Spline Arch
A Pneumatic Chambers System Applied in a Temporary Membrane Structure

A Master's Thesis submitted for the degree of
"Master of Engineering"

supervised by
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México, November 27, 2015

Affidavit

I, **JUAN JOSE RAMIREZ ZAMORA**, hereby declare

1. that I am the sole author of the present Master's Thesis, "Spline Arch. A Pneumatic Chambers System applied in a Temporary Membrane Structure", 60 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and
2. that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

Vienna, 27.11.2015

Signature

ACKNOWLEDGEMENTS

“The design of a membrane lightweight structure is only governed by forces and imagination, there are no rules, just possibilities”…… (Jürgen Hennicke).

I dedicate this work to my parents Eduardo (1945-2006) and Laura because of their huge support and love throughout my life.

Without Lili´s support and love, this work would have been impossible to do.

I thank the Technical University of Vienna crew, especially:

Robert Roithmayr

Julian Lienhard

Jürgen Hennicke

Catherina Purrucker

I also thank software sponsors of the master program:

Technet (Dieter Ströbel, and Juergen Holl)

Formfinder (Robert Roithmayr)

Dlubal

I thank Serge Ferrari team for their valuable economic contribution, and especially Thomas Dreyfus and Francois Fournier because of their support at all times.

Thanks to the program I have met very special people that became very close friends of mine and who opened their homes during my stay in Europe:

Prof. Michael Schultes

Carolin Lotz

Christian Bartl

Finally, I thank my office crew, who helped to me towards the last months to finish this work.

ABSTRACT

This work is about the feasibility of using slender elements combined with pneumatic chambers as support structural elements in temporary membrane structures applications.

Spline term in this work refers to those straight and slender elements that are bent to form a continuous spatial curve and identical moment of inertia (second moment of area) about any axis perpendicular to its centroidal axis. It is common to use arches as supporting elements in membrane lightweight structures. There are very few applications of spline elements combined with membranes in medium – large span structures. Applications in small spans are more common for example in camping tent systems where spline elements are stiff enough to take external forces. For medium (16 – 32 m) span structures it is common that spline elements need in-plane bracing systems formed by masts and cables, in order to increase stiffness and strength properties to be able to bare real external forces (wind or snow). It is included the use of pneumatic chambers instead of mast-cables system in order to make spline arches stiff enough to withstand real external forces.

Analytical models whereas realistic load conditions were developed using specialized software (EASY, RSTAB). Several configurations of size and position of pneumatic chambers were evaluated. Different materials and their properties were also analyzed to be adequate for spline element requirements. Additional vertical truss elements and cables were required to stabilize the hybrid arch.

As a result of this work, it is demonstrated analytically that it is possible to use pneumatic chambers and additional vertical truss elements and cables to give enough stiffness to spline slender arches under real loading conditions. A basic detailing proposal for connections and erection process is presented.

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CHAPTER 1. INTRODUCTION

It is very important to design and develop structural systems as simple and lightweight as possible in order to be environmentally friendly. This work is intended to be another step in the structural engineering field in terms of lightweight structures development. There are actually some structural applications involving spline elements combined with compression and tension (mast-cables) elements used in lightweight membrane structures in order to develop stiff enough and force resisting systems to solve supporting frame components. There´s theoretical and experimental knowledge about pneumatic chambers or cushions used in structural engineering and it is the idea of this work to use it in order to develop this hybrid spline-pneumatic chambers structural element.

The aim of this work is to evaluate the feasibility of the use of pneumatic chambers as a real alternative being implemented in spline elements in order to make them stiff enough to withstand buckling and to resist forces arising from pre-bending erection stresses and from real loading conditions.

The hypothesis of this work is that "Implementation of pneumatic chambers in specific zones of a spline element generates stable arch support system for medium-span temporary deployable membrane structure".

An analytical case study is developed. The solution of the structural system and demonstration of the stability, applied to 16 m span membrane structure supported by spline arch, are expected results. It is also included a proposal of the election process.

This work was inspired by some of the work of Sigrid Adriaenssens, Frei Otto, B. Oñate and B. Kröplin, and Julian Lienhard involving topics about pneumatic structures, membrane lightweight structures, spline structures and pre-stressed systems. In the document "Numerical methods for the Analysis of buckling and post-buckling behavior of arch structures", there is a full review of methods about the analytical calculation of buckling and post-buckling. In the document "stressed spline structures" Sigrid Adriaenssens (Doctor of Philosophy Thesis) gathers theoretical and practical information concerning spline arches behavior including linear and nonlinear buckling load analysis. It also includes some application cases of membrane structures supported by splines with branched and straight splices. In the document "Feasibility study of medium span spliced spline stressed membranes," she demonstrates that GFRP (Glass fiber reinforced plastic) material is adequate to be used for medium-span spline supported membrane structures. In the same document, she recognizes that there is a lot of work to do about improving buckling behavior of that system, and that is the point of this work; the use of pneumatic chambers added to the spline element in order to have better buckling behavior.

This proposed study methodology is most congruent to fulfill the research objectives including:

1. - State of the art of spline structural elements.
2. - Study of buckling on spline structural elements.
3. – Form finding of hybrid system
4. - Structural analysis including buckling analysis of the hybrid system using software.
5. - Drawings and detail.

CHAPTER 2. STATE OF ART AND THEORETICAL BACKGROUND

The erection process is a very important part of the construction of any structure and when talking about membrane lightweight structures, it plays a very special role. Spline structures have been studied by some authors because of their flexibility and possibility to change the form during the erection process without exceeding allowable stresses of materials. A brief description of the main concepts of spline structures is reviewed in this chapter. Form finding of the spline arch is also developed in this chapter.

2.1 SPLINE STRUCTURAL SYSTEMS

The term “spline structure” is adopted in this work, Adriaenssens (2000), to describe those structural elements that initially are straight and then bend into a spatial continuous curve having an identical moment of inertia (second moment of area) about any perpendicular axis to its centroidal axis. Such an element gets special importance when it is considered in the main structural supporting system of membrane structures because of some advantages listed below:

- lightweight.
- Usable as a part of the erection process and then keep it as structural supporting system element once the structure is operating normally.
- The ability to take a different shape depending on the construction stage because of its flexibility.
- Combined with bracing systems, it can provide enough stiffness to the system in order to be capable to bare external loads like wind or snow.
- It can be extremely slender because it is continuously restrained from buckling by the membrane.

Some special properties need to be fulfilled by spline elements. Firstly, it needs to be flexible enough in order to be curved and keep deformations into the elastic range, this flexibility deals with material and geometry properties of the section. Once it is bent into the final shape, it has to resist forces arising from loading conditions (wind or snow), and finally it has to be stiff enough to withstand buckling. In this thesis the main aim is to propose a hybrid system composed by spline element plus pneumatic chambers inplane bracing system providing additional bending stiffness and demonstrating the feasibility of its use it in real medium span projects.

Figures 1, 2 and 3 show some examples of the small span and medium span applications of spline arches with different materials including aluminum, steel, and bamboo.

In the case of figure 1, the camping tent, a small amount of energy is needed to curve originally straight aluminum strips into its final shape.

Figure 1: Camping tent, aluminum spline support structure. http://www.grupoblascabrera.org/web/planetar/con_cup/concup10.htm last accessed on 29.07.2015

Figure 2: Bamboo truss frame system. http://www.ecohabitar.org/la-guadua-una-maravilla-natural-de-grandes-bondades-y-prometedor-futuro last accessed on 29.07.2015

In the case of figure 2 there are two ways to get the curved shape of the top and bottom chord of the truss. The first one is by a thermal deformation process, and the second one by using molds and drying bamboo poles right there just after cutting them. In this case, the amount of energy needed to get curved shape is bigger than the case of the camping tent aluminum spline structure.

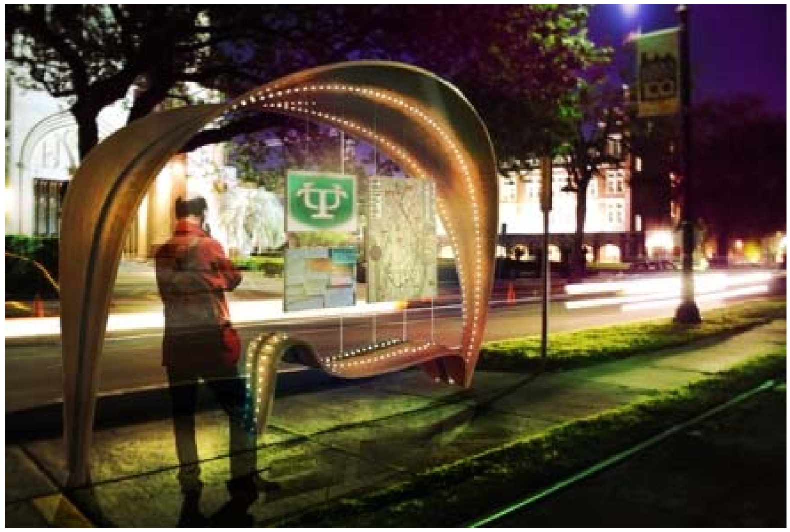

Figure 3: Tram station design. Wooden panels cover system. Steel pipe spline structure. https://www.behance.net/gallery/3542735/Cobra-Stop last accessed on 03.08.2015

In the case of the tram station, figure 3, the required energy to get the curved shape of the steel elements is also higher than the bamboo truss structure.
ASTM A-36 Steel, aluminum, bamboo, GFRP and CFRP were evaluated, see table 1. Adequate materials for bending-active structures offer a ratio of $\sigma/E \geq 2.5$. According to table 1, steel is not adequate for bending active structures.

Table 1 : Material properties

MATERIAL	E (GPa)	$\sigma_{allowable\_bending}$ (Mpa)	Volumetric weight (kN/m^3)	(σ / E) ratio
ALUMINIUM	70	330	26.5	4.71
ASTM A-36 STEEL	200	248	77	1.24
BAMBOO	19.13	213	6.9	11.13
GFRP	15	160	24.9	10.67
CFRP	165	2800	24.9	16.97

2.2 LINEAR AND NON-LINEAR BUCKLING ON SPLINE STRUCTURES

When some members or the entire structure reach yield, ultimate strength, fracture, collapse or exceed specific maximum deflection, it is considered that structures can fail. Buckling refers to an event where a structural element subjected to compression forces deviates from a behavior of elastic shortening within the original geometry undergoing large deformations involving a change in shape for a very small increase in load. There are many other forms of instability that are generally also referred to as buckling. Lateral – torsional buckling is out of plane buckling phenomenon, but it is considered in this thesis that pre-stressed membrane gives lateral bracing to be stiff enough to avoid it.

Figure 4 shows the relation between axial load (P) and rotational deformation (θ) in a stiff bar fixed in the base. It is clear that for small values of θ, linearized solution and exact solution (non-linear) are almost the same. For higher values, nonlinear behavior has to be taken into consideration.

Applying basic sum of moments in the base:

$$\sum M_A = 0 \qquad \dots\dots\dots\dots (1)$$

$$PL\sin\theta + FL\cos\theta - k\theta = 0 \qquad \dots\dots\dots\dots (2)$$

$$P = \frac{k\theta - FL\cos\theta}{L\ \sin\theta} \qquad \dots\dots\dots\dots\ (3)$$

For small values of θ, $\sin\theta \cong \theta\ and\ \cos\theta = 1$ and then ec. (3) results like:

$$\theta = \frac{FL}{k - PL} \qquad \dots\dots\dots\dots.(4)$$

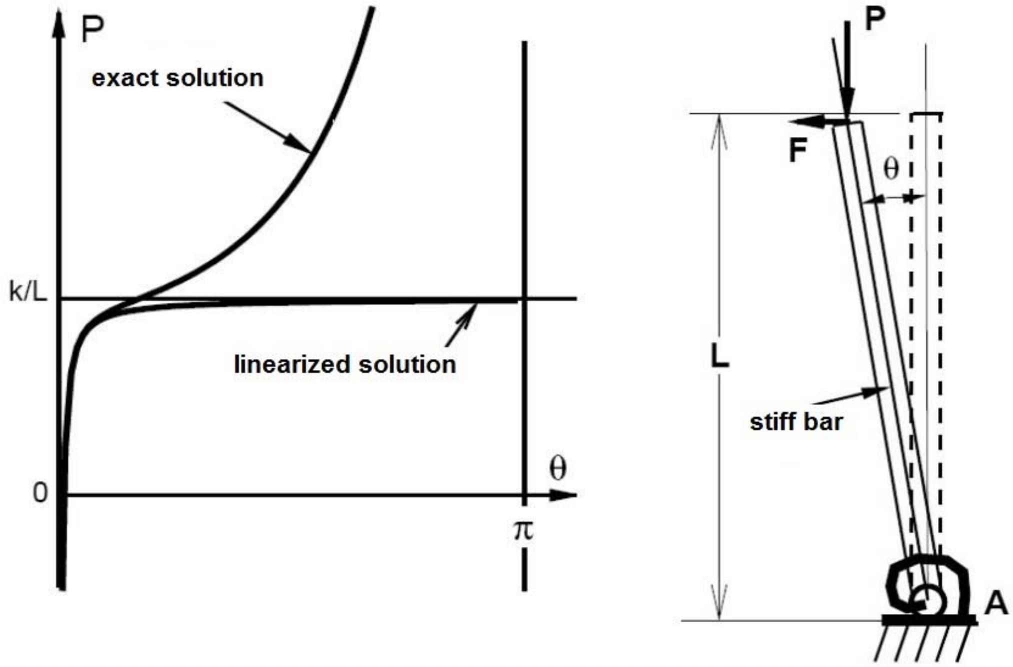

Figure 4: Linearized and exact solution P-θ curve for a stiff bar, http://es.slideshare.net/nevam21/exact-solutions-for-buckling-of-structural-members, last accessed on 05.05.15

Euler (1744), taken from Adriaenssens 2000, developed basic buckling equations restricted to elements that meet the next characteristics:

- Straight
- Prismatic
- Centrally loaded
- Pin-ended
- Slender enough to buckle before reaching yielding limit of the material
- Deformations of the member are small enough that the curvature can approximate with respect to position along the member (linear behavior)

The basic buckling equation solved as an eigenvalue problem of the governing differential equation of the element:

$$P_c = \frac{\pi^2 EI}{L^2} \qquad \dots \dots \dots \dots (5)$$

Where P_c is the minimum axial load (critical Euler load) that has to be applied to the element to produce buckling. E refers to the modulus of elasticity of the material and I – moment of inertia of the section.

It is used in this thesis equation (5) to predict the required load to induce buckling to the spline element. Once the buckling load is known, the next step is to evaluate the shape of the buckled element. A slender spline element can be bent far beyond the critical Euler load and remaining in stable equilibrium (Adriaenssens 2000), so it will be of special interest in this thesis to evaluate the shape of the spline in its buckled deformed state.

There is a possibility to describe the shape of the elastic buckling of a pin-ended bar with an allowance for large deformations (called "the elastica") when the exact differential equation governing buckling is used, rather than the equation that approximates the curvature of the member by $\frac{d^2y}{dx^2}$. Equation 6 shows the exact expression of the curvature.

$$\frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}} \qquad \dots\dots\dots\dots(6)$$

An open source script written in grasshopper for rhino as a tool to obtain the shape of "the elastica"is used in this thesis. The script is based on the work of M. E. Pacheco "The elastic rod", where the differential equation is solved exactly in terms of Jacobi´s elliptic functions finding the solution with an iterative method, varying some parameters of the elliptic functions.

2.2.1 Grasshopper definition description

Elastic bending script by Will McElwain (created on February 2014)

Input data:

PtA - First anchor point (required)

PtB - Second anchor point (optional, though 2 out of the 4--length, width, height, angle--need to be specified)

Pln - Plane of the bent rod/wire, which bends up in the +y direction. The line between PtA and PtB (if specified) must be parallel to the x-axis of this plane

** 2 of the following 4 need to be specified **

Len - Length of the rod/wire, which needs to be > 0

Wid - Width between the endpoints of the curve [note: if PtB is specified, in addition, and distance between PtA and PtB <> width, the end point will be relocated

Ht - Height of the bent rod/wire (when negative, curve will bend downward, relative to the input plane, instead)

Ang - Inner departure angle or tangent angle (in radians) at the ends of the bent rod/wire.

* Following variables only needed for optional calculating of bending force, not for the shape of the curve.

E - Young's modulus (modulus of elasticity) in GPa (=N/m^2)

I - Second moment of area (or area moment of inertia) in m^4 (cross-section-specific.

<u>Output data:</u>

Out - only for debugging messages

Pts - the list of points that approximate the shape of “the elastica”

Crv - the 3rd-degree curve interpolated from those points (with accurate start & end tangents)

L - The length of the rod/wire

W - The distance (width) between the endpoints of the rod/wire

H - The height of the bent rod/wire

A - The tangent angle at the (start) end of the rod/wire

F - The force needed to hold the rod/wire in a specific shape (based on the material properties & cross-section). The critical buckling load (force) that makes the rod/wire bend can be found at height=0

Figure 5 shows grasshopper definition interface.

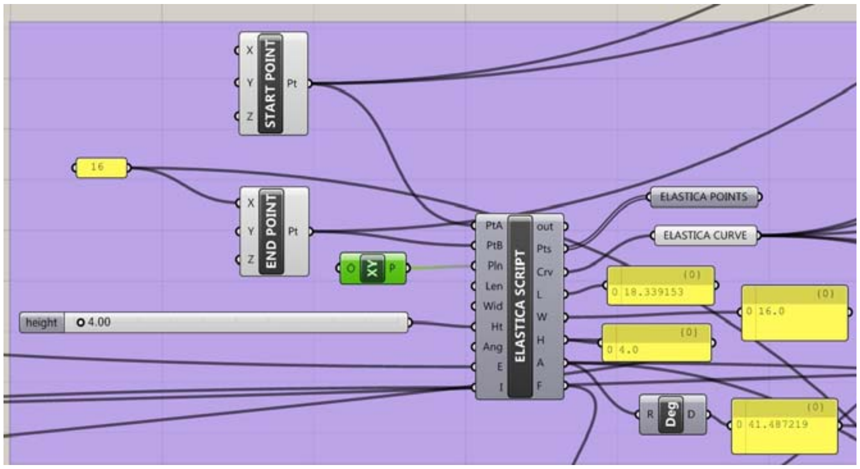

Figure 5: Elastica script definition in grasshopper, Grasshopper (2014) plugin for Rhinoceros (2015)

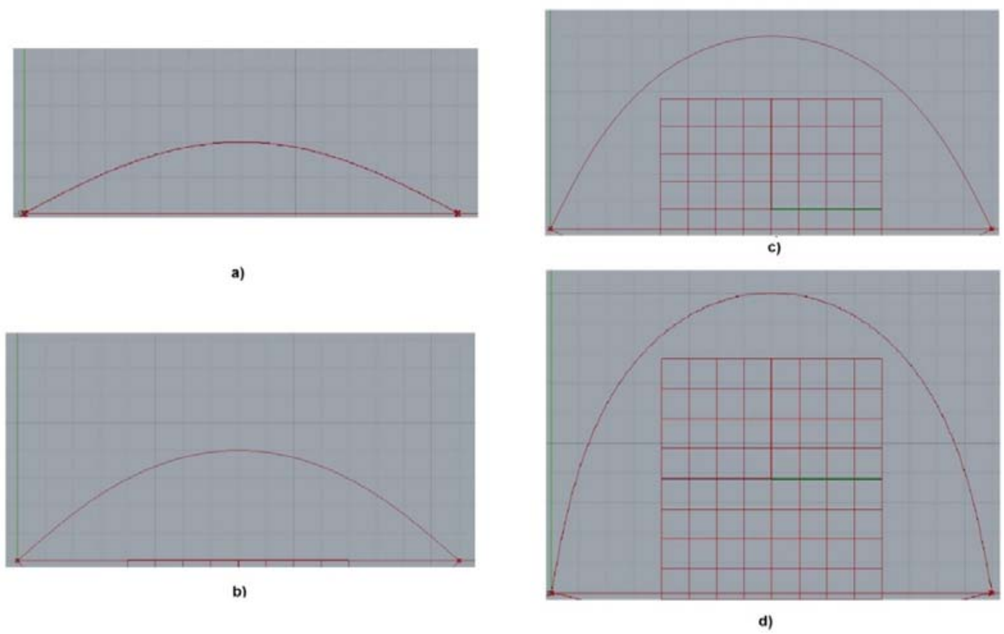

Figure 6: Elastica for different parameters of width and height, Rhinoceros (2015) interface

Figure 6 a): PtA: (0,0)

PtB: (0,16)

Pln: X-Y Plane

Ht: 2 m

Figure 6 b): PtA: (0,0)

PtB: (0,16)

Pln: X-Y Plane

Ht: 4 m

Figure 6 c): PtA: (0,0)

PtB: (0,16)

Pln: X-Y Plane

Ht: 7 m

Figure 6 d): PtA: (0,0)

PtB: (0,16)

Pln: X-Y Plane

Ht: 10 m

16 m width and 4 m height will be considered in this thesis.

2.3 PNEUMATIC STRUCTURES BEHAVIOR AND MATERIALS, AIR SUPPLY SYSTEMS

In this work, the concept of pneumatic structure is adopted to those flexible membranes that derive their stability from air pressure.

There is a large variety of materials which can be used in pneumatic structures that depend mainly on the size of the project. For small structures, natural (cotton) or synthetic membranes are used. For large scale projects, stronger membranes materials are used, such as glass fiber, nylon or polyester, protected by a Teflon or vinyl layer. In large, permanent structures, such as stadiums, the best choice is regularly glass fiber with Teflon, although it is inflammable, it presents less deformability.

The larger the structure gets, the larger the tension to which it is submitted, which forces us to add a cable system to transfer tension and force to specific points. There are also two general types of pneumatic structures, depending on how they are inflated.

Air supported: They have only one fabric layer. Spaces created are domes or similar forms. For human comfort, the pressure inside must be just a little higher than the external pressure, so it is considered a low-pressure system, more vulnerable to wind factor. They normally need air lock doors to minimize loss of air pressure. Air supported structures can adopt either convex (positive differential pressure) or concave (negative differential pressure). They require a continuous air supply.

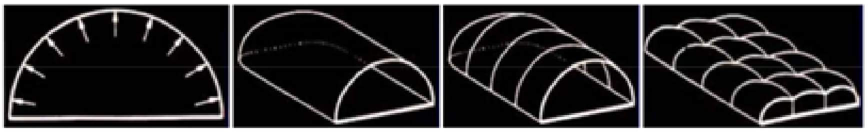

Figure 7: Air supported pneumatic structures.
https://riunet.upv.es/bitstream/handle/10251/30383/Estructuras%20neum%C3%A1ticas.pdf?sequence=1 last accessed on 06.08.2015

Air inflated (High pressure): With 100 or 1000 times more pressure than air supported, they consist on closed, and hermetical volumes (consisting of two layers of fabric) inflated with high-pressure systems. As pressurized volumes are totally airtight, inner space created by the structure can have normal air pressure. They require air pressure from 2 to 70 meters of water, producing 20 to 700 kPa pressure. This should be enough to resist gravity and other lateral loads. They also need a constant air supply.

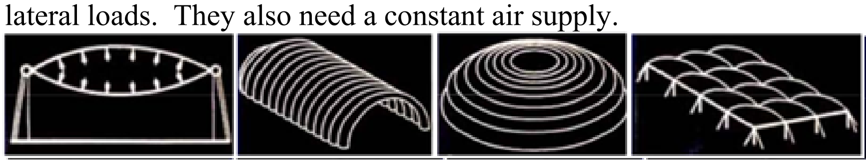

Figure 8: Air inflated pneumatic structures.
https://riunet.upv.es/bitstream/handle/10251/30383/Estructuras%20neum%C3%A1ticas.pdf?sequence=1 last accessed on 06.08.2015

One important advantage of pneumatic structures is the possibility to cover large spaces without any middle support. This simplicity also allows to set up, disassemble and transport easily. Pneumatic chambers generate loads in these structures, due to the air pressure that is held inside. At the same time, this air pressure increases stiffness on the main structure.

Membrane's weight varies from 0.61 kg/m^2 in small structures to 4.88 kg/m^2 in coated textiles for large dimension decks such as stadiums. This load is very low compared to wind and snow. The taller the structure, the more pressure the chambers need to support wind loads, as the wind is not regular and stronger within height increases. In snow matters, the more pronounced the curvature, the easier the snow can be removed by the wind without stiffness and strength problems. If the curvature gets more plain, the chambers will need more pressure to support the weight. Forces in the membrane can be reduced by adding cables strategically positioned by the geometry generated.

The design process of tension structures, including air support systems, can be divided into three subprocesses, form finding, structural analysis and cutting pattern generation. The result of form finding subprocess is a shape of equilibrium for a certain stress distribution and boundary conditions. Then from this geometry, structural analysis is developed and cutting pattern is made. It is common in practice to separate and not interact these three subprocesses and it leads in built structures to highly inhomogeneous stress distributions that can be seen in wrinkles and measured in stresses that are bigger than required. An enhanced design concept is based on 5 steps: defining the shape of equilibrium (form finding), generating the cutting patterns, reassembling and pre-tensioning the cutting patterns, structural analysis of reassembled structure and evaluation of structural behavior.

2.3.1 The tensegrity and tensairity principle, tensairity beam and air beam concepts

Tensegrity structures base their behavior in the constructive separation of tension and compression forces acting on their elements (mast and cables), the tensegrity principle of discontinuous compression and continuous tension is applied in several sculptures , but it is not suited for technical applications. The Tensairity structural concept takes the tensegrity principle of constructive separation of tension and compression in cables and struts adding low-pressured air membrane element (air beam) for stabilization.

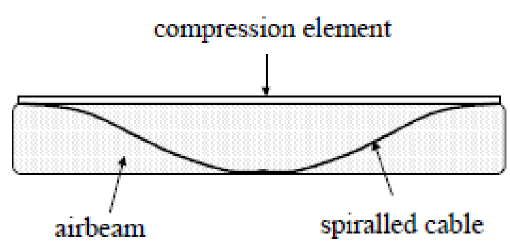

Figure 9: Tensairity beam. R.H. Luchsinger 2004

The main functions of the compressed air chamber are the pretension of the cables and stabilization of compressed element against buckling. In this work, the concept of tensairity is applied in terms of the usage of pneumatic chambers and a cable attached to the spline arch in order to increase stiffness and reduce buckling possibilities, but configuration and position of chambers are developed for spline arch structures.

2.4 PNEUMATIC CHAMBERS POSITION AND MATERIAL

As a result of previous sections of chapter 2, the way to get the form of "the elastica" was developed, which is basically a kind of arch (the compression part of the system), and the basis of the hybrid system, which will be used for the support of tensile membrane structures. It is common in medium scale projects (16-32 m) to use slender arch systems mixed with simple cable bracing in the plane of the arch to increase in plain stiffness and prevent asymmetric distortion produced by external loadings like wind or snow. Prestressed membrane gives lateral bracing effect to the system. This thesis is intended to demonstrate the feasibility of the use of pneumatic chambers instead of cable bracing.

2.4.1 Spline materials and geometric sections

In the grasshopper tool, previously described, the possibility to choose between materials and sections in order to select an adequate combo based on the next criteria was included:

- Maximum bending stress limited to 1/3 $\sigma_{allowable\_bending}$
- $\frac{\sigma}{E} ratio \geq 2.5$
- Maximum depth or diameter of the section to give as much as possible bending stiffness to the arch

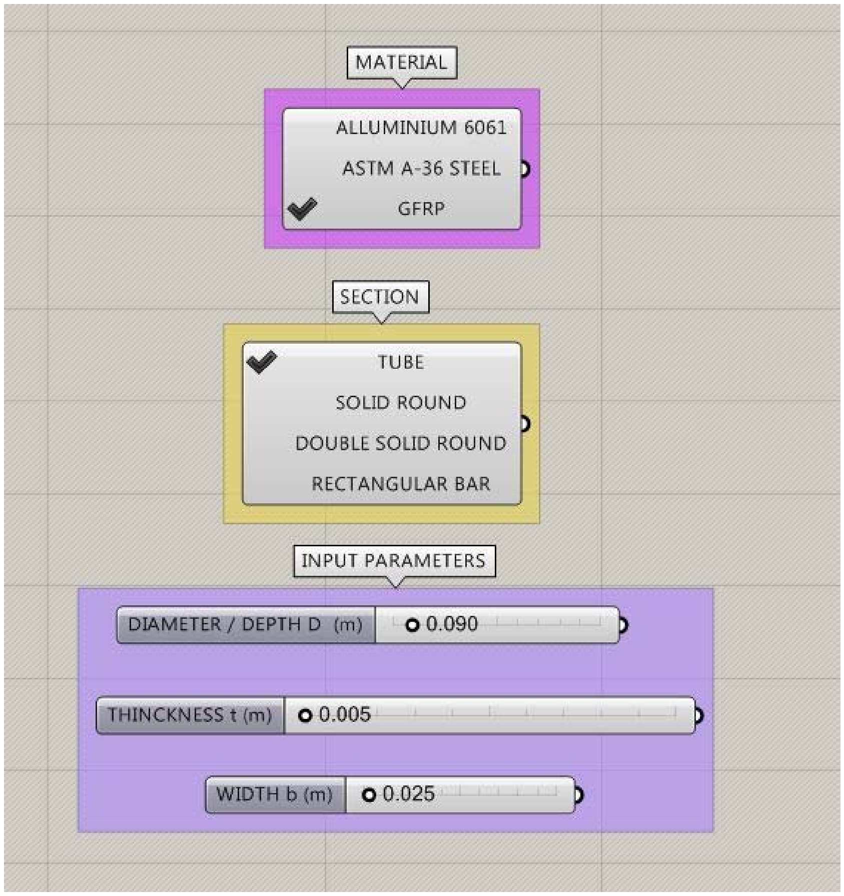

Figure 10: Materials and geometric sections of spline element. Grasshopper plugin

Applying simple theory of bending (eq. 7)

$$\frac{R}{r} = \frac{E}{\frac{1}{3}\sigma_{\text{allowable\_bending}}} \qquad \dots\dots\dots\dots(7)$$

Where R – radius of curvature, and r outside radius of the tube or depth/2

The radius of curvature is not constant in the spline arch so it was necessary to evaluate that expression for the critical value of R. It is clear that acting bending stress is maximum when R is minimum, and it is minimum at the center (midpoint) of the arch, see figure 11.

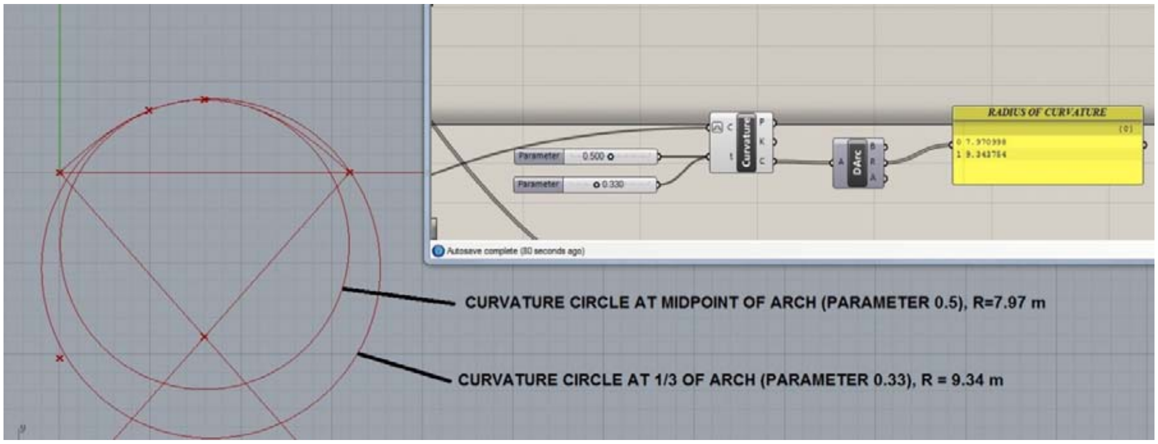

Figure 11: Variation of R along the spline arch, Grasshopper plugin

So, from the geometric shape defined previously R = 7.97 m and it has to be the same for all materials and sections and then it is necessary "r" to be calculated.

Table 2: Required diameter of the section for different materials

MATERIAL	R (m)	r (m)	required diameter D (mm)
ALUMINIUM	7.97	0.013	25
ASTM A-36 STEEL	7.97	0.003	7
BAMBOO	7.97	0.030	59
GFRP	7.97	0.028	57
CFRP	7.97	0.045	90

Table 2 shows the maximum value of r for the radius of curvature R = 7.97 m. So if we take into consideration criteria points described previously CFRP material fits better on our project demand.

2.4.2 Setup of the pneumatic chambers and materials

The next step in this work was to develop an adequate configuration of the position and size of the pneumatic chambers. Based on force polygon method an additional module was introduced in the grasshopper tool.

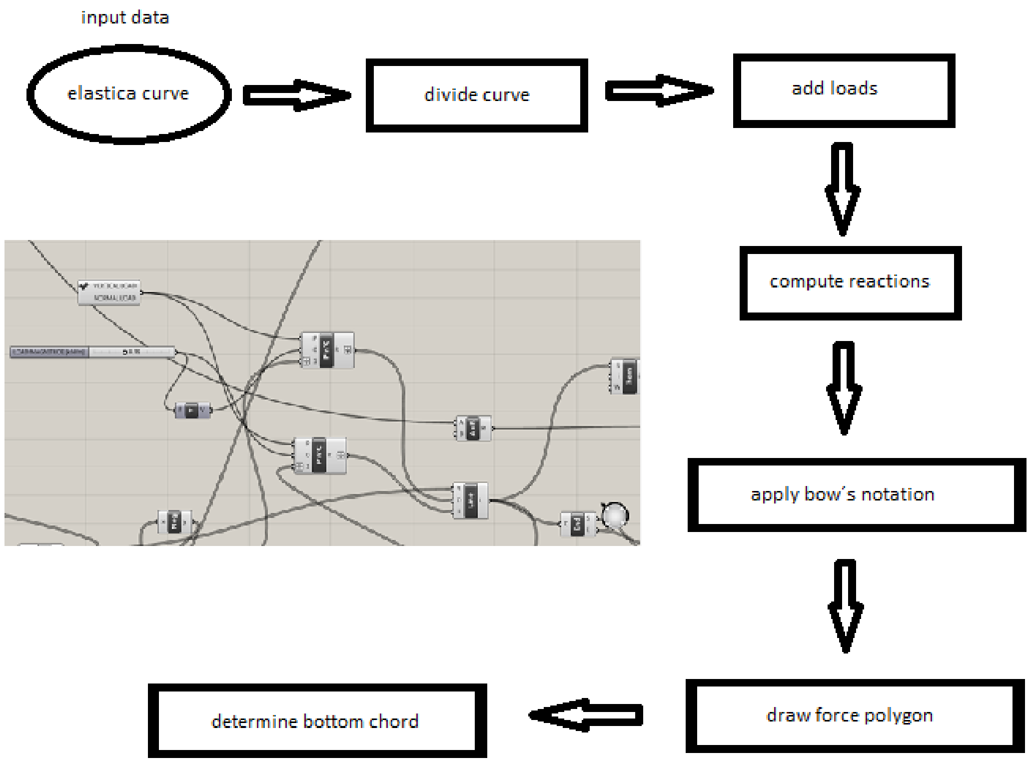

Figure 12: Force polygon method definition in grasshopper tool

It was supposed that the hybrid arch will behave like a truss with an arbitrary top chord (elastica shaped spline) and to be defined bottom chord and vertical truss members. A constant force bottom chord was considered.

According to the method, the next steps have to be fulfilled:

- Draw top chord diagram to scale (form diagram)
- Add tributary joint loads
- Compute reactions
- Add letters between truss bars (apply Bow´s notation)
- Draw force (load – reactions) polygon
- Draw lines on the force polygon parallel to the segments of the top chord using Bow´s notation
- Draw a circular arch assuming bottom chord constant force
- Draw vertical truss members directions from top and bottom chords lines intersections

- Draw bottom chord and vertical truss members into the form diagram

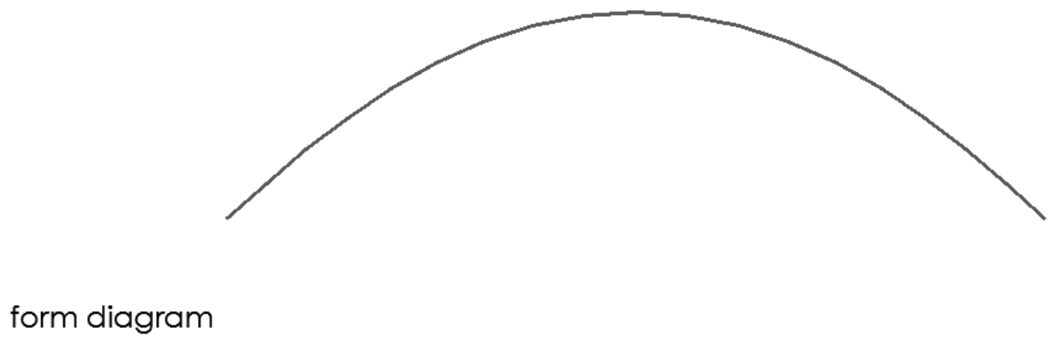

Figure 13: Top chord form diagram, Rhino

It was supposed a uniform vertical load acting on the arch to estimate tributary joint loads. Spline arch length is 18.34 m and it was divided into 18 parts, tributary length for each joint is almost 1 m.

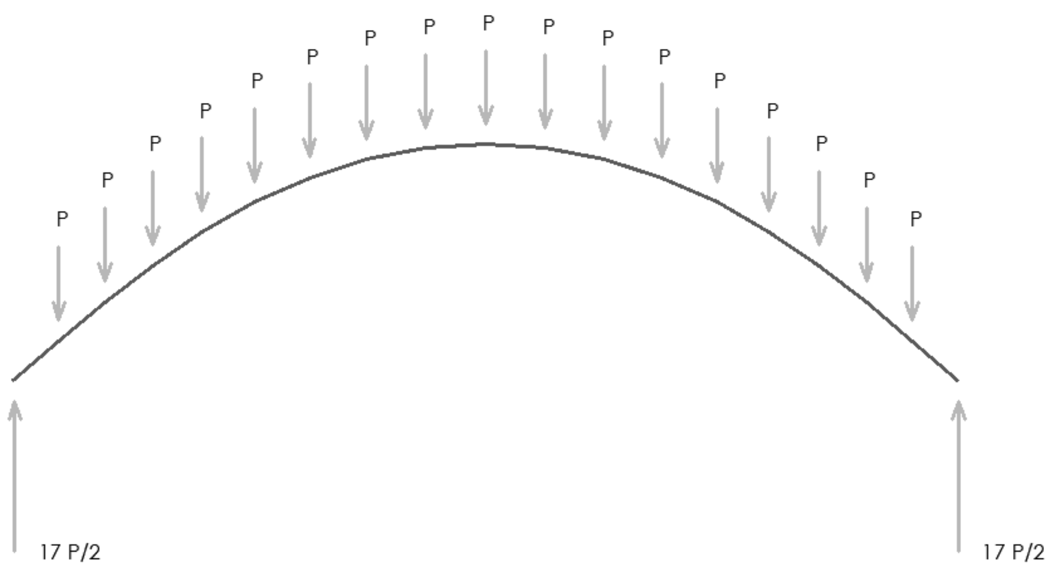

Figure 14 : Load distribution consideration, Rhino

Once load distribution is proposed then reactions must be computed. Loads are managed in terms of "P", and reactions computed also in terms of "P", see figure 13.

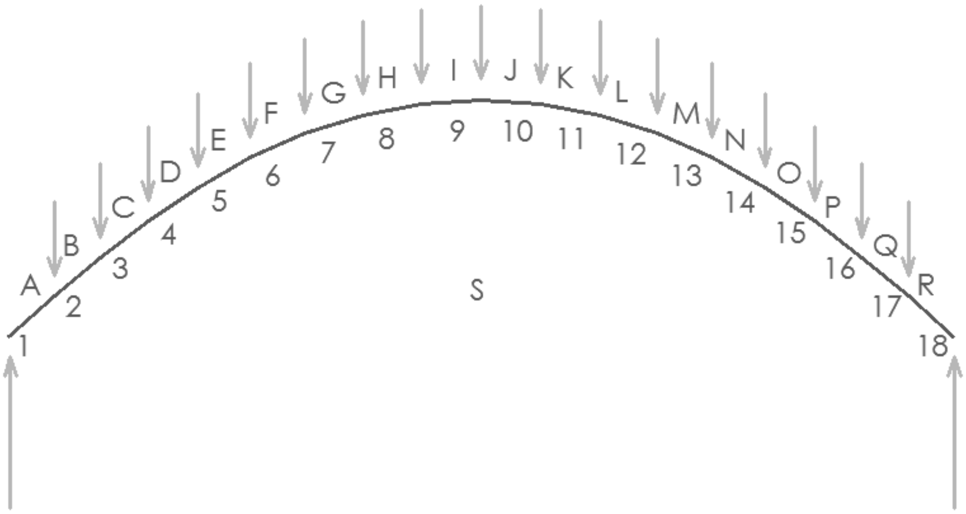

Figure 15: Bow´s notation in form diagram, Rhino

Figure 14 shows bow´s notation applied in the form diagram.

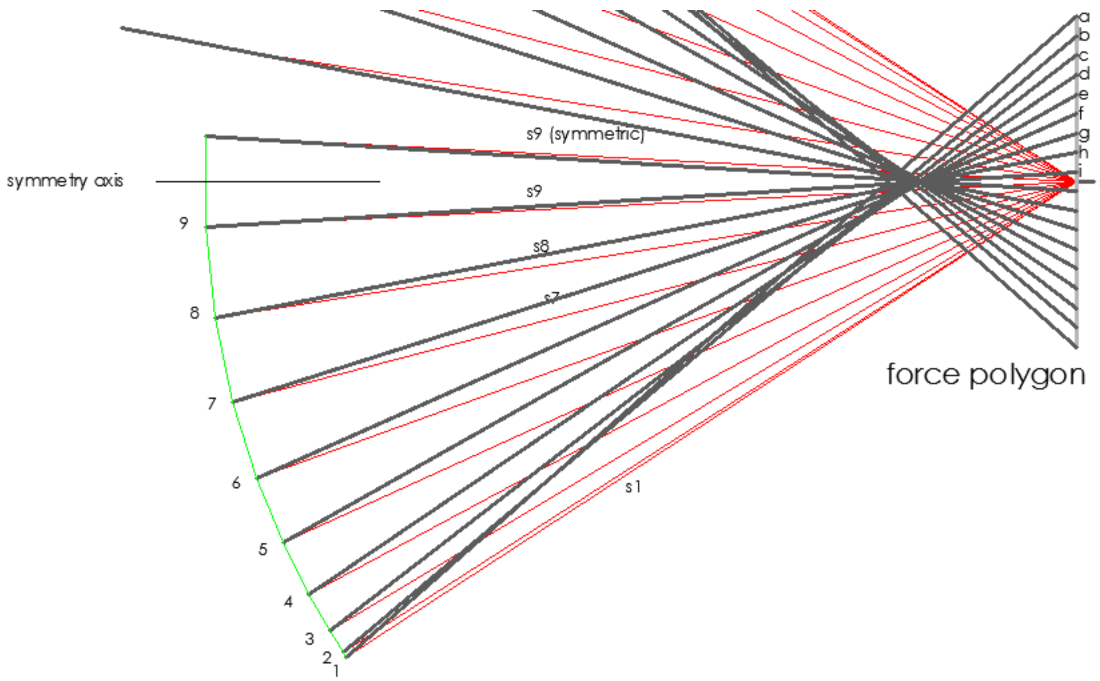

Figure 16: Force polygon, Rhino

We can see in figure 16 bottom chord (in red lines) vector directions and also can measure force for a certain value of “P” load. It is also shown in figure 16 (in green lines), vertical truss members vector directions. It is demonstrated in figure 16 that the value of load “P” does not really matter in terms of bottom chord and vertical truss members configuration.

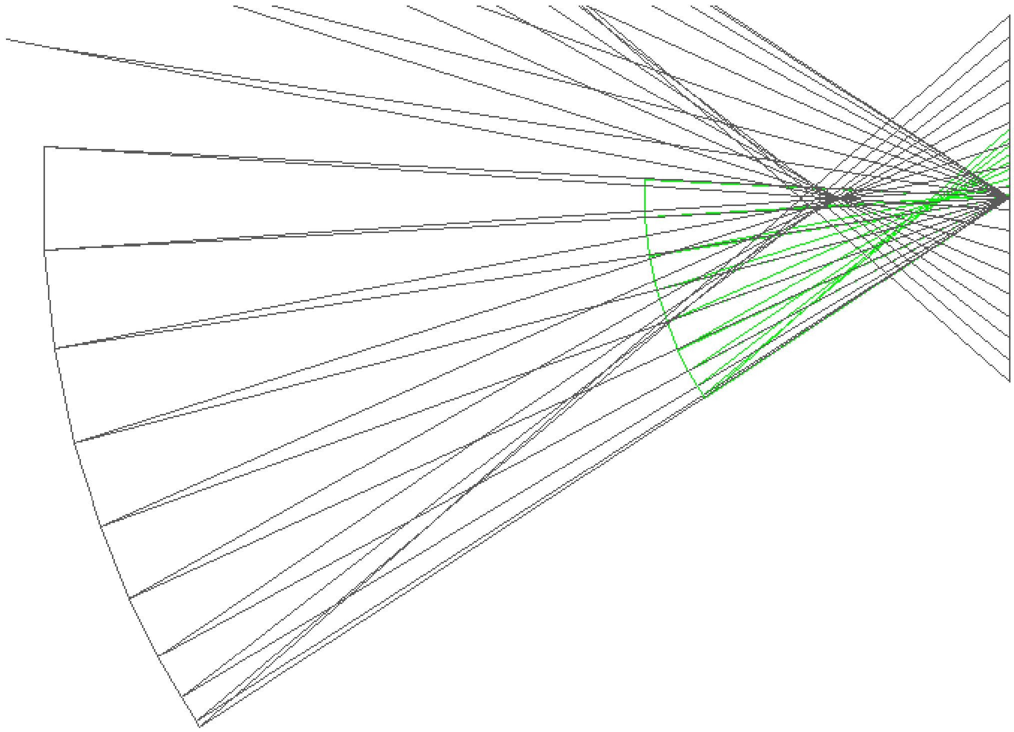

Figure 17: Force polygon for two different values of “P” load, Rhino

In figure 16, the superposition of two force polygons is shown with a different value of “P” load. Lines in green with a small value of “P” and gray ones with a higher value of “P”. It is clear that gray lines and green ones have the same direction and different sizes. If we translate these directions to the formed polygon, it results in exactly the same shape (showed in figure 17).

In conclusion, shape shown in figure 17 will be taken in this work as the basic form to be used to develop form finding of the hybrid system.

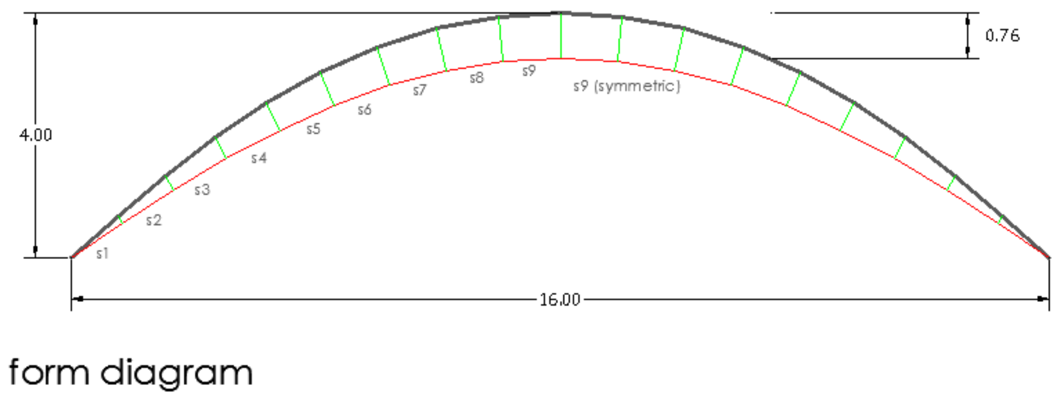

Figure 18: Final shape for bottom chord and vertical truss members, Rhino

Just for comparison purposes critical buckling load was evaluated working like an arch without pneumatic system, taking radial loading. The depth of the arch, the elastic stiffness (EA) and bending stiffness (EI) governs the buckling behavior of an arch. It is demonstrated (Adriaenssens 2000) that linearized theory (Simitses 1976, taken from Adriaenssens 2000) agree well with non-linear analysis results when pre-buckling deformations are relatively small in comparison with central rises of arches. According to Pi (1988), taken from Adriaenssens 2000, it is recommended not to exceed

$$\frac{a}{\sqrt{\frac{I_x}{A}}} > 4.6 \quad \dots\dots\dots\dots\dots (8)$$

in order to control pre-buckling deformations. Then Simitses (1976) (taken from Adriaenssens) total linear buckling load is given by:

$$N_t = EI\,\left(\frac{\pi^2}{(0.5s)^2} - \frac{1}{R^2}\right)\alpha \quad \dots\dots\dots\dots (9)$$

Where a - central rise,
Ix – second moment of area or moment of inertia
A – area of cross section
s – arc length
R – radius of curvature

Evaluating equations 8 and 9 for our spline arch:

$$\frac{a}{\sqrt{\frac{I_x}{A}}} = 132.8 > 4.6$$

And N_t = 4.53 kN, that means if the arch is submitted to a compression force of 4.53 kN or higher it will buckle. If its geometry condition is considered, it means the arch will be unstable with an average uniform load of 0.38 kN/m, a very low value considering applications in real life. For this reason, bending stiffness has to be increased and pneumatic chambers are considered in the system. Some configurations and position were tested, see figures 18 to 21.

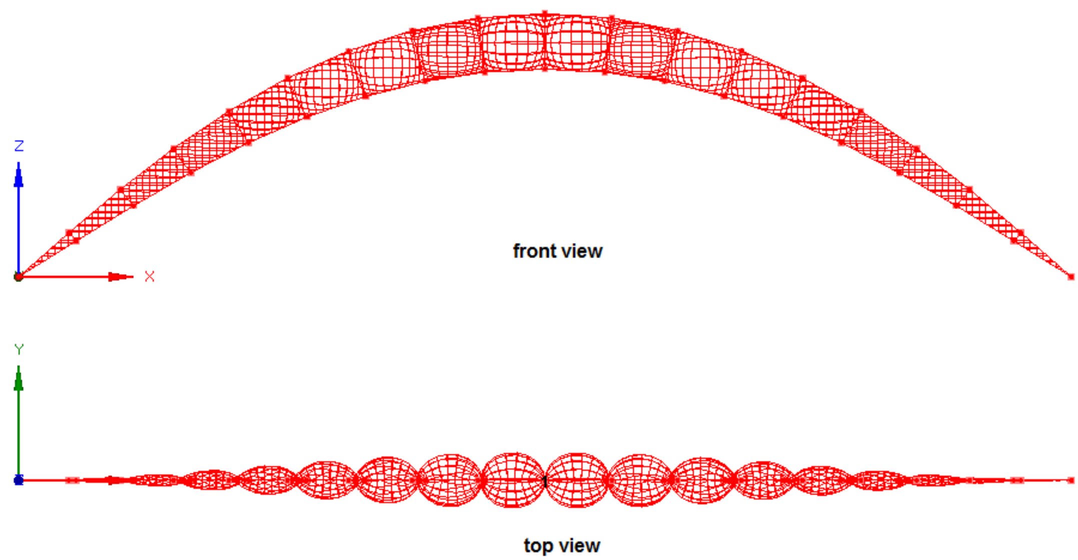

Figure 19: Position 1 of pneumatic chambers (form-finding EASY software)

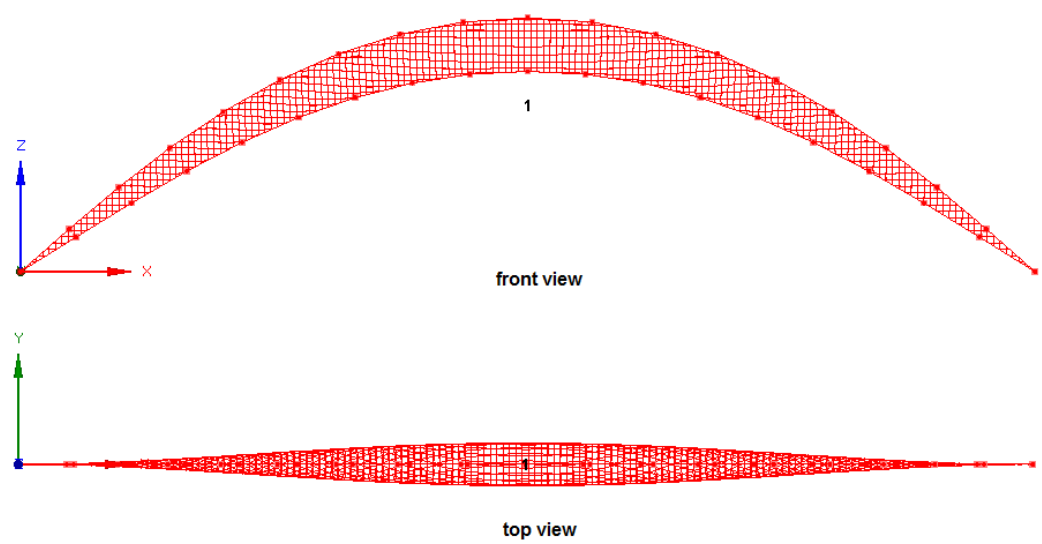

Figure 20 : Position 2 of pneumatic chambers (form-finding EASY software)

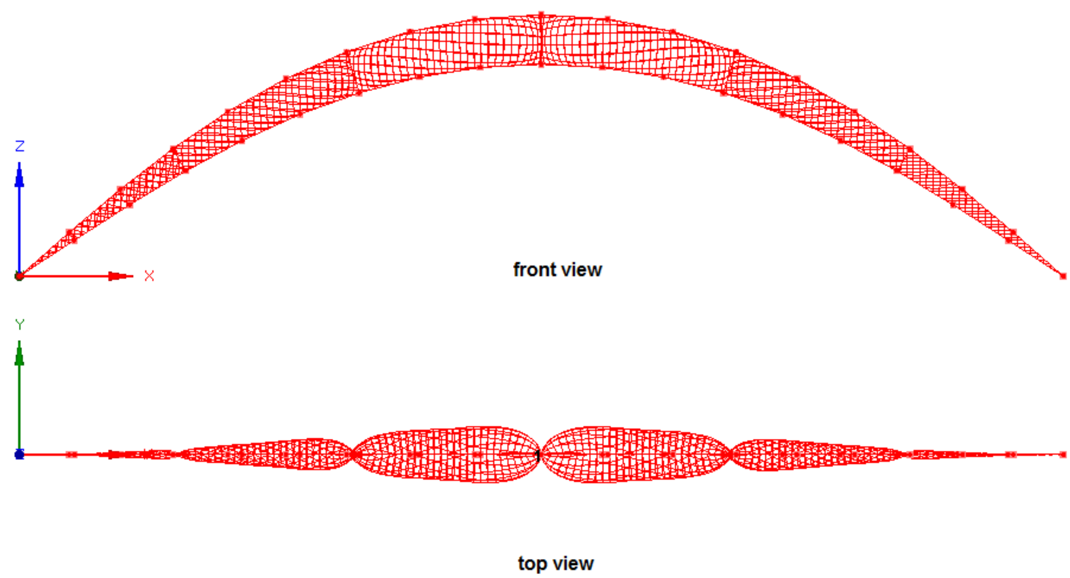

Figure 21: Position 3 of pneumatic chambers (form-finding EASY software)

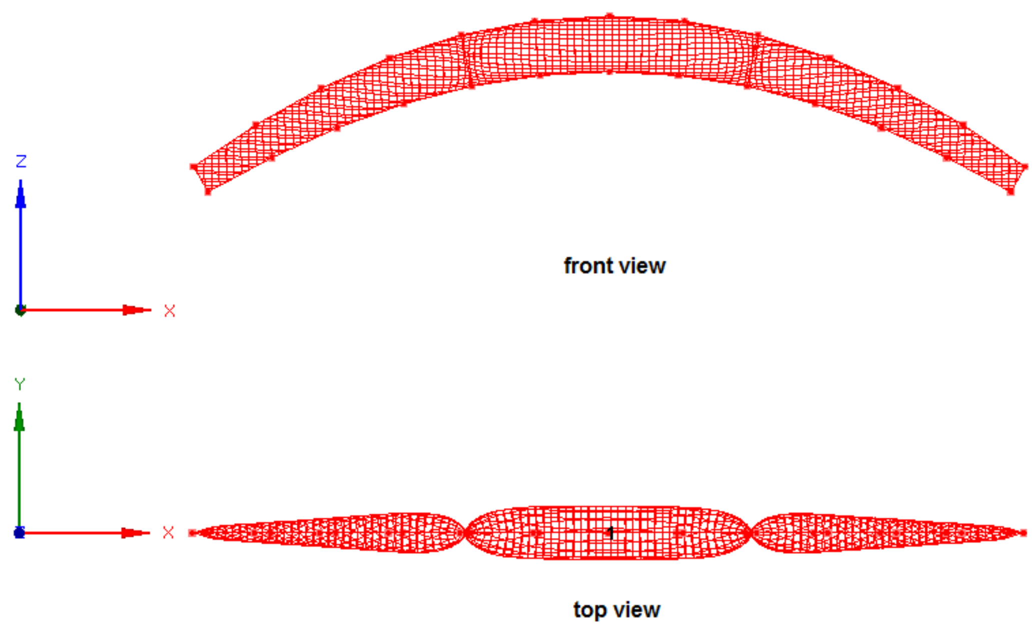

Figure 22: Position 4 of pneumatic chambers (form-finding EASY software)

Those four possibilities were analyzed in terms of following criteria points:

1. Ease to install and to connect to the spline and cable elements

2. Independent chambers, in case one of the fails or looses air pressure
3. Adequate contribution of stabilization and pretension of cable (bottom chord)

In terms of the first point, it is easier to install and connect as few chambres as possible the system includes, because of that it is ideal to have just one chamber (position 2) see figure 20, but it is not considered in position 2 that independent chambers must be considered for security reasons, so positions 3 and 4 are better possibilities. Finally, it was demonstrated that chambers one and six of position 3 have a very low contribution to the stiffness and stabilization of the system, so it was considered position 4 into the analysis.

CHAPTER 3. ANALYSIS OF HYBRID SYSTEM

Structural analysis process deals with material properties, forces considerations, and geometrical modeling assumptions. The hybrid system implies a pneumatic part, spline arch, and cables. Specially designed software (EASY) is used to develop form and statical analysis for pneumatic chambers coupled with spline arch in this chapter. Dlubal RSTAB software is also used to make section assignments and results interpretation easier.

3.1 FORM FINDING OF HYBRID SYSTEM USING SOFTWARE (EASY)

The form-finding process was carried out with EASY software using force density method. In the volume form-finding module, it is considered that the net is generated by force densities either inner pressure or volume. The main attributes in EASY that control pneumatic structures are: volume (V), pressure (P) and temperature (T). Relations of those attributes are governed by Boyle-Mariotte law:

$$(P + P_0)V = m\,R\,T \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

Where m = mass, R = gas constant value.

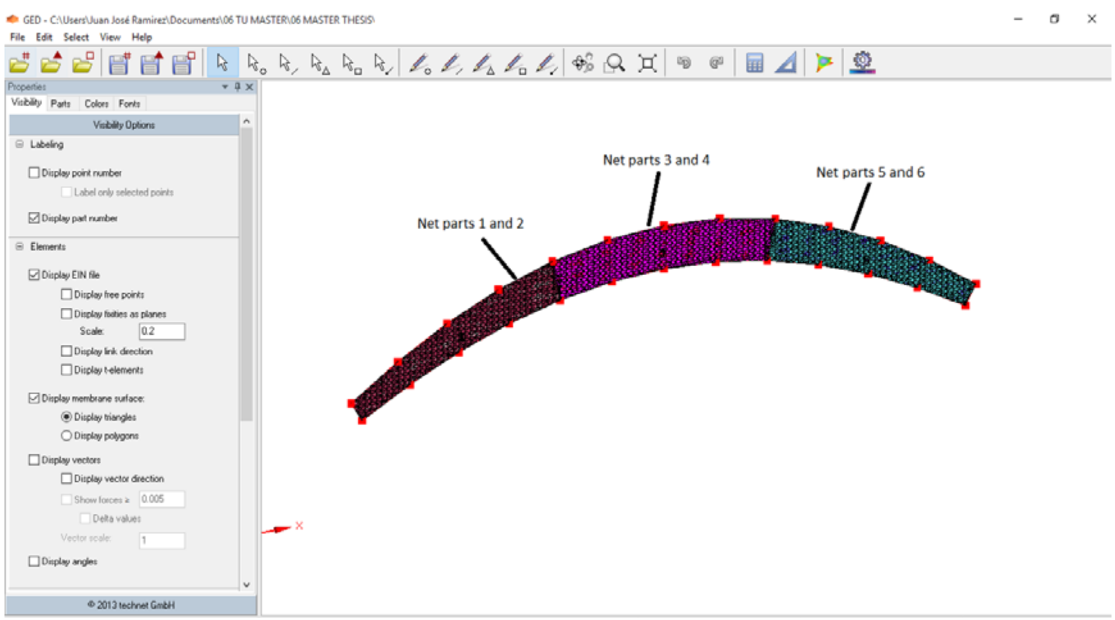

Figure 23: Net parts for pneumatic chambers (Graphical editor GED, EASY software)

Firstly, a "path file" in Rhino + Grasshopper was generated (see chapter 2, figure 18) and then resulting file was imported in edit form ready to draw two layer border lines to define the flatten net structures of the membrane (see figure 23) that later will be inflated into a closed cushion chamber. There are no material properties in this volume form finding process. The model is defined only by force densities.

To get control of the form, it can be chosen with fixing volume or inner pressure. If the volume is selected, then the inner pressure will be calculated by the program and if inner pressure is selected, then the volume will be calculated.

In this work, the volume was defined and then inner pressure was calculated (see figure 24). The inflated result model was a form of equilibrium defined only by force densities, see figure 26.

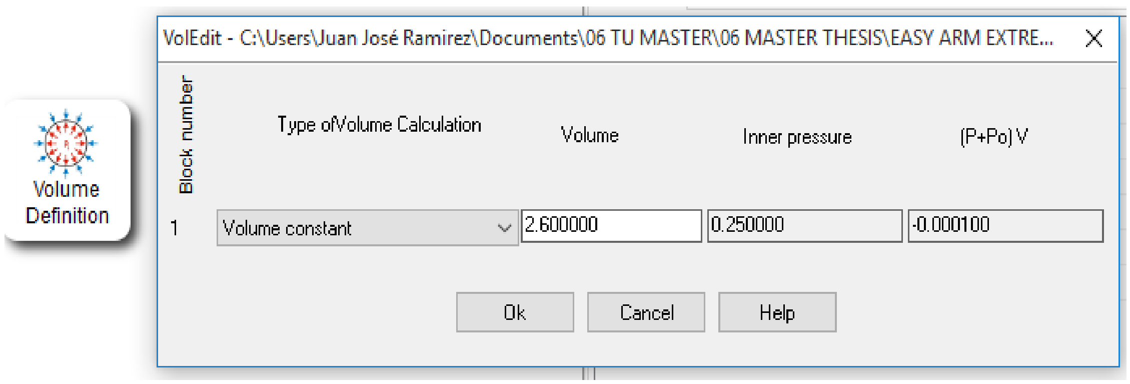

Figure 24: Setting up volume parameters (VolEdit module, EASY software)

In this work volume value was fixed to 2.6 m^3. And calculated value for inner pressure is 4.12 kN/m^2 (see figure 25).

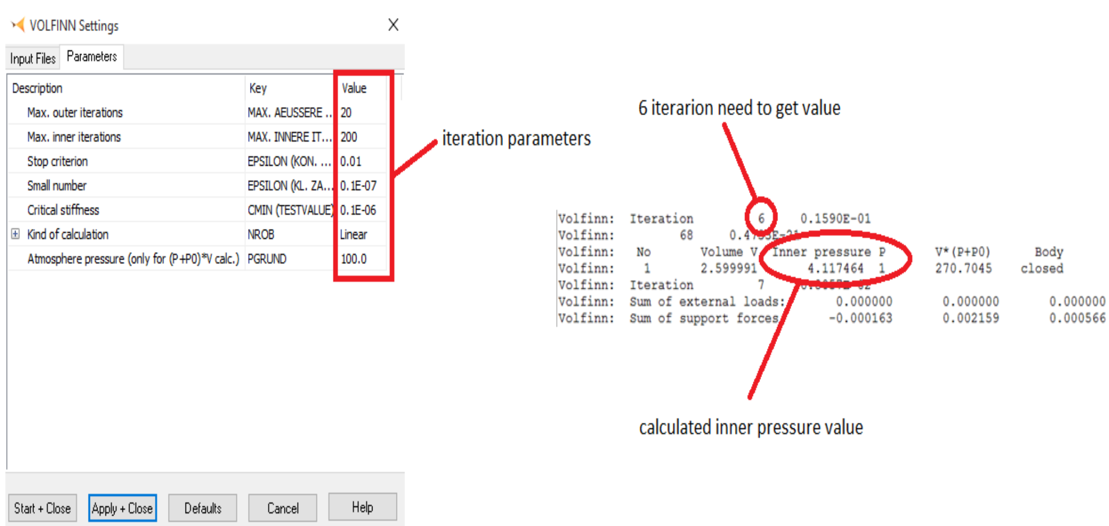

Figure 25: Calculation of inner pressure (Volfin module, EASY software)

Volfin module is in charge of volume inner pressure calculations and some iteration parameters need to be edited in order to get adequate results.

Figure 25 shows that 6 iterations were required to satisfy convergence criteria, and to find out inner pressure value.

The final shape is shown in figure 26. The result; a 3 independent pneumatic chamber with a total volume (of the three chambers) of 2.6 m^3 and inner pressure for each chamber of 4.12 kN/m^2.

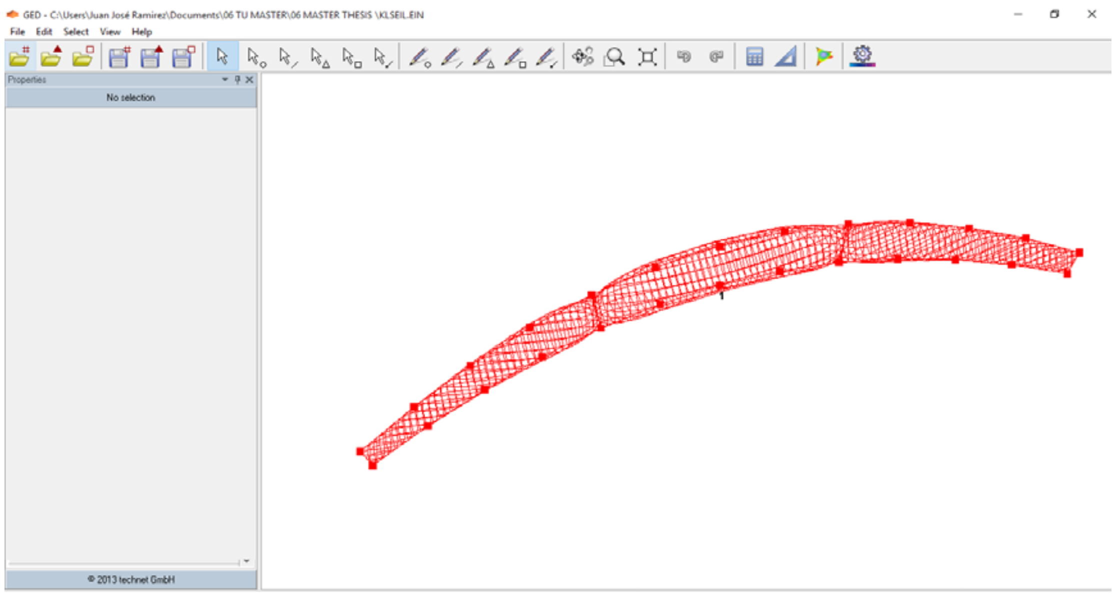

Figure 26: Inflated model (GED, EASY software)

3.2 STRUCTURAL ANALYSIS

Statical analysis process was used to get the stress in the membrane, cable and spline element and to perform buckling analysis later. Once inflated, form-finding model was obtained. It was necessary to add cable and spline additional elements to the structure. It is also needed in EASY to define tension – compression members and only tension members (see figures 27 and 28).

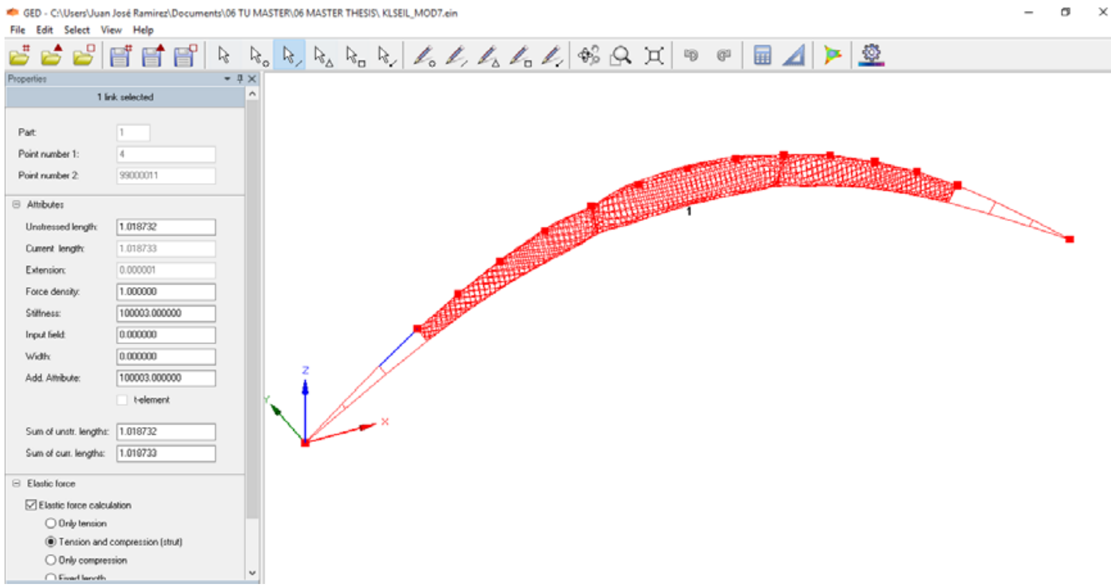

Figure 27: Inflated model with additional elements and members definition (GED, EASY software)

Each element was defined in terms of its "elastic force calculation" properties, and then spline arch (top chord) was defined as tension compression (strut) member, cable (bottom chord) and also vertical truss members were defined as only tension members and inflated chambers as membrane link elements.

When every element is correctly defined in terms of its "elastic force calculation" properties, then the model is ready to be imported into the Beam editor, to setup materials, section properties, boundary conditions, and loads.

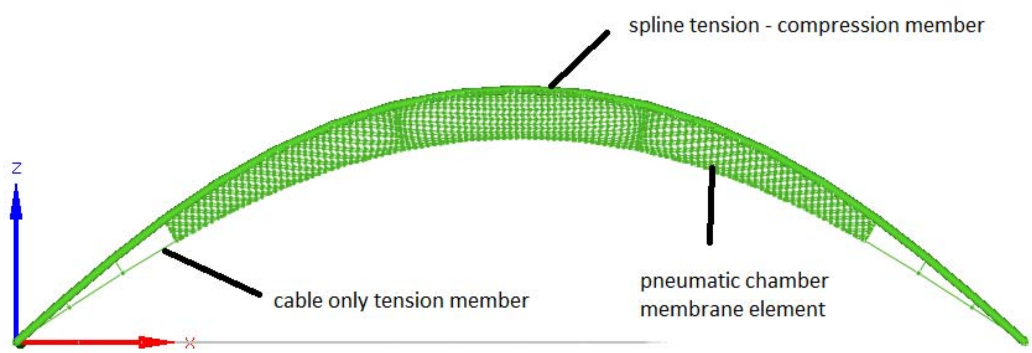

Figure 28: Geometry components hybrid system (Beam editor, EASY software)

3.2.1 Materials and sections properties

In chapter 2, it was decided to use Glass Fiber Reinforced Plastics (CFRP) pipe for the spline arch element because its properties fit very well on spline structure needs in terms of modulus of elasticity (E = 165 GPa), allowable stresses (σ_M = 2800 MPa), low weight (Vol. Weight = 24.9 kN/m^3) and resistance to corrosion. Section diameter of the pipe is 90 mm and wall thickness = 5 mm.

For the cable element, it is considered to use galvanized steel with a modulus of elasticity E = 160 GPa, and Vol. Weight = 80 kN/m^3. 13 mm diameter cable was considered.

In the case of the membrane that will be used for pneumatic chambers, PVC coated polyester fabric (Ferrari Precontraint 502) is considered. It is common that warp direction is a little bit stiffer than weft direction with different modulus of elasticity E_{warp} = 0.8 MN/m, E_{weft} = 0.6 MN/m, assuming cross stiffness E_c = 0.3 MN/m and shear modulus E_s = 0.03 MN/m. In terms of weight, 550 g/m^2 and 0.6 mm thickness was used.

The easiest way to setup properties of materials and sections of cables and struts is exporting the model to RSTAB. Membrane properties can be easily edited also in the Beam Editor.

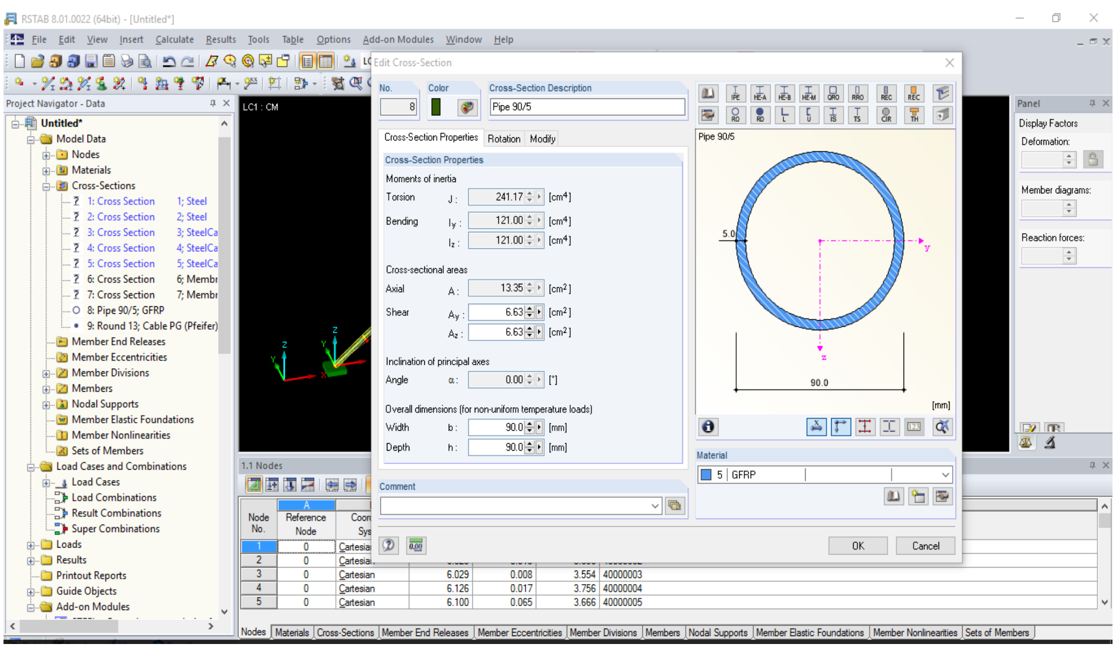

Figure 29: Cross sections and material properties definition of spline element (Dlubal RSTAB software)

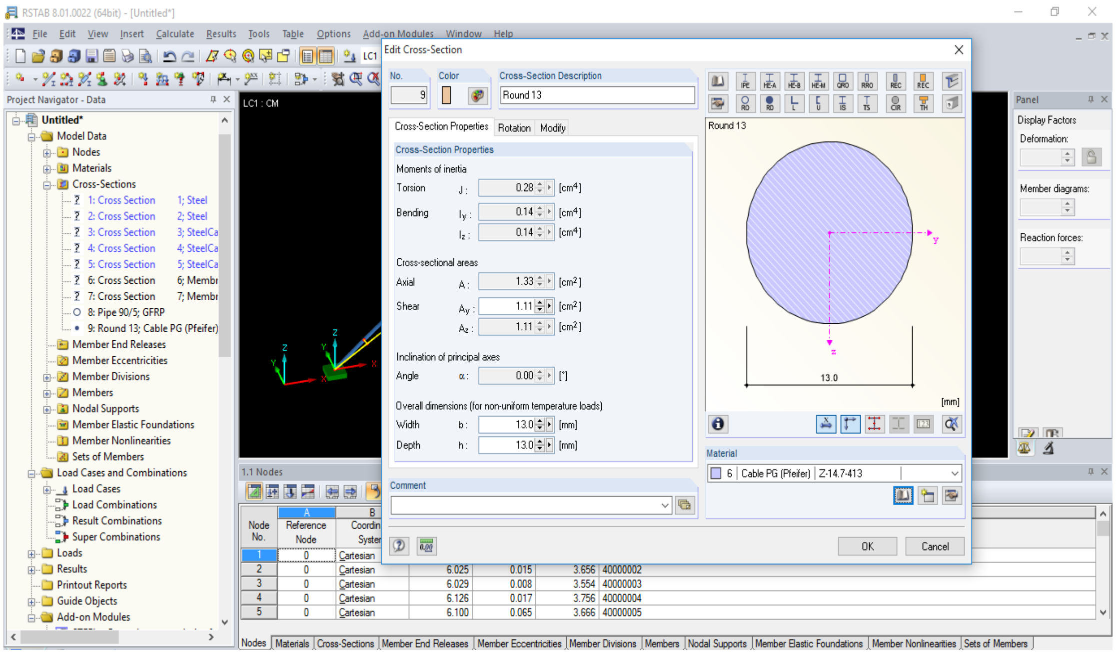

Figure 30: Cross sections and material properties definition of cable element (Dlubal RSTAB software)

Membrane properties are assigned in the Easy beam module see figure 31.

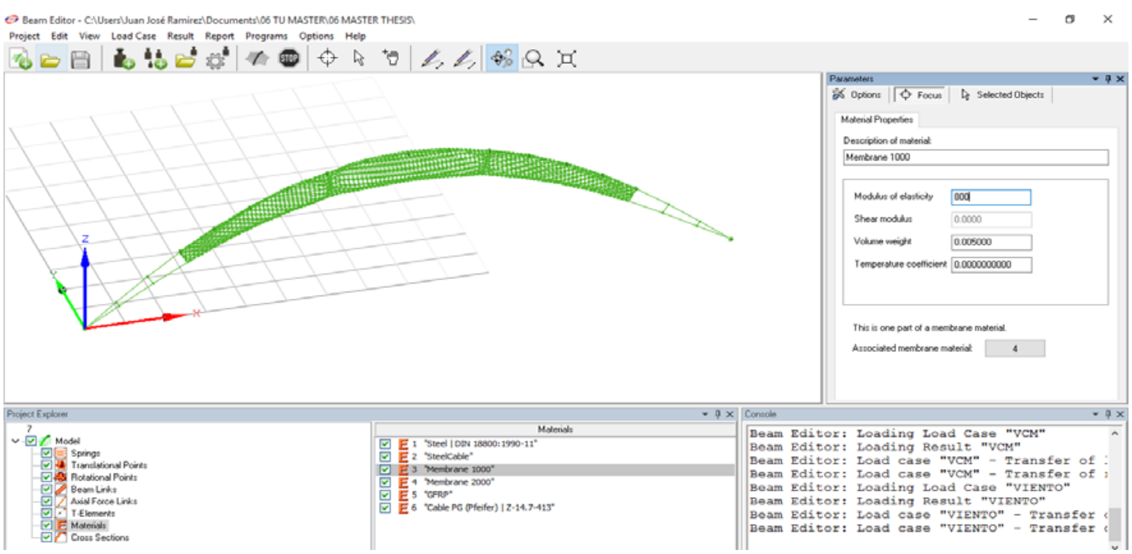

Figure 31: Membrane properties definition (Beam module, EASY software)

3.2.2 Loading considerations and boundary conditions

Once properties and sections are assigned to the model there are considered realistic loading conditions in this work. In LOAD CASE 1, self-weight + pre-stress (1 kN/m) is considered. For wind loading it is supposed a basic pressure p = 0.5 kN/m^2 acting in three different attack angles 0 degrees, 45 degrees and 90 degrees, so LOAD CASE 2: self-weight + pre-stress + Wind$_{0\ \text{degrees}}$, LOAD CASE 3: self-weight + pre-stress + Wind$_{45\ \text{degrees}}$, and LOAD CASE 4: self-weight + pre-stress + Wind$_{90\ \text{degrees}}$.

Because it is not possible to model tension membrane structure + pneumatic spline arch in EASY software it was necessary firstly to model supported membrane without spline arch, then calculate reactions under load cases described before and finally load spline arch model with those previously calculated reactions.

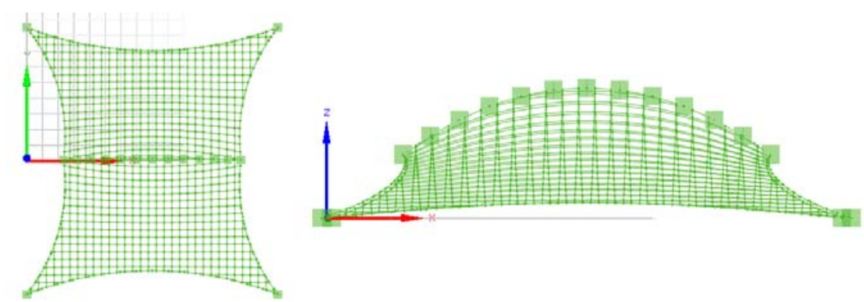

Figure 32: Membrane model (side elevation and plan view), to evaluate loads in arch (Beam module, EASY software)

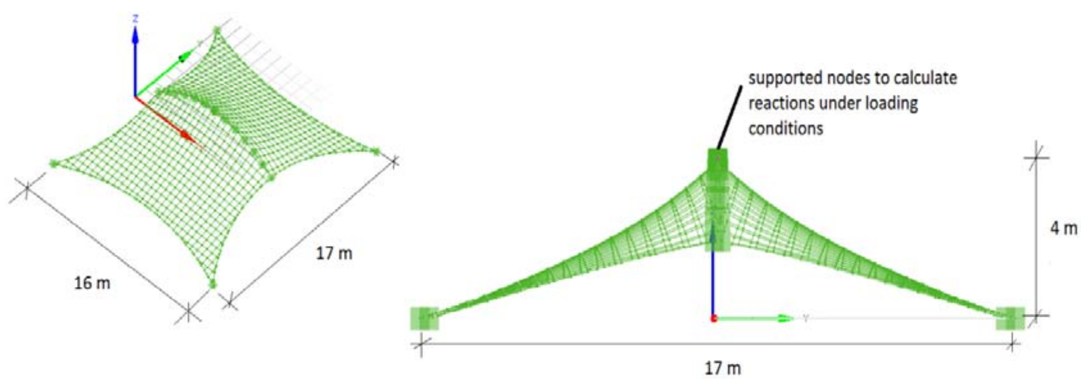

Figure 33: Membrane model (front elevation and perspective view), to evaluate loads in arch (Beam module, EASY software)

Using EASY software helps to assign automatically calculated wind loads. Depending on the geometry obtained by the Form finding tool, then there are defined pressure and suction zones with constant pressure values acting normally to the surface. It is assumed in this work a constant pressure of 0.5 kN/m^2 acting as suction (-) or pressure (+) in terms of the zones defined by software under different attack directions, see figures 34, 35 and 36.

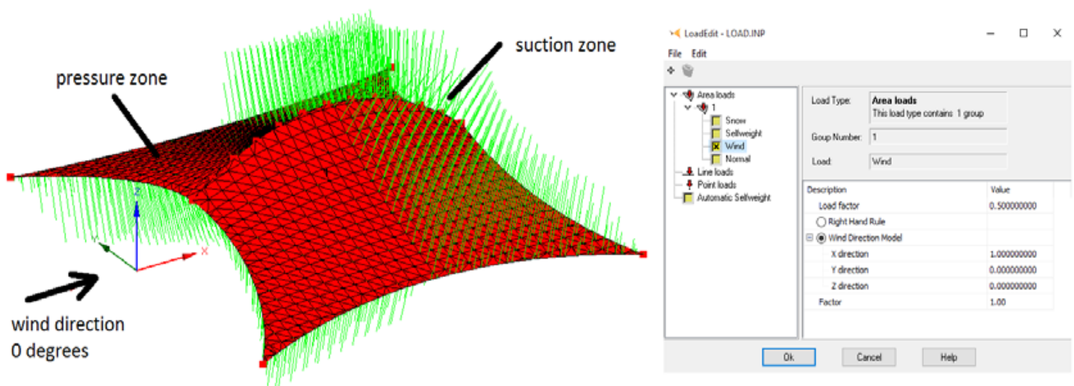

Figure 34: Force vectors under wind loading at 0 degrees direction (GED module-left side, STATICAL ANALYSIS module-right side, EASY software)

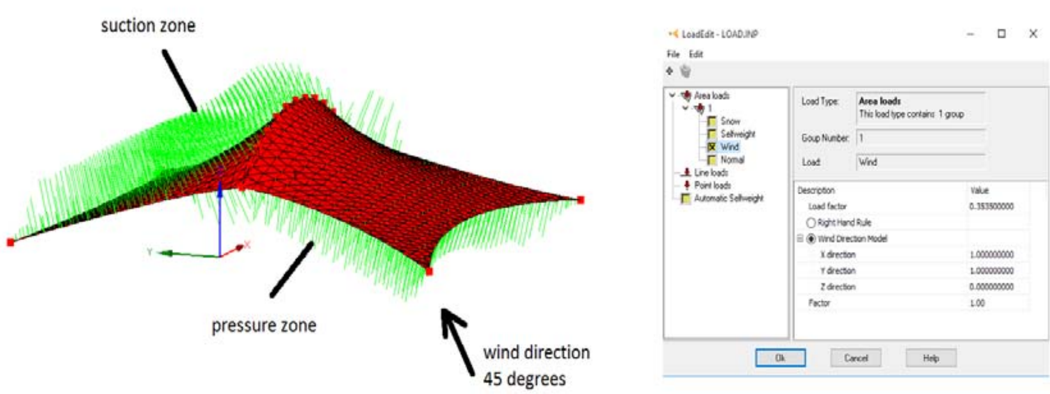

Figure 35: Force vectors under wind loading at 45 degrees direction (GED module-left side, STATICAL ANALYSIS module-right side, EASY software)

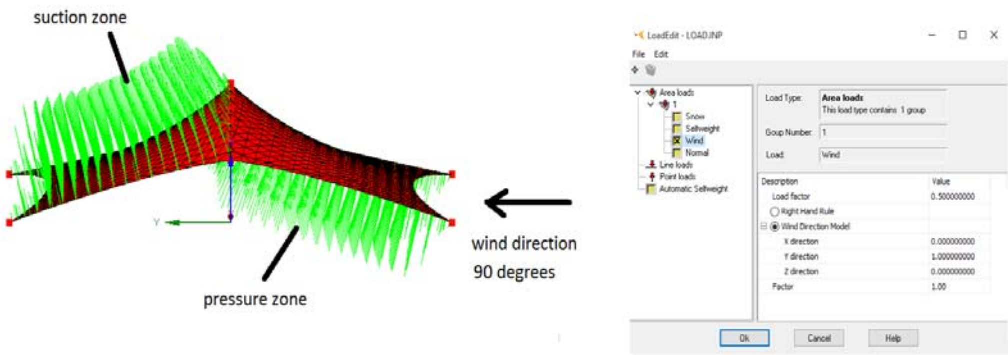

Figure 36: Force vectors under wind loading at 90 degrees direction (GED module-left side, STATICAL ANALYSIS module-right side, EASY software)

The next step is to calculate reactions and nodal support forces under each load case with EASY Beam and then export the model to DLUBAL RSTAB software to generate summary tables. See tables 3, 4, 5 and 6. First column in those tables corresponds to the number of support node (see figure 37 to find each node), next three columns support forces in “X”, “Y” and “Z” directions respectively in kN and last three columns are moments around respective axis in kN m.

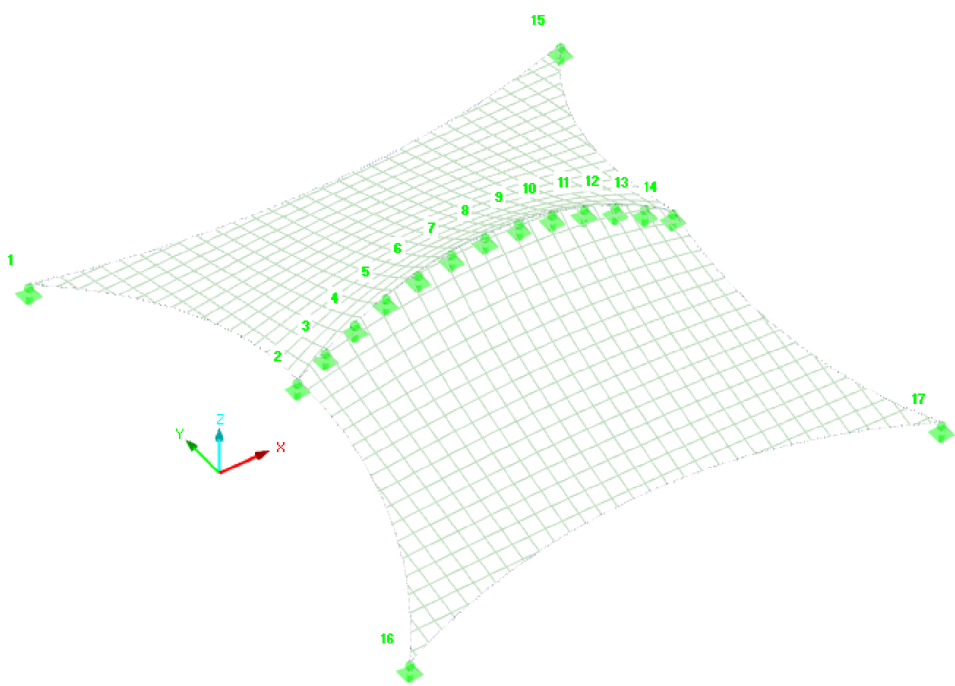

Figure 37: Support nodes numbering (DLUBAL RSTAB software)

Table 3: Nodal support forces under LOAD CASE 1

Node No.	Support Forces [kN]			Support Moments [kNm]		
	$P_{X'}$	$P_{Y'}$	$P_{Z'}$	$M_{X'}$	$M_{Y'}$	$M_{Z'}$
2	3.11	0.03	-3.00	0.00	0.00	0.00
3	0.29	-0.06	-0.73	0.00	0.00	0.00
4	0.15	0.03	-1.05	0.00	0.00	0.00
5	0.49	0.02	-1.16	0.00	0.00	0.00
6	0.37	-0.03	-1.38	0.00	0.00	0.00
7	0.00	0.00	-1.54	0.00	0.00	0.00
8	0.00	0.01	-1.56	0.00	0.00	0.00
9	0.00	0.00	-1.54	0.00	0.00	0.00
10	-0.37	-0.03	-1.38	0.00	0.00	0.00
11	-0.49	0.02	-1.16	0.00	0.00	0.00
12	-0.15	0.03	-1.05	0.00	0.00	0.00
13	-0.29	-0.06	-0.73	0.00	0.00	0.00
14	-3.11	0.03	-3.00	0.00	0.00	0.00

Table 4: Nodal support forces under LOAD CASE 2

Node No.	Support Forces [kN]			Support Moments [kNm]		
	$P_{X'}$	$P_{Y'}$	$P_{Z'}$	$M_{X'}$	$M_{Y'}$	$M_{Z'}$
2	13.87	0.20	-20.81	0.00	0.00	0.00
3	1.35	-0.09	-2.85	0.00	0.00	0.00
4	1.27	0.06	-5.68	0.00	0.00	0.00
5	2.13	-0.02	-7.02	0.00	0.00	0.00
6	1.36	0.07	-7.57	0.00	0.00	0.00
7	0.30	-0.05	-6.20	0.00	0.00	0.00
8	0.11	-0.01	-2.24	0.00	0.00	0.00
9	0.09	0.00	-0.88	0.00	0.00	0.00
10	-0.32	-0.03	-0.90	0.00	0.00	0.00
11	-0.50	0.06	-0.84	0.00	0.00	0.00
12	0.19	0.03	-1.13	0.00	0.00	0.00
13	-0.40	-0.23	-0.92	0.00	0.00	0.00
14	-12.37	0.22	-7.90	0.00	0.00	0.00

Table 5: Nodal support forces under LOAD CASE 3

Node No.	Support Forces [kN]			Support Moments [kNm]		
	$P_{X'}$	$P_{Y'}$	$P_{Z'}$	$M_{X'}$	$M_{Y'}$	$M_{Z'}$
2	10.86	49.70	-10.13	0.00	0.00	0.00
3	0.13	-1.42	-3.23	0.00	0.00	0.00
4	0.46	-2.36	-3.54	0.00	0.00	0.00
5	1.15	-2.39	-3.70	0.00	0.00	0.00

6	0.35	-2.30	-4.16	0.00	0.00	0.00
7	-0.01	-2.24	-4.44	0.00	0.00	0.00
8	0.00	-2.18	-4.45	0.00	0.00	0.00
9	0.02	-2.25	-4.44	0.00	0.00	0.00
10	-0.33	-2.32	-4.16	0.00	0.00	0.00
11	-1.10	-2.44	-3.67	0.00	0.00	0.00
12	-0.36	-2.40	-3.25	0.00	0.00	0.00
13	0.22	-1.46	-2.24	0.00	0.00	0.00
14	-10.31	50.47	-8.39	0.00	0.00	0.00

Table 6: Nodal support forces under LOAD CASE 4

Node	Support Forces [kN]			Support Moments [kNm]		
No.	$P_{X'}$	$P_{Y'}$	$P_{Z'}$	$M_{X'}$	$M_{Y'}$	$M_{Z'}$
2	14.02	63.52	-10.95	0.00	0.00	0.00
3	0.01	-1.78	-3.65	0.00	0.00	0.00
4	0.52	-2.91	-4.43	0.00	0.00	0.00
5	1.40	-2.95	-4.75	0.00	0.00	0.00
6	0.40	-2.83	-5.36	0.00	0.00	0.00
7	-0.04	-2.78	-5.71	0.00	0.00	0.00
8	0.00	-2.69	-5.72	0.00	0.00	0.00
9	0.04	-2.78	-5.71	0.00	0.00	0.00
10	-0.40	-2.83	-5.36	0.00	0.00	0.00
11	-1.40	-2.95	-4.75	0.00	0.00	0.00
12	-0.52	-2.91	-4.43	0.00	0.00	0.00
13	-0.01	-1.78	-3.66	0.00	0.00	0.00
14	-14.02	63.52	-10.96	0.00	0.00	0.00

It can be seen in all the previous tables that nodes first and last presents higher values than the rest of the nodes, that can be explained because a border cable is modeled in order to represent better real conditions and membrane detailing. Boundary conditions in this work refer to the kind and location of supports.

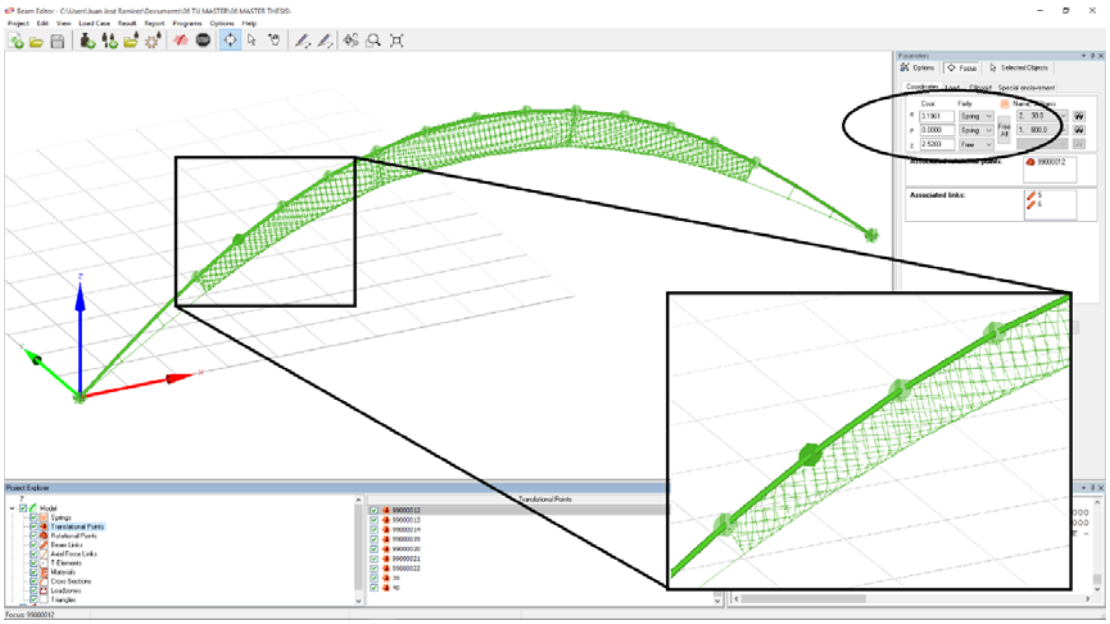

Figure 38: Boundary conditions, lateral restriction (BEAM Editor, EASY software)

In figure 38 it is shown that lateral restriction effect of the pre-stressed membrane is taken into consideration assigning spring properties to the supports. In EASY software it is possible to define certain stiffness of the spring in each degree of freedom so it is considered in this work a spring stiffness in “X” direction with a value of 0.03 MN/m, then spring stiffness in “Y” direction with a value of 0.8 MN/m, and finally a value of zero for the rest of the degrees of freedom.

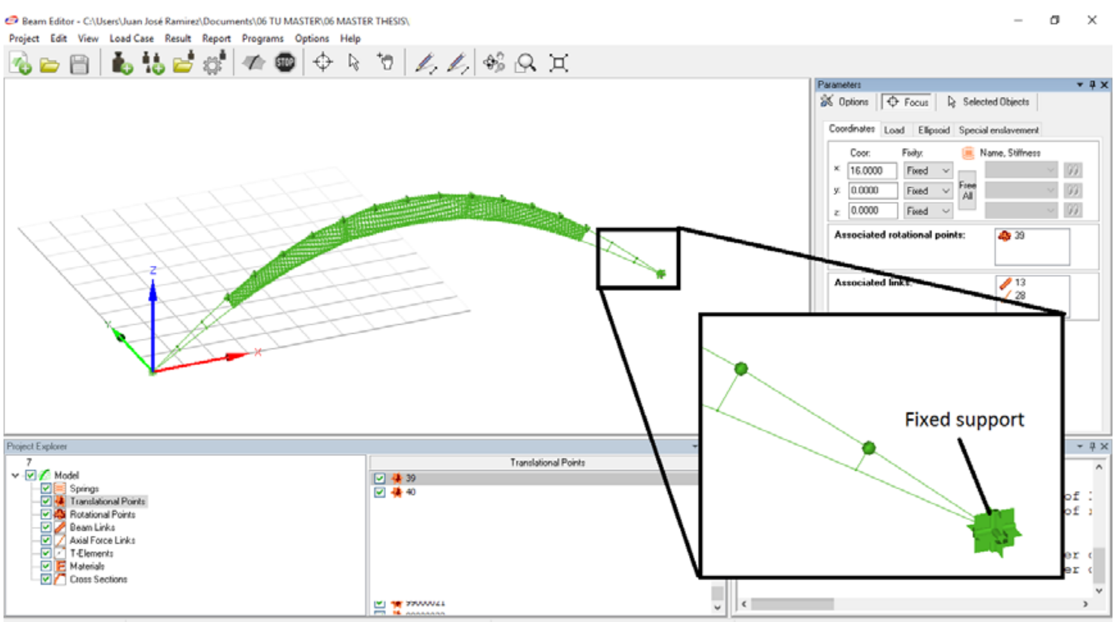

Figure 39: Boundary conditions, fixed support node (BEAM Editor, EASY software)

Figures 39 and 40 show support considerations in the first and last nodes of the arch. A totally fixed support condition is considered in both cases, restricted to translation and rotation. It is evaluated in the analysis section if moment release at this elements is needed in terms of bending moment values.

3.2.3 Spline arch system without pneumatic chambers under real loading conditions

Just for comparison purposes and to be sure that the spline arch without pneumatic chambers is not stable under real loading conditions, behavior of the same membrane used to evaluate loads with a spline arch supporting element is modeled and reviewed. see figure 41.

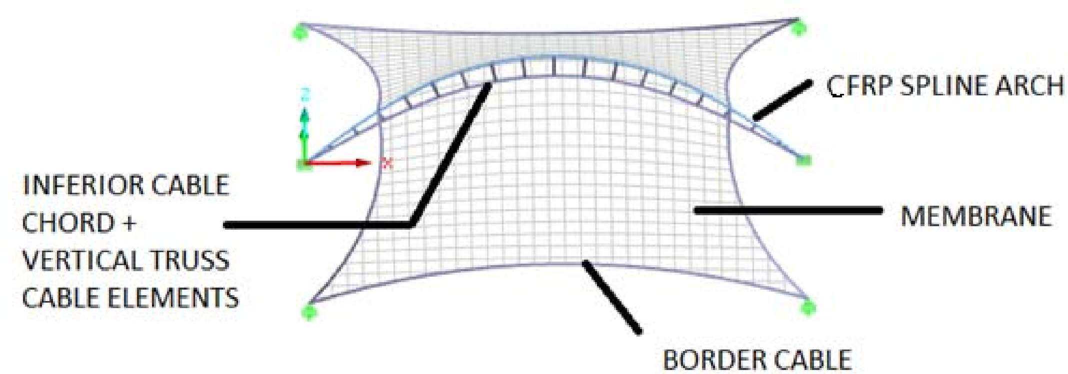

Figure 40: Spline arch truss system without pneumatic chambers (BEAM Editor, EASY software)

We considered, in this model, an inferior cable chord and vertical cable truss elements to increase in plain stiffness of the support arch. Model is loaded with the same LOAD CASES considered for the calculation of reactions on joints supported membrane model. See figures 42, 43, 44 and 45.

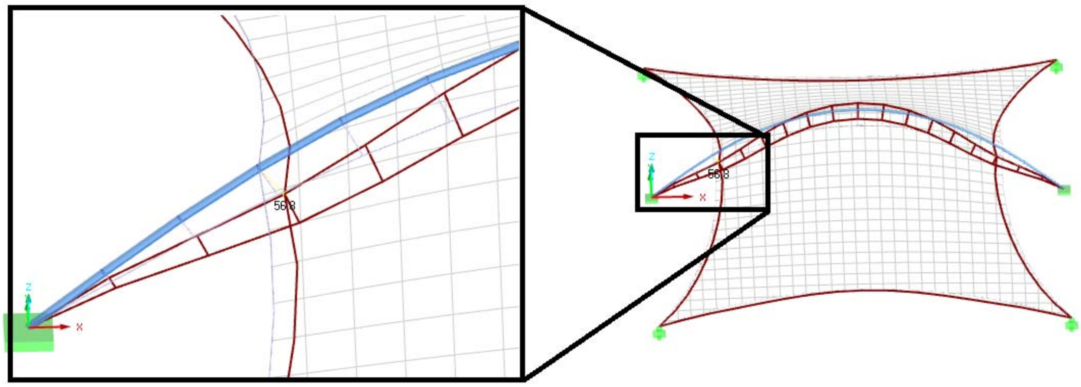

Figure 41: Spline arch truss deformed shape LOAD CASE 1 (DLUBAL RSTAB software)

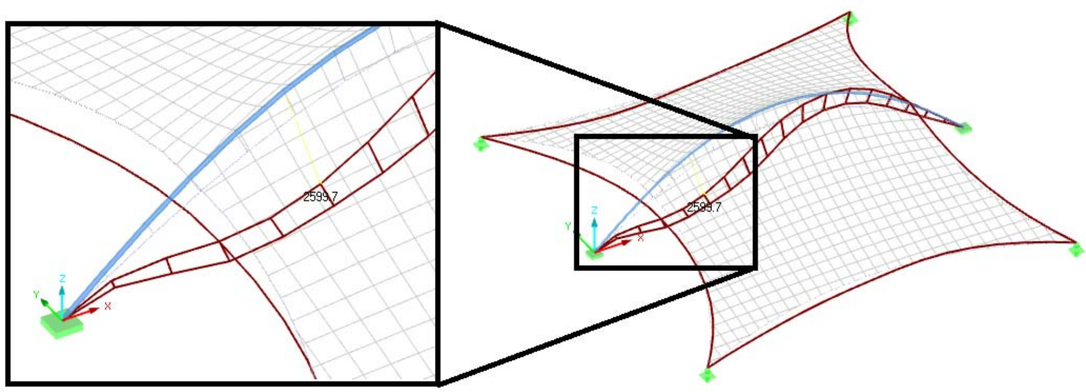

Figure 42: Spline arch truss deformed shape LOAD CASE 2 (DLUBAL RSTAB software)

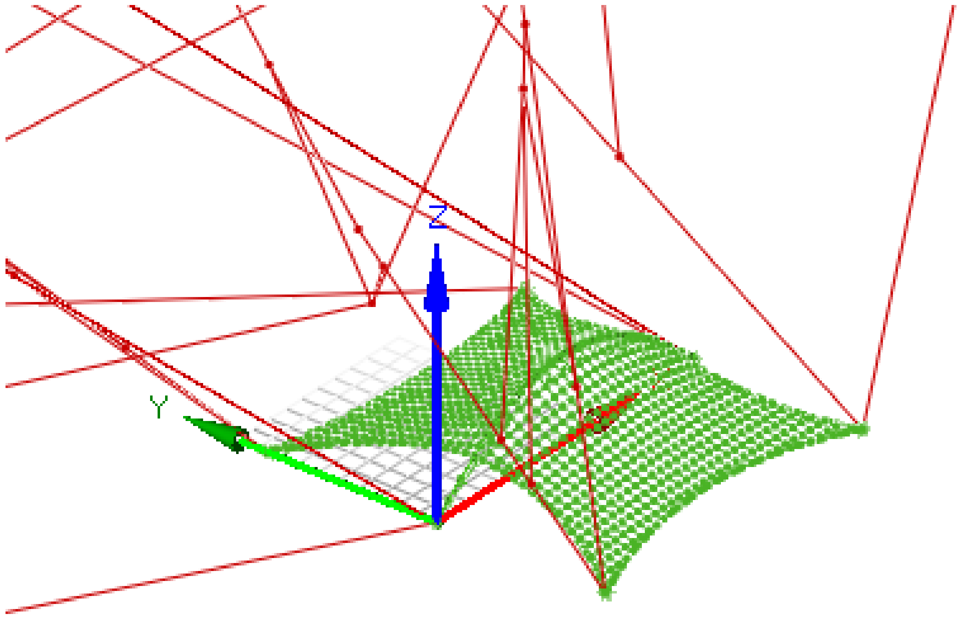

Figure 43: Spline arch truss deformed shape LOAD CASE 3 (BEAM Editor, EASY software)

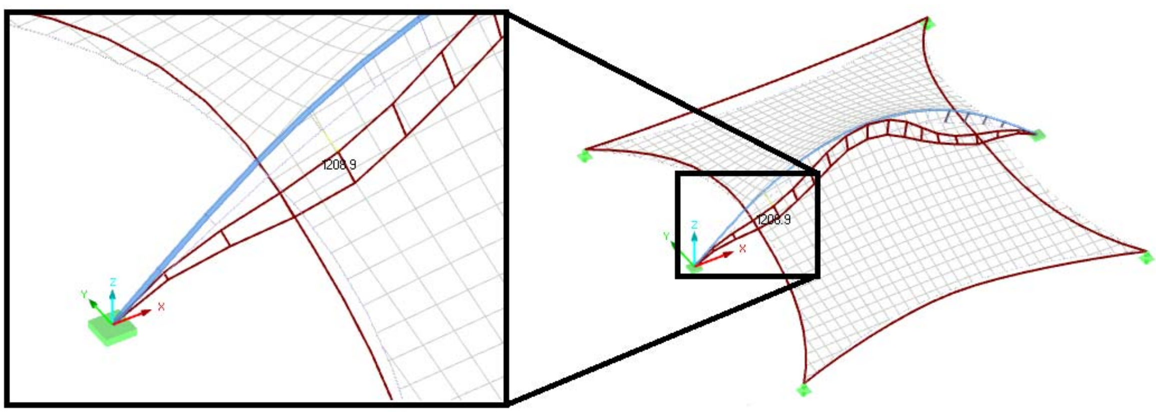

Figure 44: Spline arch truss deformed shape LOAD CASE 4 (DLUBAL RSTAB software)

For LOAD CASE 1 (Self-weight + pre-stress), it can be seen that structure is stable and maximum deformation value is 56.8 mm. When wind load acts at 0 degrees (LOAD CASE 2) there is convergence in software calculations, but deformation is very big reaching 2599.7 mm value what makes internal space unable to be there. In LOAD CASE 3 (wind acting in a 45 degree-direction), there is no convergence in calculations, which means instability of the structure. Finally in LOAD CASE 4 (the wind in 90 degrees attack direction) there is also convergence in calculations but deformation is also big reaching 1208.9 mm value. In conclusion, it is demonstrated that spline arch also with cable truss elements is not stiff enough to provide adequate and stable support to the membrane.

3.2.4 Spline arch system with pneumatic chambers under real loading conditions

Figure 45 shows deformed shape of the hybrid system subjected to LOAD CASE 1, it can be seen that maximum deformation reached 88.3 mm.

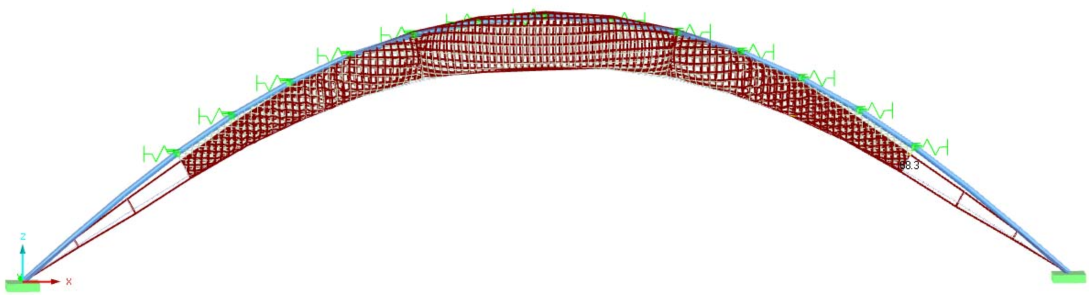

Figure 45: Spline arch plus pneumatic chambers LOAD CASE 1 (DLUBAL RSTAB software)

For LOAD CASE 2 there was not convergence and because of that no results to be shown.

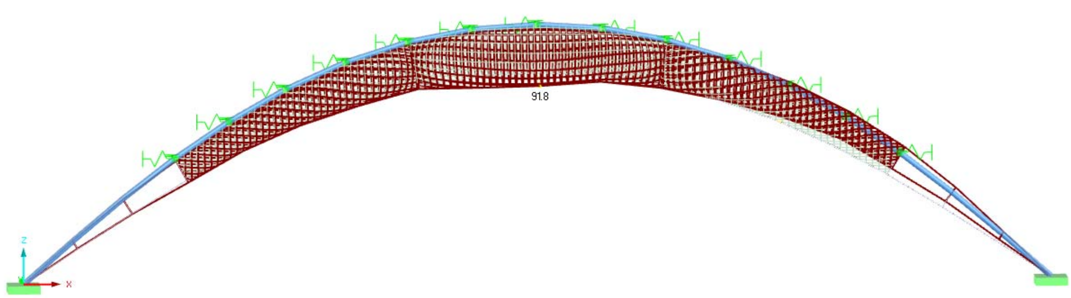

Figure 46 : Spline arch plus pneumatic chambers LOAD CASE 3 (DLUBAL RSTAB software)

For LOAD CASE 3 structure is stable and maximum deformation value reached is 91.8 mm. In the case of 90 degrees of wind attack direction (LOAD CASE 4), there is also no convergence with this configuration. It is considered in this work that when there´s no convergence in the model, the structure is not stable and stiffness must be incremented.

So after several configurations, in figure 47 a stable minimal element proposal can be seen.

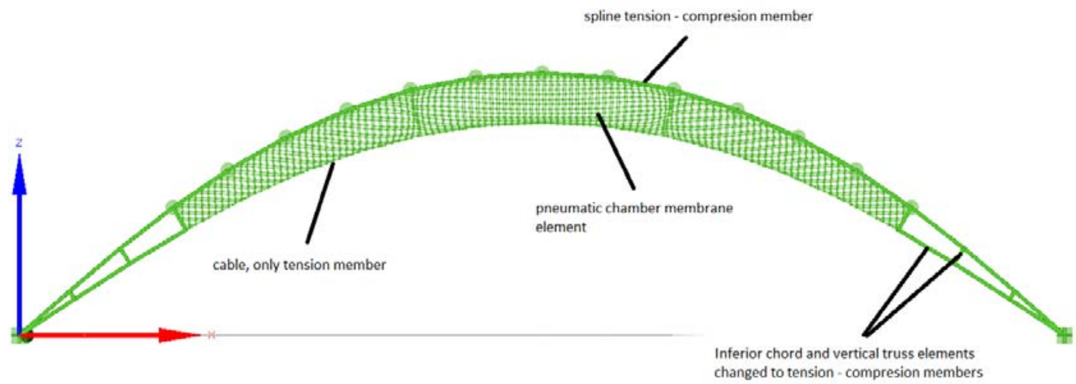

Figure 47: Final geometry components hybrid system (Beam editor, EASY software)

In the rest of this work, it is demonstrated that structure configuration showed in figure 47 is stable and also adequate to resist real external wind loading conditions (load cases 1 to 4). In figures 48 to 51 it can be seen deformed shape and maximum deformations under load cases 1 to 4 respectively.

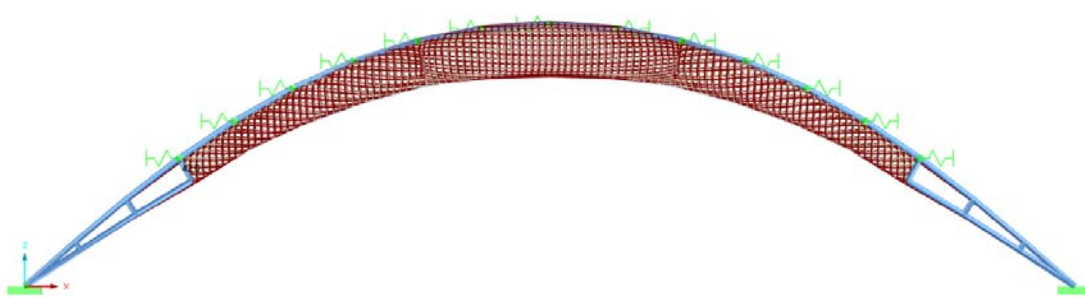

Figure 48: Spline arch plus pneumatic chambers (Final configuration) LOAD CASE 1 (DLUBAL RSTAB software)

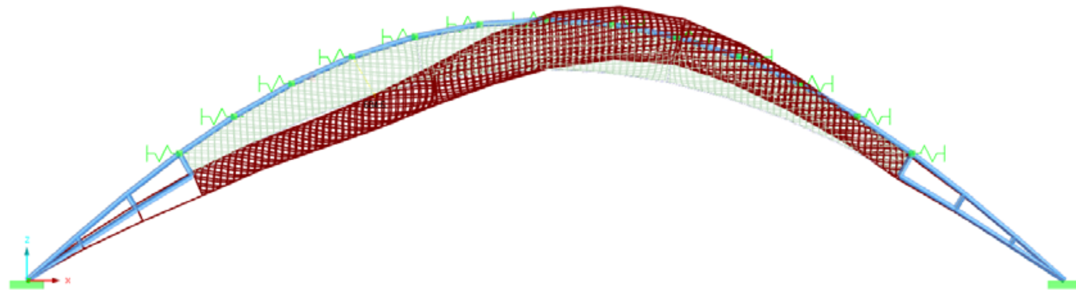

Figure 49: Spline arch plus pneumatic chambers (Final configuration) LOAD CASE 2 (DLUBAL RSTAB software)

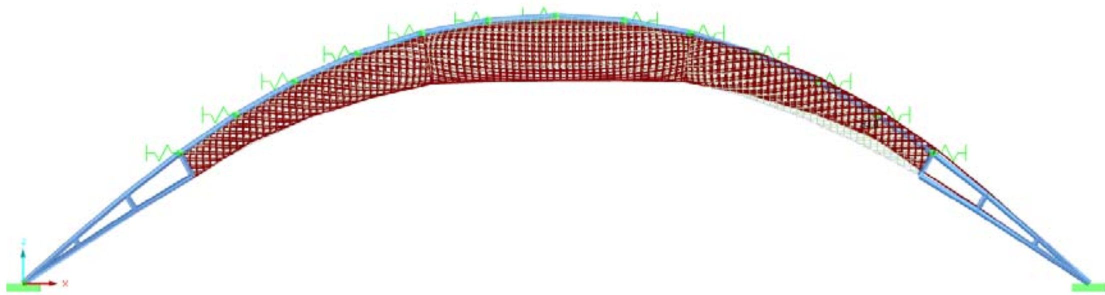

Figure 50: Spline arch plus pneumatic chambers (Final configuration) LOAD CASE 3 (DLUBAL RSTAB software)

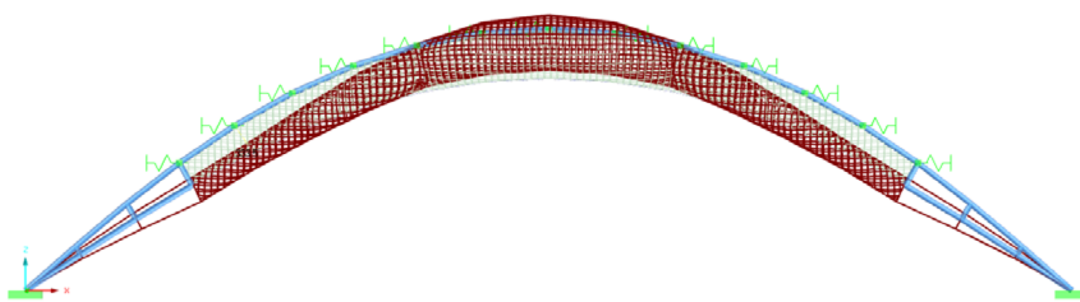

Figure 51: Spline arch plus pneumatic chambers (Final configuration) LOAD CASE 4 (DLUBAL RSTAB software)

Table 7 shows maximum deformations reached by the final structure configuration for each load case.

Table 7: Maximum deformations

LOAD CASE	Maximum deformation (mm)

1	46
2	225
3	74
4	77

3.3 ANALYSIS OF RESULTS AND CONCLUSIONS

3.3.1 Axial force diagrams on frames and cables

Axial force diagrams for spline arch, truss elements and cables are presented in this section.

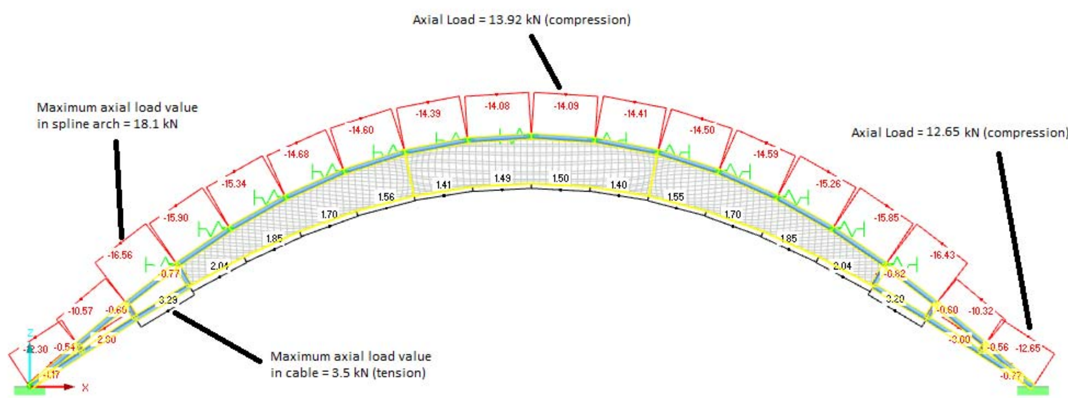

Figure 52: Axial force diagram (kN) LOAD CASE 1 (DLUBAL RSTAB software)

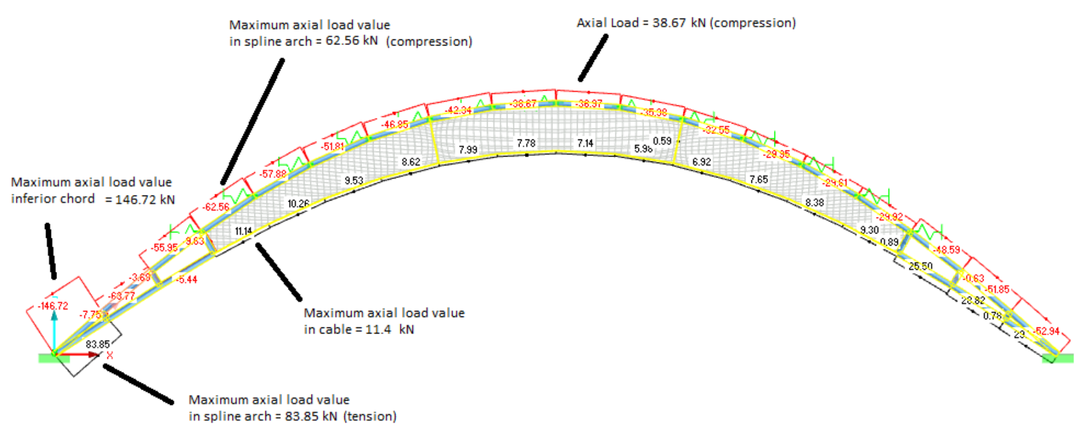

Figure 53: Axial force diagram (kN) LOAD CASE 2 (DLUBAL RSTAB software)

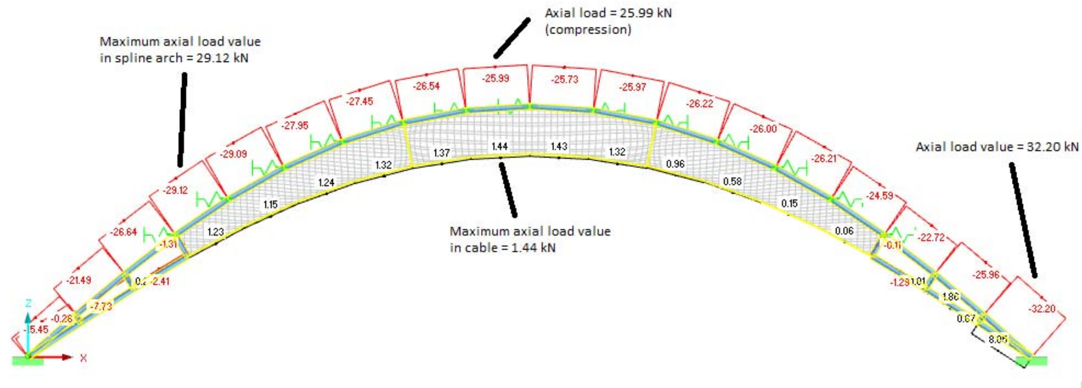

Figure 54: Axial force diagram (kN) LOAD CASE 3 (DLUBAL RSTAB software)

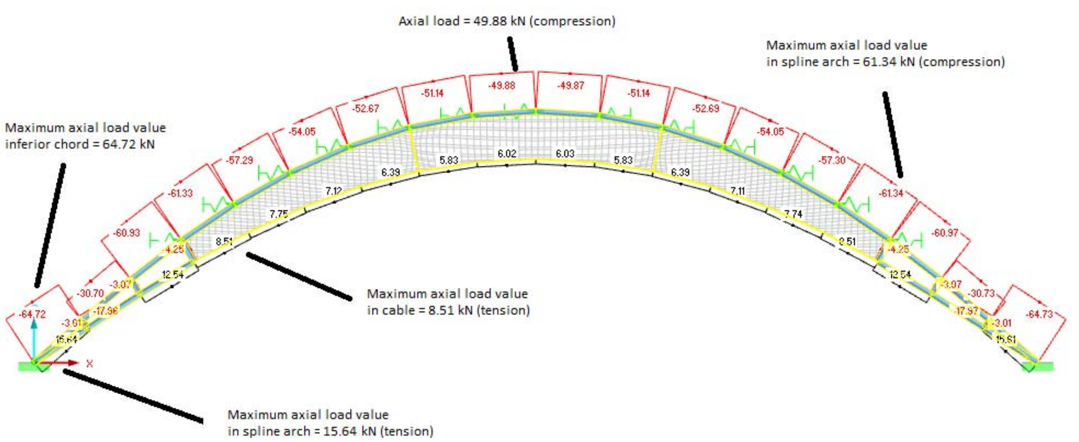

Figure 55: Axial force diagram (kN) LOAD CASE 4 (DLUBAL RSTAB software)

3.3.2 Bending moment diagrams on frames

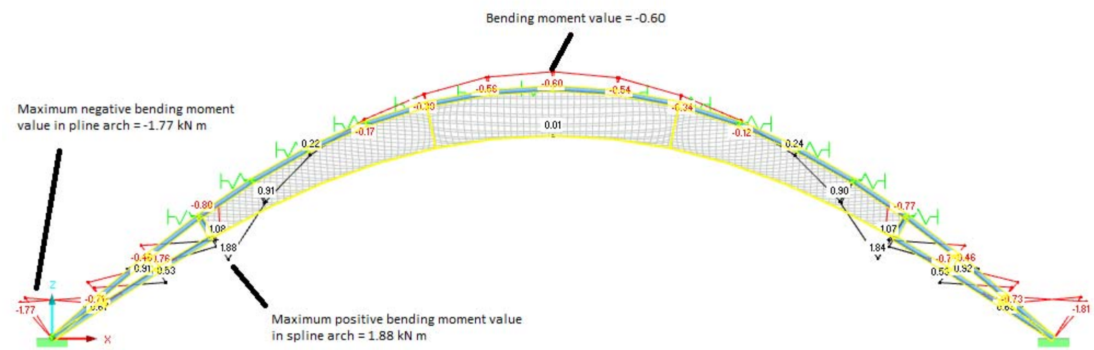

Figure 56: Bending moment diagram (kN m) LOAD CASE 1 (DLUBAL RSTAB software)

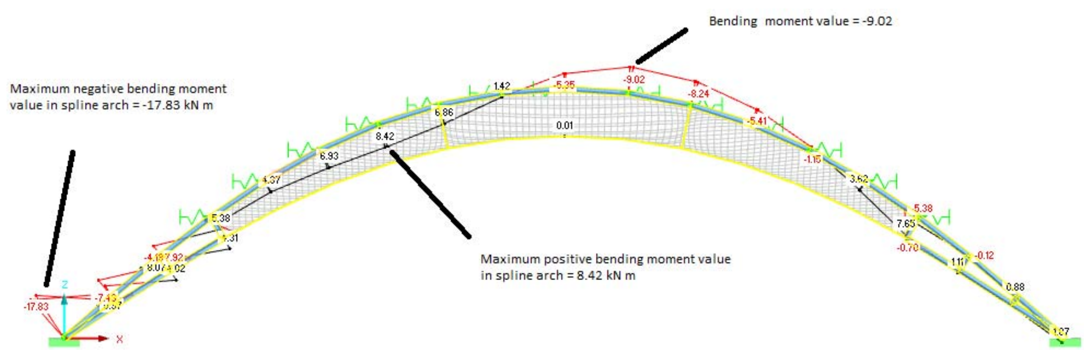

Figure 57: Bending moment diagram (kN m) LOAD CASE 2 (DLUBAL RSTAB software)

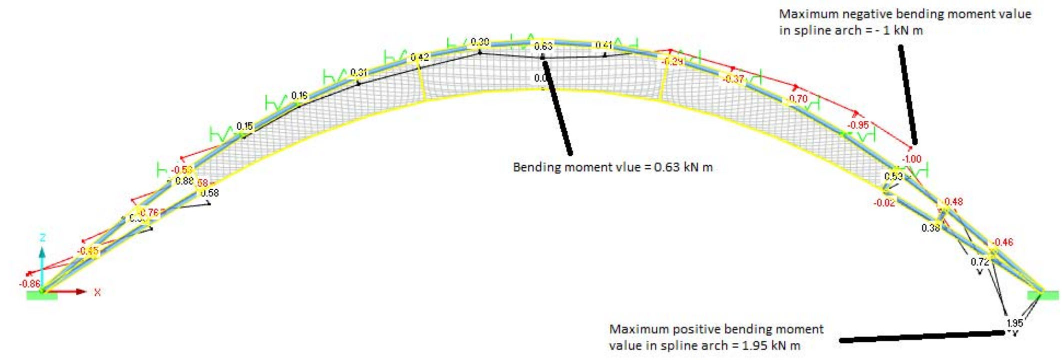

Figure 58: Bending moment diagram (kN m) LOAD CASE 3 (DLUBAL RSTAB software)

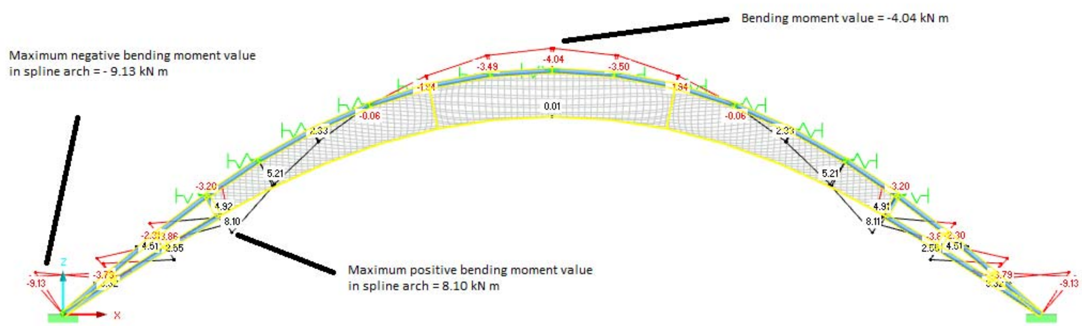

Figure 59: Bending moment diagram (kN m) LOAD CASE 4 (DLUBAL RSTAB software)

It is evaluated spline element under combined compression and bending moment stresses with equation 11.

$$c = \frac{\sigma_{axial}}{\sigma_{allowable\ axial}} + \frac{\sigma_{bending}}{\sigma_{allowable\ bending}} < 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Where: $\sigma_{axial} = \frac{Axial\ force}{Cross\ section\ area}$, $\sigma_{bending} = \frac{Bending\ moment}{Section\ modulus}$

In the pre-bending state, there is a maximum moment value in the middle of the arch of 6.07 kN m, that is considered into calculations, see figure 60.

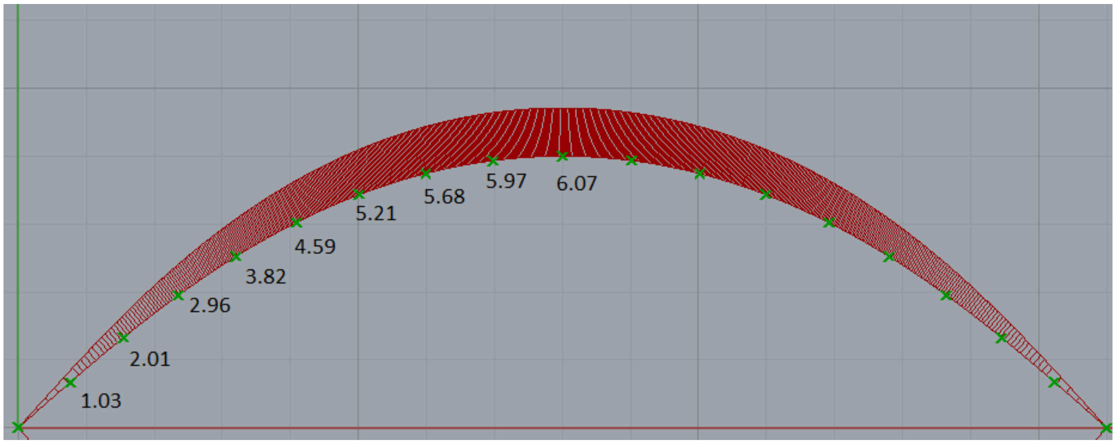

Figure 60: Pre-Bending moment diagram (values in kN m, negative and symmetric) ”elastica” grasshopper tool

After checking results, it is clear that LOAD CASE 2 is critical in the analysis and design process. So it is considered to evaluate eq. 9 in 4 critical zones, see figure 61 and table 8.

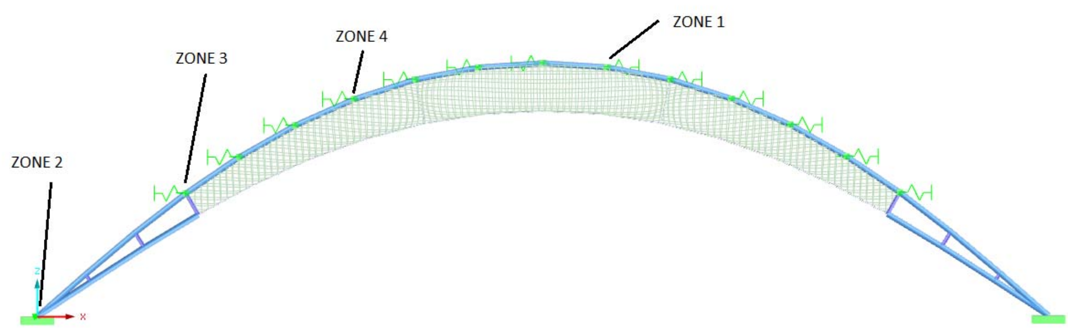

Figure 61: Critical zones of analysis (DLUBAL RSTAB software)

Table 8: Combined stresses check

	pre bending moment	bending moment	total bending moment	$\sigma_{bending}$	pre axial load	axial load	total axial load	σ_{axial}	σ bend. / σ allowable bend.	σ axial / σ allowable axial	c
ZONE	(kN m)	(kN m)	(kN m)	(Mpa)	kN	kN	kN	(Mpa)			
1	-5.97	-9.02	-15.0	-557.5	2	38.7	40.7	30.5	0.38	0.30	0.68
2	0	-17.83	-17.8	-663.1	2	7.8	9.8	7.3	0.45	0.07	0.52
3	-2.964	8.1	5.1	191.0	2	53.0	55.0	41.2	0.13	0.41	0.54
4	-5.21	8.42	3.2	119.4	2	57.0	59.0	44.2	0.08	0.44	0.52

Table 8 shows that interaction for the critical combination for any zone is less than 1.

3.3.3 Buckling analysis

In chapter 2 it is clear that spline element works in a controlled buckled state to get the deformed shape "elastica" and then in plain stiffness is increased by the pneumatic system in this work. Simitses (1976), taken from Adriaenssens 2000, found an expression to evaluate elastic buckling load considering loads that remain normal to the deflected reference axis working spline element alone, without pneumatic system:

$$N_t = EI\left(\frac{\pi^2}{(0.5s)^2} - \frac{1}{R^2}\right)\alpha \text{ .... } (12)$$

Where s – arc length,

$$R = \frac{l_c}{2\sin\alpha}\text{................}(13)$$

And for α See figure 62.

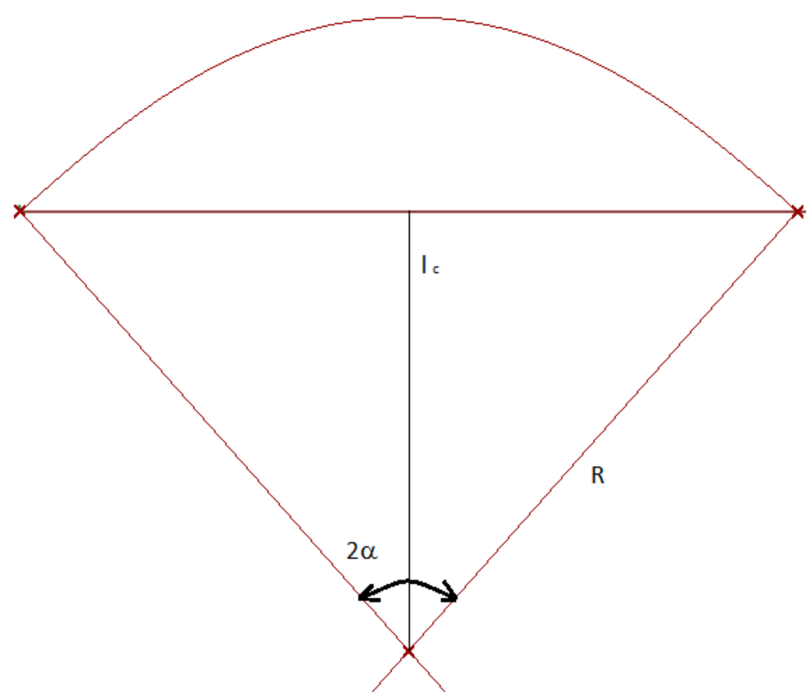

Figure 62: Elastic buckling expression drawing, Microsoft paint

After doing calculations, N_t = 18.69 kN. If we compare this value with total axial load (table 8) acting on CFRP element under real loading conditions, it is clear that without pneumatic system contribution to the stiffness, spline arch would be unstable and buckle in terms of Simitses linear buckling load criteria. In this thesis, buckling criterion was taken into consideration in terms of convergence of calculations in EASY software. In other words, it is considered that if no convergence means an unstable system and buckling assuming if convergence reached it means stable system and no buckling assumption.

3.3.4 Axial forces in membrane

Member internal forces tables were generated and exported to excel worksheets to evaluate values. Higher values = 21.3 kN/m were founded in elements near to the cable inferior chord. Considering that allowable stress in the membrane is 11.2 kN/m, then in that zone it will be necessary to put additional membrane reinforcement, see details in chapter 4.

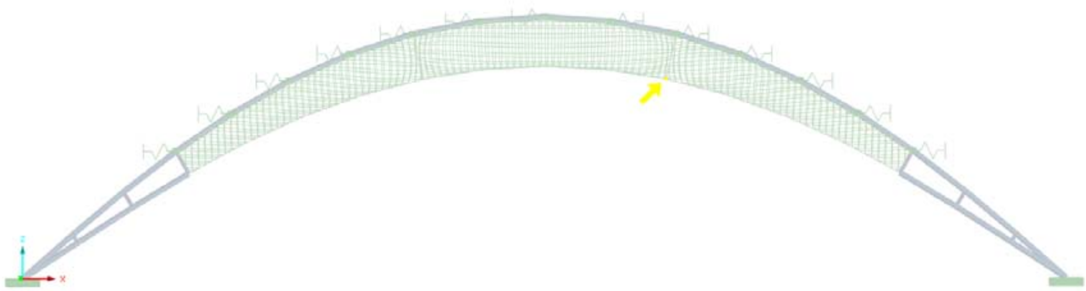

Figure 63: Membrane elements exceeding allowable stresses (DLUBAL RSTAB software)

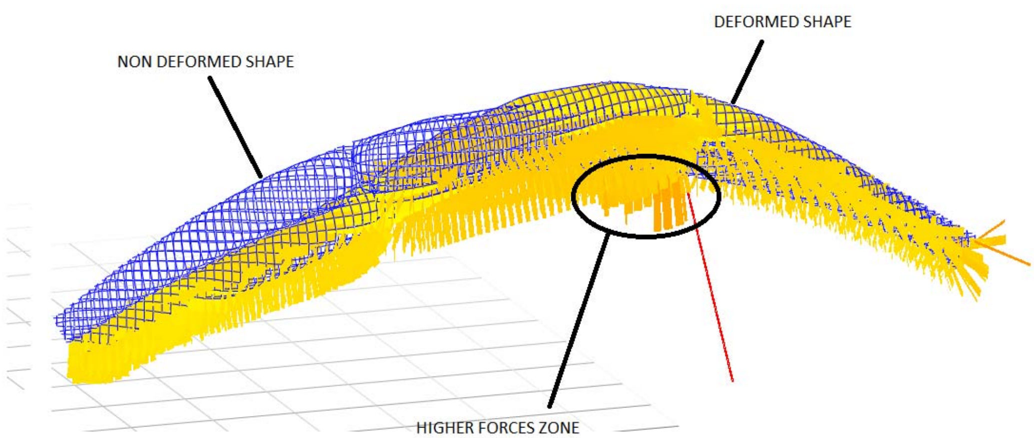

Figure 64: Membrane axial force diagram w direction (Beam Editor EASY software)

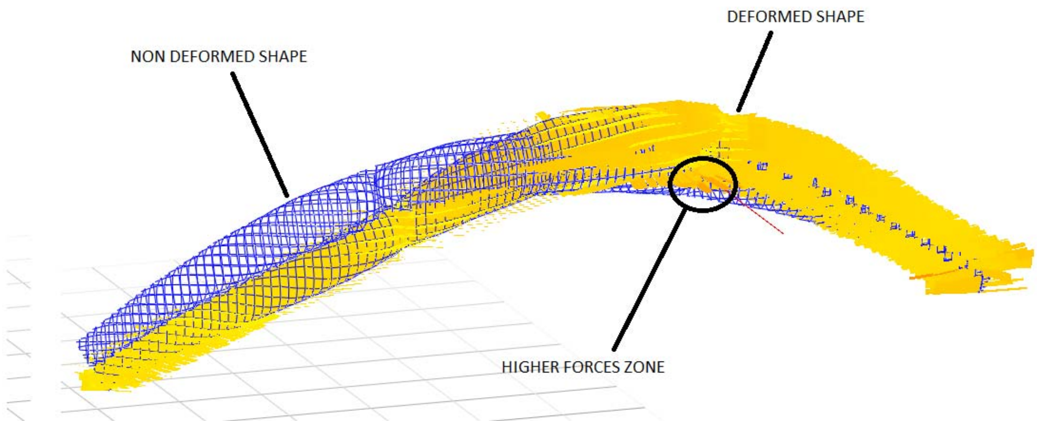

Figure 65: Membrane axial force diagram V direction (Beam Editor EASY software)

CHAPTER 4. DETAILING OF HYBRID SYSTEM

Most of the times special analysis considerations in models have to be reflected in structural detailing process. There are so many ways to connect structural elements in models and results from one way or another can be substantially different, for example if in the model it is supposed that the arch is moment released in one direction and not released in another one, in the support nodes, it is needed to take into consideration physically in fabrication process and connections details must be defined. It is intended in chapter 4 to propose basic drawings and connection details of the most important parts of the system.

4.1 STRUCTURAL DRAWINGS

It is not the aim of this work to design and optimize structure under certain design code, but the feasibility to be used as an alternative to support temporary membrane lightweight structures. In Appendix A, structural general drawings are presented in order to get a clear idea of how to use it.

4.2 ERECTION PROCESS PROPOSAL

Erection process is one of the main reasons of the development of this work because it was inspired in the advantage that spline structures represent to the fast installation process. Transportation of materials is considered to be easy and short space required, it is considered the maximum length of CFRP pipes = 2.04 m, then system is composed of 7 pipes of 2.04 m length and 2 pipes of 1.49 m. The first step is to put all pipes in the ground and attach them together following the sequence showed in figure 66.

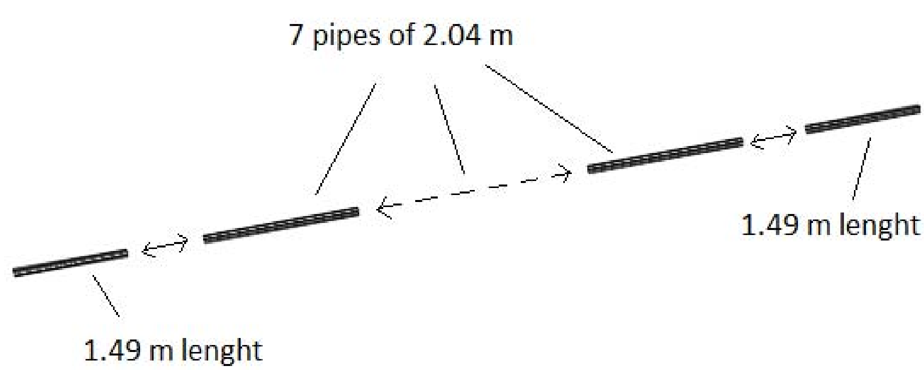

Figure 66 : Putting pipes together (erection process), Microsoft paint

Once 9 pieces of pipes are joined together (see section 4.3 for connections and details), the next step is to put special connections at the beginning and ending pipes to connect a cable and induce axial forces and produce buckling on spline element (total length = 17.26 m). When spline element is buckled enough, supports must be fixed in the ground and a security cable must be attached to avoid uncompressing of spline element. The next step is to connect special aluminum profiles proposed to attach pneumatic chambers and fabric layer (see section 4.3). Now it is the moment to attach non-inflated pneumatic chambers on the correct position to start inflation process. When pneumatic chambers are inflated then inferior chord cable and vertical truss elements must be connected to provide the system enough stiffness to be able to attach membrane and start erection process. See figures 67 to 70.

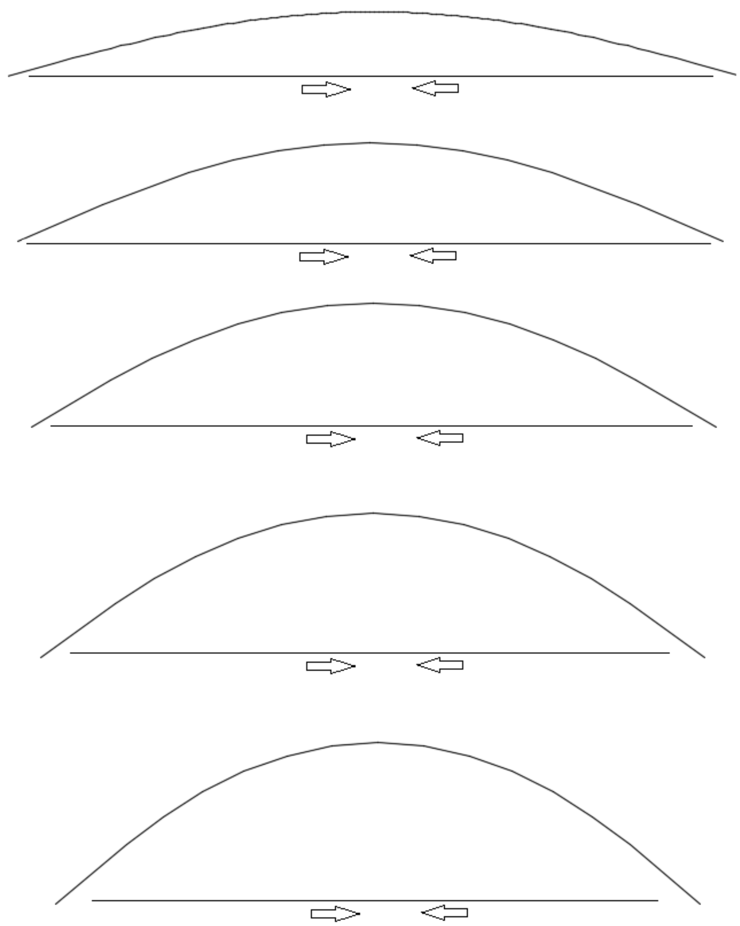

Figure 67: Putting axial load to generate final shape of the spline pipe (erection process), Microsoft paint

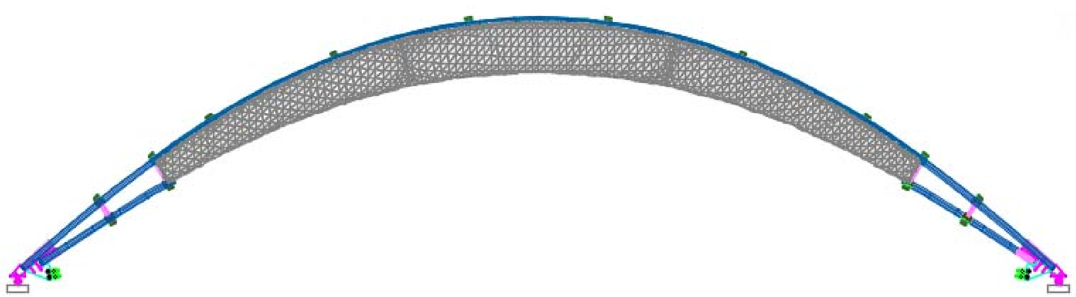

Figure 68: Hybrid system ready to attach membrane (erection process), Autocad 2014

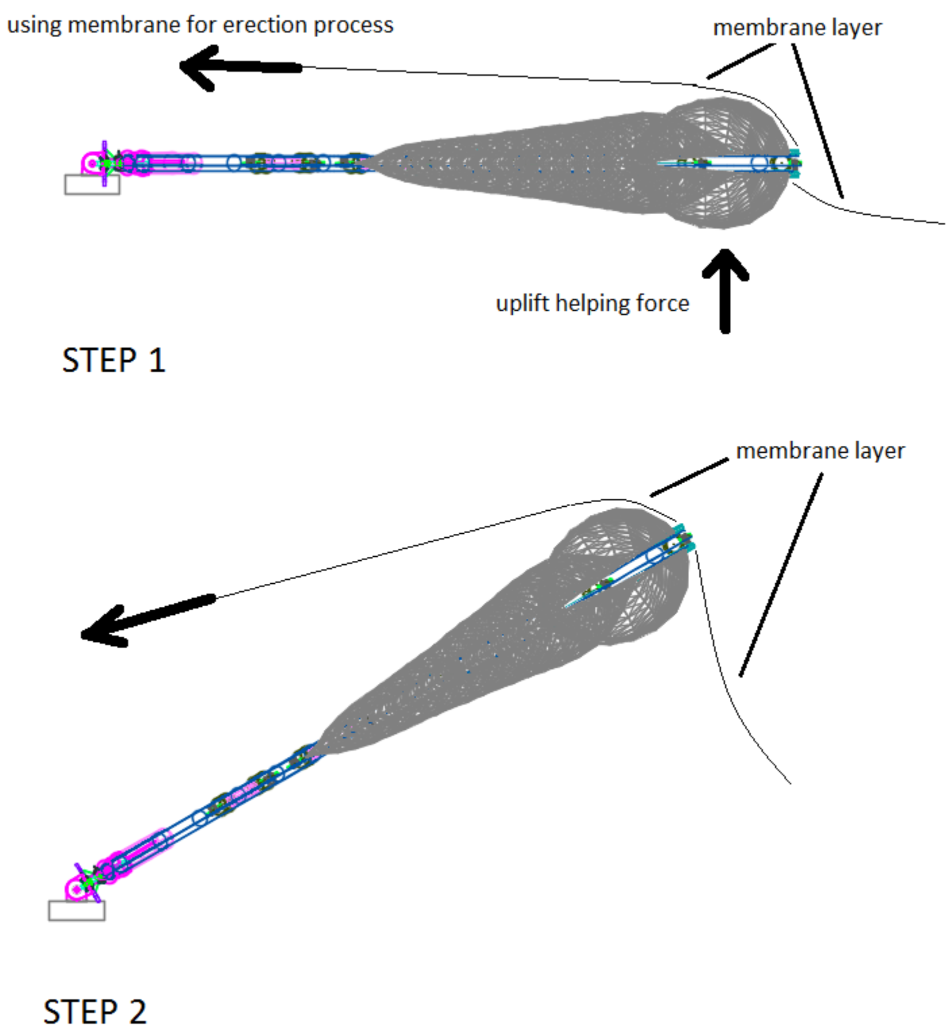

Figure 69: Lifting hybrid system steps 1 and 2 (erection process), Autocad 2014

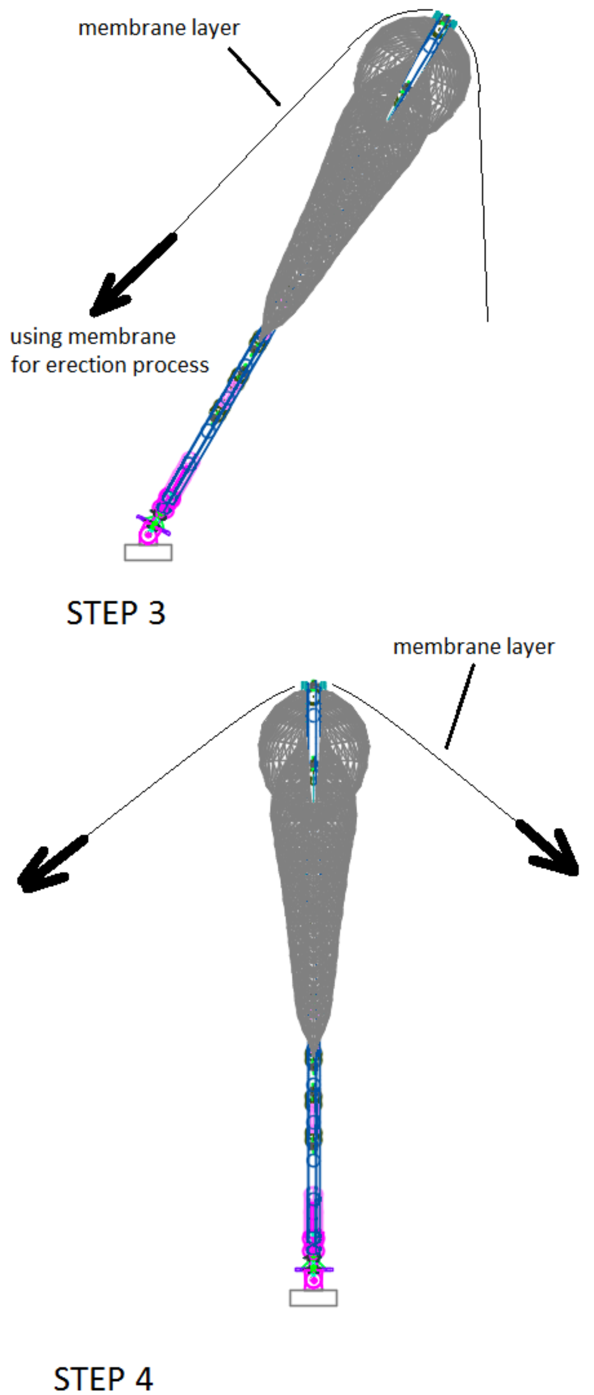

Figure 70: Lifting hybrid system steps 3 and 4 (erection process), Autocad 2014

4.3 CONNECTIONS AND DETAILS PROPOSAL

As a part of the development of the system, connections detailing proposal has been taken into consideration in this work. Structural detail 1 is about the connection between two CFRP pipes, see figure 71.

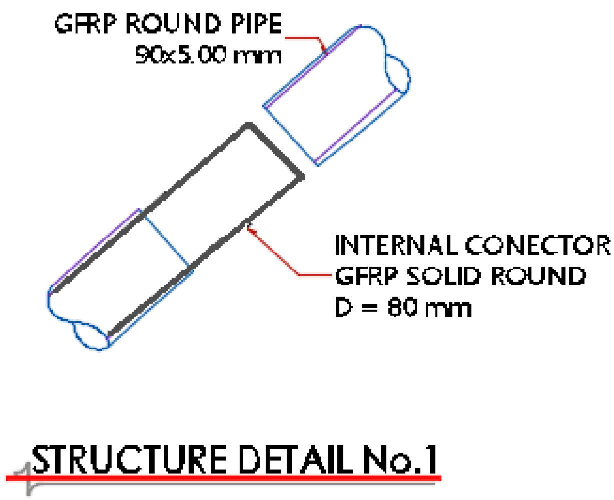

Figure 71: Structure detail 1, Autocad 2014

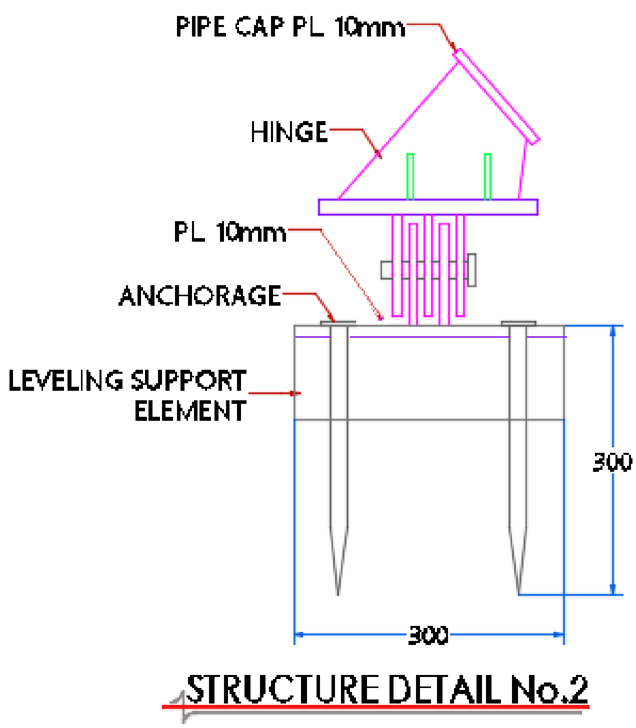

Figure 72: Structure detail 2, Autocad 2014

In order to make possible erection process as it was planned, it is necessary to make a hinged connection in the supports, see figure 72.

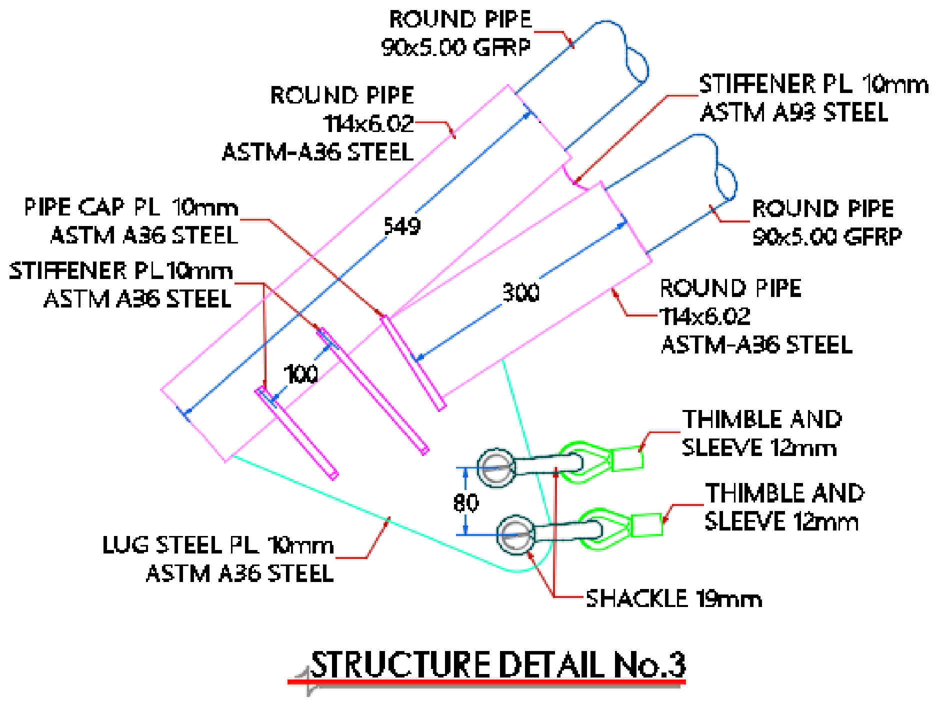

Figure 73: Structure detail 3, Autocad 2014

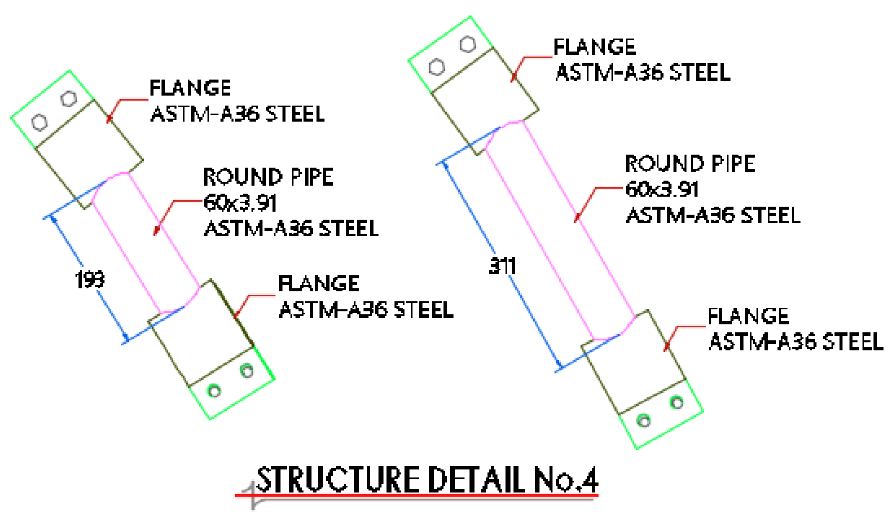

Figure 74: Structure detail 4, Autocad 2014

Detail number 3 applies for both ends of the spline arch, and detail 4 refers to the connection between spline arch and vertical truss elements. In the case of detail 5, as it was discussed in chapter 3, it is considered for two main purposes. The first to increase the resistance of joining points of spline arch and the second to be used as a reinforcement in the zones where interaction stresses exceed 1.

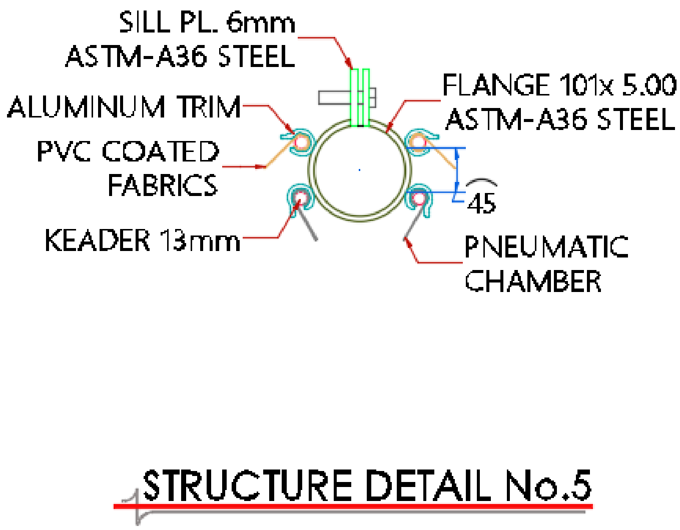

Figure 75: Structure detail 5, Autocad 2014

CHAPTER 5. CONCLUSIONS AND FURTHER WORK

Real wind loading conditions have been considered during the development of this work in terms of a constant pressure of 0.5 kN/m^2. This value is a good approach of an average value in Mexico talking about temporary structures. It is demonstrated the feasibility of the use of the pneumatic system to increase the stiffness of the spline element from an analytical point of view considering a convergence criteria to evaluate the stability of all the structure.
Because of software tools (used in this work) limitations, it was not possible to model both membrane cover and hybrid pneumatic system together, and it was necessary to assume spring support system in some nodes along the spline arch to give lateral support effect provided by pre-tensioned membrane. It is also not considered in the model pre-bending stresses produced by initial buckling process needed to get the "elastica" shape in the spline element. Pre-bending stresses and pre-compression ones has been considered in the analysis phase in chapter 3 as additional stresses added to resultant forces acting on the system. The use of some additional vertical truss members to control stability of the system was necessary in order to reduce deformations and to give additional bending stiffness to the arch near from the supports nodes.
It is proposed in this work and erection process sequence.

From the point of view of analysis modeling tools could be improved and a wind tunnel simulation may have been done to evaluate wind effects in a better way and pressure distribution over the surface of the fabric cover. Different heights should be tested to compare behavior results in the analytical model. An experimental program for testing a real scale model must be performed. Finally, a design process must be developed under certain codes and standards to regulate hybrid structures.

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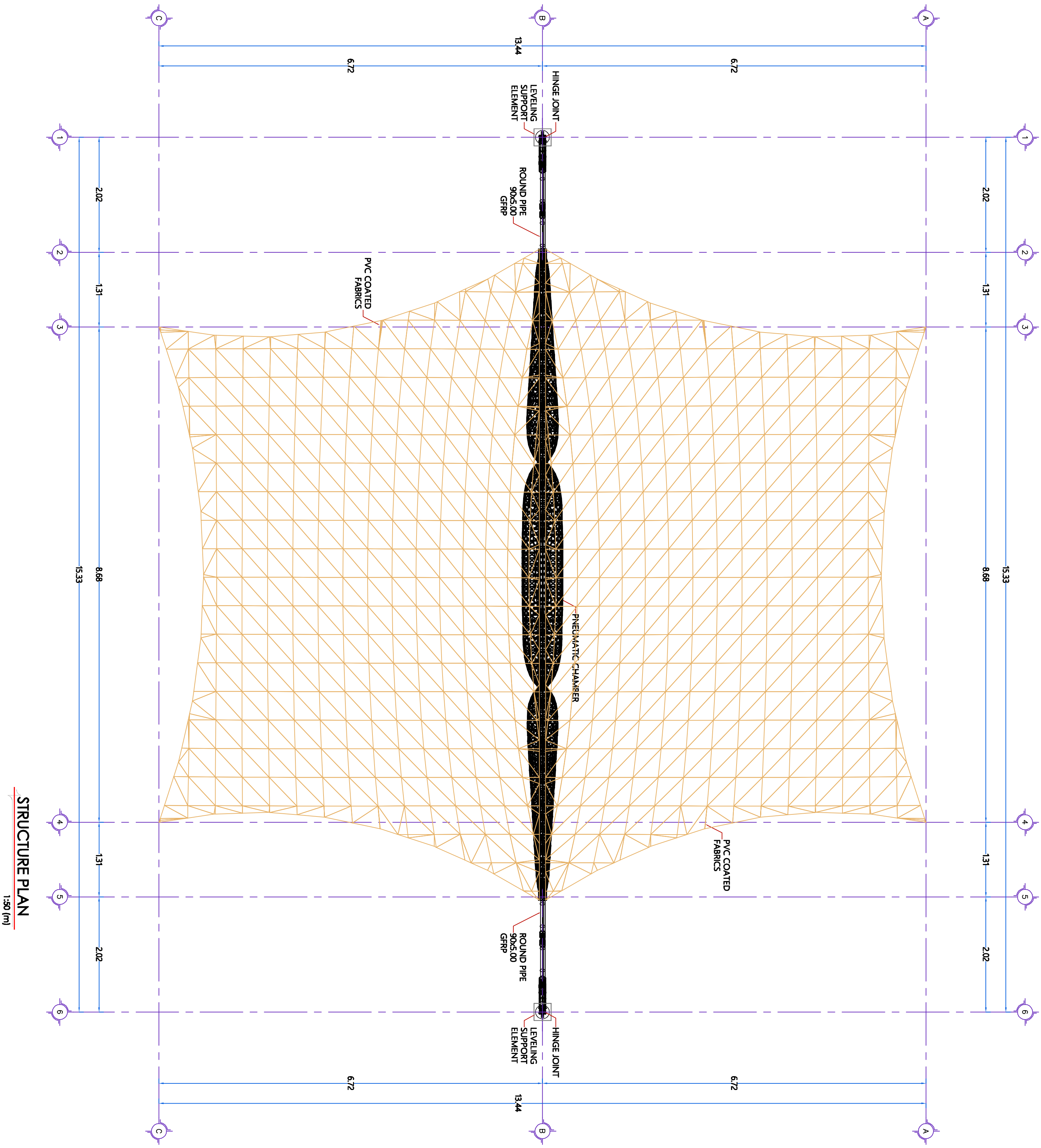
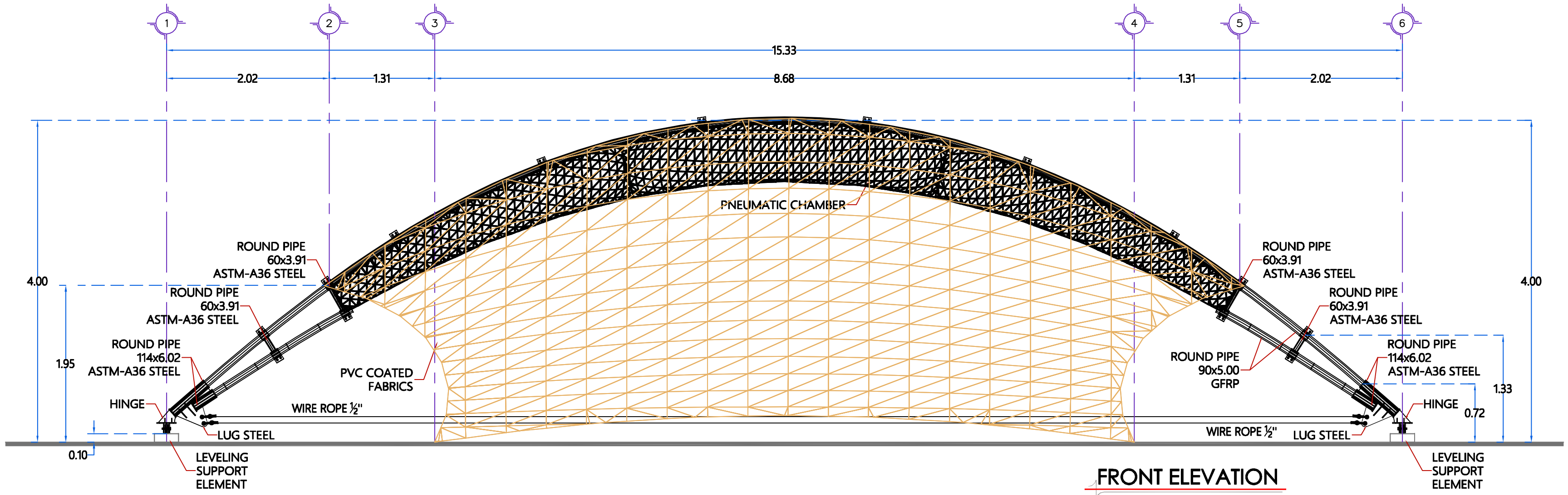
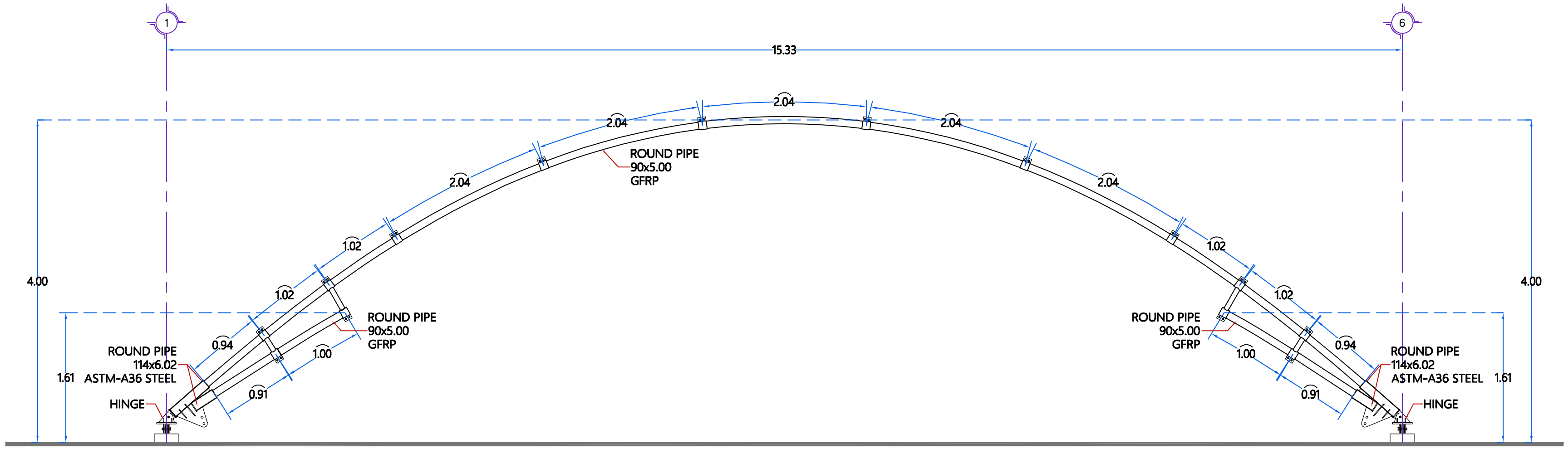


13.44
6.72
6.72
HINGE JOINT
LEVELING SUPPORT ELEMENT
ROUND PIPE 90x5.00 GFRP
PVC COATED FABRICS
PNEUMATIC CHAMBER
2.02
1.31
8.68
15.33
STRUCTURE PLAN
1:50 (m)



FRONT ELEVATION

1:50 (m)



FLANGE'S DISTRIBUTION

1:50 (m)