



MSc Economics

http://www.ub.tuwien.ac.at/eng

Housing and the Redistributive Effects of Monetary Policy

A Master's Thesis submitted for the degree of "Master of Science"

> supervised by Michael Reiter

Philipp Viktor Hergovich 0854309





MSc Economics

Affidavit

I, Philipp Viktor Hergovich,
hereby declare
that I am the sole author of the present Master's Thesis,

Housing and the Redistributive Effects of Monetary Policy

43 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

Vienna, June 18, 2014.	
	Signature

Contents

1	Intr	oduction	5
2	Emp	pirical Findings	7
3	The	model	g
	3.1	Households	Ĝ
		3.1.1 Workers	10
		3.1.2 Capitalists	12
	3.2	Bonds	13
	3.3	Production	14
	3.4	Government	15
	3.5	Pricing and Monetary Policy	15
4	Solv	ring the Model	17
5	Cali	bration	20
	5.1	Related Literature	20
	5.2	Aggregate Ratios	20
	5.3	Parameter Values	21
6	Res	ults	2 3
	6.1	Steady State Results	23
		6.1.1 Unconstrained	24
		6.1.2 Downpayment Constraint	24
	6.2	Redistribution through Monetary Policy	26
		6.2.1 Strongest Reactions	26
		6.2.2 Utility Comparison	29
	6.3	Redistributive effects of a markup shock	30
7	Con	clusion and Outlook	31
\mathbf{A}	App	pendix	33
	A.1	Derivation of Tobin's Q and F.O.C for Capital	33
	A.2	Derivation of the First Order Conditions Worker Households	34
	A.3	Derivation of the First Order Conditions Capitalists	36
	A 4	Optimal Pricing	37

List of Figures

1	Heterogeneity over the life cycle	22
2	Lifecycle paths for labor, consumption, housing owned and rented,	
	and debt without downpayment constraint	25
3	Lifecycle paths for labor, consumption, housing owned and rented,	
	and debt with downpayment constraint	26
4	Impulse Responses to a Contractionary Monetary Shock	27
5	Welfare Comparision after a Contractionary Monetary Policy shock	29
6	Welfare Comparision after a positive Markup shock	30
7	Welfare Comparision after a positive Markup shock, different re-	
	action parameters	31
\mathbf{List}	of Tables	
1	Baseline calibration of the model	23
2	Extreme Impacts of a contractionary Monetary Policy shock	27

Abstract

This thesis lays out a general equilibrium model, of which the main purpose is to measure redistributive effects between generations. To do so it builds up a lifecycle structure to introduce heterogeneity within the workers in the model, such that shocks affect different cohorts in a varied way. It separates housing from other consumption goods, and subjects agents to a downpayment constraint, which implies that they can only borrow, up to a certain fraction of their housing wealth. Redistributive effects of monetary policy shocks, which affect the model economy via the nominal interest rates of bonds, and inflation, are studied using Impulse Response Functions. The lifetime utility is employed as a measure of welfare redistribution. The analysis shows that the beneficiaries of these shocks and the subsequent reactions are elderly, wealthy households.

I would like to thank my advisor, Michael Reiter, who developed the model in its basic structure, and the toolkit I use for solving the model numerically. Moreover, I would like to thank him for helpful comments and guidance during the process of crafting this thesis. Furthermore, Dirk Krüger gave me some valuable comments on possible future directions of this thesis. I would also like to thank my colleagues from the MSc program for helpful discussions and comments, especially Ashim Dubey, Laslzo Tetenyi and Gabriel Ziegler.

1 Introduction

A house is the biggest and most important asset in the portfolio of many people since housing wealth makes up about one half of the entire net worth of households in the US (25.4 trillion dollars out of 52.9 trillion dollars), according to Iacoviello (2011). Owned housing can be considered as one of the most commonly held assets, next to transportation assets (cars) checking & savings accounts. For the median household, housing is the largest asset-position, once we abstract from people owning their own businesses (Kapteyn and van Soest, 2006). So housing is not only a consumption good which gives utility, it is also an investment decision, especially since Mortgages and Mortgage Backed Securities are interesting financial products for both sides, the institution that gives the mortgage as well as the holder of the mortgage. In this thesis I refrain from modelling mortgages explicitly and also abstract from default. A thorough treatment of this matter can be found in Garriga et al. (2013). Housing is also a highly debated political issue and this not just since the recent financial crisis. In his famous speech about home-owners, President George W. Bush declared it a major goal of his presidency, to enable everyone to own a home. ¹ He explicitly mentioned downpayment rates, a parameter which is of great importance for the time period a household spends at the constraint in this paper. But how are macroeconomic changes, especially inflation, affecting the ownership plans of households? This is one of the central questions, to be discussed in this thesis.

Numerous authors have worked on housing choice under various focuses and have developed models to address questions of economic interest. Introducing housing in a standard model will not yield huge effects, since with perfect markets, housing just serves as an explicitly modelled consumption good. There is a huge discussion in the literature about the role of housing in economic terms. Does it just serve as a mere consumption good, or can it also be used for investment purposes and wealth accumulation? Buiter (2008) argues that housing wealth lacks an important criterion usually ascribed to wealth, namely that it has an income effect on consumption, and thus should not be considered as wealth.

Since just modelling an additional consumption good is often not the desired outcome, one needs to introduce additional frictions to assign a more prominent role to housing. This is usually done via credit constraints, which means a certain fraction of the house has to be paid for by the buyer, and cannot be bought on credit.

¹http://archives.hud.gov/remarks/martinez/speeches/presremarks.cfm

Including a sluggish good and the downpayment-constraint into a model can also serve to create macroeconomic fluctuations. This has been shown by Kocherlakota (2000). In his model the production side (namely agriculture) is subjected to a credit constraint, and this then serves as an amplifier for shocks. Also when the consumers are stricken by a borrowing constraint, shocks will be amplified.

Buying a house is thought of as a huge investment, especially for young households (Wang, 2009) and is thus distortive to the consumption smoothing motive, which is common in standard macroeconomic models. It can be used to model more realistic consumption patterns over the lifecycle, as is one outcome of the paper by Iacoviello and Pavan (2013). In their paper, the authors create a lifecycle model and compare the outcomes of it to the data. Their model outcomes achieve quite a good fit with the data in both aspects, the life cycle and the business cycle. Lifecycle properties of housing are also studied in Yang (2006) who's model outcome matches the data quite well. In his model he distinguishes between housing consumption and non-housing consumption, has a stochastic death probability. He refrains from modelling the rental market explicitly and has transaction costs in his model, which create regions of inaction. The model presented in this Master Thesis resembles these two in several features, but tries to include monetary policy into it. By this, it might be possible to also asses the question of redistributive effects of monetary policy, since agents are heterogeneous and thus affected differently by actions of the central bank. Ideally, the model should replicate some of the empirical findings by Mian and Sufi (2010), most importantly that economies with higher leveraged households face more serious economic downturns and slower recoveries, compared to economies with lower leverage.

A thorough treatment of the empirically observed, redistributive effects of inflation can be found in Doepke and Schneider (2006a). In their study, the authors study the effects of inflation through the channel of nominal assets and argue that there happens a revaluation of nominal wealth due to changes in inflation. So who benefits and who loses from this effect? The biggest beneficiary of inflation is the government, since it usually has a relatively big and negative asset position. In the current model-setup of this thesis, there is no government debt, so this finding cannot be adressed by the model. Furthermore, young households gain when surprise inflation hits the economy, while elderly household lose. However, when building their findings in an OLG model (Doepke and Schneider, 2006b), they model inflation as a redistribution shock in the first place, while in our model, inflation starts with an increase in the markup charged by firms. They also only

consider six cohorts, and refrain from distinctively modelling nominal versus real assets.

Another strand of literature is dedicated to the redistributive effects of monetary policy, a selection of which includes the empirical study by Brunnermeier and Sannikov (2012), and the theoretical models by Williamson (2008) or Gahvari and Micheletto (2012). Williamson considers a segmented market model with connected and unconnected agents, who differ in whether or not they receive a transfer in the form of fiat money, to derive responses of aggregate variables. Gahvari and Micheletto use an Overlapping Generations structure, considering only two periods. Their main goal is not to asses the redistributive effects in a quantitative way, but rather to show which effects of monetary policy can be offset by nonlinear taxes. The model in this thesis is to our knowledge the first attempt, to quantify redistributive monetary policy effects for an OLG model with lifecycle elements.

The thesis is organized as follows. In section 2 I discuss a few datasources and further empirical findings, by other authors, as well as giving a short characterization of the median household in the US. Section 3 lays out the model, on which the analysis is based, while section 4 describes some steps necessary for the solving the model in detail. More of the calculations can be found in the appendix. Section 5 discusses the parametrization of the model, while the results are presented in section 6, separated between steady state results and Responses to shocks. Section 7 concludes.

2 Empirical Findings

Stylized facts of consumption have been widely described in the literature, but there are some discrepancies in these descriptions. That consumption of non-durable goods is hump shaped over the lifecycle and closely tracks disposable income, is nowadays a widely accepted view in economic literature. There have, however, been disputes about this view in the 90s, where e.g. Attanasio (1999) argued that once the consumption data has been appropriately adjusted for household size effects, the hump shape disappears. More recent papers ((Fernndez-Villaverde and Krueger, 2011), (Yang, 2006)) showed that even after controlling for demographics, the hump shape remains. While these newer papers agree that consumption is hump shaped over the lifecycle, they differ when it comes to consumption of durable goods. Yang argues that it is not humpshaped, and builds a General Equilibrium Model which matches this feature, Fernndez-Villaverde and Krueger (2011) claim that durable consumption is hump shaped as well.

Both conflicting papers use data from the Consumer Expenditure Survey (CEX) to address the question of how the consumption profiles are shaped. This datasource consists of a rotating panel, where each household is interviewed every three months over five quarters. Each quarter, a fifth of the households is replaced by a new set of households, and the sample size for each observation period is about 5000 households. Yang only uses the cross section of 2001, while in their other paper, Fernndez-Villaverde and Krueger (2007) apply a pseudopanel approach for the time period from 1980 to 2001 (excluding the years 1982 and 1983). They use the age of an interviewed reference person to construct cohorts of 5 year length, and after evaluating their means follow the cohort through the sample. The estimation is then done, controlling for different household sizes and time effects. Yang additionally uses Survey of Consumer Finance (SCF) data and creates a shorter pseudopanel for 1983 to 1998. The Survey of Consumer Finances is an interview survey, which is conducted every three years. It contains data on housing, income and net worth percentiles, and the holding of financial assets (among others), differentiated by age, employment status, and other personal characteristics. Using data from the SCF, Yang estimates the service flows from housing, again controlling for time effects and differing household sizes. These different approaches can explain the different results obtained in the papers.

Later literature built up on those empirical findings and added its own conclusions from extended data, and still maintains the view that the consumption of housing over the lifecycle is hump shaped. Iacoviello and Pavan (2013) report a hump shaped housing consumption using data from the 1983 SCF, and Browning et al. (2013) report a hump shaped housing expenditures profile, using a Danish panel-data-set.

Going to the data as well, I use the most recent issue of the Survey of Consumer Finances (SCF) for 2010 (Bricker et al., 2012), to describe some key characteristics of the median US household, which then shall serve as a benchmark and a guideline for the calibration of the model. For the 2010 survey, 6492 families were interviewed in five consecutive quarters. From their answers 307 variables have been constructed, which measure the households financial choices and consumption characteristics. The median might be the better measure compared to the mean, since certain characteristics are tilted towards the higher percentiles e.g. wealth.

The median household in the US in 2010 has an income of around 55 thousand dollars per year, and owns a house of about \$ 125,000 value. The median mortgage is zero. The reasons for this is that around 67.3% of interviewed families own their primary residence, which leaves about one third of the survey participants

without a mortgage option. Now turning the attention to the mean mortgage, the value is around \$ 113,000 . Compared to the ownership rate, only around 50 % of the families hold stocks, counting even indirect holdings via retirement funds. The median in dollar, among families who do hold stocks, is around \$ 29,000. These numbers point towards the conclusion, that houses are the most important asset, for the median US household.

3 The model

In the model, the economy's population is split into two types of households, workers and capitalists. This split seems to have been done first by Goodwin (1967), where he uses it to determine the income distribution between capitalists and workers. In our model, it serves a similar purpose. We want to model a group of people, for which the biggest asset is the house they own. For them the incentive to bequest is lower than for the rich families, as has been empirically adressed by Arrondeln and Laferrere (1998). Since wealth declines for non-rich families and approaches zero towards the end of the lifetime, it is arguably justified to represent these people with a lifecycle structure, while the capitalists are modelled dynastically as a representative, infinitely lived agent. The members of the group of Worker's differ with respect to their productivity (and therefore their wage) depending on which period they are born, and with respect to a preference parameter λ . An aggregate production function uses capital and labor to produce a single output good, and is exposed to uncertain productivity. The output good can be used for consumption and investment. Both types of agents hold housing and Workers elastically supply labor to the economy. Workers cannot hold capital, but can save in bonds, while Capitalists can save in bonds, capital, and firms. Firms have a common production technology but imperfect competition with sticky prices since they are subject to Calvo Staggering (Calvo (1983)). Therefore firms will earn profits which will serve as another source of income for Capitalists.

3.1 Households

The worker households consist of Overlapping Generations living for 240 periods (quaterly data), while capitalists are infinetly lived. The major difference in terms of housing between those two groups is that Workers can either own their houses, or rent them from capitalists, while capitalists own their own houses plus the rental houses for the workers. Both choose consumption, housing and hours

worked in order to maximize their lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{T} \beta^t \lambda_t u(c_t, l_t, h_t)$$

subject to their respective budget constraints. Capitalists and Workers also differ in their discount factor with β for Worker Households and $\hat{\beta}$ for capitalists. A general remark on notation: variables associated specifically with capitalists, will be denoted with a hat.

3.1.1 Workers

The workers period utility function is time seperable and of the form

$$u(c, l, h^{R}, h^{O}) = log(c) + \eta log(1 - l) + \eta_{H} \log(((h^{R})^{\frac{\sigma - 1}{\sigma}} + \xi(h^{O})^{\frac{\sigma - 1}{\sigma}})^{\frac{\sigma}{\sigma - 1}})$$
(3.1)

The utility function is additively separable in consumption, leisure (small l represents the hours worked, endowment normalized to 1) and housing. Notice that housing consists of two components h^R and h^O , which represent the units of housing rented and owned, respectively. As it is standard in the literature (see e.g. Iacoviello and Pavan (2013)) the units of housing owned by the household enter the utility function with a higher coefficient than housing rented. This seems quite intuitive, since usually people value property more than just being able to use something. Notice that the logarithmic form of the housing, makes σ to be the elasticity between housing owned and housing rented. With this specification, each cohort will choose to rent and to own housing, which seems puzzling at first glance. However, if one thinks of a cohort as aggregation over many households of this age, the result becomes plausible, since no age group will choose either option exclusively.

Worker households can save in bonds and houses and are forced to participate in a public Pay-as-you-go pension system.

Workers are "born" into our model at the age of 20 and live on for 60 years. They retire after 40 years at the age of 60, from which time period onwards they receive a pension, which is their only source of income next to interest payments from possible savings, which will turn out to play a huge role in the behaviour of the agents.

Non-retired Workers supply labor elastically to the firms and receive a wage which is the same independent of the firm (and this specific firm's prices). Notice that the wage is not equal to the marginal product of labor because of monopolis-

tic competition. They receive earnings from bonds that are maturing in period t, where the payoff depends on the quantity of bonds they are holding and the price of these bonds. In the baseline calibration the fraction of bonds that are maturing is 1, which means that households can only trade one period bonds. One period in the baseline calibration represents one quarter of a year. They spend their aggregate income on consumption, rental payments, purchases of new bonds and buying new housing or maintaining their current stock. This yields the following Household Budget Constraint

$$p_t^B b_{i,t} + h_{i,t}^O p_t^H + c_{i,t} + r_t^H h_{i,t}^R =$$

$$(((1 - \tau) w_t \zeta_i l_{i,t} + \mathcal{I}_{Ri} \psi_t +$$

$$((\mu + r^B) v_t^B + (1 - \mu) p_t^B) b_{i,t-1} +$$

$$(1 - \delta_H) h_{i-1,t-1}^O p_t^H)))$$

$$(3.2)$$

The left hand side (LHS) of equation 3.2 depicts the spending of the Worker, the right hand side (RHS) the various sources of income. \mathcal{I}_{Ri} is an Indicator function, which takes the value 1 if the worker is retired, and therefore receives pension-payments ψ_t . The amount of pensions paid to retired workers is endogenously determined in the model and is described in greater detail in section 3.4. The parameter τ is a tax on wages, which is collected in order to finance the pension benefits. To introduce another interesting friction in the model, there is a downpayment-constraint introduced. It takes the form $[b_{i,t} - \nu \mathbb{E}(p_{t+1}^H)h_t^O]$.

The form of the borrowing constraint is widely used in the literature (Doepke and Schneider, 2006b; Yang, 2006). Other ways to introduce them are also considering the future lifetime earnings (Iacoviello and Pavan, 2013), which is more applicable for credits than for mortgages. It is consistent with practices in the mortgage market, where buyers have to finance a certain fraction of the value of their house out of their own pocket. The parameter ν represents the Loan-to-Value-ratio (LTV), which is further described in the Calibration-section 5. The Workers are the group which introduce the Overlapping-Generations structure in our model. They differ with respect to their productivity ζ and in an additional component to the discount factor λ . Both of these parameters are exogenously given and further described in the calibration section. Since there is a time dependent discount factor, the Marginal Utility of Consumption will be different for each cohort.

3.1.2 Capitalists

The period utility function of the capitalists consists of almost the same items as the one for the Workers, only the rented housing is missing and we assume log utility in housing. This reflects the fact that capitalists are less risk averse than the households, when it comes to housing decisions. This assumption can be motivated by the house not being the biggest asset, since capital will play a more prominent role in their investment decision. Notice that because there is no Overlapping Generations structure for capitalists, one can think of them as a single representative agent.

$$\hat{u}(\hat{c}, \hat{l}, \hat{h}^O) = \log(\hat{c}) + \eta \log(1 - \hat{l}) + \eta_H \log((\hat{h}^O))$$
(3.3)

Including $\log(1-l)$ in the capitalists' utility function is innocuous, since the capitalists will choose not to provide any labor. Therefore l=0, which leaves them with $\log(1)=0$ as the disutility from working. The budget constraint for the capitalists is defined similarly to the one for the Workers, but in aggregate terms. It is given by equation 3.4. Their income is whatever is left from the production process, after the wages have been paid to the workers, plus the income from bonds and renting out housing to the workers. They spend it on investment in capital and housing, and for consumption purposes. Capitalists are also enabled to use bonds for transferring wealth over the periods, and they will hold the opposite position of the aggregate of all worker cohort's bond-holdings. Their are no returns to capital paid in our model, since all the capital is held by the homogeneous capitalist household, which collects the profit of the firm.

$$Y_{t} - I_{t}^{K} - w_{t}L_{t} + r_{t}^{H}H_{t}^{R} + p_{t}^{B}B_{t} + p_{t}^{H}(H_{t} - (1 - \delta_{H})H_{t-1}) - I_{t}^{H} = \hat{c}_{t} + p_{t}^{H}(\hat{h}_{t}^{O} + H_{t}^{R} - (1 - \delta_{H})(\hat{H}_{t-1} + H_{t-1}^{R})) + ((\mu + r^{B})v_{t}^{B} + (1 - \mu)p_{t}^{B})B_{t-1}$$

$$(3.4)$$

Capitalists can additionally to their housing make investments in capital, and receive the profits of the firm. Capitalist are not constrained by a downpayment-requirement, since their default risk can be seen as to be zero. Variables with a hat represent the choice of the capitalist, regarding consumption and housing owned. As part of the income process, the capitalist rents out housing to Worker households, which is denoted by an upper-case H^R , to stress that it represents the aggregate position of rental housing in the model economy. Similarly for the asset position B, the capitalists hold the opposite position of what the aggregated household sector holds, which is guaranteed by the First Order Conditions.

Solving their household problem gives a set of First Order Conditions, most of which are standard. Derivations can be found in the appendix A.3. The First Order Conditions for the worker households are more intersting and will be derived in more detail in 4.

$$\hat{u}_{\hat{h}_{+}^{Q}} = p_{t}^{H} \hat{u}_{\hat{c}_{t}} - \hat{\beta}(1 - \delta_{H}) p_{t+1}^{H} \hat{u}_{\hat{c}_{t+1}}$$
(3.5)

$$\hat{u}_{\hat{c}_t}[p_t^H - r_t^H] = \hat{\beta}(1 - \delta_H)\hat{u}_{\hat{c}_{t+1}}p_{t+1}^H$$
(3.6)

$$\hat{u}_{\hat{c}_t} p_t^B = \hat{\beta} \hat{u}_{\hat{c}_{t+1}} ((\mu + r^B) v_{t+1}^B + (1 - \mu) p_{t+1}^B)$$
(3.7)

Individual utility shifters λ are absent, since there is no life-cycle structure for capitalists.

3.2 Bonds

Worker and Capitalist households can save in bonds, which are besides housing the only way to transfer wealth to the next period. The return of the bond is given by

$$(\mu + r^B)v_t^B + (1 - \mu)p_t^B \tag{3.8}$$

The first bracket is the fraction that matures, represented by μ , plus the nominal coupon times v^B where v^B can be thought of as the value of the bond. In the deterministic steady state, it is equivalent to the value of a real one period bond, but it is affected by nominal quantities of the economy.

The real over the nominal value evolves according to following process

$$\log(v_t^B) = \rho_B \log(v_{t-1}^B) - \chi \log\left(\frac{\pi_t}{\pi^*}\right) - \varepsilon_t^B$$

The process has a structure similar to a classical AR-1 process with one additional term. The term $\chi \log(\frac{\pi_t}{\pi^*})$ represents the impact of monetary policy on the real and nominal value of bonds. χ is the degree of "nominality", if it is zero the bond is entirely real. Otherwise the bond is affected by monetary policy. ε_t^B is a direct shock to the value of the bond.

The second term in brackets in 3.8 is the value of the long term bonds which are still in the portfolio and have not paid off in the period under consideration.

3.3 Production

The production technology for the output good of the economy is a standard Cobb Douglas production function, with an exogenously given productivity level z_t .

$$F(z_t, K_t, L_t) = z_t K_t^{\alpha} L_t^{(1-\alpha)}$$
(3.9)

 z_t is exogenous and stochastic, following an Autoregressive Process of order 1 in logarithms.

$$\log(z_t) = \rho_Z \log(z_{t-1}) + \varepsilon_t^Z$$

where

$$\varepsilon_t^Z \sim Normal(0,1)$$

 ρ_Z is the coefficient of autocorrelation and governs the persistency of a productivity shock ε_t^Z . The firm faces a Calvo mechanism, which doesn't allow it to adjust its prices at will. The objectives of the firm are described in more detail in section 3.5.

 K_t is the aggregate capital in the economy which evolves according to the following Law of Motion:

$$K_t = (1 - \delta_K)K_{t-1} + \phi(\iota_t, \delta_K, \eta_I)K_{t-1}$$
(3.10)

The Law of Motion is standard, the only potentially unfamiliar term is the convex adjustment cost function $\phi()$.

$$\phi(I, \delta_K, \eta_I) = \iota - (\iota - \delta_K)^2 / (\eta_I \delta_K) \tag{3.11}$$

It depends on the investment ratio $\iota^K \equiv \frac{I_t}{K_{t-1}}$ (how much is invested in terms of the capital stock of the previous period), the depreciation rate and a parameter value η_I .

It is assumed that there is a competitive capital accumulation sector, which is constrained by the Law of Motion for capital 3.10.

This gives rise to the following First Order Condtion and Value of installed capital (See Appendix for derivations).

$$Q_t = \frac{1}{\phi_I'(\iota_t^K, \delta_K, \eta_I)} \tag{3.12}$$

Equation 3.12 defines Tobin's Q, a sufficient statistic for investment decisions (Tobin (1969)). It is the inverse of the marginal cost of investing one unit of the output good in the production process for capital.

L stands for the labor input, which is the aggregate labor input over all the cohorts of workers. The optimal combination of production factors is given by

$$\frac{w_t}{r_t^K} = \frac{F_L(z_t, kl_t)}{F_k(z_t, kl_t)}$$
 (3.13)

where kl_t is the Capital-labor ratio. Note that as a timing convention similar to ι it is defined as $\frac{K_{t-1}}{L_t}$

This was the production of the output good, the production of housing is similar, but we assume it is not requiring any labor input. Housing dynamics are as well governed by a a Law of Motion with adjustment costs, which again gives rise to a price for housing of similar structure as Tobin's Q above.

$$H_t = (1 - \delta_H + \phi(\iota_t^H, \delta_H, \eta_J))H_{t-1}$$
(3.14)

$$p_t^H = 1./\phi_I(\iota_t^H, \delta_H, \eta_J) \tag{3.15}$$

3.4 Government

The Government plays a very limited role in the model economy. It is just collecting taxes τ on wages and on the homeowners, and redistributing it lump sum to the pensioneers. It is running a balanced budget in each period. This means, that the lump sum transfer cannot exceed the collected taxes, in each period.

$$\psi_t = (\tau w_t L_t + p_t^H h_{T,t-1}^O (1 - \delta_H)/T) \frac{T}{(T - T_{ret} + 1)}$$
(3.16)

On the right hand side of equation 3.16, the amount of the pension payment is defined. The term in the brackets represents what is collected from taxes on wages and on homeowners, and the fraction outside divides this up, such that each retired worker receives the same amount. T_{ret} stands for the time period when Workers retire.

3.5 Pricing and Monetary Policy

There is a continuum of firms in the economy normalized to the zero one interval (Gali, 2008), and firms are indexed by $i \in [0, 1]$. Due to constant returns

to scale in the production function, it is possible to aggregate the firm, and think of the continuum as sub-departments of one firm, both in production and pricing. As is standard in New Keynesian models the price level is defined as $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$.

Since a fraction of θ has to keep their price constant each period (this can be thought of as infinitely high adjustment costs), and due to inflation there is a unique optimal price p*, all "firms" who have the chance to adjust their prices will set their new price to p*. Let $S(t) \subset [0,1]$ represent the firms not optimizing in period t. Then at each period in time the aggregate price level P_t is given by

$$P_t = \left[\int_{S(t)} P_{t-1}(i)^{1-\varepsilon} di + (1-\theta)(P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

which is obtained by just plugging in for the definition of P_t . Using the fact that the mass of firms not allowed to reallocate is θ

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Dividing both sides by P_{t-1} and then taking the resulting equation to the power of $1 - \varepsilon$ gives in steps

$$\frac{P_t}{P_{t-1}} \equiv \Pi_t = \frac{\left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{P_{t-1}}$$
$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

The last equation, is the relationship which is used to describe the development of inflation in the model economy.

Because of Calvo staggering and the resulting sticky prices, firms cannot optimally adjust to the price level. Optimal pricing is therefore given by the following three equations

$$V_{Y,t} = Y_t + \theta \left(\hat{\beta} \frac{\hat{U}_{c_{t+1}}}{\hat{U}_{c_t}} \right) \pi_{t+1}^{-1} V_{Y,t+1}$$
(3.17)

$$V_{MC,t} = (\mathcal{M} + z_{\mu_t}) Y_t RMC_t + \theta(\hat{\beta}\hat{U}_{ct+1}/\hat{U}_{ct}) V_{MC,t+1}$$
 (3.18)

$$V_{MC,t} = P_t^* V_{Y,t} (3.19)$$

Derivations of this can be found in the appendix A.4. The first two equations (3.17 and 3.18) are value functions, which govern the Output and Cost structure of the firm, while the third is an optimality condition. z_{μ} is a shock to the markup, which is charged by the firm.

 z_{μ} is an exogenous shock to the markup charged by the firm which follows an AR-1 process.

$$z_{\mu_t} = \rho_{\mu} z_{\mu_{t-1}} + \varepsilon_t^{\mu}$$

The Central Bank conducts monetary policy according to a Taylor rule, which is depending inflation and the interest rate in the previous period. It is given by equation 3.20

$$\log\left(\frac{R_t}{R^*}\right) = \rho_R \log\left(\frac{R_{t-1}}{R^*}\right) + (1 - \rho_R)\gamma_\pi \log\left(\frac{\pi_t}{\pi^*}\right) + z_{Mt}$$
 (3.20)

R stands for the short term interest rate of a real, one period bond, which is determined in our model by the consumption Euler equation of the capitalists.

4 Solving the Model

When solving the model, the first step is to solve for the deterministic steady state. This section discusses in greater detail the most interesting equations of the model, namely the equations that govern the behaviour of a worker household over the lifecycle. These steady state relationships can be obtained by considering the maximization problem of the household, subject to its intertemporal budget constraint. But since we also want to impose a borrowing constraint, this becomes part of the agent's maximization problem. Considering all of this, one can set up the following Lagrangian and solve for the first order conditions. The Lagrangian is given by

$$\mathcal{L}: \max_{c_{t}, l_{t}, h_{t}^{T}, h_{t}^{O}, b_{t}} \mathbb{E}_{0} \sum_{t=0}^{T} \beta^{t} \lambda_{t} u(c_{t}, l_{t}, h_{t})$$

$$+ \kappa_{t} [p_{t}^{B} b_{i,t} + h_{i,t}^{O} p_{t}^{H} + c_{i,t} + r_{t}^{H} h_{i,t}^{R} - (((1 - \tau)w_{t} \zeta_{i} l_{i,t} - \mathcal{I}_{Ri} \psi_{t} - ((\mu + r^{B})v_{t}^{B} + (1 - \mu)p_{t}^{B})b_{i-1,t-1} - (1 - \delta_{H})h_{i-1,t-1}^{O} p_{t}^{H})) - r_{t}^{H} h_{i,t}^{R})]$$

$$+ \gamma_{t} [b_{i,t} - \nu \mathbb{E}(p_{t+1}^{H})h_{t}^{O}]$$

where κ_t and γ_t are the Lagrange multipliers.

The strategy to solve this, is to make a transformation of variables. Since the inequality constraint of the downpayment-constraint is likely to cause troubles, I transform the variable into the newly defined variable X

$$X_t \equiv \nu \mathbb{E} p_{t+1}^H h_t^O + b_{i,t}$$

This now reduces the downpayment constraint to $X_t \geq 0$ which is easier to handle, since the left hand side is now a constant, and not a time varying outcome of the model. Notice that this implies that the networth of the household is bounded from below by a fraction $1 - \nu$ of the house value. Plugging in for b_t and taking the derivatives yields the following First Order Conditions ²

Labor Supply:

$$u_{l_{i,t}} = -(1-\tau)w_t\zeta_i u_{c_{i,t}} \tag{4.1}$$

Rental Housing:

$$u_{h_{i\,t}^{R}} = r_{t}^{H} u_{ci,t} \tag{4.2}$$

Owned Housing:

$$u_{h_{i,t}^{O}} = u_{c_{i,t}} (p_{t}^{H} - p_{t}^{B} \nu p_{t+1}^{H}) + \frac{\lambda_{t+1}}{\lambda_{t}} u_{c_{i+1,t+1}} \beta$$

$$\times \left[((\mu + r^{B}) v_{t+1}^{B} + (1 - \mu) p_{t+1}^{B}) \nu \mathbb{E} p_{t+1}^{H} - (1 - \delta_{H}) p_{t+1}^{H} \right]$$

$$(4.3)$$

The Consumption Euler Equation now includes a Lagrange Multiplier:

$$\lambda_t u_{c_{i,t}} p_t^B - \gamma_{i,t} = \beta \lambda_{t+1} u_{c_{i+1,t+1}} ((\mu + r^B) v_{t+1}^B + (1 - \mu) p_{t+1}^B)$$
(4.4)

The intuition behind most of these equations is readily explained. Equation 4.1 equates the utility loss of working another unit, to the consumption utility which is foregone since the agent cannot spend the additional wage. This relationship will be used to determine the labor choice for each cohort. The amount of housing rented for each cohort is given by equation 4.2. The longest equation is the one governing the household's choice concerning owned housing, given by equation 4.3. It equates the marginal utility of an additional unit of owned housing, to the consumption that could be realized today with the price paid for housing, but one has to subtract the amount of housing that could have been bought tomorrow with this consumption. So it symbolizes the trade-off between buying housing today or tomorrow. As is typical for an Euler-Equation, also the marginal utility of next period enters, discounted once by the overall discount factor for worker-household β , and by the change in relative importance of util-

²The whole derivations can be found in the appendix

ity $\frac{\lambda_{t+1}}{\lambda_t}$. This correction factor has been introduced by Attanasio (1999), and estimated, amongst ohters, by Cagetti (2003). It then represents the benefit of holding the bond for one period and spending the collected return on consumption, but diminished by the depreciated size of my house at the new value. So for next period's marginal utility, the order is more or less reversed, the future benefits in terms of wealth of owning an additional unit of housing today enter with a minus sign, while the opportunity costs from investing this amount in a bond enter with a positive sign.

The Consumption Euler Equation is almost standard (again additionally discounted by λ) but it still includes a Lagrange Multiplier γ . The interpretation of it is straightforward: as long as the multiplier is zero, the standard euler equation holds, since the household is not constrained. As soon as the constrained binds, the value of γ becomes positive, and thus the marginal utility of consuming today is valued higher, compared to consuming tomorrow. This is the economic interpretation of being borrowing constrained: you wish you could consume more today. The constrained households behaviour is governed by the downpayment constraint, which now holds with equality.

Together with the First Order Conditions from the capitalist households 3.1.2, the pricing and monetary policy equations and the production side, these equations govern the behaviour of the model economy.

While the economic meaning of those conditions is fairly easy to grasp, solving it proves to be tricky. There is still a Lagrange Multiplier in the First Order Conditions. For a solution it is important that the Kuhn-Tucker condition is always fulfilled, which in this model can be written as

$$\gamma_t X_t = 0$$

So when solving the problem numerically, it is important to check for which cohorts the constraint is binding, and therefore the multiplier is greater than zero. If this is the case, the value of housing and the bond-holdings of this cohort are linearly related to each other by the loan-to-value ratio (LTV) ν . If, however, the constraint is not binding, the Euler Equation for the worker households is standard. When looking at shocks to the linearized solution, it is assumed that the shocks are small enough to not push cohorts to the constraint, an assumption which is easily justified by linearity.

5 Calibration

5.1 Related Literature

The model is very rich, in the sense that it contains many different features, some of which are not exploited in this thesis. Finding the right parametrization is a difficult task, as there is no model with the same structure. The closest is probably Iacoviello and Pavan (2013), where monetary policy is absent, as well as the existence of a dynastic capitalist, but they include housing transaction costs, which are absent in our current model.

Another comparable model is the one by Garriga et al. (2013). It also considers housing in a general equilibrium framework and is interested in the redistributive effects of monetary policy. The main difference is that they do not use an Overlapping Generations Model, but explicitly model mortgages. They stress the difference between holding a fixed rate mortgage (FRM) and an adjustable rate mortgage (ARM), when a monetary shock hits the economy. It is also interesting to study the model of Gornemann et al. (2012), where they introduce a Mortensen Pissarides Labor Market (see e.g.) and imperfect consumption insurance, and therefore have heterogenous agents which differ in their productivity, wealth, and employment status. So for the calibration of this model, I follow a similar strategy, which is to calibrate some parameters in order to match certain ratios observed in the data. Popular candidates are the capital to GDP ratio, the owned-housing to GDP ratio, as well as the investments in those two stock variables. Some other crucial parameters are taken directly from the literature, or standard economic theory.

5.2 Aggregate Ratios

The capital to output ratio is a popular choice for calibration, although the literature has found quite different target values (e.g. Claire (1991): 2.5, Garriga et al. (2013): 7.06, Iacoviello and Pavan (2013): 2.2). The high rate in Garriga et al. (2013) can partially be explained by their definition of GDP, in which they exclude investments to housing, salaries to government employees among others, and therefore use a measure for output which is on average 74% of GDP. The capital to output ratio of this model in the baseline calibration is 2.069, which is close to what other papers find. The housing to GDP ratio in related literature is about 1.4 in Iacoviello and Pavan (2013), 1.8 in Iacoviello (2011) cite or 5.2 Garriga et al. (2013), whereas this model is calibrated to give a housing to output ratio of 1.17, and it is again at the lower end.

For the investment to output ratios, comparable values are taken from Iacoviello and Pavan (2013), who find 0.2 for investment in capital and 0.07 for investment in housing, or what is found in Garriga et al. (2013), which are 0.156 and 0.054 respectively. The resulting values of this model are 0.2151 and 0.0598.

Some parameters are taken from comparable literature, for example the utility gain from owning over renting a house is 1.2, similar to Iacoviello and Pavan (2013). This value is used to match the homeownership rate in the US economy, which was 64.8% ³ in the first quarter of 2014, according to the US Census Bureau. The rate in our model is 66.3 %, so this feature is captured quite well by the model.

5.3 Parameter Values

The calibration results can be seen in table 1. The structure of the lifecycle productivity (right panel of Figure 1) is similar to the results of Hansen (1993), who finds that the number of efficiency units supplied peaks around the age of 54. The parameter ζ_i is crucial for the wage of a household, and thereby affecting the consumption choice. It also affects aggregate output, since it indirectly enters the production function via the aggregate labor supply. The drop to 0.1 in retirement is just an innocuous normalization, since agents do not supply labor once the are retired, and therefore neither receive wage income nor contribute to aggregate labor supply. Another crucial paramter is λ_i , which is a weighting of the utility over the lifecycle. As mentioned above, this factor was introduced by Attanasio (1999), who estimated preference parameters which would reconcile the observed consumption and income profiles with their counterparts from lifecycle models. He finds that in order to shift the consumption peak to the earlier lifetime, one needs to take into account demographic factors. Iacoviello and Pavan (2013) interpret these as correction factors for family sizes in the household. When entering the model at the age of 20, the household size is normalized to 1. It steadily increases until the age of 40, which can be interpreted as having children living in the household. It then starts to decline slowly (children moving out) until it settles around 1.1 at the age of 80, when the household dies.

An interesting parameter in this model is of course the size of the downpayment necessary for buying a house, or differently speaking, up to what fraction of the value of the house, the agent can borrow. ν represents the Loan-to-value ratio (LTV), which has been decreasing over time, as can be seen in the data. It is set to 0.9 in the baseline calibration, which is higher than some counterparts in

³http://www.census.gov/housing/hvs/files/qtr114/q114press.pdf

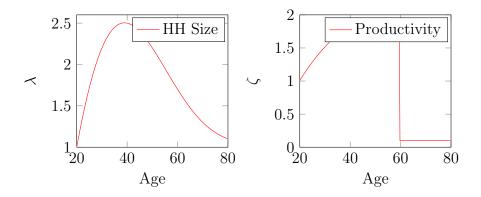


Figure 1: Heterogeneity over the life cycle

the literature (Iacoviello and Pavan (2013), Yang (2006) although he argues that his choice is very low), and also what is in the plain data ⁴. However, as is argued in reports by federal reserves and private banks (DiMartino and Duca (2007), McGill (2007)), this statistic may underestimate the true value, since it fails to account for "piggyback" mortgages. This term describes the practice to take out a second mortgage, to overcome the down-payment requirement for the first loan. "... the widespread popularity of second mortgages (piggybacks) in recent years, which are not included in traditional loan-to-value calculations, has made these LTV datapoints particularly misleading and almost irrelevant" (McGill (2007), page 30). The authors report an LTV value of 91 % based on a survey among their clients.

The discount rate for Workers is set to 0.989, while the general discount rate for worker households is calibrated to match asset positions observed in the data. The resulting value is 0.988. Note that worker households also discount with the age dependent utility preference parameter λ . η is calibrated to set the number of hours worked in the economy, efficiency adjusted, to one third of the total time endowment to the worker, which is normalized to unity. The other parameter values are quite standard, like for instance the capital share of $\alpha = 35\%$, or taken from related literature.

Another non-standard parameter is ξ , which governs the degree of nominality of the bond, which is the part of the bond that reacts to inflation. A value of 1 means that an increase of inlation by 1% translates to a 1% decrease in the value of the bond (c.f. 3.2).

 $^{^4\}mathrm{The}$ "Federal Housing Finance Agency, Monthly Interest Rate Survey" reports in Table 10 values of around 80%

Parameters	Variable	Values	
Number of cohorts	Т	240	
Retirement age in cohorts	T_{ret}	160	
Length of period in years	years	0.25	
Discount factor (workers)	β	calibrated to be 0.989846	
Discount factor (capitalists)	$egin{array}{c} eta \ \hat{eta} \end{array}$	$0.96^{0.25}$	
Real over nominal value	v^B	0.1	
Fraction maturing	μ	1	
Fraction of returns that react to inflation	χ	1	
Nominal coupon in Steady State	r^{X}	0.0102578	
Efficiency of owned housing	$rac{\xi}{\delta_K}$	1.2	
Capital depreciation rate	δ_K	$1 - 0.9^{0.25}$	
Housing depreciation rate	δ_H	$1 - 0.95^{0.25}$	
Output share of capital	α	0.35	
Payroll tax	τ	0.15	
Loan to value ratio	ν	0.9	
Weight of leisure in utility	$\mid \hspace{0.5cm} \eta \hspace{0.5cm} \mid$	calibrated to be 2.50788	
Weight of housing in utility	η_H	0.15	
Adjustment costs parameter capital	η_I	6	
Adjustment costs parameter housing	η_J	4	
Elasticity of substitution owned/rental	σ	2	
Autocorr. technology shock	$ ho_Z$	$0.8^{0.25}$	
Autorcorr. housing utility shock	$ ho_H$	$0.8^{0.25}$	
Persistency inflation in bond value	ρ_B	$0.99^{0.25}$	
Autocorr. monetary policy shock	$ ho_M$	0	
Autocorr. markup shock	$ ho_{\mu}$	0	
Influence past interest rate	$ ho_R$	$0.25^{0.25}$	
Probability of keeping the price	θ	0.75	
Demand elasticity	ε	7	
Reaction of monetary policy to inflation	γ_{π}	2.5	
Efficiency units of labor	ζ	see text	
Utility shifter	λ	see text	

Table 1: Baseline calibration of the model

6 Results

6.1 Steady State Results

A numerical solution of this model consists of a path for several variables, of the entire lifecycle for a worker, namely consumption, housing owned and housing rented, the assets held, the labor supply and the marginal utilities of consumption. Besides that, the steady state values also encompass housing and assets of the capitalist, aggregate output, capital, labor supply and housing, along with the

investment ratios in capital and housing. There is vast literature on housing and consumption profiles, as mentioned above (section 1). The model is able to reproduce the hump shaped consumption and housing profiles, which have been described in this literature. There is no discussion of the results for the capitalist, since there is no lifecycle structure and no constraint, and thus mimics the standard result of a consumer with exogenous income, since the capitalist does not supply labor in equilibrium.

6.1.1 Unconstrained

As a benchmark, the model is first solved without imposing a downpayment constraint for the worker household. The resulting paths over the lifecycle, can be seen in figure 2.

Both the consumption and the housing profile exhibit a hump shaped pattern over the lifecycle. Their path look like scaled versions of one another, as should be expected, since housing is just a specific consumption good with this model specification. Therefore, there exists an optimal ratio between those two goods, in the utility function. The same holds true for housing rented. In the last period, the household sells all the housing owned, and rents it out, an artificial feature introduced in the absence of a bequest motive. Labor exhibits a somewhat counter-factual trajectory, although it is similar to the findings in Iacoviello and Pavan (2013). The hump should face the other direction, and adjusting for efficiency unit does not entirely offset this counterfactual feature. Labor supply drops to 0 at the age of 60, when the worker retires. An interesting observation to make is by how much debt overshoots owned housing, which might serve as a collateral. This is depicted in the middle subpanel in the right column, where housing refers just to housing owned. The debt to housing ratio reaches a value of 3.0086, when the agent is 46.5 years old. This counter factually high debt ratio is a justification to introduce a constraint on borrowing, for reasons of plausibility.

6.1.2 Downpayment Constraint

The next case is the one where the downpayment constraint is actually enforced. The time paths for the constrained case are depicted in 3. Worker households start out with zero debt, buy a house and take a small loan against it from their first wage payment. They then start to save for overcoming the downpayment requirement and buying a bigger house, while continuously increasing consumption and decreasing labor. With the bigger house comes bigger debt, and households hit the debt limit at the age of 36 (cohort 65) and stay there until the age of 51

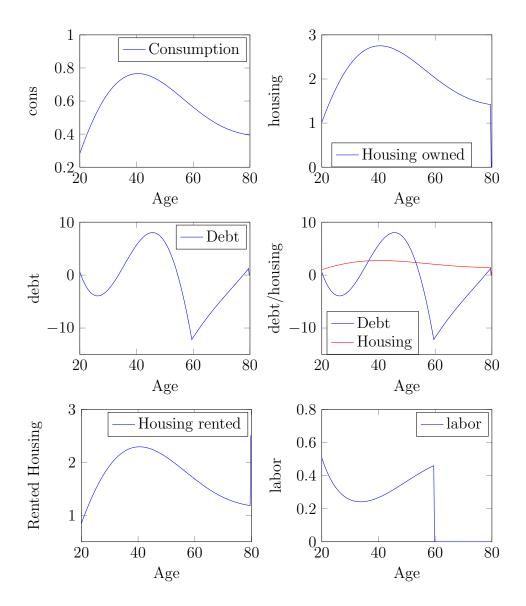


Figure 2: Lifecycle paths for labor, consumption, housing owned and rented, and debt without downpayment constraint

(cohort 124). Upon hitting the debtlimit, households would like to consume even more, and thus they reduce housing and spend it on consumption, since the high marginal utility of consumption in this phase of their life brought them to the constraint in the first place. As soon as the households overcome the debt limit, they start building huge asset positions to save for retirement, while decreasing the size of their houses and consumption. Notice that the selling of all owned housing in the final period is exogenously imposed, to avoid bequests.

Labor supply again exhibits a similar initial decrease, as seen in figure 2, but stays constant while the household is at the debt limit. Adjusted for labor-efficieny, the supplied labor increases in this period, which is the intuitive reaction to being borrowing constrained.

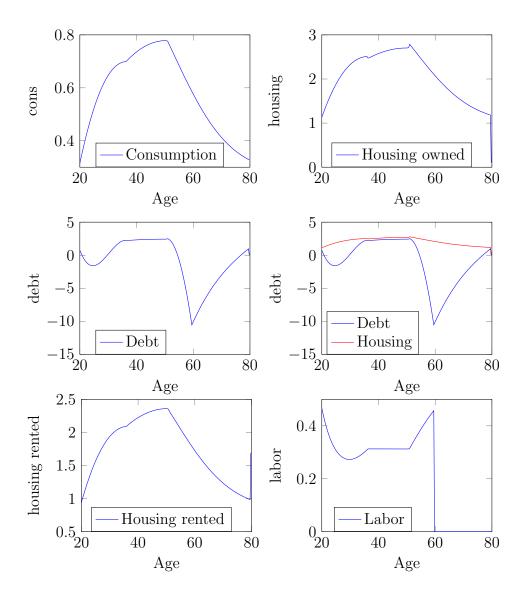


Figure 3: Lifecycle paths for labor, consumption, housing owned and rented, and debt with downpayment constraint

6.2 Redistribution through Monetary Policy

The main feature of using an OLG model is the existence of heterogenous agents. As shown in the previous section, different cohorts hold different positions in assets, which allows to measure how the same shock to the economy has different effects. This is done using impulse response functions (IRFs). Figure 4, has been created using a one time, contractionary monetary policy shock, with no propagation to the next period.

6.2.1 Strongest Reactions

This graph(figure 4) shows that studying all IRFs is not feasible. The interesting features of this graph can be seen in the very first period, which show that there

Figure 4: Impulse Responses to a Contractionary Monetary Shock

period

are on reactions in both directions, people selling housing and people acquiring more. This holds true for housing and bondholdings, but the on-impact-reactions of consumption and labor are all negative, implying that the substitution effect is dominant on impact. The agent whose housing owned goes down most on impact, is the agent in his last period of being borrowing constrained at the age of 51. This is intuitive, because the debt peaks at this point in time, and therefore a strong reaction is needed to be able to save for requirement. A summary over the biggest and second biggest beneficiaries and losers of a contractionary monetary policy shock for each variable can be seen in table 2.

Variable	_	-	+	++
assets	51	51	50	51
	(126)	(127)	(123)	(124)
consumption	80	49	60	60
	(240)	(119)	(161)	(160)
labor	27	27	49	49
	(31)	(30)	(117)	(118)
housing rented	80	79	50	51
	(240)	(239)	(123)	(124)
housing owned	51	50	60	60
ilousing owned	(124)	(123)	(161)	(160)

First row: Age of Cohort with strongest or second strongest positive or negative reactions, Second row: Cohort ID

Table 2: Extreme Impacts of a contractionary Monetary Policy shock

This table depicts the strongest and second strongest reactions to an increase in the interest rate due to a monetary intervention over all cohorts. For all this analysis, the assumption that the shock does not push people from, or to, or even over the downpayment constraint. Since the Impulse responses are linear in the size of the shock, this assumption is not overly restrictive for this exercise, since the shock could be scaled down to be arbitrarily close to zero.

The assets of the cohorts react in such a way, that people increase their positions, while other cohorts sell assests. The strongest reactions take place around the time the borrowing constraint stops to be binding. People who are constrained want to reduce their debt, because the interest rate will make it harder to repay. One quarter after leaving the constrained state, people would decrease their bondholdings. This is around the time where the household's debt peaks, thus a higher interest rate implies higher costs of refinancing, combined with a negative wealth effect.

Consumption declines for every cohort on impact, which is simply given by the Euler Equation, since next periods consumption becomes cheaper and the resulting substitution effect. The strongest negative reaction occurs for households in their last period, a result which is artificial, since people go with debt into the last periods, and have to sell their houses at the end of their life. The second strongest downturn is experienced by a cohort somewhere in the middle of the constrained period, since they are experiencing higher interest payments for the debt. The cohort which is benefiting the most is the retiring cohort at the age of 60, since this is when the savings are really high, and thus it means more income. This is supported by the fact, that the second strongest consumption increase can be observed by the following cohort, also at age 60 (cohort 161, compared to 160).

For the labor responses, only the Impulse Reponses for the cohorts up to retirement age where considered, since all others are zero anyway. As with consumption, all responses are negative. This is because the marginal utility of consumption decreases with higher interest rate, and via this link the utility of labor also decreases, leading to a decrease in labor supply for each cohort. Other effects, e.g. via the housing choice, are not strong enough to overcome this relationship. The higher interest rate also affects the firm, since it has to pay higher prices. It thus reduces output, due to its assumed market power. The changes in labor supply are the only reactions, which are not centered around retirement age, death, or leaving/entering the constrained time of a households life.

Housing rented and housing owned reacts differently to a monetary policy shock. While the strongest positive reactions for rented housing occur just before exiting the downpayment constraint, for onwed housing it occurs right after retirement. This can be due to the bondholdings, which are lowest (meaning highest debt), just before the household exits the downpayment constraint. As they react strongly positive there, housing

6.2.2 Utility Comparison

After Identifying the strongest absolute reactions on impact, another interesting question is who is made of better or worse off by the shock. This evaluation of redistributive effects is done in terms of utility, by comparing the lifetime utility for each cohort in the steady state, with its corresponding lifetime utility after the shock hits. Notice that now the size of the shock matters, since the reactions are directly proportional to the size of the shock. For this comparision (and also for the previous absolute changes) a shock of size 0.01 is used.

The lifetime utilities are all negative, a consequence of the parametrization of the model. However, this is not an issue, since the values are only compared to different bundles, plugged into the same utility function. The result of this exercise can be seen in figure 5. It depicts for age cohort the change in lifetime utility, between the regular steady state lifecycle, and the new lifecycle after the shock.

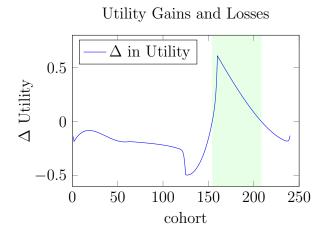


Figure 5: Welfare Comparision after a Contractionary Monetary Policy shock

The monetary policy shock raises the interest rate, therefore people holding bonds are better off. The green shaded area shows cohorts who benefits from the monetary policy intervention, namely people aged 58 to 72 (corresponding cohorts are 155 to 208). Their asset position and the resulting wealth affect boosts their utility, while younger cohorts who also hold positive asset positions still lose from this reform, since they have to pay higher refinancing costs over their lifecycle. Households older than 72 are affected negatively, because the pensions declined, due to initially lower labor supply. Although their asset positions are still positive

up to the age of 77, the positive wealth effect is not strong enough to overcome the lower pensions.

6.3 Redistributive effects of a markup shock

The previous considerations were based on a shock to monetary policy. The next thing to consider is a Markup Shock, which raises the price level, and thus increases the distortion in the production sector. The original distortion comes from the monopolistic power of the firm. The increase in the price level additionally triggers an immediate counteraction of the Central Bank, via the Taylor Rule. The markup shock used for this analysis is 0.1. How do the redistributive effects of this look like, in terms of utility? The answer can be seen in figure 6. The green area, again depicts the winning cohorts, which are now the cohorts aged 60 to 64, or the cohorts 160 to 177. These people gain utility, because the reaction of the Central Bank to the higher price level, namely the increase of the interest rate, helps them to offset the price level increase. They are able to do so, because the wealth effect is affecting them strongly, due to their high savings. All others are hit badly by the higher price level, and the higher interest rate. This is especially true for constrained households, since although the shock does not propagate to the next period, it does via the recursive formula of the price level. Thus the nominal level of bonds stays below its steady state level for the entire period. This reduces the lifetime utility of the younger cohorts, since the savings they will acquire will be less valuable.

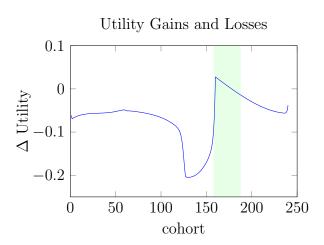


Figure 6: Welfare Comparision after a positive Markup shock

An interesting exercise that comes to mind, is to check for different effects when the reaction of the Central Bank differs. When lowering the parameter of the reaction to the price level in the Taylor rule, γ_{π} , from 2.5 to 1, the difference is

enormous. All cohorts are now worse off by the markup shock, and the resulting higher inflation. The effect for each cohort can be seen in the left panel of 7. The right panel shows the changes in utility, when the Central Bank adopts a stricter course against inflation. The parameter γ_{π} is raised to 4. This improves the welfare situation, it makes more cohorts, (from 160 to 193) better off. The young cohorts still lose in each situation.

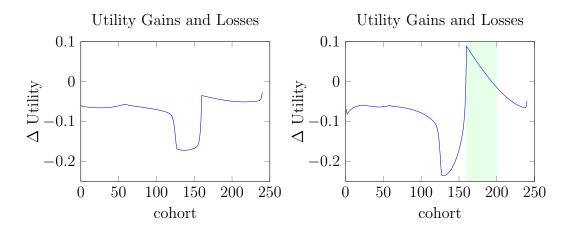


Figure 7: Welfare Comparision after a positive Markup shock, different reaction parameters

7 Conclusion and Outlook

This master thesis lays out a stochastic general equilibrium model, which addresses the housing and consumption choice of borrowing constrained agents, over their lifecycle. It separates workers from capitalists, to create a share of agents, whose biggest asset is the house they are living in. Due to endogenous downpayment constraints, the worker households have to acquire funds before buying a house, and operate at the constraint for a long period of their lifecycle. The model replicates the hump shaped consumption and housing profiles, which are observed in the literature, and replicates some additional benchmark ratios from the data. After describing the steady state results of the variables over the lifecycle, several shocks and their effects on these paths are considered. A first result is that, not surprisingly, the strongest reactions are taken by people who are around the times of the lifecycle, when their behavior changes significantly. We see the strongest reactions around the time when they move away from the borrowing constraint or retire. Parameters which determine these events, are thus crucial for the results of the model. For the comparision of welfare, the lifetime utility is computed, for each cohort, in steady state and after the shock hit the economy. A contractionary monetary policy shock raises the interest rate, and makes people with high savings better off, while leaving the young worse off. When a markup shock hits the economy and leads to a rise of the price level, a crucial parameter for welfare changes is how strongly the Central Bank reacts via the Taylor Rule. If the reaction is weak, all the lifetime utility of all cohorts is affected negatively, but more evenly than with a strong reaction. A strong reaction implies again a higher interest rate, thus leaving the old cohorts better off. Setting this parameter lower than 1 leads to indeterminate solutions. Thus the model cannot directly confirm or object a finding by Brunnermeier and Sannikov (2012), who finds that surprise inflation should make young and constrained households better off.

It is, to my best knowledge, the first attempt to quantify the effects of monetary policy and other shocks in a full-scale lifecycle model. Therefore, there are naturally plenty of opportunities for improvements. It can be seen as a shortcoming of the model that the stock of housing owned by the worker households changes every period, and the workers own a house in the first period. Introducing transaction costs for adjusting housing or a minimum house size for owning might resolve these issues, and would create regions of inaction. The homotheticity of the utility function creates a ratio of housing (owned and rented) and consumption which are optimal for the household to acquire, instead of trading off one for the other. Differently formed utility functions would be a suitable way to add this interesting feature into the model. Finding only small reactions to a markup-shock can be due to the maturity of nominal bonds, which is set to 1 period. For bonds with longer duration the effect would be larger, since in the benchmark setup the bonds can be refinanced each period. So with longer duration, effects would grow stronger. The comparison between the effects of short run bonds and long run bonds constitutes an interesting exercise for future research. One could explore the effects of TFP shocks in differently leveraged economies, to test a result of Mian and Sufi (2010), who find that more leveraged economies suffered a slower recovery from recessions. Yet another fruitful exercise, would be to compare the redistributive effects under a stricter down-payment constraint, with the one produced in the current setup. They should be smaller, since people are less indebted, which might have consequences for the influence of monetary policy on aggregate variables, in the sense that the potency is weaker.

A Appendix

A.1 Derivation of Tobin's Q and F.O.C for Capital

The Law of Motion for Capital takes the form

$$K_t = (1 - \delta_K)K_{t-1} + \phi(\iota_t, \delta_K, \eta_I)K_{t-1}$$

Now the problem of the firm can be written in terms of the following recursive value function

$$V(K_{t-1}) = \max_{I_t, K_t} R_t^K K_{t-1} - I_t + \Lambda_{t,t-1} V(K_t)$$

subject to

$$K_t = (1 - \delta_K)K_{t-1} + \phi(\iota_t, \delta_K, \eta_I)K_{t-1}$$

where R_t^K is the return on capital it should charge and $\Lambda_{t,t-1}$ is the discount factor between periods t and t+1. Notice that the value function is assumed to be time invariant.

The resulting Lagrangian is then given by

$$\mathcal{L} = R_t^K K_{t-1} - I_t + \Lambda_{t,t-1} V(K_t) + Q_t ((1 - \delta_K) K_{t-1} + \phi(\iota_t, \delta_K, \eta_I) K_{t-1} - K_t)$$

Taking the derivative with respect to investment and setting this equal to zero yields

$$\frac{\partial \mathcal{L}}{\partial I_t} = -1 + Q_t \phi'(\iota_t, \delta_K, \eta_I) K_{t-1} = 0$$

Rearranging the terms gives an expression for Q_t

$$Q_t = \frac{1}{\phi_I'(\iota_t^K, \delta_K, \eta_I)}$$

The F.O.C for Capital are obtained by taking the derivative with respect to K_{t+1} and setting it equal to zero

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = \Lambda_t V'(K_{t+1}) - Q_t = 0$$

Now using the fact that

$$V'(K_{t}) = \frac{\partial V(K_{t})}{\partial K_{t}} = R_{t+1}^{k} + \Lambda_{t+1,t} V'(K_{t+1}) \frac{\partial K_{t+1}}{\partial K_{t}}$$
$$= R_{t+1}^{k} + \Lambda_{t+1,t} V'(K_{t+1}) \left[1 - \delta_{K} + \phi \left(\frac{I_{t}}{K_{t}} \right) - \phi' \left(\frac{I_{t}}{K_{t}} \right) \frac{I_{t}}{K_{t}} \right]$$

and our relationship for Q_t , we obtain

$$V'(K_{t+1}) = R_{t+1}^k + Q_{t+1} \left[1 - \delta_K + \phi \left(\frac{I_{t+1}}{K_{t+1}} \right) \right] - \frac{I_{t+1}}{K_{t+1}}$$
$$Q_t = \Lambda_{t+1,t} \left[R_{t+1}^k + Q_{t+1} \left[1 - \delta_K + \phi \left(\frac{I_{t+1}}{K_{t+1}} \right) \right] - \frac{I_{t+1}}{K_{t+1}} \right]$$

A.2 Derivation of the First Order Conditions Worker Households

The Worker Household chooses consumption, housing (renting or owning) and their laborsupply to maximize lifetime utility, where they live until period T.

$$\mathbb{E}_0 \sum_{t=0}^{T} \beta^t \lambda_{i+t} u(c_t, l_t, h_t^O, h_t^R)$$

The utility function of the household is given by

$$U(c, l, h^R, h^O, \eta_H) = log(c) + \eta log(1 - l) + \eta_H \log(((h^R)^{(\sigma - 1)/\sigma} + \xi(h^O)^{(\sigma - 1)/\sigma})^{\sigma/(\sigma - 1)})$$
(A.1)

and they have to observe a budget constraint

$$p_t^B b_{i,t} + h_{i,t}^O p_t^H + c_{i,t} + r_t^H h_{i,t}^R =$$

$$(1 - \tau) w_t \zeta_i l_{i,t} + \mathcal{I}_{Ri} \psi_t +$$

$$((\mu + r^B) v_t^B + (1 - \mu) p_t^B) b_{i,t-1} +$$

$$(1 - \delta_H) h_{i-1,t-1}^O p_t^H)))$$
(A.2)

and a down-payment constraint, which is given by

$$-b_{i,t} \le \nu \mathbb{E}_{p_{t+1}^H H_t^O} \tag{A.3}$$

For solving the model, it is convenient tor transfer this into a new variable, namely:

$$X_t \equiv \nu \mathbb{E}_{p_{t+1}^H H_t^O} + b_{i,t} \tag{A.4}$$

which now reduces the constraint to $X_t \geq 0$

$$\mathcal{L}: \max_{c_{t}, l_{t}, h_{t}^{r}, H_{t}^{O}, X_{t}} \mathbb{E}_{0} \sum_{t=0}^{T} \beta^{t} \lambda_{t} u(c_{t}, l_{t}, h_{t})$$

$$+ \kappa_{t} [p_{t}^{B} X_{t} - \nu \mathbb{E}_{p_{t+1}^{H} H_{t}^{O}} + h_{i,t}^{O} p_{t}^{H} + c_{i,t} + r_{t}^{H} h_{i,t}^{R} - (1 - \tau) w_{t} \zeta_{i} l_{i,t} - \mathcal{I}_{Ri} \psi_{t} - ((\mu + r^{B}) v_{t}^{B} + (1 - \mu) p_{t}^{B}) X_{t-1} - \psi p_{t}^{H} h_{t-1}^{O} - (1 - \delta_{H}) h_{i-1,t-1}^{O} p_{t}^{H})) - r_{t}^{H} h_{i,t}^{R})]$$

$$+ \gamma_{t} X_{t}$$

Now taking the derivatives with respect to all the choice variables, the following First order conditions are obtained

$$\frac{\partial \mathcal{L}}{\partial c_t} : \lambda_t u_{c_t} = -\kappa_t$$

$$\frac{\partial \mathcal{L}}{\partial l_t} : \lambda_t u_{l_t} = \kappa_t [(1 - \tau) w_t \zeta_t]$$

$$\frac{\partial \mathcal{L}}{\partial h_t^R} : \lambda_t u_{h_t^R} = -\kappa_t r_t^H$$

$$\frac{\partial \mathcal{L}}{\partial h_t^O} : \lambda_t u_{h_t^O} + \kappa_t [p_t^H - P_t^B \nu \mathcal{E} p_{t+1}]$$
$$+ \beta \kappa_{t+1} [[(\mu + r^B) v_{t+1}^B + (1 - \mu) p_{t+1}^B] \psi P_t^H - (1 - \delta_H) p_{t+1}^H] = 0$$

$$\frac{\partial \mathcal{L}}{\partial X_t} : \kappa_t p_t^B + \gamma_t - \beta \kappa_{t+1} [(\mu + r^B) v_{t+1}^B + (1 - \mu) p_{t+1}^B] = 0$$

Combining the above equations yields the desired Equations for the Household

$$-(1-\tau)w_t\zeta_i u_{c_{i,t}} = u_{l_{i,t}} \tag{A.5}$$

$$u_{h_{i,t}^R} = r_t^H u_{ci,t} \tag{A.6}$$

$$u_{h_{i,t}^{O}} = u_{c_{i,t}}(p_{t}^{H} - p_{t}^{B}\nu p_{t+1}^{H}) + \frac{\lambda_{t+1}}{\lambda_{t}}u_{c_{i+1,t+1}}\beta$$

$$\times \left[((\mu + r^{B})v_{t+1}^{B} + (1 - \mu)p_{t+1}^{B})\nu \mathbb{E}p_{t+1}^{H} - (1 - \delta_{H})p_{t+1}^{H} \right]$$
(A.7)

$$\lambda_t u_{c_{i,t}} p_t^B - \gamma_{i,t} = \beta \lambda_{t+1} u_{c_{i+1,t+1}} ((\mu + r^B) v_{t+1}^B + (1 - \mu) p_{t+1}^B)$$
 (A.8)

A.3 Derivation of the First Order Conditions Capitalists

The utility function of the Capitalists (after anticipating the equilibrium outcome that capitalists do not supply labor) is given by

$$\hat{u}(\hat{c}, \hat{h}^O, \eta_H) = \log(\hat{c}) + \eta_H \log((\hat{h}^O)) \tag{A.9}$$

which leads to the inter temporal maximization problem

$$\max \sum_{t=0}^{\infty} \hat{\beta} \hat{u}(\hat{c}_t, \hat{h}_t^O, \eta_H) \tag{A.10}$$

subject to the intertemporal budget constraint

$$Y_{t} - I_{t}^{K} - w_{t}L_{t} + r_{t}^{H}H_{t}^{R} + p_{t}^{B}B_{t} + p_{t}^{H}(H_{t} - (1 - \delta_{H})H_{t-1}) - I_{t}^{H} = \hat{c}_{t} + p_{t}^{H}(\hat{h}_{t}^{O} + H_{t}^{R} - (1 - \delta_{H})(\hat{H}_{t-1} + H_{t-1}^{R})) + ((\mu + r^{B})v_{t}^{B} + (1 - \mu)p_{t}^{B})B_{t-1}$$
(A.11)

Forming a Lagrangian with Lagrange multiplier ψ (not to be confused with the pension payments to the retired worker households) and taking the derivatives with respect to \hat{c}_t , \hat{h}_t^O , H_t^R and B_t yields

$$\frac{\partial \hat{\mathcal{L}}}{\partial \hat{c}_t} : \hat{\beta}^t \hat{u}_{\hat{c}_t} = \psi_t$$

$$\frac{\partial \hat{\mathcal{L}}}{\partial \hat{h}_t^O} : \hat{\beta}^t u_{\hat{h}_t^O} = -\psi_t p_t^H + \psi_{t+1} p_{t+1}^H (1 - \delta_H)$$

$$\frac{\partial \hat{\mathcal{L}}}{\partial B_t} : \psi_t p_t^B = \hat{\beta} \psi_{t+1} ((\mu + r^B) v_{t+1}^B + (1 - \mu) p_{t+1}^B)$$

$$\frac{\partial \hat{\mathcal{L}}}{\partial H_t^R} : \psi_t [r_t^H - p_t^H] + \hat{\beta} \psi_{t+1} p_{t+1}^H (1 - \delta_H) = 0$$

Using A.3 to substitute out the Lagrange multipliers ψ , one obtains the relations given in section 3.1.2

A.4 Optimal Pricing

There is a continuum of firms in the economy normalized to the zero one interval. (Gali, 2008) Firms are indexed by $i \in [0,1]$ As is standard in New Keynesian models the price level is defined as $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$.

Since a fraction of θ has to keep their price constant each period (this can be thought of as infinitely high adjustment costs), and due to inflation there is a unique optimal price p*, all firms who have the chance to adjust their prices will set their new price to p*. Let $S(t) \subset [0,1]$ represent the firms not optimizing in period t. Then at each period in time the aggregate price level P_t is given by

$$P_{t} = \left[\int_{S(t)} P_{t-1}(i)^{1-\varepsilon} di + (1-\theta)(P_{t}^{*})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

which is obtained by just plugging in for the definition of P_t . Using the fact that the mass of firms not allowed to reallocate is θ

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Dividing both sides by P_{t-1} and then taking the resulting equation to the power of $1 - \varepsilon$ gives in steps

$$\frac{P_t}{P_{t-1}} \equiv \Pi_t = \frac{\left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{P_{t-1}}$$
$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

The last equation, is the relationship which is used to describe the development of inflation in the model economy.

So now if we redefine P^* , and call

$$Pstar_t = \frac{P_t^*}{P_t} \tag{A.12}$$

This redefinition is necessary, to make P_t^* a stationary variable.

For the optimal pricing equations, the optimality condition of Gali (2008) is a natural starting point. However, it is convenient to use a different normalization, namely normalizing by P_t instead of P_{t-1} .

The slightly modified equation 10 of chapter 3 is

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[Q_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_t} - \mathcal{M}MC_{t+k|t} \Pi_{t,t+k} \right) \right] = 0 \tag{A.13}$$

where $\mathcal{M} = \frac{\epsilon}{\epsilon - 1}$. Since in our problem the Marginal costs are independent of the history, but just depend on the prices in the period (a consequence of the Constant Returns to Scale assumption), $MC_{t+k|t}$ reduces to MC_t .

To program infinite sums, it is necessary to transform them into recursive value functions. It is easier to consider the two sides of the equation separately.

The **Left Hand Side** is thus given by

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[Q_{t,t+k} Y_{t+k|t} \frac{P_t^*}{P_t} \right]$$
 (A.14)

Explicitly writing the Q term gives

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\hat{\beta}^k \left(\frac{\hat{U}_{ct+k}}{MUCC_t} \right) \frac{P_t}{P_{t+k}} Y_{t+k|t} \frac{P_t^*}{P_t} \right]$$

Pulling out the first period

$$Y_t \frac{P_t^*}{P_t} + \sum_{k=1}^{\infty} \theta^k \mathbb{E}_t \left[\hat{\beta}^k \frac{\hat{U}_{ct+k}}{\hat{U}_{ct}} \frac{P_t}{P_{t+k}} Y_{t+k|t} \frac{P_t^*}{P_t} \right]$$

Redefining the sum from k=1 to k=0

$$Y_{t} \frac{P_{t}^{*}}{P_{t}} + \sum_{k=0}^{\infty} \theta^{k+1} \mathbb{E}_{t} \left[\hat{\beta}^{k+1} \frac{\hat{U}_{ct+k+1}}{\hat{U}_{ct}} \frac{P_{t}}{P_{t+k+1}} Y_{t+k+1|t} \frac{P_{t}^{*}}{P_{t}} \right]$$

Pulling out $\theta\beta$

$$Y_{t} \frac{P_{t}^{*}}{P_{t}} + \theta \hat{\beta} \sum_{k=0}^{\infty} \theta^{k} \mathbb{E}_{t} \left[\hat{\beta}^{k} \frac{\hat{U}_{ct+k+1}}{\hat{U}_{ct}} \frac{P_{t}}{P_{t+k+1}} Y_{t+k+1|t} \frac{P_{t}^{*}}{P_{t}} \right]$$

Manipulating the bracket expressions

$$Y_{t} \frac{P_{t}^{*}}{P_{t}} + \theta \hat{\beta} \sum_{k=0}^{\infty} \theta^{k} \mathbb{E}_{t} \left[\hat{\beta}^{k} \left(\frac{\hat{U}_{ct+k+1}}{\hat{U}_{ct}} \frac{\hat{U}_{ct+1}}{\hat{U}_{ct+1}} \right) \left(\frac{P_{t}}{P_{t+k+1}} \frac{P_{t+1}}{P_{t+1}} \right) Y_{t+k+1|t} \frac{P_{t}^{*}}{P_{t}} \right]$$

Now, assuming that \mathbb{E}_t was \mathbb{E}_{t+k} from the beginning onwards, we can now pull out most of the discount factor we want to have

$$Y_{t} \frac{P_{t}^{*}}{P_{t}} + \theta \hat{\beta} \frac{\hat{U}_{ct+1}}{\hat{U}_{ct}} \left(\frac{P_{t+1}}{P_{t}}\right)^{-1} \sum_{k=0}^{\infty} \theta^{k} \mathbb{E}_{t} \left[\hat{\beta}^{k} \frac{\hat{U}_{ct+k+1}}{\hat{U}_{ct+1}} \frac{P_{t+1}}{P_{t+k+1}} Y_{t+k+1|t} \frac{P_{t}^{*}}{P_{t}} \right]$$

which is now a recursively written value function. Using the definition of $Pstar_t$

$$Y_{t}Pstar_{t} + \theta \hat{\beta} \frac{\hat{U}_{ct+1}}{\hat{U}_{ct}} \left(\frac{P_{t+1}}{P_{t}}\right)^{-1} \sum_{k=0}^{\infty} \theta^{k} \mathbb{E}_{t} \left[\hat{\beta}^{k} \frac{\hat{U}_{ct+k+1}}{\hat{U}_{ct+1}} \frac{P_{t+1}}{P_{t+k+1}} Y_{t+k+1|t} Pstar_{t} \right]$$

Now turning to the Right Hand Side

$$\sum_{k=0}^{\infty} \theta^{k} \mathbb{E}_{t} \left[Q_{t,t+k} Y_{t+k|t} \mathcal{M} M C_{t} \frac{P_{t+k}}{P_{t}} \right]$$
(A.15)

Again, writting out the term Q

$$\sum_{k=0}^{\infty} \theta^{k} \mathbb{E}_{t} [\hat{\beta}^{k} \left(\frac{\hat{U}_{ct+k}}{\hat{U}_{ct}} \right) \frac{P_{t}}{P_{t+k}} Y_{t+k|t} \mathcal{M}MC_{t} \frac{P_{t+k}}{P_{t}}]$$

Notice that the two price fractions cancel out.

$$\mathcal{M}Y_{t}MC_{t} + \sum_{k=1}^{\infty} \theta^{k} \mathbb{E}_{t} \left[\hat{\beta}^{k} \frac{\hat{U}_{ct+k}}{\hat{U}_{ct}} Y_{t+k|t} \mathcal{M}MC_{t} \right]$$

Redefining k

$$\mathcal{M}Y_{t}MC_{t} + \sum_{k=0}^{\infty} \theta^{k+1} \mathbb{E}_{t+k+1} \left[\hat{\beta}^{k+1} \frac{\hat{U}_{ct+k+1}}{\hat{U}_{ct}} Y_{t+k+1|t} \mathcal{M}MC_{t} \right]$$

and again assuming that the Expectation is taken in each period, it is possible to pull out θ and β . This leads to

$$\mathcal{M}Y_{t}MC_{t} + \theta \hat{\beta} \sum_{k=0}^{\infty} \theta^{k} \mathbb{E}_{t+k+1} \left[\hat{\beta}^{k} \frac{\hat{U}_{ct+k+1}}{\hat{U}_{ct}} Y_{t+k+1|t} \mathcal{M}MC_{t} \right]$$

Again Expanding the fractions

$$\mathcal{M}Y_t M C_t + \theta \hat{\beta} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_{t+k+1} \left[\hat{\beta}^k \left(\frac{\hat{U}_{ct+k+1}}{\hat{U}_{ct}} \frac{\hat{U}_{ct+1}}{\hat{U}_{ct+1}} \right) Y_{t+k+1|t} \mathcal{M} M C_t \right]$$

Now taking out each expression in period t+1 from the expectation and the sum gives

$$\mathcal{M}Y_t M C_t + \theta \hat{\beta} \frac{\hat{U}_{ct+1}}{\hat{U}_{ct}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_{t+k+1} \left[\hat{\beta}^k \frac{\hat{U}_{ct+k+1}}{\hat{U}_{ct} + 1} Y_{t+k+1|t} \mathcal{M} M C_t \right]$$

References

- ARRONDELN, L. AND A. LAFERRERE (1998): "Capitalist versus Family Bequest: an Econometric Model with Two Endogenous Regimes," *Unpublished Working Paper*.
- Attanasio, Orazio P, e. a. (1999): "Humps and Bumps in Lifetime Consumption," *Journal of Business & Economic Statistics*, 17, 22–35.
- BRICKER, J., A. KENNICKELL, K. MOORE, AND J. SABELHAUS (2012): "Changes in U.S. Family Finances from 2007 to 2010: Evidence from the Survey of Consumer Finances," *Federal Reserve Bulletin*, 98.
- Browning, M., M. Grtz, and S. LethPetersen (2013): "Housing Wealth and Consumption: A Micro Panel Study," *Economic Journal*, 0, 401–428.
- Brunnermeier, M. K. and Y. Sannikov (2012): "Redistributive monetary policy," *Proceedings Economic Policy Symposium Jackson Hole*, 331–384.
- Buiter, W. H. (2008): "Housing Wealth Isn't Wealth," NBER Working Papers 14204, National Bureau of Economic Research, Inc.
- CAGETTI, M. (2003): "Wealth Accumulation over the Life Cycle and Precautionary Savings," Journal of Business & Economic Statistics, 21, 339–53.
- Calvo, G. (1983): "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics*, 12, 383–398.
- CLAIRE, D. (1991): "Capital stock and capital output ratios: a comparison between the United States, United Kingdom and the Federal Republic of Germany," *International Institute for Applied Systems Analysis*, 29.
- DIMARTINO, D. AND J. DUCA (2007): "The Rise and Fall of Subprime Mortgages," Economic Letter, Federal Reserve Bank of Dallas.
- DOEPKE, M. AND M. SCHNEIDER (2006a): "Inflation and the Redistribution of Nominal Wealth," *Journal of Political Economy*, 114, 1069–1097.
- ———— (2006b): "Inflation as a Redistribution Shock: Effects on Aggregates and Welfare," NBER Working Papers 12319, National Bureau of Economic Research, Inc.
- FERNNDEZ-VILLAVERDE, J. AND D. KRUEGER (2007): "Consumption over the Life Cycle: Facts from Consumer Expenditure Survey Data," *The Review of Economics and Statistics*, 89, 552–565.

- Gahvari, F. and L. Micheletto (2012): "Monetary Policy and Redistribution: What can or cannot be Neutralized with Mirrleesian Taxes," Tech. rep.
- Gali, J. (2008): Monetary Policy, Inflation and the Business Cycle, Princeton University Press.
- GARRIGA, C., F. E. KYDLAND, AND R. SUSTEK (2013): "Mortgages and Monetary Policy," NBER Working Papers 19744, National Bureau of Economic Research, Inc.
- GOODWIN, R. (1967): "A Growth Cycle," Socialism, Capitalism and Economic Growth. Essays presented to Maurice Dobb., Camebridge University Press, 54–58.
- Gornemann, N., K. Kuester, and M. Nakajima (2012): "Monetary policy with heterogeneous agents," Tech. rep.
- Hansen, G. (1993): "The cyclical and seuclar behaviour of the labour input: comparing efficiency units and hours worked," *Journal of Applied Econometrics*, 8.
- IACOVIELLO, M. (2011): "Housing Wealth and Consumption," Board of Governors of the Federal Reserve System.
- IACOVIELLO, M. AND M. PAVAN (2013): "Housing and debt over the life cycle and over the business cycle," *Journal of Monetary Economics*, 60, 221–238.
- Kapteyn, A. and A. van Soest (2006): "Savings, Portfolio Choice, and Retirement Expectations," *Unpublished Working Paper*.
- Kocherlakota, N. (2000): "Creating Business Cycles Through Credit Constraints," Quarterly Review, Federal Reserve Bank of Minneapolis.
- McGill, D. E. A. (2007): "Mortgage Liquididty du Jour: Underestimated No More," Sector Review, Creidte Suisse.
- MIAN, A. AND A. SUFI (2010): "Household Leverage and the Recession of 2007 to 2009," NBER Working Papers 15896, National Bureau of Economic Research, Inc.

- TOBIN, J. (1969): "A General Equilibrium Approach To Monetary Theory," Journal of Money, Credit and Banking, 1, 15–29.
- Wang, X. (2009): "Down Payment Constraint and Wealth Accumulation," *Unpublished Manuscript*.
- WILLIAMSON, S. D. (2008): "Monetary policy and distribution," *Journal of Monetary Economics*, 55, 1038–1053.
- YANG, F. (2006): "Consumption Over the Life Cycle: How Different is Housing?" Working Paper 635, Federal Reserve Bank of Minneapolis.