MSc Economics

Cointegration Analysis of the Monetary Model of Exchange Rate Determination

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Affidavit

I, Felipe Cotta d'Ávila e Silva

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Cointegration Analysis of the Monetary Model of Exchange Rate Determination

31 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

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Abstract

This study mostly rejects the flexible-price monetary model of exchange rate determination as a valid tool for establishing the drivers behind exchange rate movements in the case of the US Dollar, the Euro, the British Pound the Swiss Franc with respect to the Brazilian Real, in the period 1999:Q1 and 2016:Q4. The procedure applied in the analysis is Johansen maximum likelihood estimation to establish the cointegrating relations and to calculate the vector error correction model. Even though substantial evidence for cointegration between the variables is found, the restrictions implied by the model on the proportionality between variables are soundly rejected.

1 Introduction

After the collapse of the Bretton Woods system in the 1970s, exchange rates have been allowed to float freely between most of the countries in the developed world. In tandem with this fact, economists have been greatly interested in examining the underlying drivers behind exchange rate determination. Natural candidates for this analysis are quantities such as money stocks, real income, price levels and interest rates. These variables are stringed together in a framework known as the monetary model of exchange rate determination, which has been continuously developed since the 1950s. This work stands in the by now decade-old tradition of exchange rate analysis using statistical techniques to assess the validity of the monetary model.

The statistical technique adopted for analyzing common relations between times series has been developed in the last four decades, and is subsumed under the banner of "cointegration procedures". The cointegration framework allows us to gather evidence if different time series, when considered together, lead to a common path, which can be interpreted as the return to equilibrium. This tool lends itself fruitfully to the analysis of the relationship of the variables in the monetary model of exchange rate determination, since the model posits that the exchange rate is a consequence of interactions between money stocks, domestic production levels, price levels and interest rates. In other words, cointegration analysis permits us to make inference about the statistical evidence that exchange rates are in fact a result of interactions between these variables.

This work attempts to update the existing body of research in different ways. First, the period under scrutiny comprises the years since the introduction of the Euro as an accounting currency (1999-2016). Second, we analyze the model taking an emerging market, Brazil, as a reference, and expand the examination to the Euro, the Pound and the Swiss Franc, besides the usual consideration of the Dollar. As for the cointegration method itself, we employ the Johansen maximum likelihood procedure, and use mainly the trace and statistics to infer about the comovements of the series.

In the remaining sections of this work we start by providing a brief literature review of the subject at hand. Following the review, the theoretical model is presented, as well as the empirical model for the main cointegration analysis. Section 5 introduces the data, and provides basic analysis of the individual series. Section 6 provides the main part of the work, summing up the findings of the cointegration procedure. Finally, section 7 discusses and draws a conclusion on the results.

2 Literature Review

In the 1970s the Bretton Woods era came to an end, and with it the peg among several major world currencies. The flexibilization of the exchange rate market provided the impetus for the literature on exchange rate. Within the wide range of topics within this literature, the monetary model in particularly has received continued focus in the last four decades. The procedure employed for assessing the empirical support in favor of variations of the monetary approach to exchange rate has undergone rapid changes, both in its theoretical specification, as well as in the statistical methods involved. In general, it is safe to say that the evidence for the model is still not definitive, as the results have varied greatly depending on the period, the macroeconomic variables and the countries used, as well as on the statistical technique applied.

Among the first statistical results, the seminal paper by Frenkel (1979 [7]) considers a version of the monetary model which takes into account expected inflation to assess the exchange rate behavior between the German mark and the American dollar, during the German hyperinflation period (approx. 1920 to 1923). The statistical technique used is maximum likelihood estimation of a distributed lag model. Frenkel's results provide some early evidence to the validity of the monetary model, even though the technique might be considered rudimentary by the modern reader.

In 1983 Meese and Rogoff's article ([15]) challenged the positive results from the literature up to that point. In their study, they show that the model's macroeconomic variables do not perform better than a simple random walk in predicting future exchange rates. Their results dealt a serious blow to the confidence that the monetary model provided a valid description of the underlying drivers of exchange rate.

Renewed interest in the monetary approach to exchange rate came with the development of cointegration techniques in the 1980s by Robert Engle and Clive Granger (1987 [6]), and their extension to more general setups by Johansen (1988 [10]). Cointegration analysis rapidly became the standard technique for assessing the validity of the model. Some of the more recent results are reviewed below.

Rapach and Wohar (2002 [19]) examine the model in a longer data span than had been done so far, considering a period of approximately one century of data from developed countries. Taking the USA as reference, they find varying degrees of support for the monetary model for eight countries, out of their initial 14. Their results, coupled with the fact that they use a rather restrictive form of the already simple monetary model, imply that no strong support for the monetary approach can be derived from their study.

Zhang and Lowinger (2005 [22]) analyze quarterly data for the period 1973 to 1999 for four major economies (Germany, Japan, the United Stated and the United

Kingdom). They apply Johansen's procedure to test the validity of the model containing money supply, real income and interest rates. They find that whereas using the United States as reference tends to lead to a rejection of the model, substantial evidence is amassed in the exchange rate between the other three currencies (mark/pound, yen/pound and yen/mark).

Still taking the US as a reference, Islam and Hasan (2006 [9]) assess the monetary model for the dollar-yen case in the period between 1974 and 2003. In the study, they also perform a Gregory and Hansen cointegration test, along with the usual application of the Johansen methodology. The authors infer from the Gregory and Hansen cointegration that major structural breaks are absent from the data, and find evidence for long-run comovements in between macroeconomic variables and the exchange rate from the Johansen tests. In a simple RMSE comparison between the estimated error correction model and random walks with and without drift, the model outperforms the random walks in one through four quarter-ahead forecasts.

In an expanded version of the monetary model, Wilson (2009 [21]) shows that the exchange rate between a broad currency index, including 27 countries, and the US dollar (the reference) displays significant comovements when taking into account the typical monetary variables of the model, as well as fiscal variables. His results show that it might also be fruitful to consider the monetary model as a starting point. Supplementing the model with additional variables might improve the credibility of the model. However, it is hard to argue that the results provide evidence for the monetary model *per se*, as their findings support rather expanding the model. As for the expanded model, it would be useful to ground the choice of fiscal variables theoretically, instead of selecting them *ad hoc*. Another key issue of the paper, which the author does not address, is the robustness of the results when the composition of the index is changed.

Liew, Baharumshah and Puah (2009 [12]) deal with the evidence for the monetary approach to exchange rate in the case of an emerging market and a non standard reference country, namely the Baht to Yen exchange rate. In their paper, the Johansen procedure is applied to estimate trace and max. eigenvalue statistics, and likelihood ratio tests for testing the hypotheses implied by the flexible-price model. They find strong evidence in support of cointegration between the series, and their results lend support to the monetary restrictions tested. In short, their findings mean that monetary variables might substantially improve understanding of the behavior of the exchange rate in the Baht-Yen case.

Basher and Westerlund (2009 [1]) provide a short panel-based evaluation of the model for 18 OECD countries in the Post-Bretton Woods era, using the USA as reference. They show that taking into account structural breaks and cross-sectional dependence amongst countries changes the inference from no support to substantial support for the monetary model. The results imply that structural breaks are indeed a significant issue when testing the relationships of the model.

Still in the tradition of panel analysis, Cerra and Saxena (2010 [3]) consider the largest panel to date, including 98 countries, to approach the validity of the monetary model. In the same line as Basher and Westerlund (2009), special attention is given to the issue of cross-sectional dependence. They check both for cointegrating relationships between the variables and the out-of-sample forecast performance relative to a random walk, and find substantial evidence for the model in both cases.

In another long-span study in the Rapach and Wohar (2002) tradition, De Bruyn, Gupta and Stander (2013 [5]) consider the evidence for the monetary model for the exchange rate between the South African rand and the American dollar. In the paper, they use a simple version of the monetary model and perform a series of different cointegration tests, using OLS, FMOLS, dynamic OLS and the Johansen Maximum Likelihood estimators. Even though support for cointegration is found, the restrictions implied by the model are rejected. Their forecast comparison show, however, that the model might still be a useful tool in predicting exchange rate behavior, when comparing to a random walk. The evidence for the model in the South African case is dubious at best.

Bhanja, Arif and Aviral (2015 [2]) provide a framework which is closer to this work. In their piece, they analyze the exchange rate between the Indian rupee and four major currencies (dollar, yen, pound and euro) and its relationship with macroeconomic fundamentals, as implied by the flexible-price monetary model. Cointegration is found amongst the model variables when accounting for structural breaks in the data. Thus, their results lend support to the monetary model in the case of an emerging market.

For a brief overview of other important results, see Ucan, Akin and Aytun (2014: pp. 362-363 [20]). Now we turn to some results more relevant for this work, where the authors deal with the Brazilian case. Moura, Lima and Mendonça (2008 [16]) consider a panoply of different specifications of the monetary model. Their study is mostly concerned with the predictive performance of the power (compared to a random walk), and provide no significant insight to the validity of the model restrictions. With this caveat in mind, they find that the flexible-price exchange model performs worse than a random walk with drift, and that a Taylor-rule based model provides the best predicting specification. Cuiabano and Divino (2010 [4]) perform single-equation Engle Granger cointegration tests, and estimations based on a generalized method of moments framework and find some evidence for the model. Uz and Dalan (2009 [8]) consider four emerging markets in their analysis, amongst which also Brazil. When examining the countries individually, they find very scant support for the model, but substantially more evidence is gathered for the model in a panel-based approach. The results are, therefore, ambiguous.

3 Theoretical Model

This work is concerned with a simple version of the flexible-price monetary model of exchange rate determination. The specification relates the exchange rate to a set of underlying macroeconomic variables to each other, namely money supply, real income and interest rates. The derivation follows Wilson (2009 [21]) and de Bruyn, Gupta and Stander (2012 [5]) closely.The intuition behind the model is, first, that the interaction between money supply and demand in a country causes its price level. At a second moment, the model then posits that differences in the price levels will be adjusted via the exchange rate, leading to an effective equality of prices (a "no arbitrage condition"m see Rapach and Wohar 2002 [19]).

The starting point of the model is, thus, the LM-curve, which relates the real demand for money to real domestic income and interest rate.

$$\frac{M_t^d}{P_t} = L(Y_t; I_t) \tag{1}$$

where M_t denotes nominal demand for money, P_t the price level, Y_t real income and $I_t = 1 + i_t$, with interest rate i_t. In equilibrium, the nominal money demand M_t^d must be equal to the nominal money supply M. Imposing the equilibrium condition, and rearranging terms, the equation becomes:

$$P_t = \frac{M_t}{L(Y_t; I_t)} \tag{2}$$

Next, a functional form is assumed for the real money demand function:

$$L(Y_t; I_t) = \frac{aY_t^b}{I_t^c} \tag{3}$$

where a, b and c are real constants. Equation (2) is then expressed in observable quantities:

$$P_t = \frac{M_t I_t^c}{aY_t^b} \tag{4}$$

The next important assumption of the model is that purchasing power parity (PPP) holds at all times.

$$S_t = \frac{P_t}{P_t^*} \tag{5}$$

where S_t is the exchange rate, expressed as the price of one unit of the reference country's currency (Brazilian real) in terms of the home currency (e.g. dollars per real). Asteriscs denote the variables in the reference country. Thus, using equation (4) for the reference country, and assuming that the constants are equal in both cases, we can substitute terms in equation (5):

$$S_t = \frac{M_t(Y_t^*)^b I_t^c}{M_t^* Y_t^b (I_t^*)^c} \tag{6}$$

Taking logs, denoting the logarithms of the variables with lowercase letters (e.g. $\log Y_t = y_t$), and using the approximation $\log I = \log(1+i) = i$:

$$s_t = (m_t - m_t^*) - b(y_t - y_t^*) + c(i_t - i_t^*) \tag{7}$$

Equation (7) is known as the restricted form of the flexible-price model. The model derives its name from the fact that the assumption of perfect substitutability between capital and goods in both countries is also made. This implies that the relative price level $\frac{P_t}{P_t^*}$ is perfectly flexible, and no overshooting in the exchange rate is allowed (see also Frenkel 1979).

For the econometric procedures in this study, equation (7) is reformulated, allowing also for a constant and a stochastic error term u_t, assumed to be white noise:

$$s_t = \mu + \delta_m m_t + \delta_{m^*} m_t^* + \delta_y y_t + \delta_{y^*} y_t^* + \delta_i i_t + \delta_{i^*} i_t^* + u_t \tag{8}$$

In this framework, varying levels of evidence for the model might arise. In the most basic form, we should find some sort of relationship, in the form of at least one cointegrating vector between the exchange rate and the macroeconomic variables. The main part of the testing procedure will, however, be concerned with the more substantial evidence for the model, where certain forms of the long-run relationship are tested (as in Liew, Baharumshah and Puah 2009 [12]):

$\mathcal{H}_1 : \delta_m = -\delta_m^* = 1$
$\mathcal{H}_2 : \delta_y + \delta_y^* = 0$
$\mathcal{H}_3 : \delta_i + \delta_i^* = 0$

And the more stringent hypotheses:

$\mathcal{H}_4 : \mathcal{H}_1 \cap \mathcal{H}_2$
$\mathcal{H}_5 : \mathcal{H}_1 \cap \mathcal{H}_3$
$\mathcal{H}_6 : \mathcal{H}_2 \cap \mathcal{H}_3$
$\mathcal{H}_7 : \mathcal{H}_1 \cap \mathcal{H}_2 \cap \mathcal{H}_3$

4 Empirical Model

In this section the method used to gather evidence in favor (or against) the monetary model of exchange rate is described. For what follows, it is assumed that the reader is familiar with the augmented Dickey-Fuller (ADF) test, and its derivation is omitted. The model of central interest in this study is the multivariate model pioneered by Johansen (1988 [10]). In what follows the Johansen cointegration procedure will be described, as well as the restriction tests that will be used.[1]

In the last section the following relationship was derived:

$s_t = \mu + \delta_m m_t + \delta_{m^*} m_t^* + \delta_y y_t + \delta_{y^*} y_t^* + \delta_i i_t + \delta_{i^*} i_t^* + u_t$

The central point of the analysis is to determine if and how s_t is related to the explanatory variables. First, if s_t follows a stationary process, the variables do not have to be related at all, as exchange rate alone already follows a stable path. The more interesting case arises when s_t follows an integrated process. Under this result, at least one of explanatory variables also has to follow an integrated process of the same order. We are then interested in exploring if a combination between exchange rate and integrated explanatory variables could lead to a stationary process. The following definition of cointegration is used in the remainder of this work (from Pfaff 2008, p. 79 [18]):

"**Definition 4.2.** An $(K \times 1)$ vector of variables Z_t is said to be cointegrated if at least one nonzero K-element vector β_i exists such that $\beta_i' Z_t$ is trend-stationary. β_i is called a cointegrating vector. If r such linearly independent vectors β_i (for $i = 1, \ldots, r$) exist, we say that $\{Z_t\}$ is cointegrated with cointegrating rank r. We then define the $(K \times r)$ matrix of cointegrating vectors $\beta = (\beta_1, \ldots, \beta_r)$. The r elements of the vector $\beta' Z_t$ are trend-stationary, and β is called the cointegrating matrix."

In light of this definition, it is clear that if exchange rate cointegrates with some (or all) of the explanatory variables of the model, their joint movement displays a stable relationship. In other words, the monetary model can provide insights as to which macroeconomic variables help explaining exchange rate fluctuations.

We now turn to the statistical subtleties of the cointegration analysis. Consider the Vector Autoregressive (VAR) process of order p implied by our setup:

$Z_t = M + \sum_{i=1}^{p} \Pi_i Z_{t-p} + U_t, t = 1, \ldots, T$

In our case, we have:

[1] This section was based on Pfaff (2008 [18]) and Lütkepohl (2005 [13]), but can be found in virtually any recent Time Series textbook.

$$Z_t = \begin{bmatrix} s_t \\ m_t \\ m_t^* \\ y_t \\ y_t^* \\ i_t \\ i_t^* \end{bmatrix}; U_t = \begin{bmatrix} u_t \\ u_t^m \\ u_t^{m^*} \\ u_t^y \\ u_t^{y^*} \\ u_t^i \\ u_t^{i^*} \end{bmatrix}; M = \begin{bmatrix} \mu \\ \mu_m \\ \mu_m^* \\ \mu_y \\ \mu_y^* \\ \mu_i \\ \mu_i^* \end{bmatrix}$$ where the error vector ($U_t = [u_t, \ldots, u_t^{i^*}]'$) stacks the errors of the individual series, and the intercept vector M stacks the intercepts of the individual series. A typical Π_l matrix with the impact from lag l of each series on itself and the others is given by:

$$\begin{bmatrix} \delta_s^l & \delta_m^l & \delta_{m^*}^l & \delta_y^l & \delta_{y^*}^l & \delta_i^l & \delta_{i^*}^l \\ \pi_{s,m}^l & \pi_{m,m}^l & \pi_{m^*,m}^l & \pi_{y,m}^l & \pi_{y^*,m}^l & \pi_{i,m}^l & \pi_{i^*,m}^l \\ \pi_{s,m^*}^l & \pi_{m,m^*}^l & \pi_{m^*,m^*}^l & \pi_{y,m^*}^l & \pi_{y^*,m^*}^l & \pi_{i,m^*}^l & \pi_{i^*,m^*}^l \\ \pi_{s,y}^l & \pi_{m,y}^l & \pi_{m^*,y}^l & \pi_{y,y}^l & \pi_{y^*,y}^l & \pi_{i,y}^l & \pi_{i^*,y}^l \\ \pi_{s,y^*}^l & \pi_{m,y^*}^l & \pi_{m^*,y^*}^l & \pi_{y,y^*}^l & \pi_{y^*,y^*}^l & \pi_{i,y^*}^l & \pi_{i^*,y^*}^l \\ \pi_{s,i}^l & \pi_{m,i}^l & \pi_{m^*,i}^l & \pi_{y,i}^l & \pi_{y^*,i}^l & \pi_{i,i}^l & \pi_{i^*,i}^l \\ \pi_{s,i^*}^l & \pi_{m,i^*}^l & \pi_{m^*,i^*}^l & \pi_{y,i^*}^l & \pi_{y^*,i^*}^l & \pi_{i,i^*}^l & \pi_{i^*,i^*}^l \end{bmatrix}$$ where an individual π_{var_1,var_2}^l represents the impact of the l^{th} lag of variable 1 on variable 2, and δ_{var}^l represents the impact from the l^{th} lag of a given variable on the exchange rate s_t.

In order to estimate the rank of the matrix β which contains the cointegrating vectors $\beta_i (i = 1, \ldots, r)$ in its columns, the equation above is reformulated in its Vector Error Correction (VEC) form, by taking Z_{t-1} from each side, and rearranging terms:

$$\Delta Z_t = \Gamma_1 \Delta Z_{t-1} + \cdots + \Gamma_{p-1} \Delta Z_{t-p+1} + \Pi Z_{t-p} + M + U_t$$

where Δ represents the first difference, and:

$$\Gamma_i = \Pi_1 + \cdots + \Pi_i - I, i = 1, \ldots, p-1$$

$$\Pi = \Pi_1 + \cdots + \Pi_p - I$$

with $(K \times K)$ identity matrix I. Each Γ_i matrix is composed by a sum of coefficient matrices Π_i, and therefore captures the long run impact of the (lagged) variables. This form is called the *long run* form, and is particularly interesting for our analysis, since the flexible-price model is mainly concerned with long-run behavior.

The term ΠZ_{t-p} is called the (long-run) error correction term, as it is responsible for correcting the system back to its long-run equilibrium path, whenever it deviates. Given that every individual time series is at most $I(1)$, all differences of the vector Z_t are stationary, as well as the intercept M and the error vector U_t (by construction). Thus, it follows that the error-correction term must be stationary, since otherwise the differenced term ΔZ_t on the left hand side would not be stationary, which contradicts what was just expounded. If the term PiZ_{t-p} is stationary, the matrix Pi must induce linear combinations of the nonstationary variables in Z_t which are in turn stationary. It is thus clear that the properties of the matrix Π are of central interest in the cointegration analysis at hand.

The $K \times K$ matrix Π can be divided in three categories with respect to its rank (let $rank(\Pi) = r$):

1. $r = 0$
2. $r = K$
3. $0 < r < K$

In the first case, the matrix must cointaing only zeros, which implies that the only vector capable of rendering the vector Z_{t-p} stationary is a vector of zeros. Thus, the variables do not cointegrate, and the monetary model is incapable of adding explanatory power to the behavior of the exchange rate. In the second case, there is also no cointegration, since all variables are individually stationary for a full rank matrix.

We are particularly interested in the case where $0 < r < K$. In this case, matrix Π can be rewritten as $\Pi = \alpha\beta'$, where both α and β are $(K \times r)$ matrices. The α matrix is called the adjustment matrix, and its entries indicate how fast the long-run equilibrium is reached. The matrix β contains the cointegrating vectors in its columns, as in definition 4.2.

The eigenvalues necessary for the Johansen tests are not calculated from the Π matrix itself, but on a transformation of the series based on deriving the canonical covariation of the series under scrutiny. The transformations ensure that the eigenvalues are nonnegative real numbers ($\lambda \in \mathbb{R}_+$). It must also be noted that, in general, the α and β matrices are not well defined, since we could have some invertible matrix $\Lambda \in \mathbb{R}^{r \times r}$ such that $\alpha\Lambda\Lambda^{-1}\beta = \Pi$. This clearly means that there are different matrices that could be used (e.g. $\tilde{\alpha} = \alpha\Lambda$ and $\tilde{\beta} = \beta(\Lambda^{-1})')$. Besides ensuring positive eigenvalues, the procedure also pins down how the two matrices should be estimated. Assuming Gaussian error terms in U_t we estimate eigenvalues and α, β by maximum likelihood. [2]

In this work two tests as proposed by Johansen for the estimation of the number of the cointegrating vectors are examined: the trace and the maximum eigenvalue test. Both tests require the estimated eigenvalues to be ranked from $\hat{\lambda}_1$, the highest, to $\hat{\lambda}_K$,

[2] The full details can be found at Johansen 1995, in particular chapter 6; also Lütkepohl 2005, chapter 7 for a textbook treatment.

the smallest:
$\hat{\lambda}_1 > \hat{\lambda}_2 > \cdots > \hat{\lambda}_K$

The trace test assesses the null hypothesis that the rank of the Π matrix is less or equal than a predetermined r_0, against the hypothesis that it is strictly greater than r_0 (but equal or less than K, evidently). The procedure involves sequentially testing the hypothesis starting at $r \leq r_0 = 0$, and ending in $r \leq r_0 = K - 1$ (which gives the alternative that $r = K$). Succinctly, at the l^{th} step, for $l = 0, \ldots, K - 1$:
$\mathcal{H}_0 : rank(\Pi) \leq l$
$\mathcal{H}_1 : rank(\Pi) > l$

The test is based on a likelihood ratio test statistic given by:
$\mathcal{J}_{trace} = -T \sum_{j=r_0+1}^{K} \log(1 - \hat{\lambda}_j)$

Where $\hat{\lambda}_j$ stands for the j^{th} largest estimated eigenvalue. The procedure means, for instance, that failing to reject in the first step (where $r_0 = rank(\Pi) = 0$) implies no cointegration; if the null is rejected in the first step ($rank(\Pi) = 0$, but not in the second step, then our matrix has rank one; and so on. The procedure stops whenever the null cannot be rejected, for we then have evidence that we found the rank of the Π matrix.

The test derives its name from the fact that, asymptotically (after Lütkepohl 2005 [13], p. 332-333):
$\mathcal{J}_{trace} \xrightarrow{d} trace(\mathcal{D})$
where, in our case, we will allow for a constant in the cointegrating relationship, which implies:
$\mathcal{D} = A'B^{-1}A$
With:

$$A = \int_0^1 \begin{bmatrix} W_{K-r_0}(s)' \\ 1 \end{bmatrix} dW_{K-r_0}(s)'$$

$$B = \int_0^1 \begin{bmatrix} W_{K-r_0}(s) \\ 1 \end{bmatrix} \begin{bmatrix} W_{K-r_0}(s)' \\ 1 \end{bmatrix}' ds$$

where (e.g.) $W_{K-r_0}(s)$ means a $(K - r_0)$-dimensional standard Wiener process.

The maximum eigenvalue test is similar to the trace test, with the difference that only two alternative cointegrating dimensions are tested against each other. Again, the procedure is realized sequentially, starting with the comparison between no cointegration ($r_0 = 0$) and one cointegrating vector ($r_0 = 1$). At the l^{th} step ($l \leq K - 1$):

$\mathcal{H}_0 : rank(\Pi) = l$

$\mathcal{H}_1 : rank(\Pi) = l + 1$

The test statistic in this case is given by:

$$\mathcal{J}_{LR}(j, j+1) = -T(1 - \hat{\lambda}_{j+1})$$

where we are in fact comparing likelihoods of the j^{th} and the $(j+1)^{th}$ eigenvalues. The test statistic is once more distributed in a nonstandard manner:

$$\mathcal{J}_{LR}(j, j+1) \xrightarrow{d} \lambda_{max}(\mathcal{D})$$

where $\lambda_{max}(\mathcal{D})$ is the maximum eigenvalue estimated from the $\mathcal{D}$ matrix above.

In light of the trace and maximal eigenvalue tests, inferences can be made about the rank of the matrix Π. The next step is then to estimate the VEC model, α and β in particular. A full derivation of the estimators is beyond the scope of this work. A brief exposition of the estimators is presented in Pfaff (2008 [18], pp. 78-82). Instead, the procedure for testing restrictions in the parameters is briefly described (a detailed discussion can be found in Johansen 1995 [11], chapter 13).

After obtaining the estimates of the unrestricted model, some restrictions on the cointegrating relationship are imposed, and the likelihood of both models is compared. Restrictions on the cointegrating relationships are expressed as restrictions on the β matrix. Since we would like the restrictions to hold in any possible cointegrating relation, we impose the restrictions on all vectors in the β matrix (see Pfaff 2008 [18], chapter 8). Thus, given a free parameter vector $\psi \in \mathbb{R}^{s \times r}$, we construct a matrix $R \in \mathbb{R}^{K \times s}$ with the restrictions to be imposed, such that $\beta = R\psi$ and s determines how many of the variables are being restricted ($r \leq s \leq K$). Thus, we would like to test whether the proportionalities between the explanatory variables hold empirically:

$\mathcal{H}_1 : \delta_m = -\delta^*_m$

$\mathcal{H}_2 : \delta_y = -\delta^*_y$

$\mathcal{H}_3 : \delta_i = -\delta^*_i$

$\mathcal{H}_4 : \mathcal{H}_1 \cap \mathcal{H}_2$

$\mathcal{H}_5 : \mathcal{H}_1 \cap \mathcal{H}_3$

$\mathcal{H}_6 : \mathcal{H}_2 \cap \mathcal{H}_3$

$\mathcal{H}_7 : \mathcal{H}_1 \cap \mathcal{H}_2 \cap \mathcal{H}_3$

And the corresponding restriction matrices, accounting for a constant in the cointegration, are:

$$R_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; R_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$R_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; R_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$R_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; R_6 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$R_7 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

As mentioned, a likelihood-ratio test is then constructed, which is asymptotically χ^2 distributed, with $r(K-s)$ degrees of freedom. This concludes the elucidation of the empirical methodology. In the next section, the data will be presented and pre-analyzed.

5 The Data

5.1 General Presentation

In this study we are concerned with data from Brazil, the United States, the Euro-zone (EU), the United Kingdom (UK) and Switzerland, in the period between the first quarter of 1999 (1999:Q1) and the last quarter of 2016 (2016:Q4). This period was chosen due to the facts that (1) the Euro was effectively launched as a unit of account in 1999, and (2) Brazil relinquished any form of peg to the Dollar in that year, and adopted a free-floating regime. The data was obtained from freely available sources in order to ensure the reproduceability of the results. The variables used in this study were:

- Nominal exchange rate using Brazilian real as a reference (i.e how many Dollars, Euros, Pounds or Francs one Real costs);
- Money supply M1;
- Real income measured by real gross domestic product (GDP)
- Nominal interest rates, proxied by the leading rates in these countries and region[3]

Monthly data was obtained whenever available[4], and individual 3-month period averages were taken to compose quarterly data. A simple seasonal adjustment was applied to the M1 and GDP series, using the following specification[5] [6]:
$\mathcal{F}(x_t) = \sum_{i=0}^{3} \frac{x_{t-i}}{4}$

With the exception of the nominal interest rate series, all series were logarithmized. The nominal interest series include values very close to, or smaller than, zero, in particular for the developed countries. Therefore, they were used in the original form. The Brazilian interest rate was high enough so that the dicrepancies between $\log(1+i)$ and i were at times more than one full percentage point. Thus, Brazilian nominal interest rate was also used in logs.

A visualization of the data is provided in figure 1. The plots display the four log-normalized series (exchange rate, money supply, real income and $1+i$ for the interest

[3]That means: the effective federal funds rate for the US, the 3-month EURIBOR for the EU, the 3-month LIBOR CHF for Switzerland, the official bank rate for the UK and the SELIC rate for Brazil.

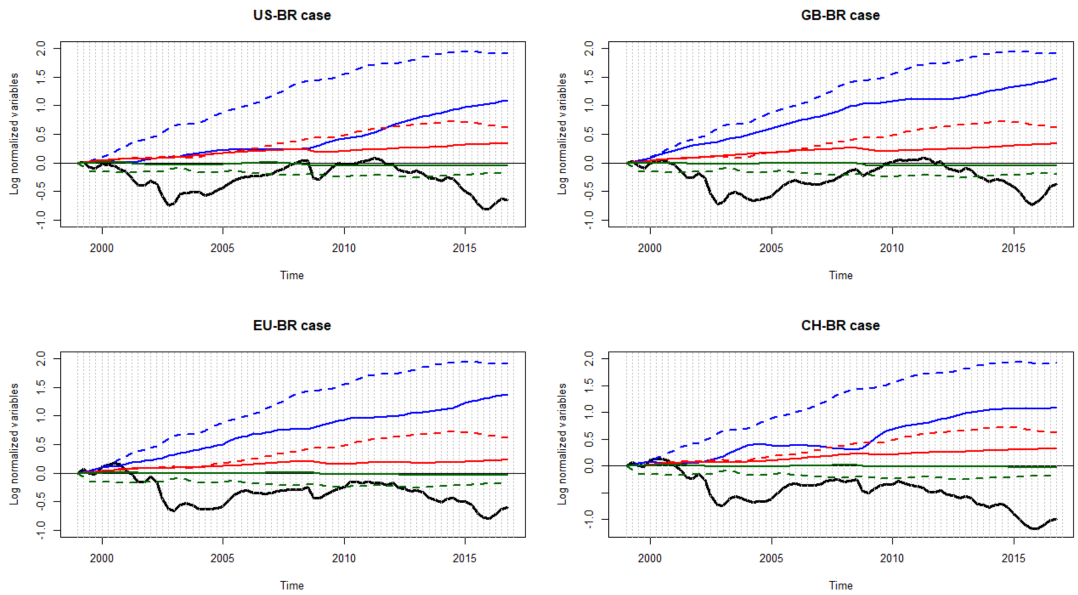
[4]Which was the case for all data sets with the exception of US, EU, UK and Swiss GDPs, which were available only as quarterly data.

[5]With the exception for the three first values for the Euro, since the series starts in 1999. For these values, the average was taken only with respect to the existing previous values, meaning that the first value is used as originally reported.

[6]It is important to note that the Swiss Franc has not been under a free-floating regime for the whole period. The Franc was only allowed to float freely against the Euro after january 2015.

rate) for each country, with the Brazilian variables shown in dashed lines as reference in all cases.

Figure 1: Log Normalized Variables



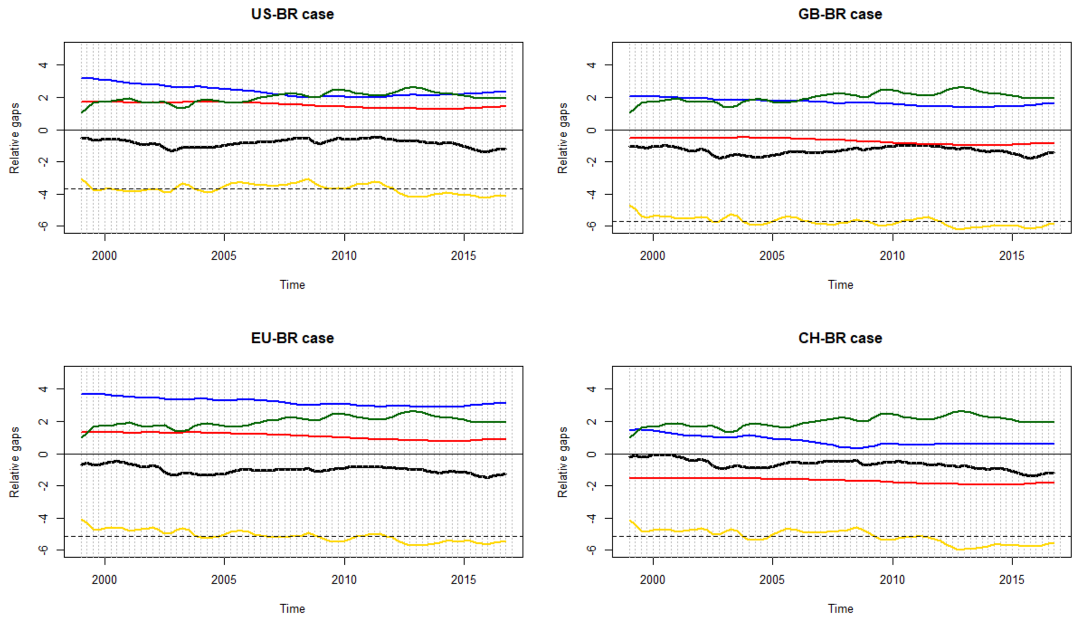
US represents the United Stated, GB Great Britain, EU the Euro-Zone, CH Switzerland, BR Brazil. The black line represents the log-normalized exchange rate, blue lines represent log-normalized M1, red lines represent log-normalized real GDP, dark green lines represent one plus interest rate. Dashed lines represent the variables for Brazil.

We can see from the graphs that in all cases the Brazilian currency lost value since 1999. We can also see that whereas Brazilian money supply and GDP grew more than other countries, the interest rate also decreased consistently more in Brazil. The graphs appear to suggest a trending behavior in money supplies and real income. In the next section statistical tests will be performed to confirm or reject these suggestions.

When the relative gap between home (e.g. American) and reference (Brazilian) money supply increases (decreases), the theoretical model suggests that the real income and interest rate differentials will react in a proportionate manner, that is the gap between real incomes should increase (decrease) as well, or the gap between interest rates should decrease (increase), or both. If the differentials do not balance out each other, the exchange rate is expected to depreciate (appreciate). Figure 2 provides better intuition to the differentials.

The "residual" of the theoretical model, $s_t - (m_t - m_t^*) - (y_t - y_t^*) + (i_t - i_t^*)$, when the parameters in the original model are $b = c = 1$ (see equation (7)) is also displayed in figure 2 as a golden line. This "residual" provides some graphical cues as to whether we can expect the variables to be cointegrated or not. A stationary "residual" series would imply that the variables are cointegrated, and that the model has

Figure 2: Differentials of Log Variables



US represents the United Stated, GB Great Britain, EU the Euro-Zone, CH Switzerland, BR Brazil. The black line represents the log exchange rate, the blue lines represent log differential of M1, the log differential of real GDP is in red, the dark green line represents the differential between interest rates. The golden line represents $s_t - (m_t - m_t^*) + (y_t - y_t^*) - (i_t - i_t^*)$, the "residual" implied by the model in the case where $b = c = 1$. A dashed black line represents the mean of the "residual" series.

some validation. The graph suggests that stationarity might hold. More importantly, we see little evidence of a particular trend in the joint relation. Intuitively, this suggests that there is no trend in the cointegrating relationship. We rely on this suggestion, and on previous studies (see Wilson 2009 [21]), and do not allow for a trend in the cointegrating relations.

The next stage is to analyze the behavior of the individual series, as a first step towards a more systematic consideration of the comovements.

5.2 Stationarity Analysis

As mentioned before, it is important to know if the individual time series at hand are stationary or not. For this purpose, a sequence of augmented Dickey-Fuller (ADF) tests will be performed, as well as Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests.[7]

The general formulation for the ADF test is given by:

$\Delta y_t = \nu_0 + \nu_1 t + \nu_2 y_{t-1} + \sum_{i=1}^{k} \zeta_i \Delta y_{t-i} + w_t$

[7] The results in this study were obtained using the packages "urca" (authored by Bernhard Pfaff, Eric Zivot and Matthieu Stigler), "vars" (by Bernhard Pfaff and Metthieu Stigler), "tsDyn" (Antonio Fabio di Narzo, Jose Luis Aznarte and Matthieu Stigler) and "tseries" (Adrian Trapletti, Kurt Hornik and Blake LeBaron) in the software R.

And that of the KPSS test is:
$y_t = \xi D_t + x_t + w_t$ where D contain model deterministics (a constant, and a trend, if it is the case) and x_t is a pure random walk. While the null hypothesis of the ADF is the presence of a unit root (that is, $\nu_2 = 0$), the null of the KPSS is stationarity (which means that the variance of the error of the random walk x_t is not zero; if the series x_t has no variance, it is a constant, and the series is thus [trend] stationary). Thus, it might be fruitful to regard the same problem of stationarity from these different perspectives.

The truncation lag k in the ADF test was determined by sequentially testing the coefficients of the lagged terms. We first determine the maximal lag value $\hat{k}$, and estimate the ADF equation above using OLS. We then test the coefficient of the highest lagged term $\delta_{\hat{k}}$. If it is significant at 5% significance level, we proceed to the ADF-test on the equation with $\hat{k}$ lags in the truncation. If the coefficient is not significant, we reduce one lag, and repeat the procedure. The procedure is repeated until the first significant coefficient, or the zero lower bound on the lag order is reached. This procedure is described in more detail in Ng/Perron (1995 [17]). The presence of a trend component is tested by verifying the joint hypothesis that $\nu_1 = \nu_2 = 0$, given that whenever $\nu_2 = 0$ cannot be rejected. If the F-test-type restriction cannot be rejected, we proceed to the same procedure with a constant. If again the null of joint zero coefficients is not rejected, then the model is a random walk. The deterministics for these cases are denoted "Trend", "Drift" and "None", for the respective cases of a trend with a constant, only a constant, and neither.

The maximal lag order $\hat{k}$ used for the procedure described above, as well as for the lags entering the KPSS test, is determined by Schwert's rule of thumb:
$\hat{k} = [4(\frac{T}{100})^{\frac{1}{4}}]$
where T is the length of the original series $\{y_t\}_{t=1}^{T}$, and the square brackets [] signifies rounding up the result to the closest integer.

Table 1 reports the results from the procedure. Each series is coded in the form $a.bb$, where a stands for which variables (s means log exchange rate, m means log M1, y log real GDP and i the nominal interest rate, in logs only for Brazil), and bb stands for the country (us stands for the US, gb for the United Kingdom, eu for the Euro-Zone and br for Brazil). The table reports the lag which minimizes AIC in the recursive procedure above, the maximal lag found with the recursive procedure[8], which deterministics were included in the model, the ADF test statistic for number of lags reported in the "Lag max" column and the result of the KPSS test. Significance at 5% is denoted with an asterisk. Whenever a variable is found to be nonstationary, the procedure is repeated for the lagged series, and the results are reported in parenthesis. The last column

[8]Tests at 5% significance.

identifies the order of integration implied for each series. [9]

As we cansee from table 1, the ADF and KPSS tests do not always agree. In the level case, the tests disagree with respect to the variables $s.us, i.us, i.eu, s.gb, i.ch$ and in the differenced case with respect to $m.gb, y.gb, m.br, y.br$. In these cases, if both tests fail to reject the null, we expand the tolerance to 10%. If one of the tests yield, then the decision suggested by the other is accepted (e.g. if both the ADF and the KPSS fail to reject at 5%, but the ADF rejects at 10%, then we accept the decision of the KPSS test). If both of them reject the null at 5%, the tolerance is reduced to 1%, and a similar procedure as the previous one is enacted. If the ambiguity cannot be resolved in this manner, the ADF test is unilaterally preferred to the KPSS.

The order of integration in the table is the final result of the whole procedure. It shows that no variable is considered stationary in levels, whereas three are considered $I(2)$: Brazilian GDP, British M1 and American M1. The result regarding Brazilian GDP alone deals already a serious blow to our hypothesis of joint movements between the variables, since an $I(2)$ variable at the right hand side of equation (8) implies that the exchange rate cannot be $I(1)$. This means that Brazilian real income cannot enter the relationship. We note, however, that the ADF test in the differenced $y.br$ series with the truncation lag suggested by AIC would not have a unit root (test statistic -2.12). In this case, Brazilian GDP would be $I(1)$, and we could still make further assessments about the flexible-price monetary model. Thus, in light of this information, and for the sake of the argument, we consider $y.br \sim I(1)$.[10]

Given the $I(2)$ behavior of American and British M1 expounded above, it is clear that the flexible-price monetary model for the Real-Dollar and Real-Pound couples has, at best, limited explanatory potential. In the remaining sections, these two variables will be removed from the equation. The equation for these cases, then, becomes:

$$s_t = \mu + \delta_{m^*} m_t^* + \delta_y y_t + \delta_{y^*} y_t^* + \delta_i i_t + \delta_{i^*} i_t^* + u_t \tag{9}$$

Also the dimensions of all vectors for the Johansen procedure is reduced by one element. The restrictions to be tested also changed, to the extent that the matrices lose dimensions, and the hypotheses $\mathcal{H}_1$,$\mathcal{H}_4$,$\mathcal{H}_5$,$\mathcal{H}_7$ and the corresponding restrictions R_1, R_4, R_5 and R_7 become obsolete. The remaining matrices are given as follows:

[9] A Ljung-Box test for serial autocorrelation was also performed on the residuals of the model selected for the ADF test. No serial correlation was found in any residual.

[10] No similar arguments can be made for $m.gb$ and $m.us$, which are definitely considered to be $I(2)$.

Table 1: Results for Stationarity Analysis–Univariate Case

	AIC Lag	Lag max	Deterministics	ADF Test	ADF Dif.	KPSS Test	KPSS Dif.	Order
$s.us$	2(1)	2(1)	None	0.46	-6.25*	0.30	0.14	I(1)
$m.us$	2(1)	2(4)	None	2.52	-0.51	2.32*	0.70*	I(2)
$y.us$	1(1)	2(1)	None	2.55	-3.88*	2.33*	0.27	I(1)
$i.us$	1(1)	1(0)	None	-1.99*	-3.50*	1.99*	0.12	I(1)
$s.eu$	1(1)	1(1)	None	0.22	-5.78*	0.96*	0.09	I(1)
$m.eu$	2(1)	3(4)	Drift	-0.61	-3.76*	1.88*	0.13	I(1)
$y.eu$	2(1)	2(1)	None	1.86	-3.76*	2.13*	0.53*	I(1)
$i.eu$	1(1)	1(0)	Drift	-3.55*	-4.55*	2.39*	0.15	I(1)
$s.gb$	1(1)	1(1)	None	-0.06	-5.78*	0.36	0.12	I(1)
$m.gb$	1(1)	1(4)	None	1.98	-0.92	3.60*	0.40	I(2)
$y.gb$	1(1)	1(4)	None	1.55	-1.83	3.20*	1.05*	I(1)
$i.gb$	1(1)	1(0)	None	-1.32	-4.27*	0.98*	0.13	I(1)
$s.ch$	1(1)	1(1)	None	0.27	-5.97*	1.94*	0.09	I(1)
$m.ch$	2(1)	2(1)	None	2.26	-4.10*	2.36*	0.15	I(1)
$y.ch$	2(1)	2(1)	Drift	-1.32	-4.23*	2.46*	0.20	I(1)
$i.ch$	1(1)	3(3)	Drift	-3.62*	-3.54*	1.84*	0.08	I(1)
$m.br$	2(2)	2(1)	Trend	1.42	-6.15*	0.54*	0.17*	I(1)
$y.br$	1(1)	1(4)	None	1.09	-0.93	3.64*	0.37	**I(1)**
$i.br$	3(2)	2(0)	None	0.10	-4.39*	1.64*	0.46	I(1)

AIC Lag denotes the number of lags in the ADF which minimizes AIC (result for differenced series in brackets); Lag max denotes the number of lags found by sequentially testing significance on coefficients of lagged variables; deterministics means how the univariate series was modelled; ADF and KPSS Test display the test statistics result from the respective tests; ADF and KPSS Dif. show the test statistics for the differenced series; Order shows the final decision about the order of integration of each individual variable. Asterisks denote significance at 5% level. Even though the tests show Brazilian GDP as $I(2)$ considering the truncation lag procedure, we consider it as $I(1)$ via AIC criterion.

$$R'_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; R'_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$R'_6 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

6 Cointegration Analysis

6.1 Johansen Cointegration Procedure

Once we have established which variables cannot be taken into the equation, and that no series is stationary in levels, we proceed to the question whether the variables show significant signs of comovements or not. The first step is to determine the lag order of the VAR equation:

$Z_t = M_0 + \sum_{i=1}^{p} \Pi_i Z_{t-p} + U_t$

For model selection, we consider the Hannan-Quinn (HQ) criterion for up to four lags (by Schwert's rule), as it has better asymptotic properties than the AIC criterion (see Lütkepohl 2005 [13], chapter 7). By this method, we select 2 lags in all cases.[11]

The next step in the analysis is to determine the rank of the Π matrix, that is, the dimension of the cointegrating space. In order to do that, we rely on the trace and maximal eigenvalue statistics explained before. Tables 2 to 5 report the results of the tests for each system (tables reported following Islam and Hasan 2006 [9], p. 137).[12]

Table 2: Johansen Trace and Max. Eigenvalue Statistics – Dollar-Real case

Dollar-Real Case		Test Statistics		5% Critical Values	
$\mathcal{H}_I$	$\mathcal{H}_\mathcal{A}$	Max. Eigenvalue	Trace	Max Eigenvalue	Trace
$r = 0$	$r > 1$	65.75	183.19	40.30	111.01
$r = 1$	$r > 1$	47.15	117.45	34.40	84.45
$r = 2$	$r > 2$	29.63	70.29	28.14	60.16
$r = 3$	$r > 3$	23.62	40.66	22.00	41.07
$r = 4$	$r > 4$	9.42	17.04	15.67	24.60
$r = 5$	$r = 6$	7.63	7.63	9.24	12.97

Eigenvalues: $0.61; 0.49; 0.35; 0.29; 0.13; 0.10; 0.00$.

[11] AIC would have resulted in lag order 4 for all models, with the exception of Great Britain, for which it coincides with the HQ criterion. We also note that tests for serial correlation fail to reject the null that the residuals are uncorrelated, for the VEC model using 2 lags in each case.

[12] The null hypothesis is formulated with a slight abuse of notation. Usually, we would have written $\leq$. We are implying that the recursive selection method is already being implemented.

Table 3: Johansen Trace and Max. Eigenvalue Statistics – Euro-Real case

Euro-Real Case		Test Statistics		5% Critical Values	
$\mathcal{H}_I$	$\mathcal{H}_\mathcal{A}$	Max. Eigenvalue	Trace	Max Eigenvalue	Trace
$r = 0$	$r > 1$	67.97	233.92	46.45	131.70
$r = 1$	$r > 1$	56.95	165.95	40.30	102.14
$r = 2$	$r > 2$	44.32	109.00	34.40	76.07
$r = 3$	$r > 3$	37.35	64.69	28.14	53.12
$r = 4$	$r > 4$	14.90	27.33	22.00	34.91
$r = 5$	$r > 5$	7.58	12.43	15.67	19.96
$r = 6$	$r = 7$	4.85	4.82	9.24	9.24

Eigenvalues: $0.62; 0.56; 0.47; 0.41; 0.19; 0.10; 0.07; 0.00$.

Table 4: Johansen Trace and Max. Eigenvalue Statistics – Pound-Real case

Pound-Real Case		Test Statistics		5% Critical Values	
$\mathcal{H}_I$	$\mathcal{H}_\mathcal{A}$	Max. Eigenvalue	Trace	Max Eigenvalue	Trace
$r = 0$	$r > 1$	64.32	177.71	40.30	102.14
$r = 1$	$r > 1$	50.70	113.38	34.40	76.07
$r = 2$	$r > 2$	25.70	62.68	28.14	53.12
$r = 3$	$r > 3$	20.63	36.98	22.00	34.91
$r = 4$	$r > 4$	8.74	16.35	15.67	19.96
$r = 5$	$r = 6$	7.60	7.60	9.24	9.24

Eigenvalues: $0.60; 0.52; 0.31; 0.26; 0.12; 0.10; 0.00$.

Table 5: Johansen Trace and Max. Eigenvalue Statistics – Franc-Real case

Franc-Real Case		Test Statistics		5% Critical Values	
$\mathcal{H}_I$	$\mathcal{H}_\mathcal{A}$	Max. Eigenvalue	Trace	Max Eigenvalue	Trace
$r = 0$	$r > 1$	79.94	224.88	46.45	131.70
$r = 1$	$r > 1$	54.34	144.94	40.30	102.14
$r = 2$	$r > 2$	28.03	90.60	34.40	76.07
$r = 3$	$r > 3$	25.49	62.57	28.14	53.12
$r = 4$	$r > 4$	19.43	37.08	22.00	34.91
$r = 5$	$r > 5$	10.90	17.66	15.67	19.96
$r = 6$	$r = 7$	6.76	6.76	9.24	9.24

Eigenvalues: $0.68; 0.54; 0.33; 0.31; 0.24; 0.14; 0.09; 0.00$.

Table 6: Number of cointegrating relations

	Trace	Max. Eigenvalue
US-BR	3	4
EU-BR	4	4
GB-BR	4	2
CH-BR	5	2

Table 6 summarizes the number of cointegrating relations (r) inferred in each case. Clearly, the variables display significant comovements. This result is paramount to saying that the flexible-price monetary model has indeed some validity as a first approximation to the behavior of the exchange rate for the countries under examination (in the period studied).

The results for the estimated adjustment matrices α are reported in the tables 7 and 8, the cointegrating vectors β in tables 9 and 10, and the error correction model for the exchange rate equation is summarized for each currency pair in table 11. From table 11 we notice that only the first error correction term enters the equation significantly for all currency pairs.[13] In all these cases, the coefficient is less than one, and it enters the equation with a negative sign. These results are consistent with the theory (see Pfaff 2008 [18], chapter 8). If the coefficients were greater than one, the error correction terms would have an explosive behavior, which is inconsistent with stationarity. In the case where the error correction term resulted in a positive number, the implication would be that a positive shock to equilibrium is responded by an even larger deviation, and not a return to the steady state.

[13] With one exception, where the third error correction term also has a significant coefficient, in the Euro-Real case.

Table 7: Adjustment Matrices for the US-BR and EU-BR cases

US-BR	ect_1	ect_2	ect_3	**EU-BR**	ect_1	ect_2	ect_3	ect_4
$s.us$	-0.84	0.64	-3.19	$s.eu$	-0.74	1.45	-0.90	3.65
$m.us$	-	-	-	$m.eu$	0.01	-0.37	0.12	0.00
$m.br$	-0.02	-0.01	0.24	$m.br$	0.01	0.11	-0.07	0.06
$y.us$	0.00	0.09	-0.38	$y.eu$	0.00	0.06	-0.01	-0.18
$y.br$	0.04	0.04	-0.01	$y.br$	0.04	-0.10	0.04	-0.26
$i.us$	-0.01	-0.04	0.17	$i.eu$	0.01	0.01	-0.03	0.39
$i.br$	-0.03	-0.19	2.40	$i.br$	-0.29	-1.42	0.25	0.83

$a.bb$ in the columns stands for a, the (log) variable (exhange rate s, money supply m, real income y and interest rate i), and bb, the country (the United States us, the Euro Zone eu, Great Britain gb, Switzerland ch and Brazil br). ect_i stands for Error Correction Term i, where $1 \leq i \leq r$ (r is the cointegration rank). These are the adjustment coefficients for each equation.

Table 8: Adjustment Matrices for the GB-BR and CH-BR cases

GB-BR	ect_1	ect_2	ect_3	ect_4	**CH-BR**	ect_1	ect_2	ect_3	ect_4	ect_5
$s.gb$	-0.65	0.11	-3.93	0.17	$s.ch$	-0.91	0.45	0.64	-0.59	0.16
$m.gb$	-	-	-	-	$m.ch$	-0.01	-0.29	-0.07	-1.24	0.43
$m.br$	0.00	-0.05	0.24	-0.03	$m.br$	0.02	0.01	-0.11	0.72	-0.07
$y.gb$	0.00	0.03	-0.31	-0.10	$y.ch$	0.00	-0.01	0.03	-0.28	-0.03
$y.br$	0.03	0.02	-0.18	-0.20	$y.br$	0.04	0.10	0.06	0.37	-0.28
$i.ch$	0.00	-0.05	0.26	0.03	$i.ch$	0.00	0.04	0.03	0.49	-0.10
$i.br$	-0.19	-0.08	0.06	0.09	$i.br$	-0.26	-1.47	0.26	-8.30	1.03

$a.bb$ in the columns stands for a, the (log) variable (exhange rate s, money supply m, real income y and interest rate i), and bb, the country (the United States us, the Euro Zone eu, Great Britain gb, Switzerland ch and Brazil br). ect_i stands for Error Correction Term i, where $1 \leq i \leq r$ (r is the cointegration rank). These are the adjustment coefficients for each equation.

Table 9: Cointegrating Vectors for the US-BR and EU-BR cases

US-BR	β_1	β_2	β_3	EU-BR	β_1	β_2	β_3	β_4
$s.us$	1.00	0.00	0.00	$s.eu$	1.00	0.00	0.00	0.00
$m.us$	-	-	-	$m.eu$	0.00	1.00	0.00	0.00
$m.br$	0.00	1.00	0.00	$m.br$	0.00	0.00	1.00	0.00
$y.us$	0.00	0.00	1.00	$y.eu$	0.00	0.00	0.00	1.00
$y.br$	-1.82	-3.23	-0.37	$y.br$	-2.55	1.66	3.06	0.37
$i.us$	-3.63	3.00	-0.60	$i.eu$	-9.44	3.83	12.17	-1.10
$i.br$	0.26	-1.25	-0.27	$i.br$	-0.23	0.84	2.25	0.14
μ	0.02	-0.01	0.00	$i.br$	0.02	-0.03	-0.03	-0.01

$a.bb$ in the columns stands for a, the (log) variable (exhange rate s, money supply m, real income y and interest rate i), and bb, the country (the United States us, the Euro Zone eu, Great Britain gb, Switzerland ch and Brazil br). μ stands for the constant in the cointegrating relation. Entries are the coefficients of the proportionalities between variables. Vectors are normalized as in Johansen (1995 [11]).

Table 10: Cointegrating Vectors for the GB-BR and CH-BR cases

GB-BR	β_1	β_2	β_4	β_4	CH-BR	β_1	β_2	β_3	β_4	β_5
$s.us$	1.00	0.00	0.00	0.00	$s.eu$	1.00	0.00	0.00	0.00	0.00
$m.us$	-	-	-	-	$m.eu$	0.00	1.00	0.00	0.00	0.00
$m.br$	0.00	1.00	0.00	0.00	$m.br$	0.00	0.00	1.00	0.00	0.00
$y.us$	0.00	0.00	1.00	0.00	$y.eu$	0.00	0.00	0.00	1.00	0.00
$y.br$	0.00	0.00	0.00	1.00	$y.br$	0.00	0.00	0.00	0.00	1.00
$i.us$	-42.67	48.62	7.36	-9.83	$i.eu$	-1.08	15.59	-1.78	-1.67	4.90
$i.br$	3.27	-2.07	-0.53	0.97	$i.br$	0.83	0.63	0.47	0.0.2	0.59
μ	-0.01	0.00	0.00	-0.01	$i.br$	0.02	0.00	-0.02	0.00	0.00

$a.bb$ in the columns stands for a, the (log) variable (exhange rate s, money supply m, real income y and interest rate i), and bb, the country (the United States us, the Euro Zone eu, Great Britain gb, Switzerland ch and Brazil br). μ stands for the constant in the cointegrating relation. Entries are the coefficients of the proportionalities between variables. Vectors are normalized as in Johansen (1995 [11]).

Table 11: VEC Estimations for Exchange Rate Equation

	γ_s	γ_m	γ_{m^*}	γ_y	γ_{y^*}	γ_i	γ_{i^*}	ect_1	ect_2	ect_3	ect_4	ect_5
US-BR	0.36	-	-1.25	3.50	-1.94	-9.68	0.07	-0.84	0.64	-3.19	-	-
	(0.13)*		(1.26)	(1.52)*	(1.73)	(2.61)*	(0.14)	(0.16)*	(0.40)	(1.81)		
EU-BR	0.29	-0.48	-0.22	-1.88	-2.65	2.20	0.07	-0.74	1.45	-0.90	3.65	-
	(0.17)	(0.70)	(1.13)	(4.28)	(1.72)	(3.73)	(0.16)	(0.19)*	(0.92)	(0.33)*	(3.07)	
GB-BR	0.28	-	0.03	6.46	-2.57	-6.43	0.15	-0.65	0.11	-3.93	0.17	-
	(0.14)*		(1.07)	(2.87)*	(1.51)	(3.13)*	(0.14)	(0.14)*	(0.28)	(2.33)	(0.74)	
CH-BR	0.36	0.89	-0.25	7.33	-2.60	-7.54	-0.09	-0.91	0.45	0.65	-0.59	0.17
	(0.15)*	(1.18)	(1.22)	(5.91)	(1.95)	(4.43)	(0.16)	(0.19)*	(0.73)	(0.56)	(5.11)	(1.06)

The coeffiecients γ_s, γ_m, γ_{m^*}, γ_y, γ_{y^*}, γ_i, γ_{i^*} refer to the coefficients of the lag variables (exchange rate, home money supply, reference money supply, home real income, reference real income, home interest rate, reference interest rate, respectively), the coefficients ect_i ($i \in \{1, \ldots, 5\}$) refer to the coefficient of the error correction term (where there are only as many error correction terms as cointegrating relationships amongst the variables). An asterisk denotes significance at 5% level. The standard deviation of each coefficient estimator is given in parenthesis. The method used was maximum likelihood estimation.

6.2 Testing the Model restrictions

We then proceed to testing the model restrictions by likelihood-ratio tests, using again the number of cointegrating relations implied by the trace statistic. The results on table 12 show that only in the Dollar-Real case some of the hypothesized restrictions of the model cannot be rejected, as well as in the Euro-Real interest rate case. In all other cases any form of restriction is very strongly rejected in favor of the alternative. This implies that, whereas the variables stressed by the model do show significant comovements with the exchange rate, the model itself does not find any support in the observed data.

Table 12: Restriction test results

Null Hypothesis	**US-BR** (χ^2)	**EU-BR** (χ^2)	**GB-BR** (χ^2)	**CH-BR** (χ^2)
$\mathcal{H}_1 : \delta_m = -\delta_m^*$	-	20.26(0.00)	-	27.71(0.00)
$\mathcal{H}_2 : \delta_y = -\delta_y^*$	4.08 (0.25)	21.89(0.00)	36.60(0.00)	34.10(0.00)
$\mathcal{H}_3 : \delta_i = -\delta_i^*$	4.48(0.21)	8.48(0.08)	41.10(0.00)	21.02(0.00)
$\mathcal{H}_4 :, \mathcal{H}_1 \cap \mathcal{H}_2$	-	25.32(0.00)	-	47.08(0.00)
$\mathcal{H}_5 :, \mathcal{H}_1 \cap \mathcal{H}_3$	-	36.68(0.00)	-	50.39(0.00)
$\mathcal{H}_6 :, \mathcal{H}_2 \cap \mathcal{H}_3$	13.17(0.024)	43.66(0.00)	57.29(0.00)	52.09(0.00)
$\mathcal{H}_7 :, \mathcal{H}_1 \cap \mathcal{H}_2 \cap \mathcal{H}_3$	-	50.08(0.00)	-	70.25(0.00)

The χ^2 test statistic is reported in the columns, and the corresponding p-value in parenthesis.

7 Discussion and Conclusion

This study attempts to establish evidence in favor, or against, the flexible-price model of exchange rate determination. The model in question posits a determined relation among the exchange rate between two countries, their relative money supplies (proxied by M1 money stock), their relative real income (proxied by GDP) and their relative interest rates (proxied by the leading market rate). The countries taken into account in this work are the United States, the Euro-Zone, the United Kingdom, Switzerland, with Brazil being used as a reference in all cases. The period under scrutiny is the free-float era of the Brazilian Real, between 1999:Q1 and 2016:Q4.

After a description of the theoretical model, the Johansen procedure for testing cointegrating relations is presented as the main tool for analyzing the research question. As a first step towards the examination of potential comovements amongst the variables, each variable is tested for its order of integration. With very few exceptions (British and American money supplies, which are $I(2)$), all variables are found to be integrated of order one.

The cointegration procedure finds support for the model only at the most basic level, meaning that the variables do display some form of common movement. However, the restrictions implied by the theoretical specification of the model are soundly rejected in 17 out of the 20 cases. In two cases, the Dollar-Real and the Pound-Real cases, the variables $I(2)$ (money supplies) had even to be removed from the model. The results from this study show that the flexible-price model of exchange rate determination is, at best, a first approximation to finding the real drivers of exchange rate.

However, McNown and Wallace (1994 [14]) argue that inconsistencies with respect to the model restrictions are not necessarily an indication that the model is wrong; simply that it does not consider the full set of variables involved in EXR. This offers an avenue for further exploration, which would involve increasing the number of variables under consideration. Another logical extension of this work would involve testing for structural breaks. The presence of structural breaks muddles the inference, and tends to lead to rejecting the null even if it is valid. The findings of the cointegration results could be checked by different means, such as dynamic OLS or fully modified OLS (as in de Bruyn, Gupta and Stander 2012 [5]). Lastly, a performance comparison between the model estimated in this study and a random walk would also increase the comparability of our findings to previous work in this subject.

We conclude by reiterating that even though the results seem to discredit the flexible-price monetary model, the question of whether it is a good approximation to modelling exchange rate behavior is not yet over. As empirical and data-gathering methods evolve, a continuous reassessment of the question is likely to take place.

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Appendices

A Data Sources

All datasets used in this work are constrained to the period between January 1999 and December 2017.

The following data was obtained from the Federal Reserve Bank of St. Louis database:

- Brazilian M1 money supply (monthly, no seasonal adjustment), code

  MYAGM1BRM189N

- US American M1 money supply (monthdly, no seasonal adjustment), code

  M1NS

- Euro M1 money supply (monthly, no seasonal adjustment), code

  MYAGM1EZM196N

- Swiss real gross domestic product (quarterly, national currency, reference basis 2010, no seasonal adjustment), code

  CLVMNACNSAB1GQCH

- British real gross domestic product (quarterly, national currency, reference basis 2010, no seasonal adjustment), code

  CLVMNACNSAB1GQUK

- US American real gross domestic product (quarterly, reference basis 2009, seasonally adjusted), code

  GDPC1

- US American effective federal funds rate (monthly), code

  FEDFUNDS

- Euro Area real gross domestic product (quarterly, reference basis 2010, no seasonal adjustment), code

  CPMEURNSAB1GQEA19

British M1 money supply (monthly, national currency, not seasonally adjusted) was obtained from the database of the Bank of England (code: LPMVWYE), as well as the monthly official bank rate (out of the report "Three Centuries of Macroeconomic Data").

Swiss M1 money supply (monthly, not seasonally adjusted) was obtained from the database of the Swiss National Bank, as well as the monthly LIBOR CHF 3-month rate.

Brazilian nominal gross domestic product was obtained from the database of the Banco Central do Brazil (Brazilian Central Bank, monthly, not seasonally adjusted, code 4380), and was adjusted using the INPC (ndice nacional de preco ao consumidor, Brazilian consumer price index, monthly, not seasonally adjusted, code 188). Brazilian nominal interest rate was proxied by the SELIC (Sistema Especial de Liquidacao e de Custodia, the central bank's target rate) rate, collected as well from the Banco Central do Brazil webpage (monthly, code 4189).

The EURIBOR 3-month rate was accessed at the European Central Bank database (monthly), series key: FM.M.U2.EUR.RT.MM.EURIBOR3MD_HSTA.

The monthly nominal exchange rate between the currencies was obtained from the Pacific Exchange Rate Service, provided by the Sauder School of Business from the University of British Columbia, using Brazilian Real as base currency.

(http://fx.sauder.ubc.ca/data.html, last accessed May 30 2017).