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MSc Economics

Self-Enforcing Climate Change Treaties: A Game Theoretic Approach

A Master's Thesis submitted for the degree of
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supervised by

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65 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

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Abstract

Climate change represents a prime example of a negative externality on a global scale. The failure of existing international agreements points out the need for treaties to be self-enforcing. In the language of game theory, this corresponds to a subgame perfect Nash equilibrium in an underlying climate change game. A dynamic game introduced by Dutta and Radner (2004) is used to model the situation. In the basic version, countries only choose emission levels every period, which yield immediate private benefits through their production functions but future global costs by increasing the level of greenhouse gases.

Besides a review of the model as well as of existing extensions, my own contributions are threefold: First, I show that certain properties of this dynamic game stem from the fact that there exists an associated repeated game which is strategically equivalent. In particular, equilibrium strategy profiles are equivalent up to a simple transformation that accounts for the different strategy domains in the dynamic and in the repeated game. Furthermore, equilibrium payoff sets are equivalent up to a linear transformation reflecting the exogenously given greenhouse gas level in period zero (which is not accounted for in the repeated game).

Second, I investigate numerically how the benchmark outcome, a myopic Markov equilibrium, changes once linearity of the climate change cost function is replaced by convexity. By approximating the value function with Chebyshev polynomials, I find that equilibrium emissions are no longer constant but convex and decreasing in the greenhouse gas level.

Third, I introduce trade decisions into the model in order to allow for trade sanctions as punishment device instead of sharp emission increases, which might not be feasible due to short-term irreversibilities. I characterize the best equilibrium under trade sanctions and show under which conditions trade sanctions can achieve a better equilibrium outcome than emission increases.

Chapter 1

Introduction

This thesis discusses game theoretic perspectives on climate change. There is broad scientific consensus that climate change poses a significant threat to mankind and that this is very likely due to the observed and projected increase in anthropogenic greenhouse gas (GHG) concentrations (see IPCC, 2007). Determining the level of globally optimal emission levels has been subject to extensive economic analysis, an excellent overview is Stern (2007)[1]. As these estimates are subject to a considerable amount of uncertainty and ethical questions such as how to discount future generations' utility, the extent of Pareto optimal abatement is debatable. However, there is general consent that current emission levels are too high.

Climate change represents a prime example of a negative externality on a global scale. While benefits from emissions are private, associated costs are global. The question that arises is how to design an international treaty that mitigates or even solves this dilemma. The failure of existing agreements has pointed out the need for treaties to be self-enforcing. At this point, game theory gets into the focus of analysis. Climate change is modeled as a non-cooperative game, where the world's countries are the players of the game and a self-enforcing treaty refers to a subgame perfect Nash equilibrium. While some authors (see Barrett, 2003; Finus, 2001) modeled the situation as static one-shot or pure repeated game,

[1] There is a large literature dealing with the economics of climate change, other prominent examples are Nordhaus (2008) and Fankhauser (1995).

the dynamic nature of the problem calls for a representation as a stochastic or dynamic game. Over the last decade, Dutta and Radner (2004, 2006a,b, 2009, 2012) presented variants of a dynamic climate change game. The GHG level in the atmosphere, measured in CO_2 equivalents, is a state variable. Every period, the sum of all countries' emissions is added to the stock and a constant small fraction of this stock dissolves (a crude approximation of actual physical processes). Each country benefits from its emissions through a concave production function, the costs stemming from climate change are represented through a cost function that is linear in the stock of GHG[2].

The thesis is structured as follows: Chapter 2 presents the benchmark dynamic game introduced by Dutta and Radner (2004), in which the costs associated with climate change are linear for the sake of analytical tractability. The case of fixed technology is explored in detail; I show that in this case one can equivalently represent this dynamic game as a repeated game. Furthermore, exogenous population and capital growth are analyzed following Dutta and Radner (2006b, 2012). In Chapter 3, I illustrate numerical results on the convex cost case. A particular myopic equilibrium, where countries do no coordinate their behavior, is analyzed numerically. Based on these results, an attempt is made on answering the question, what an increase in the convexity of the climate change cost function implies for emissions and payoffs. In Chapter 4, I introduce trade decisions in order to allow for trade sanctions as an alternative mechanism to enforce cooperation. The best equilibrium under trade sanctions is characterized and possible Pareto improvements due to the introduction of trade sanctions are assessed. Finally, Chapter 5 presents a conclusion and discussion of the results. Lengthy proofs are dislocated to Appendix A.

[2]A similar class of dynamic games that covers global warming has been treated by Dockner et al. (1996). There, costs associated with the stock (in this case the GHG level) are convex and the benefits from emissions linear. A literature in its own right has developed around the study of differential games, i.e. dynamic games in continuous time. Important contributions applicable to climate change are Dockner and van Long (1993), Rowat (2006) and Wirl (2007).

Chapter 2

Climate Change as a Dynamic Game: the Linear Cost Case

2.1 The Basic Model

The structure of the basic model introduced by Dutta and Radner (2004) is as follows: There are n players of the game, denote the set of players by $N = \{1, 2, ..., n\}$[1]. Time is discrete and the game is infinite-horizon, hence $t = 0, 1, 2, ...$. Each period, every country $i = 1, ..., n$ decides upon the (nonnegative) amount of energy $e_i(t) \geq 0$ it uses for its economic activities. This implies immediate private benefits through a twice differentiable and strictly concave production function $Y_i(e_i(t))$, where $Y_i : \mathbb{R}_+ \to \mathbb{R}_+$. Y_i is strictly increasing up to a point $\bar{e}_i > 0$ (at that point, the price of energy exceeds its marginal benefits)[2]. This production function implicitly reflects for each level of $e_i(t)$ the corresponding optimal level of other inputs; in this section, capital, production technology (except as it relates to energy usage) and population are constant. Given a country's current emission factor $f_i(t)$, its emission of GHG is represented as

$$a_i(t) = f_i(t)e_i(t), \qquad i = 1, .., n. \tag{2.1}$$

[1]One can think of them either as countries of the world or of regions. In both cases, this model does not consider the decision-making process within a country/ region.

[2]All the results would also hold if $Y_i'(e_i) > 0 \ \forall e_i \in \mathbb{R}_+$ and $\lim_{e_i \to \infty} Y_i'(e_i) = 0$.

One can consider reductions in $f_i(t)$ as using cleaner energy, equivalently, one can think of a smaller emission factor as using energy more efficiently or simply as an increase in the share of less energy-intensive goods and services in gross domestic product (GDP). Global emissions $z(t)$ are simply the sum of every country's emissions:

$$z(t) = \sum_{i=1}^{n} a_i(t).$$

The current stock of GHG in the atmosphere is denoted by $g(t) \in G = \mathbb{R}_+$. This refers to the excess GHG due to human activities, the total amount of GHG is then given by $g(t) + \bar{g}$, where $\bar{g}$ is the level of GHG around 1800. The law of motion for GHG is given by a linear difference equation,

$$g(t+1) = \eta g(t) + z(t), \tag{2.2}$$

where $0 < \eta < 1$. This is of course just an approximation to actual physical processes, see IPCC (2007) for details. It is assumed that costs stemming from climate change are linear in the GHG stock for each country: $c_i g(t)$, where $c_i > 0$ for $i = 1, .., n$. This is a critical and questionable assumption, as a convex cost function seems to be a better approximation to reality (see Stern, 2007). Nevertheless, the linear cost case is a good starting point. In particular, it allows for closed-from solutions.

A country may reduce its emission factor every period (the reduction will be effective in the following period), the associated costs are linear in the change, i.e. equal to $k_i[f_i(t) - f_i(t+1)]$, where $k_i > 0$ $\forall i$. Again, the linearity is chosen mainly for the sake of analytical tractability as increasing marginal abatement costs are probably more realistic. There is also a lower bound m_i for technological improvement, hence the constraint on the emission factor is given by

$$0 < m_i \leq f_i(t+1) \leq f_i(t) \quad \forall\ t = 0, 1, 2, ... \tag{2.3}$$

The stage-game payoff for a country is

$$v_i(t) = Y_i[e_i(t)] - c_i g(t) - k_i[f_i(t) - f_i(t+1)].$$

Assuming a common discount factor δ, the total payoff for a country is then given by

$$V_i = (1-\delta)\sum_{t=0}^{\infty} \delta^t v_i(t), \quad i = 1, .., n.$$

Thus, a country's current utility depends positively and directly on its current energy usage through its production function, while past emissions (which incorporate energy usage and past emission factors) of all countries have a negative impact on its current utility through the cost associated with the stock of GHG.

The state of the system at the beginning of period t is characterized by the $(n+1)$-dimensional vector $s(t) = [f(t), g(t)]$, where $f(t) = [f_1(t), ..., f_n(t)]$. In general, a strategy for a country is a map from the entire set of ex post histories (past actions and states as well as the current state) to the set of feasible actions (by (2.3), the set of feasible emission factors depends on the current state). A Nash equilibrium is, as usual, a profile of strategies such that no country can gain by deviating unilaterally. It is subgame-perfect, if it induces a Nash equilibrium in every subgame, i.e. also at nodes that will never be reached given the strategy profile. A strategy profile is referred to as a Markov strategy if players condition only on the current period and the current state of the system; a stationary Markov strategy is a map from the set of states to the set of actions (i.e. additionally, players do not condition on the current period). A (stationary) Markov Perfect Equilibrium (MPE) is subgame perfect Nash equilibrium (SPNE), where all players use (stationary) Markov strategies (see Mailath and Samuelson, 2006).

Finally, the concept of a global Pareto optimum (GPO) is introduced as a benchmark. Let $x = (x_1, ..., x_n)'$ be a vector of strictly positive numbers. A GPO corresponding to x is then a profile of strategies that maximizes the weighted sum of total country payoffs, $v = \sum_{i=1}^{n} x_i v_i$, which can be regarded as global welfare.

2.2 Benchmark Outcomes

In this section, two benchmark outcomes are analyzed and compared: the myopic Nash equilibrium, denoted by BAU (Business As Usual), and the GPO.

2.2.1 Business As Usual

The BAU equilibrium is a stationary MPE[3], where in addition energy usage and emission factors are constant after the first period. The name is chosen since it appears to correspond to what we currently observe in the world. It will be shown that BAU equilibrium strategies are of the form:

$$e_i(t) = E_i[f_i(t)],\ f_i(t+1) = F_i[f_i(t)],\ t \geq 0,\ i = 1, .., n.$$

Thus, countries condition their actions only on their own emission factor, neither on other players' emission factors nor on the GHG stock. Every country follows a myopic strategy, equating private marginal benefits and costs. Intuitively, the marginal benefit of using an extra unit of energy today is $Y_i'[e_i(t)]$, the marginal cost equals $c_i \delta f_i(t)(1+\delta\eta+(\delta\eta)^2+...)$. Formally, define the function E_i implicitly by

$$Y_i'[E_i(f_i)] = \delta w_i f_i, \quad w_i = \frac{c_i}{1-\delta\eta}. \tag{2.4}$$

To ensure that $E_i(\cdot)$ is indeed a well-defined function, a version of the Inada conditions is imposed[4]:

$$\lim_{e_i \to 0} Y_i'(e_i) > \delta f_i(0) \frac{\sum_{i=1}^n x_i w_i}{x_i}$$

[3]Note that although it is assumed that players choose (stationary) Markov strategies in equilibrium, the result is a strategy profile that is optimal in the full strategy set - there are no superior strategies, Markov or otherwise. We are thus not calculating an equilibrium in the game in which players are restricted to Markov strategies but Markov strategies that are an equilibrium of the full game (Mailath and Samuelson, 2006).

[4]This will be needed to ensure existence of an interior solution for the GPO. It implies that $\lim_{e_i \to 0} Y_i'(e_i) > \delta f_i(0) w_i$, which is needed here.

Note that since $Y_i'(\cdot)$ is strictly decreasing, $E_i(f_i)$ is strictly decreasing in f_i. For the choice of emission factor, define an auxiliary function $Z_i(y)$ and the function $F_i(f_i)$:

$$Z_i(y) = k_i y + \frac{\delta}{1-\delta}\left(Y_i[E_i(y)] - \delta w_i y E_i(y)\right) \tag{2.5}$$

$$F_i(f_i) = \arg\max_y \{Z_i(y) : m_i \leq y \leq f_i\} \tag{2.6}$$

If there is more than one (in fact, there will be at most two) maximizing value, pick the lowest. Differentiating (2.5) and using (2.4) yields $Z_i'(y) = k_i - \frac{\delta^2 w_i}{1-\delta} E_i(y)$, hence $Z_i'(y)$ is strictly increasing. The maximizer of a strictly convex function defined on a closed interval must be one of the two endpoints, hence $F_i(f_i) \in \{m_i, f_i\}$. If $Z_i'(m_i) \geq 0$, then $F_i(f_i) = f_i \ \forall \ f_i \geq m_i$. If $Z_i'(m_i) < 0$, then $\exists y_i^0 > m_i$ s.t. $F_i(f_i) = m_i$ for $m_i \leq f_i \leq y_i^0$ and $F_i(f_i) = f_i$ for $f_i > y_i^0$. In other words, $F_i(f_i)$ is a "bang-bang" policy, the emission factor is constant after the first period.

Definition 1. The profile $(E, F) = (E_i, F_i)_{i=1}^n$ is called a BAU strategy profile, with associated emissions denoted by $a_i^\star = f_i E_i(f_i)$.

Let $V_i(f, g)$ denote country i's total discounted payoff when the system is at state (f, g) and each country uses its BAU strategy.

Theorem 1 (Dutta and Radner, 2004)**.** *The BAU strategy profile is a stationary MPE. After the first period, each country i's emission factor and energy input are constant, and the emission factor is equal to either m_i or $f_i(0)$. The value function for country i is*

$$V_i(f, g) = (1-\delta)\left(Y_i[E_i(f_i)] - w_i g - k_i[f_i - F_i(f_i)] + u_i - \delta w_i \sum_{j=1}^n f_j E_j(f_j)\right),$$

$$u_i = \frac{\delta}{1-\delta}\left(Y_i[E_i(F_i(f_i))] - \delta w_i \sum_{j=1}^n F_j(f_j) E_j[F_j(f_j)]\right). \tag{2.7}$$

Proof. See Appendix A. □

2.2.2 Global Pareto Optimum

The GPO strategy profile is very similar to the BAU strategy profile as energy usage and emission factors are constant after the first period. Furthermore, every emission factor $f_i(t) \in \{m_i, f_i(0)\}\ \forall\ t \geq 0$. However, externalities are accounted for according to the weighting vector x, where $x_i > 0$ for $i = 1, ...n$. Let $w = (w_1, ..., w_n)'$ and define $\hat{E}_i$ (implicitly), $\hat{Z}_i$ and $\hat{F}_i$:

$$\begin{aligned} Y_i'[\hat{E}_i(f_i)] &= \frac{\delta(x \cdot w) f_i}{x_i}, \text{ where } w \cdot x = \sum_{i=1}^{n} x_i w_i = \frac{\sum_{i=1}^{n} x_i c_i}{1 - \delta\eta} \\ \hat{Z}_i(y) &= k_i y + \frac{\delta}{1-\delta} \left(Y_i[\hat{E}_i(y)] - \delta \frac{x \cdot w}{x_i} y \hat{E}_i(y) \right) \\ \hat{F}_i(f_i) &= \arg\max_y \left\{ \hat{Z}_i(y) : m_i \leq y \leq f_i \right\} \end{aligned} \tag{2.8}$$

Again, differentiating (2.8) shows that $\hat{Z}_i$ is strictly convex, thus $\hat{F}_i(f_i) \in \{m_i, f_i\}$, depending on whether $\hat{Z}_i(m_i)$ or $\hat{Z}_i(f_i)$ is greater.

Definition 2. The profile $(\hat{E}, \hat{F}) = (\hat{E}_i, \hat{F}_i)_{i=1}^n$ is called a GPO strategy profile, with associated emissions denoted by $\hat{a}_i = f_i \hat{E}_i(f_i)$.

Theorem 2 (Dutta and Radner, 2004)**.** *Given a strictly positive weighting n-vector x, the global Pareto optimum is achieved by the GPO strategy profile. After the first period, each country i's energy usage and emission factor are constant, the emission factor is $\in \{m_i, f_i\}$. The value function is given by*

$$V(f, g) = (1 - \delta) \sum_{i=1}^{n} \left(Y_i[\hat{E}_i(f_i)] - w_i g - k_i[f_i - \hat{F}_i(f_i)] + \hat{u}_i - \delta w_i \sum_{j=1}^{n} f_j \hat{E}_j(f_j) \right),$$

$$\hat{u}_i = \frac{\delta}{1 - \delta} \left(Y_i[\hat{E}_i(\hat{F}_i(f_i))] - \delta w_i \sum_{j=1}^{n} \hat{F}_j(f_j) \hat{E}_j[\hat{F}_j(f_j)] \right).$$

Proof. The proof works the same way as the proof for the BAU equilibrium and is thus omitted here. □

2.2.3 Comparison of BAU and GPO profiles

Are GPO emissions always lower than BAU emissions? Intuitively, internalizing the negative externality should lead to lower emissions. However, it will be shown that this is not necessarily true since a lower emission factor has two opposing implications. Nevertheless, for a fixed emission rate (hence for the first period), the answer is always yes as the next lemma shows:

Lemma 1. *$\forall$ strictly positive x, $\forall$ $f_i > 0$, $\forall$ $i = 1..., n$: $E_i(f_i) > \hat{E}_i(f_i)$.*

Proof.

$$\begin{aligned} & x_i w_i < \sum_{j=1}^{n} x_j w_j \\ \iff & Y_i'[E_i(f_i)] = \delta f_i w_i < \delta f_i \frac{x \cdot w}{x_i} = Y_i'[\hat{E}_i(f_i)] \\ \iff & E_i(f_i) > \hat{E}_i(f_i) \end{aligned}$$

The last equivalence statement holds because Y_i is strictly concave. □

Corollary 1. *A BAU equilibrium is not globally Pareto optimal.*

Proof. Suppose we are looking for a constrained GPO, in which we have to pick the BAU emission factors in every period. By Lemma 1, every country would use strictly less energy every period. This would correspond to a strictly higher global welfare as the interior solution $\hat{E}_i(f_i)$ is unique. The value of the unconstrained GPO is clearly higher as the value of the constrained GPO since the objective function is the same and the constraint set of the former problem is a superset of the constraint set of the latter problem. Thus, a BAU equilibrium is not globally Pareto optimal. □

On the one hand, a lower emission factor leads directly to lower emissions for fixed energy use; however, on the other hand, it also promotes higher energy use as the associated costs are smaller. Formally these opposing effects are captured by the following equation:

$$\frac{d\hat{a}_i}{df_i} = \frac{d\left(f_i\hat{E}_i(f_i)\right)}{df_i} = \hat{E}_i(f_i) + f_i\hat{E}_i'(f_i)$$

Recall that $\hat{E}_i'(f_i) < 0\ \forall f_i$. If follows that:

$$\frac{d\hat{a}_i}{df_i} > 0 \Leftrightarrow \frac{d\log\hat{E}_i(f_i)}{d\log f_i} = \hat{E}_i'(f_i)\frac{f_i}{\hat{E}_i(f_i)} > -1 \tag{2.9}$$

Note that this condition is an upper bound for the elasticity of $\hat{E}_i$ with respect to f_i (the elasticity refers to the absolute value of this expression). It turns out that if the elasticity of the globally optimal energy use is < 1 (inelastic), then GPO emissions are always lower than BAU emissions.

Theorem 3. *Suppose that* (2.9) *holds. Then* $F_i(f_i) = m_i \Rightarrow \hat{F}_i(f_i) = m_i$. *Hence, given the same initial state, GPO emission factors are equal or lower than BAU emission factors in every period.*

Proof. See Dutta and Radner (2004). □

Recall that $F_i(f_i) \in \{m_i, f_i\}$ (the same holds for $\hat{F}_i$). Hence, the only other possibility is $F_i(f_i) = f_i$, in which case either $\hat{F}_i(f_i) = m_i$ or $\hat{F}_i(f_i) = f_i$ may hold.

Corollary 2. *If* (2.9) *holds and given the same initial state, GPO emissions are lower than BAU emissions in every period.*

Proof. Since (2.9) holds, we know from Theorem 3 that the GPO emission factor is equal to or lower than the BAU emission factor. In the first case, Lemma 1 proves the result. In the second case, the GPO emissions would be even lower than in the first case by (2.9), hence the positive gap between BAU and GPO emissions even larger. □

2.3 Fixed Technology

The case of fixed emission technology, explored by Dutta and Radner (2006a, 2009), allows for a complete characterization of the set of SPNE. It is a special case of the model described in Section 2.1 ($f_i(0) = m_i$), thus the results of Section 2.2 do hold here as well. Notation can be simplified by letting countries chose their emissions a_i directly and by defining $h_i(a_i(t)) = Y_i(\frac{a_i(t)}{f_i})$, because f_i is just a constant now. In order to characterize the full SPNE set, it is necessary to restrict action spaces to compact intervals $A_i = [0, \bar{a}_i]$, i.e. to impose an upper bound on emissions (energy usage). It makes sense to require $\bar{a}_i \geq f_i \bar{e}_i$ (thus $h_i'(\bar{a}_i) \leq 0$) - as the technological/ physical upper bound on emissions $\bar{a}_i$ is approached, the price of energy[5] is expected to eventually exceed the marginal benefit of energy usage.

I will follow an approach alternative to the one pursued by Dutta and Radner (2009): For fixed emission technology, I will show that one can equivalently represent this dynamic game (referred to as $D(\delta)$) as pure repeated game ($R(\delta)$). Once this has been proven, solving the model is a simpler task as repeated games with perfect monitoring have been extensively explored[6]. To do so, define the stage game payoff for $R(\delta)$ by

$$u : \prod_{j=1}^{n} A_j \to \mathbb{R}^n, \quad u_i(a) = h_i(a_i) - \delta w_i \sum_{j=1}^{n} a_j, \quad i = 1, ..., n, \tag{2.10}$$

where $w_i = \frac{c_i}{1-\delta\eta}$ as usual. Note that the stage game payoff depends on the discount factor δ (this is not the case in the canonical repeated game). Thus one cannot immediately apply all the results on repeated games. The unique Nash equilibrium of the stage game is $a^\star$, where each country i emits its BAU level implicitly defined by $h_i'(a_i^\star) = \delta w_i$. The GPO emission level $\hat{a}$ is implicitly defined by $h_i'(\hat{a}_i) = \delta \frac{x \cdot w}{x_i}$. BAU and GPO emissions are decreasing in the discount factor δ, the climate change cost coefficient c_i and the fraction of GHG remaining

[5] Remember that the price of energy is incorporated into $h_i(\cdot)$, not to confuse with the constant marginal cost associated with climate change (c_i).

[6] See e.g. Mailath and Samuelson (2006) for results on repeated games with perfect monitoring

in the atmosphere η. A country i's min-max value is clearly given by

$$\underline{u_i} \equiv \min_{a_{-i}\in A_{-i}} \max_{a_i\in A_i} u_i(a) = h_i(a_i^\star) - \delta w_i(a_i^\star + \sum_{j\neq i} \bar{a}_j).$$

A strategy σ_i^R for a country i in $R(\delta)$ is then a mapping from the set of histories $\mathcal{H}_\mathcal{R} = \cup_{t=0}^{\infty}\mathcal{H}_R^t$, where $\mathcal{H}_R^t = A^t$ and $\mathcal{H}_R^0 = \{\emptyset\}$, to the action space A_i, and $\sigma^R = \times_{i=1}^n \sigma_i^R$. The total payoff induced by σ^R is

$$U_i(\sigma^R) = (1-\delta)\sum_{t=0}^{\infty} \delta^t u_i(a^t(\sigma^R)).$$

In the dynamic game D, the set of period t ex post histories $\mathcal{H}_D^t = (A\times G)^t \times G$ appears to be much larger. However, since the game is deterministic, the set of feasible nodes in the dynamic game reduces to $A^t \times G$, where G is the set of period 0 ex post histories (the initial GHG levels) and the dependence on G^t has been dropped because $\forall\ 0 < s \leq t$, it holds that $g(s)$ is a function of past emissions and the initial GHG level, to be precise $g(s) = \eta^s g(0) + \sum_{k=0}^{s-1}\eta^{s-k-1}z(k)$[7]. In particular, at any ex post history h_D^t, the tree of feasible nodes for the continuation game is characterized by $\cup_{t=0}^{\infty}A^t = \mathcal{H}_R$. Denote a strategy profile in D by $\sigma^D : \mathcal{H}_D \to A$. The payoffs are the fixed-technology analogs of Section 2.1 and restated here for convenience (the GHG law of motion leads directly to line (2.11)):

$$\begin{aligned} v_i(\sigma^D,t) &= h_i(a_i(\sigma^D,t)) - c_i g(\sigma^D,t), \quad \text{where} \\ g(\sigma^D,t) &= \eta^t g(0) + \sum_{s=0}^{t-1}\eta^{t-1-s}z(\sigma^D,s) \quad \text{and} \quad z(\sigma^D,s) = \sum_{j=1}^{n} a_j(\sigma^D,s), \qquad (2.11) \\ V_i(\sigma^D) &= (1-\delta)\sum_{t=0}^{\infty}\delta^t v_i(\sigma^D,t) \quad \text{and} \quad V(\sigma^D) = (V_i(\sigma^D))_{i=1}^n. \end{aligned}$$

Theorem 4 will state that it suffices to analyze the equilibria of the pure repeated game as this is strategically equivalent to the dynamic game. Before stating the result, the notion of applying a strategy profile in $D(\delta)$ to $R(\delta)$ is made precise.

Definition 3. For a fixed strategy profile σ^D and any ex post history h_D^t (and

[7]This is the solution of the linear difference equation (2.2). Recall the expression for global emissions, $z(k) = \sum_{j=1}^{n} a_j(k)$.

associated GHG level g_t), define the *projection of the continuation strategy profile* $\sigma^D|_{h_D^t}$ on $\mathcal{H}_R$, denoted by $\sigma^R_{h_D^t}$, such that $\forall\ h^\tau = (a(s))_{s=0}^{\tau-1}$,

$$\sigma^R_{h_D^t}(h^\tau) = \sigma^D|_{h_D^t}\left((a\,(s))_{s=0}^{\tau-1}, \left(\eta^s g_t + \sum_{k=0}^{s-1} \eta^{s-k-1} z(k)\right)_{s=1}^{\tau} \right).$$

This definition implies that for a given single strategy profile in $D(\delta)$, every ex post history in $D(\delta)$ defines a separate strategy profile for $R(\delta)$. Hence a single strategy profile in $D(\delta)$ leads via projection to as many strategy profiles in $R(\delta)$ as there are ex post histories in the dynamic game. Though this might seem complicated, it will be subsequently shown that it is actually very simple to use the following theorem to get a SPNE strategy profile in $D(\delta)$ from a single SPNE strategy profile in $R(\delta)$, and vice versa.

Theorem 4. *A strategy profile σ^D is a SPNE in the dynamic game D if and only if for all ex post histories h_D^t and associated GHG level g_t, the projection of the continuation strategy profile $\sigma^D|_{h_D^t}$ on $\mathcal{H}_R$, denoted by $\sigma^R_{h_D^t}$ is a Nash Equilibrium in the pure repeated game $R(\delta)$. Furthermore,*

$$V\left(\sigma^D|_{h_D^t}\right) = U(\sigma^R_{h_D^t}) - (1-\delta) w g_t. \tag{2.12}$$

Proof. To simplify notation, let $\mathbf{a} = (a(t), a(t+1), ...)$ be the outcome path induced by the strategy profile $\sigma^D|_{h_D^t}$ (equivalently, by $\sigma^R_{h_D^t}$) and g_t the associated GHG level.

$$\begin{aligned}
V_i\left(\sigma^D|_{h_D^t}\right) &= (1-\delta)\sum_{s=t}^{\infty} \delta^{s-t}\left(h_i(a_i(s)) - c_i g(s)\right)\\
&= (1-\delta)\sum_{s=t}^{\infty} \delta^{s-t}\left(h_i(a_i(s)) - c_i\left(\eta^{s-t} g_t + \sum_{k=t}^{s-1}\eta^{s-1-k} z(k)\right)\right)\\
&= (1-\delta)\sum_{s=t}^{\infty} \delta^{s-t}\left(h_i(a_i(s)) - \delta w_i z(s)\right) - (1-\delta) w_i g_t \qquad (2.13)\\
&= U_i(\sigma^R_{h_D^t}) - (1-\delta) w_i g_t.
\end{aligned}$$

To get the second term of (2.13), note that

$$\sum_{s=t}^{\infty} \delta^{s-t} c_i \sum_{k=t}^{s-1} \eta^{s-1-k} z(k) = c_i \sum_{k=t}^{\infty} z(k) \sum_{s=k+1}^{\infty} \delta^{s-t} \eta^{s-1-k}$$
$$= \sum_{k=t}^{\infty} z(k) \delta^{k-t+1} \frac{c_i}{1-\delta\eta} = w_i \sum_{s=t}^{\infty} \delta^{s-t+1} z(s).$$

The theorem then follows from (2.12) and the definition of a SPNE. □

Remark. Two things should be noted: First, although $\sigma^R_{h^t_D}$ is required to be only a plain Nash equilibrium in the repeated game, the characterization holds if and only if $\sigma^R_{h^t_D}$ is a SPNE in $R(\delta)$, as $\sigma^R_{h^t_D}$ must induce a Nash equilibrium in every subgame of $R(\delta)$. Second, the crucial point of this dynamic game is the linearity of the cost and transition function. The theorem fails once linearity is abandoned.

Theorem 4 directly implies that for any SPNE strategy profile σ^D in $D(\delta)$ and any initial GHG level g_0, there exists an associated SPNE strategy profile σ^R in $R(\delta)$ such that outcome paths are equivalent and payoffs related by (2.12). Corollary 3 shows that the converse is also true.

Corollary 3. *Suppose σ^R is a SPNE strategy profile in $R(\delta)$. Then there exists an associated SPNE strategy profile σ^D in $D(\delta)$ such that for any initial GHG level g_0 the induced outcome paths $\boldsymbol{a}$ in $R(\delta)$ and $D(\delta)$ are equivalent and total payoffs are a linear translate, i.e. $V\left(\sigma^D\right) = U(\sigma^R) - (1-\delta) w g_0$.*

Proof. Fix a SPNE strategy profile σ^R in $R(\delta)$. Define σ^D such that for all initial GHG levels and all feasible nodes in the tree of the supergame, the two strategy profiles agree and for all nodes that are not feasible, prescribe BAU play. Observe that all nodes in the subgame starting at a node that is not feasible are themselves not feasible. Formally, let

$$\sigma_i^D\left(\left(\left((a_j(s))_{j=1}^n\right)_{s=0}^{t-1}, (g(s))_{s=0}^t\right)\right) = \begin{cases} \sigma_i^R\left(\left(\left((a_j(s))_{j=1}^n\right)_{s=0}^{t-1}\right)\right), & \text{if F holds,} \\ a_i^\star, & \text{else,} \end{cases}$$

where F is the statement: $\forall\ s = 1, ..., t,\ g(s) = \eta^s g(0) + \sum_{k=0}^{s-1} \eta^{s-k-1} z(k)$.

By definition, no player can push play off the feasible nodes, where subgame perfection holds because of Theorem 4. At any history that is not feasible, σ^D prescribes BAU play forever, which obviously constitutes an equilibrium. □

Corollary 4 immediately follows and states the properties of the set of SPNEs.

Corollary 4. *The SPNE payoff correspondence $\Xi : G \to \mathbb{R}^n$ of D is given by $\Xi(g) = E - \{(1-\delta)wg\}$, where E is the compact set of SPNE of R. In particular, consider any SPNE, any period t and any history h_D^t, then the payoff vector for the continuation strategies must be of the form $u-(1-\delta)wg(t)$, where $u \in E$ and $g(t)$ is the GHG level in period t.*

Proof. Follows immediately from Theorem 4, compactness is a standard result (see e.g. Mailath and Samuelson, 2006, p.39). □

Dutta and Radner (2006a, p.198) proved a similar result[8] directly for the dynamic game, commenting on the structure of the SPNE payoff correspondence $\Xi(g)$, i.e. that it is a linear translate of $\Xi(0)$, as being of "*surprising simplicity*". The proof of Theorem 4 shows that this structure is a direct consequence of linear climate change costs.

2.3.1 BAU Reversion

While the *second best* problem refers to finding the SPNE that maximizes $u \in E$ given a weighting vector x, the *third best* problem corresponds to maximizing u given x under the restriction that u is an element of a certain subset (referred to as T) of the set of SPNE E. Grim trigger, i.e. playing the myopic stage game Nash equilibrium forever in case of defection, is a particularly simple strategy. In this context, define the third best optimum (TBO) as a profile of strategies such that u given x is maximized subject to BAU reversion. T is non-empty, i.e. the TBO exists, since $a^\star$ (the unique stage-game Nash equilibrium) with associated

[8] They did not show that this compact set E is actually the set of SPNE payoffs of the repeated game.

payoff $u^\star$ is clearly sustainable as a SPNE under the threat of BAU reversion. Formally, the maximization problem is given by:

$$\begin{aligned}
&\max_{a\in A, u\in T\subseteq E} \sum_{i=1}^{n} x_i \left((1-\delta) \left(h_i(a_i) - \delta w_i \sum_{j=1}^{n} a_j \right) + \delta u_i \right) \\
&\text{s.t. for } i = 1, ..., n : (1-\delta) \left(h_i(a_i) - \delta w_i \sum_{j=1}^{n} a_j \right) + \delta u_i \\
&\qquad \geq (1-\delta) \left(h_i(a_i^\star) - \delta w_i \left(a_i^\star + \sum_{j\neq i} a_j \right) \right) + \delta u_i^\star \qquad (2.14)
\end{aligned}$$

Note that the incentive constraint (2.14) must hold for all $a_i \in A_i$, not only for $a_i^\star$, but it is clear that BAU emissions maximize the deviation profit (the right-hand side). Furthermore, the BAU reversion payoff is $u_i^\star = h_i(a_i^\star) - \delta w_i \sum_{j=1}^{n} a_j^\star$. It follows that the solution to the problem is given by a vector of emissions $\tilde{a}$, with $\tilde{u}$ denoting the resulting payoff, that is constant over time. Using the definition of $\tilde{u}$ and $u^\star$, the constraint can be simplified to

$$h_i(\tilde{a}_i) - \delta w_i \left(\tilde{a}_i + \delta \sum_{j\neq i} \tilde{a}_j \right) \geq h_i(a_i^\star) - \delta w_i \left(a_i^\star + \delta \sum_{j\neq i} a_j^\star \right).$$

Is it possible to sustain the GPO emissions under the threat of BAU reversion, at least asymptotically (i.e. when countries are sufficiently patient)? As the discount factor is now varying, the dependence of GPO ($\hat{a}(\delta)$) and BAU ($a^\star(\delta)$) emission levels as well as of the stage game payoffs on δ is stressed. Moreover, note that $w_i(\delta) = \frac{c_i}{1-\eta\delta}$ is also a (continuous) function of δ. The constraint can be written as

$$u_i \left(\hat{a}(\delta), \delta \right) \geq (1-\delta) u_i \left(a_i^\star(\delta), \hat{a}_{-i}(\delta), \delta \right) + \delta u_i \left(a^\star(\delta), \delta \right). \qquad (2.15)$$

Note that the incentive constraint itself, as a function of δ, is continuous (also at $\delta = 1$) as $\hat{a}(\delta)$, $a^\star(\delta)$ as well as $u(\cdot, \delta)$ are. To see this, observe that $h_i'(\cdot)$ is a C^1 function and apply the implicit function theorem. Hence, a necessary condition for asymptotic sustainability of the GPO is that the GPO solution weakly Pareto-dominates the BAU outcome at $\delta = 1$, i.e. that the asymptotic incentive constraint

$$\begin{aligned} u_i\left(\hat{a}(1),1\right) &= h_i\left(\hat{a}_i(1)\right) - \frac{c_i}{1-\eta}\sum_{j=1}^{n}\hat{a}_j(1) \\ &\geq u_i\left(a^\star(1),1\right) = h_i\left(a_i^\star(1)\right) - \frac{c_i}{1-\eta}\sum_{j=1}^{n}a_j^\star(1) \end{aligned} \tag{2.16}$$

holds $\forall\ i$. If there was a country i such that $u_i\left(\hat{a}(1),1\right) < u_i\left(a^\star(1),1\right)$, then the constraint would clearly be violated for sufficiently patient countries, i.e. for $\delta \in [\underline{\delta}, 1)$ for some $\underline{\delta} < 1$. On the other hand, suppose that (2.16) is fulfilled with strict inequality, then the incentive constraint (2.15) will hold for sufficiently high δ by continuity of the constraint in δ.

Theorem 5. *Suppose the GPO emission level $\hat{a}(\delta)$ strictly Pareto dominates the BAU level $a^\star(\delta)$ asymptotically (when $\delta = 1$), i.e.* (2.16) *holds with strict inequality. Then the GPO is sustainable as SPNE (subject to BAU reversion) for sufficiently patient countries, i.e. $\exists\underline{\delta} < 1$ s.t. $\forall\delta \in [\underline{\delta}, 1)$ the result holds.*

Proof. Follows directly from above considerations (i.e. from continuity of the incentive constraint). □

Remark. In general, it is not sufficient that the GPO *weakly* Pareto dominates the BAU equilibrium asymptotically. Whether this weaker condition suffices depends on the specific production functions and parameters used. If the constraint holds asymptotically with equality for some country, it is unclear whether the difference between the left-hand and the right-hand side of the inequality turns positive or negative if the discount factor is decreased. Note that this contrasts with the intuition from repeated games analysis, where less patience always makes it more difficult to sustain an efficient outcome subject to Nash reversion.

2.3.2 Best Equilibria

Although the dynamic game can be represented as a repeated game, one cannot employ any of the standard folk theorems for asymptotic analysis since the stage game payoff depends on the discount factor. Hence, a payoff that is feasible for a fixed discount factor, say δ_0, might not be feasible anymore for some $\delta' > \delta_0$. In particular, the more patient countries are, the lower will be the efficient payoffs. However, at the same time, higher patience facilitates achieving the efficient frontier in an equilibrium.

The second-best optimum (SBO), given a weighting vector x, is similar to the TBO except that punishment strategies need not be of the BAU reversion form. Of course, the punishments itself must be equilibria. The most efficient punishments are those that give the deviator, say i, its lowest equilibrium payoff $\underline{v}^i$. The maximization problem is similar to the TBO problem (the constraint has been simplified as it is clear that BAU maximizes the deviation profit and i's worst equilibrium serves as best punishment):

$$\max_{a\in A, u\in E} \sum_{i=1}^{n} x_i \left((1-\delta) \left(h_i(a_i) - \delta w_i \sum_{j=1}^{n} a_j \right) + \delta u_i \right) \text{ s.t. } \forall i:$$
$$(1-\delta)\left(h_i(a_i) - \delta w_i a_i\right) + \delta u_i \geq (1-\delta)\left(h_i(a_i^\star) - \delta w_i a_i^\star\right) + \delta \underline{v}_i^i.$$

It follows that the SBO has a simply structure: It is generated by a vector of constant emissions $\tilde{a}$ independent of the GHG level. According to Dutta and Radner (2006a, p.199), the punishment for i consists of all other countries $j \neq i$ emitting an amount a_j^H that is higher than the BAU level for one period, followed by a constant rate $a(x_{-i})$ that is the solution to an i-less SBO (i.e. the best equilibrium when the weight of country i is set to zero). In general, it is not clear whether the high rate a_j^H coincides with the min-max action profile (the upper bound on emissions $\bar{a}_j$).

2.4 Population Change

A natural generalization of the basic model, covered by Dutta and Radner (2006b), is the introduction of population change. In this section, technology is treated as fixed for simplicity.

Let $P_i(t)$ denote the population of country i in period t. Assume further that the population evolves according to a linear difference equation,

$$P_i(t) = \lambda_i P_i(t) + (1 - \lambda_i) S_i,$$

where the parameter $\lambda_i \in (0, 1)$ indicates the speed of adjustment and S_i is i's asymptotically approached (”steady state”) population level. It follows immediately that the solution to this linear difference equation is given by

$$P_i(t) = \lambda_i^t P_i(0) + (1 - \lambda_i^t) S_i.$$

Note that more complicated (exogenous) population dynamics can also be covered by this approach, however the case treated here is more tractable. The payoff in period t for i is given by

$$v_i(t) = h_i[a_i(t), P_i(t)] - c_i P_i(t) g(t),$$

where h_i has the same properties as defined previously (C^2, strictly concave, strictly increasing up to a certain point) for every fixed $P_i(t)$. Note that the climate change cost is proportional to the population in a country i. The state of the system at the beginning of period t is now given by the $(n + 1)$-dimensional vector $s(t) = [P(t), g(t)]$, where $P(t) = [P_1(t), ..., P_n(t)]$ and $g(t)$ is the GHG level.

2.4.1 BAU with Population Change

The BAU equilibrium is a stationary MPE, where each country conditions only on its own population level. The marginal benefit of emitting an extra unit is given by $h_{i1}(a_i, P_i(t))$, where $h_{i1}(\cdot, \cdot)$ denotes the derivative of h_i with respect to its first argument. The marginal cost is

$$\delta c_i \left(P_i(t+1) + \delta\eta P_i(t+2) + (\delta\eta)^2 P_i(t+3) + ...\right) = \delta c_i \sum_{s=1}^{\infty} (\delta\eta)^{s-1} P_i(t+s)$$
$$= \delta c_i \sum_{s=1}^{\infty} (\delta\eta)^{s-1} \left(\lambda_i^{s-1} P_i(t+1) + (1-\lambda_i^{s-1}) S_i\right) = \delta c_i \left(\frac{S_i}{1-\delta\eta} - \frac{S_i - P_i(t+1)}{1-\lambda_i\delta\eta}\right)$$
$$\equiv \delta w_i(P_i(t+1)). \tag{2.17}$$

The cumulative cost function $w_i(\cdot)$ is the analogue to the constant w_i in the fixed population case, its argument is the population level of country i when the climate change costs start to become effective (one period after the emissions that cause them are produced). The cumulative costs are increasing in the discount factor δ, the GHG dissolving rate η and in current as well as in steady state population. If the population is growing ($P_i(0) < S_i$), then faster adjustment (low λ_i) leads to higher cumulative costs (and vice versa). All of that is intuitive.

Theorem 6. *Let (g, P) be the state of the system* [9] *. Let each i use its Markovian strategy $a_i^\star(P_i)$ (that conditions only on own population) defined by*

$$h_{i1}(a_i^\star(P_i), P_i) = \delta w_i(P_i'). \tag{2.18}$$

Then this is a MPE (referred to as BAU) and the value function for i is given by

$$V_i^\star(g, P) = u_i^{i\star}(P_i) + \sum_{j \neq i} u_i^{j\star}(P_i, P_j) - (1-\delta) w_i(P_i) g,$$

$$\text{where} \quad u_i^{i\star}(P_i) = (1-\delta)\left(h_i\left(a_i^\star(P_i), P_i\right) - \delta w_i(P_i') a_i^\star(P_i)\right) + \delta u_i^{i\star}(P_i') \tag{2.19}$$
$$\text{and} \quad u_i^{j\star}(P_i, P_j) = (1-\delta)\left(-\delta w_i(P_i') a_j^\star(P_j)\right) + \delta u_i^{j\star}(P_i', P_j'). \tag{2.20}$$

Proof. By the one-shot deviation principle for MPE, it must hold that

$$V_i^\star(g, P) = \max_{a_i} \left\{ (1-\delta)\left(h_i(a_i, P_i) - c_i P_i g\right) + \delta V_i^\star\left(\eta g + \sum_{j \neq i} a_j^\star(P_j) + a_i, P'\right) \right\}.$$

[9] The prime will denote the subsequent period.

The only terms involving a_i are $(1-\delta)h_i(a_i, P_i)$ and $-(1-\delta)\delta w_i(P_i')a_i$, hence (2.18) holds. Furthermore, (2.19) and (2.20) hold (writing out the value functions immediately leads to the result). The remaining terms determine the cumulative costs,

$$w_i(P_i) = c_i P_i + \delta\eta w_i(P_i'). \tag{2.21}$$

The cumulative marginal cost function (2.17) fulfills this equation: Total marginal costs are current marginal cost plus discounted future marginal costs. □

It is noteworthy that any profile of strategies with the property that a country's current action depends only on its own population is of the BAU form.

2.4.2 GPO with Population Change

The GPO profile is straightforward given the previous results and the fact that the marginal costs are given by

$$w(P) = \sum_{i=1}^{n} x_i w_i(P_i).$$

Theorem 7. *Given a strictly positive welfare vector x, the GPO strategy for each i is a function $\hat{a}_i(P)$ defined by*[10]

$$x_i h_{i1}(\hat{a}_i(P), P_i) = \delta w(P')$$

and the associated GPO value function is given by

$$\hat{V}(g, P) = \sum_{i=1}^{n} x_i u_i(P) - (1-\delta)w(P)g,$$
$$\textit{where} \quad u_i(P) = (1-\delta)\left(h_i(\hat{a}_i(P), P_i) - \delta w_i(P_i')\sum_{j=1}^{n}\hat{a}_j(P)\right) + \delta u_i(P').$$

Proof. The proof is omitted as it uses the same method as in the BAU case. □

[10] To be precise, one has to assume that the functions h_i are such that this solution exists for $P_i \in [P_i(0), S_i]$.

As in the fixed population case (Lemma 1), it follows that BAU emissions strictly exceed GPO emissions.

2.4.3 Effect of Population Size on Emissions

How do BAU and GPO emission levels change in population size? To answer this question, first recall that $h_i(\cdot,\cdot)$ is a C^2 function and note that $h_{i12}(\cdot,\cdot)$ denotes its cross-partial derivative.

Theorem 8. *The BAU emission function $a_i^\star(P_i)$ is C^1, the sign of its derivative is equal to the sign of*

$$h_{i12}(a_i^\star(P_i), P_i) - \frac{c_i \lambda_i \delta}{1 - \lambda_i \delta \eta}. \tag{2.22}$$

Likewise, the GPO emission function $\hat{a}_i(P)$ is C^1 and it is increasing (decreasing) in own population size if and only if the BAU emission function is increasing (decreasing). Moreover, GPO emissions are always strictly decreasing in any another country $j \neq i$'s population.

Proof. See Appendix A. □

Note that a country's BAU strategy prescribes the same emission level regardless of the population levels of other countries. The introduction of exogenous population change does not pose a specific extra difficulty in finding the "good equilibria", i.e. the ones that one would want to support through a self-enforcing treaty. As i's population approaches the steady-state level S_i, the incentive constraint converges monotonically to the one in the fixed population case.

2.5 Capital Growth

In this section, the introduction of exogenous capital accumulation is investigated. The material presented in this section is based on Dutta and Radner (2012). The motivation of this extension stems from the observed difficulty of getting fast-growing economies (e.g. China and India) to sign a climate-change treaty.

Let $K_i(t)$ denote country i's capital level in period t and $K(t)$ the vector of capital levels. Imposing again fixed emission technology (and fixed population), the state of the system is thus given by the $(n+1)$-dimensional vector $s(t) = [g(t), K(t)]$. The utility for country i in period t is given by

$$v_i(t) = a_i(t)^{\beta_i} K_i(t)^{\gamma_i} - c_i g(t), \tag{2.23}$$

where the production function[11] takes an explicit form. The coefficients β_i and γ_i are both $\in (0,1)$ and not required to sum to one (the constant returns to scale case). Crucially, capital is assumed to grow geometrically at a constant rate:

$$K_i(t+1) = \theta_i K_i(t), \tag{2.24}$$

where $\theta_i > 1$. The capital stock will thus grow unboundedly and one has to impose a condition to preserve boundedness of the solution: $\delta\theta_i^{\frac{\gamma_i}{1-\beta_i}} < 1 \ \forall i$[12].

2.5.1 BAU and GPO with Capital Growth

The derivation of the two benchmark solutions, namely the myopic MPE referred to as BAU and the GPO, can be conducted by the same techniques used in the earlier sections of this chapter. In the BAU equilibrium, each country conditions only on its own capital level. This is similar to the population change BAU equilibrium, where each country only conditions on its own population level.

[11] This production function does not have a maximum, contrary to the description of the basic model in Section 2.1, as there is no term subtracted that represents the price of energy. This slight modification does not affect the outcome of the model, see footnote 2 on p. 3.

[12] The BAU solution, derived in the following, is $a_i^\star(K) = constant * K^{\frac{\gamma_i}{1-\beta_i}}$. Plugging it back in the production function and substituting (2.24) for next period's capital gives an undiscounted growth rate of $\theta_i^{\frac{\gamma_i}{1-\beta_i}}$. One has to require that this term, discounted with the common discount factor, is strictly smaller than one for boundedness of the solution.

Theorem 9. *Let (g, K) be the state of the system. Let each i use its Markovian strategy $a_i^\star(K_i)$ (that conditions only on own capital level) defined by*

$$a_i^\star(K_i) = \left(\frac{\beta_i}{\delta w_i}\right)^{\frac{1}{1-\beta_i}} K_i^{\frac{\gamma_i}{1-\beta_i}}. \tag{2.25}$$

Then this is a MPE (referred to as BAU) and the value function for i is given by

$$V_i^\star(g, K) = \phi_i^i K_i^{\frac{\gamma_i}{1-\beta_i}} - \sum_{j \neq i} \phi_i^j K_j^{\frac{\gamma_j}{1-\beta_j}} - (1-\delta) w_i g,$$

where ϕ_i^i and ϕ_i^j are strictly positive constants $\forall\ i, j$.

Proof. See Appendix A. □

The proof implies that BAU is the unique equilibrium with the property that a country conditions only on its own capital level. Moreover, the form of the value function indicates that if there is a country i growing at a rate strictly higher than the growth rate of any other country, this country i will eventually dominate in terms of utility (both its own and all other countries' utility functions). This observation is crucial; it will turn out to be impossible - without changing the rules of the game - to support a cooperative solution as an equilibrium. The next theorem states the GPO solution under capital growth.

Theorem 10. *Let (g, K) be the state of the system, let x be a strictly positive n-vector. Then in the GPO solution, given weighting vector x, every country i's optimal choice $\hat{a}_i(K_i)$ is a function of i's current capital level only,*

$$\hat{a}_i(K_i) = \left(\frac{x_i \beta_i}{\delta x \cdot w}\right)^{\frac{1}{1-\beta_i}} K_i^{\frac{\gamma_i}{1-\beta_i}},$$

and the GPO value function is given by

$$\hat{V}(g, K) = \sum_{i=1}^{n} \hat{\phi}_i K_i^{\frac{\gamma_i}{1-\beta_i}} - (1-\delta) x \cdot w g,$$

where $\hat{\phi}_i$ is a strictly positive constant $\forall$ i,

$$\hat{\phi}_i = \frac{(1-\delta)(1-\beta_i)x_i\left(\frac{x_i\beta_i}{\delta x\cdot w}\right)^{\frac{\beta_i}{1-\beta_i}}}{1-\delta\theta_i^{\frac{\gamma_i}{1-\beta_i}}}.$$

Proof. Omitted (similar to the BAU equilibrium proof). □

Again, BAU emission levels are strictly higher than GPO emission levels. Indeed, given the explicit production function, an exact ratio for these two quantities can be derived easily:

$$\frac{\hat{a}_i(K_i)}{a_i^\star(K_i)} = \left(\frac{x_i w_i}{x\cdot w}\right)^{\frac{1}{1-\beta_i}} \in (0,1). \tag{2.26}$$

Observe that this ratio is independent of the capital stock, hence GPO emissions are a constant fraction of BAU emissions. Put another way, a uniform emission cut would be required to move from the myopic equilibrium to the GPO solution. This leads to the next definition:

Definition 4. A uniform cut in emissions is a capital-dependent emission policy $\tilde{a}_i(K_i)$ that is a constant fraction of the BAU emission policy, i.e.

$$\tilde{a}_i(K_i) = \kappa_i a_i^\star(K_i), \quad \text{where } \kappa_i \in (0,1).$$

Does the presence of capital growth exacerbate the tragedy of the commons? As (2.26) shows, the relative difference between the two benchmark solutions does not dependent on the capital level. Yet, the absolute difference of GPO and BAU emissions is growing at a rate of $\theta_i^{\frac{\gamma_i}{1-\beta_i}}$.

2.5.2 Uniform Emission Cuts under Capital Growth

The simplest way of supporting a Pareto-improving outcome (Pareto improving relative to the stage game Nash equilibrium, which is in this case the BAU equilibrium) as an equilibrium payoff is through the threat of Nash reversion (here BAU reversion). Theorem 11 is a negative result: if there is one country i such that its *effective growth rate* $\theta_i^{\frac{\gamma_i}{1-\beta_i}} > \theta_j^{\frac{\gamma_j}{1-\beta_j}}$ $\forall j \neq i$, then it is not possible to support any uniform emission cut that involves i. The reason for this negative finding is that at one point in time, far enough in the future, country i's utility will be dominated by a term that depends only on its own actions, hence the other countries simply cannot provide any incentive to detain i from playing its BAU action.

Theorem 11. *Suppose there is a unique country i that features the maximal effective growth rate, i.e. $\theta_i^{\frac{\gamma_i}{1-\beta_i}} > \theta_j^{\frac{\gamma_j}{1-\beta_j}}$ $\forall j \neq i$. Then no uniform emission cut that involves i can be supported as a SPNE under the threat of BAU reversion.*

Proof. See Appendix A. □

As a consequence of Theorem 11, the GPO cannot be supported as a SPNE under the threat of BAU reversion. Furthermore, note that the proof of this theorem also implies that no uniform emission cut policy - that involves the unique country featuring the maximal effective growth rate - can be supported as a SPNE even if punishments are more severe than BAU reversion (thus possibly not credible anymore) but do not become unboundedly large compared to the BAU levels: For any fixed $\kappa_j \in \mathbb{R}_+$, i.e. even for $\kappa_j > 1$ (country j suffers itself from punishing i), not even the threat of reverting to the severe κ_j-policy (that may not be an equilibrium strategy for j) forever can deter i from deviating. It seems reasonable to assume that feasible emissions are bounded from above by a multiple of BAU emissions (that depend on own capital level). Given this reasonable assumption, it follows that there does not exist a SPNE where i emits less than in the "bad" BAU equilibrium, as not even the prospect of being min-maxed forever discourages i from playing its BAU strategy.

2.5.3 Foreign Aid

The last subsection showed that capital growth makes it difficult to support equilibria that are better than the BAU equilibrium. If the threat of increasing emissions cannot deter the country featuring the maximal effective growth rate from playing its BAU strategy, can better equilibria be sustained by the introduction of foreign aid? First, define a feasible foreign aid policy in this context by requiring that all transfers sum to zero in each period. One can think of an international aid agency (e.g a United Nations institution) that conducts these transfers.

Definition 5. A *feasible foreign aid policy* is a sequence of emission levels

$$((\mu_i(t))_{i=1}^{n})_{t=0}^{\infty} \text{ s.t. } \sum_{i=1}^{n} \mu_i(t) = 0 \ \forall t.$$

For a country i, $\mu_i(t)$ is added to its utility in period t given by (2.23). It will be shown that for sufficiently patient countries and equal country weighting, an aid-induced GPO strategy profile can be sustained as a SPNE.

Definition 6. An *aid-induced GPO strategy profile* has two components:

1. At period 0 and in every period t with no previous unilateral deviation (in any period $s < t$), each country i plays it GPO strategy $\hat{a}_i(K_i(t))$ and receives/ transfers an amount of $\mu_i(t)$.

2. In the event of a unilateral deviation in period s, each country i plays its BAU strategy $a_i^\star(K_i(t))$ for all future periods $t > s$ without any transfers.

Recall that a discount factor is feasible if it is low enough to ensure boundedness of the solution, i.e. if $\delta\theta_i^{\frac{\gamma_i}{1-\beta_i}} < 1$ holds $\forall i$. A GPO profile with equal country weighting refers to equal country weights, i.e. $x_i = 1/n \ \forall i$.

Theorem 12. *There is a cut-off discount factor* $\hat{\delta} < \max_i \left\{\theta_i^{\frac{\gamma_i}{\beta_i-1}}\right\}$ *and a feasible foreign aid policy such that the aid-induced GPO strategy profile with equal country weighting constitutes a SPNE for all feasible discount factors above* $\hat{\delta}$. *Furthermore, for every country* i, *donor as well as recipient, lifetime payoffs inclusive of foreign aid strictly Pareto dominate BAU lifetime payoffs.*

Proof. The proof will only be sketched here, the algebraic details can be found in Dutta and Radner (2012): The key to the result is the observation that the individual incentive constraints (that include transfers in case of no deviation) can be replaced by a single aggregate incentive constraint that excludes transfers since they sum to zero. This simplification works because of the foreign aid mechanism that allows arbitrary "utility transfers" between countries, given that they sum to zero in every period. Reducing the proof to its essence, the theorem is true because the GPO strategy profile with equal country weighting is by definition better from a global perspective than the BAU strategy profile. Hence the aggregate incentive constraint is fulfilled, in turn the individual incentive constraints by a transfer scheme that does not have to be explicitly derived to prove the theorem. □

Three observations should be noted: First, once equal country weighting is abandoned, the theorem does not hold anymore in general. Problems emerge if the weight of a high-growth country (thus high BAU emissions) is lower than $1/n$, since the transfers than simply cannot make up for the loss stemming from the emission cut. Second, one can equivalently prove (see Dutta and Radner, 2012) that equal country weighting and sufficient patience imply that a convex combination of BAU and GPO emissions can be sustained. Third, the foreign aid mechanism has to be taken with a grain of salt. It implies that high-growth countries have to be subsidized by low-growth countries. Given that high-growth countries will eventually feature an economy that is arbitrarily large compared to low-growth countries in this model, this mechanism might not even be feasible from an economic point of view, not to mention the political aspects.

Chapter 3

The Convex Cost Case

In Chapter 2, the costs associated with the current level of GHG g in the atmosphere were assumed to be linear in g. While the linearity allows for closed-form solutions, a convex cost function might be a better approximation to reality according to Stern (2007). Therefore, I numerically investigate the structure of the BAU equilibrium (and the GPO solution) in the general model with one-period payoff given by

$$v_i(t) = a_i(t)^{\alpha_i} - c_i g(t)^{\beta_i}, \text{ where } \alpha_i \in (0,1), \beta_i \geq 1.$$

3.1 Numerical Solutions of Dynamic Games

When analyzing dynamic games numerically, it is common to focus on MPE as this reduces computational complexity considerably. In this case, players condition their actions only on the current state, i.e. strategies are functions from the set of states to the set of actions. By the one-shot deviation principle, a MPE equilibrium is characterized by a set of simultaneously satisfied Bellman equations (one for each player $i = 1, ..., n$):

$$V_i(g) = \max_{a_i \in A_i} \left\{ a_i(t)^{\alpha_i} - c_i g(t)^{\beta_i} + \delta V_i \left(\eta g + \sum_{j=1}^{n} a_j \right) \right\}$$

In the terminology of computational methods, the unknowns are the *policy functions* $a_i(g)$ and associated *value functions* $V_i(g)$. Following Fackler and Miranda (2002), for continuous value functions this setup can be exploited by approximating the value function with linear combinations of known basis functions $\phi_k(\cdot)$,

$$V_i(g) \approx \sum_{k=1}^{m} r_{ik}\phi_k(g).$$

This solution concept is known as *collocation method*, belonging to the class of *projection methods*. It stems from the Weierstrass Theorem, which says that any continuous real-valued function defined on a bounded interval can be uniformly approximated as closely as desired by a polynomial function. One requires the in total $n * m$ unknown coefficients r_{ik} to satisfy the Bellman equations at m states $g_1, ..., g_m$, the *collocation nodes*. In practice, one first specifies the basis functions ϕ_k and the number of collocation nodes m. In this example, eight standard Chebyshev polynomial basis functions and collocation nodes are used to form the approximant[1]. Furthermore, the relevant state space interval has to be specified. I use a numerical routine provided by Fackler and Miranda (2002) to find the BAU equilibrium in the case of convex costs associated with the level of GHGs.

3.2 Solving for the BAU Equilibrium

In the linear cost case (chaper 2), the BAU equilibrium is a particularly simple MPE in the sense that BAU emissions are constant (independent of the GHG level g). However, there are also other MPE, a simple example is given by Dutta and Radner (2009): below some critical GHG level $\tilde{g}$, countries emit only as much such that $\tilde{g}$ is not exceeded; above $\tilde{g}$, countries emit BAU levels. Hence, the numerical solution may provide us with a MPE, but we might not know whether

[1]Both numerical theory and empirical experience suggest that instead of evenly spaced nodes and monomials, Chebyshev polynomials and collocation nodes should be used. Chebyshev polynomials are given by $\phi_1(g) = 1, \phi_2(g) = g, \phi_3(g) = 2g^2 - 1, ..., \phi_k(g) = 2g\phi_{k-1}(g) - \phi_{k-2}(g)$ and Chebyshev nodes are more closely spaced near the endpoints of the state space interval. For details see Fackler and Miranda (2002).

this corresponds to the BAU equilibrium that equates private marginal benefits and private marginal costs. I resolved this problem by starting with the linear cost case ($\beta = 1$), where we have an analytic solution, $a_i^\star(g) = a_i^\star = \left(\frac{\alpha}{\delta w_i}\right)^{\frac{1}{1-\alpha}}$ and $V_i(g) = \frac{(a_i^\star)^\alpha - \delta w_i n a_i^\star}{1-\delta} - w_i g$. Providing the numerical routine with this policy and value function as initial guesses, the routine stops after the first iteration as expected (of course, as the initial guess is already the solution). Figure 3.1 illustrates BAU emissions and the BAU value function in the relevant GHG region. Note that the dotted line represents the steady state relationship $a_i = \frac{1-\eta}{n} g$, thus the point where the BAU emission function and the dotted line cross is the steady state of the system.

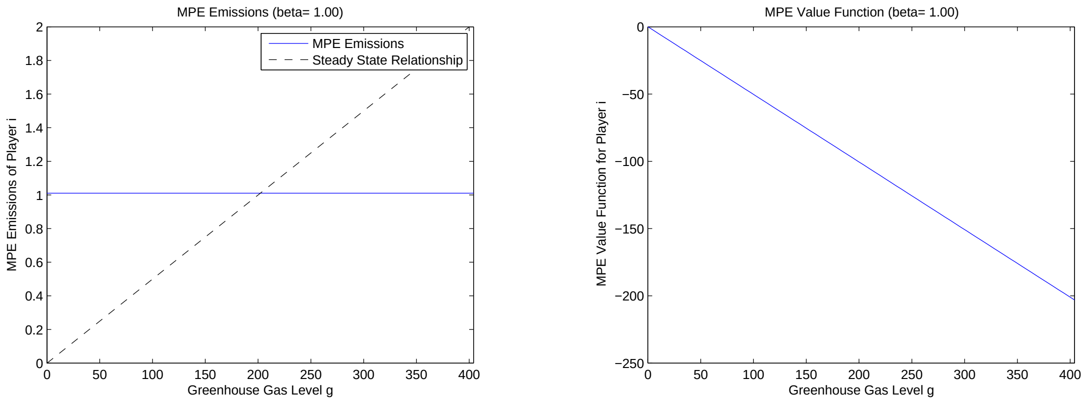

Figure 3.1: BAU Emissions (Left) and Value Function (Right), $\beta = 1$

Starting with the linear cost case does not only serve as a check for the numerical routine, but also achieves another purpose: The numerical routine is quite sensitive to the initial guesses of policy and value functions. Neither the naive guess of zeros nor the steady state values of the GPO[2] work as initial guesses for the convex cost case, in both cases the numerical routine does not converge. Apparently, the initial guesses are not close enough to the solution. However, using the solution from the linear cost case as initial guess for the case with a small amount of convexity ($\beta = 1.05$) and iterating this procedure in small increments of increasing convexity (i.e., use the $\beta = 1.05$ - solution as initial guess for the $\beta = 1.10$ - case and so on) does the job. Figures 3.2 and 3.3 show that for various levels of convexity, the BAU emissions appear to be decreasing and convex in the

[2] The steady state of the GPO can be easily computed, as the Envelope Theorem applies to this single function maximization problem.

GHG level. Moreover, BAU value functions seem to be decreasing and concave in g. Note that the respective intervals around the steady state are plotted - the scales are not comparable, higher convexity leads to lower BAU emissions (and thus a lower steady state GHG level).

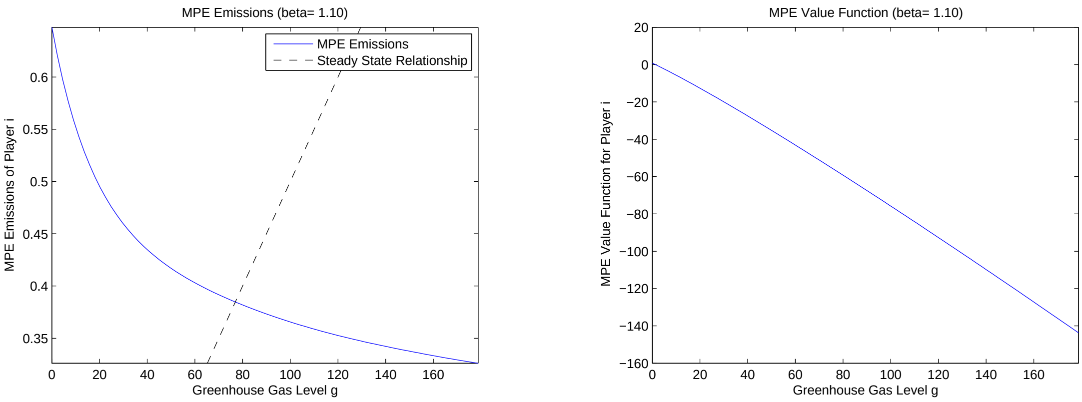

FIGURE 3.2: BAU Emissions (Left) and Value Function (Right), $\beta = 1.1$

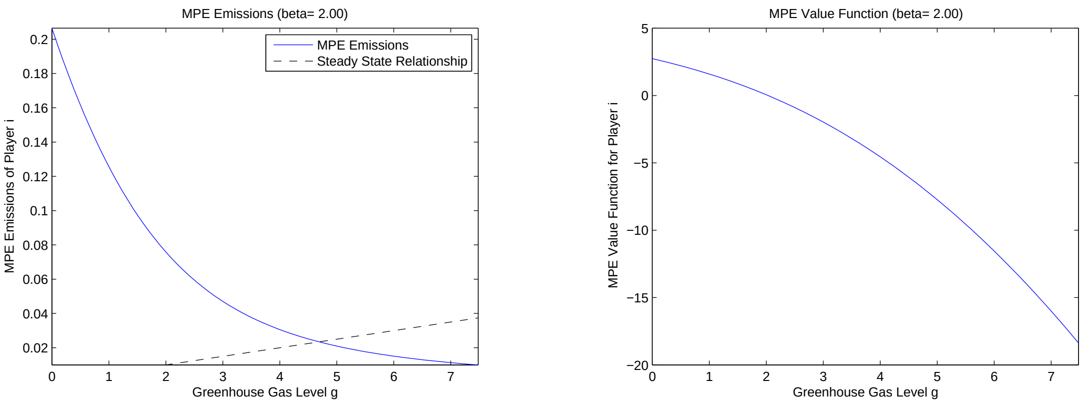

FIGURE 3.3: BAU Emissions (Left) and Value Function (Right), $\beta = 2$

To assess accuracy of the numerical solution, the residuals of this approximation are computed on a refined grid. For the linear case, these residuals are of order 10^{-14} - a pure numerical error, as the solution is indeed correct. For the convex cost case, the residuals are still acceptably low. The choice of a high discount factor ($\delta = 0.99$) and a large fraction of GHG surviving in the atmosphere ($\eta = 0.99$) leads to residuals of order 10^{-3}, respectively 10^{-5} (see Figure 3.4), lower discount factors and higher dissolving rates would lead to even smaller residuals. Note that the approximation error must equal zero at the Chebyshev nodes by design of this method and exhibits similar oscillations between the nodes, a property that is typical of Chebyshev residuals when the underlying model is smooth and effectively unconstrained.

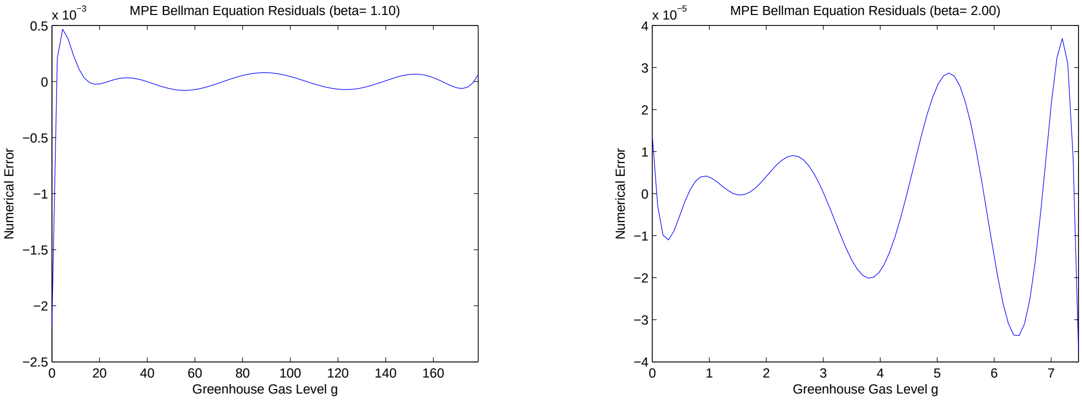

FIGURE 3.4: Approximation Error for $\beta = 1.1$ (Left) and $\beta = 2$ (Right)

Figure 3.5 illustrates BAU emissions and value functions for various levels of convexity. As β increases, BAU emissions appear to be decreasing, irrespective of the current level of GHGs. The total discounted payoff stemming from BAU behavior and current GHG level g exhibits an interesting feature: It seems that, given any two different exponents $\beta_H > \beta_L$, if g is close enough to zero, then the BAU payoff is higher for β_H. One could interpret that as a mitigation of the tragedy of the commons for faster increasing marginal costs associated with climate change. However, there exists a GHG level $\bar{g}(\beta_H, \beta_L) > 0$ such that for all $g > \bar{g}(\beta_H, \beta_L)$, the BAU payoff is higher for β_L. Above a threshold GHG level, the drastically higher costs of climate change that are due a greater exponent are not compensated anymore by lower emissions.

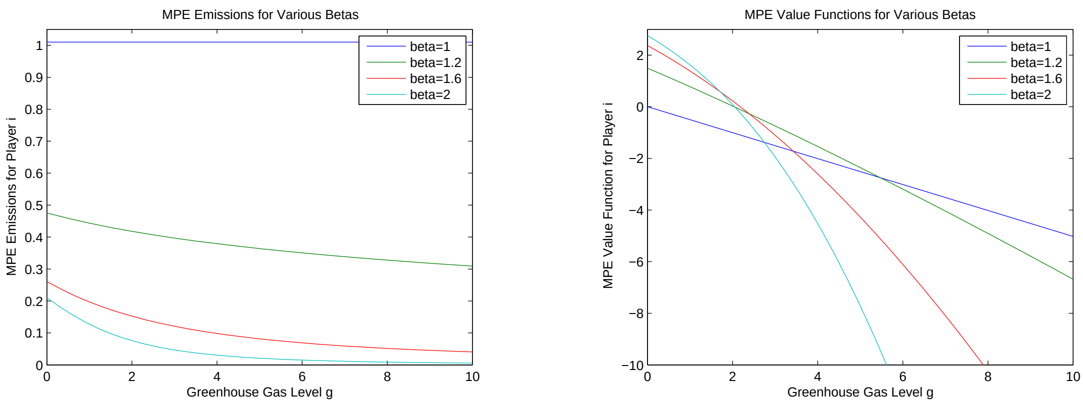

FIGURE 3.5: Comparison of BAU Emissions (Left) / Value Functions (Right)

Chapter 4

Trade Sanctions

In this chapter, I explore the introduction of trade decisions in order to allow for trade sanctions. We have seen that with sufficiently patient actors, any GPO can be supported as a SPNE, at least provided that the GPO is preferred to the BAU equilibrium. However, the most effective sanction for a deviating country i seems to be unrealistic: For one period, all other countries $j \neq i$ emit at a very high rate (higher then the BAU emission rate!), before proceeding to a quota that allows the other countries $j \neq i$ to emit at a rate moderately higher than the GPO rate at the expense of country i forever. In reality, these sharp increases and decreases in emissions might not be feasible due to short-term irreversibilities. I propose trade sanctions as an alternative mechanism to enforce cooperation.

4.1 A Simple Climate Change Model with Trade

A standard result in economics says that free trade is mutually beneficial, at least at the countrywide level (see Dixit and Norman, 1980). There might be groups within a country that actually lose from trade; however, I abstract from these issues by looking at every country i's aggregate welfare[1].

[1]"*Domestic redistributive instruments weaker than lump sum transfers (...) suffice for superiority of free trade over autarky.*" (Dixit and Norman, 1980, p. 80)

Suppose the baseline model from Chapter 2, for simplicity without technological change, already incorporates the gains from trade[2]. Each country $i \in N = \{1, 2, ..., n\}$ chooses two actions simultaneously every period $t = 0, 1, 2, ...$: Besides specifying its emissions $a_i(t)$ for the current period, it also announces a subset $p_i(t) \subseteq N \setminus \{i\}$[3] of all other countries, where $j \in p_i$ if and only if i is willing to trade with j without any barriers in addition to those that might be already in place[4]. Equivalently, one can interpret $j \in p_i$ as the absence of additional trade barriers imposed by i on j, hence $j \notin p_i$ does not necessarily imply that there is no trade at all between i and j. Formally, action pairs, which are elements of *state-independent* action sets, are given by

$$(p_i, a_i) \in \mathcal{P}(N_{-i}) \times \mathbb{R}_+, \ \forall\, i \in N.$$

For each country i, define a trading partner set p^i (not to confuse with p_i):

$$p^i = \{j \in N_{-i} \mid j \in p_i \wedge i \in p_j\}$$

Trade between i and j stops if either country choses to stop trading, $j \in p^i \Leftrightarrow i \in p^j$. Furthermore, assume that output $h_i(a_i)$ is multiplied by a trade factor $T_i(p^i) \in (0, 1]$. The trade factor should be weakly increasing with respect to set inclusion, trading with the whole world should give maximum value one and complete autarky should hurt each country (of course, output cannot be forced to equal zero). To summarize,

$$0 < T_i(\emptyset) < 1, \quad T_i(N_{-i}) = 1 \quad \text{and}$$
$$p^i \subseteq \hat{p}^i \Rightarrow T_i(p^i) \leq T_i(\hat{p}^i), \ \forall\, p^i, \hat{p}^i \subseteq N_{-i}, \ \forall\, i \in N.$$

[2]Dutta and Radner (2009) calibrated this model such that the BAU equilibrium path matches available data and estimates. The model is meant to describe the current situation in the world, hence it should already include the benefits of free trade as we currently observe them. Note that the term free trade does not refer to complete trade liberalization here in the sense that there are no barriers to trade at all, but rather to the present conditions in the real world.

[3]In the following, the notation N_{-i} will be used to denote $N \setminus \{i\}$.

[4]Of course, one could represent the trade decision as a continuum or as discrete choice with more than just two options to allow for gradual trade sanctions. Nevertheless, the binary choice is a good starting point and enhances both analytical tractability and intuitive understanding.

Simplifying notation to $T_i(p^i(p)) = T_i(p)$, where $p = (p_1, ..., p_n)$, the payoff in period t given actions (p, a) and GHG level $g(t)$ is

$$v_i(t) = T_i(p(t))h_i(a_i(t)) - c_i g(t).$$

In contrast to the model without trade, i's payoff is now directly (i.e. in the current period) affected by the other countries' actions through p_{-i}. Moreover, asymmetric punishments are possible in the sense that if i wants to punish j by a trade sanction, all other countries $k \neq i, j$ are not affected.

Remark. Additive trade factors would constitute an intuitive special case, i.e. $\forall\ i \in N$, $h_i(a_i)$ would decrease by a fraction $\phi_{i,j} \in [0, 1)$ if trade between country i and j were to stop. Then $T_i(p) = (1 - \sum_{j \in N_{-i} \setminus p^i} \phi_{i,j})$, where $0 < \sum_{j \in N_{-i}} \phi_{i,j} < 1\ \forall\ i \in N$ is required. $\phi_{i,j} = \phi_{j,i}$ will not hold in general as this would only make sense if all countries are of equal size. However, this special case is naive in the sense that it ignores substitution effects: If i stops trading with j, it will probably compensate by trading more with a country k that is similar to j.

That the introduction of trade sanctions actually increases the scope of action, meaning that all the equilibria of the basic game are also equilibria in the game including trade decisions, is made precise in Lemma 2. However, note that in the following the focus will be on equilibria that can be sustained by the threat of trade sanctions as these punishments might be considered to be more credible.

Lemma 2. *The set of SPNE payoffs of the climate change game with trade $\bar{\Xi}(g)$ contains the set of SPNE payoffs of the climate change game without trade $\Xi(g)$ for all GHG levels g.*

Proof. Fix any g, $v \in \Xi(g)$ and let σ be the associated strategy profile such that $V(\sigma, g) = v$. Define a strategy $\bar{\sigma}$ for the game with trade in the obvious way: For any history $\bar{h}^t$ in the game with trade, there is a unique history h^t of the game without trade such that h^t agrees with $\bar{h}^t$ on past emission levels as well as on past and current states. Let $\bar{\sigma}_i(\bar{h}^t) = (\sigma_i(h^t), N_{-i})\ \forall\ i$ and all histories $\bar{h}^t$, i.e. choose the same emission strategy and in addition free trade with all countries. This is clearly a SPNE: The choice of p_i does not affect the continuation value.

Trading with all countries maximizes the stage game payoff for any emission level a_i, thus the maximization problem is the same as in the game without trade. Hence there is no profitable one-shot deviation, $v \in \bar{\Xi}(g)$. $\square$

4.1.1 The Stage Game

Analogously to Section 2.3, it suffices to analyze the repeated game (a straightforward modification of Definition 3, Theorem 4 as well as Corollaries 3 and 4 proves this). The stage game payoff is given by

$$u_i(p, a, \delta) = T_i(p)h_i(a_i) - \delta w_i \sum_{j=1}^{n} a_j.$$

There are now more stage game Nash equilibria: Look at i's best reply to p_{-i} (i's best reply does not depend on a_{-i}). A higher trade factor is always better, hence setting $p_i = N_{-i}$ always constitutes an element of a best reply. Excluding a group of countries $S \subseteq N_{-i}$ from p_i is a best reply if and only if $T_i(N_{-i} \setminus S, p_{-i}) = T_i(N_{-i}, p_{-i})$. In particular, this equality will hold if $i \notin p_j \ \forall\ j \in S$, i.e. if excluding S does not change the trading partner set (but it may also hold if i does not gain from trading with j). It follows that for fixed p_{-i} (and δ), there might be more than one optimal p_i but only one optimal $a_i^\star(p_{-i}, \delta)$[5] implicitly defined by $T_i(p)h_i'(a_i^\star) = \delta w_i$ as all optimal p_i give the same trade factor. Hence, a Nash equilibrium is an action profile (a, p) such that no country can improve its trade factor by adding more countries to p_i and emissions are given by $a^\star(p, \delta) = (a_i^\star(p_{-i}, \delta))_{i=1}^n$. In particular, any p such that $i \in p_j \Leftrightarrow j \in p_i \ \forall\ i, j$ with the associated $a^\star(p, \delta)$ is a stage game Nash equilibrium. Note that best response emissions are strictly increasing in the *trade factor* and weakly increasing (with respect to set inclusion) in the *trading partner set*. As in the previous chapters, they are strictly decreasing in the discount factor. Furthermore, I will continue referring to them as BAU emissions. The GPO action profile is such that GPO emissions $\hat{a}(\delta)$ are defined by $h_i'(\hat{a}_i) = \delta \frac{x \cdot w}{x_i}$ (as in Section 2.3) and free trade prevails.

[5]In the sequel, the notation $a_i^\star(p^i, \delta)$ will also be used, where p^i is the argument of the trade factor (the trading partner set), e.g. $p^i = N_{-i}$ or $p^i = \emptyset$.

4.2 Best Equilibria under Trade Sanctions

Define the TBO with respect to trade sanctions as the optimal equilibrium (relative to a weighting vector x) subject to the constraint that the punishment for a unilateral deviation of i only involves changes in p_{-i}, i.e. that the other countries do not change their prescribed emission levels. Hence, the most severe punishment for the defecting country i is that all other countries stop trading with i forever. Formally, the optimal strategy profile then consists of $n+1$ phases:

- **Norm**: each country i emits $\tilde{a}_i$ and trades with all other countries.
- **Punishment of** i: i emits its best response $a_i^\star(\emptyset, \delta)$ and is not willing to trade with any country ($p_i = \emptyset$). All other countries $j \neq i$ continue emitting $\tilde{a}_j$ and only trade with each other, i.e. $p_j = N_{-j-i}$.

Whenever there is a unilateral deviation of i in any state, play switches to punishing i. First, note that there is no incentive to change the trade decision in any phase, since $i \in p_j \Leftrightarrow j \in p_i \;\forall\; i, j$ holds. Second, i cannot gain from deviating when already being punished (this follows trivially because i is playing a best response). Third, optimal deviations for i are $(N_{-i}, a_i^\star(N_{-i}, \delta))$ in the norm phase and $(N_{-i-j}, a_i^\star(N_{-i-j}, \delta))$ when j is punished. It follows that i's incentive constraint corresponding to a deviation from the norm is given by[6]

$$\begin{aligned} u_i(N_{-i}, \tilde{a}) \geq& (1-\delta) u_i(N_{-i}, a_i^\star(N_{-i}), \tilde{a}_{-i}) + \delta u_i(\emptyset, a_i^\star(\emptyset), \tilde{a}_{-i}) \\ \Longleftrightarrow h_i(\tilde{a}_i) - \delta w_i \tilde{a}_i \geq& (1-\delta)\Big[h_i\left(a_i^\star\left(N_{-i}\right)\right) - \delta w_i a_i^\star\left(N_{-i}\right)\Big] \\ &+ \delta\Big[T_i(\emptyset) h_i\left(a_i^\star\left(\emptyset\right)\right) - \delta w_i a_i^\star\left(\emptyset\right)\Big] \end{aligned} \tag{4.1}$$

and i's incentive constraint referring to j's punishment phase is

$$\begin{aligned} & u_i(N_{-i-j}, \tilde{a}_{-j}, a_j^\star(\emptyset)) \geq (1-\delta) u_i(N_{-i-j}, a_i^\star(N_{-i-j}), a_j^\star(\emptyset), \tilde{a}_{-i-j}) + \delta u_i(\emptyset, a_i^\star(\emptyset), \tilde{a}_{-i}) \\ \Longleftrightarrow \quad & T_i(N_{-i-j}) h_i(\tilde{a}_i) - \delta w_i\left(\tilde{a}_i + a_j^\star\left(\emptyset\right)\right) \geq (1-\delta)\Big[T_i(N_{-i-j}) h_i\left(a_i^\star\left(N_{-i-j}\right)\right) \\ & - \delta w_i\left(a_i^\star\left(N_{-i-j}\right) + a_j^\star\left(\emptyset\right)\right)\Big] + \delta\Big[T_i(\emptyset) h_i\left(a_i^\star\left(\emptyset\right)\right) - \delta w_i\left(a_i^\star\left(\emptyset\right) + \tilde{a}_j\right)\Big]. \end{aligned} \tag{4.2}$$

[6] For clarity of exposure, the dependence of u_i and $a_i^\star$ on δ is omitted and $u_i(p^i, a)$ refers to i's utility resulting from emissions a and i's trading partner set p^i.

Asymptotically, the constraints reduce to

$$h_i(\tilde{a}_i) - \frac{c_i}{1-\eta}\tilde{a}_i \geq T_i(\emptyset)h_i(a_i^\star(\emptyset)) - \frac{c_i}{1-\eta}a_i^\star(\emptyset) \quad \text{and}$$
$$T_i(N_{-i-j})h_i(\tilde{a}_i) - \frac{c_i}{1-\eta}(\tilde{a}_i + a_j^\star(\emptyset)) \geq T_i(\emptyset)h_i(a_i^\star(\emptyset)) - \frac{c_i}{1-\eta}(a_i^\star(\emptyset) + \tilde{a}_j).$$

The constraints are again continuous functions of δ (as in Section 2.3), hence for sufficiently patient countries it suffices that the asymptotic constraints are fulfilled with strict inequality. This gives rise to an intuitive observation, stating a minimum amount of emission reductions that is asymptotically sustainable:

Lemma 3. *There exists a cut-off discount factor $\underline{\delta} < 1$ such that for all $\delta \in [\underline{\delta}, 1)$, every norm emission profile $\tilde{a}$ such that $\tilde{a}_i \in [a_i^\star(\emptyset, 1), a_i^\star(N_{-i}, 1)]$ $\forall$ $i \in N$ is a SPNE supported by trade sanctions.*

Proof. Let $\tilde{a}_i \in [a_i^\star(\emptyset, 1), a_i^\star(N_{-i}, 1)]$ $\forall i \in N$. That a deviation from the norm is not profitable follows from

$$u_i(N_{-i}, \tilde{a}, 1) \geq u_i(N_{-i}, a_i^\star(\emptyset, 1), \tilde{a}_{-i}, 1) > u_i(\emptyset, a_i^\star(\emptyset, 1), \tilde{a}_{-i}, 1). \tag{4.3}$$

The first inequality holds since u_i is increasing in a_i up to $a_i^\star(N_{-i}, 1)$, the last inequality since $T_i(N_{-i}) = 1 > T_i(\emptyset)$ by assumption. That a deviation from punishing j is not profitable follows from

$$\begin{aligned} u_i(N_{-i-j}, \tilde{a}_{-j}, a_j^\star(\emptyset, 1), 1) &\geq u_i(N_{-i-j}, a_i^\star(\emptyset, 1), a_j^\star(\emptyset, 1), \tilde{a}_{-i-j}, 1) \\ &\geq u_i(\emptyset, a_i^\star(\emptyset, 1), a_j^\star(\emptyset, 1), \tilde{a}_{-i-j}, 1) \geq u_i(\emptyset, a_i^\star(\emptyset, 1), \tilde{a}_{-i}, 1). \end{aligned}$$

The first two inequalities hold for reasons similar to those explained above. The last inequality follows since $a_j^\star(\emptyset, 1) \leq \tilde{a}_j$ by assumption. To be precise, one needs strict inequalities in the asymptotic incentive constraints such that the constraints hold also for some high $\delta < 1$[7]. The lemma follows directly from continuity of the incentive constraints (at $\delta = 1$). □

[7] This can be assured for example by requiring that $T_i(N_{-i-j}) > T_i(\emptyset)$ $\forall i, j$. It is hard to imagine a situation where this is not the case (i.e. where equality holds), as this would impose that there is a country in the world that does not gain from starting to trade with the whole world, except for a single country.

Remark. Without additional assumptions, one cannot go further: For all $\tilde{a}$ such that $\tilde{a}_i < a_i^\star(\emptyset, 1)$ for some i, there exist trade factors close enough to one such that $\tilde{a}$ is not a SPNE for any discount factor. To see this, observe that in (4.3) this implies that $u_i(N_{-i}, \tilde{a}, 1) < u_i(N_{-i}, a_i^\star(\emptyset, 1), \tilde{a}_{-i}, 1)$, whereas $u_i(N_{-i}, a_i^\star(\emptyset, 1), \tilde{a}_{-i}, 1) \to u_i(\emptyset, a_i^\star(\emptyset, 1), \tilde{a}_{-i}, 1)$ as $T_i(\emptyset) \to 1$. Hence, the asymptotic incentive constraint is violated, which implies that such a norm emission profile $\tilde{a}$ is not a SPNE for any $\delta \in (0, 1)$.

Computing the best equilibrium under trade sanctions is a cumbersome task for heterogeneous countries, as the optimization exercise involves n incentive constraints for each country. In particular, without further assumption, whether i's incentive to deviate is greater in the norm phase or in the punishment phase of another country j is undetermined: The short-term gain from deviating is (weakly) greater under the norm regime, since the optimal deviation is larger ($a_i^\star(N_{-i}) \geq a_i^\star(N_{-i-j})$) and pays off more ($T_i(N_{-i}) \geq T_i(N_{-i-j})$). On the other hand, the long-term incentives[8] to deviate for i cannot be ranked - if some other country j is punished, the threat of no trade hurts i less (since it has already stopped trading with j), but i might be additionally threatened by the reversion of j to its norm strategy (if and only if $\tilde{a}_j > a_j^\star(\emptyset)$). Nevertheless, we learn something from this analysis: if low norm strategy emissions are sustainable (i.e. $\tilde{a}_j \leq a_j^\star(\emptyset)$, which is reasonable for patient countries by Lemma 3), then i's incentives to deviate are *asymptotically* greater in the punishment phase of another country and the constraint corresponding to a deviation from the norm can be ignored. In the next subsection, it will be shown that the restriction to homogeneous countries allows for a simpler characterization of the best equilibrium under trade sanctions.

4.2.1 Symmetric Countries

In this subsection, I assume symmetric countries and equal country weighting. Define $h_0 = h_i$, $c_0 = c_i$, $w_0 = w_i$ as well as $T_0 = T_i$ and let the argument of the

[8] Of course, a necessary condition for a norm profile to be sustainable as a SPNE is that there are long-term losses from deviating in every phase, i.e. all asymptotic constraints are fulfilled.

trade factor and of BAU emissions be the cardinality of the trading partner set (e.g. $n-2$ instead of N_{-i-j}). While BAU levels $a_0^\star(p^i)$ and the GPO level $\hat{a}_0$ is symmetric, a priori it is not clear whether the optimal equilibrium $\tilde{a}$ is symmetric. It turns out, however, that this is actually the case:

Lemma 4. *For symmetric countries and equal country weighting, the TBO with respect to trade sanctions ($\tilde{a}$) is symmetric.*

Proof. Let a' be any asymmetric SPNE emission vector, w.l.o.g. $a'_1 \neq a'_2$ and define a'' such that $a''_1 = a'_2$, $a''_2 = a'_1$ and $a''_i = a'_i$ $\forall i \geq 3$. Clearly, a'' is also a SPNE emission vector that gives the same sum of total country payoffs as a'. Let $\tilde{a}$ be a strictly convex combination of a' and a''. Then the sum of total country payoffs under $\tilde{a}$ is strictly greater by strict concavity of the objective function (follows from strict concavity of $h_0(\cdot)$), it remains to show that $\tilde{a}$ fulfills the incentive constraints. Recalling that $a_0^\star(\cdot)$ is a fixed function of the cardinality of the trading partner set, the norm constraint (4.1) can be written as

$$h_0(a_i) - \delta w_0 a_i \geq constant$$

and the constraint (4.2) for i when j is currently punished as

$$T_0(n-2)h_0(a_i) - \delta w_0(a_i - \delta a_j) \geq constant.$$

The left-hand sides of these inequalities are concave functions of a. Since these inequalities must hold for $a = a'$ and $a = a''$ (otherwise they would not be equilibrium emissions), they must also hold for any convex combination of them, hence for $a = \tilde{a}$. It follows that the TBO under trade sanctions is necessarily symmetric. □

This observation considerably simplifies the procedure of pinning down the optimal equilibrium, as there are now just two incentive constraints in total (in the general asymmetric case, there are n^2), one corresponding to the norm phase and one to the punishment phase. Any candidate a_0 for the TBO must satisfy these two constraints. Rewriting them such that all terms involving a_0 are on the left-hand side and defining functions f_N and f_P[9], this gives

$$f_N(a_0) \equiv h_0(a_0) - \delta w_0 a_0 \geq \kappa_N, \text{ where} \tag{4.4}$$
$$\kappa_N \equiv (1-\delta)\left[h_0\left(a_0^\star(n-1)\right) - \delta w_0 a_0^\star(n-1)\right] + \delta\left[T_0(\emptyset)h_0\left(a_0^\star(0)\right) - \delta w_0 a_0^\star(0)\right],$$

$$\text{and} \quad f_P(a_0) \equiv T_0(n-2)h_0(a_0) - (1-\delta)\delta w_0 a_0 \geq \kappa_P, \text{ where}$$
$$\kappa_P \equiv (1-\delta)\left[T_0(n-2)h_0\left(a_0^\star(n-2)\right) - \delta w_0 a_0^\star(n-2)\right] + \delta T_0(0)h_0\left(a_0^\star(0)\right).$$

If the GPO value $\hat{a}_0$ fulfills these two constraints, then trivially it is the TBO under trade sanctions. If not, pick the lowest a_0 such that both constraints are fulfilled.

Theorem 13. *For symmetric countries and equal country weighting, the TBO with respect to trade sanctions $\tilde{a}$ is*

(i) the GPO value, i.e. $\tilde{a}_i = \hat{a}_0\ \forall i$, if $f_N(\hat{a}_0) \geq \kappa_N$ and $f_P(\hat{a}_0) \geq \kappa_P$.

(ii) If this is not the case, then $\tilde{a}_i = \max\{a_0^N, a_0^P\}\ \forall i$, where a_0^N is implicitly defined by $f_N(a_0^N) = \kappa_N$ and a_0^P by $f_P(a_0^P) = \kappa_P$.

Proof. The theorem follows from Lemma 4 and above considerations. To see that a_0^N and a_0^P are well-defined, note that $f_j(0) < \kappa_j$, $f_j(a_0^\star(n-2)) > \kappa_j$[10] and that $f_j(a_0)$ is continuous and strictly increasing for $a_0 \in [0, a_0^\star(n-2)]$, $j = N, P$. □

For symmetric countries and equal country weighting, in the model without trade the GPO is a SPNE for sufficiently patient countries, even if the set of equilibria is restricted to those supported by BAU reversion. Is this also true under the restriction to trade sanctions as a punishment device? The answer is no, in

[9]The subscript N (P) corresponds to the norm (punishment) constraint.

[10]To be precise, for the norm constraint we only know that $f_N\left(a_0^\star(n-1)\right) > \kappa_N$ and have to impose that this holds for $a_0^\star(n-2)$ as well as an assumption. However, a close look at (4.4) reveals that this is just a technical assumption that holds for all reasonable functional forms and parameter values.

general. However, for sufficiently patient countries Theorem 13 can be further simplified. Corollary 5 says that it suffices to look at the punishment constraint.

Corollary 5. *For symmetric countries and equal country weighting, there exists a cut-off discount factor $\bar{\delta} < 1$ such that for all $\delta \in [\bar{\delta}, 1)$, the TBO with respect to trade sanctions $\tilde{a}_i = \tilde{a}_0\ \forall i$, where*

$$\tilde{a}_0 = \begin{cases} \hat{a}_0(\delta), & \text{if } f_P(\hat{a}_0(\delta)) \geq \kappa_P(\delta), \text{where } \hat{a}_0(\delta) \text{ is the GPO value,} \\ a_0^P, & \text{if } f_P(\hat{a}_0(\delta)) < \kappa_P(\delta), \text{where } a_0^P \text{ is defined by } f_P(a_0^P) = \kappa_P. \end{cases}$$

Proof. See Appendix A. □

Note that by Lemma 3, $\hat{a}_0(\delta) \geq a_0^\star(0, \delta = 1)$ implies that the GPO value is a SPNE under trade sanctions. Hence, $f_P(\hat{a}_0(\delta)) \geq \kappa_P(\delta)$ is guaranteed to hold in that case (again, only asymptotically) and one does not have to check this inequality. In other words, Corollary 5 is useful if the countries exhibit sufficient patience and if, additionally, the GPO emission level $\hat{a}_0(\delta)$ is smaller than the asymptotic BAU level under full trade sanctions $a_0^\star(0, \delta = 1)$.

4.3 Pareto Improving Trade Sanctions

The use of trade sanctions to enforce cooperation was justified by the claim that sudden emission increases may not be technologically feasible and thus trade sanctions may be considered to be a more credible punishment device. Yet, one might also ask whether the threat of trade sanctions can lead to a Pareto improvement relative to the threat of emission increases. To answer this question, I compare the TBO under trade sanctions with the TBO with respect to BAU reversion. BAU reversion is a particularly simple special case of using the threat of emission increases to enforce the cooperative outcome. However, the strategy profiles corresponding to the trade sanction equilibria are of comparable simplicity. In this regard, it makes sense to compare these two types of TBOs.

To foster understanding of the key factors and parameters affecting this question, I will assume that countries are symmetric in this subsection. Moreover, I will use an explicit functional form for the production function:

$$h_0(a_0) = a_0^\alpha, \text{ where } \alpha \in (0,1).$$

First, the possibility of a Pareto improvement necessitates that the GPO solution cannot be achieved by the threat of BAU reversion. This means that the incentive constraint (see Section 2.3.1) fails for $\hat{a}_0$:

$$u_i(\hat{a}) < (1-\delta)u_i(a_i^\star, \hat{a}_{-i}) + \delta u_i(a^\star),$$

which yields by homogeneity of countries,

$$f_B(\hat{a}_0) < f_B(a_0^\star), \quad \text{where } f_B(a_0) \equiv h_0(a_0) - \delta w_0 \left(1 + \delta\left(n-1\right)\right) a_0. \tag{4.5}$$

Clearly, countries must be impatient, otherwise the GPO will be achieved by the threat of BAU reversion (this inequality is violated). Furthermore, numerical computations revealed that (4.5) tends to hold if there are rather few countries (n is low) and if the output elasticity of energy is low (α closer to zero than to one). The more countries there are, the more a country's own utility depends on what other countries emit: if n is large, $a_0^\star$ will be far away from $\hat{a}_0$ and hence $u_0(a^\star)$ from $u_0(\hat{a})$; this discrepancy cannot be compensated for by the one-shot gain term $u_0(a_i^\star, \hat{a}_{-i})$, even if δ is quite low[11], because private utility is mainly driven by other countries actions $\hat{a}_{-i}$. Similarly, the larger the exponent α, the larger is the gap between $a_0^\star$ and $\hat{a}_0$. Figure 4.1 illustrates this observation: in the shaded region, parameters are such that (4.5) holds, i.e. that the GPO is not achieved by the threat of BAU reversion. The region is shrinking towards the origin as the number of countries increases, but is still non-empty for large n. Although there are nearly 200 countries in the world, one might also think of climate change negotiations being conducted by groups of countries.

[11]Of course, for every parameter constellation, one can pick a δ close enough to zero such that the GPO cannot be sustained as SPNE. However, letting $\delta \to 0$ is unrealistic in the context of this model.

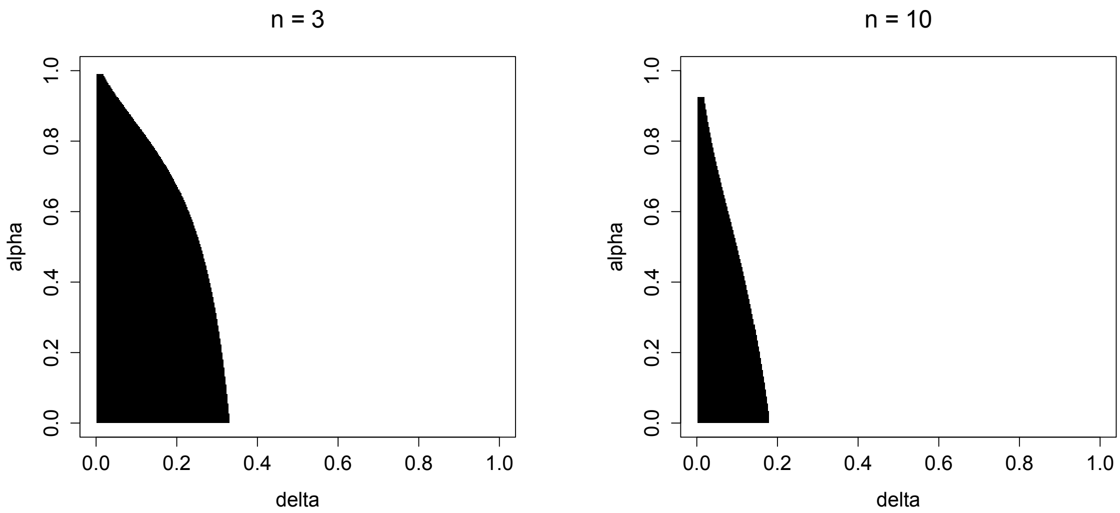

FIGURE 4.1: Parameter Region where GPO Is not Achieved by BAU Reversion

If the GPO cannot be achieved by BAU reversion, the BAU reversion TBO (denoted by $\tilde{a}_0$) is such that $f_B(\tilde{a}_0) = f_B(a_0^\star)$ for some $\tilde{a}_0 < a_0^\star$. To see that $\tilde{a}_0$ is well defined, note that f_B is strictly concave and has its unique maximum $\in (\hat{a}_0, a_0^\star)$ (follows by definition of the BAU and GPO emission levels). In that case, the use of trade sanctions leads to a Pareto improvement if and only if the two constraints corresponding to deviations from the prescribed emission levels are fulfilled with strict inequality for $\tilde{a}_0$, i.e. $f_N(\tilde{a}_0) > \kappa_N$ and $f_P(\tilde{a}_0) > \kappa_P$. This follows because if both the punishment and the norm constraint hold with strict inequality for $\tilde{a}_0$, then they must also hold for some $a_0 \in [\hat{a}_0, \tilde{a}_0)$, which leads to a strictly higher payoff for each country. Formally, the use of trade sanctions leads to a Pareto improvement relative to BAU reversion if and only if

$$f_N(\tilde{a}_0) > \kappa_N, \text{ which simplifies to} \tag{4.6}$$

$$h_0(a_0^\star(n-1)) - \delta w_0 n a_0^\star(n-1) > T_0(0) h_0(a_0^\star(0)) - \delta w_0 \left(a_0^\star(0) + (n-1)\,\tilde{a}_0\right),$$

$$\text{and} \quad f_P(\tilde{a}_0) > \kappa_P. \tag{4.7}$$

The interpretation of (4.6) is straightforward: Deviating from the norm phase is punished harder under trade sanctions than under BAU reversion. The latter condition (4.7) says that this must also be true for i when another country j

is currently being punished. For large n, it may be reasonable to assume that $T_0(n-2) \to 1$, in which case (4.7) simplifies to

$$f_N(\tilde{a}_0) > \kappa_N + \delta w_0 \left(a_0^\star(0) - \tilde{a}_0\right).$$

This is very similar to (4.6), but can hardly be justified for small n. Hence, I did not use this simplification for the numerical analysis. Figures 4.2, 4.3 4.4 display the parameter regions where the use of trade sanctions leads to a Pareto improvement compared to BAU reversion, this area is painted green. While in the dark green region, trade sanctions can even achieve the GPO, in the light green region they cannot do so (but they can still lead to a better SPNE than BAU reversion). On the other hand, in both blue areas the TBO under BAU reversion is better than the best SPNE under trade sanctions. Again, whereas dark blue indicates that the GPO can be achieved by BAU reversion, in the light blue (cyan) region this is not the case (but BAU reversion is still better than trade sanctions)[12]. Finally, in the gray region the GPO can be sustained by both methods[13]. Table 4.1 summarizes the illustration.

		GPO achieved?
	trade sanctions achieve better equilibrium	yes
		no
	BAU reversion achieves better equilibrium	yes
		no
	BAU reversion and trade sanctions do equally well	yes

Table 4.1: Legend for BAU Reversion and Trade Sanctions Comparison

This visualization illustrates the following points:

- The more a country (region) suffers from autarky (the lower $T_0(0)$ is), the more likely it is that trade sanctions lead to a Pareto improvement (compared to BAU reversion). Of course, the deterrence is higher if punishments are more severe.

[12]Hence, the black region in Figure 4.1 corresponds to the union of the light blue and the light as well as dark green area in Figures 4.2, 4.3 and 4.4.

[13]The attentive reader might ask what happened to the missing sixth region, where the TBO is the same for BAU reversion and trade sanctions but the GPO is not achieved. This region seems to be a one-dimensional line at the border between the light green and the light blue zone and is thus not visible.

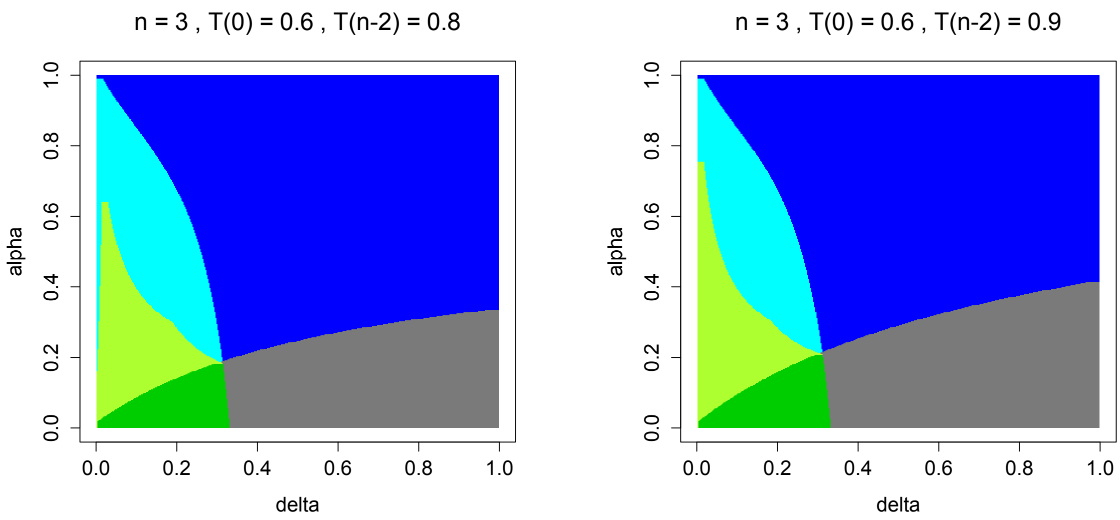

FIGURE 4.2: Pareto Improvement by Trade Sanctions: $n = 3$, $T_0(0) = 0.6$

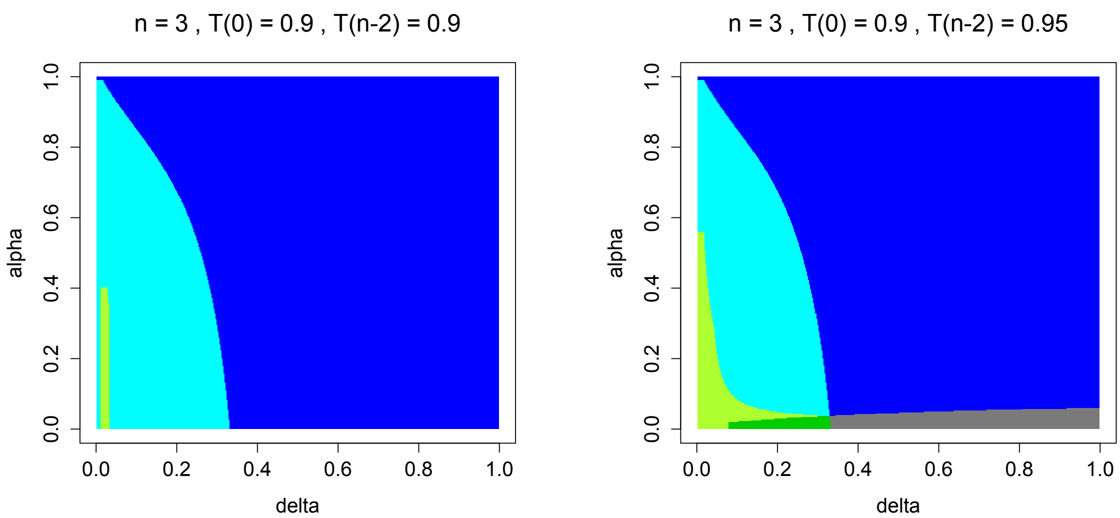

FIGURE 4.3: Pareto Improvement by Trade Sanctions: $n = 3$, $T_0(0) = 0.9$

- For fixed losses from autarky, the more a country suffers from losing just one trade partner (the lower $T_0(n-2)$ is), the less likely it is that trade sanctions lead to a Pareto improvement, as a deviation by i when j is currently punished leads to a less severe punishment of i.

- The higher the output elasticity of energy (α), the less likely it is that trade sanctions lead to a Pareto improvement. When α is high, BAU and GPO emissions are far apart. Thus, the incentive to deviate is high under trade sanctions as the gain from increasing own emissions is more likely to exceed the loss stemming from a lower trade factor. On the other hand, BAU

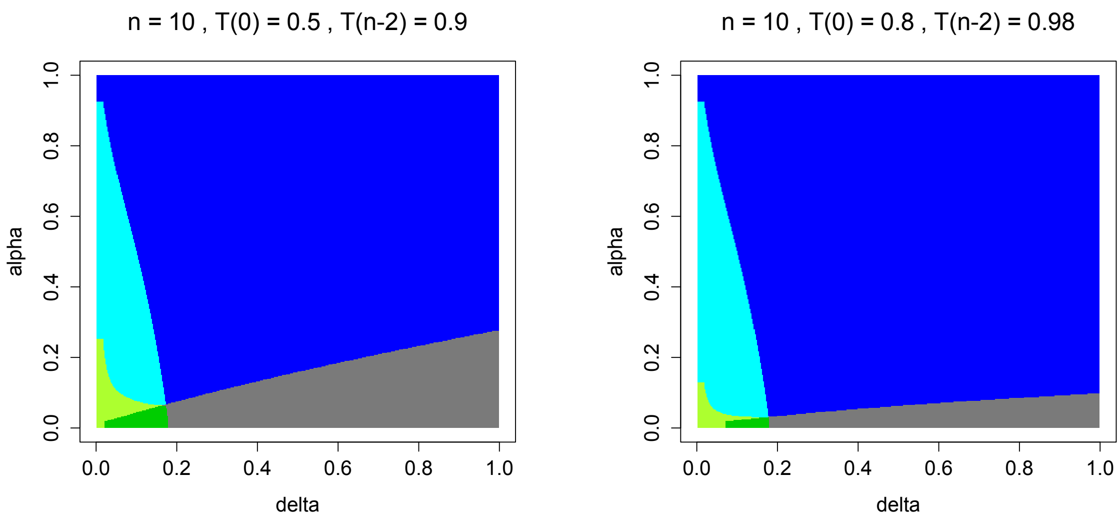

FIGURE 4.4: Pareto Improvement by Trade Sanctions: $n = 10$

reversion is actually more likely to achieve the GPO when α is high (as discussed earlier).

- Contrary to intuition, the more countries there are in a trade agreement, the less likely it is that trade sanctions lead to a Pareto improvement. This observation stems from the specification of the payoff function: as n gets larger, a country's utility is more and more dominated by other countries' emissions, the influence of the trade factor diminishes.

By increasing n to 193 (the current number of United Nations member states), the possibility for a Pareto improvement ceases to exist. Yet, one should not conclude from this simple numerical example that trade sanctions are unnecessary. First, as pointed out above, one can as well think of regions of the world as actors in this global game. Second, this elementary model represents a thought experiment; it is not intended to serve as a quantitative approximation of the real world. Third, the introduction of trade sanctions was initially justified because the alternative punishment device, sharp and sudden emission increases and decreases, might not be technologically feasible.

Chapter 5

Conclusion

This thesis addressed the question on how to design a self-enforcing international climate change treaty. In the language of game theory, the situation is modeled as a dynamic game following approaches by Dutta and Radner. In addition to a review of their contributions, I showed that in the special case of fixed emission technologies, this dynamic game can equivalently be represented as a repeated game. This observation simplifies equilibrium analysis considerably. Furthermore, I explored numerically how a benchmark myopic equilibrium (BAU) that equates private marginal benefits and costs changes once linearity of the climate change cost function is replaced by convexity. While there is a high degree of uncertainty in climate change cost estimates, convexity instead of linearity might be a better approximation to the real world according to existing literature. I found that BAU emissions are no longer constant, but decreasing and convex in the GHG level g; total BAU payoffs are decreasing and concave in g. Finally, I expanded the scope of action by allowing for trade decisions (and thus trade sanctions). Motivated mainly due to the fact that the alternative punishment device, sharp emission increases and decreases, might not be technologically feasible, I characterized the best equilibrium under trade sanctions. Moreover, I specified parameter constellations that yield a Pareto improvement due to the use of trade sanctions relative to the use of emission increases as a threat to enforce cooperation.

In order to preserve analytic traceability, the model used in this thesis is simplistic. Hence, one should rather regard it as a thought experiment than as an appropriate approximation to reality. Nevertheless, if one approves of the view that the world is currently in a bad equilibrium, this game-theoretic analysis may demonstrate the following points: Those countries in the world that would benefit from a move from the current myopic equilibrium to a new cooperative equilibrium featuring lower emission quotas should be able to design a self-enforcing treaty that either enforces or at least shifts emissions towards the GPO. Since the structure of the new equilibrium strategies would be very simple, the fact that only little progress is made towards such international agreements may point to issues within countries. For example, short-term horizons of policy makers, implying a low discount factor δ, aggravate the tragedy of the commons.

As there is a vast amount of uncertainty in estimates of consequences due to anthropogenic climate change, one of the most important issues for further research is to augment this model with uncertainty. This amounts to modeling the situation not as a dynamic game, but as a non-degenerate stochastic game. In particular, uncertainty in the evolvement of the level of GHGs in the atmosphere (the transition function) as well in climate change cost coefficients and possibly exponents should be explored. Furthermore, the implication of abandoning linearity in emission technology improvement costs represents another area for future research.

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Appendix A

Proofs

Proof of Theorem 1. The one-shot deviation principle applies to stationary MPE, hence we have to show that the following equation holds for $i = 1, .., n$:

$$V_i(f, g) = \max_{\substack{e_i \in \mathbb{R}_+ \\ f_i' \in [m_i, f_i]}} \left\{ (1-\delta) \left[Y_i(e_i) - c_i g - k_i(f_i - f_i') \right] + \delta V_i \left(f', \eta g + \sum_{j=1}^{n} f_j e_j \right) \right\},$$

where $V_i(f, g)$ is given by (2.7) and the BAU strategy is a maximizer. Simple algebra reveals that the expression inside the curly brackets can be split in two additive terms, where the first depends only on e_i and the second only on f_i. Dropping constants and dividing by $(1-\delta)$, the first part reduces to $Y_i(e_i) - \delta w_i e_i f_i$, which is clearly maximized by setting $e_i = E_i(f_i)$ (2.4). Again, dropping constants, dividing by $(1-\delta)^2$ and rearranging, the second term is given by

$$\begin{aligned} k_i f_i' + \frac{\delta}{1-\delta} \left(Y_i[E_i(f_i')] - \delta w_i f_i' E_i(f_i') \right) \\ + \frac{\delta}{1-\delta} \left(k_i F_i(f_i') + \frac{\delta}{1-\delta} \left(Y_i[E_i(F_i(f_i'))] - \delta w_i F_i(f_i') E_i(F_i(f_i')) \right) \right) \\ = Z_i(f_i') + \frac{\delta}{1-\delta} Z_i(F_i(f_i')) \quad \text{(A.1)} \end{aligned}$$

We have to prove that the maximizer f_i' of the last expression, subject to the constraint $f_i' \in [m_i, f_i]$, equals $F_i(f_i)$. Using (2.5), (2.6) and the subsequent discussion of its properties, this can be proved by exhaustive case distinction:

i Suppose $Z_i'(m_i) \geq 0$: Since $m_i \leq f_i'$, $F_i(f_i') = f_i'$. But then (A.1) reduces to $\left(1 + \frac{\delta}{1-\delta}\right) Z_i(f_i')$, which is maximized by definition (2.6) by $F_i(f_i) = f_i$.

ii Suppose $Z_i'(m_i) < 0$, $f_i \leq y_i^0$: Since $f_i' \leq f_i \leq y_i^0$, $F_i(f_i') = F_i(f_i) = m_i$. Then (A.1) equals $Z_i(f_i') + \frac{\delta}{1-\delta} Z_i(m_i)$, which is again maximized by definition (2.6) by $F_i(f_i) = m_i$.

iii Suppose $Z_i'(m_i) < 0$, $f_i > y_i^0$: (A.1) is a strictly convex function, because Z_i is strictly convex and F_i is either a constant or an increasing step function. Thus the maximizer must be $\in \{m_i, f_i\}$. Now, f_i clearly maximizes the first term $Z_i(f_i')$, since $F_i(f_i) = f_i$ ($Z_i(f_i) > Z_i(m_i)$ in this case). But f_i also maximizes the second term since $Z_i(F_i(f_i)) = Z_i(f_i) > Z_i(m_i) = Z_i(F_i(m_i))$.

This proves that the maximizer of (A.1) equals $F_i(f_i)$ in all cases. Hence, the BAU strategy profile (E, F) maximizes the value function. Simple algebra shows that acting according to (E, F) forever indeed gives the value function (2.7). The theorem is proved. □

Proof of Theorem 8. The function $f : \mathbb{R}^2 \to \mathbb{R}$ that is evaluated at zero in the first order condition,

$$h_{i1}(a_i, P_i) - \delta w_i(P_i') = 0,$$

is C^1 and its partial derivative w.r.t. its first argument is not zero (anywhere) by the strict concavity assumption on $h_i(\cdot, P_i)$ for any fixed P_i. Hence the implicit function theorem can be applied, this leads to

$$\frac{\partial a_i^\star(P_i)}{\partial P_i} = \frac{1}{h_{i11}(a_i^\star(P_i), P_i)} \left(\frac{c_i \lambda_i \delta}{1 - \lambda_i \delta \eta} - h_{i12}(a_i^\star(P_i), P_i) \right) \tag{A.2}$$

Recall that h_{i11} is strictly negative, this gives (2.22). Equivalently, the implicit function theorem implies expression (A.2) for $\frac{\partial \hat{a}_i(P)}{\partial P_i}$. For any $j \neq i$ it follows that

$$\frac{\partial \hat{a}_i(P)}{\partial P_j} = \frac{1}{h_{i11}(\hat{a}_i(P), P_i)} \left(\frac{x_j}{x_i} \frac{c_j \lambda_j \delta}{1 - \lambda_j \delta \eta} \right).$$

This expression is always strictly negative, again because of strict concavity. □

Proof of Theorem 9. Again, we look at the Bellman equation given by

$$V_i^\star(g,K) = \max_{a_i} \left\{ (1-\delta)\left(a_i^{\beta_i} K_i^{\gamma_i} - c_i g\right) + \delta V_i^\star(g',K') \right\}, \tag{A.3}$$

$$\text{where} \qquad (g',K') = \left(\eta g + a_i + \sum_{j\neq i} a_j^\star(K_j), (\theta_i K_i)_{i=1}^n \right). \tag{A.4}$$

Collecting terms that involve a_i and differentiating gives as a necessary condition for an optimum that $\beta_i a_i^{\beta_i - 1} K_i^{\gamma_i} = w_i \delta$, (2.25) follows immediately. Substituting (2.25) for a_i and equating all K_i-terms gives

$$\phi_i^i K_i^{\frac{\gamma_i}{1-\beta_i}} = (1-\delta)\left(\left(\frac{\beta_i}{\delta w_i}\right)^{\frac{\beta_i}{1-\beta_i}} K_i^{\frac{\gamma_i}{1-\beta_i}} - \delta w_i \left(\frac{\beta_i}{\delta w_i}\right)^{\frac{1}{1-\beta_i}} K_i^{\frac{\gamma_i}{1-\beta_i}} \right) + \delta \phi_i^i \theta_i^{\frac{\gamma_i}{1-\beta_i}} K_i^{\frac{\gamma_i}{1-\beta_i}}.$$

Dividing both sides by $K_i^{\frac{\gamma_i}{1-\beta_i}}$ and rearrenging yields

$$\phi_i^i = \frac{(1-\delta)(1-\beta_i)\left(\frac{\beta_i}{\delta w_i}\right)^{\frac{\beta_i}{1-\beta_i}}}{1-\delta\theta_i^{\frac{\gamma_i}{1-\beta_i}}}. \tag{A.5}$$

Applying the same steps to the K_j-terms results in

$$\phi_i^j K_j^{\frac{\gamma_j}{1-\beta_j}} = (1-\delta)\left(-\delta w_i \left(\frac{\beta_j}{\delta w_j}\right)^{\frac{1}{1-\beta_j}} K_j^{\frac{\gamma_j}{1-\beta_j}} \right) + \delta \phi_i^j \theta_j^{\frac{\gamma_j}{1-\beta_j}} K_j^{\frac{\gamma_j}{1-\beta_j}},$$

$$\text{and} \qquad \phi_i^j = \frac{(1-\delta)\delta w_i \left(\frac{\beta_j}{\delta w_j}\right)^{\frac{1}{1-\beta_j}}}{1-\delta\theta_j^{\frac{\gamma_j}{1-\beta_j}}}. \tag{A.6}$$

The denominators in (A.5)and (A.6) are strictly positive by the boundedness assumptions. The only terms left are

$$-(1-\delta)w_i g = (1-\delta)(-c_i g) + \delta(1-\delta)(-w_i \eta g),$$

which lead to the definition of the discounted marginal cost expression $w_i = \frac{c_i}{1-\delta\eta}$. □

Proof of Theorem 11. Let $(\tilde{a}_i(K_i) = \kappa_i a_i^\star(K_i))_{i=1}^n$ be a uniform emission cut policy involving i, i.e. $\kappa_i \in (0,1)$. By arguments very similar to the ones made precise in the proof of Theorem 9, the value for i stemming from such a uniform emission cut policy, characterized by the n-vector κ, is

$$\tilde{V}_i(\kappa, g, K) = \tilde{\phi}_i^i(\kappa_i) K_i^{\frac{\gamma_i}{1-\beta_i}} - \sum_{j \neq i} \tilde{\phi}_i^j(\kappa_j) K_j^{\frac{\gamma_j}{1-\beta_j}} - (1-\delta) w_i g,$$

$$\text{where} \quad \tilde{\phi}_i^i(\kappa_i) = \frac{(1-\delta)(\kappa_i^{\beta_i} - \beta_i \kappa_i) \left(\frac{\beta_i}{\delta w_i}\right)^{\frac{\beta_i}{1-\beta_i}}}{1 - \delta \theta_i^{\frac{\gamma_i}{1-\beta_i}}}$$

$$\text{and} \quad \tilde{\phi}_i^j(\kappa_j) = \frac{(1-\delta)\delta w_i \kappa_j \left(\frac{\beta_j}{\delta w_j}\right)^{\frac{1}{1-\beta_j}}}{1 - \delta \theta_j^{\frac{\gamma_j}{1-\beta_j}}} \quad \forall j \neq i.$$

Note that i maximizes its total discounted payoff by setting $\kappa_i = 1$, i.e. by playing its best response that is independent of other countries' actions and equals i's BAU strategy. The threat of BAU reversion means that a punishment of i would consist of playing the BAU equilibrium for all future periods, i.e. all other countries j punishing i by setting $\kappa_j = 1$. The net difference in total discounted payoffs for i between acting according to a uniform emission cut policy κ, where $\kappa_i < 1$, and deviating by playing its BAU strategy (the most profitable deviation) equals

$$\underbrace{\left(\tilde{\phi}_i^i(1) - \tilde{\phi}_i^i(\kappa_i)\right)}_{\text{constant } >0} K_i^{\frac{\gamma_i}{1-\beta_i}} - \delta \sum_{j \neq i} \underbrace{\left(\tilde{\phi}_i^j(1) - \tilde{\phi}_i^j(\kappa_j)\right)}_{\text{constant } >0} K_j^{\frac{\gamma_j}{1-\beta_j}}.$$

As $K_i^{\frac{\gamma_i}{1-\beta_i}}$ grows at a higher rate than all other countries' effective capital levels by assumption, i.e. $\theta_i^{\frac{\gamma_i}{1-\beta_i}} > \theta_j^{\frac{\gamma_j}{1-\beta_j}}$ $\forall j \neq i$, the last expression will eventually become positive, i.e. there is a period t where deviating is profitable for i. $\square$

Proof of Corollary 5. By Lemma 3, $\exists \underline{\delta} < 1$ s.t. $\forall \delta \in [\underline{\delta}, 1)$, any $a_0 \in [a_0^\star(0, \delta = 1), a_0^\star(n-1, 1)]$ is a SPNE under trade sanctions, which implies that both constraints are fulfilled. Hence, to prove the Corollary it suffices to prove existence of a $\bar{\delta} \in [\underline{\delta}, 1)$ s.t. $\forall \delta \in [\bar{\delta}, 1)$, $f_P(a_0) \geq \kappa_P \Rightarrow f_N(a_0) \geq \kappa_N$, $\forall a_0 \in (0, a_0^\star(0, 1))$.

Thus, take any $a_0 \in (0, a_0^\star(0,1))$ and suppose $f_P(a_0) \geq \kappa_P$. Then

$$
\begin{aligned}
& f_N(a_0) = h_0(a_0) - \delta w_0 a_0 = f_P(a_0) + (1 - T_0(n-2))h_0(a_0) - \delta^2 w_0 a_0 \\
& \geq \kappa_P + (1 - T_0(n-2))h_0(a_0) - \delta^2 w_0 a_0 = \kappa_N \\
& + (1 - T_0(n-2))h_0(a_0) + \delta^2 w_0 (a_0^\star(0,\delta) - a_0) \qquad \text{(A.7)} \\
& + (1-\delta)\left[u_0\left(n-2, a_0^\star\left(n-2,\delta\right)\right) - u_0\left(n-1, a_0^\star\left(n-1,\delta\right)\right)\right]. \qquad \text{(A.8)}
\end{aligned}
$$

There are two cases:

- $T_0(n-2) = 1$ (recall that $T_0(n-1) = 1$ by assumption): Then $f_N(a_0) \geq \kappa_N + \delta^2 w_0(a_0^\star(0,\delta) - a_0) > \kappa_N$, since for $\delta < 1$ it holds that $a_0^\star(0,\delta) > a_0^\star(0,1) > a_0$. Hence we can simply set $\bar{\delta} = \underline{\delta}$.

- $T_0(n-2) < 1$: Then line (A.7) is strictly positive and line (A.8) strictly negative. Asymptotically (for $\delta = 1$), we get

$$f_N(a_0) \geq \kappa_N + \underbrace{\left(1 - T_0\left(n-2\right)\right) h_0(a_0)}_{\lambda_1 > 0} + \underbrace{\frac{c_0}{1-\eta}(a_0^\star(0,1) - a_0)}_{\lambda_2 > 0}.$$

  Note that while $\lambda_1 \to 0$ as $a_0 \to 0$ and is strictly increasing in a_0, $\lambda_2 \to 0$ as $a_0 \to a_0^\star(0,1)$ and is strictly decreasing in a_0. As a consequence of this observation and by continuity of the constraints in δ, there exists a $\delta' < 1$ s.t. for all $\delta \geq \delta'$ and for all $a_0 \in (0, a_0^\star(0,1))$, $f_N(a_0) \geq \kappa_N$. Defining $\bar{\delta} = \max\{\underline{\delta}, \delta'\}$ finishes the proof.

$\square$