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TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

D I P L O M A R B E I T

Herleitung von Rechnungsgrundlagen für die Pensionsversicherung in Österreich

Ausgeführt am
Institut für Stochastik und Wirtschaftsmathematik
Forschungsgruppe Finanz- und Versicherungsmathematik
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der Technischen Universität Wien

unter der Anleitung von
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Mag. Dipl.-Ing. Dr. Reinhold Kainhofer

durch
Alexander Schubert

Wien, im November 2015

TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

MASTER THESIS

Derivation of an Austrian Annuity Valuation Table for the Pension Insurance

Written at the
Institute of Statistics and Mathematical Methods in Economics
Research Unit of Financial and Actuarial Mathematics
Faculty of Mathematics and Geoinformation
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Abstract

This work introduces the fundamentals of a stochastic approach in modern life and pension insurance mathematics. The idea of generation annuity valuation tables for the pension insurance is introduced using the example of the Austrian annuity valuation table "AVÖ 2008-P – Rechnungsgrundlagen für die Pensionsversicherung – Pagler & Pagler" released in the year 2008. Furthermore the required data to develop generation tables will be defined and the available data in Austria will be presented. I derive new tables inclusive all probabilities and inclusive the trend for mortality reduction which are needed for the calculation of liablities and discuss the process of development and the difficulties because of the lack of data. Finally net present values are calculated with the new annuity table and compared to the tables named above.

Zusammenfassung

In dieser Arbeit werden die Grundlagen eines stochastischen Zuganges zur modernen Lebens- und Pensionsversicherungsmathematik vorgestellt. Es wird das Konzept von Generationentafeln am Beispiel des Österreichischen Rententafelwerks "AVÖ 2008-P – Rechnungsgrundlagen für die Pensionsversicherung – Pagler & Pagler" vorgestellt, welche im Jahre 2008 veröffentlicht wurden. Des Weiteren werden die notwendigen Daten für die Erstellung von Generationentafeln definiert und die dafür in Österreich erhältlichen Daten vorgestellt. Ich erstelle ein neues Tafelwerk inklusive sämtlicher für die Bewertung von Sozialkapital erforderlicher Wahrscheinlichkeiten und inklusive des Trends der zukünftigen Sterblichkeitsverbesserung und erläutere den Prozess der Entstehung und der Schwierigkeiten aufgrund fehlenden Datenmaterials. Zu guter Letzt werden Barwerte mit dem neuen Tafelwerk errechnet und mit dem oben genannten Tafelwerk verglichen.

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1 Preface

In 2008 the Austrian annuity valuation table "AVÖ 2008-P – Rechnungsgrundlagen für die Pensionsversicherung – Pagler & Pagler" (in the following AVÖ 2008P) was published, based on data up to 2007. There seems to be no need to update the valuation table but the Actuarial Association of Austria (AVÖ) wants to have the possibility to observe the underlying data to react if significant changes occur. The motivation for this work is to collect the required data, to calculate rough probabilities and to specify the technics to smooth and extrapolate them. After a short introduction to valuation tables in Section 2, the mathematics of stochastic processes in Section 3 and a more detailed explanation of the AVÖ 2008P in Section 4, a discussion of the available data and the determination of the rough probabilities will follow in Section 5. The derivation of the extrapolated table will be shown in Section 6.

2 Introduction

Annuity valuation tables for the pension insurance are used to calculate premiums and reserves for pension funds as well as balance sheet liabilities such as pension or severance payments or jubilee premium expenses. The nature of such liabilities vary from annuities (pension liabilities) to lumb sums (severance payments or jubilee premiums) and usually is of long duration. There exist a wide range of events that release a payment: retirement, disability, death or just surviving a special date as an active. Therefore such valuation tables consist of much more than mortality probabilities. Additionally they contain disability probabilities, marriage probabilities, age differences between spouses and distinguish mortality probabilities between actives, disabled, retired and widowed. See Figure 2.1 and Table 2.1 for the possible transitions and probabilities between the different states.

The mortality strongly depends on the age, the generation, the gender, the region where someone lives, the way of life, social status and so on. As a consequence of this each country has its own mortality tables. Normally there exist separate tables for males and females though unisex-tables become even more numerous.

There is available data for mortality calculations from population censuses and from social insurance institutions. From census data a periodic live table is calculated which contains the mortality for each age and gender for a fixed period. As mentioned before, the duration of such long-term liabilities does not approve periodic live tables, since no mortality reduction is taken into consideration. Thus, generation life tables are employed for the calculation. Generation life tables consist of mortality tables for each age-group which include the future expected change in mortality.

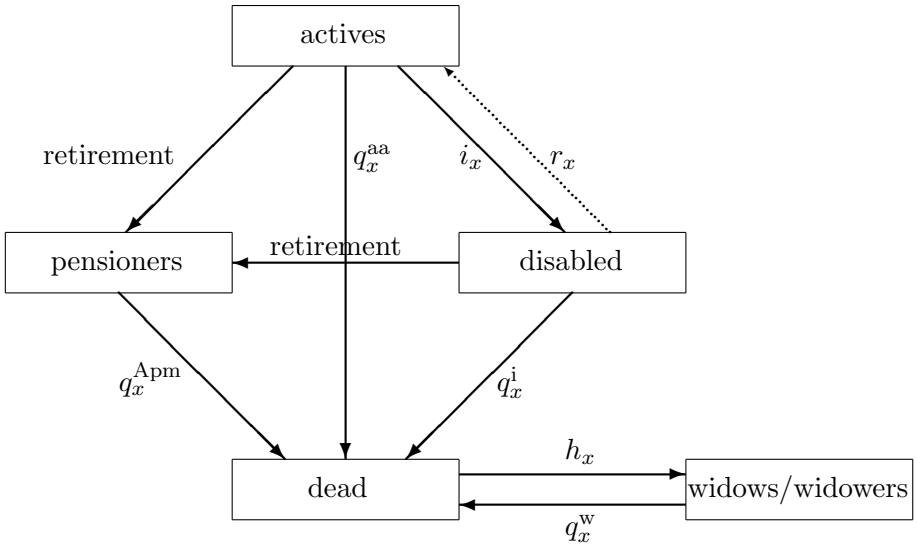

Figure 2.1: Elimination order.

From actives	
q_x^{aa} and q_y^{aa}	transition probability to death
i_x and i_y	transition probability to disabled
retirement	transition to pensioners
From disabled	
q_x^{i} and q_y^{i}	transition probability to death
r_x and r_y	transition probability to actives (not included in the annuity tables)
retirement	transition to pensioners
From pensioners	
q_x^{Apm} and q_y^{Apf}	transition probability to death
From death	
h_x and h_y	transition probability that a deceased leaves surviving dependants
From dependants	
q_x^{w} and q_y^{w}	transition probability to death
remarriage	lapse of dependants pensions because of remarriage (not included in the annuity tables)

Table 2.1: Transition possibilities and probabilities.

3 Fundamentals

In the following chapter the stochastic model as described in [14] is used and also the implementations of [21]. There the advanced reader also finds all proofs.

3.1 Stochastic Processes

Definition 3.1.1 (Probability space, stochastic process)**.** Let $(\Omega, \mathcal{F}, \mathbb{P})$ denote a probability space which fulfills Kolmogorov's Axiom.
Be (S, S) a measurable space (i.e. S a set and S a σ-algebra over S) and T a set.
A family $\{X_t : t \in T\}$ of random variables

$$X_t : (\Omega, \mathcal{F}, \mathbb{P}) \to (S, S),\ \omega \mapsto X_t(\omega) \tag{3.1}$$

is called stochastic process over $(\Omega, \mathcal{F}, \mathbb{P})$ with state space S.

Example 3.1.2 (Poisson process). The Poisson process $N = (N_t)_{t \geq 0}$ is a *counting process* with state space $\mathbb{N}$ and is often used for modeling the number of claims of insurers. This process happens in continuous time. The homogeneous Poisson process has following properties:

(a) $N_0 = 0$ almost certain.

(b) N has independent static increases.

(c) For all $t > 0$ and all $k \in \mathbb{N}$ holds: $\mathbb{P}[N_t = k] = e^{-\lambda t} \frac{(\lambda t)^k}{k!}$.

Definition 3.1.3 (Expectations)**.** For a random variable X in $(\Omega, \mathcal{F}, \mathbb{P})$ and $\mathcal{B} \subset \mathcal{F}$ a σ-algebra we denote with

- $\mathbb{E}[X]$ the expectation of the random variable X,
- $\mathbb{V}[X]$ the variance of the random variable X,

$$\mathbb{V}[X] = \mathbb{E}[(X - \mathbb{E}(X))^2] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

- $\mathbb{E}[X|\mathcal{B}]$ the conditional expectation of X regarding $\mathcal{B}$.

3.2 Markov Processes

Definition 3.2.1 (Markov chain)**.** Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, let S be a non-void finite (or more general, a countable) set, let T be an interval of $\mathbb{Z}$, and let $\{X_t\}_{t \in T}$ be a stochastic process with random variables $X_t : \Omega \to S$, for all $t \in T$. If for all n in $\mathbb{N}$, times $t_1 < t_2 < \cdots < t_{n+1}$ in T and states $i_1, \ldots, i_{n+1}$ in S with $\mathbb{P}[X_{t_1} = i_1, \ldots, X_{t_n} = i_n] > 0$, it holds that

$$\mathbb{P}[X_{t_{n+1}} = i_{n+1} | X_{t_1} = i_1, \ldots, X_{t_n} = i_n] = \mathbb{P}[X_{t_{n+1}} = i_{n+1} | X_{t_n} = i_n], \tag{3.2}$$

then $\{X_t\}_{t \in T}$ is called a *Markov chain* with *state space* S.

Remark 3.2.2. Property (3.2) is called the *Markov property* of the Markov chain $\{X_t\}_{t\in T}$. The set S is called the *state space* and the elements of S are called *states*.

The conditional probabilities depend only on the last state, but not from the way on which the Markov chain reached this state. So the probability of future events depends only on the present, but is independent of the past, provided the present is known.

Example 3.2.3. Suppose that $S \neq 0$ is a countable set, $T = \mathbb{N}_0$, and for the S-valued stochastic process $\{X_t\}_{t\in T}$ the X_t with $t \in T$ are independent. The independence of the $\{X_t\}_{t\in T}$ causes that, for all n in $\mathbb{N}$, times $t_1 < t_2 < \cdots < t_{n+1}$ in T and states $i_1, \ldots, i_{n+1}$ in S, it holds that

$$\mathbb{P}[X_{t_1} = i_1 \ldots, X_{t_n} = i_n] = \prod_{k=1}^{n} \mathbb{P}[X_{t_k} = i_k].$$

Hence, it follows that the S-valued stochastic process $\{X_t\}_{t\in T}$ is a Markov chain.

Example 3.2.4. In addition to Example 3.2.3 we assume that S is a countable group. We fix a starting point $s_0 \in S$ and define the sums

$$S_m := s_0 + \sum_{k=1}^{m} X_k, \quad \forall m \in \mathbb{N}_0.$$

A popular choice for S is $\mathbb{Z}^d$. Note that S does not have to be Abelian.

Lemma 3.2.5. The *random walk* $\{S_m\}_{m\in\mathbb{N}_0}$ is a Markov chain.

But does a stochastic process, which is derived from a Markov chain $\{X_t\}_{t\in T}$ by considering the current state X_t as well as the former state X_{t-1}, also fulfill the Markov property?

Proposition 3.2.6. Let $\{X_t\}_{t\in T}$ be a Markov chain. Then, for all $m, n \in \mathbb{N}$ with $m \geq 2$, times $t_1 < t_2 < \cdots < t_m < t_{m+1} < \cdots < t_{m+n}$ in T in T, states $i \in S$ and subsets $A \subset S^{m-1}, B \subset S^n$ with $\mathbb{P}[(X_{t_1}, \ldots, X_{t_{m-1}}) \in A, X_{t_m} = i] > 0$,

$$\begin{aligned}&\mathbb{P}[(X_{t_{m+1}}, X_{t_{m+2}}, \ldots, X_{t_{m+n}}) \in B | (X_{t_1}, \ldots, X_{t_{m-1}}) \in A, X_{t_m} = i] \\ &= \mathbb{P}[(X_{t_{m+1}}, X_{t_{m+2}}, \ldots, X_{t_{m+n}}) \in B | X_{t_m} = i].\end{aligned}$$

Remark 3.2.7. Proposition 3.2.6 generalizes the Markov property (3.2) by expressing in a more general way that the probabilities of future events, given the present, do not depend on past events.

Lemma 3.2.8. Let $\{X_t\}_{t\in T}$ be a Markov chain with state space S. Let $t_{\min}$ denote the minimal element of T, if it exists, otherwise we set $t_{\min} := -\infty$. Then, the stochastic process $\{\tilde{X}_t\}_{t\in T\setminus\{t_{\min}\}}$ with $\tilde{X}_t := (X_{t-1}, X_t)$ for $t \in T\setminus\{t_{\min}\}$, is a Markov chain with state space S^2.

Lemma 3.2.8 shows a 2-component stochastic process. The next step is a k-component stochastic process which incorporates $k-1$ steps of the past.

Lemma 3.2.9. Let $\{X_t\}_{t\in T}$ be a Markov chain with state space S and let $t_{\min}$ be the minimal element of T, if it exists, otherwise we define $t_{\min} := -\infty$. Then, the k-component stochastic process $\{\tilde{X}_t\}_{t\in\tilde{T}}$ with $\tilde{X}_t := (X_{t-k+1}, X_{t-k+2}, \ldots, X_t)$ for $t \in \tilde{T} := T\backslash\{t_{\min}, t_{\min}+1, \ldots, t_{\min}+k-2\}$, is a Markov chain with state space S^k.

Definition 3.2.10. Let $\{X_t\}_{t\in T}$ be a Markov chain with state space S. For times $s \neq t$ in T and for $i, j \in S$ with $\mathbb{P}[X_s = i] > 0$, the conditioned probability $\mathbb{P}[X_t = j|X_s = i]$ is denoted by $p_{ij}(s,t)$. We call $p_{ij}(s,t)$ the *transition probability* from state i to state j.

Proposition 3.2.11 (Chapman–Kolmogorov equation)**.** If we suppose that $\{X_t\}_{t\in T}$ is an S-valued Markov chain, the times $s \leq t \leq u$ are in T, and the states i, k are in S, with $\mathbb{P}[X_s = i] > 0$. Then,

$$p_{ik}(s,u) = \sum_{j\in S} p_{ij}(s,t)p_{jk}(t,u). \tag{3.3}$$

Remark 3.2.12. If $\mathbb{P}[X_t = j] = 0$ for a $j \in S$, then $p_{jk}(t,u)$ is not defined by Definition 3.2.10. However, $p_{ij}(s,t) = 0$ in this case, hence the product in considered as zero.

Definition 3.2.13. We call

$$P(s,t) = (p_{ij}(s,t))_{i,j\in S}, \qquad s \leq t \text{ in } T,$$

the *family of (stochastic) transition matrices*, if for all times $s < t < u$ in T, the following conditions hold:

(a) $p_{ij}(s,t) \geq 0$ for all $i, j \in S$,

(b) $\sum_{j\in S} p_{ij}(s,t) = 1$ for all $i \in S$,

(c) $p_{ij}(s,s) = \delta_{ij}$ for all $i, j \in S$,
which is equivalent to $P(s,s) = I$, where I is the unit matrix,

(d) $P(s,u) = P(s,t)P(t,u)$,
which is equivalent to $p_{ik}(s,u) = \sum_{j\in S} p_{ij}(s,t)p_{jk}(t,u)$, for all $i, k \in S$.

Definition 3.2.14. A Markov chain $\{X_t\}_{t\in T}$ has the transition matrices $P(s,t)$ for $s \leq t$ in T, if

$$p_{ij}(s,t) = \mathbb{P}[X_t = j|X_s = i],$$

for all states $i, j \in S$ and for all times $s \leq t$ in T with $\mathbb{P}[X_s = i] > 0$.

Example 3.2.15. Let $S = \{1, 2\}$. If $X_t = 2$ for all $t \in \mathbb{N}_0$ (which is equivalent to $\mathbb{P}[X_t = 1] = 0$, for all $t \in \mathbb{N}_0$), then the transition probabilities $p_{12}(s,t)$ and $p_{11}(s,t)$ are not determined by the Markov chain.

Lemma 3.2.16. If $\{X_t\}_{t\in T}$ is a Markov chain and if T is a discrete interval of Z, then there always exists a family of stochastic transition matrices which fulfills Definition 3.2.14.

To expresses the finite-dimensional distributions of a Markov chain in terms of the transition matrices we need:

Proposition 3.2.17. $\{X_t\}_{t\in T}$ is a Markov chain if and only if there exist transition matrices $(P(s,t))_{s,t\in T, s\leq t}$ with

$$\mathbb{P}[X_{t_1} = i_1, \dots, X_{t_n} = i_n] = \mathbb{P}[X_{t_1} = i_1] \prod_{k=1}^{n-1} p_{i_k i_{k+1}}(t_k, t_{k+1}), \tag{3.4}$$

for all $n \in N$, states $i_1, \dots, i_n$ in S and times $t_1 < t_2 < \cdots < t_n$ in T.

Lemma 3.2.18. Let $\{X_t\}_{t\in T}$ be a Markov chain with state space S and transition probabilities $(p_{ij}(s,t))_{i,j\in S}$, for $s \leq t$ in T. Let $\Phi : S \mapsto \tilde{S}$ be a surjective function. If for all states k, l in $\tilde{S}$ and times $s \leq t$ in T it holds that

$$\sum_{j\in\Phi^{-1}(l)} p_{ij}(s,t) = \sum_{j\in\Phi^{-1}(l)} p_{i'j}(s,t) \qquad \forall i, i' \in \Phi^{-1}(k), \tag{3.5}$$

then $\{\Phi X_t\}_{t\in T}$ is a Markov chain with state space $\tilde{S}$, which has for all states k, l in $\tilde{S}$ and times $s \leq t$ in T the transition probabilities

$$\tilde{p}_{kl}(s,t) := \sum_{j\in\Phi^{-1}(l)} p_{ij}(s,t),$$

where $i \in \Phi^{-1}(k)$.

Definition 3.2.19. An S-valued Markov chain $\{X_t\}_{t\in T}$ is called time-homogeneous if for all $i, j \in S$ and $s \leq t, u \leq v \in T$ with $t - s = u - v$ and $\mathbb{P}[X_s = i] > 0$, $\mathbb{P}[X_u = i] > 0$

$$\mathbb{P}[X_t = j | X_s = i] = \mathbb{P}[X_v = j | X_u = i].$$

Remark 3.2.20. In case of a time-homogeneous Markov chain, the transition probabilities depend only on the time difference.

Proposition 3.2.21 (Product of independent Markov chains)**.** Let $\{X_t\}_{t\in T}$ and $\{Y_t\}_{t\in T}$ be two independent Markov chains with state spaces S_X, S_Y , respectively, and transition matrices $P^X(s,t)$, $P^Y(s,t)$, respectively, for times $s \leq t$ in T. Then $\{(X_t, Y_t)\}_{t\in T}$ is a Markov chain with state space $S = S_X \times S_Y$ and the transition matrices

$$P(s,t) = \big(p_{(i,j)(k,l)}(s,t)\big)_{(i,j)(k,l)\in S} \qquad s \leq t \,\text{in}\, T, \tag{3.6}$$

where $p_{(i,j)(k,l)}(s,t) = p^X_{ik}(s,t) p^Y_{jl}(s,t)$.

Remark 3.2.22. By induction it can be shown that Proposition 3.2.21 can be generalized to the product of any finite number of independent Markov chains with the same interval T.

3.3 General Markov Model for Elimination Order

Considering $|S| = n$ possible, mutually exclusive states for a beneficiary aged x, it is now possible to calculate stochastic deviations from the model.

Definition 3.3.1 (Biometric state space for the annuity table)**.** The biometric state space S for the derivation of this annuity table is

$$S = \{\text{a, i, p, w, o, d, rem, end}\}$$

where the biometric states are:

a	active
i	disabled
p	old age pensioner
w	widow or widower
o	orphan
d	dead
rem	remarried
end	end of pension payments

i.e. there are $n = 8$ possible biometric states for each gender.

Remark 3.3.2. Normally pension payments except limited disability pension payments or orphan pension payments are promised as lifelong payments.

Definition 3.3.3. The transition probabilities of a x-aged active are:

$p_{\mathrm{a,a}}(x, x+1)$	probability to stay active
$p_{\mathrm{a,i_1}}(x, x+1)$	probability to get disabled
$p_{\mathrm{a,p}}(x, x+1)$	probability to retire
$p_{\mathrm{a,d}}(x, x+1)$	probability to die
$p_{\mathrm{a,w}}(x, x+1)$	probability to die and having a y-aged surviving widow or a x-aged surviving widower
$p_{\mathrm{a,o}_k}(x, x+1)$	probability to die and having x_k-aged resp. y_k-aged surviving orphans $k = 1, 2, \ldots$

Definition 3.3.4. The transition probabilities of a x-aged disabled in his l^{th} year of disability are:

$p_{\mathrm{i}_l,\mathrm{i}_{l+1}}(x, x+1)$	probability to stay disabled
$p_{\mathrm{i}_l,\mathrm{a}}(x, x+1)$	probability of reactivation
$p_{\mathrm{i}_l,\mathrm{p}}(x, x+1)$	probability to retire
$p_{\mathrm{i}_l,\mathrm{end}}(x, x+1)$	probability to reach payment end
$p_{\mathrm{i}_l,\mathrm{d}}(x, x+1)$	probability to die
$p_{\mathrm{i}_l,\mathrm{w}}(x, x+1)$	probability to die and having a y-aged surviving widow or a x-aged surviving widower
$p_{\mathrm{i}_l,\mathrm{o}_k}(x, x+1)$	probability to die and having x_k-aged resp. y_k-aged surviving orphans $k = 1, 2, \ldots$

Definition 3.3.5. The transition probabilities of a x-aged old age pensioner are:

$p_{\mathrm{p},\mathrm{p}}(x, x+1)$	probability to stay old age pensioner
$p_{\mathrm{p},\mathrm{end}}(x, x+1)$	probability to reach payment end
$p_{\mathrm{p},\mathrm{d}}(x, x+1)$	probability to die
$p_{\mathrm{p},\mathrm{w}}(x, x+1)$	probability to die and having a y-aged surviving widow or a x-aged surviving widower
$p_{\mathrm{p},\mathrm{o}_k}(x, x+1)$	probability to die and having x_k-aged resp. y_k-aged surviving orphans $k = 1, 2, \ldots$

Definition 3.3.6. The transition probabilities of a y-aged widow resp. a x-aged widower are:

$p_{\mathrm{w},\mathrm{w}}(x, x+1)$	probability to stay widow/widower
$p_{\mathrm{w},\mathrm{end}}(x, x+1)$	probability to reach payment end
$p_{\mathrm{p},\mathrm{d}}(x, x+1)$	probability to die
$p_{\mathrm{p},\mathrm{rem}}(x, x+1)$	probability to remarriage

Remark 3.3.7. Normally widow/widower pensions end with remarriage. So the event of remarriage is additional to reaching the contractual payment end or death an event which ends the pension payments.

Definition 3.3.8. The transition probabilities of a x-aged orphan are:

$p_{\mathrm{o},\mathrm{o}}(x, x+1)$	probability to stay orphan
$p_{\mathrm{o},\mathrm{d}}(x, x+1)$	probability to die
$p_{\mathrm{o},\mathrm{end}}(x, x+1)$	probability to reach end of payment phase

Remark 3.3.9. Normally orphan pension payments end reaching the age of 18. In special cases as orphans which are permanently unable to work or which are in higher education the payment end can enlarged to higher ages (for example lifelong or until the end of academic studies).

The black boxes in Figure 3.1 stand for the possible biometric states. The green arrows are the transition possibilities of actives due to Definition 3.3.3, the red arrows stand for the transition possibilities of old age pensioners due to Definition 3.3.5, the yellow arrows denote the transition possibilities of widows/widowers due to Definition 3.3.6, the purple arrows identify the transition possibilities of orphans due to Definition 3.3.8 and finally the blue

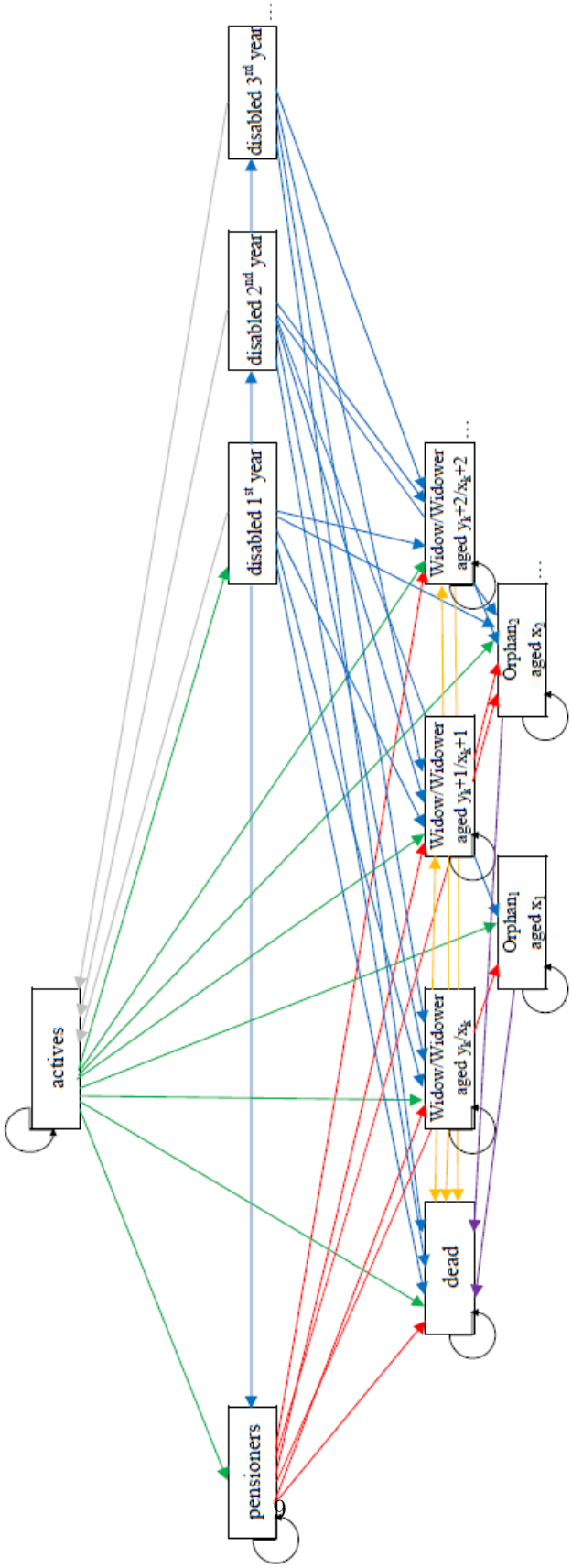

Figure 3.1: General Markovian elimination order.

and grey arrows are the transition possibilities of disabled due to Definition 3.3.4. The black cirlce arrows denote the transition possibilities to stay in one state.

Remark 3.3.10. Because of simplification reasons the states *remarriage* and *pension payment end* and the corresponding transition possibilities are not shown.

Remark 3.3.11. International studies show that the probability of reactivation strongly depends on the duration of disability, i.e. a disabled in the first year of disability has a greater chance of reactivation. With advanced disability duration the probability to get back in the working life is increasing and tends to zero.

3.4 Simplified Markov Model for Elimination Order

Because of reasons of decisiveness and available data to derive annuity tables (see 5.1) we have to simplify the general Markovian elimination order. This means that some of the biometric states respectively state transitions will be left out. Unconsidered states and transitions are:

- Orphans: Both the available data and the economic effect of consideration of orphans in the elimination order enable us to neglect this state. The statistics only give numbers of actual, new and leaving orphan pension beneficiaries but give no connection to the death cases that were the origin of this state (in contrast to the transition to widows/widowers). Until 2009 the gender of the orphans was not recorded in the statistics. Normally orphan pensions are limited to few years and the mortality rates in these ages are very small which recommends simple finance mathematical calculation of orphan net present values. In practice the entitlement to orphan pensions is considered in the calculations by increasing the widows'/widowers' entitlement, for example by 5 or 8%.

- Remarriage: The number of losses of widow/widower pension entitlement because of remarriage is so small that significant statements cannot be done. For example there was no single case of stopping of widow/widower pension payments in 2011 because of remarriage (see annual report 2011 of the Austrian Pension Insurance Organisation). One reason for these small remarriage numbers is that someone who obtains dependants pension will not officially marry again but live in undocumented partnership.

- Reactivation: The annual report 2011 of the Austrian Pension Insurance Organisation overall shows 65 cases of male reactivation and 39 cases of female reactivation (in comparison to 98,416 male white

collar disable pensioners and 31,419 female white collar disable pensioners in 2011). If someone is not able to work any more they have to apply for limited disability pension. If this request is refused by the Austrian Pension Insurance Organisation those persons get unemployment benefits and after one year (after the expiry of unemployment insurance) welfare benefits. If repetitive requests to disability pension fail the persons have to wait until they have entitlement to old age pension. Statistics show that if people have placed their request to disability pension the will to get back in employment is decreasing rapidly (also see Remark 3.3.11). Therefore reactivation is a double-dependent probability: It depends on the age and the duration of disability. Omitting this transition simplifies the whole model where all other probabilities are only age-dependent.

- Differentiation of the state of disability in years of disability duration: If reactivation is not considered this differentiation is of no use any more.

- Direct transition probabilities from actives, disabled and old age pensioners to widows/widowers: In the Austrian Pension Statistics (see 5.1) the number of new widows/widowers and the two different states of the deceased, active or pensioner, are recorded, but it is not possible to differentiate between deceased old age pensioner and deceased disabled. In Austria there are around six thousand new white collar widows and around 1.5 thousand new white collar widowers per year. Further differentiation into 3 states of the dependant causing deceased could decrease the significance of the data. Similar to reactivation these direct transition probabilities to dependants would be double-graded: depending on the age of the deceased and the age of the new dependant. The established practice in the past was to derive marriage probabilities which are independent from the state of the deceased and to determine statistical ages of the new dependants. The transition from actives, disabled and old age pensioners to dependants is then calculated by multiplication of the corresponding death probabilities with the marriage probabilities.

Definition 3.4.1. The probability that there exists a surviving dependant after the death of a x-aged beneficiary, independent of the state of the deceased, is denoted by

$$p_{d,w}(x, x+1).$$

Remark 3.4.2. The transition from actives to dependants in the simplified model is:

$$p_{\mathrm{a,w}}(x, x+1) = p_{\mathrm{a,d}}(x, x+1) \cdot p_{\mathrm{d,w}}(x, x+1)$$

The transition from disabled to dependants in the simplified model is:

$$p_{\mathrm{i,w}}(x, x+1) = p_{\mathrm{i,d}}(x, x+1) \cdot p_{\mathrm{d,w}}(x, x+1)$$

The transition from old age pensioners to dependants in the simplified model is:

$$p_{\mathrm{p,w}}(x, x+1) = p_{\mathrm{p,d}}(x, x+1) \cdot p_{\mathrm{d,w}}(x, x+1)$$

Definition 3.4.3. The age of the new dependant is assumed as $y(x)$ respectively $x(y)$.

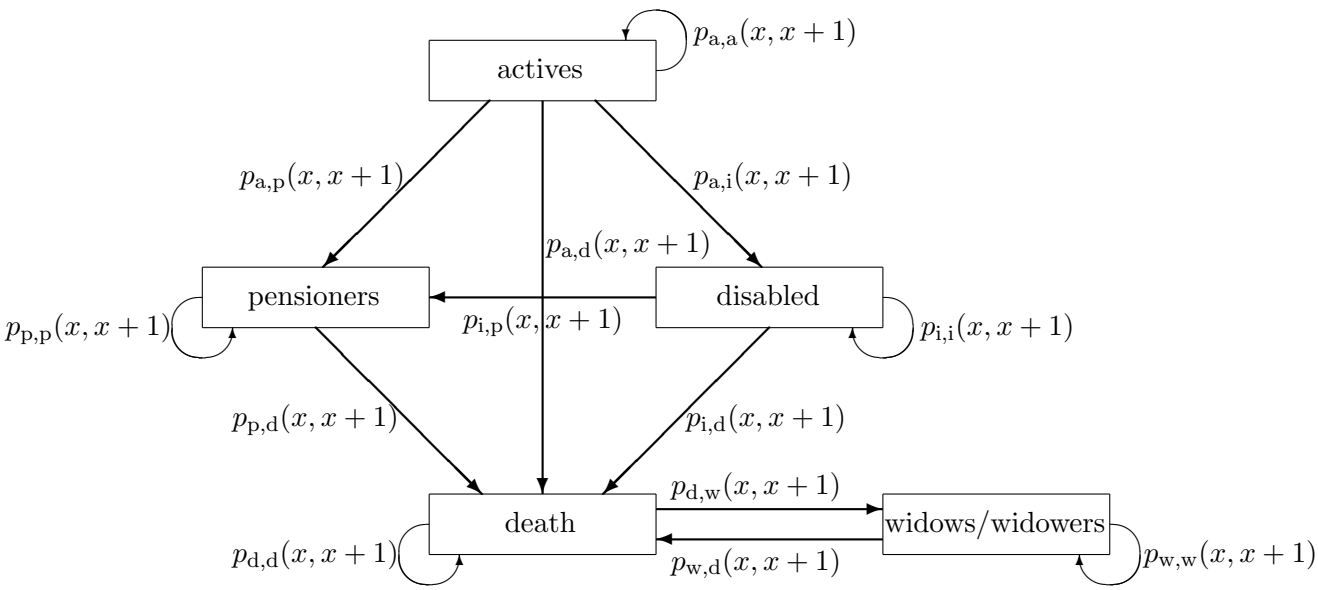

Figure 3.2: Simplified Markovian elimination order.

Definition 3.4.4 (Transition probabilities of the simplified Markov model)**.** With the Markov Model the traditional nomenclature of transition probabilities defines for x-aged beneficiaries as:

$$\begin{aligned}
i_x &:= p_{\mathrm{a,i}}(x, x+1) \\
q_x^{\mathrm{aa}} &:= p_{\mathrm{a,d}}(x, x+1) \\
q_x^{\mathrm{i}} &:= p_{\mathrm{i,d}}(x, x+1) \\
q_x^{\mathrm{Apm}} &:= p_{\mathrm{p,d}}(x, x+1) \\
q_x^{\mathrm{w}} &:= p_{\mathrm{w,d}}(x, x+1) \\
h_x &:= p_{\mathrm{d,w}}(x, x+1)
\end{aligned}$$

Where the last transition is from the original beneficiary to a surviving dependant.

And the probabilities to stay in the same state are:

$$\begin{aligned} p_x^{\mathrm{aa}} &:= p_{\mathrm{a,a}}(x, x+1) \\ p_x^{\mathrm{i}} &:= p_{\mathrm{i,i}}(x, x+1) \\ p_x^{\mathrm{Apm}} &:= p_{\mathrm{p,p}}(x, x+1) \\ p_x^{\mathrm{w}} &:= p_{\mathrm{w,w}}(x, x+1) \end{aligned}$$

3.5 Net Present Value of Future Cash Flows

Using the simplified Markov model we want to calculate the net present value of future cash flows.

Definition 3.5.1. We denote:

$N_{ij}(t)$ random number of direct transitions from state $i \in S$ to state $j \in S$, $i \neq j$, until $t \in T$ (t included).
This means for all $t \in T$ that

$$N_{ij}(t) = \sum_{s \in T, s < t} 1_{\{X_s = i, X_{s+1} = j\}}.$$

We set $N_{ij}(t) = 0$, if $t \notin T$.

$a_i(t)$ contractual payment due at time $t \in T$, if $X_t = i \in S$
e.g. an annuity due or a periodically paid premium.

$a_{ij}(t)$ contractual payment at time $t \in T$ if $X_{t-1} = i$ and $X_t = j$.
We set $a_{ij}(t) = 0$, if $t \notin T$. An example is the sum insured in case of death in the time interval $[t-1, t]$. An example of the case $i = j$ are payments in arrears.

Remark 3.5.2. Both $a_i(t)$ and $a_{ij}(t)$ are not only seen as net payments but can also include costs for example acquisition costs, collection costs or administration costs.

$A_i(t)$ random, cumulative payments until time $t \in T$ for the state $i \in S$,

$$A_i(t) = \sum_{s \in T, s \leq t} a_i(s) 1_{\{X_s = i\}}.$$

$\Delta A_i(t) =$ $A_i(t) - A_i(t-1) = a_i(t) 1_{\{X_t = i\}}$.

$A_{ij}(t)$ random, cumulative payments until time $t \in T$ for changes from state $i \in S$ to state $j \in S$.
Thus, for all $t \in T$ we have

$$\begin{aligned} A_{ij}(t) &= \sum_{s \in T, s<t} a_{ij}(s+1) 1_{\{X_s = i, X_{s+1} = j\}}, \\ &= \sum_{s \in T, s<t} a_{ij}(s) \Delta N_{ij}(s), \end{aligned}$$

with $\Delta N_{ij}(s) := N_{ij}(s) - N_{ij}(s-1)$.

$\Delta A_{ij}(t) =$ $A_{ij}(t) - A_{ij}(t-1) = a_{ij}(t) \Delta N_{ij}(t)$.

$A(t)$ random, contractual, cumulative payments until time $t \in T$,

$$A(t) = \sum_{i \in S} A_i(t) + \sum_{i,j \in S} A_{ij}(t).$$

$\Delta A(t)$ random payment at time $t \in T$,

$$\begin{aligned} \Delta A(t) &= \sum_{i \in S} \Delta A_i(t) + \sum_{i,j \in S} \Delta A_{ij}(t). \\ &= a_{X_t}(t) + a_{X_{t-1}, X_t}(t) \end{aligned}$$

r constant effective interest rate.

v discount rate with $v = \frac{1}{1+r}$. So the discount factor to discount from time $t \in T$ to time $s \in S$ with $s \leq t$ is v^{t-s}.

Definition 3.5.3 (Net present value)**.** Let $(A(t))_{t \in T}$ be a random cash flow. The net present value of the future cash flow at time $t \in T$ is defined as

$$V^+(t, A) = \sum_{u \in T, u > t} v^{u-t} \Delta A(u).$$

3.6 Prospective Reserves

Definition 3.6.1 (Prospective reserves)**.** We differentiate between three types of prospective reserves at time $t \in T$ for future cash flows, based on the current state $i \in S$ at time t with $\mathbb{P}[X_t = i] > 0$.

(a) The prospective reserves for being in the state $j \in S$ in the future, based on the current state i, are given by

$$V_i^+(t, A_j) = \mathbb{E}[V^+(t, A_j)|X_t = i].$$

(b) The prospective reserves for changing from state $j \in S$ to state $k \in S\backslash\{j\}$ in the future, based on the current state $i \in S$ are given by

$$V_i^+(t, A_{jk}) = \mathbb{E}[V^+(t, A_{jk})|X_t = i].$$

(c) The total prospective reserves based on the current state i are given by

$$V_i^+(t, A) = \mathbb{E}[V^+(t, A)|X_t = i].$$

From now on we generally refer to the total prospective reserves when we talk about the prospective reserves.
Considering the different parts of the future payments separately, we are able to calculate the overall prospective reserves.

Suppose that $i, j, k \in S$ and times $t < u$ in T with $\mathbb{P}[X_t = i] > 0$.

(a) The expected payment for being in state j at time u, based on the current state i, is given by

$$\begin{aligned}\mathbb{E}[\Delta A_j(u)|X_t = i] &= \mathbb{E}[a_j(u)1_{\{X_u=j\}}|X_t = i] \\ &= a_j(u)\ \mathbb{P}[X_u = j|X_t = i] \\ &= a_j(u)\ p_{ij}(t, u).\end{aligned}$$

(b) The expected payment for changing from state j to state k at time u, based on the current state i, is given by

$$\begin{aligned}\mathbb{E}[\Delta A_{jk}(u)|X_t = i] &= \mathbb{E}[a_{jk}(u)\Delta N_{jk}(u)|X_t = i] \\ &= a_{jk}(u)\ \mathbb{E}[\Delta N_{jk}(u)|X_t = i] \\ &= a_{jk}(u)\ \mathbb{P}[X_{u-1} = j, X_u = k|X_t = i] \\ &= a_{jk}(u)\ \mathbb{P}[X_{u-1} = j|X_t = i]\ \mathbb{P}[X_u = k|X_{u-1} = j] \\ &= a_{jk}\ (u)p_{ij}(t, u-1)\ p_{jk}(u-1, u).\end{aligned}$$

Suppose that $i, j, k \in S$, and $t \in T$ with $\mathbb{P}[X_t = i] > 0$.

(a) The prospective reserves for being in state j in the future, based on the current state i are given by

$$\begin{aligned} V_i^+(t, A_j) &= \mathbb{E}[V^+(t, A_j)|X_t = i] \\ &= \sum_{u \in T, u > t} \mathbb{E}[v^{u-t} \Delta A_j(u)|X_t = i] \\ &= \sum_{u \in T, u > t} v^{u-t}\, \mathbb{E}[\Delta A_j(u)|X_t = i] \\ &= \sum_{u \in T, u > t} v^{u-t}\, p_{ij}(t, u)\, a_j(u). \end{aligned}$$

(b) The prospective reserves for changing from state j to state k in the future, based on the current state i, are given by

$$\begin{aligned} V_i^+(t, A_{jk}) &= \mathbb{E}[V^+(t, A_{jk})|X_t = i] \\ &= \sum_{u \in T, u > t} \mathbb{E}[v^{u-t} \Delta A_{jk}(u)|X_t = i] \\ &= \sum_{u \in T, u > t} a_{jk}(u)\, v^{u-t}\, \mathbb{E}[\Delta N_{jk}(u)|X_t = i] \\ &= \sum_{u \in T, u > t} v^{u-t}\, a_{jk}(u)\, p_{ij}(t, u-1)\, p_{jk}(u-1, u). \end{aligned}$$

(c) Since the cumulative payments until time $t \in T$ are defined as

$$A(t) = \sum_{j \in S} A_j(t) + \sum_{j,k \in S} A_{jk}(t),$$

the prospective reserves at time t, given the current state $i \in S$ with $\mathbb{P}[X_t = i] > 0$, are

$$V_i^+(t, A) = \sum_{j \in S} \Big(V_i^+(t, A_j) + \sum_{k \in S} V_i^+(t, A_{jk}) \Big) \tag{3.7}$$

$$\begin{aligned} &= \sum_{u \in T, u > t} \sum_{j \in S} v^{u-t} \Big(p_{ij}(t, u)\, a_j(u) \\ &\quad + \sum_{k \in S} p_{ij}(t, u-1)\, p_{jk}(u-1, u)\, a_{jk}(u) \Big). \end{aligned} \tag{3.8}$$

Remark 3.6.2. The Markovian model gives us the possibility to calculate the reserves without explicit knowledge of the current state. It only may happen that some calculation trees in the recursion will be unneeded by knowing the current state.

3.7 Premium Reserves

Definition 3.7.1. For $t \in T$ and $i \in S$ with $\mathbb{P}[X - t = i] > 0$,

$$V_i(t, A) := a_i(t) + V_i^+(t, A)$$

is called the *premium reserve* at time t, based on the state i.

Theorem 3.7.2 (Thiele's recurrence relation for the premium reserve)**.** Suppose that $t \in T$ and $i \in S$ with $\mathbb{P}[X_t = i] > 0$, then

$$V_i(t, A) = a_i(t) + \sum_{j \in S} v\, p_{ij}(t, t+1)\big(a_{ij}(t+1) + V_j(t+1, A)\big).$$

With Thiele's recurrence relation it is possible to calculate the premium reserve $V_i(t, A)$ for any $t \in T$ by backward recursion. By choosing the last entry in the life table and setting $V_i^+(t_\omega, A) = 0$ in the premium reserve from Definition 3.7.1 we get $V_i(t_\omega, A) = a_i(t_\omega)$, for all $i \in S$. Then, we can calculate $V_i(t_\omega - 1, A), V_i(t_\omega - 2, A), \ldots, V_i(t_0, A)$ recursively for all $i \in S$.

3.8 The Principle of Equivalence

Due the principle of equivalence the expected present value of the difference between the total benefits and the premiums with respect to an individual insurance policy should equal 0 at the time of inception t_0.

Let $X_{t_0} = i$ be the initial state of the Markov chain. Since the contractual cash flow $A(t)$ consists of the paid out benefits minus the received premiums, we have the following possibilities to state the principle of equivalence,

$$a_i(t_0) + \mathbb{E}[V^+(t_0, A)|X_{t_0} = i] = a_i(t_0) + V_i^+(t_0, A) = V_i(t_0, A) = 0.$$

3.9 The Distribution Function of the Present Value of the Cash Flow A from $t \in T$ onwards

Definition 3.9.1. The present value of the cash flow A from t onwards, $t \in T$, is defined as

$$L_t(A) := a_{X_t}(t)\, 1_{\{X_t = i\}} + V^+(t, A),$$

where $V^+(t, A)$ is the present value of the future cash flow at time $t \in T$ as given by Definition 3.5.3.

Remark 3.9.2. Since

$$\mathbb{E}[a_{X_t}(t)|X_t = i] = a_i(t),$$

we can write the premium reserve for $t \in T$ and $i \in S$ with $\mathbb{P}[X_t = i] > 0$ as

$$V_i(t, A) = \mathbb{E}[L_t(A)|X_t = i] = a_i(t) + V_i^+(t, A).$$

Lemma 3.9.3 (Recurrence relation for the present value of the cash flow A from $t \in T$ onwards)**.** Suppose that $t \in T$, then

$$L_t(A) = a_{X_t}(t) + v\,\Big(a_{X_t X_{t+1}}(t+1)\;1_{\{X_t \neq X_{t+1}\}} + L_{t+1}(A)\Big).$$

Definition 3.9.4. For $t \in T$ and $i \in S$ with $\mathbb{P}[X_t = i] > 0$, we call

$$F_{i,t}(u) := \mathbb{P}[L_t(A) \leq u | X_t = i], \quad u \in \mathbb{R},$$

the *distribution function of the present value of the cash flow A from $t \in T$ onwards*, based on the current state i.

Proposition 3.9.5. For constant interest rate the recurrence relation

$$F_{i,t}(u) = \sum_{j \in S} p_{ij}(t, t+1) F_{j,t+1}\Big(\big(u - a_i(t)\big)(1+r) - 1_{\{i=j\}}\; a_{ij}(t+1)\Big)$$

with $u \in \mathbb{R}$, holds for all $i \in S$ and $t \in T$ with $\mathbb{P}[X_t = i] > 0$ and $t+1 \in T$.

3.10 Higher Moments of the Present Value of the Future Cash Flow A from $t \in T$ onwards

Theorem 3.10.1 (Recurrence relation for higher moments of the present value $L_t(A)$)**.** For $p \in \mathbb{N}$ and constant interest rate the affine recurrence relation

$$\begin{aligned}\mathbb{E}[L_t^p(A)|X_t = i] = a_i^p(t) + \sum_{k=1}^{p} \Bigg(\binom{p}{k} a_i^{p-k}(t) \sum_{j \in S} \Bigg(v^k\; p_{ij}(t, t+1) \\ \Big(a_{ij}^k(t+1) + \sum_{l=1}^{k-1} \Big[\binom{k}{l} a_{ij}^{k-l}(t+1)\; \mathbb{E}[L_{t+1}^l(A)|X_{t+1} = j] \\ + \mathbb{E}[L_{t+1}^k(A)|X_{t+1} = j]\Big]\Big)\Bigg)\Bigg) \qquad (3.9)\end{aligned}$$

holds for all $t \in T$ and $i \in S$ with $\mathbb{P}[X_t = i] > 0$.

Remark 3.10.2. If $p = 1$, this is Thiele's recurrence relation as we know it from Theorem 3.7.2.

4 Generation Annuity Tables Using the Example of the Austrian Annuity Valuation Table AVÖ 2008P

The AVÖ 2008P is a generation life table with a periodic base table and using trend functions to model the future mortality reduction.

4.1 Basis of Second Order

The static base table results as an update of the Austrian annuity valuation table "AVÖ 1999-P - Rechnungsgrundlagen für die Pensionsversicherung - Pagler & Pagler" (short AVÖ 1999P) by using data of the latest Austrian census in 2001 - adjusted using data from 2000 to 2002, published by Statistics Austria (A.1). This census provides the table ÖSt 2000/02 [7] which contains the adjusted and graduated mortality of the Austrian population in the year 2001 by sex and age up to an age of 112. Also available for the update was data from the Federal Ministry of Labor, Social Affairs and Consumer Protection (BMASK) which tested the sustainability of the Austria state pension system. Additionally Statistics Austria published a new forecast of death probabilities up to 2050. For the AVÖ 2008P these data was used to determine a new trend, new disability probabilities and new marriage probabilities in the case of death. Overall the death probabilities were not adjusted. Additionally the table was extrapolated to higher ages and cut off at the age of 120 by setting the survival probabilities to 0 above this age.

The AVÖ 2008P have the mathematical form

$$q_x^{\text{status}}(t) = \widetilde{q}_x^{\text{status}}(1982) \cdot e^{-\lambda_x \cdot (t-1982)}, \qquad t \geq 1982, \tag{4.1}$$

with the notation given in Table 4.1 (for female beneficiaries y is used instead of x).

The AVÖ 2008P are using constant age-dependent mortality reduction factors λ_x and λ_y which are monotonously decreasing to $\lambda_{120} = 0$.

4.2 Basis of First Order

There are several possibilities for model and parameter risk:

- Parameter risk:
  - The base table might be inaccurate.
- Model risk:
  - The future group of beneficiaries might differ from the actual population group which was used to generate the table.
  - The trend was determined by data of the past years. Later years show a higher trend than the first years of this period. So the actual and future trends might be underestimated.
  - The increase in the trend could even get higher in the near future (due to medical advances, in particular for old ages) or could also decline (the medical possibilities have reached the top).

$\widetilde{q}_x^{\text{status}}(1982)$	Static base table for a particular status for the year 1982, consisting of the population mortality.
$q_x^{\text{status}}(t)$	Death probability of a person of a particular status aged x in the calendar year t according to the AVÖ 2008P.
States:	
aa	Active
Apm/Apf	male/female Pensioner
w	Widow/Widower
i	Disabled
λ_x	Parameter for the yearly reduction in mortality (trend), obtained from the Austrian population data of the years 1972-2007.
i_x	Disability probability of a person aged x.
$h_{x+\frac{1}{2}}$	Marriage probability of a person aged $x + \frac{1}{2}$.
$y(x)$	Age of surviving dependant in the case of death of a person aged x.

Table 4.1: Notation used in the general form (4.1) *of the AVÖ 2008P table (for female beneficiaries y is used instead of x).*

Additionaly to those two risks there is the

- Risk of random fluctuations:
  - The table is generated by population data which is affected by statistical fluctuations, i.e. the determined empirical parameters might be inaccurate.
  - The population of the valuation is not big enough.

These risk factors can be taken into account by additional safety loadings. In contrast to annuity tables for life insurance where the insurer takes the risk of longevity (generally the life tables are guaranteed until the expiry of the policy, i.e. lifelong), pension funds and companies which calculate their balance sheet liabilities can change the underlying life tables or even have to change them. This is the reason that additional safety loadings are not made within pension insurance annuity tables which means that the new derived mortality tables are tables of second order.

5 Data and Statistical Material

5.1 Available Data

For the derivation of the valuation table it is necessary to observe the group of persons which are subject of future benefit promises. In Austria these beneficiaries typically come from one of the two large employee groups: white-collar or blue-collar workers (in the following employees or workers).

Statistical data for employees and workers are available from:

- Main Association of Austrian Social Security Organisations
  - Pension Insurance Organisation
  - Health Insurance Organisation
- Statistics Austria

Used statistics:

- Annual Pension Statistics of the Pension Insurance Organisation:
  - These statistics contain numbers of pensioners, first pension adjudications and reduction of pensioners because of death evaluated by age and by governmental insurance organisation (i.e. employees, workers, self-employed persons, farmers, state railway employees and miners, public servants, etc.).
- Statistics of death cases and surviving dependants:
  - Number of new widow/widower pensioners evaluated by age of deceased, by age of widow/widower, by gender and by employees or worker. Additionally the average age of new widows/widowers and the state (active or pensioner) of the deceased is quoted.
- Statistics of employees and workers:
  - Number of actives evaluated by age, by gender and by employees or worker.
- The graduated official General Austrian Life Tables of all censuses since 1876.
- Raw data yearly adjusted Austrian life tables from 1947 on.

5.2 Observation Period

The observation period of the available data ranges from the year 2000 to the year 2014. The single probabilities result from summation of these statistics over the years 2000 to 2014. So the mean observation year of the derived annuity table can be noted as 2007. The nearest available life table of Statistics Austria is the General Austrian Life Table ÖSt 2010/12 ([7]).

5.3 Age Determination

There are several possibilities of age determination:

- Determination of an exact age by subtraction of measurement date and date of birth (age in years, months and days): This determination generates classes of exact x-year olds.
  See 1 in Figure 5.1:
  - Living in the year t with exact age of $x-1$ years: $\overline{AD}$
  - Living in the year $t+1$ with exact age of x years: $\overline{BC}$
  - Dead of the years t and $t+1$ of the generation $x-t-1$ (exact age between $x-1$ and x years)
- Age as difference of year of observation and year of birth: Classes of persons with exact age between $x-1$ and x are generated.
  See 2 in Figure 5.1:
  - Living at beginning of year $t-1$ of generation $t-x-2$ with exact age from x to $x+1$ years: $\overline{EF}$
  - Living at beginning of year t of generation $t-x-2$ with exact age from $x+1$ to $x+2$ years: $\overline{GH}$
  - Dead of the year $t-1$ of the generation $t-x-2$ (exact age between x and $x+2$ years)

  The age structure of the underlying data of the derivation of the mortality tables pretends the age determination method. The observed groups of persons in the statistics from Austrian Social Security Organisations are structured by year of birth as at measurement date 31st December. So the age determination of these annuity tables corresponds to this method.
- The actuarial age is the - commercially to whole years - rounded exact age: Persons with exact age between $x-\frac{1}{2}$ and $x+\frac{1}{2}$ are put in one class.
  See 3 in Figure 5.1:
  - Living at beginning of year $t+1$ with exact age from $x+0.5$ to $x+1.5$ years: $\overline{IJ}$
  - Living at beginning of year $t+2$ with exact age from $x+1.5$ to $x+2.5$ years: $\overline{KL}$
  - Dead of the year $t+1$ of the generations $t-x-1$ and $t-x$ (exact age at beginning of year $t+1$ between $x+0.5$ and $x+1.5$ years)

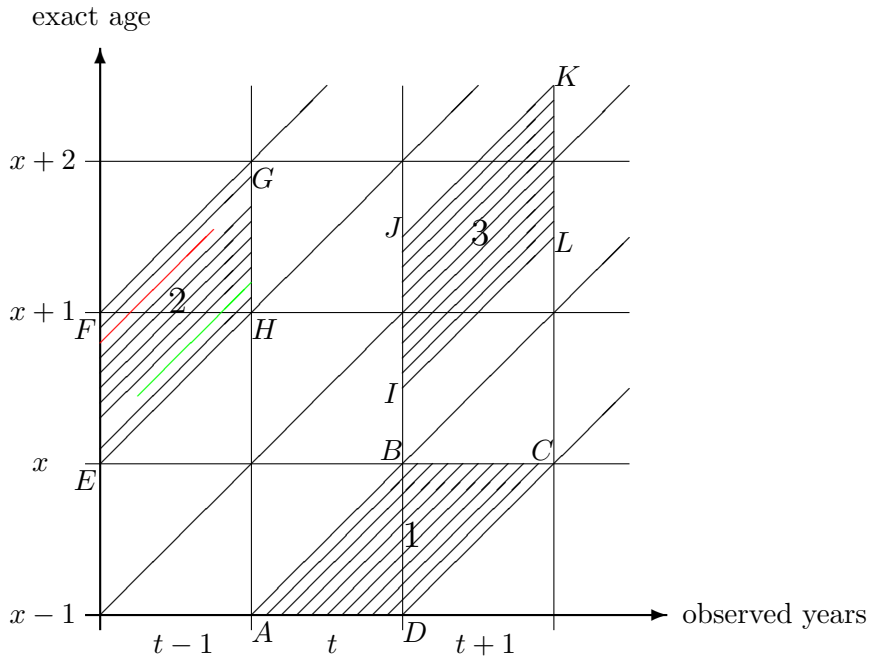

Figure 5.1: Lexis diagram.

5.4 Dependent and Independent Probabilities

The observed community of beneficiaries is a collective of "compound order", i.e. persons are affected by several causes of withdrawal. For actives withdrawal causes are death, disablement, leaving or retirement, for disabled death and reactivation and for dependants death and remarriage (see Figure 2.1). Because reactivation and loss of dependants pension because of remarriage are so marginal in Austria these withdrawal causes are not taken into account in the annuity tables. So only actives have more than one withdrawal cause. The community was observed for one withdrawal cause, even for actives only one withdrawal cause was determined, i.e. during one year an active can stay active or can die or can get disabled or can retire. So the observed partial withdrawal possibilities are independent probabilities in an actuarial sight since Karup (see [12]). For actuarial calculations we usually need dependent probabilities which are additionally linked.

5.5 Estimators for the Raw Transition Probabilities

The ideas of the next chapters are taken out of [4].

A life in Figure 5.2 that begins in the observation period (green life line) or ends in the observation period (red life line) or both (yellow life line) or a life which has age between x and $x+1$ at the begin or end of the

Age x	Observation year t minus year of birth
Living of year t	Number of
$l_x^{\mathrm{aa}}(t)$	Actives as at December 31st
$l_x^{\mathrm{i}}(t)$	Disabled as at December 31st
$l_x^{\mathrm{Apm}}(t)$	Pensioners as at December 31st
$l_x^{\mathrm{w}}(t)$	Widows/widower as at December 31st
$l_{\overline{x}}^{\mathrm{w}}(t)$	Widows/widower by the age of the deceased as at December 31st
$l_x^{\mathrm{g}}(t)$	Total population as at December 31st
Leaving in year t	Number of decrease of
$d_x^{\mathrm{aa}\to\dagger}(t)$	Actives because of death
$d_x^{\mathrm{aa}\to\mathrm{i}}(t)$	Actives because of disability
$d_x^{\mathrm{aa}\to\mathrm{Apm}}(t)$	Actives because of retirement
$d_x^{\mathrm{i}\to\mathrm{Apm}}(t)$	Disabled because of reaching retirement age
$d_x^{\mathrm{i}\to\dagger}(t)$	Disabled because of death
$d_x^{\mathrm{i}\to\mathrm{aa}}(t)$	Disabled because of reactivation
$d_x^{\mathrm{Apm}\to\dagger}(t)$	Pensioners because of death
$d_x^{\mathrm{w}\to\dagger}(t)$	Widows/widower because of death
$d_x^{\mathrm{g}\to\dagger}(t)$	Total population because of death
Entries in year t	Number of increase of
$e_x^{\to\mathrm{aa}}(t)$	Actives because of entries
$e_x^{\mathrm{i}\to\mathrm{aa}}(t)$	Actives because of reactivation of disabled
$e_x^{\mathrm{aa}\to\mathrm{i}}(t)$	Disabled because of disability of actives
$e_x^{\mathrm{aa}\to\mathrm{Apm}}(t)$	Pensioners because of retired actives
$e_x^{\mathrm{i}\to\mathrm{Apm}}(t)$	Pensioners because of retired disabled
$e_{\overline{x}}^{\to\mathrm{w}}(t)$	Widows/widowers because of death of initial beneficiaries by the age of the deceased
$e_x^{\to\mathrm{g}}(t)$	Total population because of entries

Table 5.1: Notation for data.

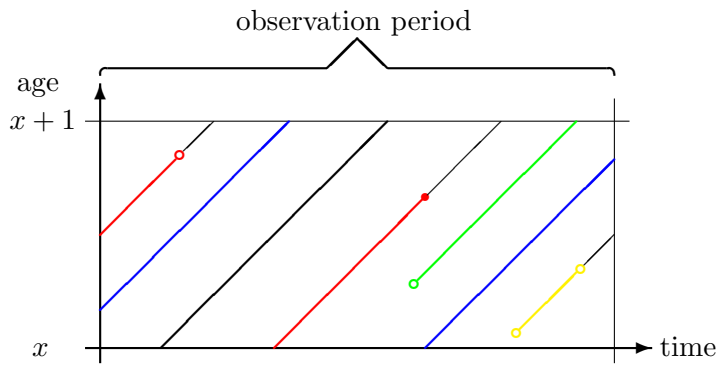

Figure 5.2: Lexis diagram detail.

observation period (blue life lines) leads to an incomplete observation. The exposure $E_x(t)$, i.e. the observed number of risks in the observation period t, is (disregarding a factor $\sqrt{2}$) the total length of all diagonal lines in the lexis diagram in the observation period. Assuming that n lives are involved in the observed rectangle and observing the live of an individual i from age $x + r_i$ to age $x + s_i$ $(0 \leq r_i < s_i \leq 1)$ the exposure is calculated by

$$E_x = (s_1 - r_1) + (s_2 - r_2) + \ldots + (s_n - r_n). \tag{5.1}$$

Remark 5.5.1. The red lines in the lexis diagram could denote death cases (line ending in a red dot) but could also denote cancellation of an insurance policy (line ending in a red circle). The green line could be a life that begins because the insurance policy is bought during the observation period.

Let d_x be the observed number of death cases with age between x and $x+1$ and I is the set of indices i of lifes for which a death case was observed.

Then

$$\begin{aligned} E_x &= \sum_{i=1}^{n}(1 - r_i + s_i - 1) \\ &= \sum_{i=1}^{n}(1 - r_i) - \sum_{i=1}^{n}(1 - s_i) \\ &= \sum_{i=1}^{n}(1 - r_i) - \sum_{i \in I}(1 - s_i) - \sum_{i \notin I}(1 - s_i). \end{aligned} \tag{5.2}$$

The idea is to equate the expected number of deaths with the observed number to derive an estimator $\hat{q}_x$.

The expected number of deaths in a narrower sense is

$$\sum_{i=1}^{n} {}_{1-r_i}q_{x+r_i} - \sum_{i \notin I} {}_{1-s_i}q_{x+s_i}. \tag{5.3}$$

Remark 5.5.2. The first term in Equation (5.3) sums up all death cases until the end of the observation period, i.e. the red line in Figure 5.2 is extended (thin black line after the red dot). The second term is the reduction sum of the extension lines (thin black line after the red and yellow circle) of all cancellation cases which are not death cases.

Definition 5.5.3 (Mortality probabilities for a fraction of a year)**.** There are 3 possibilities to assume the development of mortality during the year:

(a) Linearity of ${}_uq_x$
Assuming that the function ${}_uq_x$ is linear in u, deduces by linear interpolation between $u = 0$ and $u = 1$, that

$${}_uq_x = u\, q_x. \tag{5.4}$$

It follows that

$${}_up_x = 1 - u\, q_x. \tag{5.5}$$

Using

$$\mu_{x+t} = -\frac{d}{dt} \ln {}_tp_x, \tag{5.6}$$

where μ is the force of mortality, leads to

$$\mu_{x+u} = \frac{q_x}{1 - u\, q_x}. \tag{5.7}$$

(b) Constant μ_{x+u}
We denote the constant value of μ_{x+u} $(0 < u < 1)$ with $\mu_{x+\frac{1}{2}}$. From Equation (5.6) follows, that

$$\mu_{x+\frac{1}{2}} = -\ln p_x. \tag{5.8}$$

Also from Equation (5.6) follows, that

$$ {}_{u}p_x = \exp(-u\,\mu_{x+\frac{1}{2}}) = p_x^u. \tag{5.9} $$

(c) Linearity of ${}_{1-u}q_{x+u}$ (often called *Balducci* assumption)
By linear interpolation between $u = 0$ and $u = 1$, it holds

$$ {}_{1-u}q_{x+u} = (1-u)\,q_x. \tag{5.10} $$

It follows that

$$ {}_{u}p_x = \frac{p_x}{{}_{1-u}p_{x+u}} = \frac{1-q_x}{1-(1-u)q_x}. \tag{5.11} $$

Using Equation (5.6) we get

$$ \mu_{x+u} = \frac{q_x}{1-(1-u)\,q_x}. \tag{5.12} $$

Now we can try to simplify Equation (5.3) by using assumption (c) for the mortality during the year. W ith ${}_{1-u}q_{x+u} = (1-u)q_x$ for $0 \leq u \leq 1$ we get

$$ \sum_{i=1}^{n}(1-r_i)q_x - \sum_{i\notin I}(1-s_i)\,q_x = E_x\,q_x + \sum_{i\in I}(1-s_i)\,q_x. \tag{5.13} $$

Equalizing this term with the observed number of deaths $d_x(t)$ we get the classical estimator for the death probability

$$ \hat{q}_x = \frac{d_x}{E_x + \sum_{i\in I}(1-s_i)}. \tag{5.14} $$

Equation (5.14) works well with big data. Under the assumption that death cases happen on average in the age of $x + \frac{1}{2}$, it reduces to

$$ \hat{q}_x = \frac{d_x}{E_x + \frac{1}{2}d_x}. \tag{5.15} $$

With small sets of data Equation (5.14) has to be seen carefully: It my happen that its enumerator is bigger than its denominator, i.e. the estimator for the mortality is bigger than 1. It is also not clear how calculate confidence intervals or how to test hypotheses.

5.6 Alternative Estimators for the Raw Transition Probabilities

Let be m a positive integer and $h = \frac{1}{m}$. We want to estimate ${}_hq_x$ and use assumption (c) of Definition 5.5.3 that ${}_{h-u}q_{x+u}$ is a linear function of u, i.e. that

$$ {}_{h-u}q_{x+u} = (1 - mu)\, {}_hq_x, \qquad \text{for } 0 \le u \le h. \tag{5.16} $$

Additionally we assume that the force of mortality between ages x and $x+1$ is a periodical function with period h. So we assume, that for $j = 1, 2, \ldots, m-1$

$$ {}_{h-u}q_{x+jh+u} = {}_{h-u}q_{x+u}, \qquad \text{for } 0 \le u \le h. \tag{5.17} $$

With those two assumption one can argue, that the expected number of deaths is

$$ m\, E_x\, {}_hq_x + m \sum_{i \in I} (s_i^{(m)} - s_i)_h q_x, \tag{5.18} $$

with $s_i^{(m)} = [m\, s_i + 1]/m$ for $i \in I$ (i.e. $s_i^{(m)}$ results from s_i by rounding up to the next m^{th}). By equalization with the observed number of deaths d_x we get

$$ {}_h\hat{q}_x = \frac{h\, d_x}{E_x + \sum_{i \in I} (s_i^{(m)} - s_i)}. \tag{5.19} $$

Because of Equation (5.17) follows $p_x = ({}_hp_x)^m$. The estimator for q_x than with Equation (5.19) is

$$ \hat{q}_x = 1 - (1 - {}_h\hat{q}_x)^m. \tag{5.20} $$

In the limit $m \to \infty$ Equations (5.16) and (5.17) are equal with assumption (b) of Definition 5.5.3. From Equation (5.18) it follows that at the limit the expected number of deaths is $E_x\, \mu_{x+\frac{1}{2}}$. So we get as an estimator for the constant force of mortality

$$ \hat{\mu}_{x+\frac{1}{2}} = \frac{d_x}{E_x}, \tag{5.21} $$

and finally an estimator for q_x with

$$ \hat{q}_x = 1 - \exp(-\hat{\mu}_{x+\frac{1}{2}}) = 1 - \exp(-d_x/E_x). \tag{5.22} $$

5.7 Maximum Likelihood Estimators for the Raw Transition Probabilities

Assuming that the n observed lifes are independent, the Likelihoodfunction of the observations is

$$ \prod_{i \in I} \mu_{x+s_i}\; {}_{s_i-t_i}p_{x+t_i} \cdot \prod_{i \notin I} {}_{s_i-t_i}p_{x+t_i}. \tag{5.23} $$

Again using the assumption of a constant force of mortality, simplifies this to

$$(\mu_{x+\frac{1}{2}})^{d_x} \exp(-\mu_{x+\frac{1}{2}} E_x). \tag{5.24}$$

This expression is maximal if the forces of mortality is d_x/E_x which means that the estimator (5.21) is the Maximum Likelihood estimator of the forces of mortality. Because of invariance (5.22) is the Maximum Likelihood estimator ofq_x.

5.8 Estimators for the Raw Transition Probabilities of the New Tables 2007

Because of the great number of data sets we are content with estimators due to Equation (5.14).

It follows that the raw elimination probabilities are calculated by (also see [9])

$$\hat{q}^{\,\text{status}}_{x-\frac{1}{2}} = \frac{2\, d_x^{\text{status}\to\dagger}(t)}{l_{x-1}^{\text{status}}(t-1) + l_x^{\text{status}}(t) + d_x^{\text{status}\to\dagger}(t)}. \tag{5.25}$$

As estimators

(a) for the death probability of disabled

$$\hat{q}^{\,\text{i}}_{x-\frac{1}{2}} = \frac{2\, d_x^{\text{i}\to\dagger}(t)}{l_{x-1}^{\text{i}}(t-1) + l_x^{\text{i}}(t) + d_x^{\text{i}\to\dagger}(t)}, \tag{5.26}$$

(b) for the death probability of old age pensioners

$$\hat{q}^{\,\text{Apm}}_{x-\frac{1}{2}} = \frac{2\, d_x^{\text{Apm}\to\dagger}(t)}{l_{x-1}^{\text{Apm}}(t-1) + l_x^{\text{Apm}}(t) + d_x^{\text{Apm}\to\dagger}(t)}, \tag{5.27}$$

(c) for the death probability of dependants

$$\hat{q}^{\,\text{w}}_{x-\frac{1}{2}} = \frac{2\, d_x^{\text{w}\to\dagger}(t)}{l_{x-1}^{\text{w}}(t-1) + l_x^{\text{w}}(t) + d_x^{\text{w}\to\dagger}(t)}, \tag{5.28}$$

(d) for the death probability of the total population

$$\hat{q}^{\,\text{g}}_{x-\frac{1}{2}} = \frac{2\, d_x^{\text{g}\to\dagger}(t)}{l_{x-1}^{\text{g}}(t-1) + l_x^{\text{g}}(t) + d_x^{\text{g}\to\dagger}(t))}, \tag{5.29}$$

(e) for the death probability of actives

$$\hat{q}^{\,\text{aa}}_{x-\frac{1}{2}} = \frac{2\, d_x^{\text{aa}\to\dagger}(t)}{l_{x-1}^{\text{aa}}(t-1) + l_x^{\text{aa}}(t) + d_x^{\text{aa}\to\dagger}(t)} \tag{5.30}$$

were chosen.

It is not possible to directly derive the death probability of actives because of missing data for death cases of active employees and workers. Austrian statutory health insurance organisations (9 individual organisations for each Austrian federal state and more organsisations for self-employed persons, for farmers, for state railway workers and miners, for public servants and some company health funds) collect statistics of cases of illness but not of death cases. Due to the lack of this data we have to combine the basic values of the total population death probabilities, disabled death probabilities and disability probabilities to determine the death probabilities of actives.

From the relation between the living $l_x^{\mathrm{g}}(t) = l_x^{\mathrm{aa}}(t) + l_x^{\mathrm{i}}(t)$ and between the dead $d_x^{g\to\dagger}(t) = d_x^{\mathrm{aa}\to\dagger}(t) + d_x^{\mathrm{i}\to\dagger}(t) + d_x^{\mathrm{ai}\to\dagger}(t)$ we derive:

$$\begin{aligned}
d_x^{\mathrm{aa}\to\dagger}(t) &= d_x^{\mathrm{g}\to\dagger}(t) - d_x^{\mathrm{i}\to\dagger}(t) - d_x^{\mathrm{ai}\to\dagger}(t) \\
l_x^{\mathrm{aa}} q_x^{\mathrm{aa}} &= l_x^{\mathrm{g}} q_x^{\mathrm{g}} - l_x^{\mathrm{i}} q_x^{\mathrm{i}} - l_x^{\mathrm{aa}} i_x \tfrac{1}{2} q_x^{\mathrm{i}} \\
l_x^{\mathrm{aa}} q_x^{\mathrm{aa}} &= l_x^{\mathrm{g}} q_x^{\mathrm{g}} - (l_x^{\mathrm{g}} - l_x^{\mathrm{aa}}) q_x^{\mathrm{i}} - l_x^{\mathrm{aa}} i_x \tfrac{1}{2} q_x^{\mathrm{i}} \\
l_x^{\mathrm{aa}} q_x^{\mathrm{aa}} &= l_x^{\mathrm{g}} q_x^{\mathrm{g}} - l_x^{\mathrm{g}} q_x^{\mathrm{i}} + l_x^{\mathrm{aa}} q_x^{\mathrm{i}} - l_x^{\mathrm{aa}} i_x \tfrac{1}{2} q_x^{\mathrm{i}} \\
l_x^{\mathrm{aa}} q_x^{\mathrm{aa}} &= l_x^{\mathrm{g}} (q_x^{\mathrm{g}} - q_x^{\mathrm{i}}) + l_x^{\mathrm{aa}} q_x^{\mathrm{i}} (1 - \tfrac{1}{2} i_x) \qquad | : l_x^{\mathrm{aa}} \\
q_x^{\mathrm{aa}} &= \frac{l_x^{\mathrm{g}}}{l_x^{\mathrm{aa}}} (q_x^{\mathrm{g}} - q_x^{\mathrm{i}}) + q_x^{\mathrm{i}} (1 - \tfrac{1}{2} i_x). \qquad (5.31)
\end{aligned}$$

(f) As estimator for the disability probability

$$\hat{i}_{x-\frac{1}{2}} = \frac{2\, e_x^{\mathrm{aa}\to i}(t)}{l_{x-1}^{\mathrm{aa}}(t-1) + l_x^{\mathrm{aa}}(t) + e_x^{\mathrm{aa}\to i}(t)} \qquad (5.32)$$

was chosen.

(g) Marriage probability in the case of death:
The conditional probability, that in the case of the death of a beneficiary a surviving dependant exists, is given by

$$\hat{h}_x = \frac{e_{\overline{x}}^{\to \mathrm{w}}(t)}{d_x^{\mathrm{aa}\to\dagger}(t) + d_x^{\mathrm{i}\to\dagger}(t) + d_x^{\mathrm{Apm}\to\dagger}(t)}. \qquad (5.33)$$

(h) Age difference of spouses in the case of death:
Age differences between spouses in each observation year by age of the deceased were calculated and with this information the statistical age of the spouse $y(x)$ is tabulated.

6 Derivation of the Annuity Table

6.1 Raw Data

The used data (see 5.1) is a summation of the years 2000 to 2014.

6.1.1 Raw Death Probabilities

Since 2011 female disabled persons over age 60 and male disabled persons over age 65 are counted among old age pensioners in the annual pension statistics of the Austrian Pension Insurance Organisation. So only until 2010 it is possible to observe lifelong death probabilities of disabled persons separated from the ones for old age pensioners.

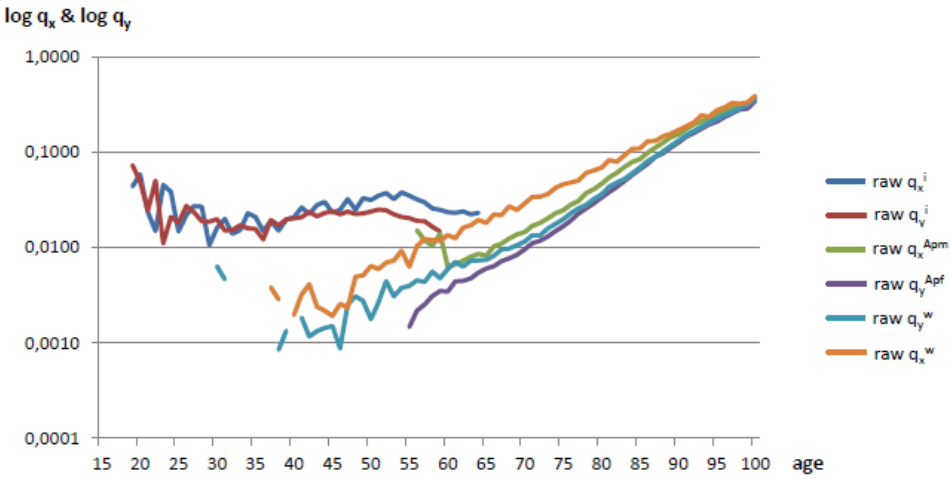

Figure 6.1: Logarithm of raw death probabilities of white collar workers.

6.1.2 Raw Disability Probabilities

It is remarkable that after the regular retirement age (60 for females and 65 for males) the raw disability probabilities decrease in such a strong manner. On the one hand the remaining active population is getting too small to give significant statements and on the other hand apparently the selection effect that only healthy and fit persons stay active after the regular retirement age produces decreasing disability probabilities.

6.1.3 Raw Marriage Probabilities

If surviving dependants have their own state old age or state disability pension or have too big salaries the state dependant pension is shortened from 60% entitlement down to 0% since the nineties. If no state dependant pension is paid because of these reasons the widow or widower is not quoted

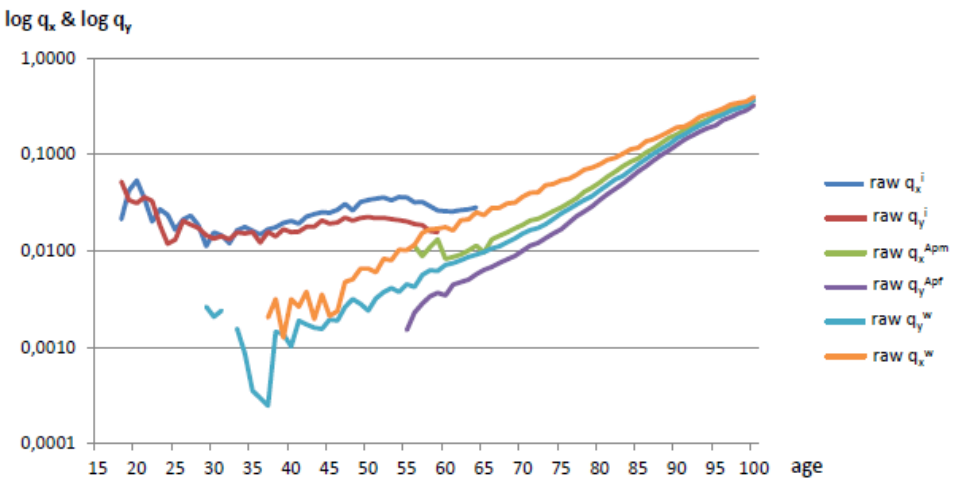

Figure 6.2: Logarithm of raw death probabilities of blue and white collar workers.

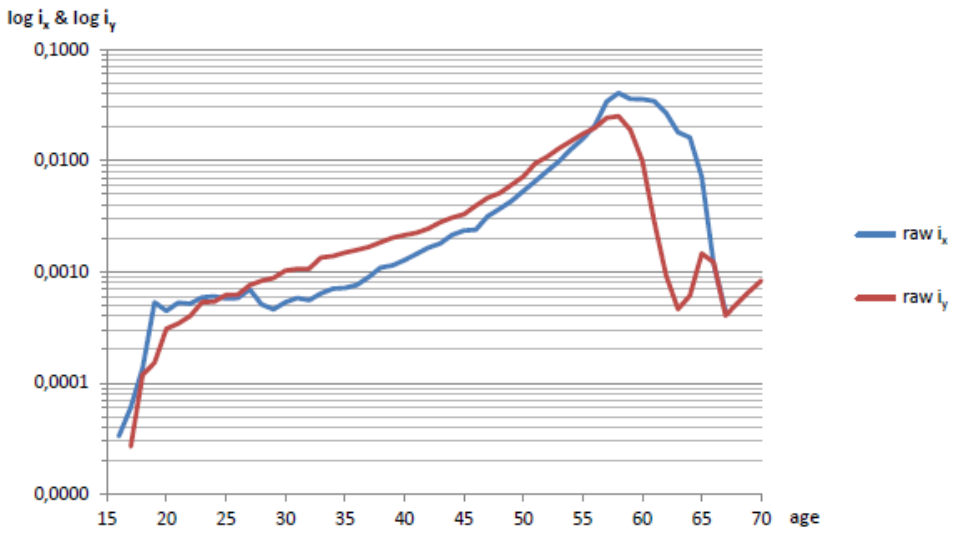

Figure 6.3: Logarithm of raw disability probabilities of male and female white collar beneficiaries.

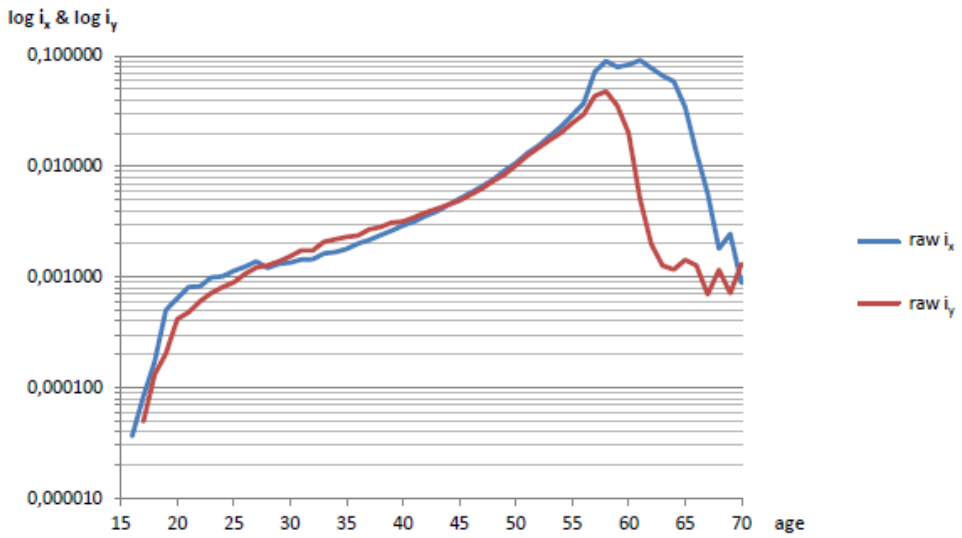

Figure 6.4: Logarithm of raw disability probabilities of male and female white and blue collar beneficiaries.

as a entry to the dependants pensions in the statistics of Austrian Pension Organisation. In the past where many married females had few or no income and the males earned most of the family income, dependant pension reductions were not acceptable in a state pension system. But today gender equality and the new role of females in the economy which causes continuous female employment, the female income would result in a too good pension coverage and therefore the reduction rules were introduced. This leads to a missing group of persons in the derivation of marriage probabilities in the case of death because normally pension promises do not take the dependant income into account for the dependant pension entitlement. So it is necessary to increase the number of new dependant pension entries especially the widow pensions in the age range from 40 to 75 and in a much weaker way the widower pensions in the age range from 50 to 65.

6.1.4 Raw Age Difference of Spouses in the Case of Death

As expected males have younger wifes and femals have older husbands. Both, the marriage probabilities and the age difference in the case of death, were determined for the hole observed group of persons and will be use in the following for both, for blue and for white collar workers.

6.2 Base Mortality Table 2007

In the first step the rough probabilities have to be graduated and here graphical, mechanical and analytical methods were developed over the years.

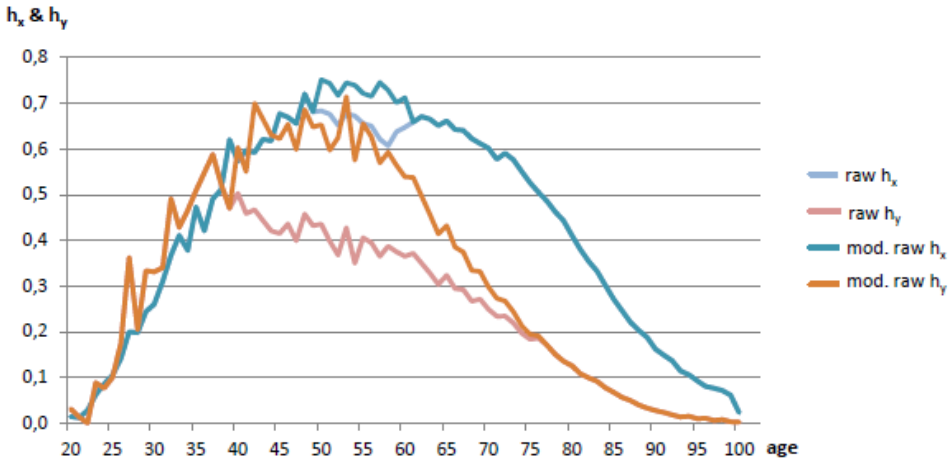

Figure 6.5: Raw marriage probabilities in the case of death of male and female beneficiaries.

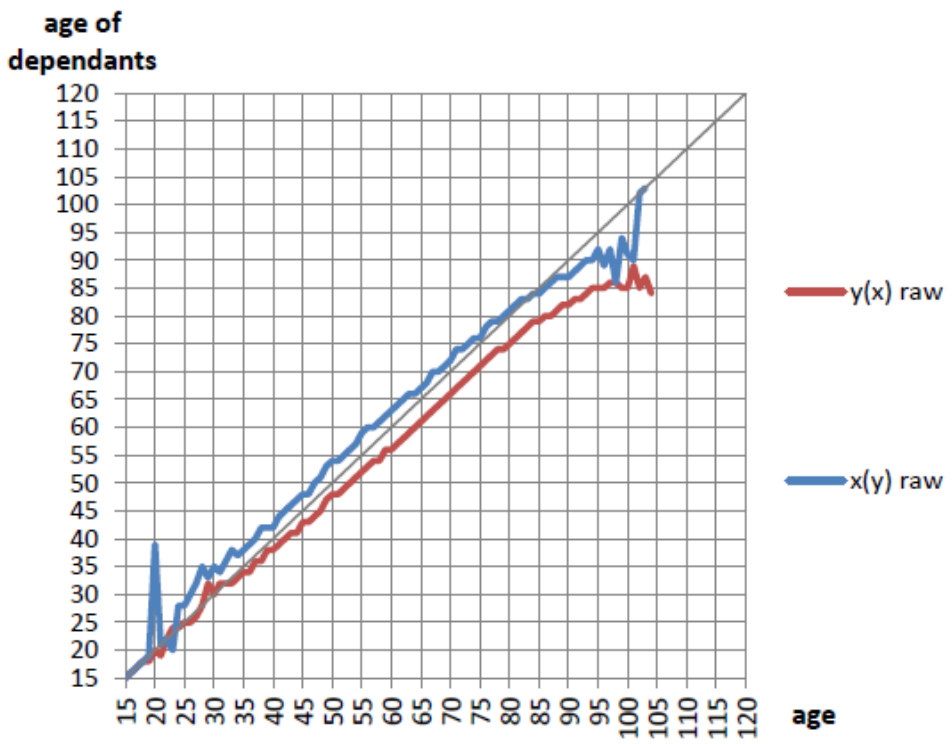

Figure 6.6: Raw age difference of spouses in the case of death of male and female beneficiaries.

6.2.1 Graduation Method of Whittacker–Henderson

The Whittacker–Henderson method (see [2],[5] and [6]) is a mechanical graduation method and became state of the art in smoothing death probabilities in many countries for example for the US "RP-2000 Mortality Tables". Its advantage is that it also provides smoothing at the extremities of the observed data. Graduation has to be a compromise between smoothness and fit and so the Whittacker–Henderson method combines the minimization of the fitting measure of the observed rough values to the graduated values with simultaneous minimization of the smoothness of the graduation curve. The aim is to graduate an observed vector $\widehat{Q} = (\hat{q}_{x_0}, \ldots, \hat{q}_{x_0+n})^{\mathrm{T}}$ to obtain a smooth vector $Q = (q_{x_0}, \ldots, q_{x_0+n})^{\mathrm{T}}$.

First we define the fitting measure

$$\sum_{k=0}^{n} (q_{x_0+k} - \hat{q}_{x_0+k})^2, \tag{6.1}$$

the smoothing measure

$$\sum_{k=0}^{n-s} (\Delta^s q_{x_0+k})^2, \quad s \in \mathbb{N}, \tag{6.2}$$

the sth differences

$$\Delta^s q_{x_0+k} = \sum_{\upsilon=0}^{s} (-1)^{\upsilon} \binom{s}{\upsilon} q_{x_0+k+\upsilon} \tag{6.3}$$

and the strict positive weights $w_k > 0$, satisfying

$$\sum_{k=0}^{n} w_k = 1. \tag{6.4}$$

Second we have to choose the positive smoothness constant g.

Remark 6.2.1. The weights w_k allow the user to evaluate their observation data for example by setting the weights w_k as the reciprocal of an estimate of the variance of the kth observation. The constant g is chosen a priori by the user. In my further cacluations I used $g = 40$ or $g = 0.5$.

The optimization of

$$\sum_{k=0}^{n} w_k (q_{x_0+k} - \hat{q}_{x_0+k})^2 + g \cdot \sum_{k=0}^{n-s} (\Delta^s q_{x_0+k})^2 \to \min \tag{6.5}$$

is unique (see[18]) and therefore provides the wanted vector Q.

The first term in in Equation (6.5) guarantees that the errors are smoothed and the second term prevents oscillation. In my further calculations I always used $w_k = w = \frac{1}{n+1}$ which [5] denotes as "unweighted" Whittaker graduation.

This method does not provide a graduation function but a matrix which k_{th} row is a mechanical graduation function of our $\hat{q}_{x_0+k}$ with $k = 0, \ldots, n$.

To handle the Whittacker–Henderson method we use following matrices and vectors

- W the $(n+1) \times (n+1)$ diagonal matrix of the w_k,
- I the $(n+1) \times (n+1)$ unit matrix,
- $\widehat{Q}$ and Q the vector of the $(n+1)$ observed resp. graduated values and
- K the $(n-s+1) \times (n+1)$ matrix of the sth differences.

Remark 6.2.2. In the ith row $(i = 1, \ldots, n-s+1)$ of the matrix K the first $i-1$ values are equal 0, the next $s+1$ values are equal the alternating binomial-coefficients $k_r = (-1)^r \binom{s}{r}$ for $r = 0, \ldots, s$ and the last $n-s-i+1$ values are all zero again.

Example 6.2.3. The matrix K with $s = 3$:

$$K = \begin{pmatrix} 1 & -3 & 3 & -1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & -3 & 3 & -1 & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & 1 & -3 & 3 & -1 & 0 \\ 0 & \ldots & 0 & 0 & 1 & -3 & 3 & -1 \end{pmatrix}.$$

Now we can formulate the minimization problem in Equation (6.5) as

$$(Q - \widehat{Q})^T W (Q - \widehat{Q}) + g(KQ)^T KQ \to \min. \tag{6.6}$$

In [5] and it is shown that this expression gets minimal if

$$(W + gK^T K)Q = W\widehat{Q}. \tag{6.7}$$

Because the matrix $W + gK^T K$ is nonsingular and positive definite [2, 5] it also holds

$$Q = (W + gK^T K)^{-1} W\widehat{Q}, \tag{6.8}$$

which is the wanted graduated vector.

The choice of s and g should consider that

- Are the observed values already smooth, g can be chosen small.
- The more smoother the curve should be,

- the higher the order of the s-differences and
- the more observed values have to be graduated, the more bigger must g be chosen.

A brief abstract of the Whittaker–Henderson algorithm:

First Step: Given the $n+1$ observed values $\hat{q}_x$, form the vector $\widehat{Q} = \{\hat{q}_x | x \in [x_0, x_0 + n]\}, \quad x_0, n \in \mathbb{N}$

Second Step: Choose the weights w_k and form the $(n+1)\times(n+1)$ diagonal matrix W.

Third Step: Choose the order of s and form the $(n-s+1)\times(n+1)$ matrix K.

Forth Step: Choose the smoothness constant $g > 0$.

Fifth Step: Calculate the $(n+1)\times(n+1)$ matrix $D = (W + gK^T K)^{-1} W$.

Sixth Step: Calculate the $(n+1)$ graduated values $Q = D\widehat{Q}$.

I used MatLab to graduate the observed death and disability probabilities with $s = 3$ and $g = 40$ resp. $g = 0.5$:

```
q = xlsread('rohwahrscheinlichkeiten.xlsx');
q = b(:,1);

n = size(b,1)-1;
s = 3;
z = n - s + 1;

W = eye(n + 1)./(n+1);

K = zeros(z,z+s);
for i = 1:z
    for j = 0:s
        K(i,i+j) = (-1)^j * nchoosek(s,j);
    end
end

g1 = 40;
g2 = 0.5;

D1 = inv(W + g1*transpose(K)*K)*W;
D2 = inv(W + g2*transpose(K)*K)*W;
```

```
P1 = D1*q;
P2 = D2*q;

xlswrite('rohwahrscheinlichkeiten.xlsx', P1,'Tabelle1','B1');
xlswrite('rohwahrscheinlichkeiten.xlsx', P2,'Tabelle1','C1');
```

6.2.2 Graduation with the Least-Squares-Method

Alternatively to the Whittaker–Henderson method we try to graduate the observed values with a least-squares model which is an example of an analytic graduation method.

Given the raw probabilities $\hat{q}_x$ for a period x_0 to $x_{\max}$ and the graduation function $f(x, a_0, a1, \ldots, a_k)$ one can calculate the graduated values q_x with

$$q_x = f(x, a_0, a_1, \ldots, a_k), \tag{6.9}$$

by choosing the parameters $a_0, a_1, \ldots, a_k$ in such a way that the sum of the least squares for each age x is minimized. Given the error for age x as $q_x - \hat{q}_x$, the function F describes the sum of the least squares

$$F(x_0, x_{\max}, a_0, a_1, \ldots, a_k) = \sum_{i=x_0}^{x_{\max}} \Big(f(i, a_0, a_1, \ldots, a_k) - \hat{q}_i \Big)^2. \tag{6.10}$$

Under the conditions that f is continuous and at least one times differentiable in all $a_0, a_1, \ldots, a_k$ it must be

$$\frac{1}{2} \cdot \frac{\partial F}{\partial a_j} = \sum_{i=x_0}^{x_{\max}} \frac{\partial f}{\partial a_j} \Big(f(i, a_0, a_1, \ldots, a_k) - \hat{q}_i \Big)^2, \tag{6.11}$$

that Equation (6.10) gets minimal for $j = 0, 1, 2, \ldots, k$. The obtained system of $k+1$ equations has to be solved for the $k+1$ unknowns $a_0, a_1, \ldots, a_k$.

As a second try of graduation in addition to the Whittacker–Henderson method a polynomial of 4th grade $q_x = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$ as graduation function which satisfies the claimed conditions was applied for our raw data.

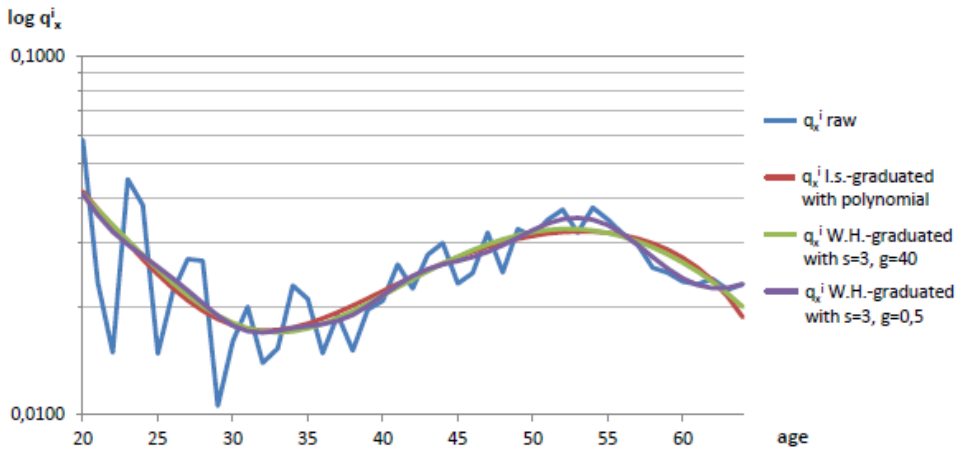

Figure 6.7: Logarithm of raw and graduated death probabilities of male white collar disabled.

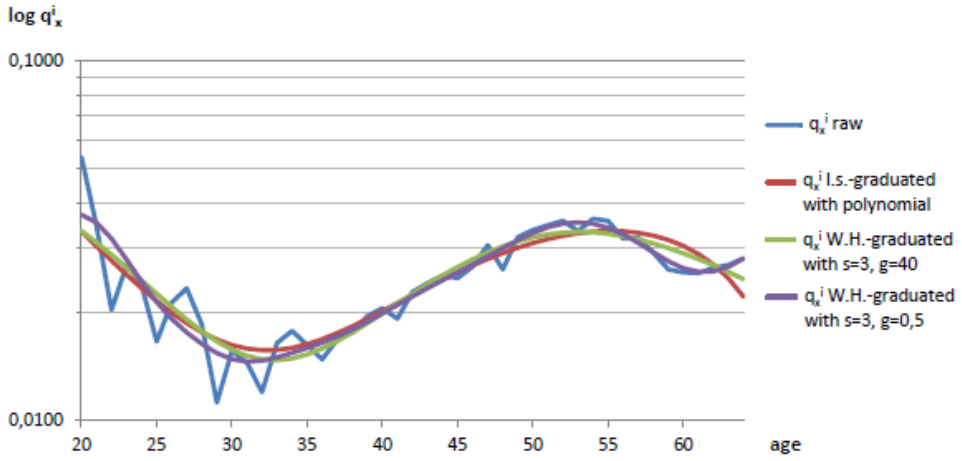

Figure 6.8: Logarithm of raw and graduated death probabilities of male white and blue collar disabled.

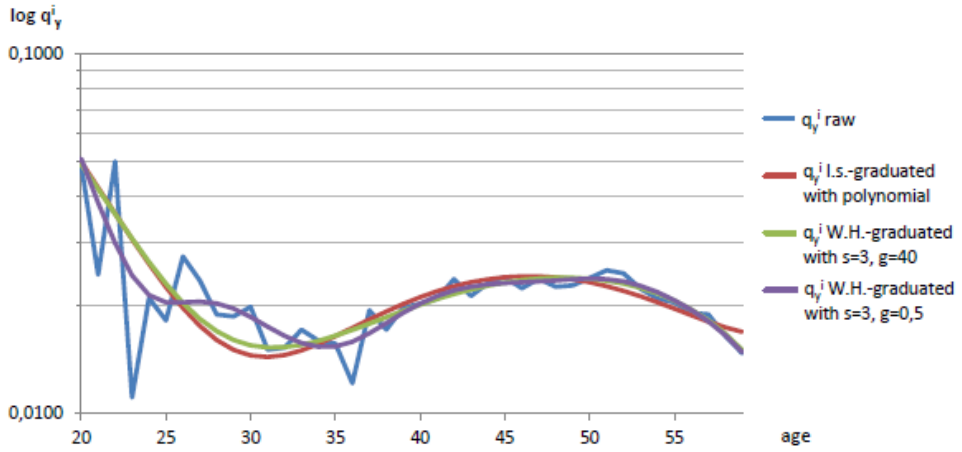

Figure 6.9: Logarithm of raw and graduated death probabilities of female white collar disabled.

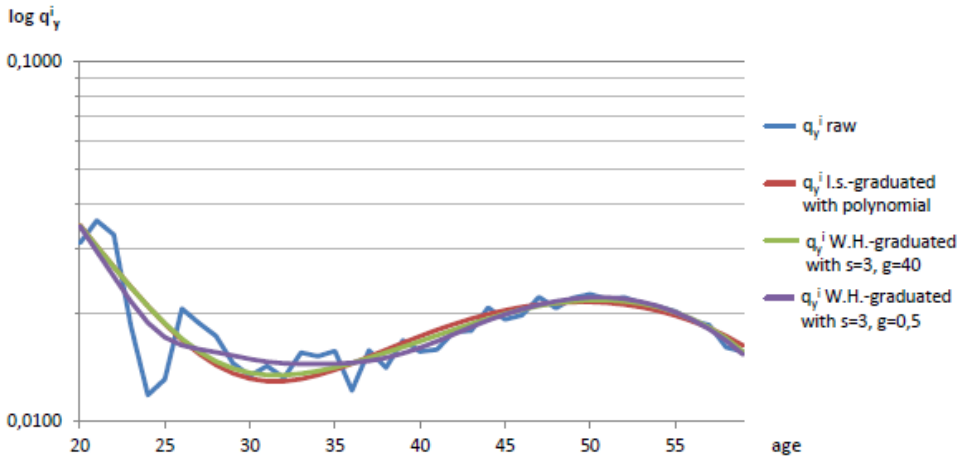

Figure 6.10: Logarithm of raw and graduated death probabilities of female white and blue collar disabled.

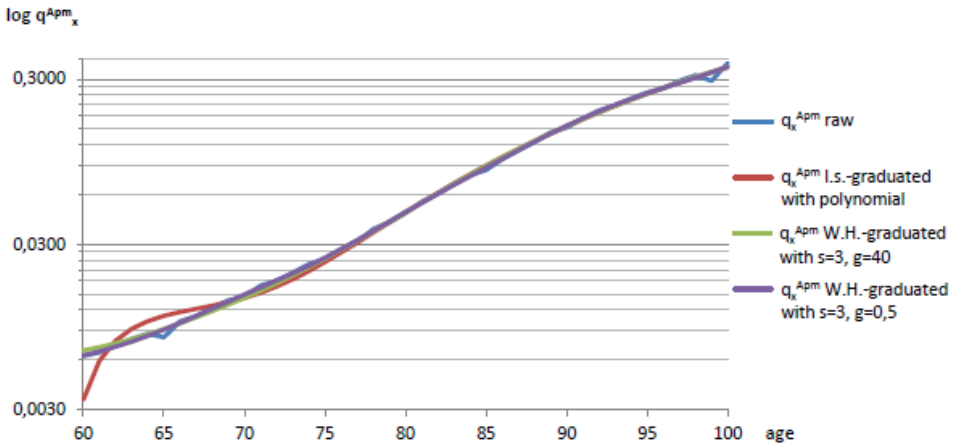

Figure 6.11: Logarithm of raw and graduated death probabilities of male white collar old age pensioners.

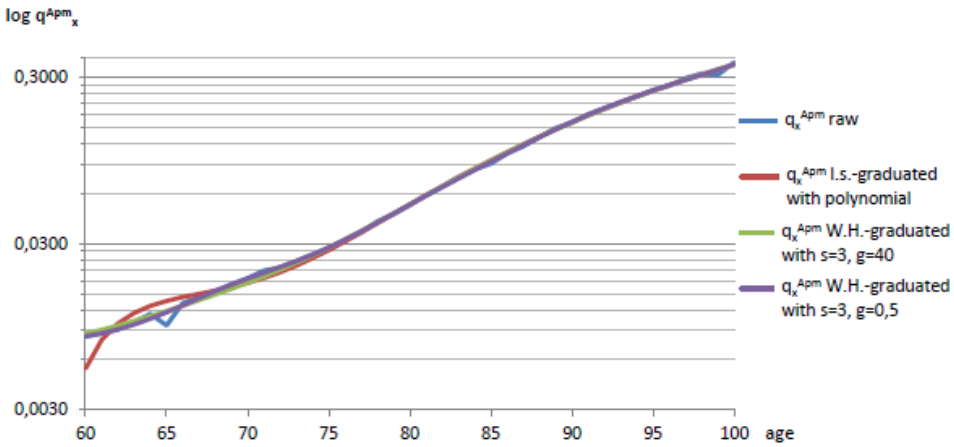

Figure 6.12: Logarithm of raw and graduated death probabilities of male white and blue collar old age pensioners.

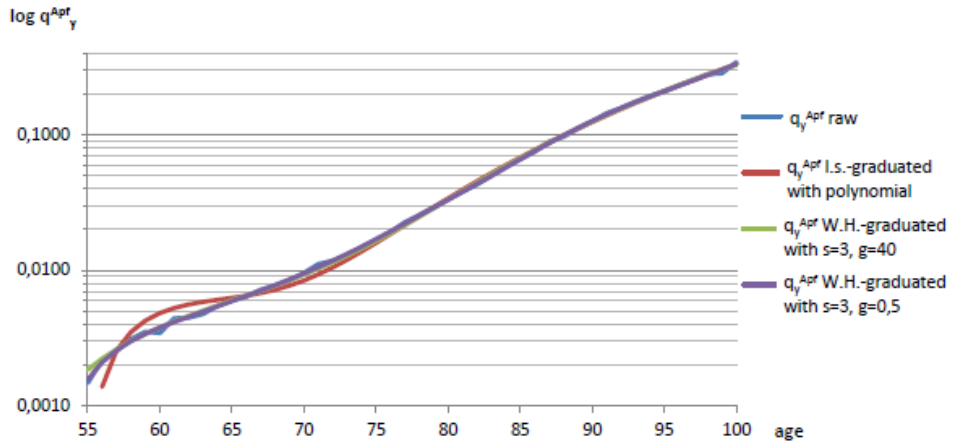

Figure 6.13: Logarithm of raw and graduated death probabilities of female white collar old age pensioners.

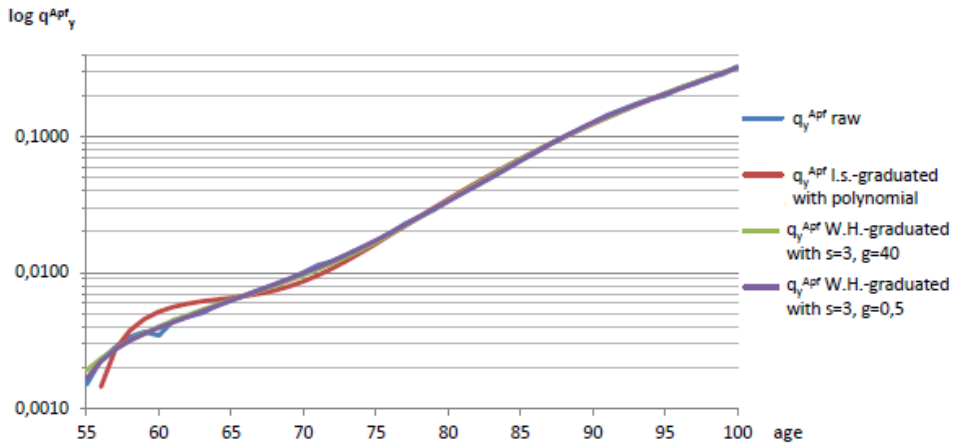

Figure 6.14: Logarithm of raw and graduated death probabilities of female white and blue collar old age pensioners.

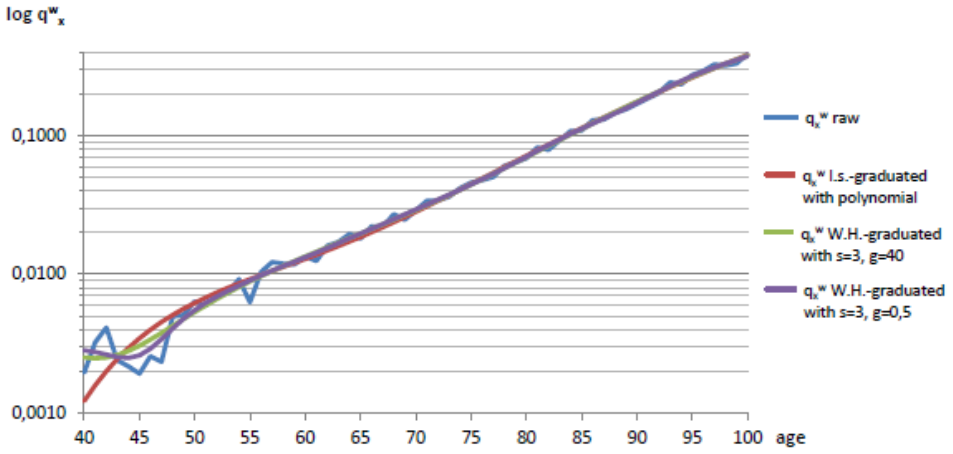

Figure 6.15: Logarithm of raw and graduated death probabilities of male white collar dependants.

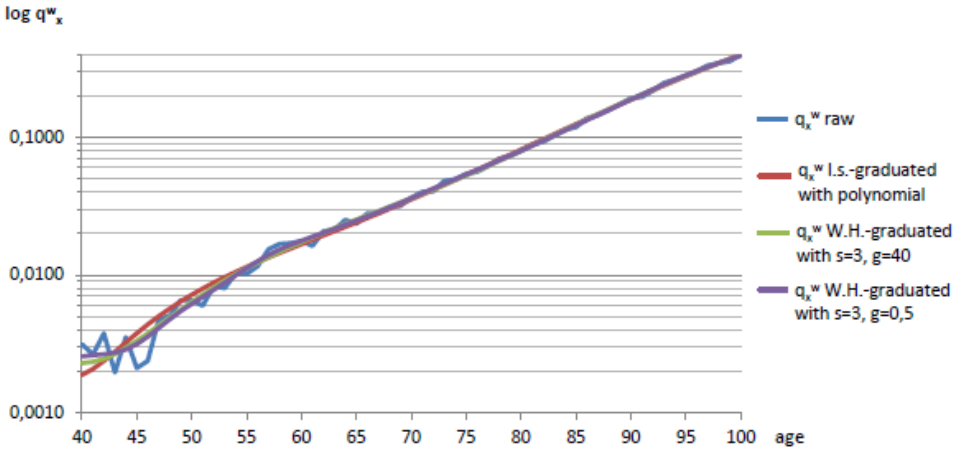

Figure 6.16: Logarithm of raw and graduated death probabilities of male white and blue collar dependants.

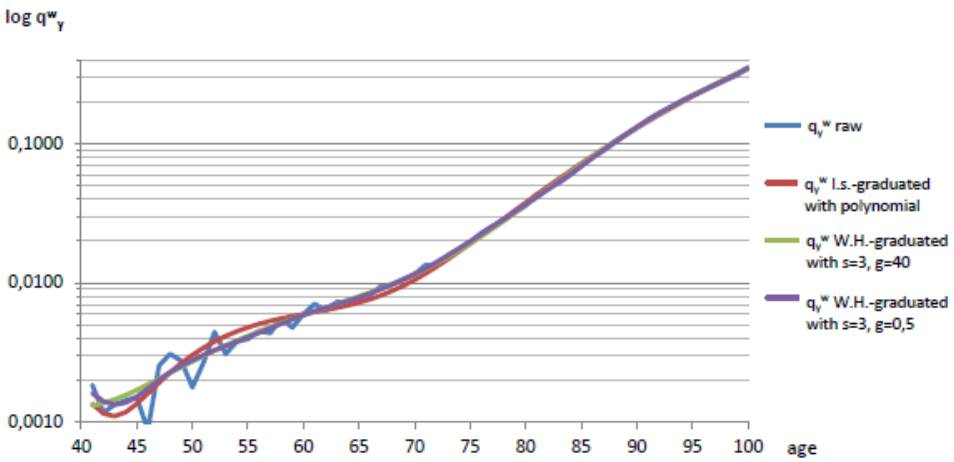

Figure 6.17: Logarithm of raw and graduated death probabilities of female white collar dependants.

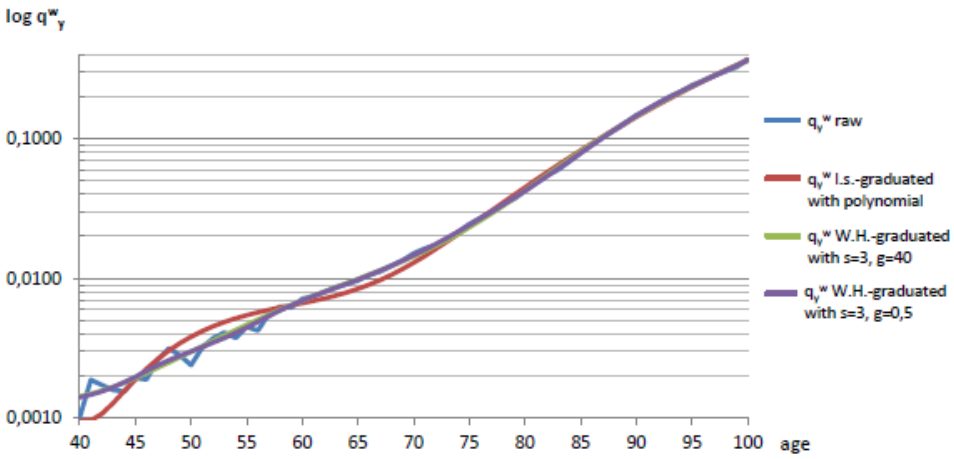

Figure 6.18: Logarithm of raw and graduated death probabilities of female white and blue collar dependants.

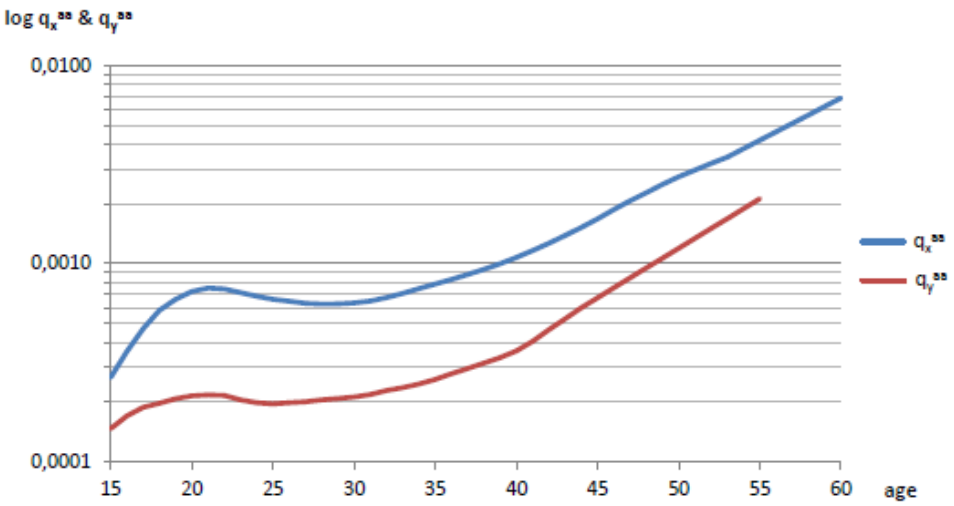

Figure 6.19: Logarithm of raw and graduated death probabilities of male and female white collar actives.

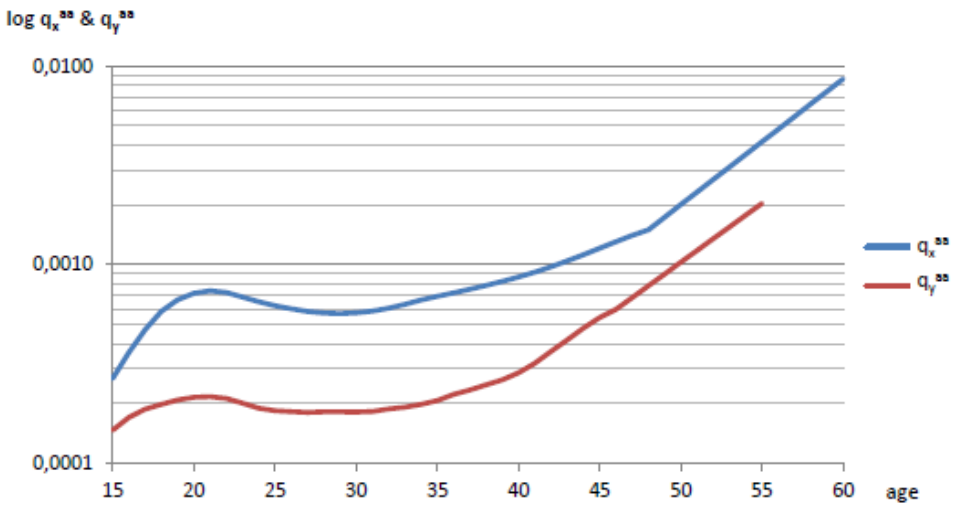

Figure 6.20: Logarithm of raw and graduated death probabilities of male and female white and blue collar actives.

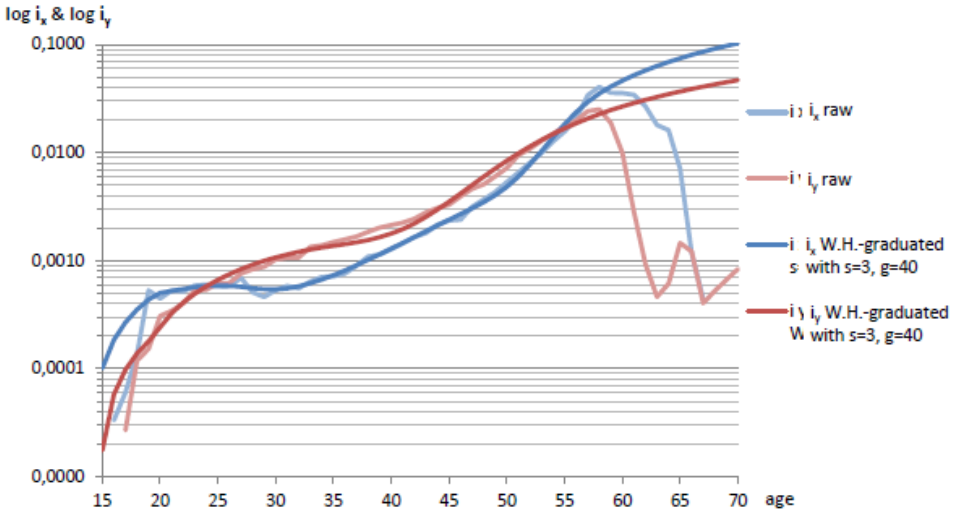

Figure 6.21: Logarithm of raw and graduated disability probabilities of male and female white collar beneficiaries.

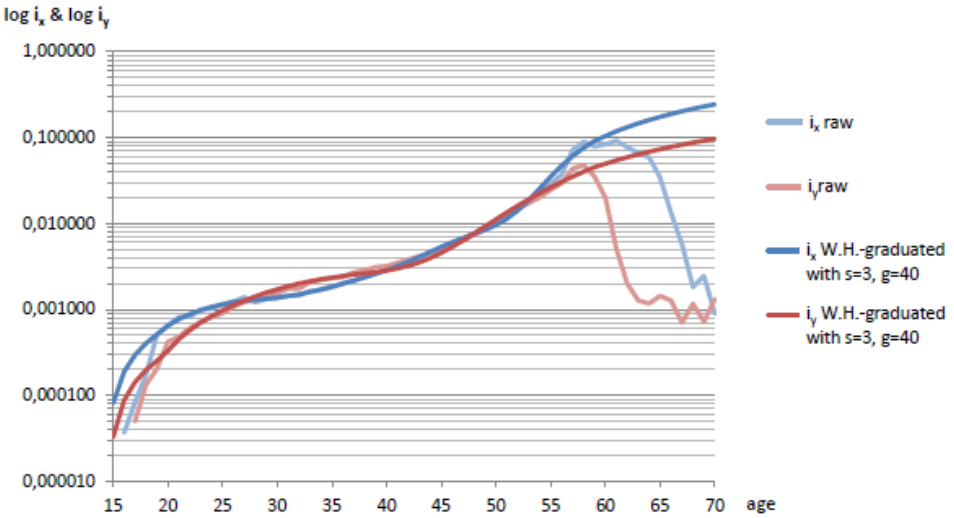

Figure 6.22: Logarithm of raw and graduated disability probabilities of male and female white and blue collar beneficiaries.

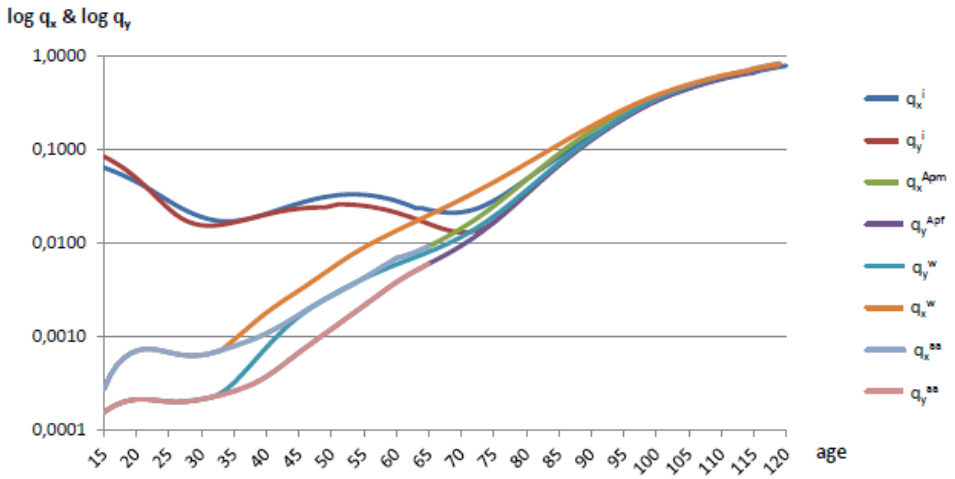

Figure 6.23: Logarithm of all graduated death probabilities of male and female white collar beneficiaries.

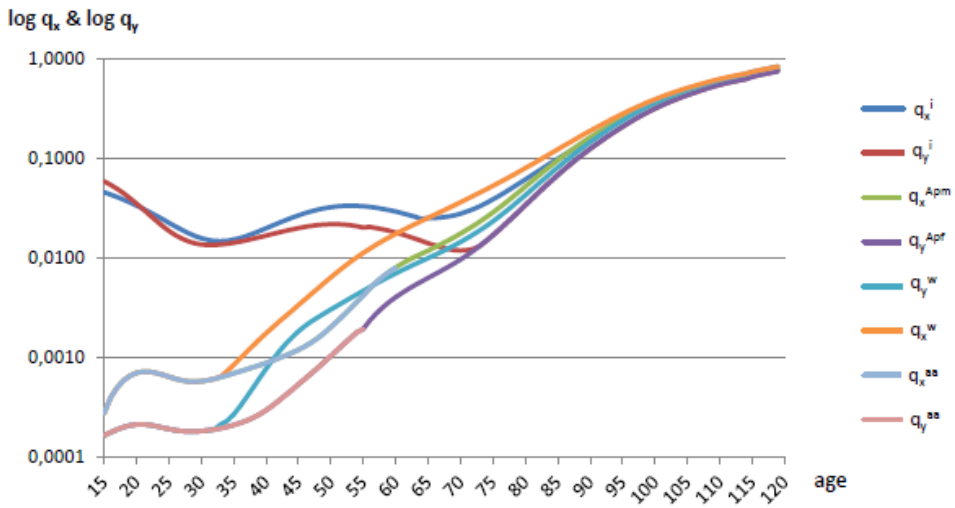

Figure 6.24: Logarithm of all graduated death probabilities of male and female white and blue collar beneficiaries.

6.3 Base Trend

Since no exact data for white and blue collar workers is available in Austria it seems the only way to use information of the whole Austrian population to derive a trend of the mortality projection.

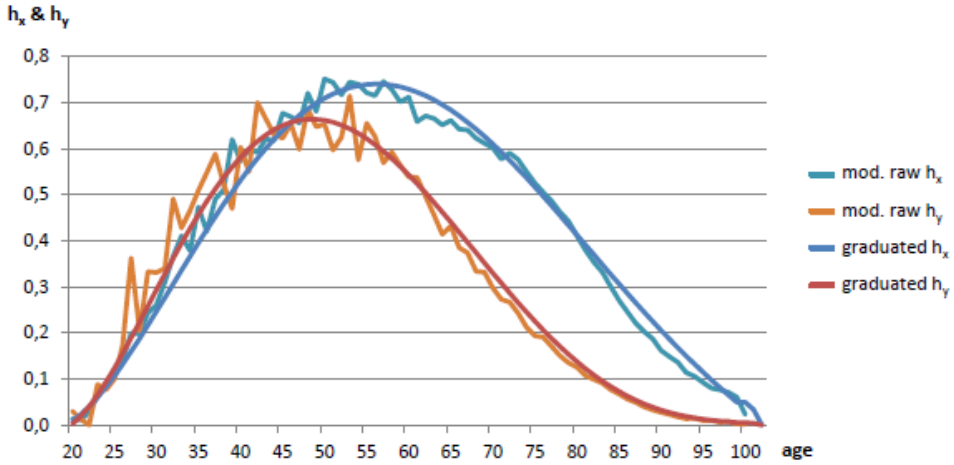

Figure 6.25: Raw and graduated marriage probabilities in the case of death of male and female beneficiaries.

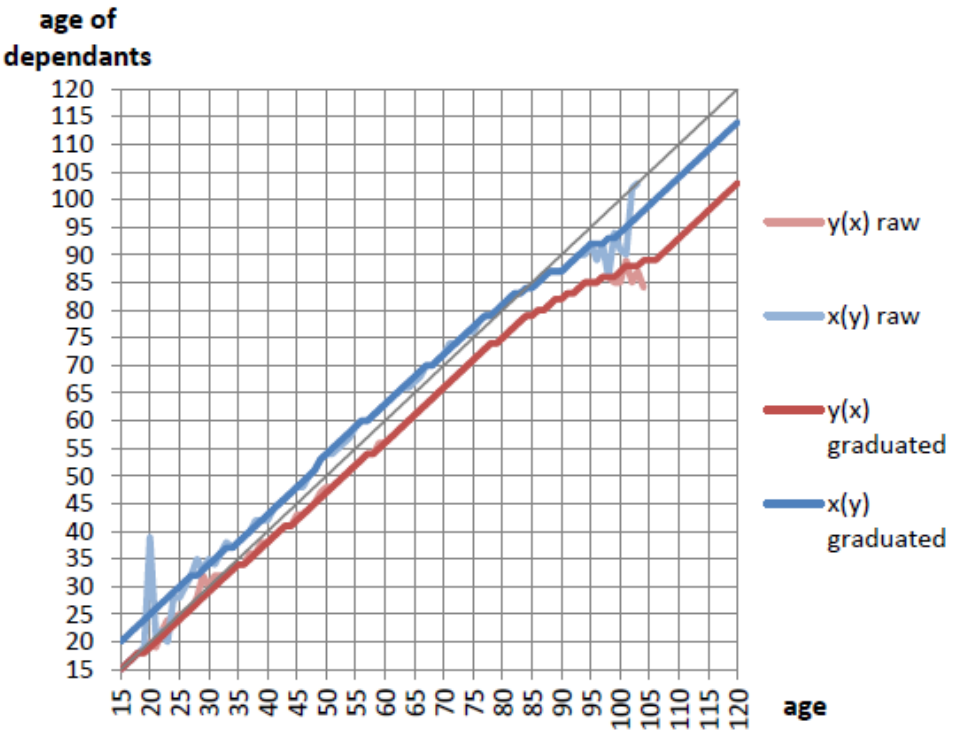

Figure 6.26: Raw and graduated age difference of spouses in the case of death of male and female beneficiaries.

6.3.1 Traditional Model

This model uses age-specific factors $F(x)$ and approximates the mortality reduction for year t by

$$\frac{q_x(t+1)}{q_x(t)} = e^{-F(x)}, \tag{6.12}$$

where the function F is fitted to the observed data. Because the factors $F(x)$ depend only on the age and are constant for all year, one can use the same trend for every year.

6.3.2 Cohort Model

In contrast to the Traditional Model this model uses factors $G(t+1-x)$ dependent from the year of birth $t+1-x$ of a person aged x in the year $t+1$. The mortality reduction for year t is than approximated by

$$\frac{q_x(t+1)}{q_x(t)} = e^{-G(t+1-x)}, \tag{6.13}$$

where the function G is again fitted to the observed data. The birth-year dependent factor $G(t+1-x)$ is the same for all ages of a person born in the year $t+1-x$. This is the weakness of this model because reality shows that the trend is also age-dependent.

6.3.3 Synthesis Model

The combination of the Traditional Model and the Cohort Model is the so called Synthesis Model where mortality reduction is approximated by

$$\frac{q_x(t+1)}{q_x(t)} = e^{-F(x)-G(t+1-x)}. \tag{6.14}$$

6.3.4 Swiss Nolfi Model

This is a special case of the Traditional Model with $F(x) = \lambda_x$ and mortality reduction is approximated by

$$\frac{q_x(t)}{q_x(t_0)} = e^{-\lambda_x \cdot (t-t_0)}. \tag{6.15}$$

Using the Swiss Nolfi Model and choosing the Öst 2010/12 for t and alternating the live tables for t_0 (Öst 2000/02 and so on) leads to mean yearly mortality reductions. Also if choosing the adjusted Austrian life table 2014 for t and alternating the adjusted live tables of the past for t_0 leads to mortality reductions. An investigation of the comparison of subsequent years was refused because the yearly fluctuation from theses not-graduated

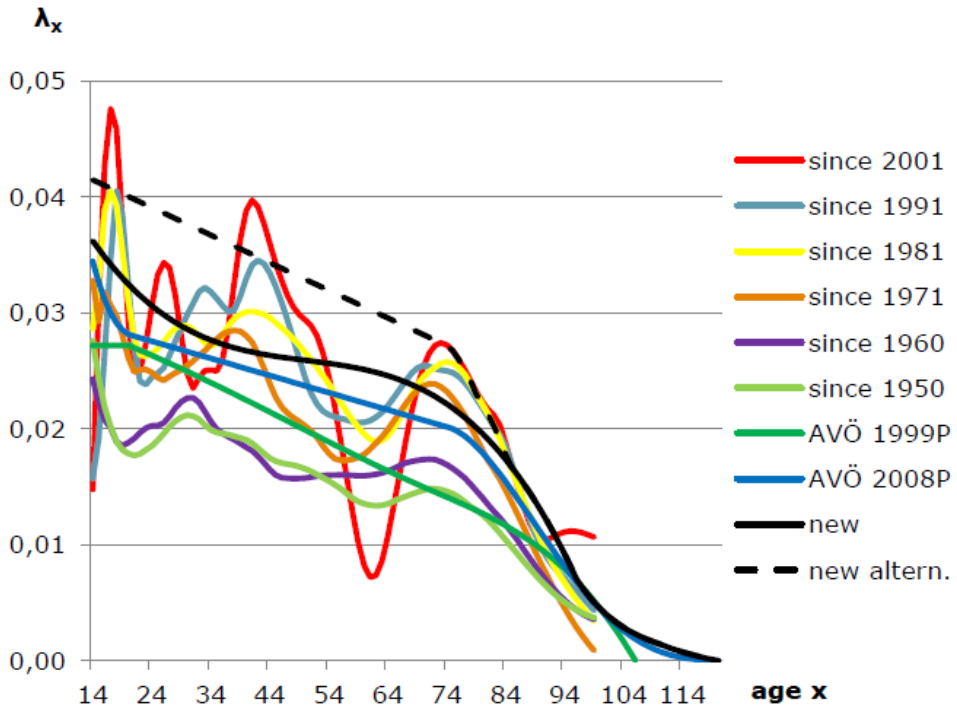

Figure 6.27: Mean yearly trend for males until 2011.

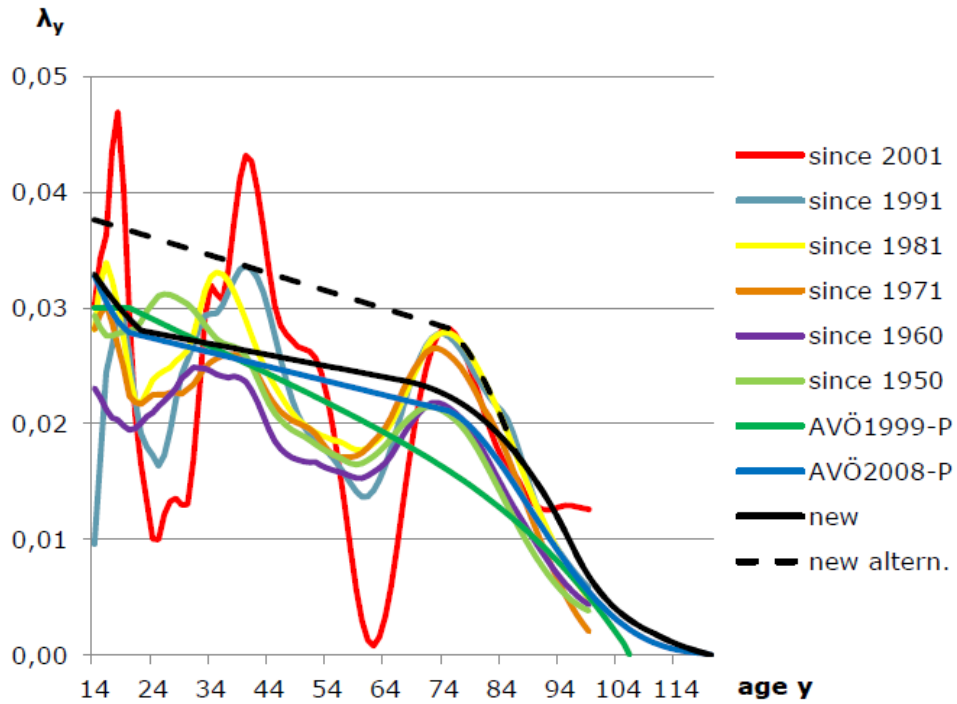

Figure 6.28: Mean yearly trend for females until 2011.

raw data life tables is to big. But choosing tables from years with differences multiples of 10 gives a similar picture of the trends.

Figure 6.27 shows the mean yearly trend for males until 2011. The new trend is graduated with a least-squares polynomial of 3rd grade from age 14 to age 96. From age 97 to age 120 the trend is extrapolated by an exponential function.

Figure 6.28 shows the mean yearly trend for females until 2011. The new trend is graduated with a least-squares polynomial of 3rd grade from age 14 to age 22 and from age 66 to age 96. Between the age of 22 and the age of 66 the trend is linearized. From age 97 to age 120 the trend is extrapolated by an exponential function.

$14 \leq x \leq 96$:	$\lambda_x = a_3 \cdot x^3 + a_2 \cdot x^2 + a_1 \cdot x + a_0$
with	$a_3 = -0,000000166837339$
	$a_2 = 0,000025182228396$
	$a_1 = -0,001331332061025$
	$a_0 = 0,050384240878873$
$96 \leq x \leq 120$:	$\lambda_x = b \cdot e^{-c \cdot x}$
with	$b = 336,205749481578000$
	$c = 0,112214833639600$

Table 6.1: Graduation and extrapolation functions of the new trend for males.

$14 \leq y \leq 22$ and	
$66 \leq y \leq 96$:	$\lambda_y = a_3 \cdot y^3 + a_2 \cdot y^2 + a_1 \cdot y + a_0$
with	$a_3 = -0,000000182703887$
	$a_2 = 0,000028505054099$
	$a_1 = -0,001455001892973$
	$a_0 = 0,048203651045934$
$22 \leq y \leq 66$:	$\lambda_y = d \cdot y + f$
with	$d = -0,000096129987402$
	$f = 0,030159484321620$
$96 \leq y \leq 120$:	$\lambda_y = b \cdot e^{-c \cdot y}$
with	$b = 584,207886446976000$
	$c = 0,114772859413805$

Table 6.2: Graduation and extrapolation functions of the new trend for females.

6.3.5 Lee–Carter Method

The following lines are based on the explanations of [11].

The original Lee–Carter method (see [15]–[16], [19]) is a bi-linear decomposition of the logarithm of the central death rate $m_x(t)$. Since we want to

directly find a trend for tabulated mortality probabilities, we try to decompose $\log q_x(t)$. Under the assumption of a constant force of mortality μ_x over each year it holds

$$m_x(t) = \mu_x(t) = -\log p_x(t).$$

Because of

$$z \approx -\log(1-z) \qquad \text{for } |z| \text{ small}$$

we can say

$$\log(-\log p_x(t)) \approx \log q_x(t).$$

So we decompose the logarithmic mortality $\log q_x(t)$ with $x = 0, \ldots, x_{\max}$ and $t = t_{\min}, t_{\min+1}, \ldots, t_{\min+d} = t_{\max}$ into a time-specific and an age-specific part:

$$\log q_x(t) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}, \tag{6.16}$$

with independent error-terms $\varepsilon_{x,t}$ satisfying $\mathbb{E}[\varepsilon_{x,t}] = 0$ and the normalization

$$\sum_{x=0}^{x_{\max}} \beta_x = 1 \qquad \text{and} \qquad \sum_{t=t_{\min}}^{t_{\max}} \kappa_t = 0. \tag{6.17}$$

Any given decomposition into α_x, β_x and κ_t with $\beta = (\beta_0, \beta_1, \ldots, \beta_{t_{\max}})^{\mathrm{t}} \neq 0$ can be normalized to these constraints. The first one can be achieved by the transformation

$$(\beta_x, \kappa_t) \mapsto \Big(\frac{\beta_x}{\sum_{x=0}^{x_{\max}} \beta_x}, \kappa_t \sum_{x=0}^{x_{\max}} \beta_x \Big)$$

and the second constraint by

$$(\kappa_t, \alpha_x) \mapsto \Big(\kappa_t - \frac{1}{d+1} \sum_{\tau=t_{\min}}^{t_{\max}} \kappa_\tau, \alpha_x + \beta_x \frac{1}{d+1} \sum_{\tau=t_{\min}}^{t_{\max}} \kappa_\tau \Big).$$

The trivial case $\beta_x = 0$ for all x would imply that the α_x are already the best estimators for the future logarithmic mortality because there is no mortality reduction. As a consequence of the normalization of κ_t, the method of moments allows to estimate the parameter α_x as the arithmetic mean over all years,

$$\widehat{\alpha}'_x = \frac{1}{d+1} \sum_{t=t_{\min}}^{t_{\max}} \log q_x(t).$$

This means that the α_x describe the mean life table over the whole time interval. The time progression, on the other hand, is completely modeled by the bi-linear term

$$(Z)_{x,t} = \log q_x(t) - \alpha_x \approx \beta_x \kappa_t.$$

So the difference between the logarithmic mortality to the mean is decomposed into a time trend κ_t and an age-specific factor β_x that determines the strength with which the time trend affects a certain age. Mathematically, such a decomposition can be achieved by the first term of a singular value decomposition (SVD) of the $m \times n$ real matrix Z into $Z = U \cdot D \cdot V^{\mathrm{t}}$. This decomposition always exists [20], and D is the $m \times n$ diagonal matrix containing the singular values of Z in descending order. These in turn are the non-negative square roots of the eigenvalues of the matrix ZZ^{t}. The matrices U and V are orthogonal $m \times m$ and $n \times n$ square matrices containing in their columns the corresponding eigenvectors of ZZ^{t} and $Z^{\mathrm{t}}Z$, respectively. Assuming that the random error terms $\varepsilon_{x,t}$ are i.i.d. normal random variables, the maximum-likelihood estimators for β_x and κ_t coincide with the least-squares estimators. The first term of the SVD of Z yields the ordinary least squares (OLS) approximation

$$Z \approx s\ \mathbf{u}\ \mathbf{v}^{\mathrm{t}}, \tag{6.18}$$

where s is the largest singular value and the column vectors $\mathbf{u}$ and $\mathbf{v}$ are corresponding normalized eigenvectors of ZZ^{t} and $Z^{\mathrm{t}}Z$. In several international investigations (e.g. [19]), this first term typically accounts for more than 90% of the variance of Z (otherwise the second term of the SVD might be used to capture effects of second order and thus an even larger part of the variance [19]). Non-normalized ML-estimators $\widehat{\beta}'_x$ and $\widehat{\kappa}'_t$ for the coefficients β_x and κ_t can easily be obtained from the vectors $\mathbf{u} = (u_0, u_1, \ldots, u_{x_{\max}})^{\mathrm{t}}$ and $\mathbf{v} = (v_{t_{\min}}, v_{t_{\min}+1}, \ldots, v_{t_{\max}})^{\mathrm{t}}$ as

$$\widehat{\beta}'_x = u_x \qquad \text{and} \qquad \widehat{\kappa}'_t = s v_t. \tag{6.19}$$

A subsequent normalization of $(\widehat{\alpha}'_x, \widehat{\beta}'_x, \widehat{\kappa}'_t)$ to conditions (6.17) as outlined above finally yields the estimated parameters $(\widehat{\alpha}_x, \widehat{\beta}_x, \widehat{\kappa}_t)$ of the Lee–Carter decomposition.

From this Lee–Carter decomposition of the mortality, one can now extrapolate to the future. The decomposition into a time-dependent trend κ_t and an age-specific factor β_x simplifies this to the extrapolation of the trend κ_t. In the simplest model, an ARIMA(0, 1, 0) time series is employed, which models the κ_t as a random walk with drift,

$$\kappa_{t+1} = \kappa_t + \Delta\kappa + \delta_t.$$

The constant $\Delta\kappa$ is the drift and the δ_t are i.i.d. homoskedastic (i.e. with similar variance) random variables with $\mathbb{E}[\delta_t] = 0$. The extrapolation simply predicts the expectation value as $\widehat{\kappa}_{t+1} = \widehat{\kappa}_t + \widehat{\Delta\kappa}$.

Rewriting the Lee–Carter decomposition and using the ARIMA(0, 1, 0) time series extrapolation for κ_t reveals after taking expectations that the estimator for the yearly trend λ_x of a person aged x is

$$\lambda_x = \widehat{\beta}_x \widehat{\Delta\kappa}. \tag{6.20}$$

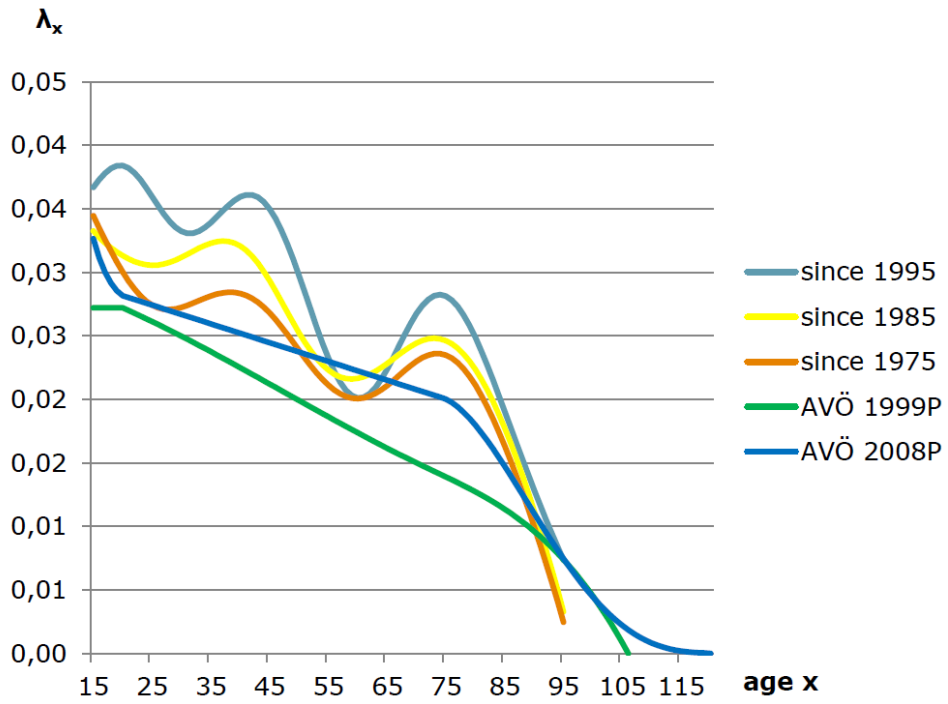

Figure 6.29: Mean yearly trend for males until 2014 with Lee–Carter method.

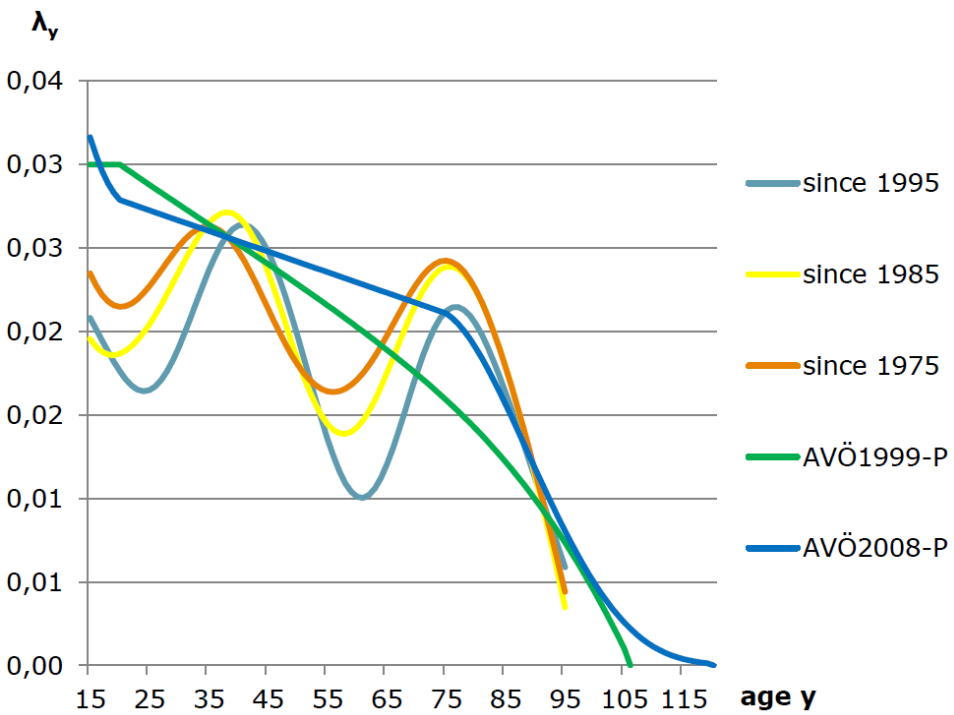

Figure 6.30: Mean yearly trend for females until 2014 with Lee–Carter method.

Figures 6.29 and Figure 6.30 show the mean yearly trend for males respectively females until 2014. The Lee–Carter values are graduated with the Whittacker–Henderson method.

7 Definitions of Net Present Values in the Pension Insurance

In this chapter the most important net present values (in the following short NPV) in the pension insurance are presented.

Definition 7.0.1 (Reduction term for periods of less than a year)**.** The age independent reduction term for payments during the fiscal year is calculated by

$$k(\tau) = \frac{\tau - 1}{2\tau} + \frac{\tau^2 - 1}{6k^2} \cdot \left(1 - \frac{r}{2}\right) r, \tag{7.1}$$

where τ is the number of payments during the fiscal year and r is the effective yearly discount rate.

Remark 7.0.2. The most common case is monthly pension payments which means $\tau = 12$. The following definitions are made under this assumption. NPVs marked with "$^{(12)}$" denote this circumstance.

Remark 7.0.3. The following definitions are made for x-aged male beneficiaries. For female beneficiaries the age y is used instead of x. Female old age pensioners are denoted by "Apf" instead of "Apm".

Definition 7.0.4 (Active annuity-due)**.** The NPV of the entitlement to a maximum of $z-x$ years lasting active annuity-due of value 1 p.a. of a x-aged active is denoted by $\ddot{a}^{\text{aa}}_{x\,\overline{z-x}|}$ and can be recursively calculated from:

$$\begin{aligned}
\ddot{a}^{\text{aa}}_{x\,\overline{z-x}|} &= 1 + (1 - q^{\text{aa}}_x) \cdot v \cdot \ddot{a}^{\text{aa}}_{x+1\,\overline{z-x+1}|} \\
&\vdots \\
\ddot{a}^{\text{aa}}_{z-1\,\overline{z-z-1}|} &= 1 + (1 - q^{\text{aa}}_{z-1}) \cdot v \cdot \ddot{a}^{\text{aa}}_{z\,\overline{z-z}|} \\
\ddot{a}^{\text{aa}}_{z\,\overline{z-z}|} &= 1.
\end{aligned} \tag{7.2}$$

For 12 monthly payments during the year:

$$^{(12)}\ddot{a}^{\text{aa}}_{x\,\overline{z-x}|} = \ddot{a}^{\text{aa}}_{x\,\overline{z-x}|} - k(12). \tag{7.3}$$

Remark 7.0.5. In practice z often denotes the retirement age.

Definition 7.0.6 (Immediate lifelong old age pension)**.** The NPV of an immediate lifelong old age annuity-due of value 1 p.a. of a x-aged old age

pensioner is denoted by $\ddot{a}_x^{\text{Apm}}$ and can be recursively calculated from:

$$\begin{aligned}
\ddot{a}_x^{\text{Apm}} &= 1 + (1 - q_x^{\text{Apm}}) \cdot v \cdot \ddot{a}_{x+1}^{\text{Apm}} \\
&\vdots \\
\ddot{a}_{x_\omega - 1}^{\text{Apm}} &= 1 + (1 - q_{x_\omega - 1}^{\text{Apm}}) \cdot v \cdot \ddot{a}_{x_\omega}^{\text{Apm}} \\
\ddot{a}_{x_\omega}^{\text{Apm}} &= 1 + (1 - \underbrace{q_{x_\omega}^{\text{Apm}}}_{=1}) \cdot v \cdot \ddot{a}_{x_\omega + 1}^{\text{Apm}} = 1 \qquad \text{with } x_\omega = 120.
\end{aligned} \tag{7.4}$$

For 12 monthly payments during the year:

$$^{(12)}\ddot{a}_x^{\text{Apm}} = \ddot{a}_x^{\text{Apm}} - k(12). \tag{7.5}$$

Definition 7.0.7 (Immediate lifelong disability pension)**.** The NPV of an immediate lifelong disability annuity-due of value 1 p.a. of a x-aged disabled is denoted by $\ddot{a}_x^{\text{i}}$ and can be recursively calculated from:

$$\begin{aligned}
\ddot{a}_x^{\text{i}} &= 1 + (1 - q_x^{\text{i}}) \cdot v \cdot \ddot{a}_{x+1}^{\text{i}} \\
&\vdots \\
\ddot{a}_{x_\omega - 1}^{\text{i}} &= 1 + (1 - q_{x_\omega - 1}^{\text{i}}) \cdot v \cdot \ddot{a}_{x_\omega}^{\text{i}} \\
\ddot{a}_{x_\omega}^{\text{i}} &= 1 + (1 - \underbrace{q_{x_\omega}^{\text{i}}}_{=1}) \cdot v \cdot \ddot{a}_{x_\omega + 1}^{\text{i}} = 1 \qquad \text{with } x_\omega = 120.
\end{aligned} \tag{7.6}$$

For 12 monthly payments during the year:

$$^{(12)}\ddot{a}_x^{\text{i}} = \ddot{a}_x^{\text{i}} - k(12). \tag{7.7}$$

Definition 7.0.8 (Immediate lifelong surviving dependant's pension)**.** The NPV of an immediate lifelong surviving dependant's annuity-due of value 1 p.a. of a x-aged widower is denoted by $\ddot{a}_x^{\text{w}}$ and can be recursively calculated from:

$$\begin{aligned}
\ddot{a}_x^{\text{w}} &= 1 + (1 - q_x^{\text{w}}) \cdot v \cdot \ddot{a}_{x+1}^{\text{w}} \\
&\vdots \\
\ddot{a}_{x_\omega - 1}^{\text{w}} &= 1 + (1 - q_{x_\omega - 1}^{\text{w}}) \cdot v \cdot \ddot{a}_{x_\omega}^{\text{w}} \\
\ddot{a}_{x_\omega}^{\text{w}} &= 1 + (1 - \underbrace{q_{x_\omega}^{\text{w}}}_{=1}) \cdot v \cdot \ddot{a}_{x_\omega + 1}^{\text{w}} = 1 \qquad \text{with } x_\omega = 120.
\end{aligned} \tag{7.8}$$

For 12 monthly payments during the year:

$$^{(12)}\ddot{a}_x^{\text{w}} = \ddot{a}_x^{\text{w}} - k(12). \tag{7.9}$$

For the age $y(x) + \frac{1}{2}$:

$$^{(12)}\ddot{a}_{y(x)+\frac{1}{2}}^{\text{w}} = \tfrac{1}{2}\Big(^{(12)}\ddot{a}_{y(x)}^{\text{w}} + {}^{(12)}\ddot{a}_{y(x)+1}^{\text{w}}\Big). \tag{7.10}$$

Definition 7.0.9 (Entitlement of an active to lifelong disability pension)**.** The NPV of the entitlement of an x-aged active to lifelong disability pension is denoted by $\ddot{a}_x^{\mathrm{ai}}$ and can be recursively calculated from:

$$\begin{aligned}
\ddot{a}_x^{\mathrm{ai}} &= i_x \cdot v^{\frac{1}{2}} \cdot \tfrac{1}{2}\left({}^{(12)}\ddot{a}_x^{\mathrm{i}} + {}^{(12)}\ddot{a}_{x+1}^{\mathrm{i}}\right) + (1 - i_x - q_x^{\mathrm{aa}}) \cdot v \cdot \ddot{a}_{x+1}^{\mathrm{ai}} \\
&\vdots \\
\ddot{a}_{z-1}^{\mathrm{ai}} &= i_{z-1} \cdot v^{\frac{1}{2}} \cdot \tfrac{1}{2}\left({}^{(12)}\ddot{a}_{z-1}^{\mathrm{i}} + {}^{(12)}\ddot{a}_{z}^{\mathrm{i}}\right) + (1 - i_{z-1} - q_{z-1}^{\mathrm{aa}}) \cdot v \cdot \ddot{a}_{z}^{\mathrm{ai}} \\
\ddot{a}_z^{\mathrm{ai}} &= 0.
\end{aligned} \tag{7.11}$$

Remark 7.0.10. The interpretation of Equation (7.11) is that if the beneficiary gets disabled the NPV of an immediate lifelong disability pension must be provided and if the beneficiary stays active the discounted NPV of the entitlement from the next year.

Definition 7.0.11 (Entitlement of an active to lifelong old age pension)**.** The NPV of the entitlement of an x-aged active to lifelong old age pension, where the payments start at age z, is denoted by ${}^{(12)}_{z-x}\ddot{a}_x^{\mathrm{aApm}}$ and can be recursively calculated from:

$$\begin{aligned}
{}^{(12)}_{z-x}\ddot{a}_x^{\mathrm{aApm}} &= (1 - i_x - q_x^{\mathrm{aa}}) \cdot v \cdot {}^{(12)}_{z-x-1}\ddot{a}_{x+1}^{\mathrm{aApm}} \\
&\vdots \\
{}^{(12)}_{z-z+2}\ddot{a}_{z-2}^{\mathrm{aApm}} &= (1 - i_{z-2} - q_{z-2}^{\mathrm{aa}}) \cdot v \cdot {}^{(12)}_{z-z+1}\ddot{a}_{z-1}^{\mathrm{aApm}} \\
{}^{(12)}_{z-z+1}\ddot{a}_{z-1}^{\mathrm{aApm}} &= (1 - i_{z-1} - q_{z-1}^{\mathrm{aa}}) \cdot v \cdot {}^{(12)}\ddot{a}_{z-1}^{\mathrm{Apm}} \\
{}^{(12)}_{z-z}\ddot{a}_z^{\mathrm{aApm}} &= 0.
\end{aligned} \tag{7.12}$$

Remark 7.0.12. This NPV is the to the age z deferred NPV of an immediate lifelong old age pension in Equation (7.5).

Definition 7.0.13 (Entitlement of a disabled to lifelong surviving dependant's pension)**.** The NPV of the entitlement to a lifelong surviving dependant's annuity-due of value 1 p.a. of a x-aged disable is denoted by ${}^{(12)}\ddot{a}_x^{\mathrm{iw}}$ and can be recursively calculated from:

$$\begin{aligned}
{}^{(12)}\ddot{a}_x^{\mathrm{iw}} &= q_x^{\mathrm{i}} \cdot h_x \cdot v^{\frac{1}{2}} \cdot {}^{(12)}\ddot{a}^{\mathrm{w}}_{y(x)+\frac{1}{2}} + (1 - q_x^{\mathrm{i}}) \cdot v \cdot {}^{(12)}\ddot{a}_{x+1}^{\mathrm{iw}} \\
&\vdots \\
{}^{(12)}\ddot{a}_{x_\omega-1}^{\mathrm{iw}} &= q_{x_\omega-1}^{\mathrm{i}} \cdot h_{x_\omega-1} \cdot v^{\frac{1}{2}} \cdot {}^{(12)}\ddot{a}^{\mathrm{w}}_{y(x_\omega-1)+\frac{1}{2}} + (1 - q_{x_\omega-1}^{\mathrm{i}}) \cdot v \cdot {}^{(12)}\ddot{a}_{x_\omega}^{\mathrm{iw}} \\
{}^{(12)}\ddot{a}_{x_\omega}^{\mathrm{iw}} &= q_{x_\omega}^{\mathrm{i}} \cdot \underbrace{h_{x_\omega}}_{=0} \cdot v^{\frac{1}{2}} \cdot {}^{(12)}\ddot{a}^{\mathrm{w}}_{y(x_\omega)+\frac{1}{2}} + (1 - \underbrace{q_{x_\omega}^{\mathrm{i}}}_{=1}) \cdot v \cdot {}^{(12)}\ddot{a}_{x_\omega+1}^{\mathrm{iw}} = \\
&= 0 \qquad\qquad \text{with } x_\omega = 120.
\end{aligned} \tag{7.13}$$

Definition 7.0.14 (Collective method)**.** Equation (7.13) is used in the case of unknow spouses, i.e. if we know neither if there exists a spouse nor the date of birth of an existing spouse. The method of using a statistically generated spouse is called *collective method*.

Definition 7.0.15 (Individual method)**.** If the spouses and their dates of birth are known, $h_x = 1$ and $y(x) = y$ hold in Equation (7.13). Then the evaluation of entitlements to dependant's pension is called *individual method*.

Definition 7.0.16 (Entitlement of an old age pensioner to lifelong surviving dependant's pension)**.** The NPV of the entitlement to a lifelong surviving dependant's annuity-due of value 1 p.a. of a x-aged old age pensioner is denoted by ${}^{(12)}\ddot{a}_x^{\mathrm{pw}}$ and can be recursively calculated from:

$$\begin{aligned}
{}^{(12)}\ddot{a}_x^{\mathrm{pw}} &= q_x^{\mathrm{Apm}} \cdot h_x \cdot v^{\frac{1}{2}} \cdot {}^{(12)}\ddot{a}^{\mathrm{w}}_{y(x)+\frac{1}{2}} + (1 - q_x^{\mathrm{Apm}}) \cdot v \cdot {}^{(12)}\ddot{a}^{\mathrm{pw}}_{x+1} \\
&\vdots \\
{}^{(12)}\ddot{a}^{\mathrm{pw}}_{x_\omega-1} &= q^{\mathrm{Apm}}_{x_\omega-1} \cdot h_{x_\omega-1} \cdot v^{\frac{1}{2}} \cdot {}^{(12)}\ddot{a}^{\mathrm{w}}_{y(x_\omega-1)+\frac{1}{2}} + (1 - q^{\mathrm{Apm}}_{x_\omega-1}) \cdot v \cdot {}^{(12)}\ddot{a}^{\mathrm{pw}}_{x_\omega} \\
{}^{(12)}\ddot{a}^{\mathrm{pw}}_{x_\omega} &= q^{\mathrm{Apm}}_{x_\omega} \cdot \underbrace{h_{x_\omega}}_{=0} \cdot v^{\frac{1}{2}} \cdot {}^{(12)}\ddot{a}^{\mathrm{w}}_{y(x_\omega)+\frac{1}{2}} + (1 - \underbrace{q^{\mathrm{Apm}}_{x_\omega}}_{=1}) \cdot v \cdot {}^{(12)}\ddot{a}^{\mathrm{pw}}_{x_\omega+1} = \\
&= 0 \qquad\qquad \text{with } x_\omega = 120. \qquad (7.14)
\end{aligned}$$

Remark 7.0.17. The entitlement of an old age pensioner to lifelong surviving dependant's pension can also calculated due to *collective method* or *individual method*.

Definition 7.0.18 (Entitlement of an active to surviving dependant's pension)**.** This NPV consists of 3 parts, dependent on the state of the (at the beginning of the year) active at the moment of transition to dead.

- The NPV of the entitlement to a lifelong widow/widower annuity-due of value 1 p.a. until age z of a x-aged active after death as active is denoted by $\ddot{a}_x^{\mathrm{aaw}}$ and can be recursively calculated from:

$$\begin{aligned}
{}^{(12)}\ddot{a}_x^{\mathrm{aaw}} &= q_x^{\mathrm{aa}} \cdot h_x \cdot v^{\frac{1}{2}} \cdot {}^{(12)}\ddot{a}^{\mathrm{w}}_{y(x)+\frac{1}{2}} + (1 - q_x^{\mathrm{aa}}) \cdot v \cdot {}^{(12)}\ddot{a}^{\mathrm{aaw}}_{x+1} \\
&\vdots \\
{}^{(12)}\ddot{a}^{\mathrm{aaw}}_{z-1} &= q^{\mathrm{aa}}_{z-1} \cdot h_{z-1} \cdot v^{\frac{1}{2}} \cdot {}^{(12)}\ddot{a}^{\mathrm{w}}_{y(z-1)+\frac{1}{2}} + (1 - q^{\mathrm{aa}}_{z-1}) \cdot v \cdot {}^{(12)}\ddot{a}^{\mathrm{aaw}}_{z} \\
{}^{(12)}\ddot{a}^{\mathrm{aaw}}_{z} &= 0. \qquad (7.15)
\end{aligned}$$

- The NPV of the entitlement to a lifelong widow/widower annuity-due of value 1 p.a. until age z of a x-aged active after death as disabled is

denoted by $\ddot{a}_x^{\text{aiw}}$ and can be recursively calculated from:

$$
\begin{aligned}
{}^{(12)}\ddot{a}_x^{\text{aiw}} &= i_x \cdot v^{\frac{1}{2}} \cdot \tfrac{1}{2}\Big({}^{(12)}\ddot{a}_x^{\text{iw}} + {}^{(12)}\ddot{a}_{x+1}^{\text{iw}}\Big) + (1 - i_x - q_x^{\text{aa}}) \cdot v \cdot {}^{(12)}\ddot{a}_{x+1}^{\text{aiw}} \\
&\vdots \\
{}^{(12)}\ddot{a}_{z-1}^{\text{aiw}} &= i_{z-1} \cdot v^{\frac{1}{2}} \cdot \tfrac{1}{2}\Big({}^{(12)}\ddot{a}_{z-1}^{\text{iw}} + {}^{(12)}\ddot{a}_z^{\text{iw}}\Big) + \\
&\quad + (1 - i_{z-1} - q_{z-1}^{\text{aa}}) \cdot v \cdot {}^{(12)}\ddot{a}_z^{\text{aiw}} \\
{}^{(12)}\ddot{a}_z^{\text{aiw}} &= 0.
\end{aligned}
\tag{7.16}
$$

- The NPV of the entitlement to a lifelong widow/widower annuity-due of value 1 p.a. until age z of a x-aged active after death as old age pensioner is denoted by $\ddot{a}_x^{\text{apw}}$ and can be recursively calculated from:

$$
\begin{aligned}
{}^{(12)}\ddot{a}_x^{\text{apw}} &= (1 - i_x - q_x^{\text{aa}}) \cdot {}^{(12)}\ddot{a}_{x+1}^{\text{apw}} \\
&\vdots \\
{}^{(12)}\ddot{a}_{z-1}^{\text{apw}} &= (1 - i_{z-1} - q_{z-1}^{\text{aa}}) \cdot {}^{(12)}\ddot{a}_z^{\text{apw}} \\
{}^{(12)}\ddot{a}_z^{\text{apw}} &= {}^{(12)}\ddot{a}_z^{\text{pw}}.
\end{aligned}
\tag{7.17}
$$

- The total NPV of the entitlement to a lifelong widow/widower annuity-due of value 1 p.a. until age z of a x-aged active is denoted by $\ddot{a}_x^{\text{aw}}$ and can be recursively calculated from:

$$
{}^{(12)}\ddot{a}_x^{\text{aw}} = {}^{(12)}\ddot{a}_x^{\text{aaw}} + {}^{(12)}\ddot{a}_x^{\text{aiw}} + {}^{(12)}\ddot{a}_x^{\text{apw}}. \tag{7.18}
$$

Remark 7.0.19. The entitlement of an active to lifelong surviving dependant's pension can also calculated due to *collective method* or *individual method*.

8 Variance of the Net Present Value

For $p = 2$ and constant interest rate the affine recurrence relation for higher moments of the present value $L_t(A)$ (3.9) is

$$
\begin{aligned}
\mathbb{E}[L_t^2(A)|X_t = i] = a_i(t)^2 + \sum_{k=1}^{2} \binom{2}{k} a_i(t)^{2-k} \sum_{j \in S} v^k \, p_{ij}(t, t+1) \\
\Bigg(a_{ij}(t+1)^k + \sum_{l=1}^{k-1} \binom{k}{l} a_{ij}(t+1)^{k-l} \, \mathbb{E}[L_{t+1}^l(A)|X_{t+1} = j] \\
+ \mathbb{E}[L_{t+1}^k(A)|X_{t+1} = j] \Bigg).
\end{aligned}
$$

If $a_{ij}(t) = 0$ for all $t \in T$ and all $i, j \in S$ this simplifies to

$$\begin{aligned}
\mathbb{E}[L_t^2(A)|X_t = i] &= a_i(t)^2 + \binom{2}{1} a_i(t)^{2-1} \sum_{j \in S} v\, p_{ij}(t, t+1)\, \mathbb{E}[L_{t+1}(A)|X_{t+1} = j] + \\
&\quad + \binom{2}{2} a_i(t)^{2-2} \sum_{j \in S} v^2\, p_{ij}(t, t+1)\, \mathbb{E}[L_{t+1}^2(A)|X_{t+1} = j] \\
&= a_i(t)^2 + 2a_i(t) \sum_{j \in S} v\, p_{ij}(t, t+1)\, \mathbb{E}[L_{t+1}(A)|X_{t+1} = j] + \\
&\quad + \sum_{j \in S} v^2\, p_{ij}(t, t+1)\, \mathbb{E}[L_{t+1}^2(A)|X_{t+1} = j]. \qquad (8.1)
\end{aligned}$$

If $a_i(t) = 0$ for all $t \in T$ and all $i \in S$ it holds

$$\begin{aligned}
\mathbb{E}[L_t^2(A)|X_t = i] &= v^2 \sum_{j \in S} p_{ij}(t, t+1) \sum_{k=0}^{2} \binom{2}{k} a_{ij}(t+1)^{2-k}\, \mathbb{E}[L_{t+1}^k(A)|X_{t+1} = j] \\
&= v^2 \sum_{j \in S} p_{ij}(t, t+1) \Bigg(\binom{2}{0} a_{ij}(t+1)^2\, \mathbb{E}[L_{t+1}^0(A)|X_{t+1} = j] + \\
&\qquad + \binom{2}{1} a_{ij}(t+1)\, \mathbb{E}[L_{t+1}(A)|X_{t+1} = j] + \\
&\qquad + \binom{2}{2} \mathbb{E}[L_{t+1}^2(A)|X_{t+1} = j] \Bigg) \\
&= v^2 \sum_{j \in S} p_{ij}(t, t+1) \times \\
&\quad \times \Big(a_{ij}(t+1)^2 + 2a_{ij}(t+1)\, \mathbb{E}[L_{t+1}(A)|X_{t+1} = j] + \\
&\qquad + \mathbb{E}[L_{t+1}^2(A)|X_{t+1} = j] \Big). \qquad (8.2)
\end{aligned}$$

The variance of $L_t(A)$ then is

$$\mathbb{V}[L_t(A)|X_t = i] = \mathbb{E}[L_t^2(A)|X_t = i] - \mathbb{E}[L_t(A)|X_t = i]^2$$

for all $t \in T$ and $i \in S$ with $\mathbb{P}[X_t = i] > 0$.

9 The New Annuity Valuation Table

9.1 Values of the Base Table 2007

Male white collar workers									
age x	i_x	q_x^{aa}	q_x^{i}	q_x^{Apm}	q_y^{w}	h_x	$y(x)$	λ_x	$\lambda_x^{\text{alternative}}$
14	0,000000	0,000144	0,063398	0,000144	0,000133	0,000000	14	0,036224	0,041486
15	0,000101	0,000272	0,059746	0,000272	0,000154	0,000000	15	0,035517	0,041248
16	0,000185	0,000388	0,056049	0,000388	0,000172	0,000000	16	0,034846	0,041009
17	0,000269	0,000492	0,052310	0,000492	0,000187	0,000000	17	0,034210	0,040771
18	0,000353	0,000581	0,048537	0,000581	0,000199	0,000000	18	0,033606	0,040533
19	0,000436	0,000651	0,044754	0,000651	0,000207	0,000000	18	0,033035	0,040295
20	0,000503	0,000701	0,041006	0,000701	0,000212	0,008022	19	0,032496	0,040056
21	0,000524	0,000727	0,037369	0,000727	0,000213	0,022696	20	0,031987	0,039818
22	0,000546	0,000733	0,033926	0,000733	0,000211	0,040237	21	0,031507	0,039580
23	0,000567	0,000722	0,030751	0,000722	0,000208	0,060304	22	0,031055	0,039342
24	0,000582	0,000700	0,027899	0,000700	0,000204	0,082573	23	0,030631	0,039103
25	0,000589	0,000675	0,025398	0,000675	0,000201	0,106739	24	0,030233	0,038865
26	0,000585	0,000651	0,023254	0,000651	0,000200	0,132507	25	0,029860	0,038627
27	0,000573	0,000633	0,021459	0,000633	0,000200	0,159602	26	0,029512	0,038388
28	0,000556	0,000624	0,019993	0,000624	0,000203	0,187759	27	0,029187	0,038150
29	0,000544	0,000623	0,018837	0,000623	0,000207	0,216732	28	0,028885	0,037912
30	0,000541	0,000632	0,017970	0,000632	0,000213	0,246286	29	0,028604	0,037674
31	0,000553	0,000650	0,017372	0,000650	0,000220	0,276199	30	0,028343	0,037435
32	0,000578	0,000676	0,017028	0,000676	0,000228	0,306266	31	0,028101	0,037197
33	0,000616	0,000707	0,016921	0,000707	0,000248	0,336293	32	0,027878	0,036959
34	0,000668	0,000744	0,017037	0,000744	0,000277	0,366098	33	0,027672	0,036721
35	0,000732	0,000785	0,017359	0,000785	0,000319	0,395512	34	0,027483	0,036482
36	0,000811	0,000830	0,017870	0,000830	0,000375	0,424380	34	0,027308	0,036244
37	0,000906	0,000879	0,018554	0,000879	0,000446	0,452557	35	0,027149	0,036006
38	0,001017	0,000935	0,019390	0,000935	0,000533	0,479909	36	0,027002	0,035767
39	0,001144	0,000999	0,020357	0,000999	0,000637	0,506316	37	0,026868	0,035529
40	0,001290	0,001074	0,021432	0,001074	0,000760	0,531665	38	0,026745	0,035291
41	0,001458	0,001163	0,022591	0,001163	0,000902	0,555857	39	0,026632	0,035053
42	0,001650	0,001268	0,023806	0,001268	0,001063	0,578803	40	0,026529	0,034814
43	0,001868	0,001392	0,025052	0,001392	0,001240	0,600421	41	0,026434	0,034576
44	0,002114	0,001535	0,026302	0,001535	0,001429	0,620641	41	0,026347	0,034338
45	0,002389	0,001697	0,027527	0,001697	0,001628	0,639403	42	0,026265	0,034099
46	0,002698	0,001876	0,028699	0,001876	0,001833	0,656655	43	0,026189	0,033861
47	0,003053	0,002070	0,029786	0,002070	0,002045	0,672353	44	0,026118	0,033623
48	0,003478	0,002274	0,030756	0,002274	0,002264	0,686463	45	0,026049	0,033385
49	0,004031	0,002487	0,031579	0,002487	0,002493	0,698958	46	0,025983	0,033146
age x	i_x	q_x^{aa}	q_x^{i}	q_x^{Apm}	q_y^{w}	h_x	$y(x)$	λ_x	$\lambda_x^{\text{alternative}}$

Male white collar workers									
age x	i_x	q_x^{aa}	q_x^{i}	q_x^{Apm}	q_y^{w}	h_x	$y(x)$	λ_x	$\lambda_x^{\text{alternative}}$
50	0,004815	0,002710	0,032222	0,002710	0,002735	0,709819	47	0,025919	0,032908
51	0,005988	0,002947	0,032658	0,002947	0,002990	0,719034	48	0,025854	0,032670
52	0,007750	0,003205	0,032865	0,003205	0,003260	0,726599	49	0,025789	0,032432
53	0,010309	0,003493	0,032831	0,003493	0,003545	0,732517	50	0,025722	0,032193
54	0,013819	0,003821	0,032551	0,003821	0,003842	0,736797	51	0,025653	0,031955
55	0,018305	0,004194	0,032032	0,004194	0,004152	0,739453	52	0,025580	0,031717
56	0,023602	0,004617	0,031289	0,004617	0,004472	0,740508	53	0,025502	0,031478
57	0,029352	0,005093	0,030342	0,005093	0,004803	0,739987	54	0,025418	0,031240
58	0,035127	0,005621	0,029219	0,005621	0,005143	0,737923	54	0,025328	0,031002
59	0,040693	0,006203	0,027944	0,006203	0,005494	0,734353	55	0,025230	0,030764
60	0,046306	0,006851	0,026540	0,006851	0,005856	0,729318	56	0,025123	0,030525
61	0,051936	0,007139	0,025024	0,007139	0,006233	0,722864	57	0,025007	0,030287
62	0,057566	0,007517	0,023407	0,007517	0,006628	0,715041	58	0,024880	0,030049
63	0,063196	0,007986	0,023348	0,007986	0,007048	0,705902	59	0,024741	0,029811
64	0,068826	0,008546	0,022461	0,008546	0,007499	0,695505	60	0,024590	0,029572
65	0,074456	0,009201	0,021735	0,009201	0,007990	0,683910	61	0,024425	0,029334
66	0,080086	0,009956	0,021206	0,009956	0,008534	0,671179	62	0,024245	0,029096
67	0,085716	0,010819	0,020906	0,010819	0,009144	0,657378	63	0,024050	0,028857
68	0,091346	0,011804	0,020869	0,011804	0,009836	0,642573	64	0,023837	0,028619
69	0,096976	0,012931	0,021129	0,012931	0,010630	0,626835	65	0,023607	0,028381
70	0,102606	0,014224	0,021720	0,014224	0,011550	0,610233	66	0,023359	0,028143
71	0,000000	0,015718	0,022677	0,015718	0,012622	0,592841	67	0,023090	0,027904
72	0,000000	0,017451	0,024033	0,017451	0,013879	0,574729	68	0,022801	0,027666
73	0,000000	0,019473	0,025821	0,019473	0,015357	0,555972	69	0,022491	0,027428
74	0,000000	0,021838	0,028074	0,021838	0,017098	0,536643	70	0,022157	0,027189
75	0,000000	0,024607	0,030824	0,024607	0,019151	0,516815	71	0,021800	0,026747
76	0,000000	0,027845	0,034101	0,027845	0,021569	0,496562	72	0,021418	0,026021
77	0,000000	0,031621	0,037937	0,031621	0,024412	0,475956	73	0,021010	0,025140
78	0,000000	0,036008	0,042359	0,036008	0,027747	0,455068	74	0,020576	0,024184
79	0,000000	0,041074	0,047396	0,041074	0,031642	0,433968	74	0,020114	0,022993
80	0,000000	0,046890	0,053074	0,046890	0,036171	0,412725	75	0,019623	0,022150
81	0,000000	0,053520	0,059417	0,053520	0,041407	0,391404	76	0,019103	0,021271
82	0,000000	0,061021	0,066448	0,061021	0,047423	0,370071	77	0,018552	0,020253
83	0,000000	0,069443	0,074189	0,069443	0,054289	0,348786	78	0,017969	0,019090
84	0,000000	0,078824	0,082658	0,078824	0,062070	0,327608	79	0,017353	0,017813
85	0,000000	0,089187	0,091873	0,089187	0,070823	0,306592	79	0,016704	0,016704
86	0,000000	0,100545	0,101847	0,100545	0,080601	0,285791	80	0,016020	0,016020
87	0,000000	0,112894	0,112594	0,112894	0,091452	0,265252	80	0,015300	0,015300
88	0,000000	0,126224	0,124122	0,126224	0,103418	0,245020	81	0,014543	0,014543
age x	i_x	q_x^{aa}	q_x^{i}	q_x^{Apm}	q_y^{w}	h_x	$y(x)$	λ_x	$\lambda_x^{\text{alternative}}$

Male white collar workers									
age x	i_x	q_x^{aa}	q_x^{i}	q_x^{Apm}	q_y^{w}	h_x	$y(x)$	λ_x	$\lambda_x^{\text{alternative}}$
89	0,000000	0,140515	0,136440	0,140515	0,116547	0,225133	82	0,013749	0,013749
90	0,000000	0,155748	0,149549	0,155748	0,130884	0,205626	82	0,012916	0,012916
91	0,000000	0,171904	0,163452	0,171904	0,146482	0,186529	83	0,012043	0,012043
92	0,000000	0,188969	0,178147	0,188969	0,163390	0,167866	83	0,011130	0,011130
93	0,000000	0,206934	0,206934	0,206934	0,181652	0,149655	84	0,010175	0,010175
94	0,000000	0,225783	0,225783	0,225783	0,201288	0,131908	85	0,009177	0,009177
95	0,000000	0,245486	0,245486	0,245486	0,222280	0,114632	85	0,008135	0,008135
96	0,000000	0,265983	0,265983	0,265983	0,244554	0,097826	85	0,007049	0,007049
97	0,000000	0,287175	0,287175	0,287175	0,267966	0,081481	86	0,006301	0,006301
98	0,000000	0,308916	0,308916	0,308916	0,292305	0,065584	86	0,005632	0,005632
99	0,000000	0,331028	0,331028	0,331028	0,317313	0,050111	86	0,005034	0,005034
100	0,000000	0,353330	0,353330	0,353330	0,342725	0,049687	87	0,004500	0,004500
101	0,000000	0,375687	0,375687	0,375687	0,368326	0,033789	88	0,004022	0,004022
102	0,000000	0,398043	0,398043	0,398043	0,393995	0,000000	88	0,003595	0,003595
103	0,000000	0,420422	0,420422	0,420422	0,419696	0,000000	88	0,003213	0,003213
104	0,000000	0,442898	0,442898	0,442898	0,445447	0,000000	89	0,002872	0,002872
105	0,000000	0,465565	0,465565	0,465565	0,471283	0,000000	89	0,002567	0,002567
106	0,000000	0,488482	0,488482	0,488482	0,497206	0,000000	89	0,002295	0,002295
107	0,000000	0,511618	0,511618	0,511618	0,523149	0,000000	90	0,002051	0,002051
108	0,000000	0,534799	0,534799	0,534799	0,548950	0,000000	91	0,001834	0,001834
109	0,000000	0,557689	0,557689	0,557689	0,574355	0,000000	92	0,001639	0,001639
110	0,000000	0,579812	0,579812	0,579812	0,599061	0,000000	93	0,001465	0,001465
111	0,000000	0,600663	0,600663	0,600663	0,622817	0,000000	94	0,001262	0,001262
112	0,000000	0,619925	0,619925	0,619925	0,645567	0,000000	95	0,001075	0,001075
113	0,000000	0,637772	0,637772	0,637772	0,667639	0,000000	96	0,000903	0,000903
114	0,000000	0,655226	0,655226	0,655226	0,689933	0,000000	97	0,000744	0,000744
115	0,000000	0,688386	0,688386	0,688386	0,724224	0,000000	98	0,000597	0,000597
116	0,000000	0,710685	0,710685	0,710685	0,749586	0,000000	99	0,000461	0,000461
117	0,000000	0,732984	0,732984	0,732984	0,774948	0,000000	100	0,000334	0,000334
118	0,000000	0,755283	0,755283	0,755283	0,800310	0,000000	101	0,000215	0,000215
119	0,000000	0,777582	0,777582	0,777582	0,825672	0,000000	102	0,000104	0,000104
120	0,000000	1,000000	1,000000	1,000000	1,000000	0,000000	103	0,000000	0,000000
age x	i_x	q_x^{aa}	q_x^{i}	q_x^{Apm}	q_y^{w}	h_x	$y(x)$	λ_x	$\lambda_x^{\text{alternative}}$

Table 9.1: The values of the Base Table 2007 for male white collar workers.

Female white collar workers									
age y	i_y	q_y^{aa}	q_y^{i}	q_y^{Apf}	q_y^{w}	h_y	$x(y)$	λ_y	$\lambda_y^{\text{alternative}}$
14	0,000000	0,000133	0,090020	0,000133	0,000144	0,000000	19	0,032919	0,037614
15	0,000018	0,000154	0,083413	0,000154	0,000272	0,000000	20	0,032176	0,037460
16	0,000058	0,000172	0,076684	0,000172	0,000388	0,000000	21	0,031473	0,037307
17	0,000098	0,000187	0,069839	0,000187	0,000492	0,000000	22	0,030809	0,037153
18	0,000138	0,000199	0,062901	0,000199	0,000581	0,000000	23	0,030184	0,036999
19	0,000179	0,000207	0,055934	0,000207	0,000651	0,000000	24	0,029596	0,036845
20	0,000242	0,000212	0,049064	0,000212	0,000701	0,002037	25	0,029044	0,036691
21	0,000325	0,000213	0,042492	0,000213	0,000727	0,018958	26	0,028527	0,036537
22	0,000410	0,000211	0,036445	0,000211	0,000733	0,040239	27	0,028045	0,036383
23	0,000494	0,000208	0,031114	0,000208	0,000722	0,065184	28	0,027948	0,036229
24	0,000579	0,000204	0,026619	0,000204	0,000700	0,093142	29	0,027852	0,036075
25	0,000664	0,000201	0,022994	0,000201	0,000675	0,123510	30	0,027756	0,035921
26	0,000749	0,000200	0,020204	0,000200	0,000651	0,155730	31	0,027660	0,035767
27	0,000833	0,000200	0,018161	0,000200	0,000633	0,189285	32	0,027564	0,035613
28	0,000915	0,000203	0,016751	0,000203	0,000624	0,223703	32	0,027468	0,035459
29	0,000994	0,000207	0,015855	0,000207	0,000623	0,258550	33	0,027372	0,035305
30	0,001070	0,000213	0,015365	0,000213	0,000632	0,293430	34	0,027276	0,035151
31	0,001140	0,000220	0,015190	0,000220	0,000650	0,327983	35	0,027179	0,034997
32	0,001205	0,000228	0,015259	0,000228	0,000676	0,361886	36	0,027083	0,034843
33	0,001264	0,000237	0,015519	0,000237	0,000708	0,394848	37	0,026987	0,034689
34	0,001318	0,000248	0,015929	0,000248	0,000801	0,426609	37	0,026891	0,034535
35	0,001369	0,000260	0,016459	0,000260	0,000915	0,456941	38	0,026795	0,034381
36	0,001421	0,000274	0,017081	0,000274	0,001049	0,485645	39	0,026699	0,034227
37	0,001478	0,000291	0,017773	0,000291	0,001204	0,512548	40	0,026603	0,034074
38	0,001550	0,000312	0,018510	0,000312	0,001379	0,537505	41	0,026507	0,033920
39	0,001645	0,000338	0,019269	0,000338	0,001572	0,560393	42	0,026410	0,033766
40	0,001775	0,000372	0,020026	0,000372	0,001782	0,581115	43	0,026314	0,033612
41	0,001956	0,000414	0,020761	0,000414	0,002007	0,599595	44	0,026218	0,033458
42	0,002202	0,000465	0,021452	0,000465	0,002247	0,615777	45	0,026122	0,033304
43	0,002531	0,000525	0,022082	0,000525	0,002502	0,629625	46	0,026026	0,033150
44	0,002960	0,000593	0,022634	0,000593	0,002778	0,641121	47	0,025930	0,032996
45	0,003506	0,000669	0,023096	0,000669	0,003083	0,650263	48	0,025834	0,032842
46	0,004184	0,000754	0,023456	0,000754	0,003428	0,657068	49	0,025738	0,032688
47	0,005006	0,000847	0,023703	0,000847	0,003821	0,661564	50	0,025641	0,032534
48	0,005980	0,000951	0,023828	0,000951	0,004270	0,663794	51	0,025545	0,032380
49	0,007109	0,001066	0,023822	0,001066	0,004776	0,663813	53	0,025449	0,032226
50	0,008392	0,001195	0,024702	0,001195	0,005339	0,661688	54	0,025353	0,032072
51	0,009821	0,001339	0,025700	0,001339	0,005955	0,657494	55	0,025257	0,031918
52	0,011385	0,001502	0,025685	0,001502	0,006619	0,651319	56	0,025161	0,031764
age y	i_y	q_y^{aa}	q_y^{i}	q_y^{Apf}	q_y^{w}	h_y	$x(y)$	λ_y	$\lambda_y^{\text{alternative}}$

Female white collar workers									
age y	i_y	q_y^{aa}	q_y^{i}	q_y^{Apf}	q_y^{w}	h_y	$x(y)$	λ_y	$\lambda_y^{\text{alternative}}$
53	0,013068	0,001686	0,025528	0,001686	0,007328	0,643256	57	0,025065	0,031610
54	0,014852	0,001892	0,025233	0,001892	0,008078	0,633405	58	0,024968	0,031456
55	0,016718	0,002125	0,024803	0,002125	0,008866	0,621876	59	0,024872	0,031302
56	0,018652	0,002386	0,024245	0,002386	0,009692	0,608781	60	0,024776	0,031148
57	0,020638	0,002678	0,023568	0,002678	0,010559	0,594237	60	0,024680	0,030994
58	0,022670	0,003013	0,022786	0,003013	0,011469	0,578366	61	0,024584	0,030841
59	0,024743	0,003393	0,021912	0,003393	0,012429	0,561291	62	0,024488	0,030687
60	0,026705	0,003785	0,020965	0,003785	0,013444	0,543139	63	0,024392	0,030533
61	0,028711	0,004187	0,019963	0,004187	0,014524	0,524038	64	0,024296	0,030379
62	0,030718	0,004602	0,018929	0,004602	0,015677	0,504114	65	0,024199	0,030225
63	0,032725	0,005033	0,017887	0,005033	0,016913	0,483497	66	0,024103	0,030071
64	0,034732	0,005485	0,016865	0,005485	0,018244	0,462313	67	0,024007	0,029917
65	0,036738	0,005967	0,015889	0,005967	0,019684	0,440687	68	0,023911	0,029763
66	0,038745	0,006487	0,014992	0,006487	0,021246	0,418743	69	0,023815	0,029609
67	0,040752	0,007061	0,014206	0,007061	0,022946	0,396602	70	0,023727	0,029455
68	0,042759	0,007705	0,013566	0,007705	0,024804	0,374381	70	0,023623	0,029301
69	0,044765	0,008440	0,013106	0,008440	0,026840	0,352194	71	0,023501	0,029147
70	0,046772	0,009291	0,012866	0,009291	0,029077	0,330150	72	0,023361	0,028993
71	0,000000	0,010286	0,012883	0,010286	0,031544	0,308353	73	0,023201	0,028839
72	0,000000	0,011461	0,013198	0,011461	0,034268	0,286904	74	0,023020	0,028685
73	0,000000	0,012855	0,013851	0,012855	0,037283	0,265894	75	0,022817	0,028531
74	0,000000	0,014513	0,014884	0,014513	0,040625	0,245412	76	0,022591	0,028377
75	0,000000	0,016486	0,016486	0,016486	0,044332	0,225539	77	0,022341	0,028223
76	0,000000	0,018830	0,018830	0,018830	0,048447	0,206350	78	0,022066	0,027539
77	0,000000	0,021605	0,021605	0,021605	0,053014	0,187910	79	0,021765	0,027048
78	0,000000	0,024875	0,024875	0,024875	0,058079	0,170280	79	0,021436	0,026403
79	0,000000	0,028708	0,028708	0,028708	0,063693	0,153512	80	0,021078	0,025603
80	0,000000	0,033173	0,033173	0,033173	0,069905	0,137651	81	0,020691	0,024696
81	0,000000	0,038338	0,038338	0,038338	0,076768	0,122731	82	0,020274	0,023717
82	0,000000	0,044268	0,044268	0,044268	0,084333	0,108782	83	0,019824	0,022687
83	0,000000	0,051025	0,051025	0,051025	0,092651	0,095823	83	0,019342	0,021614
84	0,000000	0,058661	0,058661	0,058661	0,101773	0,083864	84	0,018826	0,020457
85	0,000000	0,067224	0,067224	0,067224	0,111746	0,072909	84	0,018274	0,019222
86	0,000000	0,076751	0,076751	0,076751	0,122615	0,062950	85	0,017687	0,017937
87	0,000000	0,087277	0,087277	0,087277	0,134418	0,053973	86	0,017062	0,017062
88	0,000000	0,098832	0,098832	0,098832	0,147190	0,045954	87	0,016399	0,016399
89	0,000000	0,111448	0,111448	0,111448	0,160961	0,038862	87	0,015696	0,015696
90	0,000000	0,125167	0,125167	0,125167	0,175755	0,032654	87	0,014953	0,014953
91	0,000000	0,140035	0,140035	0,140035	0,191590	0,027282	88	0,014168	0,014168
age y	i_y	q_y^{aa}	q_y^{i}	q_y^{Apf}	q_y^{w}	h_y	$x(y)$	λ_y	$\lambda_y^{\text{alternative}}$

Female white collar workers									
age y	i_y	q_y^{aa}	q_y^{i}	q_y^{Apf}	q_y^{w}	h_y	$x(y)$	λ_y	$\lambda_y^{\text{alternative}}$
92	0,000000	0,156107	0,156107	0,156107	0,208474	0,022688	89	0,013341	0,013341
93	0,000000	0,173441	0,173441	0,173441	0,226404	0,018806	90	0,012470	0,012470
94	0,000000	0,192075	0,192075	0,192075	0,245355	0,015562	91	0,011553	0,011553
95	0,000000	0,212015	0,212015	0,212015	0,265280	0,012872	92	0,010591	0,010591
96	0,000000	0,233208	0,233208	0,233208	0,286092	0,010649	92	0,009581	0,009581
97	0,000000	0,255523	0,255523	0,255523	0,307669	0,008794	92	0,008542	0,008542
98	0,000000	0,278749	0,278749	0,278749	0,329850	0,007203	93	0,007616	0,007616
99	0,000000	0,302610	0,302610	0,302610	0,352456	0,005765	93	0,006790	0,006790
100	0,000000	0,326820	0,326820	0,326820	0,375314	0,005612	94	0,006054	0,006054
101	0,000000	0,351147	0,351147	0,351147	0,398295	0,004022	95	0,005398	0,005398
102	0,000000	0,375473	0,375473	0,375473	0,421342	0,000000	96	0,004812	0,004812
103	0,000000	0,399789	0,399789	0,399789	0,444456	0,000000	97	0,004291	0,004291
104	0,000000	0,424156	0,424156	0,424156	0,467674	0,000000	98	0,003825	0,003825
105	0,000000	0,448669	0,448669	0,448669	0,491032	0,000000	99	0,003411	0,003411
106	0,000000	0,473390	0,473390	0,473390	0,514521	0,000000	100	0,003041	0,003041
107	0,000000	0,498297	0,498297	0,498297	0,538052	0,000000	101	0,002711	0,002711
108	0,000000	0,523230	0,523230	0,523230	0,561429	0,000000	102	0,002417	0,002417
109	0,000000	0,547871	0,547871	0,547871	0,584359	0,000000	103	0,002155	0,002155
110	0,000000	0,571774	0,571774	0,571774	0,606500	0,000000	104	0,001921	0,001921
111	0,000000	0,594474	0,594474	0,594474	0,627567	0,000000	105	0,001652	0,001652
112	0,000000	0,615687	0,615687	0,615687	0,647501	0,000000	106	0,001405	0,001405
113	0,000000	0,635602	0,635602	0,635602	0,666671	0,000000	107	0,001179	0,001179
114	0,000000	0,655204	0,655204	0,655204	0,686086	0,000000	108	0,000970	0,000970
115	0,000000	0,689744	0,689744	0,689744	0,718927	0,000000	109	0,000777	0,000777
116	0,000000	0,713881	0,713881	0,713881	0,741782	0,000000	110	0,000599	0,000599
117	0,000000	0,738018	0,738018	0,738018	0,764636	0,000000	111	0,000434	0,000434
118	0,000000	0,762155	0,762155	0,762155	0,787491	0,000000	112	0,000279	0,000279
119	0,000000	0,786292	0,786292	0,786292	0,810346	0,000000	113	0,000135	0,000135
120	0,000000	1,000000	1,000000	1,000000	1,000000	0,000000	114	0,000000	0,000000
age y	i_y	q_y^{aa}	q_y^{i}	q_y^{Apf}	q_y^{w}	h_y	$x(y)$	λ_y	$\lambda_y^{\text{alternative}}$

Table 9.2: The values of the Base Table 2007 for female white collar workers.

Male blue and white collar workers									
age x	i_x	q_x^{aa}	q_x^{i}	q_x^{Apm}	q_y^{w}	h_x	$y(x)$	λ_x	$\lambda_x^{\text{alternative}}$
14	0,000000	0,000143	0,047887	0,000143	0,000151	0,000000	14	0,036224	0,041486
15	0,000080	0,000273	0,045515	0,000273	0,000163	0,000000	15	0,035517	0,041248
16	0,000188	0,000391	0,043132	0,000391	0,000175	0,000000	16	0,034846	0,041009
17	0,000295	0,000495	0,040739	0,000495	0,000187	0,000000	17	0,034210	0,040771
18	0,000402	0,000583	0,038337	0,000583	0,000198	0,000000	18	0,033606	0,040533
19	0,000509	0,000650	0,035931	0,000650	0,000206	0,000000	18	0,033035	0,040295
20	0,000643	0,000695	0,033533	0,000695	0,000211	0,008022	19	0,032496	0,040056
21	0,000765	0,000715	0,031160	0,000715	0,000211	0,022696	20	0,031987	0,039818
22	0,000876	0,000714	0,028837	0,000714	0,000208	0,040237	21	0,031507	0,039580
23	0,000977	0,000696	0,026592	0,000696	0,000203	0,060304	22	0,031055	0,039342
24	0,001066	0,000667	0,024459	0,000667	0,000196	0,082573	23	0,030631	0,039103
25	0,001144	0,000636	0,022471	0,000636	0,000189	0,106739	24	0,030233	0,038865
26	0,001208	0,000607	0,020661	0,000607	0,000184	0,132507	25	0,029860	0,038627
27	0,001256	0,000584	0,019059	0,000584	0,000181	0,159602	26	0,029512	0,038388
28	0,001294	0,000571	0,017687	0,000571	0,000179	0,187759	27	0,029187	0,038150
29	0,001330	0,000567	0,016564	0,000567	0,000179	0,216732	28	0,028885	0,037912
30	0,001372	0,000572	0,015701	0,000572	0,000181	0,246286	29	0,028604	0,037674
31	0,001427	0,000585	0,015102	0,000585	0,000184	0,276199	30	0,028343	0,037435
32	0,001499	0,000605	0,014764	0,000605	0,000188	0,306266	31	0,028101	0,037197
33	0,001591	0,000630	0,014680	0,000630	0,000211	0,336293	32	0,027878	0,036959
34	0,001703	0,000658	0,014837	0,000658	0,000229	0,366098	33	0,027672	0,036721
35	0,001835	0,000688	0,015218	0,000688	0,000266	0,395512	34	0,027483	0,036482
36	0,001988	0,000720	0,015804	0,000720	0,000322	0,424380	34	0,027308	0,036244
37	0,002161	0,000754	0,016575	0,000754	0,000397	0,452557	35	0,027149	0,036006
38	0,002357	0,000790	0,017510	0,000790	0,000495	0,479909	36	0,027002	0,035767
39	0,002585	0,000830	0,018587	0,000830	0,000616	0,506316	37	0,026868	0,035529
40	0,002855	0,000875	0,019782	0,000875	0,000764	0,531665	38	0,026745	0,035291
41	0,003184	0,000924	0,021073	0,000924	0,000939	0,555857	39	0,026632	0,035053
42	0,003591	0,000979	0,022432	0,000979	0,001138	0,578803	40	0,026529	0,034814
43	0,004088	0,001040	0,023835	0,001040	0,001355	0,600421	41	0,026434	0,034576
44	0,004676	0,001108	0,025250	0,001108	0,001585	0,620641	41	0,026347	0,034338
45	0,005340	0,001187	0,026648	0,001187	0,001820	0,639403	42	0,026265	0,034099
46	0,006055	0,001283	0,027994	0,001283	0,002053	0,656655	43	0,026189	0,033861
47	0,006795	0,001404	0,029255	0,001404	0,002285	0,672353	44	0,026118	0,033623
48	0,007564	0,001559	0,030396	0,001559	0,002518	0,686463	45	0,026049	0,033385
49	0,008429	0,001757	0,031383	0,001757	0,002759	0,698958	46	0,025983	0,033146
50	0,009574	0,002004	0,032188	0,002004	0,003014	0,709819	47	0,025919	0,032908
51	0,011336	0,002304	0,032786	0,002304	0,003288	0,719034	48	0,025854	0,032670
52	0,014213	0,002663	0,033161	0,002663	0,003588	0,726599	49	0,025789	0,032432
age x	i_x	q_x^{aa}	q_x^{i}	q_x^{Apm}	q_y^{w}	h_x	$y(x)$	λ_x	$\lambda_x^{\text{alternative}}$

Male blue and white collar workers									
age x	i_x	q_x^{aa}	q_x^{i}	q_x^{Apm}	q_y^{w}	h_x	$y(x)$	λ_x	$\lambda_x^{\text{alternative}}$
53	0,018810	0,003085	0,033306	0,003085	0,003914	0,732517	50	0,025722	0,032193
54	0,025696	0,003579	0,033225	0,003579	0,004269	0,736797	51	0,025653	0,031955
55	0,035200	0,004153	0,032930	0,004153	0,004652	0,739453	52	0,025580	0,031717
56	0,047206	0,004917	0,032441	0,004917	0,005063	0,740508	53	0,025502	0,031478
57	0,061079	0,005680	0,031786	0,005680	0,005499	0,739987	54	0,025418	0,031240
58	0,075874	0,006444	0,030994	0,006444	0,005961	0,737923	54	0,025328	0,031002
59	0,091042	0,007208	0,030092	0,007208	0,006446	0,734353	55	0,025230	0,030764
60	0,104186	0,007971	0,029106	0,007971	0,006954	0,729318	56	0,025123	0,030525
61	0,118221	0,008735	0,028054	0,008735	0,007486	0,722864	57	0,025007	0,030287
62	0,132256	0,009499	0,026949	0,009499	0,008045	0,715041	58	0,024880	0,030049
63	0,146291	0,010262	0,025795	0,010262	0,008633	0,705902	59	0,024741	0,029811
64	0,160326	0,011026	0,024596	0,011026	0,009257	0,695505	60	0,024590	0,029572
65	0,174361	0,011790	0,025086	0,011790	0,009923	0,683910	61	0,024425	0,029334
66	0,188396	0,012689	0,025126	0,012689	0,010644	0,671179	62	0,024245	0,029096
67	0,202431	0,013689	0,025379	0,013689	0,011431	0,657378	63	0,024050	0,028857
68	0,216466	0,014804	0,025875	0,014804	0,012303	0,642573	64	0,023837	0,028619
69	0,230501	0,016053	0,026643	0,016053	0,013281	0,626835	65	0,023607	0,028381
70	0,244536	0,017465	0,027711	0,017465	0,014390	0,610233	66	0,023359	0,028143
71	0,000000	0,019075	0,029108	0,019075	0,015661	0,592841	67	0,023090	0,027904
72	0,000000	0,020928	0,030864	0,020928	0,017132	0,574729	68	0,022801	0,027666
73	0,000000	0,023076	0,033007	0,023076	0,018845	0,555972	69	0,022491	0,027428
74	0,000000	0,025579	0,035566	0,025579	0,020849	0,536643	70	0,022157	0,027189
75	0,000000	0,028501	0,038569	0,028501	0,023201	0,516815	71	0,021800	0,026747
76	0,000000	0,031910	0,042044	0,031910	0,025961	0,496562	72	0,021418	0,026021
77	0,000000	0,035878	0,046018	0,035878	0,029196	0,475956	73	0,021010	0,025140
78	0,000000	0,040473	0,050518	0,040473	0,032979	0,455068	74	0,020576	0,024184
79	0,000000	0,045764	0,055569	0,045764	0,037383	0,433968	74	0,020114	0,022993
80	0,000000	0,051816	0,061197	0,051816	0,042483	0,412725	75	0,019623	0,022150
81	0,000000	0,058686	0,067426	0,058686	0,048354	0,391404	76	0,019103	0,021271
82	0,000000	0,066426	0,074278	0,066426	0,055065	0,370071	77	0,018552	0,020253
83	0,000000	0,075077	0,081775	0,075077	0,062680	0,348786	78	0,017969	0,019090
84	0,000000	0,084669	0,089939	0,084669	0,071254	0,327608	79	0,017353	0,017813
85	0,000000	0,095220	0,098787	0,095220	0,080836	0,306592	79	0,016704	0,016704
86	0,000000	0,106738	0,108338	0,106738	0,091464	0,285791	80	0,016020	0,016020
87	0,000000	0,119217	0,118607	0,119217	0,103170	0,265252	80	0,015300	0,015300
88	0,000000	0,132646	0,129610	0,132646	0,115982	0,245020	81	0,014543	0,014543
89	0,000000	0,147007	0,141359	0,147007	0,129928	0,225133	82	0,013749	0,013749
90	0,000000	0,162285	0,153864	0,162285	0,145037	0,205626	82	0,012916	0,012916
91	0,000000	0,178465	0,167134	0,178465	0,161341	0,186529	83	0,012043	0,012043
age x	i_x	q_x^{aa}	q_x^{i}	q_x^{Apm}	q_y^{w}	h_x	$y(x)$	λ_x	$\lambda_x^{\text{alternative}}$

Male blue and white collar workers									
age x	i_x	q_x^{aa}	q_x^{i}	q_x^{Apm}	q_y^{w}	h_x	$y(x)$	λ_x	$\lambda_x^{\text{alternative}}$
92	0,000000	0,195541	0,181177	0,195541	0,178872	0,167866	83	0,011130	0,011130
93	0,000000	0,213507	0,213507	0,213507	0,197657	0,149655	84	0,010175	0,010175
94	0,000000	0,232351	0,232351	0,232351	0,217705	0,131908	85	0,009177	0,009177
95	0,000000	0,252046	0,252046	0,252046	0,238988	0,114632	85	0,008135	0,008135
96	0,000000	0,272534	0,272534	0,272534	0,261433	0,097826	85	0,007049	0,007049
97	0,000000	0,293713	0,293713	0,293713	0,284905	0,081481	86	0,006301	0,006301
98	0,000000	0,315439	0,315439	0,315439	0,309211	0,065584	86	0,005632	0,005632
99	0,000000	0,337534	0,337534	0,337534	0,334117	0,050111	86	0,005034	0,005034
100	0,000000	0,359819	0,359819	0,359819	0,359385	0,049687	87	0,004500	0,004500
101	0,000000	0,382158	0,382158	0,382158	0,384821	0,033789	88	0,004022	0,004022
102	0,000000	0,404495	0,404495	0,404495	0,410318	0,000000	88	0,003595	0,003595
103	0,000000	0,426852	0,426852	0,426852	0,435844	0,000000	88	0,003213	0,003213
104	0,000000	0,449302	0,449302	0,449302	0,461417	0,000000	89	0,002872	0,002872
105	0,000000	0,471936	0,471936	0,471936	0,487067	0,000000	89	0,002567	0,002567
106	0,000000	0,494811	0,494811	0,494811	0,512798	0,000000	89	0,002295	0,002295
107	0,000000	0,517898	0,517898	0,517898	0,538547	0,000000	90	0,002051	0,002051
108	0,000000	0,541031	0,541031	0,541031	0,564168	0,000000	91	0,001834	0,001834
109	0,000000	0,563880	0,563880	0,563880	0,589429	0,000000	92	0,001639	0,001639
110	0,000000	0,585985	0,585985	0,585985	0,614057	0,000000	93	0,001465	0,001465
111	0,000000	0,606858	0,606858	0,606858	0,637821	0,000000	94	0,001262	0,001262
112	0,000000	0,626190	0,626190	0,626190	0,660672	0,000000	95	0,001075	0,001075
113	0,000000	0,644150	0,644150	0,644150	0,682908	0,000000	96	0,000903	0,000903
114	0,000000	0,661728	0,661728	0,661728	0,705345	0,000000	97	0,000744	0,000744
115	0,000000	0,694538	0,694538	0,694538	0,738671	0,000000	98	0,000597	0,000597
116	0,000000	0,716815	0,716815	0,716815	0,763893	0,000000	99	0,000461	0,000461
117	0,000000	0,739092	0,739092	0,739092	0,789114	0,000000	100	0,000334	0,000334
118	0,000000	0,761369	0,761369	0,761369	0,814336	0,000000	101	0,000215	0,000215
119	0,000000	0,783646	0,783646	0,783646	0,839558	0,000000	102	0,000104	0,000104
120	0,000000	1,000000	1,000000	1,000000	1,000000	0,000000	103	0,000000	0,000000
age x	i_x	q_x^{aa}	q_x^{i}	q_x^{Apm}	q_y^{w}	h_x	$y(x)$	λ_x	$\lambda_x^{\text{alternative}}$

Table 9.3: The values of the Base Table 2007 for male blue and white collar workers.

Female blue and white collar workers									
age y	i_y	q_y^{aa}	q_y^{i}	q_y^{Apf}	q_y^{w}	h_y	$x(y)$	λ_y	$\lambda_y^{\text{alternative}}$
14	0,000000	0,000151	0,063489	0,000151	0,000143	0,000000	19	0,032919	0,037614
15	0,000033	0,000163	0,058797	0,000163	0,000273	0,000000	20	0,032176	0,037460
16	0,000087	0,000175	0,054057	0,000175	0,000391	0,000000	21	0,031473	0,037307
17	0,000142	0,000187	0,049274	0,000187	0,000495	0,000000	22	0,030809	0,037153
18	0,000197	0,000198	0,044481	0,000198	0,000583	0,000000	23	0,030184	0,036999
19	0,000252	0,000206	0,039745	0,000206	0,000650	0,000000	24	0,029596	0,036845
20	0,000329	0,000211	0,035179	0,000211	0,000695	0,002037	25	0,029044	0,036691
21	0,000446	0,000211	0,030916	0,000211	0,000715	0,018958	26	0,028527	0,036537
22	0,000570	0,000208	0,027075	0,000208	0,000714	0,040239	27	0,028045	0,036383
23	0,000699	0,000203	0,023735	0,000203	0,000696	0,065184	28	0,027948	0,036229
24	0,000833	0,000196	0,020934	0,000196	0,000667	0,093142	29	0,027852	0,036075
25	0,000972	0,000189	0,018667	0,000189	0,000636	0,123510	30	0,027756	0,035921
26	0,001117	0,000184	0,016897	0,000184	0,000607	0,155730	31	0,027660	0,035767
27	0,001264	0,000181	0,015569	0,000181	0,000584	0,189285	32	0,027564	0,035613
28	0,001414	0,000179	0,014619	0,000179	0,000571	0,223703	32	0,027468	0,035459
29	0,001564	0,000179	0,013984	0,000179	0,000567	0,258550	33	0,027372	0,035305
30	0,001713	0,000181	0,013606	0,000181	0,000572	0,293430	34	0,027276	0,035151
31	0,001857	0,000184	0,013434	0,000184	0,000585	0,327983	35	0,027179	0,034997
32	0,001994	0,000188	0,013429	0,000188	0,000605	0,361886	36	0,027083	0,034843
33	0,002123	0,000193	0,013558	0,000193	0,000637	0,394848	37	0,026987	0,034689
34	0,002240	0,000200	0,013795	0,000200	0,000726	0,426609	37	0,026891	0,034535
35	0,002346	0,000208	0,014122	0,000208	0,000838	0,456941	38	0,026795	0,034381
36	0,002441	0,000218	0,014524	0,000218	0,000972	0,485645	39	0,026699	0,034227
37	0,002529	0,000230	0,014991	0,000230	0,001129	0,512548	40	0,026603	0,034074
38	0,002617	0,000246	0,015512	0,000246	0,001309	0,537505	41	0,026507	0,033920
39	0,002712	0,000267	0,016082	0,000267	0,001513	0,560393	42	0,026410	0,033766
40	0,002831	0,000293	0,016689	0,000293	0,001741	0,581115	43	0,026314	0,033612
41	0,002989	0,000327	0,017325	0,000327	0,001995	0,599595	44	0,026218	0,033458
42	0,003212	0,000367	0,017977	0,000367	0,002276	0,615777	45	0,026122	0,033304
43	0,003524	0,000415	0,018631	0,000415	0,002589	0,629625	46	0,026026	0,033150
44	0,003959	0,000470	0,019270	0,000470	0,002940	0,641121	47	0,025930	0,032996
45	0,004551	0,000532	0,019876	0,000532	0,003338	0,650263	48	0,025834	0,032842
46	0,005336	0,000604	0,020429	0,000604	0,003795	0,657068	49	0,025738	0,032688
47	0,006355	0,000687	0,020910	0,000687	0,004319	0,661564	50	0,025641	0,032534
48	0,007642	0,000783	0,021297	0,000783	0,004917	0,663794	51	0,025545	0,032380
49	0,009235	0,000896	0,021573	0,000896	0,005594	0,663813	53	0,025449	0,032226
50	0,011162	0,001027	0,021721	0,001027	0,006349	0,661688	54	0,025353	0,032072
51	0,013448	0,001180	0,021724	0,001180	0,007181	0,657494	55	0,025257	0,031918
52	0,016112	0,001354	0,021572	0,001354	0,008086	0,651319	56	0,025161	0,031764
age y	i_y	q_y^{aa}	q_y^{i}	q_y^{Apf}	q_y^{w}	h_y	$x(y)$	λ_y	$\lambda_y^{\text{alternative}}$

Female blue and white collar workers									
age y	i_y	q_y^{aa}	q_y^{i}	q_y^{Apf}	q_y^{w}	h_y	$x(y)$	λ_y	$\lambda_y^{\text{alternative}}$
53	0,019161	0,001553	0,021254	0,001553	0,009058	0,643256	57	0,025065	0,031610
54	0,022598	0,001778	0,020763	0,001778	0,010091	0,633405	58	0,024968	0,031456
55	0,026417	0,001920	0,020094	0,001920	0,011181	0,621876	59	0,024872	0,031302
56	0,030606	0,002334	0,020382	0,002334	0,012323	0,608781	60	0,024776	0,031148
57	0,035152	0,002749	0,019877	0,002749	0,013513	0,594237	60	0,024680	0,030994
58	0,040043	0,003166	0,019287	0,003166	0,014752	0,578366	61	0,024584	0,030841
59	0,045273	0,003585	0,018623	0,003585	0,016039	0,561291	62	0,024488	0,030687
60	0,049643	0,004008	0,017898	0,004008	0,017380	0,543139	63	0,024392	0,030533
61	0,054357	0,004435	0,017128	0,004435	0,018781	0,524038	64	0,024296	0,030379
62	0,059072	0,004871	0,016331	0,004871	0,020250	0,504114	65	0,024199	0,030225
63	0,063787	0,005318	0,015527	0,005318	0,021798	0,483497	66	0,024103	0,030071
64	0,068502	0,005783	0,014738	0,005783	0,023438	0,462313	67	0,024007	0,029917
65	0,073217	0,006273	0,013987	0,006273	0,025185	0,440687	68	0,023911	0,029763
66	0,077932	0,006799	0,013302	0,006799	0,027053	0,418743	69	0,023815	0,029609
67	0,082647	0,007374	0,012710	0,007374	0,029061	0,396602	70	0,023727	0,029455
68	0,087362	0,008016	0,012241	0,008016	0,031227	0,374381	70	0,023623	0,029301
69	0,092077	0,008745	0,011928	0,008745	0,033569	0,352194	71	0,023501	0,029147
70	0,096792	0,009587	0,011802	0,009587	0,036111	0,330150	72	0,023361	0,028993
71	0,000000	0,010575	0,011901	0,010575	0,038876	0,308353	73	0,023201	0,028839
72	0,000000	0,011744	0,012260	0,011744	0,041890	0,286904	74	0,023020	0,028685
73	0,000000	0,013137	0,013137	0,013137	0,045183	0,265894	75	0,022817	0,028531
74	0,000000	0,014801	0,014801	0,014801	0,048791	0,245412	76	0,022591	0,028377
75	0,000000	0,016789	0,016789	0,016789	0,052752	0,225539	77	0,022341	0,028223
76	0,000000	0,019158	0,019158	0,019158	0,057110	0,206350	78	0,022066	0,027539
77	0,000000	0,021969	0,021969	0,021969	0,061912	0,187910	79	0,021765	0,027048
78	0,000000	0,025286	0,025286	0,025286	0,067212	0,170280	79	0,021436	0,026403
79	0,000000	0,029174	0,029174	0,029174	0,073063	0,153512	80	0,021078	0,025603
80	0,000000	0,033697	0,033697	0,033697	0,079523	0,137651	81	0,020691	0,024696
81	0,000000	0,038919	0,038919	0,038919	0,086651	0,122731	82	0,020274	0,023717
82	0,000000	0,044895	0,044895	0,044895	0,094501	0,108782	83	0,019824	0,022687
83	0,000000	0,051678	0,051678	0,051678	0,103126	0,095823	83	0,019342	0,021614
84	0,000000	0,059309	0,059309	0,059309	0,112577	0,083864	84	0,018826	0,020457
85	0,000000	0,067824	0,067824	0,067824	0,122894	0,072909	84	0,018274	0,019222
86	0,000000	0,077249	0,077249	0,077249	0,134115	0,062950	85	0,017687	0,017937
87	0,000000	0,087606	0,087606	0,087606	0,146272	0,053973	86	0,017062	0,017062
88	0,000000	0,098916	0,098916	0,098916	0,159390	0,045954	87	0,016399	0,016399
89	0,000000	0,111201	0,111201	0,111201	0,173493	0,038862	87	0,015696	0,015696
90	0,000000	0,124495	0,124495	0,124495	0,188602	0,032654	87	0,014953	0,014953
91	0,000000	0,138839	0,138839	0,138839	0,204733	0,027282	88	0,014168	0,014168
age y	i_y	q_y^{aa}	q_y^{i}	q_y^{Apf}	q_y^{w}	h_y	$x(y)$	λ_y	$\lambda_y^{\text{alternative}}$

Female blue and white collar workers									
age y	i_y	q_y^{aa}	q_y^{i}	q_y^{Apf}	q_y^{w}	h_y	$x(y)$	λ_y	$\lambda_y^{\text{alternative}}$
92	0,000000	0,154283	0,154283	0,154283	0,221903	0,022688	89	0,013341	0,013341
93	0,000000	0,170879	0,170879	0,170879	0,240114	0,018806	90	0,012470	0,012470
94	0,000000	0,188666	0,188666	0,188666	0,259354	0,015562	91	0,011553	0,011553
95	0,000000	0,207646	0,207646	0,207646	0,279584	0,012872	92	0,010591	0,010591
96	0,000000	0,227771	0,227771	0,227771	0,300726	0,010649	92	0,009581	0,009581
97	0,000000	0,248917	0,248917	0,248917	0,322662	0,008794	92	0,008542	0,008542
98	0,000000	0,270889	0,270889	0,270889	0,345232	0,007203	93	0,007616	0,007616
99	0,000000	0,293437	0,293437	0,293437	0,368251	0,005765	93	0,006790	0,006790
100	0,000000	0,316302	0,316302	0,316302	0,391538	0,005612	94	0,006054	0,006054
101	0,000000	0,339280	0,339280	0,339280	0,414953	0,004022	95	0,005398	0,005398
102	0,000000	0,362277	0,362277	0,362277	0,438426	0,000000	96	0,004812	0,004812
103	0,000000	0,385299	0,385299	0,385299	0,461947	0,000000	97	0,004291	0,004291
104	0,000000	0,408422	0,408422	0,408422	0,485544	0,000000	98	0,003825	0,003825
105	0,000000	0,431744	0,431744	0,431744	0,509248	0,000000	99	0,003411	0,003411
106	0,000000	0,455320	0,455320	0,455320	0,533055	0,000000	100	0,003041	0,003041
107	0,000000	0,479102	0,479102	0,479102	0,556892	0,000000	101	0,002711	0,002711
108	0,000000	0,502882	0,502882	0,502882	0,580597	0,000000	102	0,002417	0,002417
109	0,000000	0,526270	0,526270	0,526270	0,603922	0,000000	103	0,002155	0,002155
110	0,000000	0,548742	0,548742	0,548742	0,626576	0,000000	104	0,001921	0,001921
111	0,000000	0,569766	0,569766	0,569766	0,648316	0,000000	105	0,001652	0,001652
112	0,000000	0,589040	0,589040	0,589040	0,669091	0,000000	106	0,001405	0,001405
113	0,000000	0,606833	0,606833	0,606833	0,689216	0,000000	107	0,001179	0,001179
114	0,000000	0,624354	0,624354	0,624354	0,709550	0,000000	108	0,000970	0,000970
115	0,000000	0,659776	0,659776	0,659776	0,741341	0,000000	109	0,000777	0,000777
116	0,000000	0,682616	0,682616	0,682616	0,764609	0,000000	110	0,000599	0,000599
117	0,000000	0,705456	0,705456	0,705456	0,787877	0,000000	111	0,000434	0,000434
118	0,000000	0,728295	0,728295	0,728295	0,811145	0,000000	112	0,000279	0,000279
119	0,000000	0,751135	0,751135	0,751135	0,834413	0,000000	113	0,000135	0,000135
120	0,000000	1,000000	1,000000	1,000000	1,000000	0,000000	114	0,000000	0,000000
age y	i_y	q_y^{aa}	q_y^{i}	q_y^{Apf}	q_y^{w}	h_y	$x(y)$	λ_y	$\lambda_y^{\text{alternative}}$

Table 9.4: The values of the Base Table 2007 for female blue and white collar workers.

9.2 The Mathematical Form of New Annuity Valuation Table

Analogue to the form of the AVÖ 2008P in Equation (4.1) the New Tables have the mathematical form

$$q_x^{\text{status}}(t) = \widetilde{q}_x^{\text{status}}(2007) \cdot e^{-\lambda_x \cdot (t-2007)}, \qquad t \geq 2007, \tag{9.1}$$

with the notation given in Table 4.1 (for female beneficiaries y is used instead of x) using $\widetilde{q}_x^{\text{status}}(2007)$ instead of $\widetilde{q}_x^{\text{status}}(1982)$.

The New Tables are also using constant age-dependent mortality reduction factors λ_x and λ_y which are monotonously decreasing to $\lambda_{120} = 0$.

9.3 Net Present Values of the New Annuity Tables

The Net Present Values NPVs are calculated under the assumption of 1.0% effective yearly discount rate r and a retirement age z of 65 for males and 60 for females.

9.3.1 NPVs of the New Table 2007 in the Year 2015

Male white collar workers										
Age	Birth	$\ddot{a}^{\text{aa}}_{x\overline{z-x}\rceil}$	${}^{(12)}\ddot{a}_x^{\text{Apm}}$	${}^{(12)}\ddot{a}_x^{\text{i}}$	${}^{(12)}\ddot{a}_x^{\text{w}}$	${}^{(12)}\ddot{a}_x^{\text{ai}}$	${}^{(12)}_{z-x}\ddot{a}_x^{\text{aApm}}$	${}^{(12)}\ddot{a}_x^{\text{iw}}$	${}^{(12)}\ddot{a}_x^{\text{pw}}$	${}^{(12)}\ddot{a}_x^{\text{aw}}$
20	1995	34,295021	50,728323	35,838331	49,557618	7,441165	8,781492	5,785006	1,995401	2,335317
25	1990	31,023576	47,942264	35,754426	46,583438	7,568414	9,084975	7,156949	2,237242	2,618433
30	1985	27,566204	44,965595	34,022004	43,396445	7,683709	9,385261	8,117197	2,494287	2,919103
35	1980	23,921383	41,803578	31,502293	39,997338	7,792491	9,684091	8,808403	2,762791	3,234736
40	1975	20,125227	38,470246	28,918801	36,437391	7,865660	10,002110	9,145628	3,031126	3,545891
45	1970	16,203556	34,981344	26,708070	32,766153	7,874392	10,371218	8,907565	3,279157	3,822188
50	1965	12,179225	31,376804	24,969232	29,026299	7,781425	10,854270	7,952523	3,466322	4,014802
55	1960	8,160172	27,681143	23,434384	25,307570	7,284636	11,720505	6,523585	3,564953	4,056255
60	1955	4,375130	23,949101	21,552300	21,665451	5,090360	14,199320	5,119554	3,531128	3,811685
65	1950	0,000000	20,193247	18,920055	18,122111	0,000000	0,000000	4,121533	3,365659	0,000000
70	1945	0,000000	16,413020	15,736324	14,720494	0,000000	0,000000	3,453285	3,108175	0,000000
75	1940	0,000000	12,740056	12,372284	11,537848	0,000000	0,000000	2,895636	2,735303	0,000000
80	1935	0,000000	9,384135	9,252123	8,683358	0,000000	0,000000	2,288332	2,236634	0,000000
85	1930	0,000000	6,601267	6,645692	6,270853	0,000000	0,000000	1,631384	1,638155	0,000000
90	1925	0,000000	4,524320	4,607156	4,370928	0,000000	0,000000	1,006322	1,020845	0,000000
95	1920	0,000000	3,092330	3,092330	2,987476	0,000000	0,000000	0,496190	0,496190	0,000000
100	1915	0,000000	2,178867	2,178867	2,092443	0,000000	0,000000	0,139965	0,139965	0,000000

Table 9.5: The NPVs in the year 2015 of the New Table 2007 for male white collar workers.

Female white collar workers										
Age	Birth	$\ddot{a}^{\mathrm{aa}}_{y\,\overline{z-y}\rvert}$	${}^{(12)}\ddot{a}_y^{\mathrm{Apf}}$	${}^{(12)}\ddot{a}_y^{\mathrm{i}}$	${}^{(12)}\ddot{a}_y^{\mathrm{w}}$	${}^{(12)}\ddot{a}_y^{\mathrm{ai}}$	${}^{(12)}_{z-y}\ddot{a}_y^{\mathrm{aApf}}$	${}^{(12)}\ddot{a}_y^{\mathrm{iw}}$	${}^{(12)}\ddot{a}_y^{\mathrm{pw}}$	${}^{(12)}\ddot{a}_y^{\mathrm{aw}}$
20	1995	31,939837	52,438919	37,093936	51,674563	4,605121	15,692799	3,653864	0,394252	0,580764
25	1990	28,490656	49,727502	38,071759	48,860148	4,669477	16,310142	4,566491	0,450278	0,657704
30	1985	24,911884	46,845723	36,361060	45,860001	4,660044	16,972163	4,995624	0,508570	0,729125
35	1980	21,177942	43,788193	33,970347	42,668589	4,595395	17,678961	5,121830	0,567243	0,792807
40	1975	17,260377	40,549414	31,704126	39,307410	4,494962	18,428994	4,826927	0,623502	0,848481
45	1970	13,177308	37,135014	29,739658	35,831410	4,298862	19,282176	4,087625	0,669187	0,882673
50	1965	8,999854	33,562573	27,921809	32,281799	3,758194	20,481829	3,099787	0,691224	0,861724
55	1960	4,706419	29,854376	26,210504	28,671160	2,461044	22,511757	2,062608	0,681353	0,770154
60	1955	0,000000	26,053616	24,187327	25,009281	0,000000	0,000000	1,221193	0,628041	0,000000
65	1950	0,000000	22,203734	21,524999	21,286981	0,000000	0,000000	0,707038	0,536761	0,000000
70	1945	0,000000	18,307609	18,181496	17,513536	0,000000	0,000000	0,458364	0,434261	0,000000
75	1940	0,000000	14,437018	14,437018	13,775894	0,000000	0,000000	0,330269	0,330269	0,000000
80	1935	0,000000	10,795047	10,795047	10,270078	0,000000	0,000000	0,227837	0,227837	0,000000
85	1930	0,000000	7,662461	7,662461	7,259021	0,000000	0,000000	0,134657	0,134657	0,000000
90	1925	0,000000	5,227812	5,227812	4,926074	0,000000	0,000000	0,064544	0,064544	0,000000
95	1920	0,000000	3,485016	3,485016	3,271765	0,000000	0,000000	0,024984	0,024984	0,000000
100	1915	0,000000	2,355269	2,355269	2,208323	0,000000	0,000000	0,007400	0,007400	0,000000

Table 9.6: The NPVs in the year 2015 of the New Table 2007 for female white collar workers.

Male blue and white collar workers										
Age	Birth	$\ddot{a}^{\mathrm{aa}}_{x\,\overline{z-x}\rvert}$	${}^{(12)}\ddot{a}_x^{\mathrm{Apm}}$	${}^{(12)}\ddot{a}_x^{\mathrm{i}}$	${}^{(12)}\ddot{a}_x^{\mathrm{w}}$	${}^{(12)}\ddot{a}_x^{\mathrm{ai}}$	${}^{(12)}_{z-x}\ddot{a}_x^{\mathrm{aApm}}$	${}^{(12)}\ddot{a}_x^{\mathrm{iw}}$	${}^{(12)}\ddot{a}_x^{\mathrm{pw}}$	${}^{(12)}\ddot{a}_x^{\mathrm{aw}}$
20	1995	32,771452	50,526535	36,838885	48,931428	12,694445	4,418864	5,816891	2,049424	2,725148
25	1990	29,471832	47,718114	36,190796	45,877365	12,915501	4,571461	7,086937	2,294149	3,056046
30	1985	26,027334	44,711560	34,195878	42,596704	13,075699	4,729520	8,024935	2,554622	3,401175
35	1980	22,412981	41,512772	31,412801	39,090288	13,201909	4,891982	8,743090	2,828477	3,759721
40	1975	18,658692	38,129568	28,571108	35,409135	13,275963	5,071060	9,152667	3,107898	4,114139
45	1970	14,798945	34,558932	26,132533	31,614015	13,259528	5,291581	9,013299	3,385025	4,438595
50	1965	10,878000	30,814320	24,202126	27,782007	13,089222	5,610519	8,163940	3,640350	4,693820
55	1960	7,016101	26,979047	22,506693	24,039458	12,537286	6,253896	6,833147	3,809049	4,787152
60	1955	3,836812	23,204097	20,541346	20,465450	9,626785	8,992393	5,473833	3,778181	4,404756
65	1950	0,000000	19,514899	17,935151	17,068207	0,000000	0,000000	4,438413	3,543111	0,000000
70	1945	0,000000	15,858795	14,885800	13,858505	0,000000	0,000000	3,669418	3,191360	0,000000
75	1940	0,000000	12,316629	11,766714	10,883149	0,000000	0,000000	2,973103	2,742455	0,000000
80	1935	0,000000	9,083867	8,894144	8,216871	0,000000	0,000000	2,268508	2,198062	0,000000
85	1930	0,000000	6,409351	6,468336	5,954226	0,000000	0,000000	1,575301	1,583979	0,000000
90	1925	0,000000	4,412573	4,520970	4,166362	0,000000	0,000000	0,957742	0,975709	0,000000
95	1920	0,000000	3,028721	3,028721	2,856987	0,000000	0,000000	0,471059	0,471059	0,000000
100	1915	0,000000	2,141037	2,141037	2,004140	0,000000	0,000000	0,133063	0,133063	0,000000

Table 9.7: The NPVs in the year 2015 of the New Table 2007 for male blue and white collar workers.

Female blue and white collar workers										
Age	Birth	$\ddot{a}^{\mathrm{aa}}_{y\,\overline{z-y}\rvert}$	${}^{(12)}\ddot{a}^{\mathrm{Apf}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{i}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{w}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{ai}}_{y}$	${}^{(12)}_{z-y}\ddot{a}^{\mathrm{aApf}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{iw}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{pw}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{aw}}_{y}$
20	1995	31,446614	52,483620	39,666546	51,213212	6,921646	13,851420	3,233891	0,372137	0,594525
25	1990	27,997049	49,774163	39,632069	48,346347	7,039706	14,406990	3,926807	0,423675	0,670847
30	1985	24,447915	46,891349	37,764570	45,284575	7,043188	15,022080	4,260882	0,477313	0,738458
35	1980	20,774693	43,828755	35,358081	42,020921	6,943035	15,705330	4,329315	0,532258	0,794006
40	1975	16,928534	40,580567	32,968818	38,578177	6,779464	16,450682	4,063543	0,586659	0,839798
45	1970	12,896030	37,149467	30,801202	35,030647	6,535074	17,290306	3,441912	0,634207	0,871550
50	1965	8,766602	33,549758	28,859720	31,422887	5,918273	18,485376	2,568277	0,663751	0,859989
55	1960	4,596138	29,813489	26,877509	27,755034	4,174773	20,819475	1,717142	0,662284	0,773516
60	1955	0,000000	26,002996	24,554258	24,063966	0,000000	0,000000	1,049251	0,611999	0,000000
65	1950	0,000000	22,167692	21,688471	20,368007	0,000000	0,000000	0,633676	0,518772	0,000000
70	1945	0,000000	18,288495	18,230938	16,672974	0,000000	0,000000	0,427378	0,416681	0,000000
75	1940	0,000000	14,430807	14,430807	13,045364	0,000000	0,000000	0,315746	0,315746	0,000000
80	1935	0,000000	10,804156	10,804156	9,671655	0,000000	0,000000	0,217216	0,217216	0,000000
85	1930	0,000000	7,697900	7,697900	6,812342	0,000000	0,000000	0,127736	0,127736	0,000000
90	1925	0,000000	5,292239	5,292239	4,630919	0,000000	0,000000	0,060867	0,060867	0,000000
95	1920	0,000000	3,566604	3,566604	3,099248	0,000000	0,000000	0,023472	0,023472	0,000000
100	1915	0,000000	2,439160	2,439160	2,112726	0,000000	0,000000	0,006863	0,006863	0,000000

Table 9.8: The NPVs in the year 2015 of the New Table 2007 for female blue and white collar workers.

9.3.2 NPVs of the New Table 2007 with the Alternative Trend in the Year 2015

Male white collar workers										
Age	Birth	$\ddot{a}^{\mathrm{aa}}_{x\,\overline{z-x}\rvert}$	${}^{(12)}\ddot{a}^{\mathrm{Apm}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{i}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{w}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{ai}}_{x}$	${}^{(12)}_{z-x}\ddot{a}^{\mathrm{aApm}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{iw}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{pw}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{aw}}_{x}$
20	1995	34,359081	51,158129	37,960572	50,259057	7,697048	8,992711	5,030142	1,768052	2,042474
25	1990	31,081473	48,379753	37,643679	47,309403	7,827676	9,305166	6,318420	2,002977	2,319165
30	1985	27,617732	45,406286	35,684610	44,139608	7,943340	9,612045	7,285407	2,258438	2,620239
35	1980	23,965767	42,240753	32,940490	40,746558	8,048475	9,914110	8,046112	2,531838	2,944258
40	1975	20,161302	38,894911	30,122846	37,174095	8,111418	10,231280	8,493870	2,812543	3,273396
45	1970	16,230336	35,382980	27,672044	33,468221	8,101774	10,594867	8,391099	3,080881	3,577886
50	1965	12,196197	31,743914	25,703188	29,672138	7,981671	11,067358	7,578598	3,295938	3,808048
55	1960	8,168561	28,003749	23,968027	25,876728	7,444213	11,921123	6,275539	3,427532	3,894199
60	1955	4,377415	24,218683	21,930315	22,143627	5,180884	14,398335	4,965970	3,428993	3,698793
65	1950	0,000000	20,404896	19,185330	18,501360	0,000000	0,000000	4,031329	3,297293	0,000000
70	1945	0,000000	16,563860	15,913053	14,994420	0,000000	0,000000	3,406076	3,069715	0,000000
75	1940	0,000000	12,825722	12,468791	11,694920	0,000000	0,000000	2,880350	2,722762	0,000000
80	1935	0,000000	9,411632	9,282024	8,733276	0,000000	0,000000	2,291672	2,240284	0,000000
85	1930	0,000000	6,601267	6,645692	6,270853	0,000000	0,000000	1,636790	1,643577	0,000000
90	1925	0,000000	4,524320	4,607156	4,370928	0,000000	0,000000	1,007453	1,022010	0,000000
95	1920	0,000000	3,092330	3,092330	2,987476	0,000000	0,000000	0,496247	0,496247	0,000000
100	1915	0,000000	2,178867	2,178867	2,092443	0,000000	0,000000	0,139965	0,139965	0,000000

Table 9.9: The NPVs in the year 2015 of the New Table 2007 for male white collar workers with the alternative trend.

Female white collar workers										
Age	Birth	$\ddot{a}^{\mathrm{aa}}_{y\,\overline{z-y}\rvert}$	$^{(12)}\ddot{a}^{\mathrm{Apf}}_{y}$	$^{(12)}\ddot{a}^{\mathrm{i}}_{y}$	$^{(12)}\ddot{a}^{\mathrm{w}}_{y}$	$^{(12)}\ddot{a}^{\mathrm{ai}}_{y}$	$^{(12)}_{z-y}\ddot{a}^{\mathrm{aApf}}_{y}$	$^{(12)}\ddot{a}^{\mathrm{iw}}_{y}$	$^{(12)}\ddot{a}^{\mathrm{pw}}_{y}$	$^{(12)}\ddot{a}^{\mathrm{aw}}_{y}$
20	1995	31,956371	52,773367	39,038352	52,110162	4,755427	15,946826	3,164241	0,318740	0,469204
25	1990	28,505312	50,077459	39,752187	49,311947	4,821431	16,578750	4,025559	0,372759	0,545248
30	1985	24,924753	47,208477	37,848650	46,321995	4,808830	17,253781	4,486903	0,430723	0,619203
35	1980	21,188896	44,159393	35,274730	43,132090	4,736820	17,970845	4,682519	0,491043	0,688674
40	1975	17,269161	40,922921	32,814494	39,760572	4,625593	18,726918	4,476316	0,551165	0,753036
45	1970	13,183530	37,502487	30,649361	36,260226	4,413472	19,580891	3,830946	0,603133	0,799053
50	1965	9,003281	33,913669	28,636000	32,672711	3,846269	20,777123	2,928308	0,633764	0,793280
55	1960	4,707419	30,177578	26,741681	29,012816	2,509202	22,801950	1,958371	0,634130	0,718507
60	1955	0,000000	26,336889	24,565518	25,293252	0,000000	0,000000	1,163289	0,591941	0,000000
65	1950	0,000000	22,436472	21,788334	21,508296	0,000000	0,000000	0,675960	0,511359	0,000000
70	1945	0,000000	18,482900	18,361848	17,670305	0,000000	0,000000	0,441369	0,418028	0,000000
75	1940	0,000000	14,548958	14,548958	13,868309	0,000000	0,000000	0,321757	0,321757	0,000000
80	1935	0,000000	10,842210	10,842210	10,305411	0,000000	0,000000	0,225137	0,225137	0,000000
85	1930	0,000000	7,666754	7,666754	7,261896	0,000000	0,000000	0,134485	0,134485	0,000000
90	1925	0,000000	5,227812	5,227812	4,926074	0,000000	0,000000	0,064544	0,064544	0,000000
95	1920	0,000000	3,485016	3,485016	3,271765	0,000000	0,000000	0,024984	0,024984	0,000000
100	1915	0,000000	2,355269	2,355269	2,208323	0,000000	0,000000	0,007400	0,007400	0,000000

Table 9.10: The NPVs in the year 2015 of the New Table 2007 for female white collar workers with the alternative trend.

Male blue and white collar workers										
Age	Birth	$\ddot{a}^{\mathrm{aa}}_{x\,\overline{z-x}\rvert}$	$^{(12)}\ddot{a}^{\mathrm{Apm}}_{x}$	$^{(12)}\ddot{a}^{\mathrm{i}}_{x}$	$^{(12)}\ddot{a}^{\mathrm{w}}_{x}$	$^{(12)}\ddot{a}^{\mathrm{ai}}_{x}$	$^{(12)}_{z-x}\ddot{a}^{\mathrm{aApm}}_{x}$	$^{(12)}\ddot{a}^{\mathrm{iw}}_{x}$	$^{(12)}\ddot{a}^{\mathrm{pw}}_{x}$	$^{(12)}\ddot{a}^{\mathrm{aw}}_{x}$
20	1995	32,820478	50,978363	38,914843	49,730433	13,182800	4,538164	5,049128	1,814887	2,363217
25	1990	29,514959	48,179835	38,055553	46,704709	13,412682	4,696285	6,254719	2,052922	2,688671
30	1985	26,065001	45,179181	35,854327	43,443891	13,573441	4,858853	7,205959	2,312004	3,037507
35	1980	22,444997	41,979872	32,864743	39,944311	13,691489	5,024199	7,996275	2,590797	3,409587
40	1975	18,684600	38,587499	29,801861	36,248970	13,745927	5,204193	8,516132	2,882358	3,788558
45	1970	14,818650	34,997626	27,131742	32,413854	13,696158	5,423571	8,509320	3,179178	4,148928
50	1965	10,891581	31,221880	24,976931	28,515033	13,477365	5,739526	7,797793	3,461825	4,450465
55	1960	7,023572	27,341056	23,084040	24,680692	12,857848	6,380449	6,587699	3,664990	4,598627
60	1955	3,839206	23,506220	20,960870	20,998478	9,828880	9,140830	5,319844	3,673047	4,278169
65	1950	0,000000	19,749434	18,234593	17,485995	0,000000	0,000000	4,347763	3,475356	0,000000
70	1945	0,000000	16,023354	15,085895	14,156921	0,000000	0,000000	3,623202	3,155494	0,000000
75	1940	0,000000	12,408522	11,874236	11,052140	0,000000	0,000000	2,960158	2,732825	0,000000
80	1935	0,000000	9,112870	8,926507	8,269792	0,000000	0,000000	2,273757	2,203608	0,000000
85	1930	0,000000	6,409351	6,468336	5,954226	0,000000	0,000000	1,581373	1,590076	0,000000
90	1925	0,000000	4,412573	4,520970	4,166362	0,000000	0,000000	0,958981	0,976998	0,000000
95	1920	0,000000	3,028721	3,028721	2,856987	0,000000	0,000000	0,471122	0,471122	0,000000
100	1915	0,000000	2,141037	2,141037	2,004140	0,000000	0,000000	0,133063	0,133063	0,000000

Table 9.11: The NPVs in the year 2015 of the New Table 2007 for male blue and white collar workers with the alternative trend.

Female blue and white collar workers										
Age	Birth	$\ddot{a}^{\mathrm{aa}}_{y\,\overline{z-y}\rvert}$	${}^{(12)}\ddot{a}^{\mathrm{Apf}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{i}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{w}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{ai}}_{y}$	${}^{(12)}_{z-y}\ddot{a}^{\mathrm{aApf}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{iw}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{pw}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{aw}}_{y}$
20	1995	31,460340	52,819177	41,434220	51,707252	7,122682	14,078170	2,791214	0,301219	0,481512
25	1990	28,008975	50,125778	41,167026	48,858764	7,243825	14,647163	3,462239	0,351047	0,557693
30	1985	24,458325	47,256640	39,125248	45,808516	7,243742	15,274753	3,831037	0,404509	0,628900
35	1980	20,783619	44,203636	36,553906	42,546413	7,133765	15,968670	3,962661	0,461045	0,691493
40	1975	16,935811	40,959032	33,991287	39,091584	6,955693	16,721383	3,774698	0,519025	0,746991
45	1970	12,901322	37,523236	31,642581	35,515738	6,691879	17,563720	3,233213	0,572331	0,790793
50	1965	8,769652	33,908311	29,521283	31,864362	6,044605	18,758324	2,430625	0,609759	0,793939
55	1960	4,597087	30,144438	27,375698	28,140166	4,250499	21,095025	1,632674	0,617837	0,723922
60	1955	0,000000	26,292862	24,916590	24,382782	0,000000	0,000000	1,000813	0,578096	0,000000
65	1950	0,000000	22,405081	21,946702	20,614624	0,000000	0,000000	0,606471	0,494973	0,000000
70	1945	0,000000	18,466775	18,411376	16,845910	0,000000	0,000000	0,411843	0,401439	0,000000
75	1940	0,000000	14,544495	14,544495	13,146177	0,000000	0,000000	0,307704	0,307704	0,000000
80	1935	0,000000	10,852047	10,852047	9,709780	0,000000	0,000000	0,214648	0,214648	0,000000
85	1930	0,000000	7,702254	7,702254	6,815424	0,000000	0,000000	0,127572	0,127572	0,000000
90	1925	0,000000	5,292239	5,292239	4,630919	0,000000	0,000000	0,060867	0,060867	0,000000
95	1920	0,000000	3,566604	3,566604	3,099248	0,000000	0,000000	0,023472	0,023472	0,000000
100	1915	0,000000	2,439160	2,439160	2,112726	0,000000	0,000000	0,006863	0,006863	0,000000

Table 9.12: The NPVs in the year 2015 of the New Table 2007 for female blue and white collar workers with the alternative trend.

9.3.3 Effect of the Alternative Trend in NPVs of the New Table 2007 in the Year 2015

Male white collar workers										
Age	Birth	$\ddot{a}^{\mathrm{aa}}_{x\,\overline{z-x}\rvert}$	${}^{(12)}\ddot{a}^{\mathrm{Apm}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{i}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{w}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{ai}}_{x}$	${}^{(12)}_{z-x}\ddot{a}^{\mathrm{aApm}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{iw}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{pw}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{aw}}_{x}$
20	1995	0,19%	0,85%	5,92%	1,42%	3,44%	2,41%	-13,05%	-11,39%	-12,54%
25	1990	0,19%	0,91%	5,28%	1,56%	3,43%	2,42%	-11,72%	-10,47%	-11,43%
30	1985	0,19%	0,98%	4,89%	1,71%	3,38%	2,42%	-10,25%	-9,46%	-10,24%
35	1980	0,19%	1,05%	4,57%	1,87%	3,29%	2,38%	-8,65%	-8,36%	-8,98%
40	1975	0,18%	1,10%	4,16%	2,02%	3,12%	2,29%	-7,13%	-7,21%	-7,68%
45	1970	0,17%	1,15%	3,61%	2,14%	2,89%	2,16%	-5,80%	-6,05%	-6,39%
50	1965	0,14%	1,17%	2,94%	2,23%	2,57%	1,96%	-4,70%	-4,92%	-5,15%
55	1960	0,10%	1,17%	2,28%	2,25%	2,19%	1,71%	-3,80%	-3,85%	-4,00%
60	1955	0,05%	1,13%	1,75%	2,21%	1,78%	1,40%	-3,00%	-2,89%	-2,96%
65	1950	0,00%	1,05%	1,40%	2,09%	0,00%	0,00%	-2,19%	-2,03%	0,00%
70	1945	0,00%	0,92%	1,12%	1,86%	0,00%	0,00%	-1,37%	-1,24%	0,00%
75	1940	0,00%	0,67%	0,78%	1,36%	0,00%	0,00%	-0,53%	-0,46%	0,00%
80	1935	0,00%	0,29%	0,32%	0,57%	0,00%	0,00%	0,15%	0,16%	0,00%
85	1930	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,33%	0,33%	0,00%
90	1925	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,11%	0,11%	0,00%
95	1920	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,01%	0,00%
100	1915	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

Table 9.13: The Effekt of the alternative trend in the NPVs in the year 2015 of the New Table 2007 for male white collar workers.

Female white collar workers										
Age	Birth	$\ddot{a}^{\mathrm{aa}}_{y\overline{z-y}\vert}$	${}^{(12)}\ddot{a}^{\mathrm{Apf}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{i}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{w}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{ai}}_{y}$	${}^{(12)}_{z-y}\ddot{a}^{\mathrm{aApf}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{iw}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{pw}}_{y}$	${}^{(12)}\ddot{a}^{\mathrm{aw}}_{y}$
20	1995	0,05%	0,64%	5,24%	0,84%	3,26%	1,62%	-13,40%	-19,15%	-19,21%
25	1990	0,05%	0,70%	4,41%	0,92%	3,25%	1,65%	-11,85%	-17,22%	-17,10%
30	1985	0,05%	0,77%	4,09%	1,01%	3,19%	1,66%	-10,18%	-15,31%	-15,08%
35	1980	0,05%	0,85%	3,84%	1,09%	3,08%	1,65%	-8,58%	-13,43%	-13,13%
40	1975	0,05%	0,92%	3,50%	1,15%	2,91%	1,62%	-7,26%	-11,60%	-11,25%
45	1970	0,05%	0,99%	3,06%	1,20%	2,67%	1,55%	-6,28%	-9,87%	-9,47%
50	1965	0,04%	1,05%	2,56%	1,21%	2,34%	1,44%	-5,53%	-8,31%	-7,94%
55	1960	0,02%	1,08%	2,03%	1,19%	1,96%	1,29%	-5,05%	-6,93%	-6,71%
60	1955	0,00%	1,09%	1,56%	1,14%	0,00%	0,00%	-4,74%	-5,75%	0,00%
65	1950	0,00%	1,05%	1,22%	1,04%	0,00%	0,00%	-4,40%	-4,73%	0,00%
70	1945	0,00%	0,96%	0,99%	0,90%	0,00%	0,00%	-3,71%	-3,74%	0,00%
75	1940	0,00%	0,78%	0,78%	0,67%	0,00%	0,00%	-2,58%	-2,58%	0,00%
80	1935	0,00%	0,44%	0,44%	0,34%	0,00%	0,00%	-1,19%	-1,19%	0,00%
85	1930	0,00%	0,06%	0,06%	0,04%	0,00%	0,00%	-0,13%	-0,13%	0,00%
90	1925	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
95	1920	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
100	1915	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

Table 9.14: The Effekt of the alternative trend in the NPVs in the year 2015 of the New Table 2007 for female white collar workers.

Male blue and white collar workers										
Age	Birth	$\ddot{a}^{\mathrm{aa}}_{x\overline{z-x}\vert}$	${}^{(12)}\ddot{a}^{\mathrm{Apm}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{i}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{w}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{ai}}_{x}$	${}^{(12)}_{z-x}\ddot{a}^{\mathrm{aApm}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{iw}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{pw}}_{x}$	${}^{(12)}\ddot{a}^{\mathrm{aw}}_{x}$
20	1995	0,15%	0,89%	5,64%	1,63%	3,85%	2,70%	-13,20%	-11,44%	-13,28%
25	1990	0,15%	0,97%	5,15%	1,80%	3,85%	2,73%	-11,74%	-10,51%	-12,02%
30	1985	0,14%	1,05%	4,85%	1,99%	3,81%	2,73%	-10,21%	-9,50%	-10,69%
35	1980	0,14%	1,13%	4,62%	2,18%	3,71%	2,70%	-8,54%	-8,40%	-9,31%
40	1975	0,14%	1,20%	4,31%	2,37%	3,54%	2,63%	-6,95%	-7,26%	-7,91%
45	1970	0,13%	1,27%	3,82%	2,53%	3,29%	2,49%	-5,59%	-6,08%	-6,53%
50	1965	0,12%	1,32%	3,20%	2,64%	2,97%	2,30%	-4,48%	-4,90%	-5,18%
55	1960	0,11%	1,34%	2,57%	2,67%	2,56%	2,02%	-3,59%	-3,78%	-3,94%
60	1955	0,06%	1,30%	2,04%	2,60%	2,10%	1,65%	-2,81%	-2,78%	-2,87%
65	1950	0,00%	1,20%	1,67%	2,45%	0,00%	0,00%	-2,04%	-1,91%	0,00%
70	1945	0,00%	1,04%	1,34%	2,15%	0,00%	0,00%	-1,26%	-1,12%	0,00%
75	1940	0,00%	0,75%	0,91%	1,55%	0,00%	0,00%	-0,44%	-0,35%	0,00%
80	1935	0,00%	0,32%	0,36%	0,64%	0,00%	0,00%	0,23%	0,25%	0,00%
85	1930	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,39%	0,38%	0,00%
90	1925	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,13%	0,13%	0,00%
95	1920	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,01%	0,00%
100	1915	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

Table 9.15: The Effekt of the alternative trend in the NPVs in the year 2015 of the New Table 2007 for male blue and white collar workers.

Female blue and white collar workers										
Age	Birth	$\ddot{a}^{aa}_{y\overline{z-y}\vert}$	$^{(12)}\ddot{a}^{Apf}_{y}$	$^{(12)}\ddot{a}^{i}_{y}$	$^{(12)}\ddot{a}^{w}_{y}$	$^{(12)}\ddot{a}^{ai}_{y}$	$^{(12)}_{z-y}\ddot{a}^{aApf}_{y}$	$^{(12)}\ddot{a}^{iw}_{y}$	$^{(12)}\ddot{a}^{pw}_{y}$	$^{(12)}\ddot{a}^{aw}_{y}$
20	1995	0,04%	0,64%	4,46%	0,96%	2,90%	1,64%	-13,69%	-19,06%	-19,01%
25	1990	0,04%	0,71%	3,87%	1,06%	2,90%	1,67%	-11,83%	-17,14%	-16,87%
30	1985	0,04%	0,78%	3,60%	1,16%	2,85%	1,68%	-10,09%	-15,25%	-14,84%
35	1980	0,04%	0,86%	3,38%	1,25%	2,75%	1,68%	-8,47%	-13,38%	-12,91%
40	1975	0,04%	0,93%	3,10%	1,33%	2,60%	1,65%	-7,11%	-11,53%	-11,05%
45	1970	0,04%	1,01%	2,73%	1,38%	2,40%	1,58%	-6,06%	-9,76%	-9,27%
50	1965	0,03%	1,07%	2,29%	1,40%	2,13%	1,48%	-5,36%	-8,13%	-7,68%
55	1960	0,02%	1,11%	1,85%	1,39%	1,81%	1,32%	-4,92%	-6,71%	-6,41%
60	1955	0,00%	1,11%	1,48%	1,32%	0,00%	0,00%	-4,62%	-5,54%	0,00%
65	1950	0,00%	1,07%	1,19%	1,21%	0,00%	0,00%	-4,29%	-4,59%	0,00%
70	1945	0,00%	0,97%	0,99%	1,04%	0,00%	0,00%	-3,64%	-3,66%	0,00%
75	1940	0,00%	0,79%	0,79%	0,77%	0,00%	0,00%	-2,55%	-2,55%	0,00%
80	1935	0,00%	0,44%	0,44%	0,39%	0,00%	0,00%	-1,18%	-1,18%	0,00%
85	1930	0,00%	0,06%	0,06%	0,05%	0,00%	0,00%	-0,13%	-0,13%	0,00%
90	1925	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
95	1920	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
100	1915	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

Table 9.16: The Effekt of the alternative trend in the NPVs in the year 2015 of the New Table 2007 for female blue and white collar workers.

9.4 Variance of the NPVs of the New Annuity Tables

Using Equations (8.1) and (8.2) the variances of all NPVs for a male beneficiary, born in 1960, were calculated.

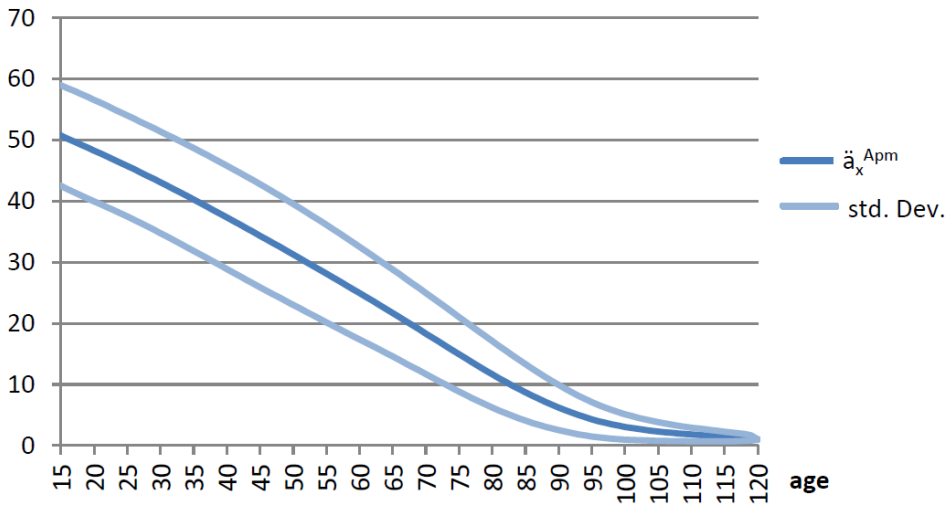

Figure 9.1: NPV and standard deviation of $\ddot{a}_x^{Apm}$.

Figure 9.10 shows how the standard deviation increases by using the *collective method* in comparison to the *individual method*.

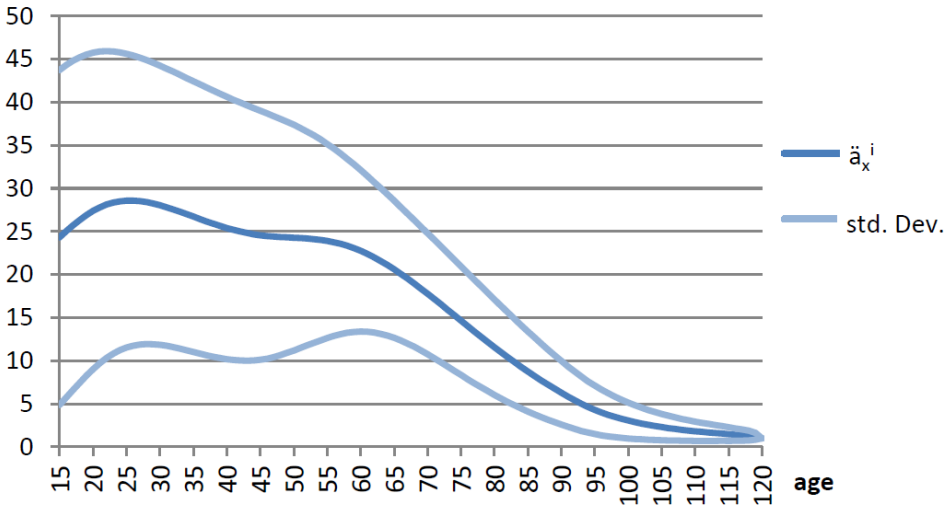

Figure 9.2: NPV and standard deviation of $\ddot{a}_x^i$.

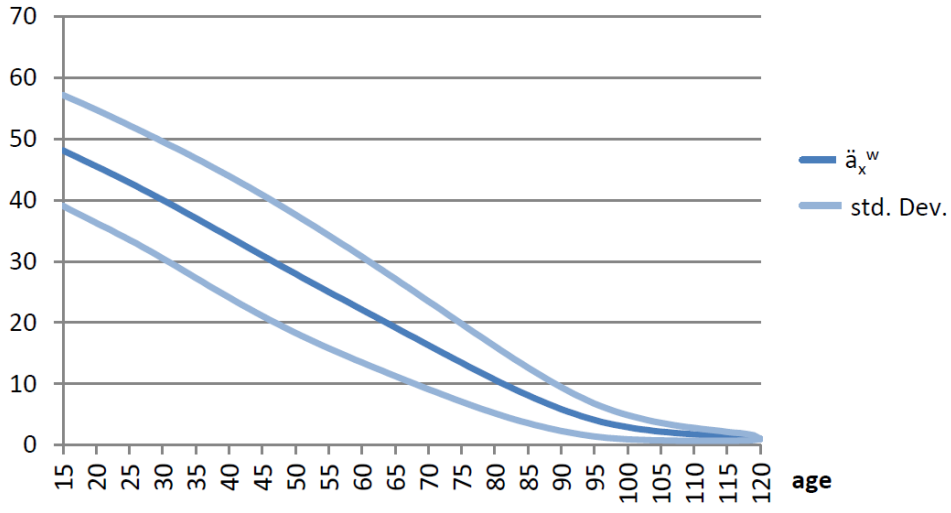

Figure 9.3: NPV and standard deviation of $\ddot{a}_x^w$.

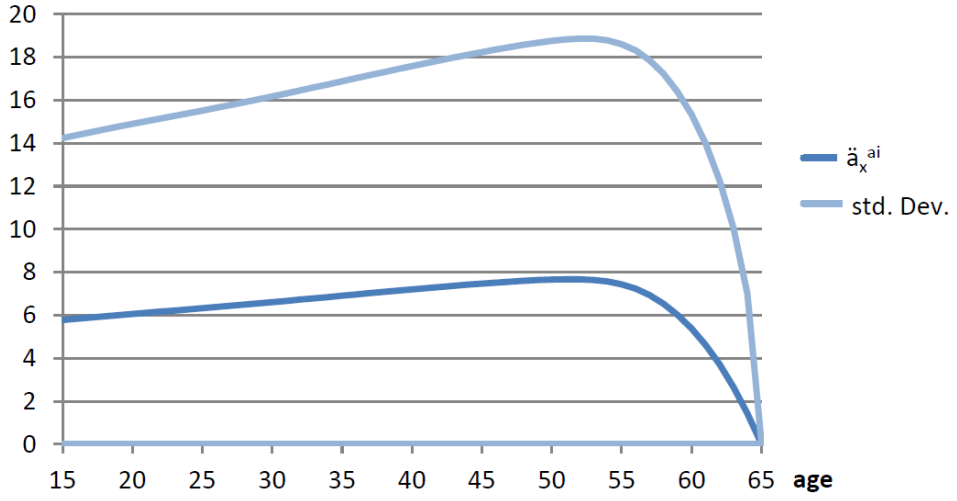

Figure 9.4: NPV and standard deviation of $\ddot{a}_x^{ai}$.

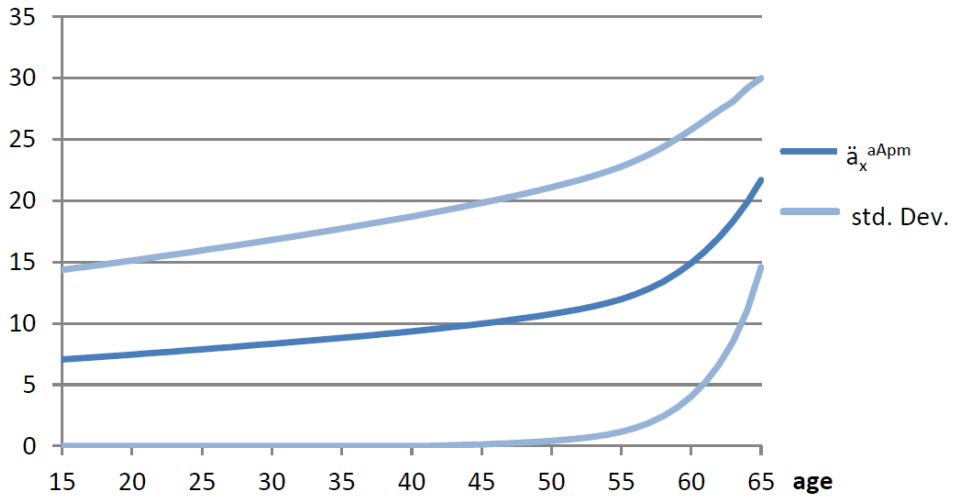

Figure 9.5: NPV and standard deviation of $\ddot{a}_x^{aApm}$.

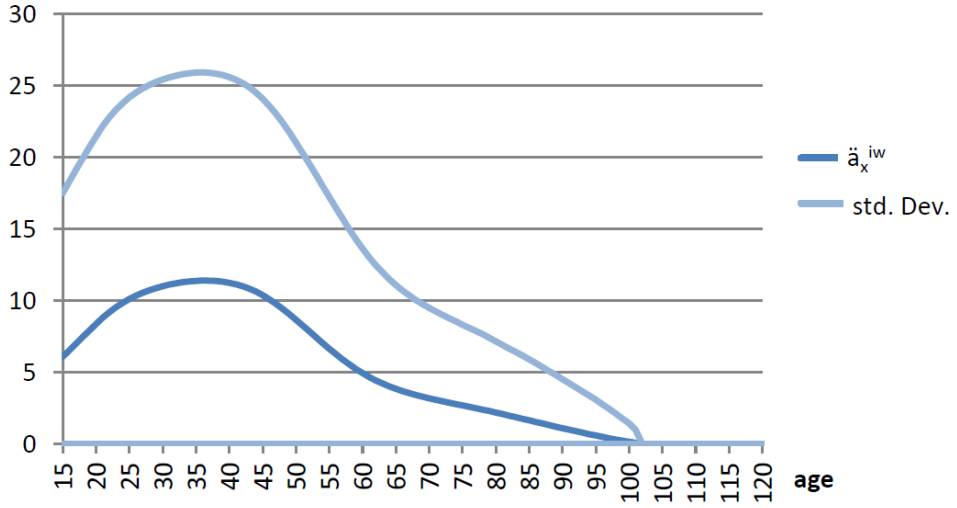

Figure 9.6: NPV and standard deviation of $\ddot{a}_x^{iw}$.

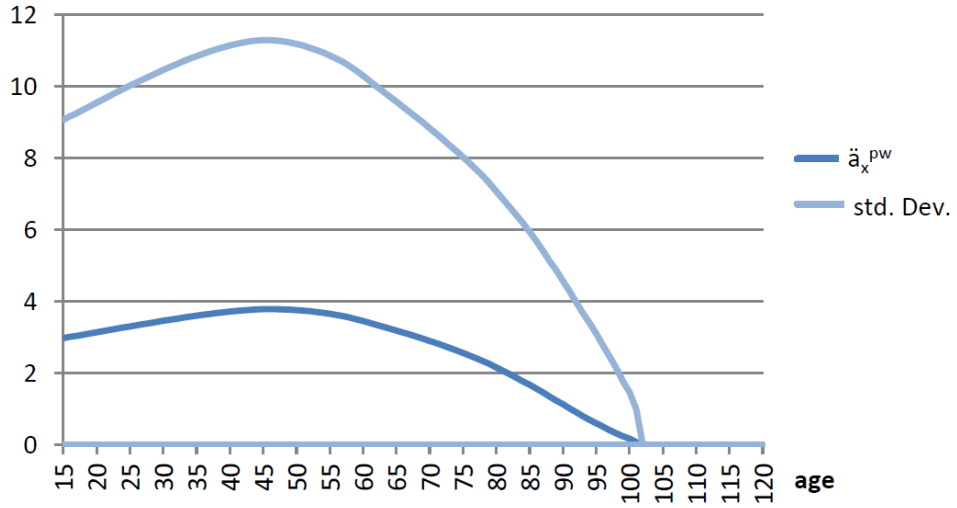

Figure 9.7: NPV and standard deviation of $\ddot{a}_x^{pw}$.

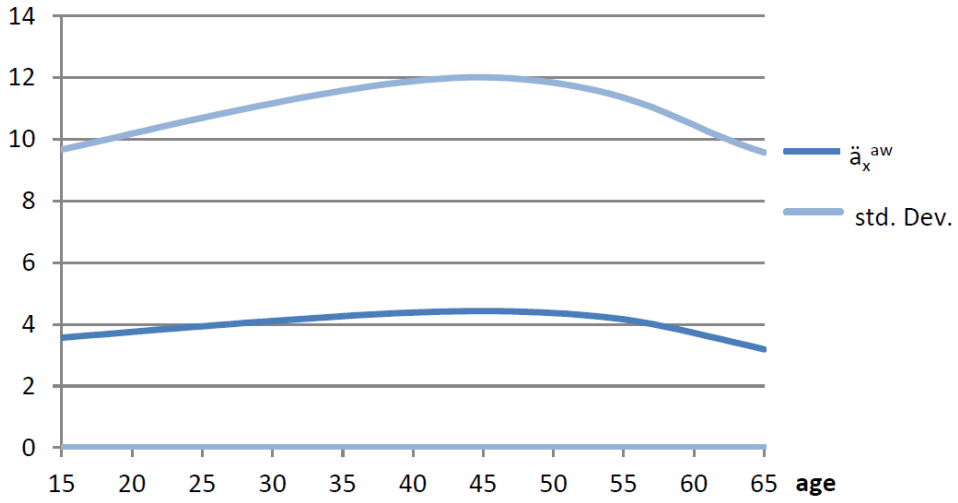

Figure 9.8: NPV and standard deviation of $\ddot{a}_x^{aw}$.

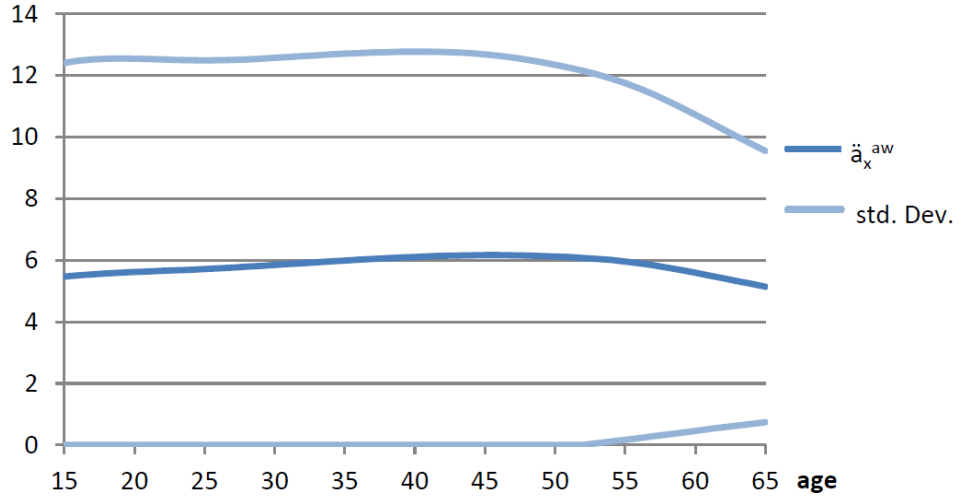

Figure 9.9: NPV and standard deviation of $\ddot{a}_x^{aw}$ *with known spouse.*

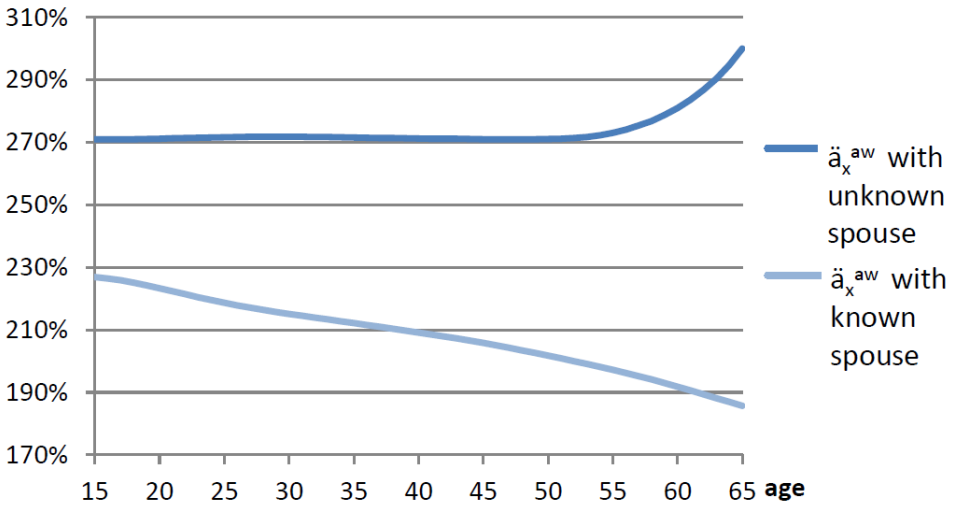

Figure 9.10: Ratio of NPV and standard deviation of $\ddot{a}_x^{aw}$ with unknown and known spouse, i.e. using collective and individual method.

10 Comparison to the AVÖ 2008P

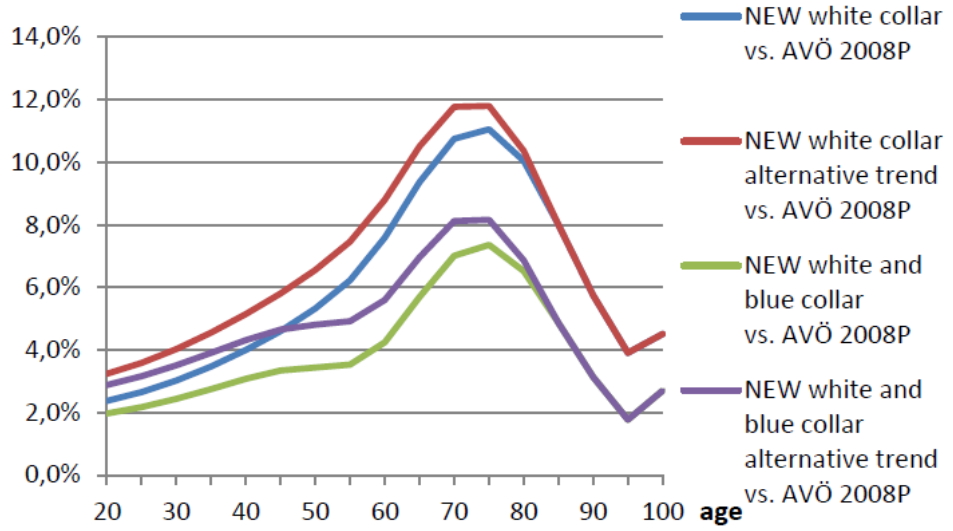

Figure 10.1: Differences in the NPV of male old age pensioners.

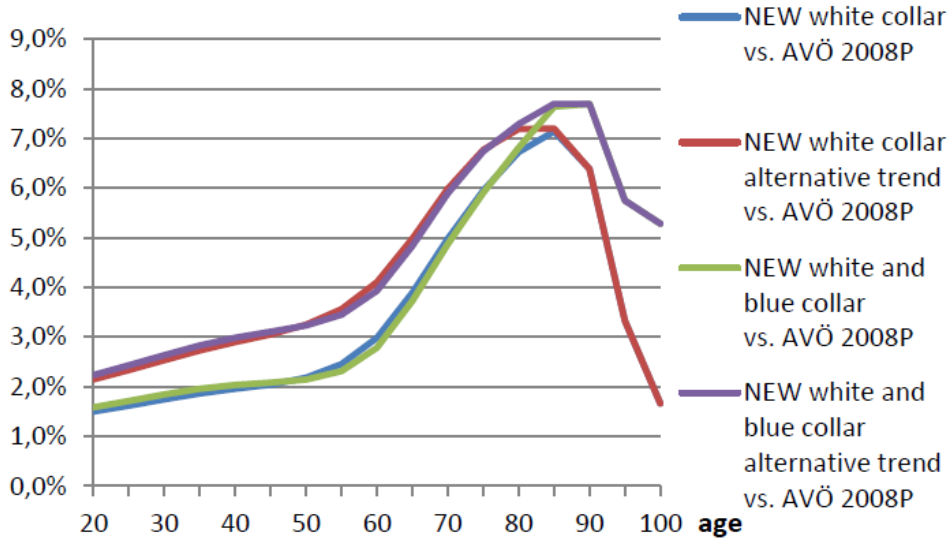

Figure 10.2: Differences in the NPV of female old age pensioners.

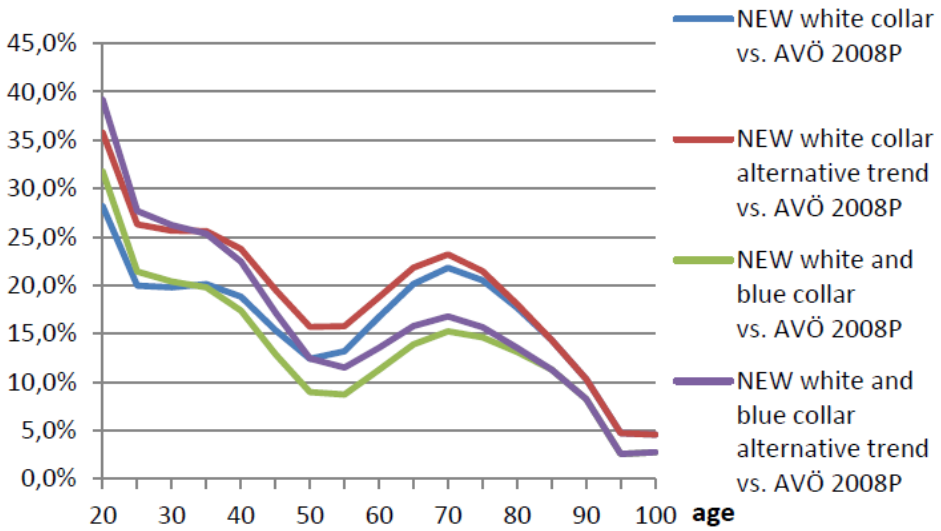

Figure 10.3: Differences in the NPV of male disabled.

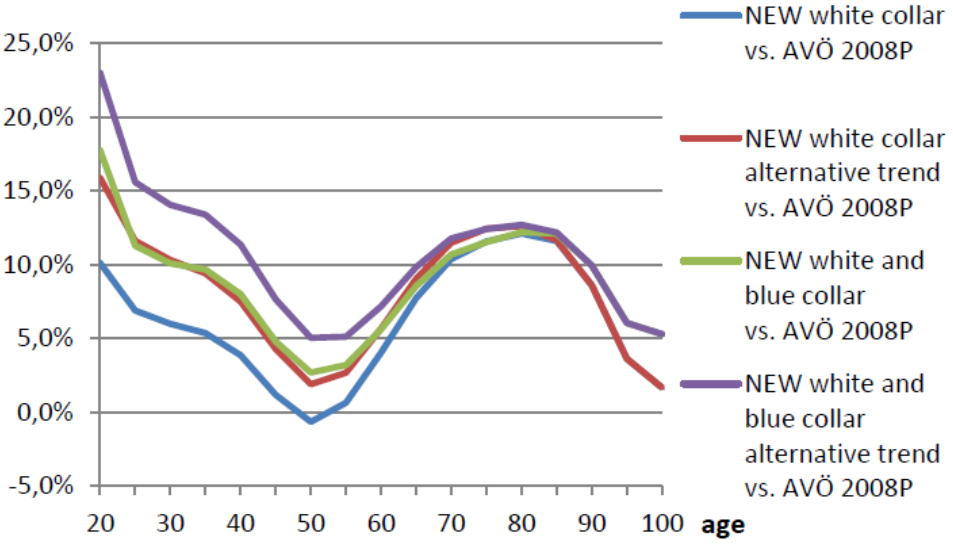

Figure 10.4: Differences in the NPV of female disabled.

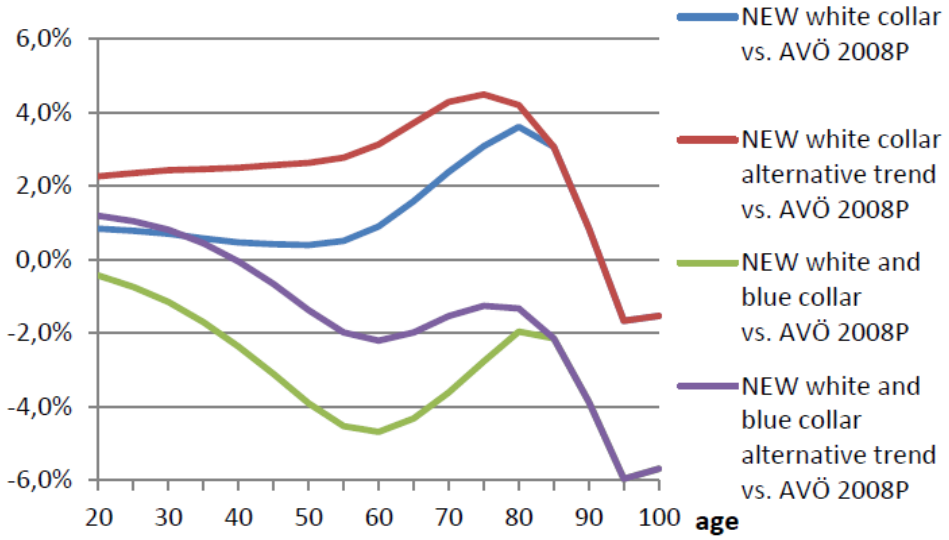

Figure 10.5: Differences in the NPV of widowers.

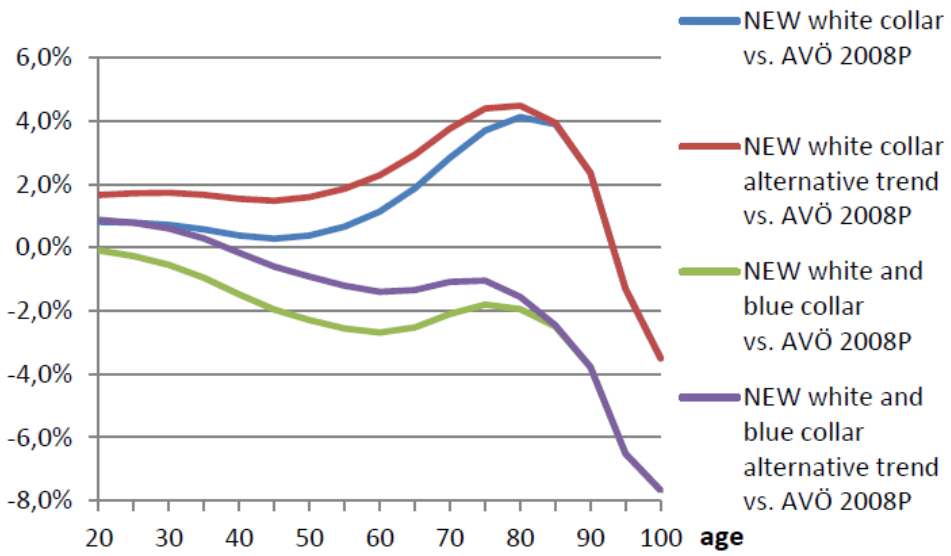

Figure 10.6: Differences in the NPV of widows.

A Appendix

A.1 Statistics Austria

A.1.1 Responsibilities and Principles of the Institution

At the beginning of 2000, the former Austrian Central Statistical Office was separated from Government Services by a new Federal Statistics Act (BStatG). It is now an independent and non-profit-making federal institution under public law and is called Statistics Austria. It is responsible for performing scientific services in the area of federal statistics. (§ 22 BStatG).

The Federal Statistics Act of 2000 defines federal statistics as a (non-personal) information system of the government providing data on the economic, demographic, social, ecological and cultural situation in Austria. This information helps administrative bodies in planning and political decision-making procedures and in controlling the measures they have taken. Moreover, data are made available to the scientific and economic community and to the general public. Federal statistics comprises the compilation of statistics of all kinds as well as analyses, prognoses and statistical models which reach beyond the interests of an individual Austrian province (§§ 1 and 2 BStatG). The statistics are decreed by international legal acts of the European Community, by federal laws and by regulations.

Access to the compiled data should be possible not only for administrative purposes but also to the scientific and economic community, to the public and to individual citizens. To enable this access, Statistics Austria provides information and expert services, in particular statistical analyses,

as well as statistical data free of charge or at a reasonable cost (§ 29 BStatG).

A.1.2 Publications: Statistical Yearbook of Austria

The "Statistical Yearbook of Austria" is a comprehensive reference book on official statistics and gives concise national information on the demographic, social and economical structure and development of Austria. The attached international part enables readers to make comparisons with European and non-European countries. All data have been clearly set out in the form of tables, many of which also include graphics presenting a quick and vivid overview of distributions and tendencies.

Chapter 3 "Health" contains information about cause of death statistics, in particular "Deaths by cause of death, age and sex".

B Curriculum Vitae

Personal Details

Name	Alexander Schubert
Date of Birth	19.3.1975
Place of Birth	St. Pölten (Lower Austria)
Nationality	Austria

Education

1981–1985	Primary school in Markersdorf
1985–1993	Secondary school in St. Pölten
June 1993	Graduation diploma
1994–2015	Study of technical mathematics at the Vienna University of Technology

Working Experience

Summer 2000	Internship at the Social Insurance Institution of Farmers
Since 2001	Actuarial employee at Valida Consulting GesmbH (former PlanPension GesmbH)

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