INSTITUT FÜR HÖHERE STUDIEN
INSTITUTE FOR ADVANCED STUDIES
Vienna

MSc Economics

Modeling favoritism with an imperfect information repeated game

A Master's Thesis submitted for the degree of
"Master of Science"

supervised by

Dr. Klaus Ritzberger

György Kozics

1325891

Vienna, 2015

MSc Economics

Affidavit

I, György Kozics

hereby declare

that I am the sole author of the present Master's Thesis,

Modeling favoritism with an imperfect information repeated game

 23 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

Vienna, June 16, 2015

Signature

Contents

Abstract

We model favoritism with an imperfect monitoring infinitely repeated game. We find that favoritism is driven by players trading favors over time. We show that favoritism is harder to maintain in larger groups, group members increase their payoff at the expense of others, and favoritism is easier to sustain in imperfect monitoring case compared to perfect monitoring. We also prove that for our game the Folk theorem does not hold if the number of players is 4, and we impose a restriction to the parameters.

1 Introduction

Favoritism, which is, according to Bramoulle and Goyal (2009) "the action of offering jobs, contracts, and resources to members of one's own social group" has a widespread impact on the economy. Although there is an extensive research on favoritism on the data side, only a few papers tried to model it with a game theoretic approach. The aim of our paper is to create a model, which can capture the mechanism how favoritism work.

Bramoulle and Goyal (2009) created a simple model, where the mechanism of favoritism is the following: player 1 do a favor for player 2 because he expects that player 2 will return the favor. In their model, people are divided in two groups, and every group can decide whether to practice favoritism or market behavior.

Our model consists of an infinitely repeated game with imperfect monitoring, where every individual can choose his own strategy. We think imperfect monitoring is important because it can capture the real world phenomenon better, and also from a game theoretical approach, the Folk theorem will not necessarily hold, so not every payoff vector could be supported by a Nash equilibrium strategy.

Our main findings are the following: favoritism is harder to maintain in larger groups, it increases the group's payoffs who practice favoritism, while decrease the payoffs of those who does not, moreover, if more than one group practice favoritism it can happen that everyone is worse off than if they follow the socially optimal strategy. Finally, we have proven that the Folk theorem does not hold, if the number of players is four, and if we have a restriction on the parameters.

To get more insight in game theory and repeated games, we used the book of Ritzberger (2002), the book of Mailath and Samuelson (2006) which is an introductory book of repeated games and attended Johannes Horner's course which was an advanced course on repeated games.

The rest of our paper is organized as follows: section 2 gives a short review of the literature of the topic, section 3 specifies our model, in section 4 we present our results, and section 5 concludes our paper.

2 Literature review

The literature on favoritism is quite extensive, however the vast majority of the papers are based on data, and there are only a few theoretical ones. The idea of our paper comes from the model suggested by Bramoulle and Goyal (2009). Their main idea is the following: the key mechanism that drives favoritism is the exchange of favors. This means that A gives a favor to B because he expects that later B will return the favor either by giving a favor to A or by giving a favor to C, who will give a favor to A.

Their model is the following: they take an infinitely repeated game with $n > 3$ players, discount rate δ, and each player is assigned to either group G or group H. At each time t nature chooses randomly with uniform distribution someone the principal and someone else the expert. Then the principal chooses someone, which can correspond to hiring an employee. If he chooses the expert, they get the output of 1, and the principal and the expert share it equally, so they both get $1/2$, everyone else gets 0. If the principal chooses someone else, then the output is $l < 1$, the principal, and the employee gets $l/2$, everyone else gets 0. The information is perfect, after nature has chosen the principal and the expert, everyone will know who are they and after the principal chosen someone, everyone will know who was chosen. If the principal chooses the expert, it is called market behavior, if he chooses someone else from his group, it is called favoritism.

Then they derive the conditions under which practicing favoritism for one group (so everyone always practices favoritism) is beneficial for the group. The main results are the following. First, favoritism is a dominant strategy if and only if $l/ > 1/2$. Second, the threshold delta for which favoritism is a subgame perfect Nash equilibrium is increasing in the group size, which means that favoritism is harder to sustain in larger groups. Third, in the group which practice favoritism the expected payoffs are larger then in the other one, and favoritism result in welfare loss.

Nitzan and Kriesler (2010) studies competitive rent-seeking clubs (CRSC) with a game-theoretic approach. In a CRSC there are heterogeneous members who are ranked according to their quality, and at time t a selector randomly sample a fixed number of members and choose the best, who gets a reward. In their paper, they examine whether the existing members would admit a new applicant and how does it depend on the new applicant's rank, and the size of the existing group. Their result is the following: even if the new applicant has a low rank, if the club is already large, he will not get applied as he decrease

the chance of the members to be sampled by the selector. Although they use a completely different model then Bramoulle and Goyal (2009), the result, that there is an upper bound on the group sizes is similar.

Imperfect private monitoring in repeated games is still an ongoing research, and the necessary and sufficient conditions for a Folk theorem in those kind of models is not yet known. To find a proof, or a counterexample for the Folk theorem in my model, we used the paper of Fudenberg and Maskin (1986) which proves it for complete information, and Fudenberg et al. (1994) and Fudenberg and Levine (1991) which proves the folk theorem in imperfect monitoring case with additional restrictions. However the assumption of these papers are not true for our model, and we have not yet found any papers that proves or disproves the Folk theorem for our case.

3 The model

Our model is based on the model of Bramoulle and Goyal (2009), with some essential changes. Instead of having two groups we consider individuals, who can use any strategy that depends on the history of the game. In our game we have imperfect monitoring: at every period t, the identity of the principal is common knowledge, the identity of the expert is only known to the expert and the principal, and after the principal has chosen someone, everyone knows whether he was chosen, but now who was chosen.

So the exact setup of our model is the following: this is an n-player infinitely repeated game with discounting, where the discount factor is δ. Every period nature randomly sets someone "principal" and sets someone else "expert" with the same probability, so the probability that player i is the expert and player j is the principal is $\frac{1}{n \cdot (n-1)}$.

The identity of the principal is a common knowledge, but the expert is only known to the principal and the expert. When nature has set the principal, he has to choose someone (for a job, or a project). If he chooses the expert, they together get a payoff of 1 and then share it among themselves, so the principal gets α, the expert gets $1 - \alpha$. If the principal chooses someone else, they get $L < 1$ and they share it again, so the principal gets βL, the chosen one gets $(1 - \beta)L$. We assume that α, β, and L is positive and smaller than 1. We also assume, that $\alpha > \beta L$, so that the principal has the incentive to choose the expert.

A trivial Nash equilibrium of this game is the following: every period the actual principal chooses the expert. This equilibrium also has the property that it is socially optimal in the sense that the overall payoff of every stage game is maximal. This means, that we can compare the payoffs of any other equilibrium to this one to see the effectiveness of the equilibrium. In the next chapter, we define a strategy profile which corresponds to favoritism, and then we calculate, for which δ it is a Nash-equilibrium.

3.1 A strategy profile of favoritism

In our model, people do a favor because they expect to get back the favor from someone. So for example if there are m people who are "friends", they could achieve favoritism with the following strategy: they are ordered from 1 to m, and if someone is the principal, he chooses the next one in the order, and the last one chooses the first one. This means, that player i does a favor to player $i + 1$

because he expects to get the favor back from player $i - 1$. In payoff terms, when player i chooses player $i + 1$ instead of the expert, he gets βL instead of α, but when player i is chosen by player $i - 1$ he gets $(1 - \beta)L$ instead of 0. If he expects that everyone acts according to his given strategy, if $(1 - \beta)L > \alpha - \beta L$, this can be profitable for him.

So we take the actual following strategy profile S: m out of the n players set the following rule: if player i is the principal $(i < m)$, he chooses player $i + 1$, irrespective of who the expert, and if player m player is the principal, he chooses player 1. All other $(n - m)$ players always choose the expert.

We have to specify what happens if someone deviates, and we also have to take into account, that we do not have perfect monitoring. We want to have the following: first, if one of the group members deviate, the group would have to fall apart, and second, if someone from outside deviates, nothing would change. For the first one: if player i sees that player $i - 1$ did not choose him, he would not choose player $i + 1$, because it is a loss for him, and he would not expect to get compensated by player $i - 1$ anymore. For the second one: as an outsider does not get compensated by anyone, he has no incentive to choose someone who is not the expert, so punishment for deviating is not necessary.

So the continuation strategy if player i does not choose player $i + 1$ is the following: after this deviation, player $i + 1$ knows, that there were a deviation, so he will also deviate, and from now on, he always chooses the expert, then, after player $i + 2$ saw that player $i + 1$ deviated, he will also deviate, and so on. This means that player i will be only punished (which means, that player $i - 1$ won't choose him), after the information of his deviation got to player $i - 1$, which requires a lot of time, if m or n is large.

Now we check whether this S strategy profile is a Nash-equilibrium. First, we calculate the expected payoffs for player one with the given strategy S if, at time 1 he is the principal and player 2 is not the expert (if he is not principal, he cannot deviate, if player 2 is expert, he has no incentive to deviate). Then, we calculate the payoffs for player one, if he deviates (so from now on he always chooses the expert, so the group falls apart, and in the end, everyone will choose the expert always). We call the strategy where player 1 deviates, and the others act according to their continuation strategy, and the group slowly falls apart S_d.

First we calculate, the expected value of the stage game for the first player, if everyone follows the S strategy profile:

$$V^* = \frac{1}{n}\left(\frac{1}{n-1}\alpha + \frac{n-2}{n-1}\beta L\right) + \frac{1}{n}\left(\frac{1}{n-1}(1-\alpha) + \frac{n-2}{n-1}(1-\beta)L\right) +$$
$$+ \frac{n-m}{n}\frac{1}{n-1}(1-\alpha)$$

Here, the first term is the value of being the principal (he has $1/n$ chance to be the principal, he chooses the second one, who can be the expert, or a non-expert), the second term is what he gets, if player m is the principal, and chooses him according to the rules, and the last term is the value of being an expert, if the outsiders are the principals.

We can simplify the previous expression to get a more compact form of V^*:

$$V^* = \frac{1}{n}\left(\frac{1}{n-1} + \frac{n-2}{n-1}L + \frac{n-m}{n-1}(1-\alpha)\right) \tag{3.1}$$

Now we introduce the notation usually used in repeated games. Assume that in the first period, someone gets A, and in the other periods, he gets B and the discount value is δ. Then for the whole game, he gets the following:

$$A + \delta B + \delta^2 B + ... = A + \frac{\delta}{1-\delta}B$$

It is more convenient to multiply the equation with $1 - \delta$, and say, that the value of the game for that player is equal to $(1-\delta)A + \delta B$. With this notation we calculate the expected value of player 1 if he does not deviate, when at time 1, he is the principal, and player 2 is not the expert:

$$W_S = (1-\delta)\beta L + \delta V^*, \tag{3.2}$$

as in the first period he chooses player 2, who is a non-expert, and in every other period, he gets V^*.

We now calculate what happens if player 1 deviates, and chooses the expert. For this, we introduce payoffs V_i, $1 < i < m$, where V_i is the expected payoff of player 1 for the stage game, if player i already knows, that there were deviation, but player $i + 1$ does not:

$$V_i = \frac{1}{n}\alpha + \frac{1}{n}\left(\frac{1}{n-1}(1-\alpha) + \frac{n-2}{n-1}(1-\beta)L\right) + \frac{n-m+i-1}{n}\frac{1}{n-1}(1-\alpha)$$

$$(3.3)$$

The only difference between V_i and V^* is in first and the last term: in the first term, he always chooses the expert, and in the last term, the people, who know about the deviation will always choose the expert instead of the next person in the group. As i increases, more people will choose always the expert, so that when player 1 is the expert he will be chosen with a higher chance. If $i = m$ then everyone knows about the deviation, so everyone always choose the expert, that means that

$$V_m = \frac{1}{n}\alpha + \frac{1}{n}(1-\alpha) = \frac{1}{n}.$$

So V_i is increasing in i from $1 < i < m-1$, and $V_m < V_1$, so after a deviation, player 1 gets more and more until the information of the deviation gets to player m when his payoff will drop.

With these payoffs, we can easily write down the expected payoff of player 1 if he chooses to deviate (we denote it with W_{dev}), and choose the expert instead of player 2. We need another variable: we use W_i for the expected payoff for the infinite game, in the state where player i already knows that there were deviation, but player $i+1$ does not. (So V's denote the stage game payoffs while W's denote the expected payoffs for the infinite game.)

$$W_{dev} = (1-\delta)\alpha + \delta W_2 \qquad (3.4)$$

here, the first term denotes the first period, where player 1 chooses the expert, the second term is for the rest periods, as here, player 2 already knows that player 1 deviated, but others do not. Now, W_2:

$$W_2 = (1-\delta)V_2 + \delta\left(\frac{n-1}{n}W_2 + \frac{1}{n}W_3\right)$$

The first term is the payoff in the actual period, which is V_2, and in the next period, we either stay in the same state, or the principal was player 2, and then player 3 will also knows about the deviation, so we go to the next state, W_3. The

rest of equations look the same:

$$W_i = (1 - \delta)V_i + \delta \left(\frac{n-1}{n} W_i + \frac{1}{n} W_{i+1} \right) \tag{3.5}$$

if $1 < i < m$ and

$$W_m = V_m = \frac{1}{n}$$

So, the S strategy profile is a Nash-equilibrium, if $W_S > W_{dev}$, so when player 1 has the incentive to not deviate from the S strategy profile. Obviously the strategy profile is symmetric, so if player 1 does not have the incentive to deviate then player $i < m + 1$ does also not have the incentive to deviate. Players who are not among the first m people, will never have the incentive to deviate, as they already always choose the expert.

Our objective is to find the threshold delta, where deviating and not deviating give us the same payoff, so $W_S = W_{dev}$. If we are above that threshold delta, the strategy profile S is a Nash-equilibrium, if we are below it, there is a profitable deviation, so then S is not a Nash-equilibrium.

Apart from calculating δ, we show the welfare effect of favoritism and we compare our results with the perfect monitoring case. Finally, we show that the Folk theorem does not hold for our model.

4 Results

In this section we present our results. First, we state how does the threshold δ^* depend on the other variables. Second, we calculate the welfare effect of favoritism. Third, we compare our model with the perfect monitoring case. And finally, we state that the Folk theorem does not hold for our model and, we prove it for $n = 4$ and a restriction on the parameters.

4.1 The threshold δ

First we calculate the lowest discount factor (δ^*) for which our S strategy profile is a Nash equilibrium. The equation that defines δ^* is the following:

$$W_S(\delta^*) = W_{dev}(\delta^*), \tag{4.1}$$

so for players in the group get the same expected payoffs when deviating, or playing their favoritism strategy.

Proposition 4.1. *Suppose, that δ^* exists. For every $\delta > \delta^*$ players will act according to the S strategy profile and for every $\delta < \delta^*$ players in the group of friends has the incentive to deviate.*

Proof. Combining (4.1) with (3.2) and (3.4), we get that

$$(1 - \delta)\beta L + \delta V^* = (1 - \delta)\alpha + \delta W_2.$$

As we know that $\alpha > \beta L$, this implies that $V^* > W_2$. So if we increase δ, the LHS gets greater than the RHS, so players have no incentive to deviate, and if we decrease delta, the RHS gets greater than the LHS, so players will deviate. $\square$

Proposition 4.1 also gives us the sufficient and necessary condition for δ^* to exist, shown in the next corollary:

Corollary 4.2. *If $W_2 < V^*$, there will exist a $0 < \delta^* < 1$, if $W_2 \geq V^*$, the S strategy profile is never a Nash equilibrium.*

Proof. If $W_2 < V^*$, proposition 4.1 has a solution for δ and it is between 0 and 1, however, if $W_2 \geq V^*$, there exists no positive δ for which the equation holds. $\square$

Now we have to calculate the exact value of W_{dev}, which is given by equation (3.4) and the recursive equations (3.5). Note that the following is only true for

9

$m \geq 3$, for $m = 2$, $W_{dev} = (1-\delta)\alpha + \frac{\delta}{n}$, as from the second period everyone among the friends will know about the deviation. First we rewrite V_i using equation (3.1):

$$V_i = V^* + \frac{1}{n}\left(\frac{n-2}{n-1}(\alpha - \beta L) + \frac{i-1}{n-1}(1-\alpha)\right) \tag{4.2}$$

Second, we have to rewrite equation (3.5), so that W_i is only present at the LHS:

$$W_i = \frac{cn}{\delta}\cdot\left((1-\delta)V_i + \frac{\delta}{n}W_{i+1}\right), \tag{4.3}$$

where

$$c = \frac{\delta}{n - \delta(n-1)}$$

is used for making calculation easier. We can solve this recursion, and we get that

$$W_{dev} = (1-\delta)\alpha + (1-\delta)\frac{n}{c}\sum_{i=2}^{m-1}c^i V_i + c^{m-2}\frac{\delta}{n}$$

Now we expand V_i according to (4.2):

$$W_{dev} = (1-\delta)\alpha + (1-\delta)\frac{n}{c}\left(V^* + \frac{1}{n}\left(\frac{n-2}{n-1}(\alpha - \beta L)\right)\right)\cdot\sum_{i=2}^{m-1}c^i +$$
$$+ \frac{1-\delta}{c}\frac{1-\alpha}{n-1}\cdot\sum_{i=2}^{m-1}c^i\cdot(i-1) + c^{m-2}\frac{\delta}{n}$$

Finally, we calculate the sums, so we get the exact value of W_{dev} which only depends on parameters:

$$W_{dev} = (1-\delta)\alpha + (1-\delta)\frac{n}{c}\left(V^* + \frac{1}{n}\left(\frac{n-2}{n-1}(\alpha - \beta L)\right)\right)\cdot\frac{c^m - c^2}{c-1} +$$
$$+ \frac{1-\delta}{c}\frac{1-\alpha}{n-1}\cdot\frac{(m-2)c^{m+1} - (m-1)c^m + c^2}{(c-1)^2} + c^{m-2}\frac{\delta}{n} \tag{4.4}$$

As c depends on δ, it is easy to see that we can not solve equation (4.1) analytically on δ for $m > 3$. We also tried using Mathematica to get some results, but it did not help. What we can do is to impose a necessary condition in the following proposition on the parameters so that the S strategy profile is a Nash-equilibrium.

Proposition 4.3. *S is not a Nash-equilibrium if $\alpha \geq L$.*

Proof. First take the case, where $m = 2$. According to corollary 4.2, it has to be the case that $W_2 < V^*$. For $m = 2$, $W_2 = 1/n$, and $V^* = 1/n + (n-2)/(n(n-1)) \cdot (L - \alpha)$. It is clear that for $m = 2$, $W_2 < V^*$ if and only if $L > \alpha$.

Second, if we increase m (and every other parameter is fixed), two (and only two) things will change. First, the Nash equilibrium payoff decreases, as there are less players outside the group who always choose the expert, and more inside, who always choose the next player, so if m increases, a group member will be chosen less often. Second, with increasing m, if a deviation happens, it will spread to the whole group slower, so there are more periods when the deviator gets more than V^* until the group falls apart and everyone gets the expected payoff of $1/n$.

Both effects make the deviation more attractive, which means that if the S strategy profile was not a Nash equilibrium in the case where $m = 2$ then it also would not be a Nash equilibrium with a higher m. So $L > \alpha$ is a necessary condition. $\qquad\square$

From the proof of proposition 4.3 we can see the effect of m on δ^*:

Corollary 4.4. *If m increases, δ^* also increases, if other variables are constant.*

Proof. As increasing m makes deviation more attractive, δ^* should increase to compensate the effect of higher m. $\qquad\square$

Lastly we prove, that m has a maximum such that S is a Nash equilibrium.

Proposition 4.5. *For $n > 2$ there exists an m^* such that for any $m > m^*$ S is not a Nash equilibrium and for any $m \leq m^*$ S is an equilibrium.*

Proof. We already proved that the partial derivative of δ^* with respect to m is positive. Now we only have to prove, that there exists an $\bar{m}$ such that S is not a Nash equilibrium.

Take the case where $m = n$. The payoff V^* is the following:

$$V^* = \frac{1}{n}\left(\frac{1}{n-1} + \frac{n-2}{n-1}L\right)$$

It is clear that $V^* > 1/n$ as $L < 1$. This means that V^* is even less, what a player would get, if the group falls apart and everyone chooses always the expert, so trivially, this can not be a Nash equilibrium.

As δ^* is increasing in m, and there exists an $\bar{m}$ such that S is not a Nash equilibrium, there has to exist a maximal m^* for which S is an equilibrium. $\qquad\square$

11

Here we should make an important remark of our findings. Suppose for a given δ, β, α and L, we calculate our m^*. This means that we found m^* number of equilibria, for group sizes of m where m goes from 1 to m^*. However, that is not all the equlibria that we have found. When we calculate whether a strategy profile is a Nash equilibrium, we look at the insiders' expected payoffs and in that payoff, the part of the payoff that comes from the outsiders, who are not part of the group, is constant and independent of deviations and punishments. This means, that for example if for $n = 5$ a group of $m = 3$ is a Nash equilibrium, then if the other 2 people forms an other group where they trade favors with each other it would also be an equilibrium. So if we have n and m^* given, then any number of groups for which the group size is smaller than m^* and everyone is a member of maximum of one group is an equilibrium.

To sum up, threshold δ^* depends on the following variables. For the equilibrium to exist, we need that $W_2 < V^*$ and $\alpha \geq L$. In our model, the partial derivative of δ^* with respect to m is positive, and also, m has a maximum value, such that S is a Nash equilibrium for any m smaller than this maximum value, and not an equilibrium for higher m's. These results are similar to the results of Bramoulle and Goyal (2009). We also found that any number of group is a Nash equilibrium provided that everyone is part of maximum 1 group and every group is not larger than the maximum value of m.

4.2 Welfare effects

As we already noted, the socially optimal strategy, where the sum of all payoffs is maximal is that everyone always chooses the expert. Moreover not only the sum of payoffs is maximal, but everyone gets the same payoff, so it is socially optimal in that sense also. In the socially optimal strategy the expected payoff for every player is $1/n$.

First take the case when favoritism exists, but there is only 1 group and fix n and m. From (3.1), the group members get

$$V^* = \frac{1}{n} \left(\frac{1}{n-1} + \frac{n-2}{n-1}L + \frac{n-m}{n-1}(1-\alpha) \right),$$

while the outsiders get

$$V_* = \frac{1}{n} \left(\alpha + \frac{n-m-1}{n-1}(1-\alpha) \right),$$

as they get α when they are the principal, and they get $1 - \alpha$ when they are the expert, and another outsider is the principal. The loss of an outsider because of favoritism is $m/(n-1) \cdot (1 - \alpha)$.

The loss of an outsider does only depend on the number of people who are in groups, but clearly it does not matter, how many groups are present in the equilibrium. So take n as given, and there are k groups and every group has m_k members such that $\sum m_k = m$. Now someone who is not a member of any group gets V_*, while someone who is a member of group i gets the following payoff:

$$V_k^* = \frac{1}{n}\left(\frac{1}{n-1} + \frac{n-2}{n-1}L + \frac{n-m}{n-1}(1-\alpha)\right),$$

Where the first two parts correspond to giving and receiving a favor, while the third part corresponds to the case, when someone is the principal who is not part of any groups. What we got is that the payoffs does not matter on the number of groups present only on the number of players who are part of one group.

However with more groups it is possible to get a lower payoff for everyone than with only one group, because if there are only one group, its size can be maximum m^*, but if we have more smaller groups, the total number of players who are part of a group can exceed m^*. For example it is possible that for an even n everyone is a part of a size 2 group, and then everyone would get a payoff of

$$V = \frac{1}{n}\left(\frac{1}{n-1} + \frac{n-2}{n-1}L\right), \tag{4.5}$$

which is clearly smaller then the socially optimal payoff for everyone as $L < 1$. So there exists some equilibria, where everyone is worse off then he would be in the social optimum.

4.3 Comparing our model with the perfect monitoring case

With imperfect monitoring, after a deviation it takes time for the group to fall apart and switch to the socially optimal strategy. With perfect monitoring, as everyone sees the deviation, after a deviation, we can immediately switch to the socially optimal strategy. So for the perfect monitoring case we create the new S_p strategy profile, which is almost the same as S, the only difference is in

the continuation strategies: if someone deviates the group immediately falls apart and switch to the socially optimal strategy.

Similarly to equation (4.1), in the perfect monitoring case S is a Nash equilibrium if there exists a $0 < \delta^* < 1$ such that

$$(1 - \delta)\beta L + \delta V^* = (1 - \delta)\alpha + \delta \frac{1}{n}$$

The only thing, that changed is the last part, instead of W_2 we have $1/n$, as the group falls apart immediately.

Obviously, the perfect monitoring case is the same as the imperfect monitoring for $m = 2$, as for $m = 2$ every group member immediately knows if there was a deviation. For higher m, we know that $W_2 > 1/n$ as after a deviation, the deviator gets more than what he got in equilibrium until everyone in his group will know about the deviation, and only when that happens, he will get $1/n$.

So for any $m > 2$, δ^* is greater for the imperfect monitoring case, which means that with imperfect monitoring favoritism is harder to sustain with imperfect monitoring. Also, as with higher m, it takes longer for the information of deviation to spread to the whole group, the difference between the threshold delta in the perfect and the imperfect monitoring case is higher.

4.4 Folk theorem

Finally, we tried to prove that the Folk theorem for our model. However it turned out to be harder than it seemed, and unfortunately we could only prove it for $n = 4$ and for a restriction on our parameters.

Theorem 4.6. *In this repeated game, if $n = 4$ and $\frac{1}{12}(1 - \alpha) + \frac{1}{4}\beta L > \frac{1}{4}\alpha$ there exists a feasible and strictly individually rational payoff vector ($u > \underline{u}$, where $\underline{u}$ is the minimax payoff) which can not be reached by any Nash-equilibrium strategy profile.*

Proof. To prove this, we assume that the theorem holds, and we find a contradiction. The expected payoff of the players for the repeated game is u_i, for $i = 1, 2, 3, 4$.

The minimax payoff for a player is $\underline{u} = \alpha/4$, as a strategy which minimaxes for example player 1's payoff is the following: nobody ever chooses player 1 even if he is the expert, and player 1 always chooses the expert.

14

We define the set A: $S \in A$ if and only if S is a strategy profile, for which $u_1 + u_2$ is maximal, such that u_3 and u_4 is a bit larger then the minimax payoff, so $u_3 = u_4 = \underline{u} + \epsilon$, where ϵ is arbitrarily small. Every element of A clearly satisfies the theorem's assumption. We will show, that any $S \in A$ is not a Nash-equilibrium. As by assumption, the Folk Theorem holds, so it is enough to look at the stage games, and assume that 1 and 2's stage game payoff is maximal while 3 and 4 get more than the minimax payoff. First, we characterize the elements of A with following lemmata.

Lemma 4.7. *For any $S \in A$, if player 1 or 2 is the expert, and player 3 or 4 is the principal, then the principal chooses the expert.*

Proof. By contradiction. Without loss of generality, assume player 3 is the principal, player 1 is the expert and player 3 chooses player 2 or 4 with probability p. Now, there are two cases.

The first is, when player 3 chooses player 2 when 1 is the expert. Now, we make a new strategy profile, where everything is the same as before, but player 3 chooses player 1 instead of player 2, when player 1 is the expert. Player 3's payoff increased, the sum of player 1 and 2's payoff increased, so we got a contradiction.

The second is, when player 3 chooses player 4 with probability q when 1 is the expert. Now, a new strategy profile would be the following: everything is the same as before, but player 3 chooses player 4 with only $q - \epsilon'$ probability, where $\epsilon' > 0$ is small enough, and player 4 would be compensated by either player 1, 2 or 3, who would choose player 4 more often when he is the expert, so that his payoff does not decrease. This way player 4's payoff remains the same, player 3's payoff is increased, as he choose the expert more often, and player 1 and 2's payoff also increase as they are chosen at least as often as before, but they are chosen more often when they are the experts, and also they choose the expert not less often. $\qquad \square$

Lemma 4.8. *For any $S \in A$, if player 3 and 4 are both the principal and the expert, with a $p > 0$ probability, the principal would choose player 1 or 2.*

Proof. By contradiction. Assume that the lemma is not true, so player 3 and 4 always choose the expert (we already know, that player 3 and 4 choose the expert if player 1 or 2 is the expert). Now we can show that there exists a strategy profile that makes player 1 better off, player 2 not worse off, while player 3 and 4 stay above the minimax payoff.

Take the following strategy profile: player 1 and 2 always choose each other, player 3 and 4 choose player 1 when 1 is the expert, player 2 when 2 is the expert, and when player 3 and 4 are both the principal and the expert then they choose player 1 with probability $0 < \epsilon'$, and choose each other with probability $1 - \epsilon'$ such that they are still above their minimax payoff. As $L, \alpha, \beta > 0$ such positive ϵ' exists. $\square$

Now we have to show, that any $S \in A$ is not a Nash-equilibrium. We will prove that whenever player 3 is the principal, player 4 is the expert, and player 3 should choose player 1 according to the strategy profile, player 3 or player 4 would have a profitable deviation.

In the previous lemmata, we found that without loss of generality, player 3 chooses player 1 with a positive probability even if player 4 is the expert. With the following lemma, we show, that if player 3 should choose the non-expert player 1, but chooses to deviate then player 1 and 2 cannot identify this deviation.

Lemma 4.9. *For any $S \in A$, whenever player 3 is the principal, player 4 is the expert and according to the strategy profile, player 3 should choose player 1 but 3 deviates and chooses the expert, player 4, player 1 and 2 cannot identify this deviation.*

Proof. The idea is the following: player 1 cannot distinguish between player 2 being the expert and player 3 choosing 2 and not deviating, and between player 4 being the expert and player 3 choosing player 4 and deviating.

Assume player 3 is the principal, player 4 is the expert. Player 1 knows that 3 is the principal, but does not know who is the expert, and does also not know who is chosen. So for player 1, the following plays are in the same information set: nature chooses player 2 as the expert, and player 3 chooses 2, and player 3 does not deviate; and nature chooses player 4 as the expert and player 3 chooses 4, and player 3 deviates.

Player 2 does also not see the deviation, as he does not know, who is the expert, and for him player 1 being the expert, and being chosen by player 3 is in the same information set as player 4 being the expert and player 1 deviating and choosing player 4. $\square$

Now take a look at continuation strategies if player 3 deviates. We can observe, that the only profitable deviation for him is to choose player 4 who is the expert instead of player 1 or player 2. We have two cases: either player 4 can not inform

player 1 and 2 about this deviation, and in that case player 4 does not have either the means nor the incentives to punish player 3, so the deviation was profitable.

The other case is that he can inform players 1 and 2 about the deviation. This case, the continuation strategy is such that player 3 gets his minimax payoff, so no players will ever choose him, and he always chooses the expert. As in the strategy profile, player 3 had the payoff of $\underline{u} + \epsilon$, where ϵ was arbitrarily small, the only way to make the deviation not profitable is to minimax him.

However, in this case player 4 gets a higher payoff than before: even if in the continuation strategy, he always chooses a non-expert, and players 1 and 2 never choose him, his payoff would be $\frac{1}{12}(1 - \alpha) + \frac{1}{4}\beta L$ (the first part is the payoff from player 3, the second is the payoff when he is the principal), which is by the theorem's assumption larger than the minimax payoff. So even if player 3 did not deviate, player 4 has the incentive to tell players 1 and 2 that player 3 deviated, and as players 1 and 2 can not verify this information, player 4 can always do it. So S is not a Nash equilibrium.

$\square$

5 Conclusion

In our paper we presented a model that explains favoritism in an imperfect monitoring repeated game. We found, that the favoritism equilibrium exists, however, if group size increases, it is harder to maintain the equilibrium, and at a point, if group size is too large, the favoritism is no longer an equilibrium. We also found, that while favoritism increase the payoffs of players who are in the group, the overall payoffs decrease, and players outside the group receive smaller payoffs. Moreover, favoritism is harder to maintain with imperfect monitoring than with perfect monitoring, as the information of deviation spreads slower in the imperfect monitoring case. Finally we have proven that the Folk theorem does not hold for $n = 4$ and a restriction on the parameters, and hope that we can extend the proof for a more general set of parameters.

References

Yann Bramoulle and Sanjeev Goyal. Favoritism. *CIRPEE Working Paper*, pages 09–41., 2009.

Drew Fudenberg and David K. Levine. An approximate folk theorem with imperfect private information. *Journal of Economic Theory*, 54(1):26–47, June 1991.

Drew Fudenberg and Eric Maskin. The folk theorem in repeated games with discounting or with incomplete information. *Econometrica*, 54(3):pp. 533–554, 1986.

Drew Fudenberg, David Levine, and Eric Maskin. The folk theorem with imperfect public information. *Econometrica*, 62(5):pp. 997–1039, 1994.

George Mailath and Larry Samuelson. *Repeated Games and Reputations: Long-Run Relationships*. Oxford University Press, 2006.

Shmuel Nitzan and Kobi Kriesler. The effect of limited search ability on the quality of competitive rent-seeking clubs. *Social Choice and Welfare*, 35(1): 81–106, June 2010.

Klaus Ritzberger. *Foundations of Non-cooperative Game Theory*. Oxford University Press, 2002.