_m(t)}{\chi_m} < \frac{p_w(t)}{\chi_w}$ w.l.o.g.

Proof: We can rewrite the production function (Eq.4.15) and budget constraint (Eq.4.16) of the trader into $p_G = \frac{p_m P_m + p_w P_w}{\chi_m P_m + \chi_w P_w}$.. Note, $G = \bar{G} + \sigma$ in case of overfertilization.

First, assume $p_G > \frac{p_w}{\chi_w}$. Then $p_w \frac{\frac{p_m}{p_w} P_m + P_w}{\chi_m P_m + \chi_w P_w} > \frac{p_w}{\chi_w}$. We can rewrite that into $\frac{p_m(t)}{\chi_m} > \frac{p_w(t)}{\chi_w}$, which disproves the assumption and we conclude $p_G < \frac{p_w}{\chi_w}$.
Second, assume $p_G < \frac{p_m}{\chi_m}$. Then $p_m \frac{P_m + \frac{p_w}{p_m} P_w}{\chi_m P_m + \chi_w P_w} < \frac{p_m}{\chi_m}$. We can rewrite that again into $\frac{p_w(t)}{\chi_w} < \frac{p_m(t)}{\chi_m}$, which disproves the assumption and we conclude $p_G < \frac{p_w}{\chi_w}$.
Consequently, $p_G \in [\frac{p_m(t)}{\chi_m}, \frac{p_w(t)}{\chi_w}]$ if $\frac{p_m(t)}{\chi_m} < \frac{p_w(t)}{\chi_w}$. Analogous we can show $p_G \in [\frac{p_w(t)}{\chi_w}, \frac{p_m(t)}{\chi_m}]$ if $\frac{p_m(t)}{\chi_m} > \frac{p_w(t)}{\chi_w}$.

Proof: trader supply switches at a price level $\frac{p_m(t)}{\chi_m}$

The traders supply function switches from downward (Fig.4.5) to upward (Fig.4.6) sloping at point $p_G = \frac{p_m(t)}{\chi_m}$.

Proof: From the production function (Eq.4.15) and budget constraint (Eq.4.16) we can derive the traders supply curve

$$p_G(G(t); P_w(t)) = \frac{p_m(t)}{G(t)} [\frac{G(t) - \chi_w P_w}{\chi_m} + p_w(A) P_w(t)]. \tag{4.75}$$

The partial derivative with respect to G is

$$\frac{\partial p_G}{\partial G} = \frac{p_m(t)}{G(t)^2} [\frac{\chi_w P_w}{\chi_m} - p_w(A) P_w(t)]. \tag{4.76}$$

So we can express

$$p_G(G(t); P_w(t)) = \frac{p_m(t)}{\chi_m(t)} - \frac{p_m(t)}{G(t)} [\frac{\chi_w P_w}{\chi_m} - p_w(A) P_w(t)] \tag{4.77}$$

$$= \frac{p_m(t)}{\chi_m(t)} - G(t) \frac{\partial p_G}{\partial G}. \tag{4.78}$$

Table 4.2: Decision variables of the model

Symbol	Description	Unit
$c_g(t)$	total production of grain	[t/a/cap]
$c_g^c(t)$	household consumption of grain	[t/a/cap]
$c_g^a(t)$	fodder consumption	[t/a/cap]
$c_a^c(t)$	consumption of animal products	[t/a/cap]
$G(t)$	fertilizer mix	[t/a/cap]
$L_g(t)$	labor in grain production	cap
$L_a(t)$	labor in animal husbandry	cap
$P_w(t)$	recycled phosphorus-fertilizer from the waste water treatment plant	[t/a/cap]
$P_m(t)$	mineral fertilizer	[t/a/cap]

If $p_G > \frac{p_m(t)}{\chi_m}$, then $\frac{p_m(t)}{\chi_m(t)} - G(t)\frac{\partial p_G}{\partial G} > \frac{p_m(t)}{\chi_m}$ and consequently $\frac{\partial p_G}{\partial G} < 0$. So the trader supply is downward sloping.
Analogous, if $p_G < \frac{p_m(t)}{\chi_m}$, we can derive $\frac{\partial p_G}{\partial G} > 0$ meaning the trader supply is upward sloping.
In the special case $p_G = \frac{p_m(t)}{\chi_m}$ the price of the fertilizer is constant, because only P_m is sold as $G(t)$.

4.6.4 Variables and parameters

The agents in the model (agricultural farmers, animal husbandry, households) will optimally choose every year what we call *decision variable*. Based on that decisions we obtain *endogenous variables*. Parameters and initial values are chosen according to existing literature or calibrated. For the calibration we use data describing the *decision variables* and *endogenous variables*. One dataset is based on Austrian data (scaled to 1.000.000 inhabitants to be nourished) from the years around 2010.

An overview of variables and parameters is given in Tables 4.2 and 4.3, and Tables 4.4 and 4.5, respectively.

Table 4.3: Endogenous variables of the model

Symbol	Description	Unit	Initial value
$E(t)$	quality of environment (amount of phosphorus in the water bodies)	[ ]	100,000
$x(t)$	other goods	[t/a/cap]	
$p_g(t)$	price for grain	EUR/t	
$p_g^c(t)$	price for grain household products	EUR/t	
$p_g^a(t)$	price for grain fodder	EUR/t	
$p_a(t)$	price for animal products	EUR/t	
$p_G(t)$	price for fertilizer mix	EUR/t	
$p_x(t)$	price for other goods	EUR/t	
$w(t)$	wages	EUR/cap	
$\pi_g(t)$	profit in grain production	EUR/a	
$\pi_a(t)$	profit in animal husbandry	EUR/a	
$\pi_G(t)$	profit of trader	EUR/a	
$\pi_w(t)$	profit of WWTP	EUR/a	
$\pi_x(t)$	profit in other sectors	EUR/a	
$u(t)$	household utility	[ ]	
$P_n(t)$	"natural" phosphorus stock in the soil	[t/a/cap]	0
$P_g^c(t)$	phosphorus in the consumed grain products	[t/a/cap]	
$P_g^a(t)$	phosphorus in the grain fodder for the animals	[t/a/cap]	
$P_g^\ell(t)$	losses of phosphorus (runoff from the grain field)	[t/a/cap]	
$P_a^g(t)$	phosphorus in the animal manure applied to grain fields	[t/a/cap]	2,100
$P_a^c(t)$	phosphorus in the consumed animal products	[t/a/cap]	
$P_h(t)$	phosphorus within the waste water from the households	[t/a/cap]	600
$P_w^\ell(t)$	losses of phosphorus from the WWTP	[t/a/cap]	
$P_w^\ell(t)$	losses of phosphorus from the WWTP	[t/a/cap]	
$P_w^{maxsupply}(t)$	maximum supply of P from WWTP	[t/a/cap]	
σ	amount of overfertilization	[t/a/cap]	
ϕ_g^c	grain yield fraction for household products	[ ]	0.75
ϕ_g^a	grain yield fraction for fodder products	[ ]	0.25

Table 4.4: Parameters of the model

Symbol	Description	Unit	Value
t	time	a	
T	time horizon	a	15
L_x	labor in other sectors	cap	999,000
L	total labor	cap	1,000,000
p_m	price for mineral fertilizer	EUR/t	2,040
p_m	price for mineral fertilizer after price increase	EUR/t	3,500
γ_c	grain consumption preference in the utility function $u(t)$	[0,1]	1
α_c	meat consumption preference in the utility function $u(t)$	[0,1]	1
ϵ	environmental quality preference in the utility function $u(t)$	[0,1]	0.1
α_g	output elasticity of fertilizer in grain production	[0,1]	0.25
α_a	output elasticity of fodder in animal husbandry	[0,1]	0.25
β_g	output elasticity of labor in grain production	[0,1]	0.60
β_a	output elasticity of labor in animal husbandry	[0,1]	0.55
ϕ_g	total factor productivity for grain	[]	7
ϕ_a	total factor productivity for animal husbandry	[]	2.9
ϕ_L	labor efficiency	[]	29

Table 4.5: Parameters of the model

Symbol	Description	Unit	Value
$\bar{P}$	maximum phosphorus intake of crops	[t/a/cap]	3,150
j	phosphorus source	$\{w, m, a^g, n\}$	
χ_a	plant availability of phosphorus from manure	[0,1]	0.8
χ_n	plant availability of phosphorus from the natural stock in the soil	[0,1]	0.4
$\chi_w(A)$	plant availability of recycled phosphorus (see Table 4.1)	[0,1]	[0.6, 0.85, 0.85, 1]
χ_m	plant availability of mineral fertilizer	[0,1]	1
ψ	tons of animal product from one ton grain (inverse FCR)		0.25
ζ_a^g	P in the manure of animals as proportion of P in fodder	[0,1]	0.75
ζ_a^c	P in the animal products as proportion of P in fodder	[0,1]	0.15
ζ_{hg}	P in the households waste water as proportion of P in consumed grain	[0,1]	0.7
ζ_{ha}	P in the households waste water as proportion P in consumed animal products	[0,1]	0.7
ℓ	proportion of phosphorus run off from the field	[0,1]	0.005
δ	regeneration rate of the environment > 1		1.001
w_g^ℓ	loss fraction of grain food waste	[0,1]	0.1
w_a^ℓ	loss fraction of animal food waste	[0,1]	0.1
w_ℓ	P run-off from waste water into water bodies	[0,1]	0.1

5 Conclusions

In this final section of the thesis, overall conclusions and discussions of the findings are provided. Furthermore, future research plans connected to the topic of this thesis are presented.

5.1 Overall conclusions

This thesis introduces the economic decision framework into the emerging field of socio-hydrology. We establish three different models to analyze optimal endogenous decisions on consumption and investment, and their two-way coupled feedbacks with water systems.
This interdisciplinary work combines socio-hydrology, economics and dynamic optimization, and consequently contributes to the literature of all three research areas. Generally, the interdisciplinary modeling approach includes limitations from a perspective of every single discipline, but also gains new insights for each discipline. To combine the different approaches we had to develop new concepts to show stylized facts observed in data.

According to Sivapalan & Blöschl (2015) we investigate socio-hydrological processes. The aim is to achieve a basic understanding of the system rather than a detailed replication of a specific case study. Nevertheless, we always relate our model framework to real-world cases and obtain general policy advices. The socio-hydrological processes are based on water systems characterized by water dynamics, i.e. water levels causing flooding events, or water quality, i.e. phosphorus in fresh water bodies.

More specifically, we analyze in the first part how societies or firms optimally invest in flood protection measures, and in the second part how households' consumption and farmers' fertilizer decisions impact the environment given the possibility of recycling phosphorus from waste water.

This thesis augments economic modeling by including profound environmental feedbacks into an economic growth model, a partial and a general equilibrium model. Moreover, modeling approaches from hydrology and material flow analysis are integrated. Last, but not least, we develop new concepts to adapt standard economic assumptions in order to describe seemingly irrational behavior like e.g. overfertilization or building capital in flood risk areas.

An important additional outcome of the thesis is the extension of mathematical optimization methods and the discussion of new mathematical phenomena. In the first model in Chapter 2 we introduced a periodic term in the state variable of a two-dimensional dynamic optimization problem with an infinite time horizon. We found that the Skiba curve, which is separating the two periodic long-term solutions, shifts in time. In the second model in Chapter 3 we expanded the Impulse Control Theory by moving from one to two state variables which leads to more complex first and second order conditions of the objective function, the Impulse-Hamiltonion. For both problems we further developed the continuation algorithmn of the Matlab®-Toolbox OCMat based on Grass & Chahim (2012); Grass *et al.* (2012); Grass & Seidl (2013); Grass (2017).

In each chapter we provide analytical and numerical solutions, and a sensitivity analysis. Instead of replicating detailed findings of the diverse applications in each chapter, we want to summarize overall conclusions.
An important outcome of each chapter is the introduction of a conceptual model which describes the economic decision framework for one or more agents including environmental feedbacks. Whereas previous literature investigates the impact of human decisions on the environment or vice versa, we bring new insights by coupling the two relations. One example of such a two-way feedback is the levee effect described in Di Baldassarre *et al.* (2013) used in Chapter 2 and 3. To our knowledge we are the first who consider anthropogenic effects on water quality, i.e. eutriphocation through overfertilization, by including two-way coupled feedbacks in a general equilibrium model.
In each case we learnt that anticipation of environmental feedbacks in economic decisions changes human behavior. E.g. firms build their plants further away from flood risk areas or farmers are driven by households demand to increase the use of recycled fertilizer products.

To sustain a decent environment, i.e. reduce flood risk or avoid eutrophication of fresh water bodies, prevention or abatement measures are required. There is always a trade-off between investing into such measures, or alternatively to invest in economic production or consumption. A different investment strategy leads to a different level of consumption and, hence, to a different level of wellbeing, denoted by a utility function. The initial endowment makes a huge difference for choosing the optimal strategy. Rich economies can afford to care about environment and are more resilient against environmental disasters, whereas poor economies suffer not only from their low economic status but also from poor environmental conditions. This often leads to a poverty trap.
Since environmental conditions cannot be changed easily, only better economic standards can induce different investment strategies leading to higher production levels and,

thus, the opportunity to invest in a safe environment.
If the economically optimal strategies of firms, households or societies do not sustain a safe environment, the government can establish incentives to introduce prevention or abatement measures. We exemplify that taxes or tolls force farmers to buy recycled phosphorus fertilizer instead of imported mineral fertilizer and avoid overfertilization. Furthermore, we present that subsidies for flood protection measures increase corresponding investments in the short term. However, in all our models we provide evidence that it is better in the long term if the government provides infrastructure like dikes and levees for flood protection or recycling technologies in waste water treatment plants, instead of giving financial incentives. Support of infrastructure has always positive long term effects on both the environment and the economy.
In Chapter 3 we obtain that also sustainable planning, i.e. investing in a safe environment, leads to economic growth. In Chapter 4 we show that additional to long-term planning a different mind set can change optimal strategies towards a more environmental friendly investment and consumption behavior. This would not only improve environmental quality, but also increase household's utility, wages and profits. It is important to understand that changing (individual) preferences can already lead to different economic decisions. These decisions are still optimal for the decision maker represented by an individual, a firm, or society, but allows for prevention and abatement measures to ensure a safe environment. Such a change in preferences can be evoked by education or well communicated information.

Like any conceptual model the proposed models in this thesis cannot perfectly reflect reality. However, considering plausible assumptions the results shed light on what might actually happen if conditions resembling the scenario settings occur. Therefore we contribute to a discussion of certain dynamics and policies in the field of socio-hydrology and beyond.

5.2 Outlook on future research

Based on findings in this thesis interesting research questions arise for the future. One could apply the method of impulse control introduced in Chapter 3 to the decision framework of a social planner who represents the whole society and can include e.g. environmental quality in their objective function. Alternatively, the idea of the partial equilibrium model could be rewritten for a household maximization problem in a flood risk area to establish a micro founded macromodel. On top, the firm's and/or household's maximization problem could be integrated in the economic growth model in Chapter 2, whereas one has to bear in mind that the mathematical complexity increases dramatically.

The general equilibrium framework introduced in Chapter 4 could be used for other materials like e.g. nitrogen instead of the closely related phosphorus. Complementary, one can introduce a price for environmental quality into the general equilibrium frame-

work to derive the willingness to pay of households or farmers for abatement in form of waste water treatment and phosphorus recycling. Further future work can include a more detailed modeling of the agricultural sector by including more cost types or economic regulators. Moreover, capturing the feedbacks of households decisions and the environment over many periods by aggregating the utility and deriving optimal long term strategies is interesting work for future research.
Additionally, finding an alternative model approach to capture the phenomenon of over-fertilization can be interesting.

For each of the proposed models, future work could find an alternative way to introduce stochasticity of floods, eutrophication events or mineral fertilizer price shocks on the global market.

More general ideas are the introduction of heterogenous agents as decision makers or to supplement the findings in the thesis with empirical studies.

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Zwaneveld, P.J., & Verweij, G. 2014. Safe Dike Heights at Minimal Costs An Integer Programming Approach. *CPB Discussion Paper*, May. ISBN 978-90-5833-643-9.

Curriculum Vitae

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Personal Information
27.10.1989 in Mistelbach, Austria
Nationality: Austria
Gender: female

Education

05/2014-03/2018	**Doctoral Programme for Water Resource Systems, PhD** Vienna University of Technology (1,0) *Thesis: Introducing the economic decision framework into socio-hydrology*
02-04/2017	Research stay abroad at University of Waterloo, Canada
01-05/2016	Research stay abroad at Tilburg University, The Netherlands
10/2011-09/2013	**Technical Mathematics – Mathematical Economics, MSc** Vienna University of Technology (1,6) *Thesis: Models of Directed Technical Change*
07-12/2012	Study abroad at Queensland University of Technology, Australia
10/2008-10/2011	**Technical Mathematics – Statistics and Math. Economics, BSc** Vienna University of Technology (1,9) *Thesis: A mixed integer problem for advanced planning and scheduling*
10/2008-06/2011	**Business, Economics and Social Sciences – BSc Courses** Vienna University of Economics and Business Administration
09/2000-06/2008	**Konrad Lorenz Bundesrealgymnasium Gänserndorf** (1,1)

Summer Schools

07/2016	Short Course Economics Thessaloniki, Dynamic Programming
09/2015	Summer-School, BoKu Wien, Production Economics
08/2015	Summer-School, TU Wien, Risk and Modelling

Scientific Conference Presentations

12/2017	HydroCarpath Conference, Vienna 2017
11/2017	ÖGOR Annual Meeting, Vienna 2017, Optimization Methods, *Invited Speaker*
09/2017	Water Resource Research Conference, Waterloo/Canada 2017
06/2017	Vienna Young Scientist Symposium, Vienna 2017, Mathematics
05/2017	JpGU-AGU joint Meeting Tokyo, Socio-hydrology
04/2017	EGU Vienna 2017, Socio-hydrology,
07/2016	ISDG Urbino, Dynamic Games and Applications
06/2016	Vienna Young Scientist Symposium, Vienna 2016, Mathematics
04/2016	EGU Vienna 2016, Panta Rhei & Socio-hydrology, *Invited Speaker*
03/2016	AAG San Francisco, Socio-Hydrology
06/2015	IUGG Prague, Socio-Hydrology, Floods
05/2015	EGU Vienna, Hydrological Extremes: From Droughts to Floods
02/2015	PhD Workshop Amsterdam, Macro-Econ. Modelling of Climate Change

Publications

- Grames J., Grass D., Kort P., and Prskawetz A. (2018): Optimal investment and location decisions of a firm in a flood risk area using Impulse Control Theory. Central European Journal of Operations Research. https://doi.org/10.1007/s10100-018-0532-0
- Grames J., Brouwer R., Van Meter K., Basu N., Prskawetz A. (2017): Optimal abatement of phosphorus eutrophication within a general equilibrium framework. WRR Conference, Waterloo
- Artner G., Bogadi A., Grames J., Hahn I., Hans P., Krebs H., Rouhi T. (Hrg.): Proceedings VSS 2017 - Vienna young Scientists Symposium; Book of Abstracts, Dipl.Ing. Heinz A. Krebs, 2352 Gumpoldskirchen, 2017, ISBN: 978-3-9504017-5-2; 162 S.
- Grames, J., Prskawetz A. (2017): Introducing the economic decision framework into socio-hydrology. HydroCarpath Conference, Vienna
- Grames J., Grass D., Kort P., and Prskawetz A. (2017): Optimal investment and location decisions of a firm in a flood risk area using Impulse Control Theory. VSS-Book of Abstracts, ISBN 978-3-9504017-5-2
- Grames J., Prskawetz A., Grass D., Viglione A., Blöschl G. (2016): Modelling the interaction between flooding events and economic growth. Ecological Economics, Volume 129, p.193-209, ISSN 0921-8009, http://dx.doi.org/10.1016/j.ecolecon.2016.06.014.
- Grames J, Prskawetz A, Grass D, Viglione A, Blöschl G. (2016): Modelling the interaction between flooding events and economic growth – the damage function. Proc VSS 2016; MAT.1:92-93
- Levy M.C., Garcia M., Blair P., Chen X., Gomes S.L., Gower D.B., Grames J., Kuil L., Liu Y., Marston L., McCord P.F., Roobavannan M., Zeng R. (2016): Wicked but worth it: student perspectives on socio-hydrology. Hydrological Processes
- Grames J., Prskawetz A., Grass D., Blöschl G. (2015): Modelling the interaction between flooding events and economic growth. Proc IAHS 2015;(92):14

Scholarships / Awards

- Outstanding Student Presentation Award, JpGU-AGU 2017, Tokyo, Japan
- TUtheTOP, High Potential Programme TU Wien (2016-2017)
- Best Speaker Award, Vienna Young Scientist Symposium (2016)
- Verbund-Frauenstipendium PhD (2016)
- Winner Science Slam, BeSt Vienna (2013)
- Students4excellence, e-fellows (2012-2013)
- Lower Austrian Top-Scholarship Abroad (2012)
- Club Alpha Top-WoMentoring (2011-2012)
- European Forum Alpbach - Scholarship (2010)
- Scholarship T^2 - Talents Austria (2008)
- Speech contest: winner 2008, participation 2007, part of the jury 2006

Employment History

Professional Career

09/2013-04/2014	**StepChange Consulting**, Analyst strategy development and business performance improvement
06-08/2010	**Erste Bank**, Department ORGA IT GCIB software implementation

Teaching, Training and Coaching

10/2011-07/2013	**Talentezentrum Drosendorf, Begabtenakademie Semmering**, Teacher (rhetoric, mathematics, economics)
02-09/2013	**il-Institute for personal development**, Coach
09/2009 -	**Trainer** for teambuilding and soft-skills (more than 100 workshop days for different organizations)
05/2006-09/2015	**Private lessons** mathematics, English
09/2014 -	Private piano lessons

Internships

07/2011	**Erste Bank**, Group Large Corporates Business Development
07/2009	**Raiffeisen Bank**, Private Customers
07/2008	**Erste Bank**, Business Monitoring and Development CEE
08/2007	**Erste Bank**, ZGM Private Customers and Marketing
07/2006	**Municipal Office** - Bad Pirawarth

Student jobs

06/2008-08/2012	**TOP-wine tavern Eschberger**, Waitress

Engagement

08/2015-08/2018	Scientific Advisory Board, European Forum Alpbach
08/2015-08/2016	Forum Alpbach Network: board (conferences, communication)
09/2011 -	Club Alpbach Lower Austria: young president 06/2012-01/2017
04/2013 -	Junger Senat der Wirtschaft Österreich: board (Senate of Economy Austria)
06/2011-06/2016	Talents Austria: vice president (2011-2012), board (2011-2013), mentor (2012/13, 2014/15, 2015/16, 2016/17, 2017/18)
03/2016 -	Regional association for Judo: board (Landesverband Niederösterreich)
06/2006 -	Union Judo Club Bad Pirawarth: board (PR, events, trainings)
04/2011-11/2017	Alumni association Konrad Lorenz Gymnasium: vice president, founder committee (2010)
09/2010-06/2012	Buddy for incoming students in Vienna
09/2005-08/2009	Legitimate student council (KLG, Lower Austria)
09/2005-07/2008	Head of a "Jungschar" (training 2005)

Professional Skills

- Trainer for Communication, Personal Development (250h course, 2009-2010) major: Leadership, Project Management, Time Management
- Moderation (events, concerts, panel discussions)
- Programming Languages: C, Matlab, Maple, R, Latex
- B – driver license (2006)

Languages

- German (native)
- English (fluently)
- Latin (basic)
- Japanese (beginner)

On the side

- Piano (band Wödscheim, event music jOlia, composing, teaching)
- Judo (2.DAN, competitions, kata, instructor, referee, tournament management)
- Tennis, Beachvolleyball, Athletics, Running, Hiking