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FÜR INFORMATIK

Optimization Challenges of the Future Federated Internet

Heuristic and Exact Approaches

DISSERTATION

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Abstract

The Internet has ossified. It has lost its capability to adapt as requirements change. A fitting practical example for ossification is the introduction of IPv6. It has been specified in 1998 to solve, among other things, the forseeable Internet address shortage. The addresses have begun to run out in 2011 and still IPv6 does not see any wide-spread usage; hacks like network address translation reduce the need to switch.

A promising approach for solving this problem is the introduction of network virtualization. Instead of directly using the single physical network, unchangeable to a large degree and working just well enough for a limited range of applications, multiple virtual networks are embedded on demand into the physical network, each of them perfectly adapted to a specific application class. Compute capabilities within the network are provided to the virtual networks, enabling them to offer their own customized topologies, routing, resource management and naming services.

There are still numerous unsolved problems regarding network virtualization, ranging from the implementation of virtualizable routers to economic aspects. In this thesis, we focus on the problem of resource allocation. All virtual networks, with all the resources they require (e.g., bandwidth), still need to fit into the available physical network. Our aim is not merely finding an arbitrary solution, we want to fit the virtual networks in a cost-optimal way. This is the core of the Virtual Network Mapping Problem (VNMP), an \mathcal{NP} -complete Combinatorial Optimization Problem.

We present several heuristic and exact approaches for solving the VNMP. As heuristic methods we investigate Construction Heuristics, Local Search, Variable Neighborhood Descent, Memetic Algorithms, Greedy Randomized Adaptive Search Procedures, and Variable Neighborhood Search. The exact approaches we develop are based on Constraint Programming and Integer Linear Programming. In addition to analyzing different solution methods and comparing their various strengths and weaknesses, we present a strong preprocessing method for VNMP instances. This preprocessing method can determine which parts of the physical network each virtual network can and cannot use. We show that preprocessing is essential for solving large VNMP instances with exact methods.

For finding a valid mapping of virtual networks into substrate networks, the preprocessing method is powerful enough to make Integer Linear Programming the solution method of choice. For low-cost solutions, the situation is more complex. Integer Linear Programming is the best method for small to medium instance sizes. If run-time is a concern, our Memetic Algorithm and Variable Neighborhood Search approaches can be used to great effect, achieving costs within 5% of the exact method. For large instances, we conclude that Variable Neighborhood Descent performs best.

Kurzfassung

Das Internet wie wir es heute kennen hat seine Fähigkeit verloren, sich an ändernde Bedingungen anzupassen. Es gilt als "erstarrt". Ein prägnantes Beispiel ist die Einführung von IPv6. Dieses Protokoll wurde schon 1998 spezifiziert, um unter anderem der bevorstehenden Internet-Adressknappheit entgegenzuwirken. Obwohl die Adressen seit 2011 zur Neige gehen, wird IPv6 noch immer nicht großflächig eingesetzt. Notlösungen wie Netzwerk-Adressübersetzung reduzieren die Notwendigkeit eines Wechsels.

Ein vielversprechender Ansatz um wieder Flexibilität in das Internet zu bringen ist Netzwerkvirtualisierung. Statt eines einzigen unflexiblen physischen Netzwerks, das eine Reihe von Anwendungen gerade noch ausreichend unterstützt, werden mehrere virtuelle Netzwerke, die voll und ganz auf verschiedene Anwendungsfälle ausgerichtet sind, in das physische Netz eingebettet.

Bevor Netzwerkvirtualisierung großflächig eingesetzt werden kann, gilt es noch eine Vielzahl von Problemen zu lösen, von der Implementierung von virtualisierbaren Routern bis hin zu wirtschaftlichen Aspekten. In dieser Dissertation konzentrieren wir uns auf Ressourcenverteilung und -belegung. Die verschiedenen virtuellen Netze, samt ihren benötigten Ressourcen (z.B. Bandbreite), müssen ein einem einzigen physischen Netz untergebracht werden. Unser Ziel ist jedoch nicht, eine beliebige Einbettung der virtuellen Netze in das physische Netz zu finden, sondern eine kosten-optimale. Das ist der Kern des Virtual Network Mapping Problems (VNMP), ein \mathcal{NP} -vollständiges kombinatorisches Optimierungsproblem.

In dieser Arbeit untersuchen wir heuristische und exakte Ansätze zur Lösung des VNMP. Die heuristischen Methoden sind Konstruktionsheuristiken, Lokale Suche, Variable Neighborhood Descent, Memetische Algorithmen, Greedy Randomized Adaptive Search Procedures und Variable Neighborhood Search. Als exakte Verfahren entwickeln wir Ansätze, die auf Constraint Programming und Integer Linear Programming basieren. Zusätzlich zur Analyse der vorgestellten Algorithmen und des Vergleichs ihrer Stärken und Schwächen präsentieren wir auch eine Vorverarbeitungsmethode für VNMP Instanzen. Wir zeigen, dass diese Vorverarbeitung ein essenzieller Schritt für die Anwendung von exakten Verfahren auf große VNMP Instanzen ist.

Nur durch die Vorverarbeitung ist es möglich, dass unser Integer Linear Programming Ansatz unabhängig von der Instanzgröße ein exzellentes Verfahren ist, wenn es um das Finden einer beliebigen Einbettung geht. Für die Suche einer kosten-optimalen Lösung ist die Wahl der besten Methode komplizierter. Integer Linear Programming liefert die besten Ergebnisse bis zu mittleren Instanzgrößen, jedoch nur unter hohem Zeitaufwand. Gilt es diesen zu minimieren, sind unsere Memetischen Algorithmen und Variable Neighborhood Search Ansätze vielversprechend. Die damit erreichten Kosten liegen nur 5% höher als die der exakten Methode. Für große Instanzen bietet Variable Neighborhood Descent die beste Lösungsqualität.

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A Solutions in Detail

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CHAPTER

Introduction

The Internet has ossified [72, 132]. This means it has lost its ability to react to changing requirements, its ability to innovate. It has fallen victim to its own success [170], and only just works [72].

Why this negativity? Without a doubt, the Internet works. Customers can expect ever increasing data transfer speeds. With higher speeds new and innovative services become possible, like video streaming. The video streaming service YouTube [181], founded in 2005, is now visited by more than a billion users and streams six billion hours of video in one single month. Other services like Twitch [99] or Ustream [173] are not satisfied with video on demand and offer true live video streams. A camera and Internet access is all that is required to instantly reach an audience of thousands. Since the advent of mobile phones with Internet connectivity, it is no longer a challenge to be online anytime, anywhere.

So, where are the big problems? If the Internet has ossified, how did it come to that? Where are the cracks that show that something is amiss? How can these problems be solved and what contribution is this thesis going to make? We will answer these questions in the following sections.

1.1 The History of a Changing Internet

The history of the Internet is one of explosive growth and change. Technologies and protocols have been discarded or modified as it became clear that they could not keep up. An excellent review can be found in [72], which forms the basis of this section.

The Internet had its beginnings as the ARPAnet, the first large-scale packet switched network. Its foundation was the Network Control Program (NCP) [35], responsible for addressing and data transport. As the ARPAnet grew, it became clear that NCP was not flexible enough. The task of addressing and data transport fell to the Internet Protocol (IP), but ensuring reliability, i.e., that sent data is actually received, was from now on achieved by the Transmission Control Protocol (TCP) on top of IP. An alternative that does not guarantee reliability, the User Datagram

Protocol (UDP) was also introduced. The switch from NCP to TCP/IP occurred on a single day in 1983, when the remaining ARPAnet nodes started using the new protocols. This procedure encompassed about 400 nodes and was probably the last time core functionality could be replaced by just moving every component to the new technology. From then on, every change has been deployed incrementally.

As the Internet grew, components failed to scale and were replaced. An early example for this is the "hosts.txt" file [111] used for name resolution in addressing. It rapidly grew infeasible to distribute a file with the names and addresses of all available servers in the Internet to each computer with access to the Internet. In 1982, this system was replaced by the Domain Name System (DNS) which in addition to solving the distribution problem also introduced namespaces which could be administrated in a decentralized fashion. Handley [72] notes that this system could have been developed years earlier but only as the scaling limit of the previous system was reached the need was pressing enough to deploy a replacement.

Other systems replacing their predecessors because they could not handle the increasing size of the Internet are link-state routing protocols [124] and the Exterior Gateway Protocol [151].

The strategy of fixing problems just in time worked rather well, until a series of congestion collapses occurred in the mid-1980s. The network was moving data at full capacity, but no useful work was done. The problem turned out to be TCP's retransmission policy, the network was flooded with data that was unnecessarily retransmitted. This problem occurred with TCP, but also UDP can cause congestion. Indeed, congestion is a consequence of trying to send more data than possible, independent of the employed protocols. Therefore, the correct solution might have been to add a layer to the protocol stack that handles congestion. This would have been a serious change to the core working principles of the Internet and in 1988 it was already too large to attempt it. Instead, a congestion control mechanism for TCP was introduced [96], not a solution for the general problem, but good enough. This change was backwards compatible, incrementally deployable and probably the first of what are called "architectural barnacles" by Peterson et al. [6], unsightly outcroppings that have affixed themselves to an unmoving architecture.

After the Border Gateway Protocol (BGP) [116] was introduced in the early 1990s to allow the commercialization of the Internet, the last major change to the core Internet was the introduction of Classless Inter-Domain Routing (CIDR) [182] in 1993, changing how addressing worked. It was basically lucky chance that made this switch possible. Firstly, it was backwards compatible and the end-hosts could use the previous system until an operating system upgrade fixed the issue eventually. Secondly, the routing hardware within the network was supplied by a single vendor, and the affected functionality was implemented in software and thus easily changed. Such a fix would be unthinkable today, as the core functionality of the Internet protocols is implemented in hardware for speed reasons.

At the time of writing, 1993 was two decades ago. What happened in the meantime? Except the explosive growth of the Internet, nothing. Numerous improvements to the Internet have been suggested, all bringing an immediate benefit. Examples include Explicit Congestion Notification [144], Integrated Services [18], Differentiated Services [26], and Mobile IP [137]. All of them failed. They might be available in small, isolated parts of the Internet [171], but never saw general adoption.

The history of the Internet shows that changes to the network only occur if there is an immediate monetary gain, or if the network is about to collapse. Achieving monetary gain by improving the core Internet is hard, because the Internet Service Providers (ISPs) need to agree on the changes. If all ISPs offer the improvement, then there is no benefit for any of them [6]. Improvements become impossible, the Internet ossifies.

In [132], the general diagnosis of ossification is further refined. First, there is intellectual ossification. Any new technology has to be compatible with the current technology from the outset. This stifles innovation. Secondly, there is infrastructure ossification. Suggested improvements are not deployed in the infrastructure, not even for testing purposes. Thirdly, there is system ossification, describing basically the same effect as the architectural barnacles [6]. Instead of fixing problems at their root, workarounds and fixes are employed to keep the system running while making it more fragile and susceptible to even more problems.

1.2 Current Problems

The Internet, the general purpose network flexible enough to evolve and meet new challenges head on, has been lost. Instead, we have the Internet, the global network working really well, as long as nothing too extreme, like Quality-of-Service (QoS) guaranty, is asked of it. How does this ossification show in practice, which problems do occur? Handley [72] offers a list of short, medium, and long term problems due to ossification.

Spam, security and Denial-of-Service (DoS) attacks are short term and immediate problems. One of the main reasons why these problems exist is that at its core, the Internet is a transport network. It efficiently transfers data from A to B. Whether B wants the data is irrelevant. As workarounds, firewalls and Network Address Translators (NATs) are used and consequently, the Internet loses transparency. Data is dropped or modified along the way for no apparent reason and the deployment of applications that need to transfer data is much more complicated than it needs to be. As a result, many applications dress up their data as HyperText Transfer Protocol (HTTP) traffic, since this is understood and accepted by most firewalls. Problems only arise if HTTP does not correspond to the communication requirements of the application.

A fitting example is the Voice-over-IP (VoIP) software Skype [162]. It uses UDP to transfer voice data since short delays are important and reliability is not an issue as long as enough packets arrive. The data has to be sent directly from one user to another for latency reasons, which is hard to do when NATs are involved. NATs change IP addresses and UDP ports and the precise mapping has to be determined by the Skype client. Techniques to do so involve contacting a remote server to determine the IP address and using heuristics to determine the port mapping. These techniques are complicated and error prone. It gets even worse if both Skype clients that want to communicate are behind NATs. Then a third party, not hidden by a NAT, has to be used as a relay station, which adds delay and reduces reliability, since the third party (another Skype client) can quit at any time. This approach fails completely if most clients are behind NATs, if the heuristics cannot determine the mappings, or if a firewall blocks UDP traffic. In those cases, Skype falls back to using TCP as a last resort. In the end, Skype works well enough to be successful, but consider the amount of engineering that was necessary. It should not have been required.

The medium term problems identified by Handley [72] are congestion control, inter-domain routing, mobility, multi-homing and architectural ossification. Congestion control is problematic, because with rising link speeds it takes longer and longer for the data transfer rate to converge to a suitable value. As for inter-domain routing, this is facilitated by BGP [116]. It is basically the glue that holds the Internet together and when it fails, connectivity is affected. In [122], the frequency of BGP misconfigurations is measured, and they have been found to be prevalent. For every misconfiguration, there is a 4% chance that it affects the connectivity, i.e., some parts of the Internet cannot be reached. Efforts to improve and secure BGP have failed. The mobility of users, or the possibility that they have multiple simultaneous connections to the Internet (multi-homing) remain open problems. An example of architectural ossification is that not even the extension paths already built into the protocols can be used. IP for instance was designed to be extensible by using IP options. However, packets without options can be handled in a hardware-accelerated manner within routers. With options, packets have to be processed in software. Using IP options would be equal to an DoS attack on the router, so those packets are highly likely to be dropped.

Address space depletion is a long term problem that will be very hard to deal with due to ossification. Handley [72] states that it was already clear in 1990 that the Internet addresses would run out. In the meantime, CIDR and NATs kept the network running. An alternative without the problem of address depletion, IPv6 [42], was specified in 1998. To this day, adoption is slow [12, 170], even though the organizations responsible for distributing the addresses have begun to run out of addresses in 2011 [84]. This is probably the best example for ossification in practice, but what are the reasons for this slow adoption? One component is, that NATs alleviate the address shortage, another, that IPv6 is complicated to implement. Maybe the problem is that IPv6 is just more of the same and does not enable the required fundamental change.

Ossification would not be a problem if the requirements were static. But they are not. According to [132], users do not only want more bandwidth. They want more reliability, predictability, manageability, configurability, security, and privacy. Improving those characteristics requires changes. The Internet was meant to be a general-purpose network, but now more than that is needed [72].

1.3 Network Virtualization: The Cure?

The Internet has stopped evolving, but how can we start the evolution up again? We have already seen that a new technology, that wants to have any hope of actually being deployed, has to have two properties. It needs to be backwards compatible and incrementally deployable. Network virtualization has been put forward as a suitable candidate [6, 12, 66, 168, 171, 172].

The basic idea of network virtualization is straight forward. Instead of using one physical network that can do everything well, use multiple virtual networks embedded in the physical network, each one specialized and perfectly adapted for a particular application. To allow for this adaptation, the nodes of the virtual networks receive compute capabilities within the routers of the physical network. Therefore, virtual networks can offer their own (and application specific) topology, routing, naming services, and resource management [171]. The alternative to adapted virtual networks, multiple physical networks, is clearly infeasible on a global scale. That sounds promising, but is it realistic? At the very least, network virtualization requires support from the physical network by offering routers that can host virtual machines (the routers of the virtual networks). This technology is already available [32]. In addition, network virtualization is actively and successfully used in large scale scientific network testbeds such as GENI [62], PlanetLab [31] or G-Lab [156], not as an enhancement to be studied, but as a central enabling technology for carrying out experiments. Virtual networks are used to partition the network testbeds so that different research groups can perform their experiments without interference from each other. As Tutschku et al. [172] state, the "virtualization of telecommunication services or applications is no longer an academic concept".

Network virtualization is not the only proposed concept to break the ossification of the Internet. Alternatives include OpenFlow [123] and Software Defined Networks [112], which allow very flexible routing that can be administrated centrally.

1.3.1 Advantages and Applications

Allowing network virtualization in the Internet offers some advantages and applications, which are currently unthinkable, become possible. As a central advantage, Berl et al. [12] mention the flexibility of the system. Virtual networks can be dynamically reconfigured, new networks can be added and old ones can be removed, suspended or discarded. The state of the network can be frozen and reverted if the need should arise. Unused parts of the network can be shut down to conserve energy. In aggregated services, it is possible to fix or replace parts transparently [27]. Some interesting possibilities also emerge on the user side of things. Turner et al. [170] describe a virtual network offering a learning environment with high quality audio and video multicast mechanisms. Format translators are available at the virtual nodes to enhance compatibility. With virtual networks, it may be possible to switch Internet Service Providers (ISPs) on the spot, like it is possible for electricity or phone providers [171].

1.3.2 Challenges

Adoption is a central problem for every new technology. How will (or should) the adoption process of network virtualization work? Anderson et al. [6] envision the following adoption process. Virtual networking will start with a single daring Next Generation Service Provider (NGSP) offering virtualization services for its own network. Customers not directly connected to the network of the NGSP can connect via standard ways through the current Internet. If the NGSP is successful, it can expand its network to reach more customers. Local ISPs may be forced to offer the same services to stay competitive. For Turner et al. [170], adoption will resemble more the introduction of the Internet. First, virtual networks will be offered as an overlay in the existing Internet. Then, a government-supported experimental backbone infrastructure will be built, which natively supports virtualization. As the last step follows the commercial operation of virtual networks.

After adoption is achieved, the tasks of the current ISPs will be carried out by two different business entities [29], the infrastructure providers (InPs) and the virtual network providers (VNPs). InPs will manage the physical infrastructure necessary for hosting the virtual networks. VNPs create virtual networks from a federation of the resources offered by the different InPs [28, 50, 170].

This structure offers a rich environment for business opportunities [170]. InPs can compete by offering better services for VNPs using their networks, such as high quality resources, management tools, operation support, and fault tolerance. VNPs can distinguish themselves by offering shorter virtual network setup times, higher quality virtual networks, or guaranteed resources. In addition, VNPs do not own any physical resources, they do not need to deploy and maintain infrastructure. Thus, there is a low barrier to entry for VNPs. There are also opportunities for network equipment vendors, as there will be demand for high performance virtualizable routers. Network virtualization is not ready to be deployed in the Internet. The authors of [29, 170] identify key research questions that still need to be answered, which we summarize below.

- **Interfacing** How can InPs and VNPs communicate, for instance about available resources or requirements?
- **Signaling and Bootstrapping** How can VNPs set up their virtual resources, if they have no communication capabilities besides the resources that they want to set up?
- **Resource Allocation** How can a VNP best fit its virtual networks into the resources it has leased from the InPs?
- **Resource Discovery** How can InPs keep track of the resources they offer and their connectivity, especially to other InPs.
- Admission Control How can it be ensured that the capacities of an InP cannot be exceeded?
- **Virtualization** How are the physical routers to be designed to allow low overhead virtualization?
- **Resource Scheduling** How can an InP efficiently distribute its resources among the interested VNPs. How long should an InP guarantee the availability of resources?
- **Naming and Addressing** How can the situation be handled that a single host may connect to multiple different virtual networks, each with different naming and addressing schemes?
- **Dynamism and Mobility** How can the dynamic nature of virtual networks and the changing location of users be efficiently handled in terms of routing?
- **Operation and Management** How can virtual networks be efficiently monitored and managed?
- **Security and Privacy** How can it be prevented that a hostile virtual network breaks out of its virtualized environment and takes control of the physical infrastructure?
- **Heterogeneity of Technology** How can the plurality of different virtualization technologies be handled efficiently?
- Economics How should the economics of virtual networks work?

Service Duplication How can the overhead caused by multiple virtual networks offering the same basic services be avoided?

In this thesis, we cannot solve all those problems. In the following section, we will outline our aim.

1.4 Scope and Structure of this Thesis

There are a lot of unsolved problems surrounding network virtualization. In this thesis, we will focus exclusively on resource allocation. We will try to answer the question of how VNPs may best utilize the resources they have acquired from the InPs so that all virtual networks they want to create actually fit and do not incur excessive operational costs. Simply put, we will study algorithms for mapping multiple virtual networks into a physical (substrate) network in a reasonable way. We call this problem the Virtual Network Mapping Problem (VNMP). The remainder of this thesis is structured as follows:

In Chapter 2, we give an overview of the relevant theory and define our experimental methodology, followed by a rigorous definition of the VNMP and its variants in Chapter 3. Related work will be discussed in Chapter 4. Chapter 5 presents the methods used to create realistic benchmark instances for the VNMP. In the following chapters, we present different algorithms for solving the VNMP. Since the VNMP is \mathcal{NP} -complete and also hard to solve in practice, we consider (meta)heuristic approaches in addition to exact solution methods. First, we will focus on heuristic methods to generate good solutions in a reasonable amount of time, later parts will focus on exact approaches to find optimal solutions. Chapter 6 contains basic Construction Heuristics, Local Search, and Variable Neighborhood Descent approaches. A Memetic Algorithm is presented in Chapter 7 while Greedy Randomized Search Procedures and Variable Neighborhood Search algorithms may be found in Chapter 8. Preprocessing techniques for VNMP instances are discussed in Chapter 9. Then we leave the heuristic solution methods behind and concentrate on exact approaches for solving the VNMP. In Chapter 10, we apply Constraint Programming and in Chapter 11 Integer Linear Programming methods. In Chapter 12, we study how the developed algorithms might be used to support a VNP when deciding where to increase capacities. Chapter 13 contains an overall comparison of the main algorithms presented and final conclusions. In Appendix A, the detailed results of all algorithms may be found.

Parts of this thesis have been presented at the following conferences and published in the corresponding proceedings (all reviewed). The Construction Heuristics, Local Search and Variable Neighborhood Descent algorithms from Chapter 6 have been published in

J. Inführ and G. R. Raidl. Solving the Virtual Network Mapping Problem with Construction Heuristics, Local Search, and Variable Neighborhood Descent. In M. Middendorf and C. Blum, editors, *Evolutionary Computation in Combinatorial Optimisation – 13th European Conference, EvoCOP 2013*, volume 7832 of *Lecture Notes in Computer Science*, pages 250–261. Springer, 2013. The Memetic Algorithm we present in Chapter 7 has been published in

J. Inführ and G. R. Raidl. A Memetic Algorithm for the Virtual Network Mapping Problem. In H. C. Lau, P. Van Hentenryck, and G. R. Raidl, editors, *Proceedings of the 10th Metaheuristics International Conference*, pages 28–1–28–10, Singapore, 2013.

The Greedy Randomized Search Procedures and Variable Neighborhood Search algorithms that can be found in Chapter 8 have been published in

J. Inführ and G. R. Raidl. GRASP and Variable Neighborhood Search for the Virtual Network Mapping Problem. In M. J. Blesa et al., editors, *Hybrid Metaheuristics*, *8th International Workshop (HM 2013)*, volume 7919 of *Lecture Notes in Computer Science*, pages 159–173. Springer, 2013.

A very early form of the Integer Linear Programming approach for solving the VNMP as presented in Chapter 11 has been published in

J. Inführ and G. R. Raidl. Introducing the Virtual Network Mapping Problem with Delay, Routing and Location Constraints. In J. Pahl, T. Reiners, and S. Voß, editors, *Network Optimization: 5th International Conference (INOC 2011)*, volume 6701 of *Lecture Notes in Computer Science*, pages 105–117, Hamburg, Germany, 2011. Springer.

The application study we perform in Chapter 12 has been published in

J. Inführ, D. Stezenbach, M. Hartmann, K. Tutschku, and G. R. Raidl. Using Optimized Virtual Network Embedding for Network Dimensioning. In *Proceedings of Networked Systems 2013*, pages 118–125, Stuttgart, Germany, 2013. IEEE.

CHAPTER 2

Theory and Methodology

2.1 Introduction

In this chapter, we give an overview of the theoretical concepts we use in this work, as well as a description of the employed experimental methodology. In Section 2.2, we introduce the basics of combinatorial optimization, give an overview of the associated complexity theory, and present some of the major principles used to solve Combinatorial Optimization Problems (COPs). Section 2.3 covers the basic definitions and associated algorithms from graph theory. The experimental setup, describing for example the computational environment and the employed statistical tests, is outlined in 2.4.

We want to state clearly that it is not in the scope of this work to even give a proper overview of the concepts that we are going to introduce in the following. We will mainly concentrate on the parts relevant for this work and give references to fill the gaps.

2.2 Combinatorial Optimization Problems and Solution Methods

2.2.1 Combinatorial Optimization Problems

Before we can start outlining different heuristic and exact solution methods for solving instances of Combinatorial Optimization Problems, we first require a definition what problems, instances of problems, and solutions are.

A problem is an abstract description of what needs to be done, usually specified by defining what data is required as input and what is requested as output. The output has to satisfy some constraints. If we are dealing with an optimization problem, the output has an attached value that either has to be minimized or maximized. An instance of a problem is a concrete set of inputs, following the rules as defined by the problem. An instance can be defined more formally as follows [15, 134]:

Definition 2.2.1 (Instance of a Problem). Given a finite tuple of variables $X = (x_1, ..., x_n)$, domains $D_1, ..., D_n$ for those variables (with $D = D_1 \times ... \times D_n$), constraints C defined on a subset of D and limiting the allowed combinations of values assigned to variables X, and an objective function $f : D \to \mathbb{R}$ that has to be minimized or maximized, a problem instance is defined as quadruple (X, D, C, f).

A problem can also be viewed as the set of all its instances. We will use the terms problem and instance interchangeably. Given a problem instance, the set of candidate solutions S is the set of all possible assignments of values to variables according to their domains, but not necessarily satisfying the constraints. S is also referred to as search space or solution space. Every $s \in S$ has an assigned objective value f(s).

The set of feasible solutions S_{feas} is a subset of S, containing all candidate solutions that fulfill the constraints. When solving a feasibility (decision) problem, we try to find any member of S_{feas} . When solving an optimization problem, we are searching for a special feasible solution; one that has the best possible objective value. In the following, we assume that smaller values are better, i.e., we focus on minimization problems. The definitions for maximization problems are analogous.

Definition 2.2.2 (Optimal Solution). A solution $s_{opt} \in S_{feas}$ is said to be globally optimal, if $\forall s \in S_{feas} : f(s_{opt}) \leq f(s)$. Note that there may be multiple globally optimal solutions with the unique globally optimal objective value.

If the domains of the variables of a problem (instance) are continuous, we are dealing with a Continuous Optimization Problem. The problems we are going to solve in this work have discrete variable domains and therefore belong to the class of Combinatorial Optimization Problems.

2.2.2 Complexity Theory

When we use an algorithm to solve a problem, we are usually interested in how the algorithm behaves in terms of run-time or memory requirements when the size of the problem (e.g., number of variables) increases. The tools available from the field of computational complexity theory [61,67,110,134,159,160] can help us to find useful answers. From this field, we get the following definitions:

Definition 2.2.3 (Time Complexity). The time complexity function of an algorithm gives the largest amount of time needed to solve problem instances of a particular size (denoted by n), for all possible values of n.

Definition 2.2.4 (Big-Oh). A function f(n) is in $\mathcal{O}(g(n))$, iff there exist constants c > 0, $n_0 > 0$ such that $\forall n > n_0 : |f(n)| \le c \cdot |g(n)|$. Informally, this means that f(n) grows asymptotically not faster than g(n) for $n \to \infty$ when neglecting scaling by a constant.

Definition 2.2.5 (Big-Theta). A function f(n) is in $\Theta(g(n))$, iff there exist constants $c_1 > 0$, $c_2 > 0$ and $n_0 > 0$, such that $\forall n > n_0 : c_1 \cdot |g(n)| \le |f(x)| \le c_2 \cdot |g(n)|$. Informally, this means that f(n) grows asymptotically as much as g(n) for $n \to \infty$ when neglecting scaling by a constant.

Definition 2.2.6 (Polynomial Time Algorithm). An algorithm runs in polynomial time, if its time complexity is in $O(n^k)$, for a constant $k < \infty$.

Definition 2.2.7 (Exponential Time Algorithm). *If the time complexity function of an algorithm cannot be bounded by a polynomial, it is called an exponential time algorithm.*

Definition 2.2.8 (Well Solved Problem). A problem is considered well solved, if a polynomial time algorithm solving the problem is known. A well solved problem is also called efficiently solvable.

In complexity theory, all efficiently solvable problems belong to the class \mathcal{P} (for polynomial time). Unfortunately, for many of the important COPs the known algorithms require exponential time. Exponential time basically means that an exponential number of solutions have to be enumerated to find the optimal solution, at least in the worst case. The problems not belonging to \mathcal{P} can be split into two groups, those for which it is easy (in \mathcal{P}) to check whether a potential solution is valid, and those for which even this check is not in \mathcal{P} . The problems for which it is easy to determine if a candidate solution satisfies all constraints form the class \mathcal{NP} (for nondeterministic polynomial). It holds that $\mathcal{P} \subseteq \mathcal{NP}$, but whether $\mathcal{P} = \mathcal{NP}$ is still an open problem. It is conjectured that $\mathcal{P} \neq \mathcal{NP}$. The "hardest" problems within \mathcal{NP} are \mathcal{NP} -complete:

Definition 2.2.9 (\mathcal{NP} -complete). A problem is \mathcal{NP} -complete, if it is in \mathcal{NP} and every problem in \mathcal{NP} is reducible to it in polynomial time.

Based on this definition, to prove that a problem A is \mathcal{NP} -complete, we first need to prove membership in \mathcal{NP} . If there is a polynomial time algorithm capable of checking whether a candidate solution for A is feasible, A belongs to \mathcal{NP} . The second condition for \mathcal{NP} -completeness is that all problems in \mathcal{NP} can be reduced to it in polynomial time. That means, for every problem B in \mathcal{NP} , there has to be a polynomial time algorithm capable of transforming any input of B into an input of A and a solution of A to a solution of B. More informally, A can solve problem B, and therefore cannot be easier to solve than B. Following the definition of \mathcal{NP} -completeness, it is sufficient to show a polynomial time algorithm capable of reducing another \mathcal{NP} -complete problem to A. The first proof determining the \mathcal{NP} -completeness of a problem was achieved by Cook [33], which of course could not use this shortcut. In addition to \mathcal{NP} -completeness, there is also the notion of \mathcal{NP} -hardness. A problem is \mathcal{NP} -hard if any problem in \mathcal{NP} can be reduced to it, but it does not necessarily lie in \mathcal{NP} itself.

To sum it all up, COPs can be split up roughly into "easy" or tractable problems (those belonging to \mathcal{P}), and "hard" or intractable problems (those being \mathcal{NP} -complete). The "hopeless" problems not even belonging to \mathcal{NP} are usually not considered. It is important to note that the (in)tractability of a problem is not in lockstep with its solvability in practice. It might be infeasible to solve a problem in \mathcal{P} (possibly due to instance size), and a problem in \mathcal{NP} might be easily solvable for the relevant instance sizes. In addition, there exist pseudo-polynomial algorithms for some \mathcal{NP} -complete problems that are very effective in practice. An algorithm is pseudo-polynomial if its run-time does not only depend polynomially on the input size, but also on the input numbers themselves. A pseudo-polynomial algorithm is allowed to run longer when for instance a cost value contained in the problem instance is changed from 100 to 1000. NPcomplete problems, for which a pseudo-polynomial algorithms exist are called weakly NPcomplete. Otherwise, they are called strongly NP-complete.

2.2.3 Multiobjective Problems and Pareto Optimality

In the previous sections, we have only considered the case of a single objective, which tells us how good a solution to a problem is. However, in reality there are frequently multiple objectives to be considered and to make matters worse, no solution might be best according to all objectives. Just think about the trade-off between quality and cost or execution time and solution quality.

The concept of Pareto optimality has been introduced to deal with these kinds of problems and is based on domination. A solution s strictly dominates a solution s', if s is at least as good as s' according to all objectives and better than s according to some objectives. The Pareto-optimal solutions are those which are not strictly dominated by another solution. An interpretation of this is that the Pareto-optimal solutions can only be improved according to one objective if another objective is made worse. The set of Pareto-optimal solutions is called the Pareto-front and is the dividing line between the best feasible and infeasible solutions to a problem.

Now that we have covered the required theory, we can start discussing solution methods for COPs. Broadly speaking, they are divided into two categories. First we will present some heuristic methods. Heuristic methods try to find good solutions in a short amount of time, but cannot give any guarantees with respect to the solution quality. The other category are the exact methods. Exact methods can give quality guarantees and even find provably optimal solutions. Since they are applied to \mathcal{NP} -complete problems, their run-time is exponential in the worst case.

2.2.4 Construction Heuristics

The first, and very basic, heuristic method we cover are Construction Heuristics. A Construction Heuristic tries to build a solution by iteratively adding components to an initially empty solution until it is complete. The selection of the components usually follows a greedy rule. That means that the component that currently seems to be the best choice (e.g., increases profit the most) is chosen, without regard for possible future consequences of this choice. In addition, once a decision has been made and a component has been added to the solution, it cannot be removed again. These properties make Construction Heuristics very fast and easy to implement and analyze. As downside, the created solutions are frequently far away from the optimal solution [16]. Depending on the problem that has to be solved, the created solutions may not even be feasible. Nevertheless, the application of a construction heuristic is in practice often the first step when solving a problem, since more sophisticated improvement heuristics may build upon it.

2.2.5 Local Search

Local Search [1, 15, 134] is a basic improvement heuristic that starts from a starting solution $s \in S$ and tries to find a solution s' in a neighborhood of s that is better than s. If an improved

Algorithm 2.1: Local Search

Input: Solution s
Output: Possibly improved solution s1while stopping criteria not met do2choose $s' \in N(s)$;3if $f(s') \leq f(s)$ then4| s = s';5end

6 end7 return s;

solution can be found, it replaces the starting solution and the search continues. A neighborhood structure is defined as follows.

Definition 2.2.10 (Neighborhood Structure). A function $N : S \to 2^S$ is called a neighborhood structure. It assigns a set of neighbors $N(s) \subseteq S$, called neighborhood, to each solution $s \in S$.

A neighborhood structure is usually not defined by explicit enumeration, but rather implicitly by giving a transformation rule how a solution is to be changed to create its set of neighbors, i.e., its neighborhood. These transformations are usually local changes, like swapping or replacing single components of the solution. The larger the neighborhood, i.e., the more solutions are reachable from an initial solution, the higher the probability of finding an improving solution in general. Of course, larger neighborhoods usually lead to higher run-time requirements.

Definition 2.2.11 (Locally Optimal Solution). If N(s) of solution s does not contain solutions better than s, s is locally optimal with respect to N. A locally optimal solution can be arbitrarily bad compared to the globally optimal solution.

The main strategic choice for Local Search, besides the definition of the used neighborhood, is the method of selecting a neighbor. There are three commonly used selection strategies. First-improvement enumerates the solutions of N(s) and stops when the first improving one has been found. With best-improvement, all solutions of N(s) are enumerated and the best solution among them is chosen. Random-neighbor randomly generates neighbors from N(s) and selects the first improving solution.

A natural stopping point of Local Search is when a local optimum has been reached. However, based on the size of the employed neighborhood, other limits such as the number of iterations, elapsed run-time, or no improvement in a certain number of iterations might be chosen. Algorithm 2.1 shows the outline of Local Search.

An important type of neighborhoods are ruin-and-recreate neighborhoods [154]. Instead of specifying transformation rules, they define how a solution is to be partially destroyed, i.e., how some components of the solution are to be removed. For recreating, any method for solving COPs can be applied, as long as it can handle partially fixed solutions. Depending on the chosen recreation method, these neighborhoods can be very powerful. Algorithm 2.2: Variable Neighborhood Descent

Input: Initial solution *s* **Output**: Possibly improved solution *s*

1 l = 1;2 while $l \leq k$ do select $s' \in N_l(s)$ by first- or best-improvement; 3 if f(s') < f(s) then 4 s = s';5 l = 1;6 end 7 else 8 9 l = l + 1;10 end 11 end 12 return s;

2.2.6 Variable Neighborhood Descent

An extension of Local Search is Variable Neighborhood Descent [74], which is shown in Algorithm 2.2. Instead of one neighborhood structure, a set $\{N_1, \ldots, N_k\}$ is utilized. An initial solution is improved by N_1 until no more improvements can be found, then N_2 is applied. If this neighborhood is not able to improve the solution, the next one is tried. If N_k fails, VND terminates. If at any point an improvement is found, the algorithm goes back to N_1 . At the end of VND, the solution is locally optimal for all considered neighborhoods. The neighborhoods are usually searched in first-improvement or best-improvement fashion.

This method is especially promising if there is a set of neighborhood structures which complement each other very well. For example, if a problem that has to be solved can be decomposed into a part that deals with route planning and in another part that deals with packing, it is natural to use one neighborhood structure that focuses on the routing and another one that focuses on the packing aspect. Usually, the neighborhood structures are ordered according to their size, with the smallest ones first.

2.2.7 Variable Neighborhood Search

The main drawback of VND is that it only focuses its search on the part of the search space around the initial solution. As such, it has a heavy focus on intensification, that means it only tries to improve the initial solution, but does not diversify the search by sampling solutions from other parts of the search space. The General Variable Neighborhood Search (VNS) [73– 75] rectifies this problem by using the very scheme of VND around VND once again, with another set of larger neighborhood structures $\mathcal{N}_1, \ldots, \mathcal{N}_k$ that are only sampled by the randomneighbor strategy. These are the so-called shaking neighborhoods and not meant for improving the solution directly, but rather to move the search to another part of the search space and leave

Algorithm 2.3: General Variable Neighborhood Search

Input : Initial solution s
Output : Possibly improved solution s

1 while stopping criteria not met do

```
l = 1:
2
       while l < k' do
3
           randomly select s' \in \mathcal{N}_l(s) // diversification;
4
           s' = \text{VND}(s') / / intensification;
5
           if f(s') < f(s) then
6
              s = s';
7
              l = 1;
8
9
           end
10
           else
              l = l + 1;
11
           end
12
       end
13
14 end
15 return s;
```

the basin of attraction of the VND's neighborhoods. The basin of attraction of a VND solution s is the set of all solutions s' which are transformed into s by VND. A move in \mathcal{N}_1 causes the least change to the current solution, while \mathcal{N}_k perturbs the current solution a lot. VNS is a very successful metaheuristic for Combinatorial Optimization Problems, for more details and a survey of applications see [76]. The general outline is presented in Algorithm 2.3.

2.2.8 Greedy Randomized Adaptive Search Procedure

The Greedy Randomized Adaptive Search Procedure (GRASP) [52, 53] is an extension of Construction Heuristics in combination with Local Search. It works by continually repeating two steps. The first step is the randomized greedy construction of a solution to the problem to be solved. A second step is applying a local improvement technique to the constructed solution. These two steps are repeated until a termination criterion (like run-time or number of iterations) is reached. The best found solution is the final result of GRASP. How the randomized greedy solution construction works is a central aspect of GRASP. It iteratively builds a solution by adding components that seem good (but not necessarily the best) according to a greedy criterion. All possible components are collected in a candidate list (CL). A restricted candidate list (RCL) is derived from the CL, usually by selecting the best k candidates, where k is a specified parameter, or all parts whose greedy evaluation lies above a certain quality threshold. The actual component that is added to the solution is selected uniformly at random from this RCL. This procedure usually leads to promising and at the same time diversified solutions for local optimization. Comprehensive overviews of GRASP can be found in [54, 147]. For hybridization techniques see [55]. The outline of GRASP is shown in Algorithm 2.4.

Algorithm 2.4: Greedy Randomized Adaptive Search Procedure

Input: Instance of a problem as quadruple (X, D, C, f)**Output**: Solution s 1 $s = \emptyset;$ 2 while stopping criteria not met do build CL from unassigned variables of X and their domains; 3 $s' = \emptyset$: 4 while s' is not a complete solution **do** 5 build RCL from CL; 6 randomly select an element s_i from RCL; 7 $s' = s' \cup \{s_i\};$ 8 $CL = CL \setminus \{s_i\};$ 9 end 10 (locally) improve s'; 11 if $s == \emptyset \lor f(s') < f(s)$ then 12 s = s';13 end 14 15 end 16 return s;

2.2.9 Genetic Algorithm

A Genetic Algorithm (GA) [80] is a nature-inspired population-based algorithm that can be used to solve Combinatorial Optimization Problems. An overview can be found in [161]. It mimics natural evolution as described by Darwin [40] and Mendel [126] by applying three different methods to a set of solutions, called population: selection, crossover, and mutation. The task of the selection procedure is to choose promising solutions from the population as a basis for creating new solutions. The crossover procedure combines two selected solutions (the parents) such that characteristics of both parents are inherited to the offspring. Mutation changes the offspring in a small way so that new solution properties may emerge. Depending on the particular GA variant, multiple offspring may be collected to form a new population (generational GA) or the offspring is immediately reinserted into the population (steady-state GA) where it replaces a solution.

Common methods for selecting solutions from the population are tournament selection and roulette-wheel selection. With tournament selection, k solutions are randomly chosen from the population and the best one is the result of the selection procedure. The parameter k controls the level intensification caused by the selection procedure, higher values of k mean that only the very best solutions have a significant chance of being selected. For k = 2, this method is called binary tournament selection. In roulette-wheel selection, every solution in the population is assigned a selection probability proportional to its solution quality, the best solutions having the highest probability of being selected.

For a description of the common crossover operators, we assume that the solutions are repre-

Algorithm 2.5: Genetic Algorithm

Input: Instance of a COP
Output: Solution s
1 P ... initial population;
2 while stopping criteria not met do
3 0 ... empty set of offspring;
4 while offspring O not sufficien

```
while offspring O not sufficient do
          if crossover condition satisfied then
5
              select parent solutions P' from P;
6
              select crossover parameter;
7
              o = crossover(P');
8
9
          end
10
          if mutation condition satisfied then
               select mutation parameters;
11
              o = mutate(o);
12
          end
13
          evaluate fitness of offspring o;
14
          O = O \cup \{o\};
15
      end
16
      P = select(P, O);
17
18
  end
19 return best solution s \in P;
```

sented by a vector of integers. To create an offspring with one-point crossover, a location within the vector of integers is selected randomly. All integers up to this location are copied from one parent, the remaining values are copied from the other parent. For two-point crossover, two locations where the source of the values changes are selected. An extreme form of this crossover type is the uniform crossover. For uniform crossover, there is a random decision at every location to determine whether the value will be copied from the first or from the second parent.

The applied mutation operator is usually point mutation. With point mutation, a single value in the vector of integers of a solution is changed to another allowed value. This change might be random, or biased by the previous value so that a result close to the previous value is more likely. Algorithm 2.5 shows a general GA template [146], which is one of many possible ways of constructing a GA. Additional information about GAs can be found in the literature, e.g., [129, 146, 178].

An extension of GA is the Memetic Algorithm. The Memetic Algorithm (MA) is a combination of GA (or other population based optimization method) and a local improvement technique [130, 131, 142]. The main idea is to use the GA to find promising regions in the search space and then use the local improvement technique to find excellent solutions in those promising regions. There is a tradeoff between the time spent in the GA and the time spent executing the local improvement technique. Without enough time for the GA, it will fail to find promising regions, without enough time for the local improvement method, the found solutions will not be excellent. Usually, the local improvement technique is only applied to a fraction of the generated solutions. Sometimes, only the very best solution in the population is improved to save on execution time. Another implementation issue surrounding Memetic Algorithms is the treatment of an improved solution. This solution could replace the solution in the population it was derived from. The other possibility is that the improved solution is discarded and the original solution is treated as if it had the solution quality of the improved solution. Both approaches have their drawbacks. If we replace solutions, the diversity in the population might suffer. In the worst case, every solution in the population is transformed into the same solution by the local improvement method. Discarding the improved solution is problematic, because we throw away a lot of work done by the local improvement method. The properties of a highly successful solution are not inserted into the population so that related, therefore it is not possible to create related, possibly even better, individuals.

2.2.10 Tree Search and Branch & Bound

We now leave the area of heuristics and focus on exact methods for solving COPs. As outlined previously, for \mathcal{NP} -complete problems we have to expect that we need to check an exponential number of solutions in the search space to find the best one. Enumerating all possible solutions only works for the very smallest instances, due to the effect of combinatorial explosion. For every variable added to a problem instance, we multiply the size of the search space by the size of the domain of the added variable. This becomes untenable very quickly, so some better approach is required.

A common concept for exact solution methods is Tree Search, named so because its execution builds a tree graph (see Definition 2.3.18). The root node represents the complete search space to be explored. Tree Search then recursively partitions the search space in mutually disjoint spaces by restricting variable domains or adding constraints. For instance, starting from the root node, we may create two children representing each one half of the search space by fixing a variable with domain $\{0, 1\}$ in one child node to 0 and in the other one to 1. These child nodes may be partitioned further by restricting other variables. In its simplest form, the search tree is explored in a depth-first fashion, which means we partition the search space until we either find a feasible solution, or can prove that given the constraints added during the partitioning, no feasible solution can exist (without having to enumerate all candidate solutions in the current sub-space). If we prove that no feasible solution exists, the current node has failed, and we need to back-track to continue the search, i.e., we need to find another unexplored part of the search space. This is done by going to the parent of the currently failed node and checking if it has children that have not been explored yet. If no more unexplored children exist, we repeat this procedure with the parent of the current node. When we reach the root node in this manner, and it too has no more unexplored children, Tree Search is finished, and in this case has proven that no feasible solution exists. It is easy to see that Tree Search is a complete search method, i.e., if there is a solution it will be found.

In this description, we have only focused on finding a feasible solution (i.e., solving a Constraint Satisfaction Problem), but could already see the strength of Tree Search: whole parts of the search space can be discarded, if we can prove that they do not contain a feasible solution.

The same applies when Tree Search is used to solve COPs, but now, once we have found a feasible solution, we can also discard parts of the search space for which we can prove that no solution better than the currently best found solution exists. This procedure is called Branch & Bound. Branching refers to the partitioning of the search space and bounding to the calculation of an upper bound for solution quality (note that this is a lower bound in case of minimization problems) for the partitions. If the upper bound on solution quality of a part of the search space not better than the currently best known solution, it can be discarded. How branching and bounding are implemented depends on the concrete method employing the Branch & Bound principle.

2.2.11 Constraint Programming

Constraint Programming (CP) [7,119,152] is an exact method following the Tree Search principle. It is mostly used to solve Constraint Satisfaction Problems but can also solve optimization problems. The branching works as described before, variables are assigned values (or their domains are reduced) to partition the search space. In CP, this is also called the labeling strategy. However, the main power of CP comes from propagation [9], also called filtering in the context of CP. Once a new node in the search tree is created, propagation is performed on the sub-problem it represents. That means that values, which cannot occur in a feasible solution, are removed from the domains of the variables. Assume variables a, b, and c with domains $D_a = D_b = \{0, 1\}$ and $D_c = \{0, 1, 2, 3\}$ respectively and a constraint a + b = c. Propagation would remove 3 from D_c , since this value clearly cannot be produced by the sum of a and b. Propagation cannot reduce the domains any further, a fixed point is reached and branching is required. Assume we branch by assigning c every value from its domain, i.e., one branch where c = 0, one where c = 1, and one where c = 2. When we consider the first and the third branch, propagation alone is sufficient to find a complete assignment to a and b. For the second branch, we end up with $D_a = D_b = \{0, 1\}$ and $D_c = \{1\}$. Here we can see that by propagation alone, we are not able to describe the set of feasible solutions. The domains suggest that the set of feasible solutions (in terms of assignments to a and b) is $\{(0,0), (0,1), (1,0), (1,1)\}$, when in reality it should be $\{(0,1), (1,0)\}$. Branching has to be used to find all feasible solutions. If propagation removes all values from the domain of a variable, then we know that the current partial assignment (variables that have been assigned a value due to branching or propagation) is inconsistent, i.e., it is not possible to assign values to the remaining variables such that all constraints are satisfied. In this case, CP backtracks and evaluates another partial assignment.

Propagation methods come in different strengths, i.e., how well they can reduce the domains of variables. This is called the consistency of a propagator. The most common consistency level is domain (or arc) consistency. Two variables are domain consistent if for every value allowed for the first variable there is a value for the second variable such that the constraint between the variables is satisfied. Other consistency levels are node consistency or path consistency.

CP is very flexible with respect to the employed variables and constraints, as long as suitable propagation and branching procedures can be defined. Regarding the types of variables, we have already seen that binary (a and b) and integer variables (c) can be handled. Real-valued variables are also possible. Set variables are going to be relevant later on. The domain of set variables is the set of all possible sets of integers. Due to combinatorial explosion, it is not

possible to represent the domain of a set variable directly. Indeed, direct representation is already problematic for integer variables with large domains. Instead, the domain is approximated by two sets, the greatest lower bound (GLB) and the least upper bound (LUB). The domain of a set variable contains all sets that can be built by using all elements of the greatest lower bound and any selection of elements from the least upper bound. It is written as [GLB .. LUB]. Intuitively speaking, the value (which is a set) of the set variable has to contain all elements of GLB and may contain elements of the LUB. This representation cannot represent every set of sets. The best representation of the domain $\{\{1,2\},\{1,3\},\{2,3\}\}$ for example is $[\emptyset .. \{1,2,3\}]$, which also includes sets such as $\{1\}$ or $\{1,2,3\}$. To improve on this, the domain of a set variable usually also contains information on the cardinality of the set, i.e., lower and upper bounds on its size. The example domain can be represented exactly by specifying a lower and upper cardinality bound of 2.

For optimization problems, CP works the same way as for satisfaction problems, but every time a feasible solution is found, the remaining search space is restricted by adding the constraint that the solution has to be better than the current one. This is usually implemented by adding a variable f to the model, which represents the value of the objective function. If propagation can remove all values from the domain of f which are better than the currently best known solution at some node in the search tree, then this node can be pruned immediately.

2.2.12 Integer Linear Programming

Integer Linear Programming is a class of methods for solving COPs. In contrast to CP, it is far less flexible with respect to the types of COPs that can be solved. Only if the COP can be modeled as shown by Equations (2.1)–(2.3), i.e., as a system of linear inequalities with a linear objective function and using integer variables, Integer Linear Programming can be applied.

$$z = \min \quad c'x \tag{2.1}$$

subject to
$$Ax \ge b$$
 (2.2)

$$\boldsymbol{x} \in \mathbb{Z}_+^n. \tag{2.3}$$

Using additional real-valued variables (Mixed Integer Linear Programming) is also allowed, but it is not possible to model an \mathcal{NP} -complete COP exclusively with real-valued variables (unless the model is exponential in size). In the following, we will consider Integer Linear Programming, but everything is applicable to Mixed Integer Linear Programming as well. To solve Constraint Satisfaction Problems, set c = 0.

Branch & Bound can be applied to solve Integer Linear Programs (ILPs). Some definitions are required before we can cover the details.

Definition 2.2.12 (Set of Integer-Feasible Solutions). *The set of integer-feasible solutions* X *of ILP* (2.1)–(2.3) *is defined as* $X = \{x \in \mathbb{Z}^n_+ \mid Ax \ge b\}$.

Following this definition, an ILP can also be written as $z = \min\{c'x \mid x \in X\}$.

Definition 2.2.13 (LP Relaxation). *Given an ILP* (2.1)–(2.3), the LP relaxation is obtained by replacing the integrality constraints (2.3) by $x \in \mathbb{R}^n_+$.

Algorithm 2.6: LP-based Branch & Bound (for minimization problems)

Input: ILP min{ $c'x : x \in X$ } **Output**: Optimal solution x^* 1 problem list $L = \{X\}; //$ initialize problem list 2 $x^* = 0; //$ best known feasible solution $\overline{z} = \infty; //$ objective of best known feasible solution while $L \neq \emptyset$ do 4 choose set X_i and remove it from L; 5 $(m{x}_{ ext{LP}}^i, \underline{z}^i) = LP(X_i);$ // get optimal LP solution and objective 6 7 if $LP(X_i) = \emptyset$ then prune X_i by infeasibility; else if $\underline{z}^i \geq \overline{z}$ then prune X_i by bound; 8 9 else if $x_{\text{LP}}^i \in X$ then // LP solution is integer if $z^i \leq \overline{z}$ then 10 $oldsymbol{x}^* = oldsymbol{x}^i_{ ext{LP}}; //$ update best known solution 11 $\overline{z} = z^i$; // update best known objective 12 13 end prune X_i by optimality; 14 15 end else $L = L \cup \{X_{i,1}, X_{i,2}\}; //$ partition search-space X_i 16 17 end 18 return x^* ;

The LP relaxation can be solved efficiently (in polynomial time) by ellipsoid [104] or interior point methods [103]. Much better in practice and more widely used is the simplex method [37], even though it can have an exponential run-time in the worst case [13]. Details on the simplex method can be found in [13]. By using this method, we can define the LP based Branch & Bound scheme (for minimization problems), as presented in Algorithm 2.6.

The problem list L is the central data structure of this algorithm. It contains all partitions of the search-space that have not yet been considered. When this list is implemented as a stack, we get the standard depth-first search (as we have outlined for Branch & Bound). In practice, there are a lot of improvements possible by intelligently choosing which partition to consider next. In addition to L, we also need to store the best known feasible solution (which is the optimal solution at the end of this algorithm), and its objective. During the execution, this objective is an upper bound on the optimal value, hence the overline and initial value of ∞ .

After the initialization, the algorithm starts in earnest by considering every search space partition (subproblem) in turn, until L is empty. For the currently considered subproblem, we solve the LP relaxation and get the optimal solution of the LP relaxation x_{LP}^i and its objective \underline{z}^i . Since we do not enforce variables x to be integer in the LP relaxation, the LP solution will be fractional in the general case. Its objective is a lower bound (hence underlined) on the objective of the optimal integer solution contained in X_i . This is the bounding step, and depending on the result, different actions are executed. If X_i is not even feasible when considering its LP-relaxation

(i.e., there is not even a fractional assignment to all variables that satisfies all inequalities), then X_i cannot contain any integer feasible solutions and can be discarded. If the best fractional solution contained in X_i is not better than the globally best known integer feasible solution, X_i can be discarded without further consideration, since no integer feasible solution that will lead to an improvement exists within it. This step has the potential of quickly recognizing large parts of the search space as uninteresting and therefore hugely increasing performance. If the LP solution happens to be integer, then we store it, if it is better than the best known solution. As a side-effect, this increases the probability of being able to prune by bound in subsequent iterations. Then we discard X_i , since we have already identified the optimal solution contained within. If none of those conditions apply, i.e., the LP solution is fractional and better than the currently best known integer solution, then X_i might contain an improving integer solution and further analysis is required.

This leads us to the branching step, where we partition X_i into two sets. This is done by selecting a variable x of the LP solution that has a fractional value v, and adding for one partition the constraint $x \leq \lfloor v \rfloor$ and for the other partition $x \geq \lceil v \rceil$. Note that if x is a binary variable (its domain is $\{0, 1\}$), it is now forced to be integer in both partitions. By partitioning X_i in that way, x_{LP}^i is no longer a feasible solution for both partitions. The optimal LP solution of both partitions is likely worse than \underline{z}^i , so in the best case it may be possible to immediately prune both partitions of X_i , while we could not prune X_i itself. Due to this branching, we can guarantee that the LP solution will be integer feasible at some point going down the search tree (or no LP solution exists and we need to track back), and the search tree depth is finite.

When modeling a COP with ILP, one aim is finding a strong model, that means that the optimal objective of its relaxation is close to the objective of the optimal integer solution. It is possible to add inequalities to a model to strengthen it, i.e., the inequalities are redundant and do not change the set of integer feasible solutions, but they remove feasible solutions of the relaxation. These inequalities might be added to the model at the beginning or added during search as necessary. An added inequality is also called cut, because it cuts away previously feasible fractional solutions. The problem of finding such a violated cut for a given LP solution is called the (cut) separation problem. It is theoretically possible to add cuts to the model until the solution of the LP relaxation yields the optimal integer solution by means of the cutting plane algorithm [38]. Usually, the cutting plane algorithm is computationally too expensive.

Far more promising is the combination of the idea of cut generation and Branch & Bound, yielding highly effective Branch & Cut algorithms. At each node in the Branch & Bound tree, cuts are separated to strengthen the LP relaxation of the current subproblem. Procedures to automatically derive certain types of generic cuts, such as Gomory cuts [68] or Lift-and-Project cuts [8], are included in the major ILP solvers. With Branch & Cut, it is also possible so solve models which are exponential in the number of required inequalities, for instance forbidding all cycles of a specific length when formulating a problem on a graph structure. The initial model does not contain these constraints. Only if a constraint is violated by a LP solution, the cut enforcing the constraint is added.

Instead of adding constraints, it is also possible to dynamically add variables to a model. For the LP relaxation, this is called column generation [64], as a new variable adds a column to the coefficient matrix A. When combined with Branch & Bound for solving ILPs, it is called

Branch & Price. With this method, it is possible to use models with an exponential number of variables. Initially, an LP containing only a small fraction of the variables is solved. An important condition here is that this small fraction of variables has to permit a feasible solution. This is problematic when even finding a feasible solution is \mathcal{NP} -complete. After the LP is solved, a variable that is currently not included in the model and may improve the solution of the LP has to be identified. This is called the pricing problem. Only variables with negative reduced costs may improve the solution, see the simplex algorithm [37] for details. By means of Danzig-Wolfe decomposition [39], it is possible transform a compact model into a model with exponentially many variables. The transformed model usually has much tighter LP bounds when compared to the original compact model which reduces the number of Branch & Bound nodes that need to be considered.

As an example for a model with exponentially many variables, consider a variant of the Vehicle Routing Problem (VRP), where packets have to be delivered to customers by a fleet of vehicles. Each vehicle starts from the central depot, delivers packets to a set of customers and returns to the depot. The total length of the driven distance by all vehicles has to be minimized. All packets that need to be delivered by a vehicle need to fit into the vehicle. One decision variable could represent a specific route of a vehicle that is valid with respect to the capacity constraint of the vehicle. There are exponentially many such variables. In the model for this VRP, we select one route for each vehicle, such that every customer is visited exactly once, and the total cost of the routes is minimized.

One point of interest of the column generation approach (besides potentially better LP bounds) is that it is possible to hide arbitrary constraints within the pricing subproblem and the LP model deals only with variables that respect those constraints. However, there are also some downsides. First of all, column generation cannot be stopped during execution and still yield a valid LP bound. Only after we have proven that no more improving variables exist is the LP bound valid. Otherwise it might be too high (for a minimization problem), which means we might then prune subproblems during Branch & Bound that may still contain improving integer feasible solutions. The cutting plane method can be stopped and we get a valid (if not as strong as possible) lower bound. Proving that no further cuts exist is only necessary when an integer solution is found, otherwise the integer solution could be invalid for the complete (exponential) model. Branching sometimes is also problematic for Branch & Price. Assume we need to chose (i.e., set to 1) one of exponentially many variables. When we branch on one of those variables, we either fix it to 1 (and are finished unless there are also other variables), or we fix it to 0 and have basically no additional restriction since there are still exponentially many other variables that may need to be generated and branched upon. As a result, the search tree is very skewed. The third major hurdle of column generation are convergence issues. Without taking care, a lot of variables will be generated that improve the LP bound only a tiny bit which leads to long convergence times. Stabilization techniques [45] are required to combat this. Despite these challenges, Branch & Price has been very successfully applied to Cutting & Packing and Network Design problems. For an overview on column generation, see [44, 118].

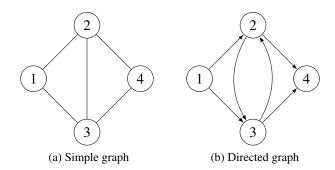


Figure 2.1: A simple graph and a directed graph.

2.3 Graph Theory

In this section, we review the basics of graph theory and related algorithms required in later chapters of this work. We follow the definitions from [176].

Definition 2.3.1 (Graph). A graph G = (V, E) with n = |V| vertices an m = |E| edges consists of a vertex set $V(G) = \{v_1, \ldots, v_n\}$ and edge set $E(G) = \{e_1, \ldots, e_m\}$, where each edge consists of two (possibly equal) vertices called its endpoints. If $e = \{u, v\} \in E(G)$, then u and v are adjacent. A loop is an edge whose endpoints are equal. Parallel edges are edges that have the same pair of endpoints. A simple graph is a graph having no loops or multiple edges.

We will use V and E as shorthand for V(G) and E(G) when G is clear from the context. We will use the word graph to denote simple graphs unless stated otherwise.

Definition 2.3.2 (Directed Graph). A directed graph G = (V, A) consists of a vertex set V(G) and arc set A(G), where each arc is an ordered pair of vertices. If $a = (u, v) \in A(G)$, u is the head and v the tail of the arc. The choice of head and tail gives an arc a direction, from head to tail. A simple directed graph is a directed graph in which each ordered pair of vertices occurs at most once as an arc.

We will use A as shorthand for A(G) when G is clear from the context.

Figure 2.1 shows an example for a simple graph and a directed graph. Both graphs have a vertex (node) set $V = \{1, 2, 3, 4\}$. The simple (undirected) graph 2.1a has edge set $E = \{\{1, 2\}, \{1, 3\}, \{2, 4\}, \{2, 3\}, \{3, 4\}\}$. The directed graph 2.1b has arc set $A = \{(1, 2), (1, 3), (2, 3), (3, 2), (2, 4), (3, 4)\}$.

Definition 2.3.3 (Dense and Sparse Graphs). A graph G is considered to be dense if $m \propto n^2$, it is sparse if $m \propto n$.

Definition 2.3.4 (Source and Target of an Arc). Given an arc $a = (u, v) \in A$, s(a) denotes the source (head) of the arc, i.e., s(a) = u while t(a) denotes the target (tail) of the arc, i.e., t(a) = v.

Definition 2.3.5 (Subgraph). A subgraph of a graph G is a graph H such that $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

Definition 2.3.6 (Degree). The degree of vertex v of a simple graph G, written $\delta(v)$, is the number of edges containing v.

Definition 2.3.7 (Incident Edges). The incident edges of vertex v of a simple graph G are the edges containing v.

Definition 2.3.8 (In-Degree). The in-degree of a vertex v of a directed graph G, written δ_v^- , is the number of arcs with v as target.

Definition 2.3.9 (Out-Degree). The out-degree of a vertex v of a directed graph G, written δ_v^+ , is the number of arcs with v as source.

Definition 2.3.10 (Shadow). The shadow of a directed graph G is an undirected graph S with the same vertex set. The edge set of S is chosen such that adjacent vertices in G are adjacent in S and vice versa.

Definition 2.3.11 (Reversal Graph). The reversal graph G_R of a directed graph G contains all vertices of G, and $(u, v) \in A(G_R)$ if and only if $(v, u) \in A(G)$.

The degree of node 2 of the simple graph in Figure 2.1a is 3, i.e., $\delta(2) = 3$. The incident edges of node 1 of this graph are $\{1, 2\}$ and $\{1, 3\}$. The in-degree of node 3 of the directed graph in Figure 2.1b is 2, i.e., $\delta_3^- = 2$. The out-degree of node 4 is 0, i.e., $\delta_4^+ = 0$. Graph 2.1a is the shadow of graph 2.1b.

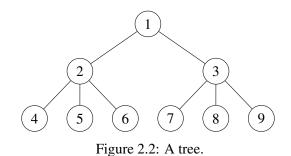
Definition 2.3.12 (Path). A path p of length k is a sequence $v_0, e_1, v_1, e_2, \ldots, e_k, v_k$ of vertices and edges such that $e_i = \{v_{i-1}, v_i\}, \forall i \in [1, k]$. A path with no repeated vertices is called simple. The source or start of p, s(p), is v_0 , the target or end, t(p), is v_k . The nodes of the path p are $N(p) = \{v_0, \ldots, v_k\}$, the edges of the path are $E(p) = \{e_1, \ldots, e_k\}$.

Definition 2.3.13 ((Simple) Cycle). A path p of length at least 2 is called a cycle, if s(p) = t(p). If p is simple (with the exception of its source and target node), the cycle is called simple.

Definition 2.3.14 (Weighted Path). Given a path p in graph G and a function $w : E(G) \to \mathbb{R}$, the weight of p is $\sum_{e \in E(p)} w(e)$. If a weight function for edges is available, the length of a path refers to its weight instead of the number of its edges.

Definition 2.3.15 (Node and Edge Disjoint Path). Two paths p_1, p_2 are node disjoint if $N(p_1) \cap N(p_2) = \emptyset$. They are edge disjoint if $E(p_1) \cap E(p_2) = \emptyset$.

The definitions of a path for directed graphs and arc disjointness are analogous. Let $p_1 = 1, (1, 2), 2, (2, 4), 4$ in graph 2.1b. Then the length of p_1 is 2, $s(p_1) = 1, t(p_1) = 4, N(p_1) = \{1, 2, 4\}$ and $A(p_1) = \{(1, 2), (2, 4)\}$. Let $p_2 = 2, (2, 3), 3$ and $p_3 = 3$. Then p_1 and p_2 are arc disjoint but not node disjoint, p_1 and p_3 are arc and node disjoint.



Definition 2.3.16 (Connected Graph). A graph G is connected if there is a path between every pair of vertices from V(G).

Definition 2.3.17 (Connected Components). The connected components of a graph G are its maximal connected subgraphs. A connected graph has one connected component.

Definition 2.3.18 (Tree). A graph G is a tree, if it is connected and the path between each pair of vertices is unique. For trees, n = m + 1. The root of a tree is a node of V(G) and often used as starting point for algorithms on trees.

Figure 2.2 shows an example tree. Node 1 is the root of the tree. Nodes 4–9 are the leafs. Nodes 2 and 3 are intermediate nodes and the children of 1. The parent of 4 is 2. Nodes 7–9 are siblings. Node 1 is ancestor of Node 7. The height (or depth) of the tree is 2 and is the length of the longest path from the root to one of the leafs.

Definition 2.3.19 (Depth-First Search). Depth-first search is a traversal order of the nodes of the tree. Starting at the root, we select one of its children and then one of the children of the child and so on, until we have reached a leaf. Then we go back in direction of the root (back-tracking). We stop at the first node that still has unexplored children and continue with one of those. The search is finished if all children of the root have been explored.

Depth-first search applied to the tree in Figure 2.2 could visit nodes in the following order (not showing nodes visited during back-tracking): 1, 3, 8, 9, 7, 2, 4, 6, 5.

Definition 2.3.20 (Biconnected Graph). A graph G is biconnected, if any vertex of V(G) or edge of E(G) can be removed and G remains connected.

Definition 2.3.21 (Articulation Point). Vertex v of graph G is an articulations point, if its removal increases the number of connected components.

Definition 2.3.22 (Bridge). *Edge* e of graph G is a bridge, if its removal increases the number of connected components.

Definition 2.3.23 (Block). A block of a graph G is a maximal connected subgraph of G that has no articulation points.

If a block has more than two vertices, then it is biconnected. If it has two vertices, the edge connecting them has to be a bridge. Two blocks in the same graph share at most one vertex, hence the blocks of a graph partition its edge set, i.e., all edges belong to exactly one block. A shared vertex has to be an articulation point, every articulation point belongs to at least two blocks.

Definition 2.3.24 (Block Tree). The block tree B is built from a connected graph G, by adding all articulation points of G to B, and one vertex for every block of G. Vertices v_1, v_2 of B are connected, if v_1 represents an articulation point of G which belongs to the block represented by v_2 .

Definition 2.3.25 (Weakly Connected Graph). A directed graph is weakly connected, if its shadow is connected.

Definition 2.3.26 (Strongly Connected Graph). A directed graph is strongly connected, if there is a path in both directions between every pair of vertices.

Definition 2.3.27 (Strongly Connected Components). *The strongly connected components of a directed graph are its maximal strongly connected subgraphs.*

The strongly connected components of a graph G can be calculated in O(m + n) by using Tarjan's algorithm [166] based on depth-first search.

Definition 2.3.28 (Strong Articulation Point). A vertex is a strong articulation point, if its removal increases the number of strongly connected components of a directed graph.

Definition 2.3.29 (Strong Bridge). An arc is a strong bridge, if its removal increases the number of strongly connected components of a directed graph.

Definition 2.3.30 (Flowgraph). A flowgraph G(s) = (V, A, s) is a directed graph with a start vertex s in V such that every vertex in V is reachable from s.

Definition 2.3.31 (Dominator). Given a flowgraph G(s), vertex u is a dominator of vertex v if all paths from s to v include u. The trivial dominators of u are s and u. D(s) is the set of non-trivial dominators in G(s).

Definition 2.3.32 (Immediate Dominator). *Given a flowgraph* G(s), *vertex* u *is an immediate dominator of* v *if* u *is a dominator of* v *and every other non-trivial dominator of* v *also dominates u. The immediate dominator is unique.*

Definition 2.3.33 (Dominator Tree). The dominator tree DT(s) of a flowgraph G(s) contains all vertices of G. There is an arc from a vertex u to a vertex v in DT(s) if u is the immediate dominator of v. DT(s) is a tree rooted at s, the dominators of a vertex in G(s) are all its ancestors in DT(s)

Definition 2.3.34 (Planar Graph). A graph G is called planar, if it can be drawn in the plane without edge crossings.

Definition 2.3.35 (Diameter of a Graph). *The diameter of a graph is the length of the longest shortest path between any pair of vertices.*

Definition 2.3.36 (Small World Graph). A graph is a small world graph, if the average degree of each node is small, but the graph also has a small diameter. The diameter grows proportionally to the logarithm of the number of vertices.

2.3.1 Dominators

Efficiently calculating dominators and the dominator tree has been an open problem for a long time. Lengauer and Tarjan [115] presented an algorithm solving this problem in $\mathcal{O}(m\alpha(m, n))$, where $\alpha(m, n)$ is the extremely slow-growing functional inverse of the Ackermann function, in 1979. Truly linear-time algorithms have been proposed by Harel [77], Alstrup [4] and Buchsbaum [22]. These algorithms either turned out to be wrong or far too complicated for a practical implementation. Georgiadis et al. [63] were able to present an implementable algorithm for finding dominators in $\mathcal{O}(m + n)$ in 2004 and Buchsbaum et al. [23] were able to correct their algorithm in 2005. The best source for actually implementing a linear time dominator algorithm seems to be the work of Buchsbaum, Georgiadis and Tarjan et al. [21] from 2008. For an easily implementable algorithm for dominators in $\mathcal{O}(n^2)$ see the work of Cooper et al. [34].

2.3.2 Strong Articulation Points

The advances made with algorithms for finding the dominators in a flowgraph enabled Italiano et al. [94,95] to formulate a $\mathcal{O}(m+n)$ algorithm for finding all strong articulation points in a directed strongly connected graph G. We will just present the main ideas here and refer to the referenced work for more details and proofs.

The first step of the algorithm is to determine for an arbitrary node s if it is a strong articulation point. This is done by removing s from G and checking if the remainder is still strongly connected, which can be done in $\mathcal{O}(m+n)$. In the second step, we calculate the dominators in G(s) and its reversal $G_R(s)$, which is also in $\mathcal{O}(m+n)$. These dominators (possibly together with s depending on the outcome of the first step) give all strong articulation points of G.

To see why this is so, consider the following argument. It is clear that every dominator has to be an articulation point, since crossing a dominator is the only way to reach the node it dominates. Removing the dominator means that the dominated nodes cannot be reached any more, which increases the number of strongly connected components, the defining property of articulation points. Therefore, we only need to be certain that we do not miss any strong articulation points, i.e., every strong articulation point has to be a dominator in either G(s) or $G_R(s)$. Assume there is a strong articulation point a and a node b, node b being in another strongly connected component than s if a were to be removed. In G, there have to be paths from s to b and from bto s. In one direction, there is only allowed to be a single path, which has to cross a, otherwise this would violate the assumption that a is a strong articulation point. If the path from s to bcrosses a, then a is a dominator in G(s). If the path from b to s crosses a, then a is a dominator in $G_R(s)$. Therefore, it is not possible to miss a strong articulation point by using the outlined algorithm. Basically the same method can be used to find all strong bridges.

2.3.3 All Pair Shortest Path

The All Pair Shortest Path Problem is defined as follows:

Definition 2.3.37 (All Pair Shortest Path Problem). Given a graph G and a function $w : E(G) \to \mathbb{R}$, determine for each pair of vertices of G the shortest path.

There are two well-known algorithms for solving this problem. Both allow negative edge weights, but no cycles of negative length. The first algorithm is Johnson's algorithm [97], solving the problem in $\mathcal{O}(n^2 \log(n) + nm)$ by essentially calculating for each vertex in the *G* the shortest path to all other vertices. The alternative is the Floyd-Warshall algorithm [56] requiring a run-time of $\mathcal{O}(n^3)$. The modern implementation of this algorithm is essentially a series of n-1 matrix multiplications [93].

Based on the run-time complexities, Johnson's algorithm is the fastest choice for sparse graphs, while the Floyd-Warshall algorithm has an advantage for dense graphs. In this work, we deal with very sparse graphs, so Johnson's algorithm is used to solve the All Pair Shortest Path Problem.

2.4 Experimental Setup

2.4.1 Computational Environment

All computational results presented in this thesis have been achieved on Intel Xeon E5540 multicore systems with 2.53 GHz and 24 GB RAM, which corresponds to 3 GB RAM per core. The implemented algorithms only utilize one core. All reported run-times are CPU-times, as opposed to wall-clock time. A memory limit of 5 GB has been used, unless otherwise specified. If an algorithm exceeds the memory limit during execution, it is aborted.

2.4.2 Statistical Tests

In the course of this thesis, we will often compare the performance of two different algorithms solving the same problem, for instance with respect to the required run-time. A natural question of course is, which algorithm is faster? When algorithm A always requires a run-time in the order of seconds for a particular set of problem instances, and algorithm B always requires a run-time in the order of hours, then we may rightly conclude (assuming that the set of problem instances is representative of the problem) that algorithm A is faster than algorithm B. However, the situation is not so clear most of the time. If there is only a difference of some percent between the average run-times of the two algorithms, it could very well be that it is caused by random chance.

In such cases, statistical hypothesis testing becomes important to distinguish between random chance and a true difference. In statistical hypothesis testing, one first states a null and alternative hypotheses. For our example, the null hypothesis would be that algorithm A is not faster than algorithm B, while the alternative hypothesis is that A is indeed faster than B. Now we need to select an appropriate statistical test to determine if we can reject the null hypothesis in favor of the alternative given a certain error probability. In our case, we are dealing with paired data.

Table 2.1: Example table showing how the results of the statistical test are presented.

Size	Α	B	
20	5.1 >	1.0 =	
30	11.0 =	10.9 =	

For every problem instance, we know the run-time of algorithm A and algorithm B. A common assumption for hypothesis testing is that the data (run-time) follows a normal distribution. In the context of this work, this assumption will not hold, as we compare run-times (and other properties) across different classes of problem instances, some more readily solved than others. As a result, we get clusters of different run-times according to the hardness of the instances. Therefore, we need a statistical test that does not assume a normal distribution and can deal with paired data: the Wilcoxon Signed-Rank test [179]. Information on how this test works exactly is readily available [100], so we will not go into detail here. It is just important to note that the null hypothesis for the Wilcoxon Signed-Rank test is that the difference between the run-times of A and B is centered at a value ≥ 0 and the alternative that it is centered at a value < 0. The main point here is that this is different from saying that the arithmetic mean (which we usually report) of the run-times of A is smaller than the mean of the run-times of B. In rare cases, this will become visible, as a mean run-time of one algorithm might be marked as the lowest run-time, even though other algorithms have lower reported mean run-times.

On the topic of presentation of the results of the statistical tests, Table 2.1 shows an example. The average run-time in seconds of algorithms A and B are compared for two different instance sizes. For size 20, we can see that B requires only 1 second, while A requires 5.1 seconds. A Wilcoxon Signed-Rank Test has shown that A-B is > 0 (with a certain error probability), hence the ">"-sign next to the reported run-time of A. B is equal to itself (we always compare to the best algorithm) and therefore marked with an "="-sign. The situation is not as clear for size 30. Again, algorithm B is faster, but now the difference between A and B is not significantly greater than 0, therefore A is also marked with an "=" sign and shaded, but slightly less so than B.

In this thesis, we always perform the Wilcoxon Signed-Rank test with a level of significance of 5%, which is the probability of mistakenly rejecting the null hypothesis. We used the Wilcoxon test available in the statistics software R [86]. For an introduction to statistical hypothesis testing, see for example [24, 150, 169].

2.4.3 Used Software

This thesis would not have been possible without utilizing the work of others. In this section we want to acknowledge the authors of the libraries and software packages which we used. As programming environment, we used Eclipse Kepler [59, 81] with the C++ Development Tooling [138]. The employed compiler was gcc-4.7.1 [65]. Considerable support was provided by the various libraries of the boost project [17, 102], especially the graph library [158]. Other libraries and applications we used to handle graphs were Stanford GraphBase [107, 108], GT-ITM [184] and nem [120]. For statistical computations we used R [86].

As solver for (Integer) Linear Programs we used CPLEX 12.5 [85]. Unless otherwise specified, we used default settings, with the exception that we solved in single-threaded mode and that the time-limit was specified in CPU time and not wall-clock time. We used GECODE [155, 167] as Constraint Programming solver.

CHAPTER 3

The Virtual Network Mapping Problem

3.1 Introduction

In Chapter 1, we have outlined the need for virtual networks within physical networks, but we did not go into detail on how this works exactly. The main aim is clear: we want to fit virtual networks into physical networks. Even this simple statement is ambiguous. It could mean that we want to find a part of the physical network that has the same structure as a virtual network, so the virtual network will fit there. Another possibility would be that we have to recreate the structure of a virtual network by using parts of the physical network. Additionally, resources should be involved in some way, since it is not possible to implement an unlimited amount of virtual networks within a physical network. This demonstrates that a rigorous definition of the problem we are going to solve in the following chapters, the Virtual Network Mapping Problem (VNMP), is required. In this chapter, we will present the definition of the VNMP. Chapter 4 will outline other definition possibilities that can be found in the literature.

3.2 Input of the VNMP

Two central components of the input of the VNMP are the physical network and the virtual networks. Another name for the physical network that we will use throughout this thesis is substrate network. The substrate network is modeled by a directed graph G(V, A) with node set V (the substrate nodes) and arc set A (the substrate arcs). The virtual networks, also called slices, will be defined by a directed graph G'(V', A') with node set V' (the virtual nodes) and arc set A' (the virtual arcs). Each separate connected component of G' represents one virtual network. Based on these definitions, we will use the terms network and graph interchangeably. We chose directed graphs to model the physical and virtual networks because this allows us to distinguish between communication directions. As a result, we can model asymmetric con-

nections, i.e., the connections that have different properties in different directions, which often occurs in practice.

Virtual networks are meant to represent a specific application or application class. This implies that they have to connect the users of the application which are located at fixed points in the substrate network. As a result, we cannot place all the virtual nodes to which the users want to connect arbitrarily within the substrate network. They need to be close to the users. There are other reasons why the ability to limit the location of virtual nodes is important. One example would be that the operator of a virtual network for video streaming wants to ensure that each country in which he operates is covered by a virtual node the users can connect to. There might also be legal reasons why some locations are undesirable. Therefore, the third major input required to define an instance of the VNMP is a set $M \subseteq V' \times V$, which specifies allowed mappings. If and only if $(k, i) \in M$, virtual node k is allowed to be located at substrate node i. By M(k), we denote the set of substrate nodes where virtual node k might be placed. Obviously, $M(k) \neq \emptyset$, $\forall k \in V'$ for a valid VNMP instance.

It is not physically possible to map an unlimited number of virtual networks into a substrate network, so we need to consider relevant resources which are used up by the virtual networks. First and foremost, virtual networks transfer data, so they require data transfer capacities, or bandwidth, from the substrate network. To be able to model this, we associate a bandwidth requirement $b_f \in \mathbb{N}^+$ with all $f \in A'$ and bandwidth capacities $b_e \in \mathbb{N}^+$ with all $e \in A$. With data transfer capacities, we are able to represent applications (i.e., virtual networks) that are just concerned with bulk data transfer. There are applications with this characteristic (e.g., Peer-to-Peer networks), but there are also many applications that require more than guaranteed transfer capacities. One might imagine a telephone conversation, where the time lag between the speaking of the caller and the listener being able to hear it is too high. Such situations are experienced as irritating by many people. Therefore, the time it takes to transfer data is a more important characteristic than the transfer capacity for Voice-over-IP applications. We take transmission delays into account by specifying an upper bound for delay $d_f \in \mathbb{N}^+$ for all $f \in A'$ and associating a transmission delay $d_e \in \mathbb{N}^+$ to all $e \in A$.

A virtual network is not simply a passive container for a specific application. It offers services to the application, be it in the form of customized routing protocols, naming services, or something else. To be able to do that, the virtual nodes require computing capacities that have to be available at the substrate node where they are hosted. So we associate CPU requirements $c_k \in \mathbb{N}^+$ to all $k \in V'$ and CPU capacities $c_i \in \mathbb{N}^+$ to all $i \in V$. The CPU capacities of the substrate nodes are not just used to host virtual nodes. In a physical network, the substrate nodes are typically represented by routers, and their main task is forwarding data. This data forwarding requires processing power, which is then unavailable for hosting virtual nodes (or vice versa). This interaction is approximately modeled by the VNMP by assuming that one unit of data (as a slight misuse of terminology, we will also call this one unit of bandwidth) from a virtual arc that crosses a substrate node requires one unit of processing capacity there. It is inconsequential whether this bandwidth is simply relayed by the substrate node or has originated from a virtual node mapped to the substrate node. Even if both, the sending and receiving virtual node are mapped to the same substrate node, CPU capacity is required to route data from one virtual node to the other. These are all the inputs that are required for a very basic form of the VNMP, when we are only interested in finding valid solutions (the exact definition of a valid solution will be discussed later). For richer forms of the VNMP, we require additional inputs. We want to map virtual networks into a physical network in a cost-optimal way. Again, we have to refer to a later part of this chapter for the definition of the objective. For now it is sufficient to state that each substrate node *i* has an associated usage $\cot p_i^V \in \mathbb{N}^+$ and every substrate arc *e* has a $\cot p_e^A \in \mathbb{N}^+$. If the resources within the substrate network are not sufficient, it might be possible to buy additional resources. For this, we have a defined price of one unit of CPU capacity $p^{\text{CPU}} \in \mathbb{N}^+$ and one unit of bandwidth $p^{\text{BW}} \in \mathbb{N}^+$.

This concludes the discussion of the input necessary to define a VNMP instance. In the next section, we will define the output of the VNMP.

3.3 Output of the VNMP

The output of a VNMP consists of two parts. First, a mapping $m : V' \to V$ such that $(k, m(k)) \in M, \forall k \in V'$. That means every virtual node has to be assigned to one of its allowed substrate nodes. The second part is the implementation of the virtual arcs. For every virtual arc f, we need to find a simple path p_f from m(s(f)) to m(t(f)) in the substrate. Such a solution is only valid, if the following resource constraints are fulfilled. First of all, there is the delay constraint. The sum of the delays of the substrate arcs that are used to implement a virtual arc f may not exceed the maximum allowed delay d_f . Then there is the bandwidth constraint. The data transferred across a substrate arc e by the implementations of virtual arcs has to be less than or equal to the bandwidth capacity b_e . As last resource constraint, we have the CPU capacity at the substrate nodes. The total CPU load caused by hosting virtual nodes and by routing data for virtual arcs is not allowed to be greater than the CPU capacity c_i . If a solution to the VNMP satisfies those constraints, it is valid.

One thing to note for this definition of output is that we require the implementing paths for virtual arcs to be simple, i.e., to not contain loops. This is not strictly necessary, as long as the substrate has enough resources and the path does not exceed the delay limit, implementing paths with loops would not be a problem. However, given a valid VNMP solution (except the simple path requirement), we can always derive a solution that only contains simple paths by removing all loops. The result will be a valid solution and require strictly less resources, so it will be better in this sense. The simple path requirement also adds optimization potential, as outlined in Chapter 9, where we will heavily depend on the simpleness of implementing paths.

3.4 Example VNMP Instance

Before we define the objective for the VNMP, we will show a small example to demonstrate the main components of the VNMP.

Figure 3.1 shows a simple VNMP instance. The virtual network G' contains two virtual nodes, showing their CPU requirement. A virtual arc connects them and is labeled with its bandwidth requirement and allowed delay. The substrate network G contains the physical network nodes

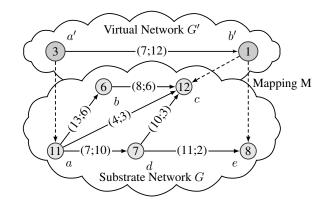


Figure 3.1: An illustrative VNMP instance.

showing their CPU resources and the available links (i.e., connections) between the nodes, labeled by their bandwidth capacity and the delay that is incurred when data is transmitted across them. The dashed lines show M, i.e., the allowed locations of the virtual nodes. This example instance has only one valid solution, as b' cannot be mapped to c, even though c has enough resources available. At least eight units of processing capacity are required at c, one for hosting b'and seven for routing the data of the virtual arc to b', so the 12 available units are sufficient. The problem is that the virtual connection cannot be implemented from a to c. The implementing path cannot use b, because it does not have enough resources to route seven units of bandwidth. The direct connection from a to c lacks the required bandwidth capacity, and the path (a, d, c)incurs too much delay. So the only valid solution is to map a' to a, b' to e and use the path (a, d, e) to implement the virtual arc between a' and b'.

3.5 Objective

Until now, we have defined how a valid solution to the VNMP looks like and which constraints it has to fulfill. However, when considering practical applications it is clear that not all valid solutions are equally desirable. We have already touched on this previously during the discussion of the simple path requirement, where we noted that wasting resources is bad. What we now need is a measure of desirability of a valid solution and there are a lot of possibilities for defining it. We could say that it is desirable to spread the load on the substrate nodes so that the CPU requirement at each node is as far away from the available capacity as possible. Another possibility would be to minimize the length of the implementing paths (measured by the number of used arcs), because that reduces the total bandwidth load on the substrate and the probability that a defect of a substrate arc will break a virtual connection. Or we could minimize the length of the implementing paths (measured by the length of the implementing of the substrate change. We chose to define a model of the operational costs of a VNMP solution, which we try to minimize. In this model, the resources of the substrate network are available to a virtual network

provider. If it does not access these resources, they do not incur any costs. In case it utilizes the virtualization capabilities of a substrate node *i*, by hosting any amount of virtual nodes there, it has to pay the usage cost p_i^V for this substrate node. For substrate arcs, the situation is the same. If any number of virtual arcs utilize a substrate arc *e*, usage cost p_e^A has to be paid. The sum total of the incurred costs gives the substrate usage cost C_u , which has to be minimized. Therefore, the objective is to fit all virtual networks into a cheap part of the substrate network, subject to the resource and mapping constraints.

This choice of objective function has some implications. First and foremost, there is no immediate connection between a solution and its cost. By paying the substrate usage cost, we select a part of the substrate that we are allowed to use to implement virtual networks. How they are implemented within this selected part does not influence the costs. As a result, it can be expected that a VNMP instance will have a lot of different optimal solutions. The weak link between a solution and its cost may be problematic for heuristic methods. In Chapter 11 we will show problems this objective causes for Integer Linear Programming approaches. Another property of the objective function is that it provides a force away from feasibility. To create a very good solution, we need to aim for high utilization of the parts of the substrate we pay for. However, if the utilization is too high, we might fail to find a valid solution at all. To increase the probability of finding a valid solution, we need to spread the utilization of substrate nodes and arcs, which will result in a solution with very high C_u .

For practical purposes, just using this objective is insufficient. It might happen that the substrate network does not contain enough resources to implement all virtual networks. If we try to solve such a VNMP instance with heuristic methods, we will just fail to find a solution. If we apply exact methods, we might be able to prove that no solution exists. This is an important piece of information, but useless for a virtual network provider as it does not give any hints as how to solve this problem. There are two ways to deal with insufficient resources: reduce the amount of resources required or add more resources. To reduce the amount of required resources, there are multiple possible approaches. We could assign penalties to virtual networks that do not get the requested resources and try to minimize the penalties. Another possibility would be to assign profits to virtual networks and try to select a subset of virtual networks that still fits into the substrate network and maximizes the profits.

In this work, we choose the second way to deal with insufficient resources: we add the possibility of increasing the amount of available resources. This is more constructive than simply rejecting or not satisfying customers and also possible in practice. A virtual network provider might be able to rent additional resources for the controlled substrate network. As a first step, we need to identify where the resources are missing. In the general case, the location and amount of missing resources is ambiguous. It might be possible to make a VNMP instance solvable by adding CPU capacity at one node, or increasing the bandwidth somewhere else in the substrate, or a combination of both. In absence of any other constraints, we assume that we want to minimize the cost of renting additional resources, that one additional unit of resource has a fixed cost, and that the amount of resources that can be bought is unlimited. One unit of CPU resources costs p^{CPU} and one unit of bandwidth p^{BW} . The total cost of additional resources is the additional resources cost C_a , which we want to minimize.

We have neglected to mention delay as resource that might be changed. This has three reasons, the first of which is the ease of change in practice. Changing the available CPU resources is simple, just add another server. Also increasing the available bandwidth is not problematic, just lease more bandwidth or activate additional fiber-optic connections if available. Changing the delay of a connection is far more complicated and usually means changing the employed mode of transmission, for instance from copper wire to fiber-optic cable. Also, the amount of CPU or bandwidth resources to be added can be controlled much better. For delay there basically is only the choice whether we want to have the delay of an electrical connection or of an optical one. The second reason why we neglect to discuss delay changes is that this is problematic to model with Integer Linear Programming approaches (see Chapter 11). For adding CPU and bandwidth, only a new variable per substrate node or arc is required to specify the amount of added resources. For delay changes, we would need to add new variables for every combination of virtual arc and substrate arc, which is a substantial amount of additional variables. The third reason for ignoring delay as a changeable resource is that changing only CPU or bandwidth capacities is sufficient to make a VNMP instance feasible, as long as there is a mapping for the source and target node of every virtual arc such that the delay bound is not exceeded. All these reasons notwithstanding, in Chapter 12 we will present an application where we also consider changes to delays.

A VNMP solution has two properties, its substrate usage cost C_u and its additional resource cost C_a . This allows us to define the following:

Definition 3.5.1 (Valid VNMP Solution). A VNMP solution is valid, if $C_a = 0$.

Definition 3.5.2 (The VNMP Satisfiability Problem VNMP-S). *Given a VNMP instance, find a valid solution.*

Definition 3.5.3 (The VNMP Optimization Problem VNMP-O). Given a VNMP instance, find a valid solution with minimal C_u . If no such solution exists, find a solution with minimal C_a .

In this work, our main aim is solving the VNMP-O. In some cases we will also consider the VNMP-S. A slight variation of VNMP-S is looking for a solution with minimal C_a , which we will do for example in Chapter 11. For some applications, we do not allow the possibility of adding additional resources. It will be clear from the context when this is the case. For a compact mathematical definition of the VNMP, see Section 11.2.

3.6 Complexity

VNMP-S is \mathcal{NP} -complete. Quite obviously, it cannot be harder than \mathcal{NP} since we can guess a solution and check if it satisfies all constraints in polynomial time. To show that it is \mathcal{NP} -hard, there are different possibilities for reducing other \mathcal{NP} -hard problems to VNMP-S. One way is shown in [5], but there are also simpler reductions, for which we will present the general ideas. Bin-packing can be reduced to VNMP-S by using virtual nodes with their CPU requirements as items to pack, substrate nodes as bins and allowing every mapping from virtual nodes to substrate nodes (i.e., every item can be packed in every bin).

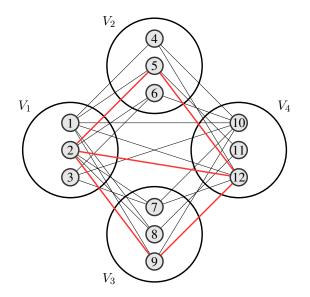


Figure 3.2: VNMP-S when just considering mapping and delay constraints.

A similar reduction can be performed by using the bandwidth requirements of virtual arcs as items to pack. Every item is a virtual network with two nodes and a virtual arc between those nodes. The bandwidth requirement of the virtual arc is the size of the item. The substrate contains one main source and one main target node. The source nodes of all virtual arcs are only allowed to be mapped to the main source node and the target nodes of the virtual arcs only to the main target node in the substrate. In addition, the substrate contains a node for every bin (the bin-nodes). Arcs from the main source node to all bin-nodes and from the bin-nodes to the main target node have the bandwidth capacity of the bin that is represented by the bin-node.

We have seen that by considering only CPU or bandwidth resources, \mathcal{NP} -hard problems can be reduced to VNMP-S. The same is possible for the delay constraint. Consider a relaxed version of the VNMP-S where only the delay and mapping restrictions are relevant. By calculating the shortest possible delay of paths between all pairs of substrate nodes, we know for each virtual arc f which mapping combinations of s(f) and t(f) are allowed. Those are the combinations for which the delay between m(s(f)) and m(t(f)) is $\leq d_f$. Once we have identified the allowed combinations, we need not consider the delay constraint any further.

Figure 3.2 shows how the remaining problem looks like. In this example, we have to implement a virtual network containing four nodes (V_1-V_4) . Each of those nodes can be mapped to three substrate nodes, for example V_1 can be mapped to substrate nodes 1–3. The edges present in the figure denote substrate connections that are feasible with respect to the delay constraint. For example, if $m(V_1) = 1$ and $m(V_2) = 4$, then there exists a delay feasible path for virtual arcs between V_1 and V_2 . Note the use of the plural here as there might be a virtual arc from V_1 to V_2 , from V_2 to V_1 , or both. An edge is only present if a delay feasible path exists for all virtual arcs. What we need to do now is to choose a substrate node for each virtual node, such that edges for all virtual arcs can be chosen. Mapping V_1 to node 1 and V_2 to node 6 would not be valid for example, since there is no edge between those nodes. We have chosen $m(V_1) = 2$, $m(V_2) = 5$, $m(V_3) = 9$, and $m(V_4) = 12$. The selected edges are colored and also show the structure of the virtual network (i.e., which virtual nodes have to be connected).

Given a clustered graph (V_1-V_4 are clusters) as shown in Figure 3.2, it is straight forward to construct a VNMP instance corresponding to it. The VNMP instance contains one virtual network, with one node for each cluster. If the graph contains at least one edge between clusters, then there is a corresponding arc between the virtual nodes. The orientation of the arc is arbitrary. The substrate network is given by the nodes within the clusters and the connections between them. The orientation of the substrate arcs has to be compatible to the corresponding virtual arc. For example, the VNMP instance derived from the graph shown in Figure 3.2 contains a virtual node for V_1 and a virtual node for V_2 . Since there are edges between those clusters, there is also a virtual arc between the virtual nodes. We orient it arbitrarily from V_1 (more precisely the virtual node representing V_1) to V_2 . In the substrate network we have nodes 1 and 4 (among others) and an arc between them. They represent a connection between V_1 and V_2 . Since we chose an orientation from V_1 to V_2 , also the substrate arc has to go from node 1 to node 4 in order to be compatible. The mapping constraint we set according to the clusters. All virtual arcs have a delay requirement of 1 and all substrate arcs have a delay of 1.

There is a transformation of the \mathcal{NP} -hard graph coloring problem [60] to a clustered graph as shown in Figure 3.2. This transformation was presented in [83, 114], we will just give the main idea here. For the graph coloring problem [60], we are given an undirected graph and a set of colors. To each node, we need to assign a color such that no pair of adjacent nodes is colored the same. For each node in the input graph, we create a cluster of nodes, one node representing a color the original node is allowed to have. For every arc in the input graph, we connect all nodes from the cluster of its source node to all compatible nodes of the cluster of its target node. Nodes are compatible if the colors they represent are different. This creates a structure as shown in Figure 3.2, which in turn can be transformed to a VNMP-S instance (only considering mapping and delay constraints). Thus VNMP-S is \mathcal{NP} -hard and the \mathcal{NP} -completeness of VNMP-O follows accordingly.

The \mathcal{NP} -completeness of VNMP-O is another reason why we defined additional resource costs C_a . We cannot expect that heuristic methods will always be able to find a valid solution to a VNMP instance if such a solution exists. Therefore, we need a way to guide the search process towards valid solutions, which is achieved by trying to minimize C_a . Another complication with respect to the delay constraints concerns the creation of a (not necessarily valid) solution to a VNMP instance. Previously, we have stated that just adding CPU and bandwidth resources is enough to ensure the existence of a feasible solution to a VNMP instance *if there is a mapping of the virtual nodes such that the delay constraint of all virtual arcs can be satisfied*. If we want to construct a feasible solution, we need to check if such a mapping exists and then use it to define the solution. Now we have shown that finding the mapping is \mathcal{NP} -complete, so we actually missed our goal that by allowing to buy additional resources, finding a feasible solution to an arbitrary VNMP instance will be easy (i.e., polynomially solvable). We circumvent this problem by using VNMP instances that allow delay feasible implementing paths for every mapping configuration (see Chapter 5). Chapter 12 contains techniques for dealing with delay changes, so that we do not need to restrict the structure of VNMP instances. Also note that using VNMP

instances that guarantee delay feasible paths for any mapping does not mean that the graph coloring aspect of VNMP can be ignored. It is very much relevant for VNMP-S and VNMP-O, since we want to find a solution with $C_a = 0$, which means that some delay feasible paths may not be allowed anymore due to bandwidth or CPU constraints.

There is one further problem caused by delay constraints, which occurs when trying to find implementing paths. When trying to find implementing paths for virtual arcs, it is easy to find a path that is valid with respect to the delay constraint by just using a shortest path calculation (with the delay used as distance). However, we are usually not interested in the shortest path, we want to find a good path that satisfies the delay constraint, for some definition of "good". If we want to find implementing paths such that the final VNMP solution is valid, one approach could be to use arcs where only few bandwidth resources have been used. Trying to find a path with the least amount of used resources on the arcs that satisfies the delay constraint is a Resource Constrained Shortest Path Problem, which is \mathcal{NP} -complete [125]. So even when we apply heuristics for finding solutions to a VNMP instance, we have to solve \mathcal{NP} -complete problems. Luckily, the Resource Constrained Shortest Path Problem can be solved in pseudo-polynomial time by Dynamic Programming approaches, for example the one presented in [69], if the arc costs are non-negative.

3.7 Ranking

For a solution to a VNMP instance we have defined two properties: its substrate usage cost C_u and its additional resource cost C_a . When solving VNMP-S, there are no complications. Depending on the variant, we either set $C_a = 0$ and try to find a valid solution, or we try to minimize C_a . Given different solutions, we also need to be able to determine which is the best. Again, this is straight forward for VNMP-S as the solution with lowest C_a is the best. In addition, the difference in C_a also gives us a measure of how much a solution is better than another one.

For VNMP-O, the situation is more complicated, since we also care about C_u . In effect, we have a lexicographic objective measure. When comparing two solutions, the solution with lower C_a is better. Only if those costs are equal, C_u decides which solution is better. For one particular instance, this still allows to decide which solution (and therefore the algorithm that created it) is better. The problem arises when we want to analyze the average performance across multiple instances. In the case of VNMP-S, we can just compare average C_a values. For VNMP-O, comparing average C_u values and declaring the algorithm with the lowest value the best is not a valid approach, since it is very easy to reduce C_u by increasing C_a . Therefore, we need a representative value that we can compare. An often used approach is to calculate this value by $KC_a + C_u$, with K greater than the maximum usage cost. This would work and also give a measure of how much an algorithm is better than another one. The main downside with this approach is that it places a huge emphasis on valid solutions, more than we deem reasonable. Assume we have two algorithms for VNMP-O to compare and ten different instances. One algorithm ignores C_u and creates ten valid solutions which utilize the complete substrate (and therefore incur the maximum C_u). The other algorithm finds very good (i.e., valid with low C_u) solutions for nine instances but fails to find a valid solution for one instance. The first algorithm would seem to be better according to the representative value, even though there are good reasons to prefer the second one in practice.

Because of this, we chose to use another approach: ranking. Given different algorithms to compare and a set of VNMP instances, we can order the algorithms, for each instance separately, according to the solutions they achieve. Based on this order, we assign ranks. The algorithm with the best solution gets rank zero, the second best rank one and so on. If two algorithms create the same result, they share the same rank. Now every algorithm has a rank R for each VNMP instance. Average ranks cannot be meaningfully compared because the total number of ranks is (potentially) different for every instance. Achieving rank one out of two ranks means that this algorithm achieved the worst result, but achieving rank one out of ten means that this algorithm achieved nearly the best result. So we need to normalize the rank by dividing it by the highest assigned rank. The created value is the relative rank R_{rel} . If $R_{rel} = 0$, the algorithm achieved the worst result. If all algorithms created the same solution (which would cause a division by zero), we assign a relative rank of zero to all algorithms. Average relative ranks across multiple instances can be compared in a meaningful way. If an algorithm achieves an average relative rank of 0.2, that means its results are among the top 20% on average. The relative rank R_{rel} will be the main metric for comparing results in this work.

The relative rank is not without its problems. The first problem is that we lose all information about how much better one solution is compared to another. Assume we have two algorithms (A and B) and two VNMP instances. Both algorithms create valid solutions for both instances. For the first instance, A's solution has half the substrate usage cost of B's. For the second instance, B's solution costs one monetary unit less than A's. Both algorithms have an average relative rank of 0.5, but clearly algorithm A should be preferred. The second problem is that the relative rank of an algorithm depends on the algorithms that are being compared. Therefore, Appendix A contains the full results of all algorithms that are going to be presented in the following chapters to allow comparison with future VNMP solution methods.

3.8 Extensions

The model of Virtual Network Mapping as presented in this chapter is greatly simplified. In this section, we are going to outline some possibilities for refining the model to better match it to the real world.

One extension could be to model virtualization overhead, i.e., depending on the number of virtual nodes mapped to a substrate node we have an additional demand for CPU resources caused by the overhead of managing the different virtual nodes. Also the delay behaviour can be improved. As a first step, instead of assuming a constant delay on an arc, we could use a normally distributed delay. Instead of a hard upper bound, virtual arcs could specify a delay limit and the maximum allowed probability of exceeding this delay. However, this is still a very limited model. The delay of network connection depends on its utilization, so a relation between used bandwidth and delay of a connection could be added. However, the relationship between delay and utilization is highly non-linear [135] and incurred delay is actually mostly a property of the routing nodes and not the links between them. Even the used hypervisor influences the delays [177]. Whether a more realistic delay model generates tangible benefits in practice remains to be seen. There is research [188] that suggests that the higher statistical moments (skewness, kurtosis, ...) are also important to characterize the behaviour of a link with respect to delay. For a valid VNMP solution, we require that we have exactly one implementing path for each virtual arc. It is possible to relax this constraint and allow an arbitrary number of paths. Assuming we already have fixed a particular mapping and relaxing the delay constraint, finding valid paths for the virtual arcs subject to the bandwidth and CPU constraints is a Multi-Commodity Flow Problem with fractional flows. This problem can be solved in polynomial time by Integer Programming or if fast solutions are required by approximation schemes [101]. Allowing multiple paths was also considered for example in [183]. For this work, we chose the restriction to a single path due to practical reasons. If we allow multiple paths, then the observed behaviour of the virtual connection will be much more erratic in terms of delay and probably harder to deal with by the employed protocols in the virtual network.

One additional enhancement to the model could be to allow redistribution of bandwidth between the directions of a connection between two substrate nodes. That would capture the flexibility inherent especially in fiber optic connections. Different wavelengths can be configured to be either used in sending or receiving direction. In this case it would also be possible to simplify the substrate network to an undirected graph where edges have a specific bandwidth capacity that can be used in arbitrary directions. As a downside, we would lose the ability to model cases when the distribution between sending and receiving capacity is fixed, for instance if the substrate resources are only rented.

We have now outlined some possibilities of modeling Virtual Network Mapping closer to the real world by including behaviours that can be observed in practice. However, we neglected an important point: Virtual Network Mapping is not a static process. For the VNMP we assume that we know all virtual networks and the available substrate network in advance. In practice, virtual network mapping will be very dynamic, new virtual networks will have to be added to the substrate network while other virtual networks are removed. Already present virtual networks might need to be reconfigured because their resource requirements have changed. Of course, also the substrate network is subject to modifications due to failures, maintenance, or because resources can be rented somewhere else much cheaper and we want to move the virtual networks there.

With all those sources of uncertainty, it might be useful to consider finding solutions that are robust when the environment changes [11, 14]. For instance, we could try to find a selection of resources in the substrate, which allows for a valid solution even if an arbitrary node or arc of the selection fails. Alternatively, we could try to find a selection that allows adding another virtual network (at least with high probability) without having to use additional parts of the substrate.

3.9 Summary

In this chapter, we have presented the definition of the Virtual Network Mapping Problem, VNMP. Virtual networks require CPU resources on the nodes, bandwidth resources on the arcs and have a maximum allowed data transfer delay. The physically available network (the substrate) offers CPU resources on the nodes, the arcs have data transfer capacities and delay is incurred when data is transferred across them. In addition, virtual nodes can only be hosted on a

Input		Description		
G(V, A)		Substrate graph		
$c_i \in \mathbb{N}^+$	$\forall i \in V$	Available CPU resources on substrate nodes		
$b_e \in \mathbb{N}^+$	$\forall e \in A$	Available bandwidth on substrate arcs		
$d_e \in \mathbb{N}^+$	$\forall e \in A$	Incurred delay on substrate arcs		
$p_i^V \in \mathbb{N}^+$	$\forall i \in V$	Cost of using a substrate node		
$p_e^A \in \mathbb{N}^+$	$\forall e \in A$	Cost of using a substrate arc		
$p^{\mathrm{CPU}} \in \mathbb{N}^+$		Cost of one additional unit of CPU capacity		
$p^{\mathbf{BW}} \in \mathbb{N}^+$		Cost of one additional unit of bandwidth capacity		
G'(V', A')		Virtual network graph		
$c_k \in \mathbb{N}^+$	$\forall k \in V'$	Required CPU resources of a virtual node		
$b_f \in \mathbb{N}^+$	$\forall f \in A'$	Required bandwidth of a virtual arc		
$d_f \in \mathbb{N}^+$	$\forall f \in A'$	Maximum allowed delay for the implementation of a virtual arc		
Ň		Set of allowed mappings of virtual nodes to substrate nodes		

Table 3.1: Input of the Virtual Network Mapping Problem.

Table 3.2: Output of the Virtual Network Mapping Problem.

Output	Description
$m:V'\to V$	Mapping of each virtual node to a substrate node
$p_f, \ \forall f \in A'$	Implementing path for each virtual arc
C_u	Substrate usage cost
C_a	Additional resource cost

specific set of substrate nodes. The aim is to find a mapping of virtual nodes to substrate nodes and a simple path in the substrate for each virtual arc subject to the resource constraints such that the operational costs are minimized. Every substrate node that is used to host virtual nodes incurs costs, as does every substrate arc that is used to implement a virtual arc. The sum of those costs is the substrate usage cost C_u . Just finding a solution that satisfies all constraints is \mathcal{NP} -complete. Therefore, it is allowed to buy additional resources so that an arbitrary solution can be made to satisfy all resource constraints. The cost of the additional resources is the additional resource cost C_a . We call a VNMP solution valid if no additional resources are necessary ($C_a = 0$). By VNMP-S we denote the problem of finding a valid solution, VNMP-O is the problem of finding a valid solution with minimal operational costs. Table 3.1 gives a summary of the input for the VNMP, Table 3.2 summarizes the output of the VNMP.

CHAPTER 4

Related Work

4.1 Introduction

In this chapter, we present an overview of the available literature on Virtual Network Mapping and relate it to our approach. Work on the VNMP can also be found under the names Virtual Network Assignment [187], Virtual Network Embedding [30], Virtual Network Resource Allocation [164], and Network Testbed Mapping [148]. The different names are a result of slightly different application scenarios. Network Testbed Mapping for instance deals with the problem of embedding virtual networks into a network testbed to share it among different research groups and their experiments. Even though the core problem is always the same, i.e., mapping virtual networks into a physical network, there is a huge diversity in the details.

In Section 4.2, we will cover the basic setup of the related work in terms of employed network model, size, and structure of virtual and substrate networks. The types of resources that have been used will be discussed in Section 4.3. In Section 4.4, we consider the different possibilities with respect to the objective. The test methodology will be the main focus of Section 4.5 and Section 4.6 presents the different solution methods employed for solving the VNMP. We conclude in Section 4.7.

4.2 Network Models

In this work, we use directed graphs to represent substrate and virtual networks. From the sampled related work, only one [164] employs directed graphs. The vast majority [30, 48, 71, 82, 139, 145, 148, 175, 180, 185–187] uses undirected graphs. In between lies the work of Lu and Turner [117], who consider undirected substrate networks but directed virtual networks. Unfortunately, there is never a reason stated as to why directed graphs were rejected in favor of undirected graphs (or vice versa). We chose directed graphs, since they allow for asymmetric resource requests, which appears to be very meaningful in practice. For instance in some video streaming applications a virtual server node a may send a virtual node b a lot of data, while

b sends only small amounts to *a*. Also, substrate connections may have different properties depending on direction, which cannot be modeled by undirected graphs. The one advantage of undirected graphs is that they can capture the situation when the Virtual Network Operator owns the substrate resources and is able for instance to split the capacity of a network link arbitrarily between the two directions. The directed model we employ could handle this with additional constraints, but we do not explore this possibility further.

The employed substrate networks are mostly synthetic, only [117, 164, 185] use substrate networks based on real topologies. Considering the sizes of the employed networks, they can be split roughly into three size classes: 10–50 nodes, 100 nodes and more than 100 nodes. Into the smallest class fall [164] (15 and 27 nodes), [117, 185] (20 nodes), [180] (40 nodes), and [30] (50 nodes). Note that every work using realistic substrate networks falls into this class. The representatives of the medium size class with 100 nodes are [48, 82, 145, 175, 187]. For the largest size, we have the work presented in [186] with 150-250 nodes and [148] with 120 and 525 nodes. The synthetic substrates are random graphs, where the probability of an edge (or arc) between two vertices was chosen either to be 50%, or less commonly 40%. As a result, the substrates are rather dense. For our work, we use substrates based on real topologies covering sizes from 20 to 1000 nodes.

The used virtual networks are usually very small synthetic random graphs with 50% chance of an edge between two nodes. The exception are the virtual networks used in [164], which are complete graphs with 2-4 nodes. Most work [30, 48, 175, 180, 185–187] uses random sizes between 2 and 10 nodes. The largest virtual networks we encountered were 2 to 20 nodes in [145], 25 nodes in [82] and 10 to 100 nodes in [148]. The only work considering realistic virtual network loads is [148]. In this case, realistic means that the topologies have been requested within a network testbed. Our work will use virtual networks between 5 and 30 nodes. In addition, they are not random graphs but rather try to capture different types of applications and their requirements with respect to topology and resources. Details will be presented in Chapter 5.

4.3 **Resources and Constraints**

Virtual networks have an impact on the substrate network. They require resources and only a finite number of virtual networks can fit into a substrate network. This is (nearly) universally acknowledged among the sampled work. The main differences lie in the considered resources.

The work presented in [187] does not use resources at all, but rather the count of virtual nodes mapped to a substrate node (node stress) and the count of virtual arcs crossing a substrate arc (link stress). The node and link stresses are not allowed to exceed a certain value. The remaining work at least considers the available bandwidth on substrate arcs, [71, 117, 164] exclusively use bandwidth as resource.

The next logical resource to consider is the CPU capacity available at the substrate nodes, which is consumed by the virtual nodes. The work presented in [30, 139, 145, 148, 175, 180, 185] considers CPU and bandwidth resources.

In addition to resource restrictions, limiting the mapping possibilities of virtual nodes is popular. One method to implement this is to assign a location to substrate nodes and virtual nodes. Virtual nodes may only be mapped to substrate nodes not too far distant. This approach is employed in [30, 117, 186]. Another way of limiting the mapping possibilities is to forbid that a substrate node hosts multiple virtual nodes of the same virtual network [48, 180, 187]. An in-depth discussion about the possibilities of restricting virtual node placement can be found in [180]. The authors mention location preference constraints, which means that a virtual node may only be mapped to a specific set of substrate nodes, location exclusion, which means that a virtual node is not allowed to be mapped somewhere and location separation, which means that two virtual nodes may not be mapped on the same substrate node or otherwise close together (e.g., on servers in the same rack).

Some less usual resources and restrictions are also covered in the literature. The power state (similar to the notion of a used substrate node in this work) is considered in [186]. Some nodes cannot host any virtual nodes in [164]. In [185], virtual nodes may be split up and mapped to multiple substrate nodes. This parallelization causes an additional overhead. The authors of [48] consider the available memory at the substrate nodes as additional resource. As a further extension, they consider the possibility of "overbooking" bandwidth resources on substrate arcs, since virtual arcs will not require their specified bandwidth constantly. A limit is placed on the probability that the bandwidth capacity of a substrate connection is exceeded. In [71, 117], the virtual networks are not even fixed. Given are data sending and receiving rates at the virtual nodes and the topology of the virtual network has to be created such that every traffic pattern conforming to these rates is possible. The authors of [71] also consider the restriction that the virtual network has to be a tree.

We have already mentioned in the previous chapter that an argument for allowing multiple paths to implement a single virtual arc can be made. This approach is used in [30, 164, 180, 183]. On the topic of the implementing paths, some work (e.g., [117, 145]) implicitly focuses on shortest paths, instead of allowing arbitrary implementations. For instance, the algorithm presented in [145] will reject a virtual network, if one of its virtual arcs cannot be implemented by using one of the $k \leq 6$ shortest paths. Longer paths which may have enough free resources are not considered. In tests with physical networks, it has been concluded that in 25% of cases the shortest path (with respect to hops) is not the shortest path with respect to transmission delay [171], so focusing only on shortest paths is really a restriction that might hurt the final solution quality. The VNMP version considered in this work utilizes CPU, bandwidth and delay as main resources. From the sampled work, only the authors of [82] and [10] considered delay as factor for virtual network mapping. However, in the model proposed in [82], virtual networks cannot specify delay constraints and furthermore it is assumed that resources are unlimited. In [10], only the mapping of virtual nodes is considered, together with communication delays for the users of the virtual nodes. No virtual connections have to be implemented.

As location constraints we allow only a specific set of substrate nodes as host for each virtual node. We do not forbid the mapping of two virtual nodes from the same virtual network to the same substrate node, because if it is the cheapest way to implement a virtual network, why should it be forbidden. If two virtual nodes of a network are not allowed to be hosted on the same substrate node, this can be realized by choosing two disjoint sets of substrate nodes as possible hosts. Only if the sets of possible nodes cannot be disjoint, our employed method for specifying location constraints fails to capture this restriction. Restrictions based on distance between virtual node and substrate node can be realized by our model. For paths implementing

virtual arcs, we consider all paths that are short enough (with respect to delay), not only the shortest ones. We only use one resource on nodes, the CPU capacity. We chose not to consider other resources on the nodes, since they are basically the same as the CPU capacity and do not require any special handling. In fact, an additional resource on the nodes would be easier to handle than the CPU capacity in our model, since routing of data also requires the CPU. This causes a much tighter coupling between node mapping and arc implementing than in other work.

4.4 Objectives

The main objectives used for Virtual Network Mapping are either maximizing revenue or minimizing cost. The work published in [139, 145, 175, 185] focuses on maximizing revenue. Virtual networks with high resource demands generate high revenue.

The cost of a virtual network implementation is usually the sum of resources used in the substrate, which is used (with some variations) in [30, 71, 82, 148, 164, 180, 187].

We have already mentioned the work of Fajjari et al. [48], where it is allowed to exceed the bandwidth capacity of a substrate arc with a certain probability. Two objectives are used, maximizing the acceptance rate (fitting as many virtual networks as possible to maximize revenue) and minimizing the probability of exceeding the available capacity.

In [186], another interesting objective is used: minimizing the electricity cost. This is similar to the usage cost we use in this work (not using a substrate component means it can be shut down, thus saving on electricity other operational costs), but exceeds our approach by also considering the time dependence of the cost of electricity.

4.5 Testing Methodology

Most algorithms presented for the VNMP are tested in an online setting. That means that a substrate network is given and random virtual networks arrive over time that have to be mapped. These virtual networks also have a limited lifetime, after which they are removed from the substrate. As a representative of this methodology, we present the test setup as used in [175]. A single substrate network is used. Virtual networks arrive in a Poisson process with an average rate of 5 requests per 100 time units. Each request has an exponentially distributed lifetime with a mean of 1000 time units. The simulation is run for 50000 time units. The main performance metric is the acceptance ratio, i.e., how many of the arriving virtual networks can be mapped into the physical network. Also the work presented in [30,48,139,145,164,180,185,186] follows this evaluation style. It is important to note that the mapping algorithms only see one virtual networks are mapped to the substrate network. However, some methods allow to store a request for some time to wait for resources to become available.

In [187], the alternative of changing already mapped virtual networks is considered. This work also contains a discussion of the associated costs when changing a virtual network implementation (migrating virtual machines,...). However, also in this case the presented algorithm only considers one virtual network at a time. It could be shown that reconfiguring already present virtual networks leads to better performance.

Unfortunately, when this testing setup is used, the total number of virtual networks present in the substrate is not reported. Based on the available resources in the substrate networks and required resources by virtual networks, we estimate the total number of virtual networks to lie between 10 and 50. Another, in our opinion interesting, parameter that is usually not reported is the time needed to map a virtual network. In [82], a required time of 1 second was mentioned.

The work presented in [117, 148] uses an offline approach, where the total load of virtual networks is known from the start. Only the algorithm presented in [148] makes use of this information, the algorithm from [117] maps the virtual networks iteratively. A completely different type of evaluation is performed in [82], where the proposed distributed algorithm is simulated by means of a multi-agent system.

In our evaluation, we will focus on the offline case, which means we know which virtual networks need to be implemented. Every heuristic and exact approach we will introduce in the following chapters will deal with all virtual networks at once. In some sense, these methods can be viewed as complementary to the methods presented in the literature. While they concentrate on immediately answering virtual network requests, and by necessity have to focus on keeping the implementations of the virtual networks spread across the substrate so that critical resources remain available for future networks, our algorithms deal with the global view and can optimize the virtual network implementations to reduce operating costs. Instead of a single substrate network, we use 210 networks spanning 7 size classes and up to 40 virtual networks to evaluate the performance of our algorithms.

4.6 Solution Methods

In the previous section, we have shown that the approaches from the literature are meant to be employed in an online setting, which means they have to be able to decide nearly immediately if a virtual network is accepted or not. As a consequence, most of the presented methods are Construction Heuristics [48, 139, 145, 175, 185–187], with quite involved strategies for determining which virtual node or arc should be implemented next. The algorithm presented in [139] for instance uses Bayesian network analysis to improve the node selection strategy. Another interesting observation is that some methods, even though they only have to implement a single (usually small) virtual network, apply decomposition techniques to the virtual network and then concentrate on implementing the decomposed parts [48, 187].

The first step away from Construction Heuristics is the algorithm presented in [48], which in addition to greedy rules also utilizes backtracking to find good solutions. In [148], simulated annealing is utilized, in [117] Local Search.

The methods published in [30, 164, 180] are based on solving multi-commodity flow problems, which is possible since implementing virtual arcs with multiple paths is allowed in their version of Virtual Network Mapping. The algorithm from [164] uses a heuristic approach, while [180] uses Integer Linear Programming. The method presented in [30] uses the LP relaxation of the Virtual Network Mapping Problem to derive the mapping via rounding procedures and solves the multi-commodity flow problem afterwards. A distributed algorithm is presented in [82].

In this work, we will cover exact and heuristic approaches to solve the VNMP. In contrast to the work in the literature, we will focus less on execution speed and more on solution quality.

4.7 Conclusion

The VNMP as solved in the literature could be summarized as follows: Given is an synthetic undirected substrate network with 50 nodes and a capacity of about 30 virtual networks. From a continuous stream of synthetic virtual network requests with a size of 2 to 10 nodes, select those that can be implemented and implement them. The VNMP as solved in this work may be summarized as follows: Given are hundreds of realistic directed substrate networks from 20 to 1000 nodes in size and a capacity of at least 40 virtual networks. For each of those networks, implement at most 40 realistic virtual networks with a size between 5 and 30 nodes.

This already shows some areas in which we go beyond what is usually considered in the literature. We deal with a sizable set of substrates derived from networks occurring in the real world. Also the virtual networks are not synthetic but based on the requirements of different applications. Instead of CPU and bandwidth capacities, which are independent of each other, we consider an interaction between them. CPU capacity has to be used at substrate nodes to transfer data. In addition, we allow virtual arcs to specify delay constraints for their implementing paths. In contrast to the majority of the works in literature, we focus on the offline VNMP. This allows us to consider more powerful solution approaches, which would require too much run-time in an online setting.

Some interesting approaches from the literature go beyond what we will do in this work, for instance [186] with its complex operational costs model or [71, 117], where the virtual network structure has do be derived based on a set of traffic demands. We also do not consider the possibility of splitting up the workload of a virtual node as done in [185].

CHAPTER 5

Towards a Realistic VNMP Benchmark Set

5.1 Introduction

In this chapter, we describe how we created the VNMP instance set we use to evaluate different solution methods. Our design goals during the development of the instance set were the following:

- Realistic substrate networks with a structure comparable to networks as they occur in the real world, i.e., no random or structured (e.g., Hub-and-Spoke) graphs.
- Wide range of substrate network sizes so that we have instances which can be solved by exact methods, but also instances for which heuristic methods are required.
- Virtual networks that cover different applications or application classes with a diverse set of requirements with respect to resources but also with respect to structure.
- Mapping restrictions that make sense in terms of the substrate network, i.e., not just random locations, but locations that cover specific parts of the substrate.
- Guaranteed existence of a valid solution.
- A way other than substrate network size to control the "hardness" of an instance. Put differently, a way to go from an instance where finding a valid solution is easy and optimality is hard to an instance where finding a valid solution is already a challenge.
- Any solution satisfying the mapping constraints can be made to satisfy the resource constraints by buying more CPU and bandwidth resources.

In the following sections we describe how we built the substrate and virtual networks to conform to these goals.

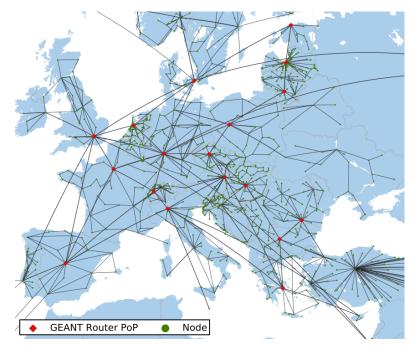


Figure 5.1: Topology map of national research networks and their interconnections by the GÉANT backbone network, as published in [106].

5.2 Substrate

Our main aim with respect to the substrate networks was that they have to have a realistic structure. Therefore, they need to be derived from real, measured topologies of the Internet, which are available in the literature. Candidates include the topology maps from the Rocketfuel project [163], the Scan-Lucient map (which contains the union of the topologies measured by the SCAN [70] and Lucent [25] Internet mapping projects) or the survivable network design library (SNDlib) [133]. In earlier work [88], we used the Rocketfuel and Scan-Lucient maps to derive substrate networks. The disadvantage with those topologies is that the Rocketfuel maps are rather small (about 100 nodes at most) and the Scan-Lucient map, while huge, is so sparse that most subgraphs are trees. The topologies of the SNDlib mostly do not exceed 50 nodes in size. Due to these reasons, we decided to look elsewhere for suitable substrate networks.

In the end, we decided to generate the substrate networks from a network of the Internet topology zoo [105], the European Research and Education Network GÉANT [106]. It contains national European research networks which are interconnected by the European Backbone Research Network. Figure 5.1 shows the structure of the complete network, the Points-of-Presence (PoPs) of the GÉANT network and their interconnects. For more details on how this topology was derived see [106].

This network proved to be very suitable for our needs. First of all, it contains 1157 nodes, which allows the creation of a wide range of substrate network sizes. Secondly, it associates with each node a geo-location (i.e., the county in which this node is located), which is very useful

for defining meaningful mapping constraints. The network also contains information about the capacity of different links. We did not utilize this information because it ranges across several orders of magnitude which is unsuitable for creating benchmark instances.

To derive substrate networks from the GÉANT map, we used the network manipulator nem-0.9.6 [120, 121]. Nem can extract sub-graphs of a given network that retain the main characteristics of the network, if the requested sub-graph size is small enough. If the target size exceeds 30% of the source network size, another approach is necessary. We selected a subset of the GÉANT map by choosing a random node from the map and then iteratively grew the selection by adding random nodes which are connected to the already selected part until the target size was reached. Since the GÉANT network is undirected, we added connections in both directions to the substrate graph. We created 30 substrates for each of the following sizes: 20, 30, 50, 100, 200, 500, and 1000 nodes. Up to (and including) size 200 nem was used. The costs p_i^V and p_e^A were assigned uniformly at random from [1, 50] and the arc delays d_e uniformly at random from [5, 40]. The other properties of the substrate (CPU and bandwidth capacity) are assigned based on the demand by the virtual networks that are added to the VNMP instance because we need to make sure that a valid solution exists. The detailed method will be discussed at a later point in this chapter.

5.3 Virtual Networks

Our main design goal for virtual networks was that they reflect different use-cases with a diverse set of resource demands. To fulfill this goal, we designed four different archetypes of virtual networks. After the creation of a substrate network as outlined in the previous section, we added ten instances of each of the four virtual network types to define a VNMP instance. The size of each virtual network was chosen uniformly at random from $[5, \min(30, 0.3 * |V|)]$. These bounds ensure that on one hand, the created virtual networks are not too small and on the other hand that they are significantly smaller than the substrate network into which they have to be mapped. Now follows a discussion of the four different virtual network archetypes.

5.3.1 Stream Network

This virtual network type models (video-)streaming services and has a random tree structure. The root node represents a streaming service which broadcasts its content consisting of multiple channels. Intermediate nodes in the distribution tree split this stream and forward only the channels that are actually watched by the customers connected to the leaf nodes of the tree. This is an example of an application that can directly benefit from custom routing protocols, as this splitting and forwarding of data is currently not possible within the network. A streaming service sends between 10 and 20 channels, each requiring between 2 and 10 units of bandwidth, which covers audio and video streaming. Every intermediate node forwards between 30% and 100% of received channels to each of its children, making sure each channel is forwarded at least once and each child receives at least one channel. The required bandwidth of each virtual arc is the product of transferred channels and the bandwidth requirement per channel. The required CPU power on each virtual node is three times the received bandwidth which models the required

processing power for unpacking and redistributing the stream. For the root node, the required CPU power is three times the sent bandwidth. Streaming applications are not delay constrained in our model, so we set the delay requirement of each virtual arc to the highest shortest path delay within the substrate network. As for the allowed mapping of the virtual nodes, we create groups of substrate nodes for the root and all leaf nodes. The node grouping procedure works as follows: Initially, substrate nodes are grouped according to their geo-location. If this creates more groups than required, some are randomly deleted so that the required number of groups is reached. Otherwise, we generate a new group by randomly moving half of the nodes of the largest group (rounding down in case of odd-sized groups) to this new group. New groups are generated until the required number of groups is reached. The location of the root node of the Stream virtual network is fixed to one of the nodes of its group. The leaf nodes can be mapped to any substrate node in their group. Intermediate nodes in the streaming tree can be mapped to any node in any group.

5.3.2 Web Network

This virtual network type models the typical web usage and has star structure. The center represents the web-server which serves the connected customers. Each customer receives between one and five units of bandwidth, the processing of which requires one unit of CPU power per received unit of bandwidth. The CPU requirement of the web-server is the sum of the transmitted bandwidth. As for the allowed mapping, the node grouping procedure is used to create mapping target candidates for each node of the virtual network. The virtual node of the web-server can only be mapped to one randomly selected substrate node of its group. All other nodes can be mapped to two randomly selected nodes of their group. As delay requirement for each arc (which connects the web-server node to one of the customer nodes) we set the smallest possible delay with which both possible locations of the customer node may be reached from the location of the web-server node. This models hard delay requirements, since users usually expect web-pages to load fast.

5.3.3 Peer-to-Peer Network

This virtual network type represents Peer-to-Peer (P2P) networks and is based on a directed small world graph [98, 149]. P2P applications do not have delay requirements (in our model), so the delays are set as they are for the Stream virtual networks. However, they have high bandwidth requirements; each arc requires between 10 and 70 units of bandwidth. The CPU requirements on each virtual node are twice the outgoing bandwidth which models compression or encryption services offered by this network type. As for the allowed mapping, the node grouping procedure is used to create groups for each of the nodes of the P2P network. Every virtual node is allowed to be mapped to any substrate node of its group.

5.3.4 Voice-over-IP Networks

This virtual network type models Voice-over-IP (VoIP) networks and is, like the P2P type, based on a directed small world graph. It requires 2 to 20 units of bandwidth for each virtual connection

and three times the outgoing bandwidth as CPU power on each virtual node, which models video compression and transcoding services. The virtual nodes can be mapped to any substrate node of their substrate node group created by the node grouping procedure. The delay requirement of a virtual connection f for this virtual network type is set to the lowest possible value such that a delay constrained path between every location of s(f) and t(f) in the substrate exists.

To sum it all up, Stream networks have high bandwidth and CPU requirements but are barely delay constrained. Web networks on the other hand have very low bandwidth and CPU requirements but stringent delay constraints. P2P networks have high bandwidth and medium CPU requirements and lax delay constraints. VoIP networks have medium bandwidth and CPU requirements and are moderately delay constrained. P2P and VoIP networks also have a complex structure.

At this point, we have built a VNMP instance by creating a substrate network and assigning costs and delays to it. Then we added ten virtual networks for each of the four different network types together with their mapping constraints. To clarify, the 40 different networks are combined into one graph, the virtual network graph G'. The only missing component are the bandwidth and CPU capacities in the substrate network. We assign them by creating a random solution to the VNMP instance as it is now and use it as a guide to define the available resources.

As the first step for creating the guiding solution, we map each virtual node to one of its allowed locations in the substrate. Note that due to the chosen delay limits of the virtual networks, it is guaranteed that implementing paths satisfying the delay constraint exist for any mapping of the virtual nodes. To set the implementing paths of the solution, we could just use delay shortest paths. However, that would introduce too much structure into the solution, meaning that it could be found easily based on the final VNMP instance. Therefore, we construct implementing paths by solving a resource constrained shortest path problem. The delays within the substrate are used as resource, as lengths we set random values between one and ten for every substrate arc. The distance values are set differently for every path calculation to increase the randomness of the final solution. Based on the created solution, we assign CPU and bandwidth resources available in the substrate so that the solution is valid. Some nodes or arcs may not have been assigned resources. Substrate nodes get assigned the average amount of CPU resources (that have already been set) within their geo-location. If all nodes of a geo-location have not been assigned CPU resources, they are assigned the average amount of assigned CPU resources within the whole substrate network. The same process is applied to assign missing bandwidth resources. Arcs within a geo-location get assigned the average of the location, if no arc has been assigned resources the average bandwidth resources within the substrate are assigned. Arcs connecting different geo-locations get assigned the average bandwidth of arcs that have been assigned a bandwidth capacity and cross a geo-location. Note that it is not possible that no arc connecting different geo-locations has been assigned a bandwidth capacity based on the guiding random solution. Now all resources have been assigned. As a final step, we increase the CPU and bandwidth capacities at each node or arc separately by 20% to 50% so that there is more room for optimization.

One design goal is still unfulfilled: a way to control the "hardness" of a VNMP instance. We do this by modifying the number of virtual networks that have to be mapped, which we will denote by the load of an instance. The instances created by the outlined procedure have a load of 1, or

Table 5.1: Properties of the VNMP instances: average number of substrate nodes (V) and arcs (A), virtual nodes (V') and arcs (A'), number of allowed mapping targets for each virtual node $(M_{V'})$ and the average total usage costs (C_u) .

Size	$ \mathbf{V} $	$ \mathbf{A} $	$ \mathbf{V}' $	$ \mathbf{A}' $	$ \mathbf{M}_{\mathbf{A}'} $	$\mathbf{C}_{\mathbf{u}}$
20	20	40.8	220.7	431.5	3.8	1536.0
30	30	65.8	276.9	629.0	4.9	2426.6
50	50	116.4	398.9	946.9	6.8	4298.1
100	100	233.4	704.6	1753.1	11.1	8539.1
200	200	490.2	691.5	1694.7	17.3	17584.2
500	500	1247.3	707.7	1732.5	30.2	44531.8
1000	1000	2528.6	700.2	1722.8	47.2	89958.4

full load. For lower loads, some virtual networks are removed. At load 0.9, one virtual network of each type has been removed. At load 0.1, the VNMP instance contains four virtual networks, one of each type. Based on the created 210 VNMP instances of full load (30 instances for seven different substrate network sizes), 1890 additional instances can be derived by reducing the virtual network load (0.1 to 0.9), yielding a total of 2100 VNMP instances. The created instances are available at [87].

5.4 Main VNMP Instance Properties

In this section, we show the main properties of the created VNMP (full load) instances, which are presented in Table 5.1. It can be seen that the created substrate graphs are very sparse. For instances of size 20 (which means that the substrate network contains 20 nodes), we observe an average of 40.8 substrate arcs. For a connected substrate, the lowest possible number of arcs is 38, which is twice the number of edges required to form a tree. This number is doubled since due to the construction of the substrate network, if two nodes are connected, they are connected in both directions. With rising instance size, the substrate networks also get marginally denser. The main point to note with respect to the virtual networks is that the number of virtual nodes to map and the number of virtual arcs to implement stays roughly constant from size 100 onward, because each virtual network has a size limit of 30 nodes. Even if the number of virtual nodes and arcs stays the same, with rising instance size we can expect that the required implementing paths for each virtual arc get longer, which complicates the problem of finding cheap and valid solutions. In addition, for larger instance sizes there are far more mapping possibilities for each virtual node. Table 5.1 also shows the complete substrate usage costs for reference, i.e., how much it would cost to use every part of the substrate network.

CHAPTER 6

Construction Heuristics, Local Search, and Variable Neighborhood Descent

6.1 Introduction

In this chapter, we will introduce Construction Heuristics (Section 6.2), Local Search (Section 6.3), and Variable Neighborhood Descent algorithms (Section 6.4) for the VNMP. We present those algorithms together since they depend on each other and also share the property that they terminate by design, i.e., they are finished at some point and do not need to be aborted by a stopping criterion like the elapsed time. The heuristic algorithms we discuss in later chapters do not have this property. The deterministic termination also offers some great opportunities to analyze the tradeoff between required run-time and solution quality, which is more complicated if the run-time is an external parameter to the algorithm. The performance of the algorithms is compared in Section 6.5, Section 6.6 concludes and gives some directions for future work. The results presented in this chapter have been published in a more compact form in [91].

6.2 Construction Heuristics

A Construction Heuristic (CH) is used to create solutions to problems by following heuristic rules that guide the construction process towards feasible solutions of high quality. During each step, a partial solution is extended by the currently most promising component. For the VNMP, we can already see that these are conflicting objectives; guiding towards valid solutions means spreading resource usage across the whole substrate to decrease the probability of having to buy additional resources, which causes C_u to be unnecessarily high. Trying to pack virtual networks densely to keep C_u low will most likely lead to high C_a as some substrate network components run out of resources, so some kind of balancing is required. To find the right balance, we first define a general outline of a CH for the VNMP. This outline defines sub-problems, which can

Alg	gorithm 6.1: Outline of the Construction Heuristic for the VNMP
I	nput : A VNMP instance I
C	Dutput : A solution to I
1 S	olution S(I);
	<i>Suse node emphasis NE</i> then
3	while not all virtual nodes mapped do
4	VirtualNode k=getVirtualNode(I,S); // virtual node selection SVN
5	SubstrateNode i=getMapTarget(I,S,k); // mapping target sel. IVN
6	S.setMapping(k,i);
7	end
8 e	
9 W	hile S incomplete do
10	while virtual arcs are implementable do
11	<pre>VirtualArc f=getVirtualArc(I,S); // virtual arc selection SVA</pre>
12	<pre>Path p=getImplementingPath(I,S,f); // path selection IVA</pre>
13	S.setPath(f,p);
14	end
15	if not all virtual nodes mapped then
16	<pre>VirtualNode k=getVirtualNode(I,S); // virtual node selection SVN</pre>
17	<pre>SubstrateNode i=getMapTarget(I,S,k); // mapping target sel. IVN</pre>
18	S.setMapping(k,i);
19	end
20 e	nd
	eturn S;

be solved by different heuristics. These heuristics can be tuned towards low C_u or low C_a and by selecting the right heuristics for the sub-problems, a CH for the VNMP can be derived that creates valid results with low C_u with high probability.

Algorithm 6.1 shows the outline of the CH. It uses the solution to four sub-problems to build a VNMP solution. The four sub-problems are:

- SVN Selecting a virtual node to map from the nodes that have not been mapped.
- **IVN** Selecting an implementation of the virtual node, i.e., a substrate node to which the virtual node is mapped.
- **SVA** Selecting a virtual arc f to implement. This arc has to be implementable, which means that it is not implemented in the current solution and s(f) and t(f) have to be mapped. Otherwise we would not know, which substrate nodes the implementing path for f has to connect.
- **IVA** Selecting an implementation of the virtual arc, i.e., the path in the substrate network.

In addition to the heuristics used to solve those four sub-problems, there is another parameter which determines the behaviour of the CH. Basically, we can decide if we want to map all virtual nodes before we start to implement virtual arcs (node emphasis NE) or if we implement virtual arcs as soon as they are implementable (arc emphasis AE). We will call this the implementation emphasis. If NE is used, Algorithm 6.1 iteratively selects substrate nodes to implement (SVN) and a mapping target for them (IVN), until all virtual nodes are mapped. Only after that, the main loop of the CH is entered. This loop is executed until the solution is complete, that means every virtual node has been mapped and every virtual arc has an implementing path. In the main loop, we first implement all virtual arcs that are currently implementable by selecting such an arc (SVA) and finding a path for it (IVA). Once no more implementable arcs remain, an additional node has to be mapped to (potentially) make more arcs implementable. The structure of Algorithm 6.1 might seem a bit counterintuitive, because it contains the node mapping twice, which is not strictly necessary. We chose this structure because it can be easily adapted to work with a partial solution as input, which has to be completed. This functionality will be required for the Local Search and Variable Neighborhood Descent algorithms discussed in the following sections. The required modification to Algorithm 6.1 is removing its first line and adding Solution S, the solution to be completed, as second input.

We implemented four heuristics for each of the four different sub-problems to solve, which are described in the following. If the strategies define no specific order of nodes or arcs (or ties occur), it is arbitrary. Now follows the discussion of the SVN heuristics.

NextVNode Selects an unmapped virtual node from V'.

- **CPUHeavy** Selects the virtual node with the highest sum of CPU requirement and connected bandwidth. The connected bandwidth is the sum of the bandwidth requirements of all virtual arcs that start, or end, at a virtual node. It is important to include this factor, since it is a (nearly) guaranteed CPU load caused by the virtual node. It slightly overestimates the CPU load because transferring data from one virtual node to another if both are hosted on the same substrate node requires CPU capacity only once but is counted twice according to this calculation model. We want to focus on virtual nodes that require a lot of resources because they are the most problematic to fit into the substrate network.
- **CPUHeavyVN** Selects with CPUHeavy from the virtual network (VN) with the highest total CPU and bandwidth requirements that still has unmapped nodes left. Concentrating on one virtual network when selecting nodes supports AE, since virtual arcs become implementable much faster.
- **DLHeavyVN** Selects with CPUHeavy from the VN with the lowest total sum of allowed delays that still has unmapped nodes left. This heuristic assumes that VNs that are highly delay constrained are the most critical to implement. Especially in concert with AE, this ensures that virtual arcs with stringent delay constraints are implemented first, when the substrate still has enough resources to allow a delay feasible path without any additional costs in terms of C_a .

Now follows the description of the node mapping heuristics. Note that for the node mapping strategies, only substrate nodes allowed by M are regarded as candidates. If no substrate node would yield a valid solution, i.e., no allowed substrate node has enough CPU resources or connected bandwidth left, then the substrate node (allowed by M) with the most free resources is chosen. This is the substrate node with the least missing resources in case no substrate node has resources left.

NextSNode Maps a virtual node to the first substrate node with enough free CPU capacity.

- **NextFree** Maps a virtual node to the first substrate node with enough free CPU capacity to support the CPU requirements and the total connected bandwidth of the virtual node (the total CPU load of a virtual node).
- **MostFree** Maps to the substrate node with the most free CPU capacity. In case of ties, the substrate node with the most free connected bandwidth is chosen as map target.
- **CheapHost** Maps to the substrate node with enough free resources (with respect to the total CPU load) and least increase of C_u , i.e., if possible to a substrate node that already hosts virtual nodes.

For the following description of virtual arc selection heuristics, keep in mind that the SVA strategies only consider implementable virtual arcs.

NextVArc Selects an arbitrary unimplemented virtual arc.

BWHeavy Selects the arc with the highest bandwidth requirement.

DLHeavy Selects the arc with the smallest delay.

RelDLHeavy Selects the arc with the smallest fraction of allowed delay to shortest possible delay between m(s(f)) and m(t(f)). This might be a more accurate measure of how delay constrained a connection actually is.

All four IVA strategies implement a virtual arc f by finding a Delay Constrained Shortest Path in the substrate from m(s(f)) to m(t(f)) via the Dynamic Program from [69]. The only difference between the strategies is the calculation of the substrate arc costs, which define the length of a path.

- **MinUse** If substrate arcs have already been used, they have a cost of 0. Otherwise, their usage cost p_e^A is assigned. If arcs do not have enough free bandwidth, or their source and target node not enough CPU capacity to host f, a penalty cost of 10^6 is applied.
- **Spread-n** The cost of a substrate arc is the sum the fraction of the arc's remaining free bandwidth that would be used by the virtual arc and the fraction of free CPU capacity the virtual arc would use on the node the substrate arc connects to. This represents the relative resource usage the virtual arc would incur on a substrate arc. Low values mean that the virtual arc has a low impact on the available resources of a substrate arc (and the node

it connects to). The relative resource usage is then taken to the power of $n \in \{0.5, 1, 2\}$ so that it is possible to evaluate the influence of different biasing strategies. We will denote them by Spread-0.5, Spread-1 and Spread-2 respectively.

These methods result in a total of 512 different CH configurations, the results of their evaluation can be found in Section 6.5. The strategies were kept simple to keep running times short as the following heuristics build on the best CH variants.

6.3 Local Search

The basic idea of Local Search (LS) is that a found solution to a problem may be improved by iteratively making small changes. The solutions immediately reachable from a starting solution S are defined by a neighborhood N(S), which can be generated by the appropriate neighborhood structure. LS starts with a solution S and replaces it with a better solution from N(S) until no more improvements can be found. For selecting the neighbor, we use the two standard strategies first-improvement (select the first improving solution) and best-improvement (select the best solution from a neighborhood).

We implemented six different neighborhood structures for the VNMP. They are ruin-andrecreate [154] neighborhoods, which means (in the context of this work) that they remove a part of a complete solution, for instance the implementation of a virtual arc, and then rebuild the solution by applying a CH. Here we need the CH with the modified structure as discussed in the previous section. The discussion of the neighborhoods will skip this rebuilding step.

RemapVarc (N_1) Removes the implementation of a virtual arc.

- **RemapVnode** (N_2) Removes a virtual node and the implementations of all adjacent virtual arcs.
- **RemapSlice** (N_3) Removes a virtual network from the solution by removing all virtual nodes and implementations of all virtual arcs of the virtual network.
- **ClearSarc** (N_4) Clears a substrate arc, which means it removes the implementation of all virtual arcs using this substrate arc.
- **ClearSnode** (N_5) Clears a substrate node, which means it removes the implementation of all virtual arcs that are crossing the substrate node and removes all virtual nodes that are mapped to the substrate node.
- **RemapVnodeTAP** (N_6) Works like RemapVnode, but instead of delegating the choice of substrate node for the removed virtual node to the CH, it explicitly tries to map the virtual node to all possible (TAP) substrate nodes.

Note that the description of the neighborhood structures only specifies how one neighboring solution is reached. How the neighborhood is used during Local Search is straight forward to derive from the description. For example, when performing Local Search with the RemapVarc

neighborhood and best-improvement, we start with an initial solution created by CH. Then we remove the implementation of a virtual arc and rebuild the solution with CH. If the created solution is better, we store it as the currently best one. This procedure is repeated for all remaining virtual arcs, always starting from the initial solution. After all virtual arcs have been tested, the best found solution replaces the initial solution and the process of finding an improving solution is repeated until no further improvements can be found.

For each neighborhood, there is a natural order in which to evaluate the neighbors, for instance the order in which virtual nodes are specified in the VNMP instance. This order is relevant when we use first-improvement. We might be able to speed up the search process if we try the most promising neighbors first. When the current solution for example is not valid, then the most promising neighbors are those which might reduce C_a . In case of RemapVnode, that means virtual nodes, which are mapped to substrate nodes that are overloaded (additional resources had to be bought there), should be tried first. For neighborhoods that focus on the substrate (ClearSarc, ClearSnode), overloaded substrate nodes or arcs should be cleared first. We will denote this neighbor ordering by OverloadingFirst. When solving VNMP-S instead of VNMP-O, it might make sense to only consider neighbors which might reduce C_a instead of just prioritizing them. We will call this strategy OnlyOverloading. The choice of not focusing on any particular neighbors will be denoted by None. Note that even when solving VNMP-S, OnlyOverloading is not as strong as OverloadingFirst since changing parts of a solution that do not directly contribute to C_a might make future improvements of C_a possible.

For an evaluation of the different neighborhoods, see Section 6.5.

6.4 Variable Neighborhood Descent

The neighborhood structures discussed in the previous section can be applied in combination within a Variable Neighborhood Descent (VND) algorithm [74]. A VND utilizes a series of neighborhoods $N_1 \dots N_k$. An initial solution is improved by N_1 until no more improvements can be found, then N_2 is applied to the solution and so on. If N_k fails, VND terminates. If an improved solution is found in some neighborhood, VND restarts with N_1 . For more information, see Section 2.2.6.

We use the neighborhood structures defined in the previous section in two variants: as described without any neighbor prioritization and with OnlyOverloading. We will denote the second variant with a prime. For example N'_6 denotes RemapVnodeTAP in OnlyOverloading configuration. The following neighborhood orderings (VND configurations) were tested:

- All (C₁): N'_1 , N'_2 , N'_3 , N'_4 , N'_5 , N'_6 , N_1 , N_2 , N_3 , N_4 , N_5 , N_6 All neighborhoods, in order of their size.
- **OnlyOverloading** (C₂): N'_1 , N'_2 , N'_3 , N'_4 , N'_5 , N'_6 Only neighborhoods in OnlyOverloading configuration.
- **Complete** (C₃): N_1 , N_2 , N_3 , N_4 , N_5 , N_6 Only complete neighborhoods.

RComplete (C₄): N_6 , N_5 , N_4 , N_3 , N_2 , N_1 Like C_3 , but in reverse order.

ImprovCompA (C₅): N_1, N_2, N_3, N_4, N_6

An improvement to Complete based on preliminary results which showed that ClearSnode does not contribute in a significant way to VND.

RImprovCompA (C₆): N_6 , N_4 , N_3 , N_2 , N_1 C_5 in reverse order.

-

ImprovOverload (C₇): N'_1, N'_2, N'_3

The neighborhoods of OnlyOverloading which find improvements based on preliminary results.

RImprovOverload (C₈): N'_3 , N'_2 , N'_1

 C_7 in reverse order.

ImprovCompB (C₉): N_2, N_4, N_5, N_6

Another selection of neighborhoods to improve Complete.

ImprovCompC (C_{10}): N_3, N_4, N_5, N_6

 C_9 , but using RemapSlice instead of RemapVnode in the hope of speeding up the algorithm while achieving similar results.

OnlyClear (C_{11}): N_4 , N_5

Only the neighborhoods that try to clear parts of the substrate.

ImprovCompD (C_{12}): N_2, N_4, N_5

A variant of C_9 which does not use RemapVnodeTAP to improve run-times.

6.5 Results

Each CH, LS and VND variant was tested on the full VNMP instance set as described in Chapter 5. In addition to load level 1, we also used 0.1, 0.5 and 0.8 to see how the different algorithms and neighborhoods react to changing levels of hardness. This gives a total of 840 instances. A time-limit of 1000 seconds was applied, which was only reached for some VND configurations for the largest instance sizes. The main objective of the evaluation was to find configurations which are suitable for solving VNMP-S or VNMP-O, but also to identify configurations which have a good tradeoff between performance and required run-time. We use the average additional resource cost C_a to evaluate the performance of an algorithm with respect to VNMP-S and the relative rank $R_{\rm rel}$ to determine the suitability for VNMP-O. The price of one unit of CPU resources $p^{\rm CPU}$ was set to one, the price of one unit of bandwidth $p^{\rm BW}$ to five.

6.5.1 Construction Heuristics

Before we can compare all presented heuristics, we needed to identify promising CH variants which can be used to generate the initial solution for LS and VND and perform the rebuilding step of the proposed ruin-and-recreate neighborhoods. To do this, we evaluated the 512 different CH configurations on all 840 VNMP instances. The following tables will show the influence of the different choices for the selection and implementation strategies.

Table 6.1 shows the average performance of the CH configurations depending on the chosen SVN strategy. Based on this data, using DLHeavyVN as the strategy for choosing a virtual node to be mapped is the obvious choice, as this strategy dominates with respect to $R_{\rm rel}$ and C_a . Note that in some cases, it is significantly better to choose an arbitrary virtual node to implement (NextVNode) instead of one that requires a lot of resources (e.g., CPUHeavy) when solving VNMP-S. NextVNode also performs surprisingly well with respect to the $R_{\rm rel}$.

The influence of the chosen IVN strategy is presented in Table 6.2. It is very clear that MostFree is the IVN strategy of choice, beating the other choices by a very large margin both with respect to R_{rel} and to C_a . The additional resource cost is very close to zero for instances with the lowest load. That means nearly every CH configuration creates a valid VNMP solution for all VNMP instances of lowest load, no matter which configuration choices are made, as long as MostFree is used as IVN strategy (these are 128 different CH configurations in total).

Regarding the influence of the SVA strategy, Table 6.3 shows that in this case the best choice really depends on the instance size and load. For small instances, focusing on delay constrained virtual arcs works best. When the load is low, it is beneficial to focus on the virtual arcs that have a delay requirement close to the best achievable delay within the substrate. For higher loads, implementing virtual arcs with low allowed delay values is more important. Notice however, that the differences in R_{rel} and C_a , while statistically significant, are very small. Beginning with instances of size 200, the critical virtual arcs are those with high bandwidth requirements.

Table 6.4 shows that the best choice of IVA strategy mostly depends on instance load. When trying to minimize R_{rel} , using MinUse for the lowest loads is advantageous, but in most cases Spread-1 performs the best. For low loads and large instances, squaring the relative resource usage of a virtual arc is advantageous. Using Spread-1 achieves the best results in nearly all cases when minimizing C_a is the objective.

The last parameter of CH is the implementation emphasis and its influence is presented in Table 6.5. Using AE gives clearly better results, both in terms of R_{rel} and C_a .

The presented results suggest the following CH configuration for the best performance: DL-HeavyVN as SVN, MostFree as IVN, DLHeavy as SVA, Spread-1 as IVA and using AE. Using this would of course be wrong. We have only presented the average performance tendencies of the different strategic choices. It may very well be that a great configuration is drowned in a sea of average or bad configurations and as it turns out, the suggested configuration is neither the best with respect to R_{rel} , nor with respect to C_a . To find the truly best configurations, we order the compared configurations with respect to their average R_{rel} (or C_a) over all VNMP instances and choose the top configuration. Note that we choose based on average performance over all instances. We have already shown that the strategy selection is very sensitive with respect to the instance size and also the load. We could have also defined the best configurations for particular size or load classes. However, this would make further discussion very hard to follow, especially

Table 6.1: Influence of the chosen SVN strategy on the performance of CH.

				$\mathbf{R}_{ ext{rel}}$				C_{a}	
Size	Load	NextVNode	CPUHeavy	CPUHeavyVN	DLHeavyVN	NextVNode	CPUHeavy	CPUHeavyVN	DLHeavyVN
20	0.10	0.479 >	0.537 >	0.521 >	0.454 =	58.5 =	61.2 >	58.4 >	44.8 =
	0.50	0.423 >	0.540 >	0.501 >	0.404 =	1815.9 >	2854.8 >	2645.4 >	1624.4 =
	0.80	0.389 >	0.512 >	0.459 >	0.356 =	6302.6 >	9385.4 >	8586.0 >	5572.5 =
	1.00	0.425 >	0.539 >	0.487 >	0.397 =	10704.1 >	14922.1 >	13752.3 >	9813.4 =
30	0.10	0.535 >	0.602 >	0.596 >	0.501 =	42.4 >	49.9 >	45.8 >	32.4 =
	0.50	0.359 >	0.518 >	0.455 >	0.321 =	2880.0 >	4934.5 >	4265.0 >	2429.7 =
	0.80	0.352 >	0.523 >	0.461 >	0.327 =	7638.8 >	13327.9 >	11817.7 >	6691.6 =
	1.00	0.379 >	0.540 >	0.467 >	0.362 =	14479.2 >	22861.1 >	20134.8 >	13488.4 =
50	0.10	0.485 >	0.516 >	0.513 >	0.434 =	70.5 >	81.1 >	< 7.77 >	67.3 =
	0.50	0.342 >	0.486 >	0.429 >	0.309 =	3121.7 >	5866.0 >	4992.9 >	2792.1 =
	0.80	0.360 >	0.504 >	0.444 >	0.329 =	8829.4 >	15524.2 >	13555.8 >	= 0.999.0 =
	1.00	0.387 >	0.534 >	0.465 >	0.377 =	17835.8 >	27937.3 >	24395.4 >	17110.2 =
100	0.10	0.530 >	0.572 >	0.557 >	0.504 =	181.6 =	225.7 >	195.5 >	185.1 >
	0.50	0.368 >	0.507 >	0.447 >	0.336 =	5813.2 >	< 9.0006	8450.1 >	5108.6 =
	0.80	0.372 >	0.518 >	0.453 >	0.348 =	17596.6 >	28750.2 >	25693.1 >	15489.5 =
	1.00	0.396 >	0.533 >	0.472 >	0.372 =	33843.5 >	50388.3 >	45585.4 >	30608.5 =
200	0.10	0.434 >	0.471 >	0.463 >	0.424 =	469.8 =	629.3 >	579.8 >	480.1 =
	0.50	0.394 >	0.498 >	0.447 >	0.376 =	6062.7 >	10290.6 >	8786.8 >	5666.0 =
	0.80	0.420 >	0.526 >	0.468 >	0.396 =	18516.0 >	28589.2 >	24682.5 >	17042.0 =
	1.00	0.442 >	0.550 >	0.492 >	0.422 =	38208.1 >	53651.3 >	47368.1 >	35975.8 =
500	0.10	0.424 >	0.464 >	0.456 >	0.402 =	2382.0 >	2773.4 >	2697.3 >	2292.6 =
	0.50	0.433 >	0.518 >	0.478 >	0.395 =	31111.8 >	42214.1 >	37565.6 >	26359.6 =
	0.80	0.449 >	0.535 >	0.493 >	0.418 =	67833.6 >	89784.4 >	79344.5 >	58674.5 =
	1.00	0.465 >	0.557 >	0.512 >	0.428 =	105206.9 >	137353.0 >	122249.9 >	91504.3 =
1000	0.10	0.432 >	0.454 >	0.443 >	0.416 =	4078.5 >	4288.3 >	4202.9 >	3916.8 =
	0.50	0.451 >	0.522 >	0.485 >	0.425 =	43036.6 >	54687.8 >	48947.4 >	37740.8 =
	0.80	0.470 >	0.540 >	0.504 >	0.445 =	8892.0 >	113597.4 >	101484.4 >	78002.9 =
	1.00	0.475 >	0.555 >	0.514 >	0.448 =	134388.2 >	170552.6 >	153323.9 >	119068.0 =

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Size	Load	NextSNode	R _{rel}	rel MostFree	CheanHost	NextSNode	C _a NextFree	a MostFree	CheanHost
20	0.10	0.626 >	0.612 >	0.337 =	0.416 >	143.0 >	76.4 >	0.0 =	3.3 >
	0.50	0.716 >	0.640 >	0.151 =	0.362 >	3941.3 >	3383.5 >	21.4 =	1594.5 >
	0.80	0.653 >	0.582 >	0.118 =	0.363 >	12345.8 >	11188.1 >	408.1 =	5904.5 >
	1.00	0.674 >	0.606 >	0.168 =	0.401 >	19100.4 >	17573.6 >	2079.9 =	10437.9 >
30	0.10	0.672 >	0.649 >	0.420 =	0.493 >	85.2 >	68.6 >	0.0 =	16.7 >
	0.50	0.631 >	0.559 >	0.115 =	0.348 >	6313.9 >	5403.6 >	14.9 =	2777.0 >
	0.80	0.621 >	0.554 >	0.108 =	0.380 >	15577.8 >	13888.6 >	520.6 =	9488.9 >
	1.00	0.637 >	0.577 >	0.138 =	0.396 >	26807.1 >	24639.0 >	3264.1 =	16253.3 >
50	0.10	0.588 >	0.525 >	0.306 =	0.529 >	191.7 >	85.9 >	0.0 =	19.0 >
	0.50	0.582 >	0.489 >	0.119 =	0.375 >	7178.8 >	5884.7 >	45.8 =	3663.5 >
	0.80	0.599 >	0.526 >	0.098 =	0.414 >	17921.9 >	15647.9 >	646.9 =	11691.6 >
	1.00	0.624 >	0.556 >	0.139 =	0.446 >	31924.9 >	28690.8 >	4162.3 =	22500.8 >
100	0.10	0.687 >	0.595 >	0.344 =	0.537 >	508.6 >	250.9 >	0.2 =	28.2 >
	0.50	0.628 >	0.519 >	0.110 =	0.401 >	12717.7 >	10092.6 >	76.6 =	6385.7 >
	0.80	0.617 >	0.532 >	0.103 =	0.439 >	33583.7 >	28642.0 >	1313.5 =	23990.2 >
	1.00	0.633 >	0.556 >	0.132 =	0.452 >	58386.7 >	51981.5 >	6827.4 =	43230.1 >
200	0.10	0.598 >	0.466 >	0.209 =	0.519 >	1294.9 >	676.9 >	0.0 =	187.1 >
	0.50	0.603 >	0.487 >	0.095 =	0.531 >	10628.5 >	7790.0 >	150.3 =	12237.3 >
	0.80	0.631 >	0.531 >	0.101 =	0.547 >	30648.3 >	25306.6 >	1759.9 =	31115.0 >
	1.00	0.650 >	0.570 >	0.157 =	0.529 >	59623.6 >	52057.9 >	9869.4 =	53652.3 >
500	0.10	0.830 >	0.546 >	0.074 =	0.297 >	6385.5 >	2971.1 >	0.1 =	788.6 >
	0.50	0.776 >	0.594 >	0.089 =	0.365 >	74085.0 >	45915.8 >	152.4 =	17097.8 >
	0.80	0.773 >	0.617 >	0.102 =	0.404 >	146362.1 >	101503.3 >	4162.6 =	43608.9 >
	1.00	0.770 >	0.628 >	0.151 =	0.413 >	208378.7 >	154279.4 >	18410.3 =	75245.7 >
1000	0.10	0.844 >	0.568 >	0.069 =	0.264 >	11005.2 >	4505.1 >	1.5 =	974.7 >
	0.50	0.818 >	0.604 >	0.092 =	0.368 >	106735.2 >	57087.9 >	306.2 =	20283.3 >
	0.80	0.812 >	0.625 >	0.119 =	0.402 >	205072.2 >	121937.6 >	5953.1 =	49013.9 >
	1.00	0.808 >	0.631 >	0.157 =	0.395 >	287114.7 >	183846.1 >	23381.0 =	82990.8 >

Table 6.2: Influence of the chosen IVN strategy on the performance of CH.

Table 6.3: Influence of the chosen SVA strategy on the performance of CH.

			ц	$\mathbf{R}_{ ext{rel}}$			U	C_{a}	
Size	Load	NextVArc	BWHeavy	DLHeavy	RelDLHeavy	NextVArc	BWHeavy	DLHeavy	RelDLHeavy
20	0.10	0.499 >	0.500 >	0.496 >	0.495 =	55.7 =	55.7 =	55.7 =	55.7 =
	0.50	0.468 >	0.469 >	0.466 =	0.466 >	2235.0 >	2242.3 >	2229.9 =	2233.4 >
	0.80	0.430 >	0.431 >	0.428 =	0.428 >	7461.8 >	7478.4 >	7448.4 =	7457.8 >
	1.00	0.463 >	0.464 >	0.460 =	0.462 >	12302.3 >	12323.7 >	12277.9 =	12288.0 =
30	0.10	0.562 >	0.563 >	0.557 >	0.552 =	42.6 =	42.7 =	42.5 =	42.7 =
	0.50	0.415 >	0.417 >	0.411 >	0.410 =	3634.8 >	3648.4 >	3619.2 >	3606.8 =
	0.80	0.418 >	0.417 >	0.413 =	0.415 >	9882.4 >	9911.4 >	9818.2 =	9863.9 >
	1.00	0.438 >	0.438 >	0.436 =	0.436 =	17765.2 >	17843.7 >	17642.8 =	17711.8 =
50	0.10	0.491 >	0.496 >	0.482 >	0.479 =	74.1 =	74.7 >	73.9 =	73.9 =
	0.50	0.394 >	0.394 >	0.389 >	0.388 =	4206.6 >	4209.1 >	4178.9 =	4178.1 =
	0.80	0.413 >	0.413 >	0.406 >	0.405 =	11553.8 >	11562.3 >	11406.0 >	11386.2 =
	1.00	0.442 >	0.444 >	0.438 =	0.439 =	21887.7 >	22054.7 >	21685.6 =	21650.7 =
100	0.10	0.543 >	0.551 >	0.536 >	0.534 =	195.6 =	196.2 =	200.0 >	196.1 >
	0.50	0.417 >	0.419 >	0.410 =	0.411 =	7321.4 >	7360.4 >	7293.4 =	7297.3 =
	0.80	0.423 >	0.426 >	0.420 =	0.422 >	21872.9 >	22097.8 >	21724.9 =	21833.7 >
	1.00	0.442 >	0.449 >	0.439 =	0.443 >	40028.6 >	40928.9 >	39623.6 =	39844.7 >
200	0.10	0.450 >	0.452 >	0.445 >	0.446 =	538.5 >	537.9 =	541.1 >	541.5 >
	0.50	0.429 >	0.432 >	0.428 =	0.427 =	7694.3 =	7726.6 =	7693.9 =	7691.4 =
	0.80	0.452 =	0.455 =	0.450 >	0.453 >	22171.7 =	22588.7 =	21969.6 >	22099.8 >
	1.00	0.474 =	0.482 >	0.474 >	0.477 >	43488.1 =	45005.0 >	43247.7 >	43462.5 >
500	0.10	0.439 >	0.434 =	0.439 >	0.435 =	2542.9 >	2517.5 =	2546.8 >	2538.1 >
	0.50	0.457 >	0.456 =	0.455 >	0.455 >	34365.5 >	34249.4 =	34333.5 >	34302.6 >
	0.80	0.474 >	0.473 =	0.474 >	0.475 >	73999.8 >	74337.1 =	73647.7 >	73652.4 >
	1.00	0.490 >	0.492 =	0.489 >	0.491 >	114180.3 >	115803.8 =	113103.6 >	113226.5 >
1000	0.10	0.437 >	0.430 =	0.438 >	0.439 >	4122.2 >	4080.5 =	4135.5 >	4148.3 >
	0.50	0.471 >	0.468 =	0.471 >	0.471 >	46167.1 >	45809.0 =	46264.5 >	46171.9 >
	0.80	0.489 >	0.487 =	0.491 >	0.491 >	95570.4 >	95466.0 =	95531.1 >	95409.3 >
	1.00	0.497 >	0.497 =	0.498 >	0.499 >	144268.3 >	145707.3 =	143710.7 >	143646.3 >

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Size	Load	Minlise	Spread-1	R _{rel}	Spread-2	MinTee	Snread-1	a Snread-0 5	Spread-2
	0.10	0.477 =	0.502 >	0.520 >	0.491 >	55.9 >	55.6 =	55.7 =	55.6 =
	0.50	0.480 >	0.461 =	0.466 >	0.462 >	2347.5 >	2195.4 =	2200.6 >	2197.1 >
	0.80	0.450 >	0.421 =	0.422 >	0.423 >	7738.1 >	7367.5 >	7357.7 =	7383.1 >
	1.00	0.487 >	0.452 =	0.452 =	0.458 >	12876.3 >	12083.8 =	12080.3 =	12151.4 >
30	0.10	0.471 =	0.589 >	0.614 >	0.561 >	45.1 >	41.8 =	41.8 =	41.8 :
	0.50	0.441 >	0.402 =	0.408 >	0.403 =	3903.8 >	3529.7 =	3544.1 >	3531.7
	0.80	0.456 >	0.399 =	0.402 >	0.404 >	10838.0 >	9520.4 =	9534.3 >	9583.2
	1.00	0.491 >	0.414 >	0.414 =	0.428 >	19943.0 >	16879.6 =	16866.5 >	17274.3 >
50	0.10	0.397 =	0.519 >	0.540 >	0.491 >	81.5 >	71.6 =	71.6 =	71.9 >
	0.50	0.427 >	0.376 =	0.383 >	0.379 >	4557.0 >	4052.4 =	4058.9 >	4104.4 >
	0.80	0.466 >	0.386 =	0.390 >	0.395 >	12881.3 >	10923.9 =	10956.1 >	11147.0 >
	1.00	0.522 >	0.407 =	0.410 >	0.425 >	25347.6 >	20428.6 =	20487.0 >	21015.5 >
100	0.10	0.407 =	0.579 >	0.623 >	0.554 >	214.4 >	190.9 =	191.7 >	191.0 >
	0.50	0.454 >	0.399 =	0.406 >	0.399 =	8022.1 >	7060.2 =	7089.2 >	7101.1 >
	0.80	0.493 >	0.395 =	0.399 >	0.403 >	25293.0 >	20642.6 =	20713.2 >	20880.6 >
	1.00	0.537 >	0.407 =	0.408 >	0.422 >	48594.2 >	36952.2 =	36992.9 >	37886.4 >
200	0.10	0.374 =	0.470 >	0.506 >	0.441 >	602.7 >	509.2 =	526.9 >	520.3 =
	0.50	0.476 >	0.408 =	0.419 >	0.413 >	8717.5 >	7304.9 =	7396.7 >	7387.0 >
	0.80	0.555 >	0.412 =	0.419 >	0.423 >	27737.9 >	20186.1 =	20348.7 >	20557.1 >
	1.00	0.621 >	0.420 =	0.425 >	0.441 >	59471.9 >	38104.1 =	38343.9 >	39283.4 >
500	0.10	0.444 >	0.429 >	0.450 >	0.424 =	2721.5 >	2455.5 >	2518.7 >	2449.5 =
	0.50	0.486 >	0.442 =	0.452 >	0.443 >	37137.4 >	33220.4 =	33569.7 >	33323.5
	0.80	0.530 >	0.451 =	0.459 >	0.457 >	83113.8 >	70507.0 =	71029.3 >	70986.8
	1.00	0.577 >	0.456 =	0.463 >	0.467 >	137133.1 >	105808.8 =	106409.7 >	106962.5
1000	0.10	0.448 >	0.427 >	0.452 >	0.418 =	4438.4 >	3971.8 >	4108.7 >	3967.6 =
	0.50	0.504 >	0.454 =	0.466 >	0.457 >	50099.2 >	44401.8 =	45145.3 >	44766.2 >
	0.80	0.540 >	0.466 =	0.477 >	0.475 >	107190.5 >	90793.7 =	91997.8 >	91994.8 >
	1.00	0.580 >	0.463 =	0.473 >	0.477 >	172246.9 >	133765.1 =	135233.4 >	136087.2 >

Table 6.4: Influence of the chosen IVA strategy on the performance of CH.

		R	rel	C	а
Size	Load	NE	AE	NE	AE
20	0.10	0.494 =	0.501 >	58.6 >	52.8 =
	0.50	0.578 >	0.356 =	3747.1 >	723.2 =
	0.80	0.585 >	0.273 =	12778.5 >	2144.7 =
	1.00	0.646 >	0.278 =	20671.6 >	3924.3 =
30	0.10	0.566 >	0.551 =	56.7 >	28.5 =
	0.50	0.538 >	0.289 =	6169.9 >	1084.7 =
	0.80	0.574 >	0.257 =	16474.9 >	3263.1 =
	1.00	0.615 >	0.259 =	28356.6 >	7125.1 =
50	0.10	0.504 >	0.469 =	99.9 >	48.4 =
	0.50	0.520 >	0.262 =	7026.1 >	1360.2 =
	0.80	0.564 >	0.254 =	18924.2 >	4029.9 =
	1.00	0.618 >	0.264 =	34720.5 >	8918.9 =
100	0.10	0.563 >	0.519 =	265.0 >	128.9 =
	0.50	0.548 >	0.281 =	12218.7 >	2417.6 =
	0.80	0.589 >	0.257 =	36500.2 >	7264.4 =
	1.00	0.622 >	0.265 =	63745.9 >	16467.0 =
200	0.10	0.494 >	0.402 =	754.4 >	325.1 =
	0.50	0.566 >	0.292 =	12417.1 >	2985.9 =
	0.80	0.608 >	0.296 =	34072.4 >	10342.5 =
	1.00	0.642 >	0.311 =	64038.9 >	23562.7 =
500	0.10	0.485 >	0.388 =	2960.5 >	2112.1 =
	0.50	0.567 >	0.345 =	51195.5 >	17430.0 =
	0.80	0.611 >	0.337 =	111432.2 >	36386.3 =
	1.00	0.637 >	0.344 =	168010.0 >	60147.1 =
1000	0.10	0.484 >	0.388 =	4731.2 >	3512.1 =
	0.50	0.565 >	0.376 =	65001.9 >	27204.4 =
	0.80	0.607 >	0.372 =	136567.3 >	54421.1 =
	1.00	0.624 >	0.371 =	203415.5 >	85250.8 =

Table 6.5: Influence of the choice of implementation emphasis on the performance of CH.

when different CH configurations are used as sub-solver within LS or VND. Because of that, we choose a selection based on the average performance over all instances.

Table 6.6 shows the ten best CH configurations with respect to achieved average R_{rel} . Due to space constraints we cannot show the full table containing all 512 configurations. The best configuration for solving VNMP-O is DLHeavyVN, MostFree, ReIDLHeavy, MinUse, and AE. We will denote this configuration by CH-O. The presented table also shows the importance of the different strategic choices for the final outcome. Least important is the choice of SVA, as we can see groups of four configurations that only differ in this strategy. For the different choices of IVA, we can at least observe some differences in R_{rel} based on the selected strategy. The 16th best configuration uses a different SVN strategy than CH-O (which is not visible in the table). The 53rd configuration is the first that does not use AE and the most important strategic choice for CH is the mapping strategy for virtual nodes. The 95 best configurations map virtual nodes to the substrate node with the most free resources. As a side note, the choice of AE explains

Table 6.6: Average R_{rel} and C_u , number of valid solutions (# Valid) and average required CPUtime (t[s]) of the top 10 CH configurations according to R_{rel} . The implementation emphasis is denoted by Em.

SVN	IVN	SVA	IVA	Em.	\mathbf{R}_{rel}	C_a	# Valid	t[s]
DLHeavyVN	MostFree	RelDLHeavy	MinUse	AE	0.091 =	1950.8 >	511	0.3
DLHeavyVN	MostFree	DLHeavy	MinUse	AE	0.092 >	1958.8 >	510	0.2
DLHeavyVN	MostFree	BWHeavy	MinUse	AE	0.092 >	1964.1 >	507	0.2
DLHeavyVN	MostFree	NextVArc	MinUse	AE	0.092 >	1955.4 >	508	0.2
DLHeavyVN	MostFree	DLHeavy	Spread-2	AE	0.101 >	702.7 >	586	0.2
DLHeavyVN	MostFree	BWHeavy	Spread-2	AE	0.101 >	700.2 >	588	0.2
DLHeavyVN	MostFree	NextVArc	Spread-2	AE	0.101 >	703.9 >	586	0.2
DLHeavyVN	MostFree	RelDLHeavy	Spread-2	AE	0.102 >	705.8 >	586	0.2
DLHeavyVN	MostFree	BWHeavy	Spread-1	AE	0.108 >	652.9 >	590	0.2
DLHeavyVN	MostFree	DLHeavy	Spread-1	AE	0.108 >	659.5 >	586	0.2

Table 6.7: Average R_{rel} and C_u , number of valid solutions (# Valid) and average required CPUtime (t[s]) of the top 10 CH configurations according to C_a . The implementation emphasis is denoted by Em.

SVN	IVN	SVA	IVA	Em.	\mathbf{R}_{rel}	C_{a}	# Valid	t[s]
CPUHeavyVN	MostFree	NextVArc	Spread-1	AE	0.127 >	263.4 =	650	0.2
CPUHeavyVN	MostFree	DLHeavy	Spread-1	AE	0.127 >	264.0 =	648	0.2
CPUHeavyVN	MostFree	RelDLHeavy	Spread-1	AE	0.127 >	264.7 =	649	0.2
CPUHeavyVN	MostFree	BWHeavy	Spread-1	AE	0.127 >	265.1 =	649	0.2
CPUHeavyVN	MostFree	BWHeavy	Spread-2	AE	0.120 >	267.9 =	648	0.2
CPUHeavyVN	MostFree	RelDLHeavy	Spread-2	AE	0.120 >	269.1 =	649	0.3
CPUHeavyVN	MostFree	NextVArc	Spread-2	AE	0.120 >	272.5 =	647	0.2
CPUHeavyVN	MostFree	DLHeavy	Spread-2	AE	0.120 >	276.4 >	647	0.2
CPUHeavyVN	MostFree	BWHeavy	Spread-0.5	AE	0.137 >	276.7 >	638	0.2
CPUHeavyVN	MostFree	RelDLHeavy	Spread-0.5	AE	0.137 >	279.5 >	636	0.3
5		2						

why the choice of SVA strategy has nearly no influence on the outcome. When using AE, there are never a lot of implementable arcs to choose from, so the order among them is not critical. In addition, since we use a SVN strategy that focuses on virtual networks (only if all nodes from one virtual network are mapped, nodes from another one are chosen), the virtual arcs will have similar properties.

The ten best CH configurations for VNMP-S can be seen in Table 6.7. The best configuration uses CPUHeavyVN, MostFree, NextVArc, Spread-1, and AE. We will denote this configuration by CH-S. In contrast to minimizing R_{rel} , we can see that a lot of different configurations achieve the same level of performance. Also, the achieved average C_a is far lower than those for the configurations minimizing R_{rel} . In contrast, the achieved R_{rel} is just a bit higher. For far more

valid solutions, CH-S only has to sacrifice a bit of the cheapness of solutions found by CH-O. The importance of the different strategic choices is the same as before.

We have now defined two good CH configurations, CH-S for solving VNMP-S and CH-O for solving VNMP-O. Both use MostFree as IVN strategy, which focuses on finding valid solutions. In the context of LS and VND, we will use the CH configurations also to rebuild a small part of the solution. If we rebuild this part with a bias towards validity, we might hamper LS and VND during the search for minimal C_u with neighborhood structures that remove the mapping of a node. Therefore, we define a third configuration, CH-R, which is used for cheaply rebuilding solutions. It is equivalent to CH-O, but uses CheapHost as IVN strategy. The full configuration of CH-R is DLHeavyVN, CheapHost, RelDLHeavy, MinUse, and AE.

6.5.2 Local Search

In this section, we will analyze the influence of the different configuration parameters of Local Search, similar to the analysis carried out in the previous section. We used CH-S and CH-O for creating initial solutions and in addition CH-R for rebuilding solutions. Together with the six presented neighborhoods, two step functions (first-improvement and best-improvement) and three neighbor priorities (None, OverloadingFirst, OnlyOverloading), this gives a total of 216 different LS configurations.

Table 6.8 shows the influence of the selected neighborhood on the performance of Local Search. For solving VNMP-O, the neighborhood structure of choice is clearly RemapVnodeTAP, the largest and most powerful neighborhood. Only for the largest instance sizes and loads, RemapSlice is better, since it can be searched much faster. For solving VNMP-S, RemapSlice is generally the best choice. For the lowest load (and all but the largest instance size), the choice of neighborhood structure does not matter, every configuration is able to find a valid solution. The worst neighborhood structure is RemapVarc, which is not surprising since it is the smallest neighborhood.

The influence of different neighbor priorities is presented in Table 6.9. The one thing that is clear from this data is that OnlyOverloading is a bad choice. It is not surprising that it does not perform well when solving VNMP-O, since LS stops once a valid solution has been found. One could have expected better performance with respect to C_a , but, as we have already pointed out, being able to change parts of the solution which are not directly causing overloaded substrate nodes or arcs is beneficial when trying to reduce C_a . The presented data shows this quite nicely. OnlyOverloading is not completely useless, we will show later that it has very attractive run-time characteristics. As for the best choice, OverloadingFirst is better than None for large instances and high loads, so focusing on overloaded parts of the substrate network first is beneficial.

In Table 6.10, we present the influence of the employed Construction Heuristics to create the initial solution and for rebuilding the solution. The employed configuration is given in pairs A-B, where A denotes the CH used to create the initial solution and B the CH for rebuilding the solution. O denotes CH-O, R denotes CH-R, and S denotes CH-S. It is clear that the idea of having a special CH configuration for the rebuilding task was successful. For solving VNMP-O and for solving VNMP-S, using CH-R achieves the best performance overall. Note how the choice of the initial solution influences the whole Local Search. When starting from a solution created by CH-O, we get the best solutions with respect to R_{rel} . By starting with a solution

			1000				500				200				100				50				30				20	Size	
1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	Load	
0.809 >	0.760 >	0.804 >	0.817 >	0.813 >	0.740 >	0.872 >	0.789 >	0.803 >	0.798 >	0.884 >	0.824 >	0.811 >	0.884 >	0.890 >	0.876 >	0.814 >	0.885 >	0.909 >	0.848 >	0.810 >	0.888 >	0.889 >	0.889 >	0.938 >	0.934 >	0.946 >	0.930 >	\mathbf{N}_1	
0.274 =	0.400 >	0.640 >	0.719 >	0.319 =	0.520 >	0.742 >	0.692 >	0.420 >	0.640 >	0.767 >	0.731 >	0.593 >	0.769 >	0.790 >	0.778 >	0.622 >	0.796 >	0.812 >	0.748 >	0.691 >	0.804 >	0.800 >	0.783 >	0.748 >	0.889 >	0.878 >	0.873 >	\mathbf{N}_2	
0.252 =	0.345 =	0.677 >	0.776 >	0.303 =	0.541 >	0.781 >	0.736 >	0.414 >	0.675 >	0.794 >	0.751 >	0.591 >	0.784 >	0.809 >	0.804 >	0.606 >	0.813 >	0.820 >	0.782 >	0.688 >	0.813 >	0.812 >	0.806 >	0.722 >	0.888 >	0.886 >	0.885 >	\mathbf{N}_3	R
0.791 >	0.716 >	0.724 >	0.657 >	0.803 >	0.704 >	0.770 >	0.624 >	0.786 >	0.717 >	0.776 >	0.663 >	0.760 >	0.787 >	0.789 >	0.772 >	0.773 >	0.802 >	0.813 >	0.748 >	0.758 >	0.809 >	0.825 >	0.822 >	0.918 >	0.901 >	0.918 >	0.898 >	\mathbf{N}_4	rel
0.462 >	0.465 >	0.597 >	0.586 >	0.529 >	0.536 >	0.642 >	0.553 >	0.490 >	0.576 >	0.654 >	0.580 >	0.584 >	0.665 >	0.661 >	0.652 >	0.556 >	0.669 >	0.674 >	0.630 >	0.629 >	0.668 >	0.670 >	0.679 >	0.756 >	0.741 >	0.738 >	0.721 >	\mathbf{N}_{2}	
0.630 >	0.510 >	0.526 =	0.486 =	0.418 >	0.391 =	0.450 =	0.511 =	0.263 =	0.385 =	0.467 =	0.524 =	0.376 =	0.472 =	0.481 =	0.517 =	0.407 =	0.490 =	0.496 =	0.502 =	0.496 =	0.521 =	0.560 =	0.532 =	0.514 =	0.643 =	0.637 =	0.697 =	\mathbf{N}_6	
7824.0 >	1727.9 >	119.6 >	1.1 >	4949.5 >	931.7 >	12.7 >	0.0 =	1178.9 >	269.8 >	66.5 >	0.0 =	331.0 >	40.2 >	2.3 =	0.0 =	529.4 >	0.0 =	0.0 =	0.0 =	94.9 >	0.0 =	0.0 =	0.0 =	230.5 >	33.8 >	0.0 =	0.0 =	\mathbf{N}_{1}	
2908.0 =	852.3 >	100.8 >	1.0 >	1927.2 =	524.1 >	9.0 >	0.0 =	710.5 >	238.1 >	55.2 >	0.0 =	243.5 >	58.5 >	7.2 >	0.0 =	131.0 >	4.6 >	0.2 >	0.0 =	100.7 >	10.4 >	0.0 =	0.0 =	120.5 >	18.9 >	0.4 >	0.0 =	\mathbf{N}_2	
2364.2 =	557.4 =	79.6 =	0.8 >	1296.0 =	366.0 =	7.3 >	0.0 =	368.4 >	161.6 >	45.7 >	0.0 =	56.8 =	2.4 =	2.3 =	0.0 =	10.3 =	0.1 >	0.0 =	0.0 =	0.1 =	0.0 =	0.0 =	0.0 =	46.5 =	6.0 =	0.0 =	0.0 =	\mathbf{N}_3	0
7699.8 >	1674.8 >	121.4 >	1.1 >	4891.4 >	935.0 >	12.5 >	= 0.0	1200.3 >	267.2 >	66.5 >	= 0.0	341.2 >	40.2 >	2.3 =	= 0.0	602.8 >	0.0 =	0.0 =	0.0 =	101.3 >	0.0 =	0.0 =	0.0 =	233.4 >	33.8 >	0.0 =	0.0 =	\mathbf{N}_4	ົລ້
4237.8 >	911.7 >	90.8 >	1.0 >	3086.1 >	705.9 >	9.6 >	0.0 =	579.9 >	231.1 >	55.0 >	0.0 =	217.8 >	75.3 >	6.7 >	0.0 =	88.9 >	4.6 >	0.2 >	0.0 =	95.9 >	10.4 >	0.0 =	0.0 =	191.8 >	30.6 >	0.4 >	0.0 =	\mathbf{N}^{22}	
7749.5 >	1165.2 >	94.3 =	0.5 =	2586.3 >	422.4 =	5.5 =	0.0 =	488.0 =	139.0 =	25.5 =	0.0 =	215.7 >	58.5 >	7.2 >	0.0 =	115.0 >	4.6 >	0.2 >	0.0 =	100.5 >	10.4 >	0.0 =	0.0 =	105.7 >	14.9 =	0.4 >	0.0 =	\mathbf{N}_6	

Table 6.8: Influence of the selected neighborhood structure on the performance of LS.

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Size	Load	None	OverloadingFirst	OnlyOverloading	None	OverloadingFirst	OnlyOverloading
20	0.10	0.779 =	= 0.779 =	0.944 >	0.0 =	0.0 =	0.0 =
	0.50	0.773 =	0.773 =	0.956 >	0.0 =	0.0 =	0.6 > 0.0
	0.80	0.777 =	0.777 =	0.944 >	17.2 =	17.2 =	34.6 >
	1.00	0.694 =	0.694 =	0.910 >	121.6 =	121.6 =	220.9 >
30	0.10	0.675 =	0.675 =	< 300.00	0.0 =	=0.0	0.0 =
	0.50	0.685 =	0.685 =	< 900.0	0.0 =	=0.0	0.0 =
	0.80	0.666 >	0.666 =	0.918 >	0.0 =	=0.0	15.6 >
	1.00	0.603 =	0.603 =	0.830 >	37.8 =	37.3 =	171.6 >
50	0.10	0.626 =	0.626 =	0.876 >	0.0 =	0.0 =	0.0 =
	0.50	0.663 =	0.663 =	0.935 >	0.0 =	0.0 =	0.4 >
	0.80	0.652 =	0.652 >	0.925 >	0.0 =	0.0 =	< 6.9
	1.00	0.539 =	0.539 >	0.810 >	190.1 =	190.9 =	357.7 >
100	0.10	0.654 =	0.654 =	0.892 >	0.0 =	0.0 =	0.0 =
	0.50	0.643 =	0.644 =	0.923 >	2.2 =	2.2 =	9.6 >
	0.80	0.630 >	0.629 =	0.922 >	19.2 =	19.1 =	99.2 >
	1.00	0.523 =	0.522 =	0.812 >	148.3 =	148.2 =	406.6 >
200	0.10	0.594 =	0.594 =	0.849 >	0.0 =	0.0 =	0.0 =
	0.50	0.624 =	0.624 =	0.923 >	44.6 =	44.6 =	68.0 >
	0.80	0.522 =	0.521 =	0.852 >	166.9 >	166.5 =	319.9 >
	1.00	0.451 =	0.450 =	0.686 >	573.6 >	570.6 =	1118.8 >
500	0.10	0.571 =	0.571 =	0.810 >	0.0 =	0.0 =	= 0.0 =
	0.50	0.609 =	0.609 =	0.910 >	7.6 =	7.5 =	13.3 >
	0.80	0.485 >	0.479 =	0.751 >	554.3 >	545.8 =	842.5 >
	1.00	0.483 >	0.470 =	0.639 >	2864.5 >	2731.5 =	3772.3 >
1000	0.10	0.589 =	0.589 =	0.843 >	0.7 =	0.7 =	1.3 >
	0.50	0.565 >	0.562 =	0.857 >	88.4 >	86.9 =	128.0 >
	0.80	0.470 >	0.459 =	0.669 >	1026.8 >	984.7 =	1433.2 >
	1.00	0.517 >	0.487 =	0.605 >	5571.1 >	4964.1 =	5856.5 >

			1000				500				200				100				50				30				20	Size	
1.00	080	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	Load	
0.671 >	0 643 >	0.680 >	0.547 >	0.639 >	0.627 >	0.698 >	0.495 >	0.605 >	0.642 >	0.693 >	0.559 >	0.648 >	0.713 >	0.718 >	0.673 >	0.652 >	0.750 >	0.746 >	0.633 >	0.683 >	0.743 >	0.735 >	0.728 >	0.799 >	0.837 >	0.866 >	0.845 >	0-0	
0.599 >	0 560 >	0.517 =	0.442 =	0.582 >	0.543 >	0.530 =	0.402 =	0.531 >	0.508 =	0.531 =	0.446 =	0.531 >	0.569 =	0.563 =	0.535 =	0.554 >	0.607 =	0.601 =	0.512 =	0.572 =	0.613 =	0.595 =	0.581 =	0.691 =	0.721 =	0.719 =	0.712 =	O-R	
0.582 >	0 587 >	0.751 >	0.648 >	0.602 >	0.671 >	0.766 >	0.586 >	0.583 >	0.708 >	0.774 >	0.655 >	0.725 >	0.784 >	0.776 >	0.731 >	0.717 >	0.804 >	0.805 >	0.694 >	0.724 >	0.780 >	0.762 >	0.768 >	0.812 >	0.845 >	0.876 >	0.852 >	0-S	R
0.480 >	0 481 >	0.686 >	0.752 >	0.469 >	0.536 >	0.764 >	0.756 >	0.500 >	0.652 >	0.791 >	0.772 >	0.610 >	0.779 >	0.803 >	0.821 >	0.615 >	0.774 >	0.806 >	0.794 >	0.708 >	0.799 >	0.836 >	0.822 >	0.793 >	0.890 >	0.884 >	0.897 >	o-S	rel
0.430 =	0430 =	0.518 >	0.691 >	0.429 =	0.446 =	0.612 >	0.700 >	0.441 =	0.522 >	0.639 >	0.688 >	0.503 =	0.632 >	0.656 >	0.696 >	0.523 =	0.641 >	0.666 >	0.681 >	0.607 >	0.680 >	0.702 >	0.682 >	0.687 =	0.767 >	0.739 >	0.764 >	S-R	
0.455 >	0 4 94 >	0.815 >	0.962 >	0.464 >	0.608 >	0.887 >	0.967 >	0.514 >	0.761 >	0.914 >	0.954 >	0.699 >	0.884 >	0.905 >	0.944 >	0.716 >	0.881 >	0.900 >	0.944 >	0.777 >	0.886 >	0.927 >	0.930 >	0.815 >	0.936 >	0.921 >	0.935 >	S-S	
7143.5 >	1469.2 >	138.7 >	0.9 >	3820.8 >	750.9 >	19.4 >	0.0 =	994.3 >	295.6 >	62.4 >	0.0 =	300.1 >	69.0 >	7.2 >	0.0 =	289.1 >	4.6 >	0.2 >	0.0 =	121.5 >	10.4 >	0.0 =	0.0 =	171.5 >	25.9 >	0.4 >	0.0 =	0-0	
6556.7 >	1257.2 >	111.5 >	0.5 =	3500.8 >	647.2 >	12.6 >	0.0 =	877.0 >	204.9 >	37.3 >	0.0 =	290.5 >	64.8 >	6.9 >	0.0 =	302.8 >	4.6 >	0.2 >	0.0 =	123.9 >	10.4 >	0.0 =	0.0 =	162.4 >	22.5 >	0.4 >	0.0 =	O-R	
6323.1 >	1278 8 >	137.3 >	0.9 >	3544.5 >	732.4 >	12.6 >	0.0 =	842.4 >	261.6 >	62.4 >	0.0 =	289.3 >	69.0 >	7.2 >	0.0 =	270.6 >	4.6 >	0.2 >	0.0 =	110.0 >	10.4 >	0.0 =	0.0 =	170.7 >	25.9 >	0.4 >	0.0 =	0-S	0
4463.0 >	10198 >	81.9 >	1.1 >	2735.3 >	631.9 >	4.1 >	0.0 =	651.8 >	208.4 >	59.1 >	0.0 =	179.4 >	25.5 >	2.3 >	0.0 =	203.8 >	0.0 =	0.0 =	0.0 =	50.1 >	0.0 =	0.0 =	0.0 =	144.1 >	22.3 >	0.0 =	0.0 =	S-O	ື
4044.9 =	8757 =	51.2 =	0.7 >	2456.7 =	494.9 =	3.7 =	0.0 =	541.4 =	129.8 =	34.1 =	0.0 =	168.1 =	21.2 =	2.1 =	0.0 =	213.0 >	0.0 =	0.0 =	0.0 =	44.8 =	0.0 =	0.0 =	0.0 =	135.8 =	18.9 =	0.0 =	0.0 =	S-R	
4252.1 >	< 9 880	86.1 >							206.5 >	59.1 >	0.0 =	178.8 >	25.5 >	2.3 >	0.0 =	198.2 =	0.0 =	0.0 =	0.0 =	43.0 =	0.0 =	0.0 =	0.0 =	143.8 >	22.3 >	0.0 =	0.0 =	S-S	

Table 6.10: Influence of the selected CH methods for finding an initial solution and for rebuilding it on the performance of LS.

		R	rel	C	'a
Size	Load	FirstImprove	BestImprove	FirstImprove	BestImprove
20	0.10	0.835 =	0.834 =	0.0 =	0.0 =
	0.50	0.835 >	0.833 =	0.2 =	0.2 =
	0.80	0.833 =	0.832 =	22.9 =	23.1 >
	1.00	0.767 =	0.765 =	154.6 =	154.8 >
30	0.10	0.754 >	0.749 =	0.0 =	0.0 =
	0.50	0.764 >	0.755 =	0.0 =	0.0 =
	0.80	0.753 >	0.748 =	5.2 =	5.2 =
	1.00	0.686 >	0.671 =	86.4 >	78.1 =
50	0.10	0.713 >	0.706 =	0.0 =	0.0 =
	0.50	0.757 >	0.751 =	0.1 =	0.1 =
	0.80	0.747 >	0.738 =	2.3 =	2.3 =
	1.00	0.635 >	0.624 =	253.4 >	239.0 =
100	0.10	0.736 >	0.731 =	0.0 =	0.0 =
	0.50	0.741 >	0.732 =	4.7 =	4.7 =
	0.80	0.732 >	0.722 =	45.8 =	45.8 =
	1.00	0.626 >	0.612 =	236.1 >	232.6 =
200	0.10	0.682 >	0.676 =	0.0 =	0.0 =
	0.50	0.727 >	0.721 =	52.4 =	52.4 =
	0.80	0.639 >	0.625 =	219.0 >	216.6 =
	1.00	0.541 >	0.517 =	788.1 >	720.5 =
500	0.10	0.654 >	0.648 =	0.0 =	0.0 =
	0.50	0.712 >	0.707 =	9.6 >	9.3 =
	0.80	0.590 >	0.554 =	676.0 >	619.1 =
	1.00	0.560 >	0.501 =	3366.8 >	2878.7 =
1000	0.10	0.677 >	0.670 =	0.9 =	0.9 =
	0.50	0.678 >	0.645 =	108.4 >	93.8 =
	0.80	0.563 >	0.502 =	1236.6 >	1059.8 =
	1.00	0.544 >	0.529 =	5506.2 >	5421.6 =

Table 6.11: Influence of the selected step function on the performance of LS.

created by CH-S, we get the best solutions with respect to C_a . The exception to this rule are instances of high load. Building the initial solution with CH-S results almost universally in better $R_{\rm rel}$, building it with CH-O is in one case significantly better than CH-S when trying to solve VNMP-S. It is surprising that CH-S is worse than CH-R as rebuilding heuristic when minimizing C_a , although the difference is small. The reason could be that, when used as rebuilding strategy, it repeats the same mistakes it made when creating the initial solution.

The influence of the step function on the performance of Local Search is shown in Table 6.11. For all but the smallest instances, best-improvement (labeled BestImprove) is significantly better than first-improvement (labeled FirstImprove) when trying to find valid solutions with low C_u , but the difference is quite small. First-improvement can keep up much longer when trying to minimize C_a , but is worse for high loads and large instances. Later we will show the difference in run-time between first-improvement and best-improvement.

Table 6.12: Average R_{rel} and C_u , number of valid solutions (# Valid) and average required CPU-time (t[s]) of the top 10 LS Configurations according to R_{rel} .

Neighborhood	Step-Function	Priority	Init.	Reb.	R _{rel}	C_{a}	# Valid	t[s]
ClearSnode	BestImprove	OverloadingFirst	CH-S	CH-R	0.092 =	202.6 >	703	115.0
ClearSnode	BestImprove	None	CH-S	CH-R	0.092 =	202.8 >	703	114.8
ClearSnode	BestImprove	OverloadingFirst	CH-O	CH-R	0.093 =	203.2 >	695	121.6
ClearSnode	BestImprove	None	CH-O	CH-R	0.093 =	203.2 >	695	121.5
ClearSnode	FirstImprove	OverloadingFirst	CH-O	CH-R	0.125 >	192.0 >	696	59.0
ClearSnode	FirstImprove	OverloadingFirst	CH-S	CH-R	0.126 >	193.7 >	701	53.9
ClearSnode	FirstImprove	None	CH-O	CH-R	0.137 >	255.2 >	693	66.3
ClearSnode	FirstImprove	None	CH-S	CH-R	0.142 >	252.4 >	694	54.2
RemapVnodeTAP	BestImprove	OverloadingFirst	CH-O	CH-O	0.234 >	540.8 >	746	265.8
RemapVnodeTAP	BestImprove	None	CH-O	CH-O	0.234 >	538.0 >	746	266.4

Table 6.13: Average R_{rel} and C_u , number of valid solutions (# Valid) and average required CPU-time (t[s]) of the top 10 LS Configurations according to C_a .

Neighborhood	Step-Function	Priority	Init.	Reb.	\mathbf{R}_{rel}	C_{a}	# Valid	t[s]
RemapVnode	BestImprove	None	CH-S	CH-R	0.428 >	64.5 =	714	27.0
RemapVnode	BestImprove	OverloadingFirst	CH-S	CH-R	0.429 >	64.8 =	714	27.0
RemapVnode	FirstImprove	OverloadingFirst	CH-S	CH-R	0.442 >	77.3 >	708	8.9
RemapVnode	FirstImprove	None	CH-S	CH-R	0.442 >	77.7 >	709	9.8
RemapVnode	BestImprove	OverloadingFirst	CH-O	CH-R	0.343 >	86.4 >	706	35.4
RemapVnode	BestImprove	None	CH-O	CH-R	0.343 >	86.4 >	706	35.5
RemapVnode	FirstImprove	None	CH-O	CH-R	0.362 >	100.7 >	696	14.2
RemapSlice	BestImprove	None	CH-S	CH-R	0.497 >	108.5 >	703	4.6
RemapSlice	BestImprove	OverloadingFirst	CH-S	CH-R	0.497 >	108.5 >	703	4.5
RemapVnode	FirstImprove	OverloadingFirst	CH-O	CH-R	0.361 >	109.0 >	698	11.9

As in the previous section, we show the 10 best LS configurations with respect to the average R_{rel} in Table 6.12. In comparison with the best CH configurations with respect to C_a , the best LS configurations according to the R_{rel} beat them with respect to C_a and also in terms of valid solutions. Of course, this has a substantial cost in terms of run-time. It can be observed that with all other settings being equal, there is no significant difference between OverloadingFirst and None. Also, the initialization CH is far less important for the best configurations than in the average case. The rebuilding CH however still has to be CH-R for the best performance, notice the large increase in relative rank for the configurations using CH-O. Observe how switching from BestImprove to FirstImprove increases the R_{rel} by 30%, but halves the required run-time. Using RemapVnodeTAP instead of ClearSnode (which by the way was not indicated to be a promising neighborhood in the slightest) doubles the required run-time.

Table 6.13 shows the 10 best LS configurations with respect to the average C_a . Again we can observe that changing the neighbor priority has no significant effect on the final result (as long as we do not use OnlyOverloading). Using first-improvement reduces the required run-time to a

third for a slight increase in average C_a . The best configurations for solving VNMP-S are generally faster than those for VNMP-O. One important thing to note is that the best configuration with respect to average C_a does not solve the most instances to validity. The best LS configuration is able to solve 764 out of the 840 instances. It uses RemapVnodeTAP, best-improvement, OverloadingFirst, CH-S for initialization, and CH-R for rebuilding.

The best CH configurations have shown that the mapping strategy for virtual nodes is essential. The same can be observed here. The best LS configuration with respect to R_{rel} tries to clear substrate nodes (by removing the virtual nodes) to reduce C_u . To minimize C_a , the more conservative approach of removing a single virtual node achieves the best results.

We will denote the best LS configuration for solving VNMP-O (ClearSnode, best-improvement, OverloadingFirst, CH-S, and CH-R) by LS-O. The best LS configuration with respect to VNMP-S (RemapVnode, best-improvement, None, CH-S, and CH-R) will be called LS-S.

6.5.3 Variable Neighborhood Descent

For Variable Neighborhood Descent, we tested all combinations of the twelve neighborhood structure configurations, two step-functions, two initialization CHs and three rebuilding CHs, yielding in total 144 VND algorithms. The different step-functions were applied to all neighborhoods simultaneously, so either all neighborhoods were searched by first-improvement or all were searched by best-improvement.

Table 6.14 shows the influence of the different neighborhood configurations on the achieved average R_{rel} . For all instances up to and including size 200, C_4 (RComplete) is the best neighborhood structure configuration. Very similar performance levels are achieved by C_1 (All), C_3 (Complete), C_9 (ImprovCompB) and C_{10} (ImprovCompC), which are all configurations that utilize the RemapVnodeTAP neighborhood structure. For the instances of the two largest size classes, reducing the number of employed neighborhood structures (ImprovCompB) is required to achieve the best levels of performance. A key factor is also that ImprovCompB uses RemapVnodeTAP as last neighborhood structure and not as first, as is done by RComplete, because RemapVnodeTAP requires a lot of time for large instances. This is because RemapVnodeTAP explicitly tries to map all virtual nodes to all possible mapping targets, and we know from Chapter 5 that for the largest instances we can expect 700 virtual nodes with an average of 50 allowed mapping targets. The configurations that use neighborhood structures in the Only-Overloading setting (C_2 , C_7 , and C_8) produce very bad results with respect to R_{rel} , as could be expected.

How different neighborhood configurations influence the achieved performance of VND with respect to average C_a is shown in Table 6.15. It is striking that the choice of configuration basically has no influence on the performance up to instance size 100. In addition, every configuration choice is able to solve every instance of size 30, with the exception of C_{11} (OnlyClear) and full load. It is interesting that instances of size 20 seem to be harder to solve than instances of size 30 or 50. For larger instances, configurations C_1 , C_3 , C_5 , C_9 , and C_{10} achieve the best performance. Note that the configurations that reverse the order of neighborhood structures (C_4 , C_6) perform far worse.

The influence of the CH choice on the performance of VND is presented in Table 6.16. The CH choice is encoded in the same way as for LS, i.e., S-R means that CH-S was used to create the

	Size Load		0.50 0.444 >	0.80 0.473 >	1.00 0.406 >	30 0.10 0.324 =	0.50 0.408	0.80 0.431 >	1.00 0.394 >	50 0.10 0.343 >	0.50 0.423 >	_	$0.80 \parallel 0.460 >$			0.80 1.00 0.10 0.50	0.80 1.00 0.50 0.80	0.80 1.00 0.10 0.50 0.80 1.00	0.80 1.00 0.10 0.50 0.80 1.00	0.80 0.10 0.50 0.80 1.00 0.10 0.50	0.80 1.00 0.50 0.80 1.00 1.00 0.50 0.50 0.50 0.50 0.50 0.50	0.80 0.10 0.50 0.80 1.00 0.10 0.10 0.10 0.50 0.50 0.50 0.50 0.50 0.50 0.50 0.50 0.50 0.50 0.10	0.80 1.00 0.50 0.50 0.80 1.00 0.10 0.50 0.50 0.50 0.50 0.50 0.50 0.50 0.50 0.50 0.10	0.80 1.00 0.50 0.80 1.00 0.10 0.10 0.50 0.50 0.50 0.50 0.50 0.50	0.80 1.00 0.10 0.50 0.80 1.00 0.10 0.50	0.80 1.00 0.50 0.80 1.00 0.50	0.80 1.00 0.50 0.80 1.00 0.50	0.80 0.10 0.50 0.50 0.80 1.00 0.50 0.50 0.80 1.00 0.10 0.50 0.80 1.00 0.10 0.50 0.80 0.50	$\begin{array}{c} 0.80\\ 1.00\\ 0.50\\ 0.50\\ 0.80\\ 1.00\\ 0.50\\ 0.80\\ 1.00\\ 0.50\\ 0.80\\ 1.00\\ 0.50\\ 0.80\\ 1.00\\ 0.50\\ 0.80\\$
	Ů Ů	0.951 >	0.959 >	0.958 >	0.889 >	0.927 >	0.941 >	0.961 >	0.890 >	0.925 >	0.962 >	0.964 >	0.905 >	0 026 ~	0.720 /	0.965 >	0.965 > 0.952 >	0.965 > 0.952 > 0.887 >	0.965 > 0.952 > 0.887 > 0.943 >	0.965 > 0.965 > 0.952 > 0.887 > 0.943 > 0.972 >	0.965 > 0.965 > 0.952 > 0.887 > 0.943 > 0.972 > 0.972 >	0.965 > 0.952 > 0.887 > 0.943 > 0.972 > 0.972 > 0.917 > 0.810 >	0.965 > 0.965 > 0.952 > 0.887 > 0.943 > 0.943 > 0.972 > 0.917 > 0.917 > 0.917 > 0.917 >	0.965 > 0.965 > 0.952 > 0.987 > 0.943 > 0.972 > 0.917 > 0.917 > 0.917 > 0.917 > 0.917 > 0.917 >	0.955 > 0.955 > 0.952 > 0.987 > 0.943 > 0.943 > 0.972 > 0.917 > 0.917 > 0.917 > 0.917 > 0.917 > 0.917 > 0.917 > 0.917 > 0.917 > 0.810 > 0.928 >	$\begin{array}{c ccccc} 0.965 & > \\ 0.965 & > \\ 0.952 & > \\ 0.887 & > \\ 0.943 & > \\ 0.972 & > \\ 0.917$	0.952 > 0.952 > 0.952 > 0.987 > 0.972 > 0.972 > 0.917	0.952 > 0.952 > 0.952 > 0.943 > 0.972 > 0.972 > 0.917	0.952 > 0.952 > 0.952 > 0.943 > 0.972 > 0.917 > 0.928 > 0.938 > 0.938 > 0.938 > 0.952 > 0.938 > 0.952 > 0.938 > 0.952 > 0.952 > 0.952 > 0.952 > 0.972 > 0.917
$\begin{array}{c c} \mathbf{C}_2 \\ \hline \mathbf{C}_2 \\ \hline 0.951 > \\ 0.959 > \\ 0.958 > \\ 0.958 > \\ 0.958 > \\ 0.958 > \\ 0.958 > \\ 0.958 > \\ 0.961 > \\ 0.961 > \\ 0.961 > \\ 0.961 > \\ 0.962 > \\ 0.962 > \\ 0.962 > \\ 0.964 > \\ 0.965$	Ũ	0.454 =	0.444 >	0.470 >	0.425 >	0.324 =	0.408 =	0.431 >	0.391 >	0.343 >	0.423 >	0.460 >	0.422 >		0.422 >	0.422 > 0.485 >	0.422 > 0.485 > 0.468 >	0.422 > 0.485 > 0.468 > 0.382 >	0.422 > 0.485 > 0.468 > 0.382 > 0.417 >	0.422 > 0.485 > 0.468 > 0.382 > 0.417 > 0.417 >	$\begin{array}{c c} 0.422 \\ 0.485 \\ 0.468 \\ 0.382 \\ 0.417 \\ 0.417 \\ 0.453 \\ 0.391 \\ \end{array}$	0.422 > 0.485 > 0.485 > 0.488 > 0.488 > 0.488 > 0.382 > 0.382 > 0.417 > 0.417 > 0.453 > 0.391 > 0.391 > 0.295 > 0.29	$\begin{array}{c} 0.422 > \\ 0.485 > \\ 0.486 > \\ 0.382 > \\ 0.417 > \\ 0.417 > \\ 0.391 > \\ 0.295 > \\ 0.417 > \end{array}$	$\begin{array}{c} 0.422 > \\ 0.485 > \\ 0.485 > \\ 0.488 > \\ 0.382 > \\ 0.417 > \\ 0.417 > \\ 0.391 > \\ 0.295 > \\ 0.295 > \\ 0.417 > \\ 0.430 = \end{array}$	$\begin{array}{c} 0.422 > \\ 0.485 > \\ 0.485 > \\ 0.382 > \\ 0.382 > \\ 0.417 > \\ 0.417 > \\ 0.391 > \\ 0.295 > \\ 0.295 > \\ 0.417 > \\ 0.430 = \\ 0.339 > \\ \end{array}$	$\begin{array}{c} 0.422 > \\ 0.485 > \\ 0.485 > \\ 0.488 > \\ 0.382 > \\ 0.417 > \\ 0.417 > \\ 0.391 > \\ 0.295 > \\ 0.295 > \\ 0.417 > \\ 0.417 > \\ 0.339 > \\ 0.339 > \\ 0.298 > \\ \end{array}$	$\begin{array}{c} 0.422 > \\ 0.485 > \\ 0.485 > \\ 0.488 > \\ 0.382 > \\ 0.417 > \\ 0.417 > \\ 0.453 > \\ 0.391 > \\ 0.295 > \\ 0.417 > \\ 0.417 > \\ 0.430 = \\ 0.339 > \\ 0.339 > \\ 0.298 > \\ 0.435 = \end{array}$	$\begin{array}{c} 0.422 > \\ 0.485 > \\ 0.485 > \\ 0.382 > \\ 0.382 > \\ 0.417 > \\ 0.453 > \\ 0.391 > \\ 0.295 > \\ 0.295 > \\ 0.417 > \\ 0.430 = \\ 0.339 > \\ 0.339 > \\ 0.298 > \\ 0.298 > \\ 0.392 > \\ \end{array}$	$\begin{array}{c} 0.422 > \\ 0.485 > \\ 0.485 > \\ 0.382 > \\ 0.382 > \\ 0.391 > \\ 0.391 > \\ 0.295 > \\ 0.295 > \\ 0.417 > \\ 0.417 > \\ 0.430 = \\ 0.392 > \\ 0.392 > \\ 0.341 > \\ 0.341 > \\ \end{array}$
	õ	0.447 =	0.443 =	0.431 =	0.381 =	0.322 =	0.407 =	0.421 =	0.379 =	0.325 =	0.407 =	0.450 =	0.364 =		0.400 =	0.400 = 0.474 =	0.400 = 0.474 = 0.457 =			$\begin{array}{l} 0.400 = \\ 0.474 = \\ 0.457 = \\ 0.363 = \\ 0.387 = \\ 0.439 = \end{array}$	$\begin{array}{l} 0.400 = \\ 0.474 = \\ 0.457 = \\ 0.363 = \\ 0.387 = \\ 0.387 = \\ 0.439 = \\ 0.357 = \end{array}$	$\begin{array}{l} 0.400 = \\ 0.474 = \\ 0.457 = \\ 0.363 = \\ 0.387 = \\ 0.439 = \\ 0.439 = \\ 0.357 = \\ 0.282 = \end{array}$	$\begin{array}{l} 0.400 = \\ 0.474 = \\ 0.457 = \\ 0.363 = \\ 0.387 = \\ 0.387 = \\ 0.439 = \\ 0.357 = \\ 0.282 = \\ 0.402 = \end{array}$	$\begin{array}{l} 0.400 = \\ 0.474 = \\ 0.457 = \\ 0.363 = \\ 0.387 = \\ 0.387 = \\ 0.439 = \\ 0.357 = \\ 0.282 = \\ 0.282 = \\ 0.402 = \\ 0.518 > \end{array}$	$\begin{array}{l} 0.400 = \\ 0.474 = \\ 0.457 = \\ 0.363 = \\ 0.387 = \\ 0.387 = \\ 0.439 = \\ 0.357 = \\ 0.282 = \\ 0.282 = \\ 0.402 = \\ 0.518 > \\ 0.508 > \end{array}$	$\begin{array}{l} 0.400 = \\ 0.474 = \\ 0.457 = \\ 0.363 = \\ 0.387 = \\ 0.387 = \\ 0.439 = \\ 0.357 = \\ 0.282 = \\ 0.282 = \\ 0.282 = \\ 0.282 = \\ 0.508 > \\ 0.508 > \\ 0.658 > \end{array}$	$\begin{array}{l} 0.400 = \\ 0.474 = \\ 0.457 = \\ 0.363 = \\ 0.387 = \\ 0.439 = \\ 0.439 = \\ 0.357 = \\ 0.357 = \\ 0.282 = \\ 0.282 = \\ 0.282 = \\ 0.282 = \\ 0.518 > \\ 0.508 > \\ 0.508 > \\ 0.558 > \\ 0.458 = \\ \end{array}$	$\begin{array}{l} 0.400 = \\ 0.474 = \\ 0.457 = \\ 0.363 = \\ 0.387 = \\ 0.387 = \\ 0.357 = \\ 0.357 = \\ 0.282 = \\ 0.282 = \\ 0.282 = \\ 0.282 = \\ 0.518 > \\ 0.508 > \\ 0.508 > \\ 0.508 > \\ 0.508 > \\ 0.508 > \\ 0.458 = \\ 0.458 = \\ 0.601 > \end{array}$	$\begin{array}{l} 0.400 = \\ 0.474 = \\ 0.457 = \\ 0.363 = \\ 0.387 = \\ 0.387 = \\ 0.357 = \\ 0.357 = \\ 0.282 = \\ 0.282 = \\ 0.282 = \\ 0.282 = \\ 0.283 > \\ 0.508 > \\$
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	O	0.634 >	0.612 >	0.613 >	0.545 >	0.424 >	0.534 >	0.576 >	0.509 >	0.427 >	0.557 >	0.603 >	0.539 >		0.518 >		0.518 > 0.637 > 0.617 >	0.518 > 0.637 > 0.617 > 0.500 >	$\begin{array}{c} 0.518 > \\ 0.637 > \\ 0.617 > \\ 0.500 > \\ 0.505 > \end{array}$	$\begin{array}{c c} 0.518 > \\ 0.637 > \\ 0.617 > \\ 0.500 > \\ 0.505 > \\ 0.602 > \end{array}$	$\begin{array}{c c} 0.518 > \\ 0.637 > \\ 0.617 > \\ 0.500 > \\ 0.505 > \\ 0.602 > \\ 0.531 > \end{array}$	$\begin{array}{c c} 0.518 > \\ 0.637 > \\ 0.617 > \\ 0.500 > \\ 0.505 > \\ 0.602 > \\ 0.531 > \\ 0.393 > \end{array}$	$\begin{array}{c} 0.518 > \\ 0.637 > \\ 0.617 > \\ 0.500 > \\ 0.505 > \\ 0.505 > \\ 0.602 > \\ 0.602 > \\ 0.393 > \\ 0.497 > \end{array}$	$\begin{array}{c c} 0.518 > \\ 0.637 > \\ 0.617 > \\ 0.500 > \\ 0.505 > \\ 0.505 > \\ 0.602 > \\ 0.602 > \\ 0.531 > \\ 0.531 > \\ 0.393 > \\ 0.497 > \\ 0.548 > \end{array}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{l} 0.518 > \\ 0.637 > \\ 0.617 > \\ 0.500 > \\ 0.505 > \\ 0.602 > \\ 0.602 > \\ 0.602 > \\ 0.602 > \\ 0.602 > \\ 0.602 > \\ 0.602 > \\ 0.617 > \\ 0.531 > \\ 0.393 > \\ 0.393 > \\ 0.393 > \\ 0.393 > \\ 0.393 > \\ 0.393 > \\ 0.393 > \\ 0.393 > \\ 0.393 > \\ 0.393 > \\ 0.519 > \\ \end{array}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{l} 0.518 > \\ 0.637 > \\ 0.617 > \\ 0.500 > \\ 0.505 > \\ 0.505 > \\ 0.602 > \\ 0.602 > \\ 0.602 > \\ 0.602 > \\ 0.602 > \\ 0.602 > \\ 0.0393 > \\ 0.393 > $
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		0.633 >	0.608 >	0.588 >	0.502 >	0.413 >	0.536 >	0.567 >	0.496 >	0.415 >	0.536 >	0.590 >	0.489 >	0.498 >		0.615 >		0.615 > 0.595 > 0.476 >	0.615 > 0.595 > 0.476 > 0.478 >		$\begin{array}{c c} 0.615 > \\ 0.595 > \\ 0.476 > \\ 0.478 > \\ 0.580 > \\ 0.483 > \end{array}$	$\begin{array}{c cccc} 0.615 > \\ 0.595 > \\ 0.476 > \\ 0.478 > \\ 0.580 > \\ 0.483 > \\ 0.372 > \end{array}$	$\begin{array}{c cccc} 0.615 > \\ 0.595 > \\ 0.476 > \\ 0.478 > \\ 0.580 > \\ 0.580 > \\ 0.483 > \\ 0.372 > \\ 0.478 > \end{array}$	$\begin{array}{c} 0.615 > \\ 0.595 > \\ 0.476 > \\ 0.478 > \\ 0.580 > \\ 0.580 > \\ 0.580 > \\ 0.580 > \\ 0.478 > \\ 0.478 > \\ 0.478 > \\ 0.478 > \\ 0.615 > \end{array}$	$\begin{array}{c} 0.615 \\ 0.595 \\ 0.476 \\ 0.478 \\ 0.580 \\ 0.580 \\ 0.580 \\ 0.580 \\ 0.483 \\ 0.483 \\ 0.483 \\ 0.483 \\ 0.483 \\ 0.478 \\ 0.478 \\ 0.615 \\ 0.510 \\ \end{array}$	$\begin{array}{c c} 0.615 \\ 0.595 \\ 0.476 \\ 0.478 \\ 0.580 \\ 0.580 \\ 0.580 \\ 0.483 \\ 0.483 \\ 0.483 \\ 0.483 \\ 0.483 \\ 0.483 \\ 0.483 \\ 0.478 \\ 0.478 \\ 0.615 \\ 0.510 \\ 0.510 \\ 0.558 \\ \end{array}$	$\begin{array}{c c} 0.615 \\ 0.595 \\ 0.476 \\ 0.478 \\ 0.580 \\ 0.580 \\ 0.580 \\ 0.483 \\ 0.483 \\ 0.483 \\ 0.483 \\ 0.483 \\ 0.483 \\ 0.478 \\ 0.510$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		0.951 >	0.959 >	0.958 >	0.889 >	0.927 >	0.941 >	0.961 >	0.890 >	0.925 >	0.962 >	0.964 >	0.905 >		0.956 >	0.956 > 0.965 >	0.956 > 0.965 > 0.952 >	$\begin{array}{c} 0.956 > \\ 0.965 > \\ 0.952 > \\ 0.887 > \end{array}$	$\begin{array}{c} 0.956 > \\ 0.965 > \\ 0.952 > \\ 0.887 > \\ 0.943 > \end{array}$	$\begin{array}{c} 0.956 > \\ 0.965 > \\ 0.952 > \\ 0.987 > \\ 0.943 > \\ 0.972 > \end{array}$	$\begin{array}{c} 0.956 > \\ 0.965 > \\ 0.952 > \\ 0.952 > \\ 0.987 > \\ 0.943 > \\ 0.972 > \\ 0.917 > \end{array}$	$\begin{array}{c} 0.956 > \\ 0.965 > \\ 0.952 > \\ 0.887 > \\ 0.943 > \\ 0.917 > \\ 0.813 > \end{array}$	$\begin{array}{c} 0.956 > \\ 0.965 > \\ 0.952 > \\ 0.887 > \\ 0.943 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.938 > \\ \end{array}$	$\begin{array}{c} 0.956 > \\ 0.965 > \\ 0.952 > \\ 0.887 > \\ 0.943 > \\ 0.917 > \\$	$\begin{array}{c} 0.956 > \\ 0.965 > \\ 0.952 > \\ 0.987 > \\ 0.943 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.928 > \\ 0.921 > \\$	$\begin{array}{l} 0.956 > \\ 0.965 > \\ 0.965 > \\ 0.952 > \\ 0.887 > \\ 0.943 > \\ 0.943 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.813 > \\ 0.813 > \\ 0.938 > \\ 0.971 > \\ 0.829 > \\ 0.704 > \\ \end{array}$	$\begin{array}{c} 0.956 > \\ 0.965 > \\ 0.965 > \\ 0.952 > \\ 0.887 > \\ 0.943 > \\ 0.972 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.911 > \\ 0.938 > \\ 0.971 > \\$	$\begin{array}{l} 0.956 > \\ 0.965 > \\ 0.965 > \\ 0.952 > \\ 0.887 > \\ 0.943 > \\ 0.972 > \\ 0.917 > \\ 0.917 > \\ 0.911 > \\ 0.829 > \\ 0.704 > \\ 0.901 > \\ \end{array}$	$\begin{array}{l} 0.956 > \\ 0.965 > \\ 0.965 > \\ 0.952 > \\ 0.887 > \\ 0.972 > \\ 0.972 > \\ 0.917 > \\ 0.917 > \\ 0.917 > \\ 0.813 > \\ 0.971 > \\ 0.971 > \\ 0.829 > \\ 0.704 > \\ 0.704 > \\ 0.901 > \\ 0.694 > \\ \end{array}$
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	õ	0.951 >	0.960 >	0.958 >	0.891 >	0.927 >	0.941 >	0.961 >	0.901 >	0.925 >	0.960 >	0.965 >	0.901 >	0.956 >		0.966 >	0.966 > 0.956 >	0.966 > 0.956 > 0.895 >	0.966 > 0.956 > 0.895 > 0.943 >	$\begin{array}{c} 0.966 > \\ 0.956 > \\ 0.895 > \\ 0.943 > \\ 0.974 > \end{array}$	$\begin{array}{c} 0.966 > \\ 0.956 > \\ 0.895 > \\ 0.943 > \\ 0.974 > \\ 0.928 > \end{array}$	$\begin{array}{c c} 0.966 \\ 0.956 \\ 0.895 \\ 0.943 \\ 0.974 \\ 0.928 \\ 0.838 \\ \end{array}$	$\begin{array}{c} 0.966 > \\ 0.956 > \\ 0.895 > \\ 0.943 > \\ 0.974 > \\ 0.928 > \\ 0.928 > \\ 0.938 > \\ \end{array}$	$\begin{array}{c} 0.966 > \\ 0.956 > \\ 0.895 > \\ 0.943 > \\ 0.974 > \\ 0.928 > \\ 0.928 > \\ 0.838 > \\ 0.938 > \\ 0.938 > \\ 0.972 > \end{array}$	$\begin{array}{c c} 0.966 > \\ 0.956 > \\ 0.895 > \\ 0.943 > \\ 0.974 > \\ 0.974 > \\ 0.928 > \\ 0.838 > \\ 0.838 > \\ 0.938 > \\ 0.972 > \\ 0.841 > \end{array}$	$\begin{array}{c} 0.966 > \\ 0.956 > \\ 0.895 > \\ 0.943 > \\ 0.974 > \\ 0.974 > \\ 0.928 > \\ 0.928 > \\ 0.838 > \\ 0.838 > \\ 0.938 > \\ 0.972 > \\ 0.841 > \\ 0.694 > \\ \end{array}$	$\begin{array}{c} 0.966 > \\ 0.956 > \\ 0.895 > \\ 0.943 > \\ 0.974 > \\ 0.974 > \\ 0.978 > \\ 0.928 > \\ 0.938 > \\ 0.938 > \\ 0.938 > \\ 0.972 > \\ 0.841 > \\ 0.694 > \\ 0.694 > \\ \end{array}$	$\begin{array}{l} 0.966 > \\ 0.956 > \\ 0.895 > \\ 0.943 > \\ 0.943 > \\ 0.974 > \\ 0.928 > \\ 0.928 > \\ 0.928 > \\ 0.938 > \\ 0.938 > \\ 0.938 > \\ 0.972 > \\ 0.841 > \\ 0.841 > \\ 0.964 > \\ 0.964 > \\ 0.907 > \\ \end{array}$	$\begin{array}{l} 0.966 > \\ 0.956 > \\ 0.895 > \\ 0.943 > \\ 0.974 > \\ 0.974 > \\ 0.978 > \\ 0.938 > \\ 0.938 > \\ 0.972 > \\ 0.841 > \\ 0.841 > \\ 0.694 > \\ 0.907 > \\ 0.907 > \\ 0.641 > \\ \end{array}$
	Ů	0.454 =	0.444 >	0.469 >	0.414 >	0.324 =	0.409 =	0.431 >	0.387 =	0.343 >	0.424 >	0.463 >	0.372 =	0.424 >	0	0.488 >	0.488 > 0.471 >	0.488 > 0.471 > 0.397 >	0.488 > 0.471 > 0.397 > 0.418 >	$\begin{array}{c} 0.488 > \\ 0.471 > \\ 0.397 > \\ 0.418 > \\ 0.455 > \end{array}$	$\begin{array}{c c} 0.488 > \\ 0.471 > \\ 0.397 > \\ 0.418 > \\ 0.455 > \\ 0.391 > \end{array}$	$\begin{array}{c c} 0.488 > \\ 0.471 > \\ 0.397 > \\ 0.418 > \\ 0.455 > \\ 0.391 > \\ 0.295 > \end{array}$	$\begin{array}{c c} 0.488 > \\ 0.471 > \\ 0.397 > \\ 0.418 > \\ 0.455 > \\ 0.391 > \\ 0.295 > \\ 0.416 > \\ \end{array}$	0.488 > 0.471 > 0.397 > 0.418 > 0.418 > 0.455 > 0.455 > 0.391 > 0.391 > 0.295 > 0.416 >	0.488 > 0.471 > 0.397 > 0.418 > 0.455 > 0.391 > 0.391 > 0.295 > 0.416 > 0.416 > 0.431 = 0.320 =	$\begin{array}{c} 0.488 > \\ 0.471 > \\ 0.397 > \\ 0.418 > \\ 0.455 > \\ 0.391 > \\ 0.295 > \\ 0.295 > \\ 0.416 > \\ 0.416 > \\ 0.431 = \\ 0.431 = \\ 0.320 = \\ 0.286 = \end{array}$	$\begin{array}{c} 0.488 > \\ 0.471 > \\ 0.397 > \\ 0.418 > \\ 0.455 > \\ 0.391 > \\ 0.295 > \\ 0.295 > \\ 0.416 > \\ 0.416 > \\ 0.431 = \\ 0.320 = \\ 0.286 = \\ 0.438 = \end{array}$	$\begin{array}{c} 0.488 > \\ 0.471 > \\ 0.397 > \\ 0.397 > \\ 0.418 > \\ 0.455 > \\ 0.391 > \\ 0.295 > \\ 0.295 > \\ 0.295 > \\ 0.416 > \\ 0.431 = \\ 0.320 = \\ 0.320 = \\ 0.325 = \\ 0.375 = \end{array}$	$\begin{array}{l} 0.488 > \\ 0.471 > \\ 0.397 > \\ 0.397 > \\ 0.418 > \\ 0.455 > \\ 0.455 > \\ 0.391 > \\ 0.295 > \\ 0.295 > \\ 0.416 > \\ 0.431 = \\ 0.320 = \\ 0.320 = \\ 0.375 = \\ 0.320 = \\ 0.320 = \\ \end{array}$
	C	0.458 >	0.440 =	0.476 >	0.426 >	0.333 >	0.411 =	0.440 >	0.399 >	0.350 >	0.432 >	0.474 >	0.381 >	0.426 >	0 501 >		0.487 >	0.487 > 0.426 >	0.426 > 0.414 >	0.487 > 0.426 > 0.414 > 0.475 >	0.487 > 0.426 > 0.414 > 0.475 > 0.475 >	$\begin{array}{c} 0.487 \\ 0.426 \\ 0.414 \\ 0.475 \\ 0.422 \\ 0.422 \\ 0.361 \\ \end{array}$	$\begin{array}{c} 0.487 \\ 0.487 \\ 0.426 \\ 0.414 \\ 0.475 \\ 0.475 \\ 0.422 \\ 0.361 \\ 0.421 \\ \end{array}$	$\begin{array}{c} 0.487 \\ 0.426 \\ 0.414 \\ 0.414 \\ 0.475 \\ 0.475 \\ 0.422 \\ 0.361 \\ 0.361 \\ 0.421 \\ 0.439 \\ \end{array}$	$\begin{array}{c} 0.487 \\ 0.426 \\ 0.414 \\ 0.414 \\ 0.475 \\ 0.475 \\ 0.422 \\ 0.361 \\ 0.421 \\ 0.439 \\ 0.439 \\ 0.373 \\ \end{array}$	$\begin{array}{c} 0.487 \\ 0.426 \\ 0.414 \\ 0.414 \\ 0.475 \\ 0.475 \\ 0.461 \\ 0.361 \\ 0.361 \\ 0.421 \\ 0.421 \\ 0.421 \\ 0.439 \\ 0.373 \\ 0.312 \\ \end{array}$	$\begin{array}{c} 0.487 \\ 0.426 \\ 0.414 \\ 0.414 \\ 0.475 \\ 0.475 \\ 0.422 \\ 0.422 \\ 0.361 \\ 0.361 \\ 0.361 \\ 0.373 \\ 0.373 \\ 0.312 \\ 0.312 \\ \end{array}$	$\begin{array}{c} 0.487 \\ 0.426 \\ 0.414 \\ 0.4175 \\ 0.475 \\ 0.475 \\ 0.422 \\ 0.422 \\ 0.421 \\ 0.421 \\ 0.361 \\ 0.421 \\ 0.421 \\ 0.421 \\ 0.434 \\ 0.373 \\ 0.312 \\ 0.312 \\ 0.414 \\ 0.414 \\ \end{array}$	$\begin{array}{c} 0.487 \\ 0.426 \\ 0.414 \\ 0.4175 \\ 0.475 \\ 0.475 \\ 0.422 \\ 0.422 \\ 0.421 \\ 0.421 \\ 0.361 \\ 0.421 $
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	<u>C</u>	0.640 >	0.682 >	0.707 >	0.759 >	0.610 >	0.651 >	0.673 >	0.667 >	0.607 >	0.670 >	0.705 >	0.681 >		0.676 >	0.676 > 0.697 >	0.676 > 0.697 > 0.687 >	0.676 > 0.697 > 0.687 > 0.685 >	0.676 > 0.697 > 0.687 > 0.685 > 0.660 >	$\begin{array}{c} 0.676 > \\ 0.697 > \\ 0.687 > \\ 0.685 > \\ 0.660 > \\ 0.660 > \end{array}$	$\begin{array}{c} 0.676 > \\ 0.697 > \\ 0.687 > \\ 0.685 > \\ 0.660 > \\ 0.712 > \\ 0.689 > \end{array}$	$\begin{array}{c} 0.676 > \\ 0.697 > \\ 0.687 > \\ 0.685 > \\ 0.660 > \\ 0.712 > \\ 0.689 > \\ 0.795 > \end{array}$	$\begin{array}{c} 0.676 \\ 0.697 \\ 0.687 \\ 0.685 \\ 0.685 \\ 0.660 \\ 0.660 \\ 0.712 \\ 0.712 \\ 0.689 \\ 0.795 \\ 0.795 \\ \end{array}$	$\begin{array}{c} 0.676 \\ 0.697 \\ 0.687 \\ 0.685 \\ 0.685 \\ 0.660 \\ 0.712 \\ 0.712 \\ 0.689 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.732 \\ \end{array}$	$\begin{array}{c} 0.676 \\ 0.697 \\ 0.687 \\ 0.685 \\ 0.685 \\ 0.660 \\ 0.712 \\ 0.712 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.732 \\ 0.732 \\ 0.708 \\ \end{array}$	$\begin{array}{c} 0.676 \\ 0.697 \\ 0.687 \\ 0.685 \\ 0.660 \\ 0.712 \\ 0.712 \\ 0.795 \\$	$\begin{array}{c} 0.676 \\ 0.697 \\ 0.687 \\ 0.685 \\ 0.660 \\ 0.712 \\ 0.712 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.770 \\ 0.770 \\ \end{array}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c} 0.676 \\ 0.697 \\ 0.687 \\ 0.685 \\ 0.685 \\ 0.712 \\ 0.712 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.795 \\ 0.798 \\ 0.708 \\ 0.708 \\ 0.708 \\ 0.708 \\ 0.770 \\ 0.848 \\ 0.770 \\ 0.848 \\ 0.770 \\ 0.771 \\ 0.848 \\ 0.771 \\ 0.771 \\ 0.719 \\ 0.710 \\$
C_3 C_4 C_5 C_6 C_7 C_8 C_9 C_{10} 0.454= 0.447= 0.634 > 0.633 > 0.951 > 0.951 > 0.951 > 0.454= 0.454 = 0.447 = 0.633 > 0.951 > 0.951 > 0.951 > 0.454 = 0.458 > 0.441 = 0.453 = 0.951 > 0.951 > 0.951 > 0.454 = 0.458 > 0.458 > 0.959 > 0.960 > 0.444 > 0.440 = 0.440 = 0.440 = 0.440 = 0.440 = 0.458 > 0.958 > 0.958 > 0.958 > 0.469 > 0.416 > 0.416 > 0.416 > 0.426 > 0.426 > 0.426 > 0.426 > 0.426 > 0.426 > 0.426 > 0.426 > 0.426 > 0.426 > 0.426 > 0.426 > 0.411 > 0.440 = 0.333 > 0.324 = 0.333 > 0.324 = 0.333 > 0.411 = 0.426 > 0.411 > 0.440 > 0.440 > 0.440 > 0.410 > 0.411 > 0.440 > 0.411 > 0.440 > 0.440 > 0.431 > 0.440 >	ç	0.648	0.682 >	0.704 >	0.654 >	0.611 >	0.643 >	0.665 >	0.610 >	0.607 >	0.671 >	0.694 >	0.621 >	0.684 >	2122	0.704 >	0.704 > 0.681 >	0.681 0.637	0.704 0.681 0.637 0.664	0.704 0.681 0.637 0.664 0.704	0.704 0.681 0.637 0.664 0.664 0.704 0.704	0.704 0.681 > 0.637 > 0.664 > 0.704 > 0.665 > 0.665 > 0.665 >	0.704 0.637 0.637 0.664 0.664 0.704 0.704 0.665 0.665 0.665 0.593	0.704 0.681 0.663 0.664 0.704 0.704 0.704 0.593 0.593 0.718 0.718	$\begin{array}{c} 0.704 \\ 0.681 \\ 0.637 \\ 0.664 \\ 0.704 \\ 0.704 \\ 0.704 \\ 0.665 \\ 0.665 \\ 0.665 \\ 0.665 \\ 0.704 \\ 0.718 \\$	$\begin{array}{c} 0.704 \\ 0.681 \\ 0.637 \\ 0.664 \\ 0.704 \\ 0.704 \\ 0.704 \\ 0.665 \\ 0.665 \\ 0.665 \\ 0.6593 \\ 0.593 \\ 0.593 \\ 0.518 \\ 0.718 \\ 0.718 \\ 0.515 \\ \end{array}$	0.704 0.637 0.664 0.704 0.704 0.704 0.665 0.665 0.665 0.593 0.718 0.718 0.720 0.720 0.720 0.721 0.721 0.721 0.781	$\begin{array}{c} 0.704 \\ 0.681 \\ 0.637 \\ 0.664 \\ 0.704 \\ 0.704 \\ 0.704 \\ 0.704 \\ 0.704 \\ 0.704 \\ 0.704 \\ 0.665 \\ 0.665 \\ 0.6593 \\ 0.593 \\ 0.593 \\ 0.593 \\ 0.593 \\ 0.593 \\ 0.593 \\ 0.593 \\ 0.515 \\ 0.515 \\ 0.515 \\ 0.515 \\ 0.640 $	$\begin{array}{c} 0.704 \\ 0.681 \\ 0.6637 \\ 0.664 \\ 0.704 \\ 0.704 \\ 0.704 \\ 0.704 \\ 0.704 \\ 0.704 \\ 0.704 \\ 0.701 \\ 0.665 \\ 0.718 \\ 0.718 \\ 0.718 \\ 0.718 \\ 0.718 \\ 0.718 \\ 0.718 \\ 0.718 \\ 0.718 \\ 0.640 \\ 0.640 \\ 0.495 \\ 0.640 \\ 0.701 \\ 0.640 \\ 0.701 $

Table 6.14: Influence of the selected VND neighborhood structure configuration on the performance of VND with respect to average $R_{\rm rel}$.

Table 6.15: Influence of the selected VND neighborhood structure configuration on the performance of VND with respect to average C_a .

Γ_0	ad			ů	\mathbf{C}_4	C S		C C	ů Č		\mathbf{C}_{10}	\mathbf{C}_{11}	\mathbf{C}_{12}
0.10		0.0 =	0.0 =		0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =
		0.0 = 0.0		0.0 =	0.0 =	0.0 =		0.0 =	0.0 =		0.0 =	0.0 =	0.0 =
		6.0 >			0.0 =	6.0 >		6.0 >	6.0 >		6.0 >	23.6 >	6.0 >
_	_	20.7 =			24.2 =	29.5 >		46.4 >	46.5 >		20.7 =	151.4 >	46.3 >
0.10		0.0 =			0.0 =	0.0 =		0.0 =	0.0 =		0.0 =	0.0 =	0.0 =
0.50		0.0 = 0.0			0.0 =	0.0 =		0.0 =	0.0 =		0.0 =	0.0 =	0.0 =
0.80		0.0 =			0.0 =	0.0 =		0.0 =	0.0 =		0.0 =	0.0 =	0.0 =
1.00		= 0.0			0.0 =	0.0 =		0.0 =	0.0 =		0.0 =	49.1 >	0.0 =
0.10		0.0 =			0.0 =	0.0 =		0.0 =	0.0 =		0.0 =	0.0 =	0.0 =
0.50		0.0 =			0.0 =	0.0 =		0.0 =	0.0 =		0.0 =	0.0 =	0.0 =
0.80		= 0.0		0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =		0.0 =	0.0 =	0.0 =
1.00		1.8 =			2.9 >	2.9 >		20.7 >	10.6 >		1.6 =	76.3 >	21.1 >
0.10	-	0.0 =			0.0 =	0.0 =		0.0 =	0.0 =		0.0 =	0.0 =	0.0 =
0.50		1.6 =			1.6 =	2.3 >		2.3 >	2.3 >		1.6 =	1.6 =	1.6 =
0.80		2.3 =			2.3 =	2.3 =		2.3 =	2.3 =		2.3 =	27.6 >	2.3 =
1.00		9.1 =			8.2 =	9.1 =		59.3 >	56.6 >		9.5 >	111.0 >	42.7 >
0.10		0.0 =			0.0 =	0.0 =		0.0 =	0.0 =		0.0 =	0.0 =	0.0 =
0.50		0.0 =			0.0 =	0.0 =		45.7 >	45.7 >	0.0 =	0.0 =	44.4 >	44.4 >
0.80		= 0.0			0.3 =	0.3 =		160.4 >	158.3 >	0.0 =	0.0 = 0.0	153.6 >	144.3 >
1.00		4.1 =			10.2 >		11.3 >	357.6 >	334.9 >	4.6 >	5.6 >		279.2 >
0.10		0.0 = 0.0			0.0 =			0.0 =	0.0 =	= 0.0	0.0 =		0.0 =
0.50		= 0.0			= 0.0			< 6.9	7.2 >	0.0 =	0.0 =		5.5 >
0.80	_	< 0.76			185.3 >			347.2 >	346.1 >	71.1 =	103.4 >		285.4 >
1.00		392.3 =		396.6 =	2393.8 >		2397.3 >	1240.5 >	1179.1 >	388.9 =	385.3 =	2838.9 >	847.2 >
0.10	-	= 0.0		0.0 =	0.0 =			0.7 >	0.7 >	= 0.0	0.0 =		0.7 >
0.50	_	24.3 >		24.1 >	68.1 >	18.8 =	68.2 >	72.9 >	72.4 >	23.8 =	23.6 =	70.5 >	61.9 >
0.80		228.2 =		265.6 =	1106.9 >	264.9 =	1111.2 >	606.6 >	478.9 >	229.5 =	227.4 =	729.3 >	416.6 >
1.00		= 7.999		1288.1 >	10411.5 >	1232.8 >	10437.2 >	2176.9 >	1913.7 >	1190.2 >	1447.0 >	5547.8 >	1480.4 >

			1000				500				200				100				50				30				20	Size	
1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	Load	
0.696 >	0.630 >	0.635 >	0.546 >	0.626 >	0.617 >	0.678 >	0.522 >	0.564 >	0.631 >	0.677 >	0.548 >	0.630 >	0.695 >	0.708 >	< 809.0	0.633 >	0.708 >	0.655 >	0.527 >	0.615 >	0.676 >	0.642 >	0.556 >	0.661 >	0.724 >	0.732 >	0.736 >	0-0	
0.500 >	0.428 >	0.411 =	0.432 =	0.468 >	0.408 >	0.429 =	0.397 =	0.381 >	0.385 =	0.420 =	0.393 =	0.398 >	0.439 =	0.443 =	0.405 =	0.412 =	0.450 >	0.421 =	0.341 =	0.398 =	0.411 =	0.386 =	0.328 =	0.440 >	0.441 =	0.417 =	0.388 =	O-R	
0.588 >	0.632 >	0.763 >	0.840 >	0.624 >	0.727 >	0.843 >	0.786 >	0.659 >	0.772 >	0.829 >	0.760 >	0.736 >	0.819 >	0.817 >	0.760 >	0.723 >	0.798 >	0.769 >	0.658 >	0.693 >	0.748 >	0.710 >	0.653 >	0.704 >	0.754 >	0.748 >	0.739 >	0-S	R
0.519 >	0.521 >	0.623 >	0.594 >	0.515 >	0.545 >	0.664 >	0.561 >	0.508 >	0.611 >	0.682 >	0.572 >	0.598 >	0.681 >	0.713 >	0.624 >	0.613 >	0.693 >	0.648 >	0.560 >	0.616 >	0.691 >	0.667 >	0.583 >	0.657 >	0.726 >	0.739 >	0.749 >	S-0	$\mathbf{R}_{\mathrm{rel}}$
0.402 =	0.384 =	0.400 >	0.490 >	0.363 =	0.349 =	0.426 =	0.437 >	0.342 =	0.382 =	0.429 >	0.415 >	0.387 =	0.434 =	0.452 >	0.428 >	0.400 =	0.434 =	0.419 =	0.367 >	0.395 =	0.427 >	0.416 >	0.372 >	0.425 =	0.447 =	0.421 =	0.395 =	S-R	
0.481 >	0.586 >	0.781 >	0.943 >	0.544 >	0.643 >	0.848 >	0.938 >	0.622 >	0.766 >	0.858 >	0.906 >	0.721 >	0.832 >	0.857 >	0.845 >	0.714 >	0.812 >	0.802 >	0.815 >	0.739 >	0.806 >	0.794 >	0.742 >	0.704 >	0.811 >	0.781 >	0.829 >	S-S	
4975.9 >	744.7 >	70.4 >	0.4 >	1640.4 >	293.5 >	8.6 >	0.0 =	230.1 >	104.6 >	27.7 >	0.0 =	37.0 >	5.5 >	2.3 >	0.0 =	13.9 =	0.0 =	0.0 =	0.0 =	5.0 =	0.0 =	0.0 =	0.0 =	45.3 >	7.3 =	0.0 =	0.0 =	0-0	
4211.0 >	436.9 >	36.1 >	0.0 =	1175.1 >	135.6 >	1.7 =	0.0 =	91.8 >	11.4 >	1.1 =	0.0 =	26.2 =	2.3 =	1.0 =	0.0 =	17.9 >	0.0 =	0.0 =	0.0 =	4.0 =	0.0 =	0.0 =	0.0 =	38.5 =	4.8 =	0.0 =	0.0 =	O-R	
4559.6 >	715.7 >	84.3 >	0.4 >	1643.4 >	292.3 >	3.8 >	0.0 =	176.5 >	89.9 >	27.7 >	0.0 =	37.2 >	5.5 >	2.3 >	0.0 =	9.6 =	0.0 =	0.0 =	0.0 =	2.5 =	0.0 =	0.0 =	0.0 =	45.1 >	7.3 =	0.0 =	0.0 =	S-0	•
2275.0 >	467.0 >	51.1 >	0.4 >	1020.1 >	274.8 >	1.3 >	0.0 =	186.6 >	89.1 >	27.7 >	0.0 =	35.9 >	5.5 >	2.3 >	0.0 =	11.2 >	0.0 =	0.0 =	0.0 =	6.5 =	0.0 =	0.0 =	0.0 =	45.0 >	7.3 =	0.0 =	0.0 =	S-O	(ล
1861.7 =	264.0 =	9.3 =	0.0 =	553.1 =	89.0 =	0.9 =	0.0 =	69.2 =	5.7 =	1.1 =	0.0 =	25.3 =	2.3 =	1.0 =	0.0 =	19.8 >	0.0 =	0.0 =	0.0 =	3.8 =	0.0 =	0.0 =	0.0 =	38.7 =	4.8 =	0.0 =	0.0 =	S-R	
2076.7 >	466.3 >																												

Table 6.16: Influence of the selected CH methods for finding an initial solution and for rebuilding it on the performance of VND.

		R	rel	C	'a
Size	Load	FirstImprove	BestImprove	FirstImprove	BestImprove
20	0.10	0.645 >	0.634 =	0.0 =	0.0 =
	0.50	0.646 >	0.633 =	0.0 =	0.0 =
	0.80	0.653 >	0.648 =	6.4 =	6.5 =
	1.00	0.602 >	0.595 =	44.3 =	41.6 =
30	0.10	0.546 >	0.532 =	0.0 =	0.0 =
	0.50	0.615 >	0.590 =	0.0 =	0.0 =
	0.80	0.634 >	0.619 =	0.0 =	0.0 =
	1.00	0.584 >	0.568 =	3.9 =	4.3 =
50	0.10	0.555 >	0.535 =	0.0 =	0.0 =
	0.50	0.629 >	0.609 =	0.0 =	0.0 =
	0.80	0.659 >	0.640 =	0.0 =	0.0 =
	1.00	0.578 =	0.588 =	12.0 =	15.8 >
100	0.10	0.618 >	0.605 =	0.0 =	0.0 =
	0.50	0.680 >	0.650 =	1.9 =	1.9 =
	0.80	0.663 >	0.637 =	4.4 =	4.4 =
	1.00	0.592 >	0.565 =	33.2 >	32.6 =
200	0.10	0.605 >	0.593 =	0.0 =	0.0 =
	0.50	0.660 >	0.638 =	18.8 =	18.8 =
	0.80	0.600 >	0.582 =	65.3 =	64.6 =
	1.00	0.524 >	0.502 =	159.7 >	147.3 =
500	0.10	0.619 >	0.594 =	0.0 =	0.0 =
	0.50	0.654 >	0.642 =	3.2 >	2.7 =
	0.80	0.590 >	0.506 =	284.1 >	164.9 =
	1.00	0.596 >	0.450 =	1610.2 >	730.3 =
1000	0.10	0.659 >	0.623 =	0.3 =	0.3 =
	0.50	0.641 >	0.563 =	70.8 >	29.5 =
	0.80	0.612 >	0.449 =	712.0 >	319.5 =
	1.00	0.562 >	0.500 =	3633.1 >	3020.2 =

Table 6.17: Influence of the selected step function on the performance of VND.

initial solution and the solutions are rebuilt by CH-R. As with LS, we can see that using CH-R for rebuilding is essential for good performance. When solving VNMP-O, it can be observed that the best choice of CH configuration for initialization depends on the instance load. For low loads, initializing with CH-O is better, for higher loads an initial VNMP solution created by CH-S leads to the best results. When minimizing C_a , using CH-S for the initial solution is almost always the best choice.

Table 6.17 shows the influence of the selected step function on the performance of VND. The results are basically the same as for LS. For better ranks, using best-improvement is essential. When trying to minimize the additional resource cost, first-improvement can keep up until size 200.

One parameter that is always of interest for VND is the contribution of the different neighborhood structures to the final solution. In Table 6.18 we present the fraction of successful executions of a neighborhood structure, i.e., the number of times a neighborhood structure found an improvement divided by the number of its executions. Note that this data is based on all

Table 6.18: Fraction of improvements found per execution of the different neighborhood structures used for VND

		1			F	ractio	n of I	mprove	ements	[%]			
Size	Load	N'_1	N'_2	N'_3	N'_4	N'_5	N'_6	\mathbf{N}_1	N_2	N_3	N_4	N_5	N_6
20	0.10	0.0	0.0	0.0	0.0	0.0	0.0	0.9	21.6	3.1	9.7	39.3	33.8
	0.50	0.0	0.0	0.0	0.0	0.0	0.0	3.8	23.7	0.8	6.1	47.2	52.1
	0.80	0.0	0.0	1.6	0.0	0.0	0.0	5.6	16.6	3.0	11.2	49.8	46.1
	1.00	2.7	3.3	9.1	0.0	0.0	0.0	0.6	22.8	11.5	9.7	40.5	52.1
30	0.10	0.0	0.0	0.0	0.0	0.0	0.0	7.5	30.9	0.9	18.8	42.6	48.3
	0.50	0.0	0.0	0.0	0.0	0.0	0.0	9.0	30.8	0.9	17.8	47.4	61.4
	0.80	0.0	0.0	0.0	0.0	0.0	0.0	7.2	24.4	3.6	21.4	49.3	63.7
	1.00	1.9	1.6	1.7	0.0	0.0	0.0	6.7	28.2	11.2	21.2	47.0	62.0
50	0.10	0.0	0.0	0.0	0.0	0.0	0.0	8.2	34.3	3.0	27.5	41.7	59.3
	0.50	0.0	0.0	0.0	0.0	0.0	0.0	6.2	27.6	1.1	26.2	48.0	74.0
	0.80	0.0	0.0	0.0	0.0	0.0	0.0	9.6	32.2	1.4	21.5	46.5	78.5
	1.00	32.9	6.3	1.2	0.0	0.0	0.0	12.1	37.4	12.7	22.7	48.7	75.1
100	0.10	0.0	0.0	0.0	0.0	0.0	0.0	9.0	35.0	2.0	35.4	45.4	75.8
	0.50	0.0	0.0	0.2	0.0	0.0	0.0	5.6	30.8	1.1	30.2	51.6	85.0
	0.80	0.0	0.0	0.4	0.0	0.0	0.0	5.5	32.4	2.0	25.3	52.8	86.0
	1.00	1.4	1.0	2.2	0.0	0.0	0.0	8.4	33.9	7.6	24.3	50.7	86.9
200	0.10	0.0	0.0	0.0	0.0	0.0	0.0	13.2	33.7	1.4	53.1	33.6	76.6
	0.50	0.0	0.0	0.2	0.0	0.0	0.0	7.9	38.8	1.0	36.4	38.3	91.1
	0.80	0.7	0.3	0.8	0.0	0.0	0.1	8.5	38.3	4.0	27.8	39.4	93.9
	1.00	8.2	4.2	1.4	0.0	0.0	0.1	8.9	41.1	10.2	23.3	38.4	94.7
500	0.10	0.0	0.0	0.0	0.0	0.0	0.0	15.5	35.3	0.6	59.6	22.9	81.5
	0.50	0.1	0.0	0.4	0.0	0.0	0.0	9.4	39.7	0.7	40.3	33.6	94.2
	0.80	1.5	1.2	1.6	0.0	0.0	0.0	10.8	43.3	6.8	37.4	45.7	98.0
	1.00	9.2	11.2	4.7	0.2	0.0	0.2	11.4	53.3	24.2	39.9	56.4	99.8
1000	0.10	0.1	0.0	0.2	0.0	0.0	0.0	17.6	34.5	0.2	60.7	16.7	85.9
	0.50	0.6	0.2	0.8	0.0	0.0	0.0	13.4	45.5	2.6	56.5	56.4	98.2
	0.80	6.6	3.8	2.6	0.0	0.0	0.0	17.0	57.2	17.7	62.2	85.6	100.0
	1.00	24.8	23.0	6.2	0.4	0.3	1.0	19.6	72.7	40.0	76.7	96.2	100.0

tested VND configurations. Not all neighborhood structures are present in all configurations and are also executed in different orders so what we show here is the average behaviour of a neighborhood structure.

The first thing noticeable from Table 6.18 is that the neighborhood structures used with Only-Overloading (marked with a prime) seldom contribute any improvements. Only for the highest loads and for the largest instance sizes do they improve solutions with a significant probability. The neighborhood structures that do not restrict themselves to parts of the substrate network which are overloaded have a far better success rate. However, there are still neighborhood structures that perform better than others. Just implementing virtual arcs in another way (N_1) does not find improvements very often. Only for instances of size 1000 is the success rate consistently above 10%. Reimplementing complete virtual networks (N_3) achieves this success rate only for instances of highest load. This is also the neighborhood structure that is most strongly influenced by the load of an instance. The best performance is consistently achieved by N_6 (RemapVnodeTAP). Note especially the 100% success rate for high load instances of size 1000.

Conf.	Step-Function	Init.	Reb.	\mathbf{R}_{rel}	C_{a}	# Valid	t[s]
C_9	BestImprove	CH-S	CH-R	0.095 =	29.6 >	772	221.4
C_1	BestImprove	CH-S	CH-R	0.099 >	31.9 >	773	226.9
C_3	BestImprove	CH-S	CH-R	0.100 >	35.0 >	773	225.3
C_9	BestImprove	CH-O	CH-R	0.114 >	36.9 >	770	221.4
C_{10}	BestImprove	CH-S	CH-R	0.117 >	32.8 >	776	232.7
C_3	BestImprove	CH-O	CH-R	0.123 >	39.3 >	766	224.7
C_1	BestImprove	CH-O	CH-R	0.124 >	43.9 >	769	226.0
C_9	FirstImprove	CH-S	CH-R	0.133 >	28.8 >	769	205.0
C_{10}	BestImprove	CH-O	CH-R	0.133 >	50.8 >	772	230.8
C_3	FirstImprove	CH-S	CH-R	0.138 >	34.0 >	769	211.6

Table 6.19: Top 10 VND Configurations according to $R_{\rm rel}$.

Table 6.20: Top 10 VND Configurations according to C_a .

Conf.	Step-Function	Init.	Reb.	\mathbf{R}_{rel}	$\mathbf{C}_{\mathbf{a}}$	# Valid	t[s]
C_1	BestImprove	CH-S	CH-S	0.620 >	15.6 =	773	271.9
C_5	BestImprove	CH-S	CH-S	0.618 >	22.2 >	769	264.4
C_{10}	BestImprove	CH-S	CH-S	0.628 >	23.2 =	774	269.5
C_9	BestImprove	CH-S	CH-S	0.616 >	23.4 >	768	268.5
C_3	BestImprove	CH-S	CH-S	0.620 >	24.6 >	768	271.8
C_1	BestImprove	CH-S	CH-O	0.403 >	28.3 >	773	275.3
C_9	FirstImprove	CH-S	CH-R	0.133 >	28.8 >	769	205.0
C_5	BestImprove	CH-S	CH-R	0.408 >	29.1 >	772	216.2
C_9	BestImprove	CH-S	CH-R	0.095 =	29.6 >	772	221.4
C_1	BestImprove	CH-S	CH-R	0.099 >	31.9 >	773	226.9

First of all, this is based on very few executions compared to the other neighborhood structures (168 and 17, compared to 2000–5000). Secondly, that means that the VND was terminated due to run-time and not because it finished. When VND is finished, every neighborhood structure has failed to improve the solution at least once. We do not present an evaluation of the time required to search each neighborhood structure because the individual times were so small that no significant conclusion can be drawn from them.

Table 6.19 shows the 10 best VND configurations based on R_{rel} . We can see that in comparison to LS, the required run-time has doubled. There is a surprising diversity of employed neighborhood structure configurations, configurations C_1 , C_3 , C_9 , and C_{10} can be observed. Far more important seems to be the use of best-improvement and CH-R as rebuilding CH. The best VND configuration for solving VNMP-O uses C_9 with best improvement, creates its initial solution with CH-S and uses CH-R for rebuilding the solution. We will denote this configuration by VND-O.

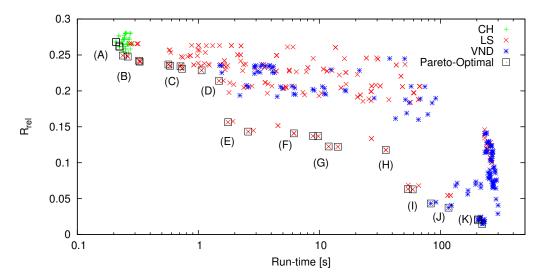


Figure 6.1: Pareto-front of the tested heuristics regarding average R_{rel} and run-time over all instances.

The 10 best VND configurations with respect to C_a are shown in Table 6.20. Even more neighborhood structure configurations are present, C_5 now also occurs. Rather surprising is that for the very best performance, CH-S has to be used for rebuilding instead of CH-R. There is even one VND configuration that uses CH-O for rebuilding before configurations using CH-R appear. It can be observed that for VND, concentrating on low average C_a costs a lot of performance with respect to the relative rank. The best VND configuration achieves a relative rank of 0.62. For VND, the top configuration for low R_{rel} (VND-O) also appears in the top 10 configurations for low C_a . This allows us to directly observe the cost (with respect to R_{rel}) of minimizing C_a . VND-O appears at the ninth position. Just by using CH-S for rebuilding instead of CH-R, the fourth position can be reached. This reduces the average C_a from 29 to 23, but increases the R_{rel} from 0.1 to 0.6.

The best configuration for solving VNMP-S uses C_1 with best-improvement and CH-S for both building the initial solution and rebuilding it. We will denote this configuration by VND-O. As with Local Search, the best configuration with respect to low C_a is not the same as the best configuration with respect to the number of instances solved. The best configuration in this regard uses C_{10} with best improvement, creates the initial solution with CH-O and rebuilds with CH-S. It is able to find a valid solution to 778 VNMP instances.

6.5.4 Comparing CH, LS and VND

Until now, we have considered the different CH, LS and VND algorithms separately. We have also mostly neglected a very important property of the presented algorithms: their required runtime. In this section, we will analyze the trade-off between the performance of the algorithms and the required run-time. We assume knowledge of Pareto-optimality (c.f. Section 2.2.3). Figure 6.1 shows the trade-off between low R_{rel} and low run-time for all tested heuristics over all instances. Note that most of the CH algorithms have been cut off, since they produce far worse results than the LS and VND configurations.

The best non-dominated construction heuristics are marked by label (A). Since we are trying to solve VNMP-O, we would expect that CH-O is present. However, this is not the case. The CH configurations at (A) are just related to CH-O in the sense that they too use DLHeavyVN as SVN strategy, MostFree as IVN strategy and emphasize the implementation of virtual arcs. The main difference to CH-O is that they use Spread-1 (faster but slightly worse) or Spread-2 as IVA strategy. CH-O itself does not occur in this graph (as Pareto-optimal CH configuration), it is dominated by the fastest LS configuration. This shows that the "best" configuration of an algorithm is not necessarily useful.

Two clusters of LS configurations are marked by (B). Since they perform on par with CHs in term of required run-time (and R_{rel}), it is not surprising that they use first-improvement and OnlyOverloading. They employ the RemapVnode neighborhood structure. The difference between the two clusters is the initialization CH, using CH-S is faster but CH-O gives better results. The three configurations at (C) use the ClearSarc, ClearSnode and RemapSlice neighborhood structures, still with first-improvement, OnlyOverloading and CH-O for initialization.

Marked with (D) is the first configuration using a complete neighborhood structure with OverloadingFirst. The employed neighborhood structure is still RemapSlice, which is not surprising because it has nearly the same size whether using OnlyOverloading or OverloadingFirst. Therefore, the increase in run-time is not large. At (E), a huge improvement in performance is evident. It is caused by switching from CH-O to CH-R as rebuilding CH. From here on out, CH-R is exclusively used as rebuilding strategy.

The configurations at (E) also mark the begin of a clearly visible pattern. (E) and (F) mark a group of three clusters, (G) and (H) mark a group of three clusters, and (I) and (J) (the dominated LS configurations) mark a group of three clusters. The three cluster groups represent different neighborhood structures, at (E) RemapSlice is used, at (G) RemapVnode and ClearSnode at (I). Analyzing one cluster group further reveals that the changes in performance levels are caused by going from CH-S to CH-O as initialization heuristic and then by using best-improvement instead of first-improvement. The differences at the lowest level are caused by using OverloadingFirst instead of None as neighborhood prioritization strategy. Putting this together, that means that for example at (G) and (H), the RemapVnode neighborhood structure is used. Starting from the fastest (and worst with respect to R_{rel}) configuration of LS, we know that it uses OverloadingFirst and CH-S for initialization. The next slower configuration switches to None. The following increase in performance is caused by using CH-O for initialization (and going back to OverloadingFirst). The last configuration at (G) again switches to None. (H) marks the same configuration, but now using best-improvement. Since this pattern repeats itself for three different neighborhood structures, we can state with some certainty the following: Using best-improvement instead of first-improvement has a significant performance penalty for a small gain in R_{rel} . Initial solutions created by CH-S let LS terminate earlier but with a slightly worse solution than when using CH-O. Using OverloadingFirst gives an edge with respect to required run-time while being R_{rel} neutral, even when using first-improvement. Note that LS-O (like

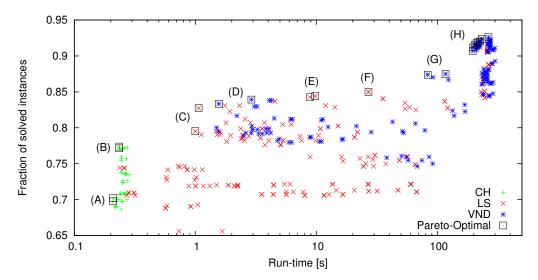


Figure 6.2: Pareto-front of the tested heuristics regarding the fraction of solved instances and run-time over all instances.

CH-O earlier) is dominated by VND configurations at (J), which achieve better results while requiring the same amount of time.

The VND configurations at (J) use C_{12} , CH-S for initialization and CH-R for rebuilding. The difference between the two Pareto-optimal configurations is once again the employed step function, with first-improvement being faster but producing slightly worse results. The cluster of VND configurations at (K) uses C_9 . The best configuration is VND-O.

In Figure 6.2 we present the same trade-off analysis, but now with respect to the number of solved instances. Again, most of the tested CH configurations have been cut off to focus on the interesting configurations. At (A), we see the fastest non-dominated CH configurations, which are relatives of CH-O. The difference lies just in the selected SVA strategy. (B) marks (relatives of) CH-S.

At (C), we see the fastest LS configurations. As was the case for minimizing R_{rel} , they use RemapSlice, first-improvement, OnlyOverloading, and initialize with CH-S. The better configuration (in terms of solved instances) uses CH-R instead of CH-O for rebuilding. At (D), we can observe a phenomenon that did not occur previously. It marks VND configurations which are fast enough to beat LS configurations by a large margin and also achieve excellent results. Fast VND configurations could also be seen in the previous figure, but they were far worse than LS in terms of performance. The neighborhood structure configuration used at (D) is C_8 . This is remarkable, since no analysis we have carried out previously indicated that C_8 is useful. Its main strength is speed, which is achieved by using three neighborhood structures in OnlyOverloading configurations. The difference in the two marked configurations at (D) is again the step-function. After the incursion of VND, (E) and (F) mark the only complete (i.e., not using OnlyOverloading) neighborhood structures able to compete in terms of solved instances: RemapVnode. At (E) first-improvement is used, at (F) best-improvement. At (G), VND is used in configuration C_{12} , which derives its advantage in execution speed from the fact that it does not utilize the RemapVnodeTAP neighborhood structure. (H) marks a mix of different configurations, all of which use RemapVnodeTAP, also containing VND-S. The slightly faster configurations use first-improvement, the slower ones best-improvement.

6.6 Conclusion & Future Work

In this chapter, we compared 512 CH, 216 LS, and 144 VND algorithms. We could show that for the VNMP, each algorithm class has its application area: CHs for finding solutions fast, VND for finding the best solutions and LS covering the range in-between, depending on the used neighborhood structures. The flexibility of LS was especially surprising, since it was able to outperform CH configurations on one end of the spectrum and VND configurations on the other. For CHs, the most important strategy is the target choice for virtual nodes, so this is a clear area of interest for future improvements. For LS we could see that best-improvement works slightly better than first-improvement, but at a significant run-time cost. Reducing the neighborhood size also reduced the performance, but brought the execution speed into CH territory. VND benefited from the reduced neighborhoods as well when searching for valid solutions. For LS, the initialization strategy has a pronounced influence on the result. The discussed VND variants produced the best results, but at a high run-time cost.

We could also show that for all of the compared algorithms, every parameter has an influence on the final outcome and in addition, synergy between settings was very important to produce the best results. Quite often it was the case that a combination of settings that produce the best results on average does not result in a configuration achieving the very best performance.

A major direction for promising future work is parameter tuning. For CH, we selected three out of 512 configurations and based all further results upon them. It might be the case that other configurations produce even better results. An indication of this is that the handcrafted CH-R works far better for rebuilding solutions than CH-O or CH-S. Searching the RemapVnodeTAP neighborhood structure more efficiently might also be a promising area for future research, leading to Very Large Neighborhood Search [2].

CHAPTER

Memetic Algorithm

7.1 Introduction

This chapter presents a Memetic Algorithm (MA) for solving the VNMP. For the basics of Genetic Algorithms (GAs), and its extension the Memetic Algorithm, see Section 2.2.9. During the following discussions, our main focus will be to answer the following questions in the context of the VNMP:

- What are suitable solution representations and crossover operators, and what is their influence?
- Does the crossover operation have a beneficial influence on the final outcome, or is mutation as the only variation operator enough for good performance? In other words, does the application of a full Genetic Algorithm bring any benefit?
- Is the time for local improvement well spent, i.e., does it make sense to use a Memetic Algorithm instead of a Genetic Algorithm?

The main motivation for developing a Memetic Algorithm for solving the VNMP is that it is a population based algorithm. In the end, we get a collection of very good solutions instead of a single one. When we simply want to implement the cheapest solution, it does not matter that we have multiple high quality solutions. In practice however, there are often factors that require consideration, but are not included in the objective. By having a collection of good solutions available, we can select according to additional external criterions. By applying a local improvement technique, we hope to speed up the process of finding good solutions.

In Section 7.2, we will discuss the relevant background with respect to Genetic Algorithms with focus on different solution representations applicable to the VNMP. The proposed Memetic Algorithm is outlined in Section 7.3. For the evaluation of the algorithms see Section 7.4. We conclude in Section 7.5. The algorithm presented in this chapter has been published in [89].

Alg	gorithm 7.1: Memetic Algorithm for the VNMP
I	nput : VNMP instance I
C	Dutput: Solution S for I
1 P	Population P;
2 II	nitializePopulation(P,I);
3 W	<pre>while !terminate() do</pre>
4	Solution p1=select(P);
5	Solution p2=select(P);
6	Solution offspring=crossover(p1,p2);
7	mutate(offspring);
8	copyArcs(offspring,p1,p2);
9	localImprovement(offspring);
10	insert(P,offspring);
11 e	nd
12 re	eturn best(P);

7.2 Background and Related Work

The problem representation plays a crucial role for the performance of a Genetic Algorithm. As an example, its influence in the context of the Travelling Salesman Problem is discussed in [113]. In particular, there is a special representation designed for problems where entities have to be grouped together (grouping problems) [49], like in the case of the VNMP virtual nodes that are mapped to the same substrate node (see the following section for details). We will utilize this representation to define a GA. Successful applications of this representation include the access node location problem [3] and the Multiple Travelling Salesman Problem [47]. However, it is not clear that this representation is always advantageous when applied to grouping problems. For instance, [51] reports a successful application of a GA to the generalized assignment problem, without using this representation. Also, its robustness is questioned in [20]. Therefore, we set to goal of analyzing the performance implications of different representations for the VNMP. For an overview on Memetic Algorithms, see Section 2.2.9.

7.3 A Memetic Algorithm for the VNMP

Algorithm 7.1 shows the Memetic Algorithm for the VNMP in pseudocode, which we will describe in this section. In Chapter 6 we have shown that the most important step while constructing a solution to the VNMP is the choice of the location of the virtual nodes in the substrate. Therefore, we designed the Genetic Algorithm to work primarily on finding good mapping targets for virtual nodes and use other algorithms to create a complete solution, i.e., to implement the virtual arcs. The main task of the GA is thus to assign virtual nodes to substrate nodes. But do we actually care where a single virtual node is mapped? It matters if it is the first node using a particular substrate node, because then the associated usage cost has to be paid. Otherwise,

	1	2	3	4	5	6	7	8	9		1	2	3	4	5	6	7	8	9
a) P1:	D	C	D	D	В	С	Α	В	A	UXD:	D	C	D	Е	A	D	Α	В	В
P2:	В	А	C	Е	A	D	D	E	В	UXD':	$\{5,$	7}	{8,9)}	{2}	{1	,3,6}	{	4}
	А		В		С		D		E		A	1	В		С		D]	E
b) P1':	{7,	9}	$\{5, 8$	3}	$\{2, 6\}$	} {1	, 3, 4	} •	{}	UXA:	{7	'}	$\{1, 9\}$)}	$\{3\}$		{}	{4	,8}
P2':	$\{2,$	5}	$\{1, 9\}$)}	{3}	{	$6,7\}$	{4	,8}	UXB:	{7	7}	{9}	$\left \right $	{}	{	$1, 3\}$	{4	,8}

Figure 7.1: Comparison of different implementations of uniform crossover for the direct (a) and grouping (b) representations.

the concrete mapping decision does not directly influence the solution cost. The one thing that matters though is that there are enough free resources, and this is determined to a large extent by the other virtual nodes also mapped to the same substrate node. Therefore, it is important to find good groups of virtual nodes that can be mapped to the same substrate node, without requiring more than the available amount of resources. This gives raise to two different ways of solving the mapping problem: finding a substrate node for every virtual node or finding a good group of virtual nodes for every substrate node. We study the two corresponding solution representations. The first representation is a simple vector that specifies the mapping target for each virtual node. We will call this representation the direct representation. The second representation focuses on the grouping aspect and represents a solution as a vector of sets, which specify the virtual nodes mapped to each substrate node. We will call this representation. Figure 7.1 shows examples of the direct and grouping representations of the same VNMP solution. P1 and P2 are solutions in direct representation, P1' and P2' are the grouping representations of the very same solutions. For instance, P1 shows that the virtual nodes 1, 3 and 4 are mapped to substrate node D. Correspondingly, P1' contains the set $\{1, 3, 4\}$ for substrate node D.

The employed representation influences how the crossover operator works. As its basic scheme, we chose uniform crossover. The simpler variants of one-point and two-point crossover were rejected based on preliminary results, which showed their inferiority.

For the direct representation, the uniform crossover works in a straight forward manner. For every virtual node the mapping target is randomly selected from one of the parents. Figure 7.1 shows a possible result of the uniform crossover of P1 and P2 in direct representation. We will call this crossover operator UXD. The marked components in the figure are the ones selected to be carried over to the offspring. UXD' shows the translation of UXD to the grouping representation for reference later on.

The uniform crossover for the grouping representation utilizes the same principle. For every substrate node, the virtual nodes mapped to it are chosen randomly from one of the parents. We will call the set of virtual nodes mapped to a substrate node a virtual node group from here on out. Due to the solution representation based on sets, two effects can occur that are not possible with the direct representation. In each solution, a virtual node is part of exactly one virtual node group. When none of those groups are selected to be present in the offspring, a virtual node

remains unmapped after the crossover operation. If both groups are selected, then the virtual node would be mapped twice, which is not allowed. The first problem can be remedied by just utilizing the mapping decision of one of the parents for all unmapped virtual nodes after the crossover procedure has finished. To solve the second problem, we override the old mapping with a newer mapping. This means that the sequence in which the groups are copied matters, since the later copy may disturb an earlier one. We will compare two different copying strategies: copying all groups of one parent, then all groups of the other (UXA), and copying the groups in order of the substrate node labels (UXB). Note that our decision to override the old mapping was arbitrary. Equally valid would be to keep the first mapping decision for a virtual node. In the end, this does not make a difference as all we will discuss in the following would still apply. Figure 7.1 shows the result of applying these crossover operators using parents P1' and P2'. For UXA, we see for example that only node 7 is mapped to substrate node A, instead of 7 and 9, which are mapped for P1'. This is because after all groups for P1' have been copied (for substrate nodes A and D), those of P2' are transferred. In particular, the group for substrate node B, containing virtual nodes 1 and 9. Node 9 therefore cannot also be mapped to substrate node

A and is removed from there; only virtual node 7 remains. For UXB, the substrate node groups are copied according to the order of substrate nodes, first the node group from P1' for substrate node A, the node group of P2' for substrate node B, and so on. We can observe the same destruction of groups as for the UXA crossover. For example, after UXB has finished, no virtual node is mapped to substrate node C, even though the group containing virtual node 3 has been copied from P2'. This is because in the following step, the group for D was copied from P1', which also contains virtual node 3, so it is removed from C. Note that for both crossover operators, some virtual nodes remain unmapped.

The main idea of the crossover operator in general is to combine important solution properties from the parents to generate superior offspring, so we want as much information from both parents to be present in the offspring as possible. In our case, that means keeping the virtual node groups intact. The marked regions in the crossover results of UXD', UXA and UXB show the groups that have survived without node removal. We can see that for UXA three groups have survived, for UXB one group has survived, and no group survived UXD. The bad performance of UXD with respect to groupings was the reason why the grouping representation was introduced in the first place [49]. However, there is also a big difference between UXA and UXB. With UXA, at least all virtual node groups of the second parent, which are selected for crossover will survive (which are half of the groups in the expected case). With UXB, only the last group that is copied is guaranteed to survive. Therefore, we use UXA when comparing the different representation possibilities for the VNMP.

After the crossover operation we apply the mutation operator, which we will call ClearSnodes mutation, with a probability of p_m . The ClearSnodes mutation clears a fraction of substrate nodes by mapping virtual nodes to substrate nodes that are not selected to be cleared, if it is allowed by the mapping constraints. In Chapter 6, we introduced a neighborhood structure based on the same principle. It was in fact the inspiration for this mutation operator. This fraction of cleared nodes is chosen uniformly at random from [0, r], but at least one node is cleared. In this work we used $p_m = 0.2$ and r = 0.2 based on preliminary results, which also showed that mutation is required for good performance.

In addition, we evaluated other, more standard approaches for mutation, like moving a virtual node to some other substrate node or swapping two virtual nodes (which are mapped to different substrate nodes). The ClearSnodes mutation was shown to be clearly superior.

Until now, we have neglected the implementation of virtual arcs. It is also a part of the solution representation, even though the crossover and mutation operators do not work on them directly. Since the arc implementation may represent a significant amount of work done by the local improvement, and the basic idea of crossover is to transfer as much information as possible from the parents to the offspring, we copy the arc implementation of the parents once the mapping for the virtual node is fixed. For every virtual arc f, we check the locations of s(f) and t(f) in the substrate graph for both parents and the offspring. If one parent utilizes the same mapping locations as the offspring, we copy its arc implementation. If both parents are compatible, the arc implementation is chosen randomly from one of the parents. If the mapping is different from both parents, the arc remains unimplemented. Unimplemented arcs will be assigned an implementation during the local improvement phase.

Since we want to check whether the time spent for local improvement actually improves the performance of the algorithm, we either use a Variable Neighborhood Descent [74] to perform local improvement, or we skip local improvement and apply a Construction Heuristic instead. The only reason for applying the Construction Heuristic is to implement all virtual arcs that have not been implemented yet to guarantee that after this step a complete solution has been generated. We selected the best Construction Heuristic for solving the VNMP-O presented in Chapter 6, CH-O, which means that virtual arc implementations are paths that cause the least increase in the substrate usage cost C_u without increasing the additional resource cost C_a . In this chapter, we will call this method CH.

As for the Variable Neighborhood Descent, we chose C_{12} as defined in Section 6.4. This configuration utilizes the neighborhood structures RemapVnode, ClearSarc, and ClearSnode (in this order). First-improvement is used to search the neighborhoods. CH-R is employed for solution reconstruction. This configuration was chosen based on the results presented in Section 6.5.4. It achieves the best possible results without using the RemapVnodeTAP neighborhood structure, which reduces the time requirements. We will denote this configuration by VND. We are also going to compare the Memetic Algorithm to VND-O, the best VND configuration for VNMP-O. The local improvement method is executed without time-limit.

The newly created and improved offspring is immediately inserted back into the population and replaces the worst solution present (steady-state GA), unless the offspring is already present in the population. Two individuals are considered to be equal, if they specify the same mapping. At this point, one GA iteration is complete, and the next one begins by utilizing a binary tournament to select the parents for the next crossover operation. Until now, we have neglected the problem of population initialization for reasons that will become obvious shortly. The main aim when initializing a population is the creation of a diverse set of good solutions. For the VNMP, there are different possibilities. One could simply randomly map virtual nodes to one of the allowed substrate nodes. Mapping virtual nodes in a way that tries to minimize the increase in C_u is another. Preliminary results showed that these approaches, while creating a very diverse set of initial solutions, do not work well, because VND requires a lot of time to improve the offspring during the initial iterations. Therefore, we chose a different approach: we create one good

solution by using VND, and then apply the mutation operator with r = 0.2 to generate all other initial solutions. This has the additional benefit that the MA will have a good solution from the start. A population size of 10 was used.

7.4 Results

As in the previous chapter, we set $p^{\text{CPU}} = 1$ and $p^{\text{BW}} = 5$ to reflect the fact that it is easier to increase the CPU power of a router than to increase the bandwidth of a network connection. With these costs, even if we are unable to find a valid solution to a VNMP instance, we are able to derive a cost effective way to be able to host the current virtual network load.

To evaluate the performance of the proposed Memetic Algorithm, we used the VNMP instance set as presented in Chapter 5 in the same configuration as in Chapter 6 to allow meaningful comparisons. That means we tested all instances of the instance set with loads 0.1, 0.5, 0.8 and 1, which creates a total of 840 test instances. A run-time limit of 200 seconds was applied for instance sizes up to 100 nodes, 500 seconds were used for the larger instances. These run-time limits were chosen to reflect practical usage scenarios. Virtual networks are meant to be dynamic. Even if we assume that they only change on an hourly basis, spending 500 seconds to find a good solution means nearly 14% of the time the solution is going to be useful has already elapsed.

We compare four different MA configurations: direct representation with CH as local improvement (D-CH), with VND as local improvement (D-VND), grouping representation using UXA and CH as local improvement (G-CH) and with VND as local improvement (G-VND). To fully compare the influence of the employed crossover operator, we also test the grouping representation with UXB and CH as local improvement (G-CH-B) and VND as local improvement (G-VND-B). Furthermore, we investigate the G-VND variant with disabled crossover (G-VND-N), i.e., one individual is chosen from the population, mutated, improved and then reinserted. Preliminary experiments showed that the performance of G-VND-N cannot be improved by increasing (or decreasing) r. Finally, to be able to analyze the improvement caused by the GA around VND, we also present the results for VND alone and in addition compare to VND-O. In this chapter, our main focus will be solving VNMP-O, so we are mainly concerned with the average relative rank the different algorithms achieve. For reference, we also show the performance of the algorithms with respect to VNMP-S, both in terms of solved instances (i.e., the solution found to an instance has an additional resource cost of zero) and in terms of average C_a .

Table 7.1 shows the average performance of the tested algorithms for different instance sizes. It can be seen that D-VND and G-VND achieve the best results for all instances up to and including size 200. For sizes 500 and 1000 VND-O performs best. However, VND-O also takes more time (a maximum of 1000 seconds was allowed in Chapter 6) than the 500 seconds allowed for all GA variants for these sizes. The GA variants based on CH achieve the best results at sizes 100 and 200. With smaller instances, local improvement with VND is better than a higher number of iterations made possible by not spending time on local improvement. However, starting with size 100, performing more iterations gets more important and CH outperforms VND. Even though the configuration of VND was selected for low run-time requirements, the number of iterations for larger instances is very low. For the largest instances, the final result is basically the one

Table 7.1: Average relative rank R_{rel} and its relation to the best result, average number of iterations (Its.) for GA based algorithms or average run-time for the other algorithms, fraction of solved instances (Solv.) in percent and average C_a for all compared algorithms per instance size.

	Size	D-CH	G-CH	D-VND	G-VND	G-CH-B	G-VND-B	G-VND-N	VND	VND-O
$R_{\rm rel}$	20	0.352 >	0.367 >	0.206 =	0.196 =	0.366 >	0.231 >	0.205 =	0.914 >	0.761 >
	30	0.442 >	0.452 >	0.232 =	0.231 =	0.445 >	0.230 =	0.247 =	0.922 >	0.727 >
	50	0.475 >	0.475 >	0.249 =	0.259 =	0.471 >	0.288 >	0.378 >	0.942 >	0.746 >
	100	0.409 =	0.408 =	0.388 =	0.411 =	0.419 =	0.364 =	0.546 >	0.969 >	0.614 >
	200	0.393 =	0.373 =	0.425 =	0.410 =	0.389 =	0.461 >	0.633 >	0.941 >	0.379 =
	500	0.438 >	0.444 >	0.609 >	0.645 >	0.402 >	0.628 >	0.757 >	0.992 >	0.143 =
	1000	0.525 >	0.547 >	0.715 >	0.729 >	0.536 >	0.715 >	0.788 >	0.664 >	0.240 =
GA: Its.	20	393268	357568	8185	8169	359589	8141	8265	0.2	0.4
Other: t[s]	30	259702	241245	3899	3854	238717	3869	3912	0.7	1.3
	50	163663	151068	1663	1671	151207	1657	1691	2.1	4.2
	100	63276	59591	314	325	59571	315	328	16.0	29.7
	200	109125	104063	333	352	103817	340	355	40.2	119.7
	500	43412	42076	94	95	42057	93	99	126.6	605.1
	1000	13631	13348	23	25	13407	24	27	397.1	828.1
Solv. [%]	20	97.5	97.5	100.0	100.0	97.5	100.0	100.0	96.7	97.5
	30	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	50	99.2	99.2	100.0	100.0	99.2	100.0	100.0	99.2	98.3
	100	95.0	95.0	100.0	100.0	95.0	99.2	99.2	95.0	97.5
	200	94.2	93.3	95.8	96.7	94.2	96.7	97.5	90.0	98.3
	500	77.5	78.3	76.7	79.2	78.3	77.5	76.7	73.3	90.8
	1000	60.0	59.2	58.3	57.5	59.2	61.7	57.5	57.5	61.7
C_a	20	9.9	8.4	0.0	0.0	7.4	0.0	0.0	13.1	4.5
	30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	50	4.9	4.9	0.0	0.0	4.9	0.0	0.0	4.9	2.1
	100	5.3	6.3	0.0	0.0	5.3	2.1	0.6	6.3	3.3
	200	5.5	4.1	3.0	3.4	5.6	7.6	4.4	19.0	1.0
	500	62.4	73.6	77.3	76.6	70.9	64.4	65.9	97.6	13.9
	1000	215.4	215.5	215.9	214.7	216.2	214.1	215.9	184.1	198.9

created during population initialization. Surprisingly, UXB achieves the same results as the algorithms using UXA. G-VND has a slight advantage compared to G-VND-B, but no clear pattern is visible. Disabling crossover (G-VND-N) however has a pronounced negative effect on the results for medium sized instances. Generally, no significant differences could be observed between direct and grouping representations. The influence of the type of local improvement is far more pronounced. The results for VND show that the combination with the GA has a significant positive effect on the achieved results.

Table 7.2 shows the average performance of the tested algorithms for different loads. For low loads, every tested GA achieves basically the same results, except G-VND-N, which performs far worse due to the disabled crossover operator. For medium load (0.5), a direct representation and CH as local improvement are essential. Interestingly, the grouping representation is only able to achieve the same level of performance by using UXB. It seems as if the additional disruption caused by the crossover operation is the key for good performance for this load case. Higher loads require a MA for the best performance. For load 0.8 the direct representation is

Table 7.2: Average relative rank R_{rel} and its relation to the best result, average number of iterations (Its.) for GA based algorithms or average run-time for the other algorithms, fraction of solved instances (Solv.) in percent and average C_a for all compared algorithms per load.

	Load	D-CH	G-CH	D-VND	G-VND	G-CH-B	G-VND-B	G-VND-N	VND	VND-O
R _{rel}	0.10	0.302 =	0.297 =	0.317 =	0.319 =	0.289 =	0.312 =	0.411 >	0.893 >	0.527 >
	0.50	0.355 =	0.382 >	0.449 >	0.454 >	0.358 =	0.432 >	0.561 >	0.981 >	0.479 >
	0.80	0.492 >	0.492 >	0.425 =	0.473 >	0.500 >	0.475 >	0.559 >	0.912 >	0.495 >
	1.00	0.584 >	0.582 >	0.423 =	0.401 =	0.582 >	0.448 >	0.499 >	0.839 >	0.562 >
GA: Its.	0.10	430129	401525	6354	6321	401711	6323	6373	5.6	41.7
Other: t[s]	0.50	82299	75242	1016	1020	74971	1010	1027	50.2	218.3
	0.80	48183	43667	537	547	43522	533	566	111.2	316.2
	1.00	37147	33257	385	393	33147	385	420	166.0	331.6
Solv. [%]	0.10	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	0.50	99.0	98.6	97.1	97.6	98.6	99.0	97.1	95.7	99.0
	0.80	88.6	88.6	88.6	88.1	88.6	90.0	88.1	85.7	91.9
	1.00	68.6	68.6	74.8	76.2	69.0	73.8	75.2	68.1	77.1
C_a	0.10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.50	0.1	0.1	0.6	0.4	0.1	0.2	0.5	0.7	0.1
	0.80	19.0	18.3	21.4	26.5	21.1	23.1	19.4	30.0	11.4
	1.00	154.3	160.3	147.3	141.6	156.1	141.5	144.0	155.0	116.3

significantly better, for load 1 the grouping representation has an advantage, but is not significantly better. Note that disabling the crossover operation (G-VND-N) results in bad solutions for every tested load case. Also, VND-O is outperformed by the MAs for every load case. Keep in mind that for the highest load, VND-O requires the same amount of time as all the tested GA variants, which have an average run-time of 328.5 seconds due to the set run-time limits. However, VND-O is able to solve more instances of the highest load than all other algorithms.

7.5 Conclusion and Future Work

In this chapter we have introduced a Memetic Algorithm for the VNMP. Based on the presented results, the MA configuration using the direct representation and VND as local improvement (D-VND) finds the best solutions. We will denote this configuration by MA-O. We could show that MA-O outperforms VND-O over a wide range of instance sizes and load cases. With reference to the main questions we set out to answer in this chapter, we could not observe any significant difference between the two tested representations if performance for specific instance sizes is relevant. For high loads, the grouping representation might offer an advantage. As for the difference between UXA and UXB, we have shown that UXB can cause performance degradations, but also seen one case where it is beneficial. We believe that analyzing the difference between those crossover variants warrants further research. Whether or not the time for local improvement increases performance, while for large instances executing more GA iterations is more important. For high loads, using local improvement is essential. We have shown that disabling crossover decreases performance in all cases. Chapter 13 offers a more in-depth comparison of MA-O with other algorithms.

We have already stated in Section 3.8 that looking into more dynamic variants of the VNMP (with changing configurations of virtual networks) is a promising area for future study. The MA presented in this chapter would be a prime candidate for solving the dynamic variant of VNMP, since it is population based. The hope here is that, when the configuration of virtual networks changes, some individuals within the population can be easily adapted to the new virtual network load. If this proves to be true, some more elaborate population management methods could be devised. For instance, multiple populations may be used: one population for finding good solutions for the current problem, another one for trying to find a solution that keeps some fraction of resources available at each substrate node or arc, and a third population that tries to minimize C_u , without regard to C_a (within reason). The aim of the first population is clear: finding a good solution that is immediately useful. The second population's task is to find good solutions, which easily accommodate additional virtual networks in case some are added to the current instance. The third population tries to find solutions which are similarly flexible in the other direction. The solutions may not be valid at the moment, but once some virtual network is removed, they might lead to valid solutions of high quality. This is of course only a rough idea, and a lot of details such as possible interactions between populations need to be addressed. A multi-objective GA [41] might be used instead of separate populations. For an overview on the design of Genetic Algorithms for dynamic problems, see [19].

CHAPTER **8**

Greedy Randomized Adaptive Search Procedure and Variable Neighborhood Search

8.1 Introduction

In this chapter, we present a Greedy Randomized Adaptive Search Procedure (GRASP) and a Variable Neighborhood Search (VNS) algorithm for solving the VNMP. Instead of simple Local Search, both algorithms make use of a Variable Neighborhood Descent algorithm with ruin-and-recreate neighborhoods [154], which makes them belong to the class of Hybrid Metaheuristics [16, 143, 165]. We consider GRASP and VNS to be promising methods for solving the VNMP, since they extend the already developed algorithms from Chapter 6, which might lead to performance improvements. Indeed, we will show that the VNS approach significantly outperforms the algorithms presented in the previous chapters.

The rest of this chapter is structured as follows: Section 8.2 presents the GRASP algorithm, Section 8.3 the VNS approach. We refer to Sections 2.2.7 and 2.2.8 for background on VNS and GRASP respectively. The results of the experimental evaluation of the proposed algorithms and their comparison to other algorithms presented in this work can be found in Section 8.4. We conclude in Section 8.5. The algorithms and results presented in this chapter have been published in [90].

8.2 GRASP

The main idea of GRASP is to repeatedly construct solutions to a problem in a greedy, but randomized, fashion and then to improve it by a local improvement method. We refer to Section 2.2.8 for a general introduction to GRASP.

A key component for a well working GRASP approach is the randomized greedy construction heuristic. Since we will concentrate on solving VNMP-O in this chapter, we use CH-O (see Section 6.5.1) as basis for randomization. In a nutshell, CH-O works as follows: As long as virtual arcs are implementable (source and target node have been mapped), the virtual arc f with the smallest fraction of d_f to shortest possible delay between m(s(f)) to m(t(f)) is implemented by the path with the least increase in C_u without increasing C_a . If no such virtual arc exists, the unmapped node with the highest total CPU requirement (CPU requirement of the virtual node and bandwidth of connected virtual arcs) is selected from the virtual network that has the smallest sum of total delay requirements. It is mapped to the substrate node with the highest amount of free CPU capacity.

We know from our experiments with Construction Heuristics (see Section 6.5.1), that the substrate node selection strategy is most influential for the overall performance of the construction heuristic. Therefore, we concentrate on randomizing this strategy and keep all other parts of the randomized construction heuristic deterministic. We introduce a parameter $\alpha \in [0, 1]$ that controls the level of randomization. When selecting a suitable substrate node for a virtual node, we collect a list of possible targets sorted by the available CPU and bandwidth, the candidate list. Let $f_{\text{Best}}^{\text{CPU}}$ denote the free CPU capacity and $f_{\text{Best}}^{\text{BW}}$ the free bandwidth capacity of the node that would have been selected by the deterministic strategy. We build the restricted candidate list by selecting all nodes *i* with $f_i^{\text{CPU}} \ge \alpha f_{\text{Best}}^{\text{CPU}} \land f_i^{\text{BW}} \ge \alpha f_{\text{Best}}^{\text{BW}}$. If $f_{\text{Best}}^{\text{CPU}}$ is negative (i.e., more resources are used than are actually available), α is replaced by $2 - \alpha$ in the relevant acceptance criterion. The mapping target is chosen uniformly at random from the restricted candidate list.

After a randomized greedy solution is generated, it is locally improved. For comparison purposes, we choose the same method that MA-O presented in the previous chapter uses. It is Variable Neighborhood Descent using configuration C_{12} as defined in Section 6.4. This configuration utilizes the neighborhood structures RemapVnode, ClearSarc and ClearSnode (in this order). First-improvement is used to search the neighborhoods. CH-R is employed for solution reconstruction. This configuration was chosen because it achieves very good results for a rather small amount of required run-time. We will call this configuration simply VND and we use it without time-limit to improve solutions generated by the randomized construction heuristic up to local optimality. If the found solution after the improvement phase is better than the best solution found so far, we keep it. Then we repeat the randomized construction and improvement steps until the time-limit is reached. The best found solution is the result of GRASP.

8.3 VNS

VNS is an improvement over VND that focuses on diversification. Where deterministic VND would be finished, VNS adds a probabilistic shaking phase. That means that a set of neighborhood structures, which are different from the ones employed by the VND and typically larger, are used to change the currently best known solution to escape its basin of attraction. We refer to Section 2.2.7 for further details.

Our proposed VNS algorithm uses a single type of shaking neighborhood structure in multiple configurations. Let this neighborhood structure be called $N^s(v)$, with $v \in [0, 1]$ as parameter controlling the shaking vigor. N^s is based on the idea of clearing substrate nodes. When $N^s(v)$

```
Algorithm 8.1: VNS for the VNMP
   Input : VNMP instance I
   Output: Solution S for I
1 Solution best=initialize(I);
2 n_{ni}=1;
3 while !terminate() do
       Solution candidate=shake(N^{s}(v_{b}n_{ni}),best);
4
5
       applyVND(candidate);
       if candidate.value <best.value then // New best solution found
6
           best=candidate;
7
           n<sub>ni</sub>=1;
8
9
       end
10
       else
11
           ++n_{ni};
           if n_{ni} > n_{max} then n_{ni} = 1
12
       end
13
14 end
15 return best;
```

is applied to a VNMP solution, N^s randomly selects $\lfloor v \cdot |V| \rfloor$ substrate nodes. All virtual arc implementations that traverse the selected nodes are removed from the solution. All virtual nodes mapped to the selected substrate nodes are mapped to a substrate node that is allowed by M but not selected. If no such node exists, the mapping remains unchanged. The resulting solution is completed and improved by VND to create the final solution of one VNS iteration. During the execution of VNS we apply N^s with different values for v. The used values are determined by two parameters, the base neighborhood size $v_{\rm b}$ and the count of iterations that have not resulted in an improvement of the best found solution $n_{\rm ni}$. At the beginning of a new iteration, $N^{s}(v_{b}n_{ni})$ is applied to the currently best found solution and the result is improved by VND. If the solution created in this manner is better than the currently best known solution, n_{ni} is reset to one, otherwise n_{ni} is increased by one. The upper limit for n_{ni} is n_{max} . If this value is exceeded, n_{ni} is reset to one. The largest shaking neighborhood searched during VNS is $N^{s}(v_{b}n_{max})$. Values for v_{b} and n_{max} have to be chosen such that $v_{b}n_{max} \leq 1$. The shaking and improvement steps are applied until the time-limit is reached. The initial solution for VNS is built by the same method as for GRASP, but without randomization, i.e., CH-O. Algorithm 8.1 shows the general outline of the proposed VNS.

8.4 Results

To test the proposed GRASP and VNS algorithms, we used the same selection of VNMP instances as in the previous two chapters, that means 210 full-load instances and 630 derived instances with loads 0.1, 0.5, and 0.8. In total, there are 120 instances for every size class and

Table 8.1: Average relative rank R_{rel} and its relation to the best result, average number of iterations (Its.), fraction of solved instances (Solv.) in percent and average C_a for different values of α per instance size.

	Size	GR-0.00	GR-0.10	GR-0.20	GR-0.30	GR-0.40	GR-0.50	GR-0.60	GR-0.70	GR-0.80	GR-0.90	GR-0.99
R _{rel}	20	0.278 =	0.216 =	0.219 =	0.235 =	0.215 =	0.331 >	0.384 >	0.445 >	0.516 >	0.618 >	0.842 >
	30	0.431 >	0.337 >	0.318 =	0.288 =	0.266 =	0.341 >	0.337 >	0.461 >	0.551 >	0.626 >	0.826 >
	50	0.549 >	0.512 >	0.463 >	0.362 >	0.311 =	0.329 =	0.402 >	0.484 >	0.536 >	0.640 >	0.868 >
	100	0.870 >	0.665 >	0.468 >	0.362 =	0.319 =	0.359 =	0.410 >	0.384 >	0.460 >	0.554 >	0.692 >
	200	0.889 >	0.737 >	0.488 >	0.361 >	0.301 =	0.301 =	0.313 =	0.339 =	0.436 >	0.531 >	0.740 >
	500	0.856 >	0.718 >	0.511 >	0.449 >	0.381 >	0.306 =	0.325 =	0.358 >	0.390 >	0.488 >	0.617 >
	1000	0.902 >	0.665 >	0.623 >	0.529 >	0.425 >	0.354 =	0.338 =	0.341 =	0.375 =	0.426 >	0.470 >
Its.	20	1998	2323	2641	2916	3142	3291	3453	3677	3751	3836	3897
	30	780	896	1034	1153	1248	1399	1534	1588	1676	1667	1664
	50	282	329	390	442	481	497	531	553	565	567	566
	100	39	47	56	62	66	71	77	82	83	87	87
	200	45	54	66	78	90	98	103	112	116	127	129
	500	15	18	22	26	30	33	36	39	41	41	42
	1000	4	5	7	7	9	10	11	12	12	12	12
Solv.	20	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.2
[%]	30	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	50	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	100	100.0	100.0	100.0	99.2	98.3	97.5	97.5	99.2	99.2	97.5	95.8
	200	95.8	99.2	99.2	99.2	98.3	97.5	97.5	96.7	95.8	95.0	91.7
	500	71.7	80.8	81.7	81.7	76.7	80.8	78.3	75.8	77.5	75.0	70.0
	1000	34.2	58.3	57.5	55.8	60.0	55.0	54.2	55.8	55.0	55.0	55.8
C_a	20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.6
	30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	100	0.0	0.0	0.0	1.3	2.5	3.7	2.8	1.5	1.5	2.5	4.2
	200	0.4	0.0	0.0	0.1	0.4	1.6	0.9	1.8	2.0	4.9	20.4
	500	120.5	92.8	50.0	26.9	47.1	32.8	55.7	54.1	76.8	89.4	83.9
	1000	708.7	227.1	251.1	206.1	245.5	273.5	251.2	311.4	263.5	336.7	277.5

210 for every load. See Chapter 5 for more detail on those instances. A time-limit of 200 seconds was applied for sizes up to 100 nodes, 500 seconds for larger instances.

Our design goal for the proposed algorithms was solving VNMP-O. Therefore, the relative rank as defined in Section 3.7 will once again be our main performance metric. Section 8.4.1 compares the performance of the GRASP approach for different values of α , Section 8.4.2 analyzes the performance of the VNS approach for different shaking neighborhood structure configurations and Section 8.4.3 shows a comparison of the best GRASP and VNS approaches, also considering previously presented algorithms.

8.4.1 GRASP

To evaluate the influence of α on the GRASP approach, we tested values for α from 0 (completely random initial solution) to 0.9 in 0.1 increments and 0.99 (very similar initial solutions). The average performance depending on the instance size can be seen in Table 8.1. Once again, we mark the relation of the results to the best observed result with = if no significant difference

could be observed, or > if the difference is indeed significant. See Section 2.4.2 for further details on the employed statistical tests.

Immediately visible in the presented data is the tendency of the best α value to rise with the instance size. For size 20, $\alpha \in [0, 0.4]$ yields the best results w.r.t. R_{rel} , while for size 1000 $\alpha \in [0.5, 0.8]$. The reason for this behaviour is that for small instances, the randomized construction heuristic does not have to make as many random choices as for the larger instance sizes. Therefore, to get the same search space coverage w.r.t. initial solutions, α has to be small for small instances. The results for the larger instances show that if α is too small, then the performance degrades, because the initial solution is far too random. Another contributing factor is that VND takes longer to optimize a very random initial solution, as can be seen by the iteration counts, which increase with rising values of α . Therefore, fewer iterations can be performed in the same amount of time.

Note that for finding valid solutions, low α values seem to be beneficial, even for large instances. The GRASP approaches presented here are already an improvement when compared to MA-O as outlined in the previous chapter. GRASP with an $\alpha \in [0.1, 0.2]$ is nearly able to find a valid solution to every instance of size 200, just a single instance remains unsolved. The solution derived for the one remaining instance has so little additional resource cost C_a , that the reported average is zero. This is by no means certain, as the results for $\alpha = 0.3$ indicate. With this configuration, also one instance remains unsolved, but with a noticeable impact on the average C_a . Also for the two largest instance sizes, the average C_a is lower than for the best MA configurations.

Table 8.2 shows the influence of α for different load cases. Again we can observe that higher values of α allow more iterations, but they do not lead to improved performance for high load. Instead, a value for $\alpha \in [0.4, 0.5]$ seems to be best suited when performance at a specific load level across different sizes is most important. Low α values are again beneficial for finding valid solutions and especially for the highest load, GRASP performs better than MA previously.

Based on these results, we select the GRASP approach with $\alpha = 0.4$ (GR-0.40) for further comparisons. We will denote this configuration by GRASP-O.

8.4.2 VNS

To analyze the influence of different shaking neighborhood configurations, we performed experiments with $n_{\text{max}} \in \{2, 5, 10\}$ and $v_b \in \{0.01, 0.05, 0.1\}$ to cover the range from very small changes with few shaking neighborhoods (i.e., few different configurations for N^s) to large changes with a lot of neighborhoods. Table 8.3 shows the performance of different neighborhood configurations based on instance size. The different configurations are labeled as "VNS $n_{\text{max}}.v_b$ ", e.g., VNS-2.05 uses $n_{\text{max}} = 2$ and $v_b = 0.05$. We can see a similar behaviour to GRASP. For smaller sizes, large shaking neighborhoods are beneficial, while large instance sizes require small neighborhoods for the best levels of performance. Smaller shaking neighborhoods lead to an increased number of iterations in the same amount of time. Also note the similarity in number of iterations between VNS-5.05 and VNS-2.10, caused by the very similar maximum shaking neighborhood sizes. Indeed, between sizes 50 and 500, there is no significant difference between the two configurations. Larger shaking neighborhoods seem to increase the chance of finding valid solutions. Indeed, VNS-10.10 is able to solve all instances of size 200.

Table 8.2: Average relative rank R_{rel} and its relation to the best result, average number of iterations (Its.), fraction of solved instances (Solv.) in percent and average C_a for different values of α per load.

	Load	GR-0.00	GR-0.10	GR-0.20	GR-0.30	GR-0.40	GR-0.50	GR-0.60	GR-0.70	GR-0.80	GR-0.90	GR-0.99
$R_{\rm rel}$	0.10	0.520 >	0.413 >	0.330 >	0.288 =	0.299 =	0.335 =	0.419 >	0.495 >	0.573 >	0.683 >	0.766 >
	0.50	0.744 >	0.586 >	0.445 >	0.384 >	0.307 =	0.313 =	0.324 =	0.354 =	0.473 >	0.574 >	0.773 >
	0.80	0.782 >	0.634 >	0.521 >	0.430 >	0.305 =	0.318 =	0.312 =	0.350 >	0.397 >	0.482 >	0.695 >
	1.00	0.682 >	0.568 >	0.470 >	0.377 =	0.356 =	0.360 =	0.380 =	0.407 >	0.423 >	0.482 >	0.654 >
Its.	0.10	1529	1737	1990	2193	2348	2490	2638	2781	2871	2917	2947
	0.50	108	142	173	208	239	267	298	322	330	333	336
	0.80	80	105	120	137	155	169	180	190	192	195	201
	1.00	90	114	126	139	151	160	167	173	175	176	173
Solv.	0.10	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
[%]	0.50	90.0	97.1	97.6	97.1	97.1	96.7	96.7	96.7	96.2	95.7	95.7
	0.80	81.0	90.5	89.0	89.5	90.0	87.6	87.6	89.0	88.6	86.7	85.2
	1.00	72.9	77.1	78.1	76.7	74.8	76.2	74.3	72.9	73.8	73.3	69.0
C_a	0.10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.50	46.5	7.7	0.4	0.5	0.5	0.5	0.5	0.5	0.6	0.7	7.1
	0.80	106.0	17.2	21.7	22.1	18.3	14.7	25.4	24.2	25.8	38.1	25.9
	1.00	321.5	158.0	149.9	111.3	150.0	162.8	151.6	185.9	170.0	209.0	192.5

The average C_a however is higher for the largest instance sizes than for GRASP-O. The results also indicate that increasing the shaking neighborhood size in multiple small steps works better than few large steps. This can be seen with configurations that have the same maximum shaking neighborhood size. VNS-10.01 and VNS-2.05 show no significant difference in R_{rel} , except for sizes 200 and 1000 where using smaller steps is significantly better. The difference is more pronounced for VNS-10.05 and VNS-5.10. Until size 50 there is no difference in performance, for larger instances using smaller steps is significantly better.

The influence of the shaking neighborhood configuration across different load cases can be seen in Table 8.4. Small shaking neighborhoods lead to the best performance. Load 0.1 is an exception, as larger shaking neighborhoods achieve the best results. As for the configurations with the same maximum shaking neighborhood size, smaller steps are a significant advantage for half of the load cases.

Based on these results, we chose VNS-10.01 for further comparison. We will denote this configuration by VNS-O.

8.4.3 Comparison

In this section, we compare our proposed algorithms GRASP-O (GR-0.4) and VNS-O (VNS-10.01) with previously presented algorithms. These are MA-O, the Memetic Algorithm for the VNMP introduced in Chapter 7, VND-O as introduced in Chapter 6, and the local improvement method VND on its own, to see improvement caused by GRASP and VNS when using it. Recall that the available run-time for MA-O was the same as for the GRASP and VNS algorithms and that MA-O also made use of VND for local improvement. The reported results of VND and VND-O are based on a time-limit of 1000 seconds. Note that we only show the average run-time for these two algorithms, since the others were run until the time-limit was reached.

Table 8.3: Average relative rank R_{rel} and its relation to the best result, average number of iterations (Its.), fraction of solved instances (Solv.) in percent and average C_a for different shaking neighborhood configurations per instance size.

	Size	VNS-2.01	VNS-5.01	VNS-10.01	VNS-2.05	VNS-5.05	VNS-10.05	VNS-2.10	VNS-5.10	VNS-10.10
$R_{\rm rel}$	20	0.389 >	0.436 >	0.396 >	0.416 >	0.240 >	0.244 >	0.340 >	0.198 =	0.163 =
	30	0.402 >	0.404 >	0.394 >	0.344 >	0.305 >	0.293 >	0.229 =	0.304 >	0.283 =
	50	0.457 >	0.390 =	0.356 =	0.396 =	0.333 =	0.368 =	0.338 =	0.390 =	0.472 >
	100	0.432 >	0.371 =	0.372 =	0.349 =	0.486 >	0.506 >	0.425 >	0.578 >	0.630 >
	200	0.460 >	0.360 =	0.316 =	0.376 >	0.490 >	0.554 >	0.479 >	0.591 >	0.685 >
	500	0.467 >	0.420 >	0.344 =	0.395 =	0.500 >	0.520 >	0.547 >	0.658 >	0.624 >
	1000	0.468 >	0.339 =	0.366 =	0.462 >	0.459 >	0.518 >	0.579 >	0.596 >	0.665 >
Its.	20	7327	7327	7345	7325	6796	5812	6822	5701	4742
	30	3721	3699	3670	3590	3132	2577	3156	2511	2010
	50	1766	1758	1664	1583	1321	1042	1327	1001	786
	100	415	389	346	311	237	181	233	169	127
	200	504	450	399	349	270	208	260	192	147
	500	157	143	124	110	85	67	83	62	48
	1000	51	44	39	33	25	20	25	18	14
Solv.	20	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
[%]	30	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	50	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	100	98.3	99.2	100.0	100.0	99.2	100.0	100.0	100.0	99.2
	200	93.3	95.8	97.5	95.8	95.0	99.2	99.2	98.3	100.0
	500	74.2	74.2	76.7	79.2	76.7	77.5	80.0	76.7	76.7
	1000	55.8	55.0	59.2	53.3	55.0	54.2	55.0	55.0	53.3
C_a	20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	100	1.2	1.3	0.0	0.0	1.3	0.0	0.0	0.0	0.6
	200	13.6	2.5	6.1	3.9	4.1	1.4	0.5	0.4	0.0
	500	97.3	100.0	68.5	76.5	86.1	91.7	87.9	86.3	107.2
	1000	333.1	281.4	311.2	317.8	357.6	315.8	314.6	339.8	360.7

For the others, we show the number of performed iterations instead. One iteration is basically one execution of VND, which takes the majority of the required run-time, and some algorithm dependent actions. For GRASP, the execution of the randomized construction heuristic, for VNS the shaking and for MA the creation of a new individual. For reference, the average run-time of these three algorithms when considering different load cases is 328.5 seconds.

Table 8.5 shows the performance of the compared algorithms in relation to each other. It can be seen that the results achieved by GRASP-O are disappointing. It is significantly outperformed by the VNS and MA algorithms. However, using GRASP around VND is significantly better than using VND alone, except for size 1000, where both perform equally well. VND-O can only be beaten or matched by GRASP-O up to size 100, then VND-O achieves significantly better results. VNS-O works far better, achieving the best solutions for sizes 30 to 200. For size 20, MA-O works marginally better. Keep in mind however, that we selected a shaking configuration for the VNS that was significantly worse for the smallest instance sizes than the alternatives, so it should be possible to at least match the MA with a different configuration. For the two largest sizes, VNS-O is beaten by VND-O, partly because the VND-O had more run-time available (and

Table 8.4: Average relative rank R_{rel} and its relation to the best result, average number of iterations (Its.), fraction of solved instances (Solv.) in percent and average C_a for different shaking neighborhood configurations per load.

	Load	VNS-2.01	VNS-5.01	VNS-10.01	VNS-2.05	VNS-5.05	VNS-10.05	VNS-2.10	VNS-5.10	VNS-10.10
R _{rel}	0.10	0.393 >	0.289 =	0.253 =	0.242 =	0.239 =	0.250 =	0.250 =	0.304 >	0.390 >
	0.50	0.464 >	0.408 =	0.360 =	0.407 >	0.415 >	0.458 >	0.436 >	0.486 >	0.516 >
	0.80	0.446 =	0.451 =	0.408 =	0.467 >	0.471 >	0.479 >	0.486 >	0.559 >	0.535 >
	1.00	0.454 =	0.407 =	0.432 =	0.449 =	0.482 >	0.528 >	0.506 >	0.546 >	0.572 >
Its.	0.10	6220	6176	6100	5992	5451	4640	5472	4542	3744
	0.50	924	905	877	853	712	550	708	526	410
	0.80	483	477	458	444	360	274	367	262	204
	1.00	339	335	329	312	257	196	256	186	141
Solv.	0.10	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
[%]	0.50	95.2	96.7	98.1	96.7	97.1	97.1	97.6	97.1	96.2
	0.80	87.1	87.1	88.6	85.7	86.7	87.1	88.6	87.1	88.6
	1.00	72.9	72.9	75.2	76.7	73.8	76.2	76.2	75.7	74.8
C_a	0.10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.50	7.2	7.0	6.9	0.6	6.9	0.4	0.4	7.5	1.3
	0.80	45.5	30.0	29.6	36.7	21.5	21.0	25.8	40.8	31.7
	1.00	201.6	183.2	183.9	190.2	228.2	212.3	204.1	195.4	234.7

also made use of it) as evidenced by the average run-times. Also, it is not a coincidence that there is no significant difference between the GRASP, VNS, MA, and VND approaches for size 1000. They all use VND as local improvement strategy, and as can be seen by the iteration count, not enough iterations could be performed to reap the benefits of the more involved heuristics within the available run-time. Based on the presented results, it seems that it is best to create one good solution and improve upon it (VNS-O), instead of creating a population of good solutions and profiting from their combination (MA-O) or trying to get lucky with randomly generated (and then improved) solutions (GRASP-O).

For solving instances at a specific load level, Table 8.6 shows that the VNS approach is the best choice across all load levels, achieving significantly better results than all of the other compared algorithms. There is no reason to use GRASP-O, it is matched or outmatched by VND-O within the same or lower run-time.

8.5 Conclusions

In this chapter, we have presented a GRASP and VNS algorithm for solving the Virtual Network Mapping Problem. We have shown that the VNS algorithm produces significantly better results than the MA and VND approaches previously introduced. Based on the presented results, we can conclude that the main idea of VNS (successively larger random moves away from local optima) works better than learning from a set of good solutions (MA) or improving good random solutions (GRASP) for the Virtual Network Mapping Problem, at least with a rather constraining run-time budget. The comparison is fair since the same local improvement strategy (VND) was used, the parameters of all algorithms have been optimized and the same time-limits were employed.

Table 8.5: Average relative rank R_{rel} and its relation to the best result, average number of iterations (Its.) or run-time, fraction of solved instances (Solv.) in percent and average C_a for different solution methods per instance size.

	Size	GRASP-O	VNS-O	MA-O	VND	VND-O
R _{rel}	20	0.476 >	0.222 =	0.192 =	0.912 >	0.753 >
	30	0.518 >	0.222 =	0.241 =	0.920 >	0.705 >
	50	0.598 >	0.214 =	0.275 >	0.930 >	0.696 >
	100	0.606 >	0.187 =	0.368 >	0.916 >	0.564 >
	200	0.577 >	0.197 =	0.413 >	0.859 >	0.372 >
	500	0.628 >	0.489 >	0.538 >	0.846 >	0.171 =
	1000	0.623 >	0.592 >	0.589 >	0.569 >	0.228 =
Its. /	20	3142	7345	8185	0.2	0.4
t[s]	30	1248	3670	3899	0.7	1.3
	50	481	1664	1663	2.1	4.2
	100	66	346	314	16.0	29.7
	200	90	399	333	40.2	119.7
	500	30	124	94	126.6	605.1
	1000	9	39	23	397.1	828.1
Solv.	20	100.0	100.0	100.0	96.7	97.5
[%]	30	100.0	100.0	100.0	100.0	100.0
	50	100.0	100.0	100.0	99.2	98.3
	100	98.3	100.0	100.0	95.0	97.5
	200	98.3	97.5	95.8	90.0	98.3
	500	76.7	76.7	76.7	73.3	90.8
	1000	60.0	59.2	58.3	57.5	61.7
C_a	20	0.0	0.0	0.0	13.1	4.5
	30	0.0	0.0	0.0	0.0	0.0
	50	0.0	0.0	0.0	4.9	2.1
	100	2.5	0.0	0.0	6.3	3.3
	200	0.4	6.1	3.0	19.0	1.0
	500	47.1	68.5	77.3	97.6	13.9
	1000	245.5	311.2	215.9	184.1	198.9

A promising direction for future work could be to further look into the behaviour of the presented algorithms for the largest instances sizes, where their performance still leaves something to be desired. The main problem is that VND, while already selected for reduced run-time requirements, takes too much time. It might be promising to try for instance LS-O as local improvement strategy, or some other Local Search variants as presented in Section 6.5.4. Even setting a time-limit for VND might be sufficient to improve performance for the largest instance sizes.

Table 8.6: Average relative rank R_{rel} and its relation to the best result, average number of iterations (Its.) or run-time, fraction of solved instances (Solv.) in percent and average C_a for different solution methods load.

	Load	GRASP-O	VNS-O	MA-O	VND	VND-O
R _{rel}	0.10	0.497 >	0.215 =	0.315 >	0.876 >	0.528 >
	0.50	0.616 >	0.294 =	0.384 >	0.913 >	0.450 >
	0.80	0.602 >	0.333 =	0.397 >	0.841 >	0.484 >
	1.00	0.586 >	0.371 =	0.400 =	0.771 >	0.532 >
Its. /	0.10	2348	6100	6354	5.6	41.7
t[s]	0.50	239	877	1016	50.2	218.3
	0.80	155	458	537	111.2	316.2
	1.00	151	329	385	166.0	331.6
Solv.	0.10	100.0	100.0	100.0	100.0	100.0
[%]	0.50	97.1	98.1	97.1	95.7	99.0
	0.80	90.0	88.6	88.6	85.7	91.9
	1.00	74.8	75.2	74.8	68.1	77.1
C_a	0.10	0.0	0.0	0.0	0.0	0.0
	0.50	0.5	6.9	0.6	0.7	0.1
	0.80	18.3	29.6	21.4	30.0	11.4
	1.00	150.0	183.9	147.3	155.0	116.3

CHAPTER 9

Preprocessing of VNMP Instances

9.1 Introduction

In this chapter, we present preprocessing techniques to use on VNMP instances. The main aim is to extract as much information as possible from those instances and possibly reduce their size or complexity before we start solving them. As an example, one can determine if a virtual arc can never use a particular substrate arc. If we know this beforehand, we can reduce the model of the problem by removing the variable that would tell us if the virtual arc uses the substrate arc. In addition, it is also possible to remove some constraints. If we can detect that a virtual arc can never cross a particular substrate node, then we can omit the flow conservation constraint for this substrate node and virtual arc (for formulations based on network flows). The following chapters on exact approaches for solving the VNMP will make use of the preprocessing techniques discussed in this chapter.

There are also other preprocessing opportunities which we will not regard any further. One is checking the consistency of the mapping possibilities, i.e., for each allowed mapping of a virtual node, is it possible to find a valid implementing path for all virtual arcs going out of (going into) that virtual node for one of the allowed mapping targets of the target (source) of the virtual arc. Since the VNMP instances are generated in a way that ensures this property, we do not check it during preprocessing. Implementing this check would be straight forward. Another preprocessing possibility would be checking if the capacities of nodes or arcs are actually constraining (e.g., all virtual arcs that could traverse a substrate arc require more bandwidth than available) and only add a constraint if it is actually possible to violate it. These checks are performed within the exact solvers discussed in the next chapters, so we do not consider them any further. Note however that they benefit from the domain reductions that we present in this chapter.

Before we can introduce the preprocessing methods for the VNMP, we require the following definitions:

Definition 9.1.1 (Set of Delay-Constrained Simple Paths). Given a directed graph G(V, A) and delays d_e , $\forall e \in A$, $P_{s,t}^d$ denotes the set of all simple paths from $s \in V$ to $t \in V$ of length at most d.

Definition 9.1.2 (Possible Nodes of a Delay-Constrained Substrate Connection). The set of possible nodes of a substrate connection from $s \in V$ to $t \in V$ with delay limit d, $PN_{s,t}^d$, is defined as $PN_{s,t}^d = \{i \in V \mid \exists p_{s,t}^d \in P_{s,t}^d : i \in p_{s,t}^d\}$.

Definition 9.1.3 (Fixed Nodes of a Delay-Constrained Substrate Connection). The set of fixed nodes of a substrate connection from $s \in V$ to $t \in V$ with delay limit d, $FN_{s,t}^d$, is defined as $FN_{s,t}^d = \{i \in V \mid \forall p_{s,t}^d \in P_{s,t}^d : i \in p_{s,t}^d\}$.

The definition of the set of possible arcs of a delay-constrained substrate connection $PA_{s,t}^d$ and the set of fixed arcs $FA_{s,t}^d$ is analogous.

Definition 9.1.4 (Domain of a Delay-Constrained Substrate Connection). The domain of a substrate connection from $s \in V$ to $t \in V$ with delay limit d, $D_{s,t}^d$, is defined as the quadruple $(PN_{s,t}^d, PA_{s,t}^d, FN_{s,t}^d, FA_{s,t}^d)$. We will refer to this also as substrate domain.

Definition 9.1.5 (Possible Nodes of a Virtual Arc). Given a VNMP instance, the set of possible nodes of a virtual arc $f \in A'$, PN_f , is defined as $PN_f = \bigcup_{s \in M(s(f)), t \in M(t(f))} PN_{s,t}^{d_f}$.

Definition 9.1.6 (Fixed Nodes of a Virtual Arc). Given a VNMP instance, the set of fixed nodes of a virtual arc $f \in A'$, FN_f , is defined as $FN_f = \bigcap_{s \in M(s(f)), t \in M(t(f))} FN_{s,t}^{d_f}$.

The definition of the possible arcs PA_f and fixed arcs FA_f for a virtual arc $f \in A'$ is analogous.

Definition 9.1.7 (Domain of a Virtual Arc). Given a VNMP instance, the domain of a virtual arc $f \in A'$, D_f , is defined as the quadruple (PN_f, PA_f, FN_f, FA_f) .

These definitions allow us to state the VNMP preprocessing problem as follows:

Definition 9.1.8 (The VNMP Preprocessing Problem). *Given a VNMP instance, calculate the virtual arc domains* $D_f, \forall f \in A'$.

Given a solution to this problem, we can remove superfluous variables and constraints from the exact VNMP models. As for solving this problem, it is immediately apparent that the VNMP Preprocessing Problem can be decomposed into multiple instances of the following problem:

Definition 9.1.9 (The Substrate Domain Problem (SDP)). *Given a source node s, a target node* t and a delay limit d, calculate $D_{s,t}^d$ for the substrate graph of a VNMP instance.

The decomposition of the calculation of D_f works as follows: Let S = M(s(f)) be the set of all allowed sources of f, T = M(t(f)) the set of all allowed targets and d_f the allowed delay of f. Then D_f is given as $(\bigcup_{s \in S, t \in T} \mathsf{PN}_{s,t}^{d_f}, \bigcap_{s \in S, t \in T} \mathsf{FN}_{s,t}^{d_f}, \bigcup_{s \in S, t \in T} \mathsf{PA}_{s,t}^{d_f}, \bigcap_{s \in S, t \in T} \mathsf{FA}_{s,t}^{d_f})$, which is the combination of all $D_{s \in S, t \in T}^{d_f}$.

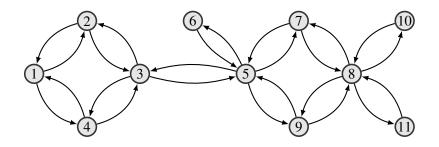


Figure 9.1: A small sample substrate network.

At first glance, solving the Preprocessing Problem in that way seems wasteful. For instance, we check if it is possible to use a substrate node for every mapping configuration, even if we have already found a mapping configuration for which it is possible. The same holds for the fixed parts. We check for every mapping configuration if we have to use a substrate arc, even if we have already found a mapping configuration where we do not have to use it. As it turns out, it is actually wasteful to skip evaluating nodes or arcs if we already know the result when solving the SDP. When nodes or arcs are skipped based on external assumptions, the solution of the SDP will be invalid for another set of assumptions. This slows down preprocessing considerably, because the solutions cannot be memoized. Section 9.6.3 shows an evaluation of this effect.

9.2 Solving the SDP

Central to the design of a solution method for the SDP is the question of how many SDP instances we need to solve. When considering the largest VNMP instances with 1000 substrate nodes, we know that each virtual node has about 50 different allowed locations in the substrate and that an instance contains about 1700 virtual arcs, which means that we can expect about 2500 SDP instances per virtual arc and 4.25 million instances in total. Section 9.6 will show that the real number is actually closer to 7.8 million. As there are only one million unique pairs of substrate nodes, we will need to solve the SDP for every pair multiple times with different delay values. We also know that the substrate graphs are sparse. Given these preconditions, we present the preprocessing procedure guided by an illustrative example.

Figure 9.1 shows a possible substrate network that will be the basis for solving SDPs. There are delays associated with the substrate, but they are inconsequential for now. Before we start to solve all SDPs, we perform a decomposition step that will allow us to derive partial SDP solutions to a whole range of SDP instances. The first step is calculating the biconnected components of the shadow of the substrate graph, as shown in Figure 9.2.

The red nodes are articulation points, all arcs labeled with the same number (and the nodes they connect) belong to the same biconnected component. As the second step, we build the block tree from this graph, by introducing a node for each biconnected component, which is then connected to all articulation points that connect the biconnected component to the rest of the network. Figure 9.3 shows the resulting block tree, the rectangular nodes represent biconnected

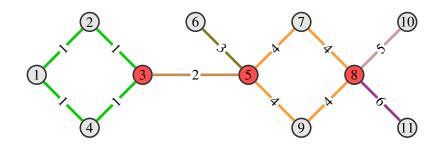


Figure 9.2: Biconnected components and articulation points of the shadow of the sample substrate network.

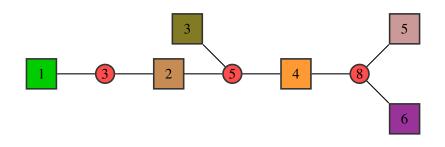


Figure 9.3: Block tree of the sample substrate network.

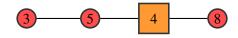


Figure 9.4: Simplified block tree.

components labeled by the component number, the circular ones are the articulation points which are labeled by their substrate node number.

This tree can be simplified further by removing all component nodes that are only connected to one articulation point (in this example nodes 1, 3, 5 and 6) and also all component nodes that represent bridges (i.e., only containing two articulation points and an edge between them), in this example component node 2. The final result can be seen in Figure 9.4.

During this decomposition, we keep track of the location of the original substrate nodes, so that we know for instance that substrate node 4 is represented by articulation point 3 in the simplified block tree, or that node 9 is represented by component node 4. We have now created a very compact representation of the original substrate graph that can be used to derive partial results to an SDP instance in the following way: Locate the representation of s and t in the simplified block tree and find a path between them. Since this path is unique, we immediately know that all traversed articulation points belong to $FN_{s,t}^d$ (and $PN_{s,t}^d$) and for every traversed bridge that the corresponding arc in the substrate belongs to $FA_{s,t}^d$ (and $PA_{s,t}^d$). We also know which components are traversed and in addition by which articulation point we have to enter a

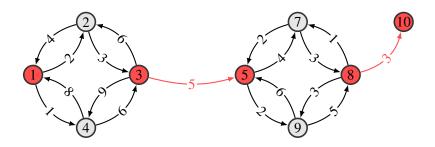


Figure 9.5: Partial domain for $D_{1,10}^{21}$.

component and by which we have to leave. If those articulation points are the same, we do not consider the component traversed. The only exception to this is when both s and t belong to the same component but are represented by an articulation point (i.e., the connected component node was removed during simplification). Then the representing articulation point is not in $FN_{s,t}^d$ (but possibly in $PN_{s,t}^d$). Due to the applied simplification, the found path in the block tree might still be incomplete. If s is not an articulation point, but the start of the path is one, then we need to add the component that s belongs to at the beginning of the path. As a refinement, if this component is a bridge, then we can immediately determine a fixed arc and do not need to consider the component. The same extension might be needed at the end of the path. The last piece of information that we can extract from the path in the simplified block tree is, that any node or arc that was not touched by the path (excluding all nodes and arcs contained in a traversed component) are definitely not in $PN_{s,t}^d$ or $PA_{s,t}^d$.

The $D_{s,t}^d$ we have calculated up to this point is a partial solution to the SDP, information concerning the traversed components is still missing. However, note that this $D_{s,t}^d$ is both independent of d and valid for all SDP instances that start and end in the same component as the SDP that is currently being solved. As such, it is a prime candidate for memoization, as in storing partial domains given start and end nodes in the simplified block tree, so that they can be used while solving future SDP instances instead of calculating paths again. In some sense, this is the solution to the "easy" part of the SDP, and the "hard" part is determining the domains within the components.

Before we start outlining different methods for calculating the domains within the components, we will continue our example by considering the SDP with s = 1, t = 10, and d = 21 for our example substrate. Figure 9.5 shows the created partial domain $D_{1,10}^{21}$.

The process for deriving this is the following: Substrate node 1 is represented by articulation point 3 and substrate node 10 is represented by articulation point 8 in the simplified block tree. We search the corresponding path in the simplified block tree (which in this case is the complete tree). At the start of the path we need to extend with component node 1. At the end we can immediately fix the arc in the substrate from node 8 to 10, since the end component is a bridge. Also, we can add substrate nodes 1, 3, 5, 8, and 10 to $FN_{1,10}^{21}$ and the arc from 3 to 5 to $FA_{1,10}^{21}$. In Figure 9.5, $FN_{1,10}^{21}$ and $FA_{1,10}^{21}$ are marked in red, all nodes and arcs of the substrate that can not belong to $PN_{1,10}^{21}$ or $PA_{1,10}^{21}$ have been removed. Now we only need to solve the domain problem

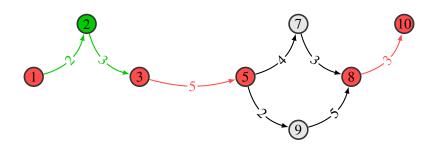


Figure 9.6: Complete domain for $D_{1,10}^{21}$.

within components 1 and 4. At this stage the specified delay limit becomes relevant. The figure contains the delay values for the arcs that might still be possible. For our path from node 1 to 10 we know the delay of all arcs currently in $FA_{1,10}^{21}$ and lower bounds for traversing components 1 and 4, determined by shortest path calculations. This allows us to derive upper bounds for the delay that we are allowed to spend while traversing the components. This bound for a particular component is calculated by taking the original delay bound d and subtracting the delay of all fixed arcs and the lower delay bound of all other traversed components. In our example, we can not spend more delay than 6 in component 1 (21 - 8 - 7) and 8 in component 4 (21 - 8 - 5). Calculating $D_{1,3}^6$ and $D_{5,8}^8$ is the last step. A discussion of how to achieve this will follow, but for this example the solution can be derived easily. We can immediately discard all arcs that go into starting nodes within the component or leave the target node. Since we are only considering simple paths, these arcs cannot be used. For component 1, the path using node 4 requires a total delay of 7, which exceeds our delay bound, so this node and all connecting arcs cannot be used. Now only one path remains, therefore its nodes and arcs have to be fixed. A similar argument applies for component 4. The final result for $D_{1,10}^{21}$ is shown in Figure 9.6. The parts of the domain that were fixed by considering the component subproblem are marked in green, all arcs that cannot be used have been removed.

9.3 The SDP for One Component

In this section we will outline techniques for deriving domains within components. More formally, we cover methods for calculating $D_{s,t}^d$, with s and t belonging to the same component, and d being the upper bound on the delay that we are allowed to spend within the component. To do this, we consider the extended component graph $G_c(V_c, A_c)$, which contains all nodes and arcs of the substrate that are contained within the same biconnected component c in the shadow of the substrate. In addition, every substrate arc is represented by a node (called an arc node) and two arcs, one going from the source of the original substrate arc to the representing node and one going from the node to the original target of the substrate arc. The delay of the substrate arc is assigned to the arc going into the representing node, the outgoing arc has no delay. Distributing the delay any other way would not make sense, because we could for example get the situation that due to the delay bound the representing node is still allowed, but the following node is not. That would mean that using an arc is allowed, but the target of this arc is not. By

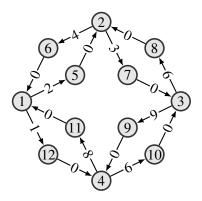


Figure 9.7: Extended component graph G_1 .

using this extension, we only need to consider $PN_{s,t}^d$ and $FN_{s,t}^d$ for G_c . $PA_{s,t}^d$ and $FA_{s,t}^d$ can be reconstructed by checking which arc nodes are possible or fixed. Figure 9.7 shows G_1 of the example substrate. Without loss of generality, we assume that $s \neq t$ and $|V_c| \geq 6$, otherwise the problem is immediately solvable.

The following techniques are split into two categories, the first one are methods for determining $PN_{s,t}^d$. These methods will be called pruning methods, because starting from V_c they remove nodes which cannot be part of $PN_{s,t}^d$. The second category are methods for calculating $FN_{s,t}^d$ from $PN_{s,t}^d$. We will call them fixing methods. The simplest pruning and fixing methods do just the bare minimum to create a valid domain for a substrate connection. All nodes of the component are added to $PN_{s,t}^d$ and only the nodes by which the component is entered and left are added to $FN_{s,t}^d$. Unsurprisingly, these methods are very fast and do not require additional memory, but of course also do not perform any additional pruning or fixing within the component. However, the final calculated domain for a substrate connection will still benefit from the information extracted from the simplified block tree and these simple pruning and fixing methods allow us to determine the additional benefit of using more elaborate methods within biconnected components.

9.3.1 Pruning by Simple Heuristics

Heuristic pruning was already applied during the calculation of $D_{1,3}^6$ for the example substrate. It works by removing all incoming arcs of the source node and all outgoing arcs of the target node in G_c . In addition, all nodes n for which $\delta_n^- = 0$ or $\delta_n^+ = 0$ can be removed, together with any incident arcs. This step is repeated until no further nodes can be removed. Figure 9.8 shows the result of applying heuristic pruning to G_1 . Note that the path across substrate node 4 cannot be excluded with this simple pruning strategy since it does not take delays into account. Therefore this pruning method is incomplete; it does not calculate $D_{s,t}^d$, but a superset of it. Its run-time is in $\mathcal{O}(n^2 + m)$, where $n = |N(G_c)|$ and $m = |A(G_c)|$.

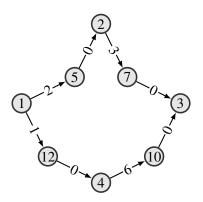


Figure 9.8: Extended component graph G_1 after heuristic pruning for $D_{1,3}^6$.

9.3.2 Pruning by All Pair Shortest Paths

In order to improve the pruning capabilities, we need to take delays into account. Therefore, we calculate the shortest possible delays between all pairs of nodes in G_c with Johnson's Algorithm, see Section 2.3.3. A node $k \in V_c$ can only be in $PN_{s,t}^d$ if the shortest delay from s to k plus the shortest delay from k to t is $\leq d$. This pruning method is still incomplete, since we do not check if the two path segments $s \to k$ and $k \to t$ are disjoint, so again we get a superset of $D_{s,t}^d$. This pruning method is sufficient to generate the complete domain $D_{1,10}^{21}$ as shown in Figure 9.6. In general, we might miss pruning opportunities that are caught by the heuristic pruning, so we include the first step of heuristic pruning discussed in the previous section; the removal of incoming arcs of the source node and of outgoing arcs of the target node. Why we chose not to use the whole heuristic pruning method will become clear in Section 9.6.8. To keep this method fast, we do not repeat the delay checking after the heuristic pruning, even though it might be able to exclude further nodes. Doing so would require a costly recalculation or at least updating of the distance matrices (for instance with the algorithm presented in [43]) based on the pruned nodes. This pruning method runs in $\mathcal{O}(n)$, but requires an initial calculation of the shortest paths taking $\mathcal{O}(n^2 \log(n) + nm)$.

9.3.3 Pruning by Integer Linear Programming

Until now, we could only solve the problem of computing $PN_{s,t}^d$ approximately. This is not surprising, since for every node in V_c , we basically need to solve a 2-disjoint paths problem, which is \mathcal{NP} -complete [57].

Definition 9.3.1 (The 2-Disjoint Paths Problem). Given a directed graph G = (V, A) and two pairs $(s_1, t_1), (s_2, t_2)$ of pairwise different vertices of G. Find two vertex-disjoint directed paths P_1 and P_2 in G, where P_1 goes from s_1 to t_1 and P_2 from s_2 to t_2 .

If we want to test whether $k \in PN_{s,t}^d$, we need to split k into two copies k' and k''. All incoming arcs of k go to k', all outgoing arcs leave from k''. Then we set $G = G_c$, $s_1 = s$, $t_1 = k'$, $s_2 = k''$ and $t_2 = t$ to get a 2-disjoint paths problem. To reduce the 2-disjoint paths problem to

the problem of determining whether $k \in PN_{s,t}^d$, we just need to do this transformation in reverse. We include an additional node k in G and add an arc from t_1 to k and from k to s_2 . By finding a simple path from s_1 to t_2 crossing k within G, we solve the 2-disjoint paths problem. As a side-note, the special case of planar graphs is in \mathcal{P} [153], but we do not restrict ourselves to planar extended component graphs. So if we want to calculate $PN_{s,t}^d$ exactly, we need to solve an \mathcal{NP} -complete problem for every node in V_c (and we have not even considered the delay constraint yet). One possibility to do this is using Integer Linear Programming (ILP) to solve the following problem:

Definition 9.3.2 (The Node Testing Problem). Given a directed graph G(V, A) with arc delays d_e , $\forall e \in A$, three nodes s, t, and k from G and a delay limit d. Find a $p_{s,t}^d \in P_{s,t}^d$, such that $k \in p_{s,t}^d$.

We now present two different ILP models based on network flows for solving the node testing problem.

The first model, denoted by (TWOFLOW), is based on the idea of finding a simple path from s to k, and a simple path from k to t, while forbidding that they share a node and enforcing that they do not exceed d. It utilizes decision variables $y_e^1 \in \{0,1\}, \forall e \in A$ to indicate if an arc is used for the first path (to k) and decision variables $y_e^2 \in \{0,1\}, \forall e \in A$ to indicate if an arc is used for the second path (to t).

(TWOFLOW)
$$\sum_{e \in A \mid s(e)=i} y_e^1 - \sum_{e \in A \mid t(e)=i} y_e^1 = \begin{cases} 1, & \text{if } i = s \\ -1, & \text{if } i = k \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in V$$
(9.1)

$$\sum_{e \in A | s(e) = i} y_e^2 - \sum_{e \in A | t(e) = i} y_e^2 = \begin{cases} 1, & \text{if } i = k \\ -1, & \text{if } i = t \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in V$$
(9.2)

$$\sum_{e \in A \mid s(e)=i} (y_e^1 + y_e^2) \le 1 \qquad \forall i \in V \qquad (9.3)$$

$$\sum_{e \in A \mid t(e)=i} (y_e^1 + y_e^2) \le 1 \qquad \forall i \in V \qquad (9.4)$$

$$\sum_{e \in A} (y_e^1 + y_e^2) d_e \le d \tag{9.5}$$

$$y_e^1 \in \{0, 1\} \qquad \qquad \forall e \in A \qquad (9.6)$$

$$y_e^2 \in \{0, 1\} \qquad \qquad \forall e \in A \qquad (9.7)$$

Equalities (9.1) and (9.2) ensure flow conservation, so that the result is one connected path, inequalities (9.3) and (9.4) limit the incoming and outgoing flow at every node, so that the result is a simple path and that the two flows cannot share an arc. Inequality (9.5) is the delay constraint. The integrality constraints (9.6) and (9.7) ensure the integrality of the final solution. Note that this model allows disconnected flow circulations. As a result, for reconstructing the

solution to the model, we need to follow the flow values from s over k to t to see which arcs (and nodes) are actually used for the path. After we have introduced the second model, we will discuss why we need to know the actual path and why we accept the overhead of having to reconstruct it instead of ensuring that no superfluous flow can be contained in the result. The presented model does not contain an objective function, since node testing is a satisfiability problem. (TWOFLOW) can be transformed to correspond to the definition of an ILP model as presented in Section 2.2.12 by adding the minimization of a constant as objective. This also holds for the following models for satisfiability problems.

The second model, denoted by (FLOWINFLOW) is based on the idea of finding a simple path from s to t which is limited by d and contains a path from s to k. It utilizes decision variables $y_e^1 \in \{0,1\}, \forall e \in A$ to indicate if an arc is used for the path from s to t and decision variables $y_e^2 \in \{0,1\}, \forall e \in A$ to indicate if an arc is used for the path from s to k.

(FLOWINFLOW)
$$\sum_{e \in A \mid s(e)=i} y_e^1 - \sum_{e \in A \mid t(e)=i} y_e^1 = \begin{cases} 1, & \text{if } i = s \\ -1, & \text{if } i = t \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in V$$
(9.8)

$$\sum_{e \in A | s(e) = i} y_e^2 - \sum_{e \in A | t(e) = i} y_e^2 = \begin{cases} 1, & \text{if } i = s \\ -1, & \text{if } i = k \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in V$$
(9.9)

$$y_e^1 - y_e^2 \ge 0 \qquad \qquad \forall e \in A \quad (9.10)$$
$$\sum y_e^1 \le 1 \qquad \qquad \forall i \in V \quad (9.11)$$

$$\sum_{e \in A | t(e) = i} y_e^1 \le 1 \qquad \qquad \forall i \in V \quad (9.12)$$

$$\sum_{e \in A} y_e^1 d_e \le d \tag{9.13}$$

$$y_e^1 \in \{0, 1\} \qquad \qquad \forall e \in A \quad (9.14)$$

$$y_e^2 \in \{0, 1\} \qquad \qquad \forall e \in A \quad (9.15)$$

Equalities (9.8) and (9.9) ensure flow conservation. The linking constraints (9.10) state that the flow from s to k has to be contained in the flow from s to t. Inequalities (9.11) and (9.12) force the path from s to t to be simple, (9.13) realizes the delay constraint. The integrality of the solution is enforced with (9.14) and (9.15). As with the previous model, (FLOWINFLOW) allows disconnected flow circulations, so the actual path from s to t over k has to be reconstructed after the model has been solved.

Why do we allow disconnected flow circulations? After all, it would be easy to add an objective to both models, stating for instance that the number of selected arcs should be minimized. However, keep in mind that we are not only interested in the solution of the model with one particular k, we need to check all $k \in V_c$, which would be very time-consuming if done in the naive way. Luckily, when we solve the model for one particular k, we can extract the path from s to t that has been found. This path proves not only that there exists a simple path from s to t using k within the delay limit, it proves this for all nodes on the path, and those nodes do not have to be checked via the ILP models. So we actually want to get long paths as a result, because they reduce the need for further ILP solver invocations. By adding constraints to remove the disconnected flow circulations, we shorten the resulting path which causes more work later on. We could go towards the other extreme and add an objective to maximize the number of selected arcs and remove the circulations by using directed connection cuts, but then we get the overhead of actually having to find a longest path and proving its optimality, which again increases the run-time requirements.

This touches on another requirement for ILP models for solving the node testing problem: rapid reconfigurability. The structure of the ILP model (number of constraints, involved variables) is constant for one particular extended component graph. The only changing components are the coefficients on the right-hand side of the (in)equalities for different values of s, t, k and d. So it is much more efficient to change the coefficients of the model to match new values of s, t, k, and d than to rebuild the whole model. A consequence of this is that models that require cuts for correctness, like the variant previously mentioned that maximizes path length, are highly undesirable, because every cut that has to be separated represents work that has to be thrown away when the model is reconfigured. Section 10.5 will offer some results on this reconfiguration aspect.

As a result of these considerations, we rejected two other modeling possibilities that we know of for the node testing problem. One model is based on the idea of having two units of flow which leave s, at k one unit is removed and at t the second one. This model has the advantage of only needing one flow variable per arc. However, it can happen that we get solutions where one unit of flow goes from s to k and then loops back to k again (so we have two units of flow entering k and one leaving) and one unit of flow directly from s to t. We would need cuts to ensure feasibility and as already explained, this is undesirable in our situation. Due to this reason, we also rejected another modeling idea: require a flow from s to t and use directed connection cuts to ensure that this flow crosses k.

9.3.4 Pruning by Path Enumeration

Until now, we have looked at the problem to calculate $PN_{s,t}^d$ from the point of view that we have to find paths that prove the membership of each node within this set. However, it also holds that $PN_{s,t}^d = \bigcup_{p_{s,t}^d \in P_{s,t}^d} N(p_{s,t}^d)$. Therefore, if we know $P_{s,t}^d$, we can immediately calculate $PN_{s,t}^d$. The problem here is of course that the size of $P_{s,t}^d$ grows exponentially with the size of the input graph (in our case the extended component graph). There are three arguments that make the enumeration of all paths nevertheless a viable approach in this situation: The biconnected components are rather small (and adding one node for each arc makes paths only longer but does not increase the number of paths), they are still relatively sparse (even though they are denser than the parts of the network that could be represented by the simplified block tree) and we do not need to store all paths, we only need to keep the union of the nodes used by the paths. To put it more precisely, for each $s, t \in V_c$, determine each value of delay d for which simple paths between s and t exist and store the union of the nodes used by those paths and the nodes used by the next smaller possible value of d. The result is $\text{PN}_{s,t}^d$. Since it is not possible to enumerate only paths from s to t, we enumerate all paths starting from s to all $t' \in V_c$ within d and calculate the corresponding $\text{PN}_{s,t'}^d$.

Algorithm 9.1 shows how the path enumeration procedure works. The most important data structure used by this algorithm is the PathSearchState. It contains information about how a particular node was reached, including: which node was reached, how much delay was incurred to reach this node and which nodes have been visited. One input of the algorithm is a collection of such states that have not been explored during the last invocation because they have exceeded the allowed delay value. These are called the continuation search states. Assume $PN_{s,t}^d$ has already been calculated. If we now have to calculate $PN_{s,t'}^{d'}$, with d' > d and possibly $t \neq t'$, then we can simply continue the algorithm with the state it ended in when calculating until d, without having to repeat the computations. For the first invocation of the algorithm for a particular start node s, the continuation search state is initialized with one PathSearchState, the node set to s, delay to 0, and the path just containing s. The algorithm has two main stages. In the first stage all paths not exceeding d, starting from the continuation search states, are enumerated and stored for each target node in ascending order of delay. In the second phase, the store for $PN_{s,t}^d$ (in the algorithm referred to as Unions) is updated. So with one invocation of this algorithm to calculate $\text{PN}_{s,t}^d$, we also get $\text{PN}_{s,t'}^{d'}$ for all $t' \in V_c$ and $d' \leq d$. As an implementation side-note, in line 28 an empty set is returned for the first invocation of the algorithm.

Within the path enumeration algorithm, the union of (nodes of) paths is a central operation and therefore critical for the overall execution speed. In preliminary runs we have seen that using a bit-vector outperforms other alternatives, like sets or hash-sets, by a wide margin. In the bit-vector representation, a path is a vector of bits with the size of $|V_c|$. All nodes used by the path are set to 1, the others are set to 0. Note that in the case of the extended component graph, this representation of a path via its nodes is unique because G_c contains nodes representing arcs in the original biconnected component. The union of paths in the bit-vector representation is a simple bitwise-or operation. Using this representation also has a very low overhead compared to the alternatives in terms of memory. However, even though we only store the path unions and not the paths themselves, the memory requirements are too high when applying this method to the largest VNMP instances. To combat this, there is a size limit for the biconnected component. If this size limit is exceeded, pruning by all pair shortest paths is performed instead of path enumeration. See Section 9.6.7 for an evaluation of this behavior.

This concludes the discussion of pruning techniques, we will now present fixing techniques to calculate $FN_{s,t}^d$ of G_c .

9.3.5 Fixing by Testing

To determine if a node $k \in V_c$ belongs to $\operatorname{FN}_{s,t}^d$, we need to check if all paths from s to t not longer than d have to use k. This is equivalent to checking if there are no paths not longer than dthat do not use k. The second statement can easily be tested by using a shortest path computation while forbidding the use of k. This is the basic idea of fixing by testing and immediately shows that determining $\operatorname{FN}_{s,t}^d$ is in \mathcal{P} , in contrast to determining $\operatorname{PN}_{s,t}^d$, which is in \mathcal{NP} . Of course, applying fixing by testing in the straight forward manner of forbidding every $k \in V_c$ in turn and Algorithm 9.1: Pruning by path enumeration

Input : Extended Component Graph G, delay limit d, continuation search states CS **Output**: For each node in G, complete set of nodes usable when reaching them within d

```
1 Queue<PathSearchState> states, Paths paths(num_vertices(G));
```

2 for *PathSearchState* continueState $\in CS$ do

```
if continueState.delay <d then
3
         states.push(continueState);
4
         CS.erase(continueState);
5
      end
6
  end
7
  while !states.empty() do
8
      PathSearchState s=states.pop_front();
9
      if s.delay>d then CS.push(s);
10
      else // found a new way to reach a node
11
         paths.at(s.node).add(s.delay,s.path);
12
         forall the outgoing arcs oa from s.node do
13
             n=target(oa);
14
             nn=outgoingNeighbor(n); // n is an arcnode
15
             if s.path.contains(nn) then continue; // would introduce a loop
16
             PathSearchState newS=s; // store new state
17
             newS.path.add(n,nn);
18
             newS.node=nn;
19
             newS.delay=s.delay+delay(oa);
20
             states.push(newS);
21
         end
22
      end
23
24 end
25 forall the nodes n in G do // update path unions
      npaths=paths.at(n); // new paths in ascending delay order
26
      if npaths.empty() then continue;
27
      union=Unions.at(n).latest(); // fetch union of previous call
28
      delay=npaths.front().delay;
29
      forall the paths p in npaths do
30
31
         delayNow=p.delay;
         if delayNow>delay then // previous delay-level complete
32
             Unions.at(n).insert(delay,union);
33
             delay=delayNow;
34
35
         end
         union.add(p);
36
      end
37
      Unions.at(n).insert(delay,union);
38
39 end
```

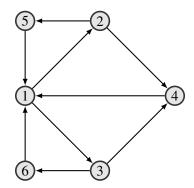


Figure 9.9: Nodes 2 and 3 are strong articulation points, but do not belong to $FN_{1.4}^d$.

checking if a path of length at most d from s to t exists would be highly inefficient. Every node belonging to $\text{FN}_{s,t}^d$ has to be contained in every path not longer than d from s to t. So if we test a node k and find an alternative path, we immediately know that only nodes within this path can belong to $\text{FN}_{s,t}^d$. If we test a node of this path, and again find an alternative path, we can iterate this argument. Only nodes belonging to the intersection of both paths can belong to $\text{FN}_{s,t}^d$. As a result, for every alternative path we find, we can exclude more than one node from $\text{FN}_{s,t}^d$ with high probability. This method runs in $\mathcal{O}(n^2 \log(n) + nm)$.

There are some possibilities to reduce the number of nodes that need to be tested even further. One of those is to calculate the strong articulation points (see Section 2.3.2) of G_c . Only strong articulation points can belong to $FN_{s,t}^d$, if delays are not considered. If a node different from s and t belongs to $FN_{s,t}^d$, then there is no path from s to t without this node. So removing this node would increase the number of strongly connected components, which means that this node is a strong articulation point. Note that the converse is not true, strong articulation points do not necessarily belong to $FN_{s,t}^d$, even if a path from s to t contains them.

An example of this can be seen in Figure 9.9. When trying to determine the fixed nodes for paths from node 1 to 4, it is necessary to use a strong articulation point (2 or 3), but they are not fixed. This example also shows that heuristics like "only cross a strong articulation point if it is really necessary, then it belongs to $FN_{s,t}^d$ " are invalid. Restricting testing to strong articulation points can potentially speed up this fixing method. However, it is no longer complete since we do not consider delays. With delays, it might happen that a node is fixed, even though it is not a strong articulation point.

Another possibility for improving the performance of the fixing method is the use of dominators (see 2.3.1). When calculating $FN_{s,t}^d$, every dominator of t (while using s as a source node) has to belong to $FN_{s,t}^d$ and only the remaining nodes of G_c have to be tested. In the results section we will analyze the fixing performance of the dominator technique on its own to be able to judge what additional solving performance it could add to the testing approach.

Note that both methods do not take the delay limit into account, so they can be strengthened by not using G_c as a basis, but rather the graph that is derived from G_c by deleting all nodes not in $\text{PN}_{s,t}^d$. Then, better pruning will improve the performance of the fixing by testing (but not its (in)completeness). However, every determination of $FN_{s,t}^d$ depends on $PN_{s,t}^d$, which precludes the possibility of precomputation, so strong articulation points or dominators have to be calculated anew every time.

9.3.6 Fixing by Path Enumeration

Fixing by path enumeration is essentially the same as pruning. It is also basically free if the pruning is done by path enumeration. Algorithm 9.1 still applies, but in addition to the union we also keep track of the intersection of the paths to calculate $FN_{s,t}^d$. In the step corresponding to line 28, the intersection returned during the first execution of the algorithm is V_c .

9.3.7 Fixing by Integer Linear Programming

For completeness sake, it is also possible to test if a node has to be fixed by using an ILP formulation, which we will denote by (FIXFLOW). It is based on the idea of finding a simple path form s to t which is limited by d and does not contain k. It utilizes decision variables $y_e \in \{0, 1\}, \forall e \in A$ to indicate if an arc is used for the path from s to t.

(FIXFLOW)
$$\sum_{e \in A | s(e) = i} y_e - \sum_{e \in A | t(e) = i} y_e = \begin{cases} 1, & \text{if } i = s \\ -1, & \text{if } i = t \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in V$$
(9.16)

$$\sum_{e \in A | s(e) = i} y_e \le \begin{cases} 0, & \text{if } i = k \\ 1, & \text{otherwise} \end{cases} \quad \forall i \in V \qquad (9.17)$$

$$\sum_{e \in A} y_e d_e \le d \tag{9.18}$$

$$y_e \in \{0, 1\} \qquad \qquad \forall e \in A \qquad (9.19)$$

Equalities (9.16) ensure flow conservation. Inequalities (9.17) forbid the use of k for the path from s to t and force the path to be simple, (9.18) forces the path to respect the delay constraint. The integrality of the solution is enforced with (9.19). As with previous formulations, the actual path has to be reconstructed after the model has been solved.

9.4 Solving the SDP for One Component Efficiently

After outlining all of the pruning and fixing methods, one part of the algorithm for solving the SDP for one component is still missing: the procedure to execute these methods. As already discussed, we can expect to have to calculate $PN_{s,t}^d$ and $FN_{s,t}^d$ for extended component graphs with the same start and end nodes for different delay values. This means that is possible (and effective) to use previous results as bounds for the current domain request. Algorithm 9.2 shows how this works in detail.

Algorithm 9.2: Calculating $PN_{s,t}^d$ and $FN_{s,t}^d$ for one component

```
Input : Extended component graph G_c, source node s, target node t, delay limit d and
          domain store Domains
  Output: PN_{s,t}^d and FN_{s,t}^d of G
1 PosNodes pn,FixNodes fn;
   // calculate pn (=PN_{s,t}^d)
2 if Domains.hasPosNodes(s,t,d) then pn=Domains.getPosNodes(s,t,d) else
      PosNodes pnUB=Domains.getPosUB(s,t,d);
3
      PosNodes pnLB=Domains.getPosLB(s,t,d);
4
      if pnUB==pnLB then pn=pnUB else
5
         PosNodes pnToTest=pnUB∧(¬ pnLB); // bit-vector operations
 6
         prune(s,t,d,pnToTest); // call one of the pruning methods
7
         pn=pnLB\/pnToTest;
8
      end
9
10 end
   // calculate fn (=FN_{s,t}^d)
11 if Domains.hasFixNodes(s,t,d) then fn=Domains.getFixNodes(s,t,d) else
      FixNodes fnUB=Domains.getFixUB(s,t,d);
12
      FixNodes fnLB=Domains.getFixLB(s,t,d);
13
      if fnUB==fnLB then fn=fnUB else
14
         RestrictGraph Gr=restrict(G<sub>c</sub>,pn);
15
         FixNodes fnToTest=pn\wedgefnLB\wedge(\neg fnUB);
16
         fix(s,t,d,fnToTest,Gr); // call one of the fixing methods
17
         fn=fnUB\/fnToTest;
18
      end
19
20 end
   // store calculated domains, return
21 Domains.setPosDomain(s,t,d,pn);
22 Domains.setFixDomain(s,t,d,fn);
23 Domains.simplify(s,t,d);
24 return pn,fn;
```

The input of the preprocessing algorithm for a component c is the extended component graph G_c , the data for the required connection (source and target nodes, maximum allowed delay) and the store for previously calculated $\text{PN}_{s,t}^d$ and $\text{FN}_{s,t}^d$ within this component.

The first step of the algorithm is calculating $PN_{s,t}^d$. If $PN_{s,t}^d$ is already present in the domain store, then there is nothing more to do. Otherwise, we use the results of previous calls as upper and lower bounds. The upper bound is a previously calculated $PN_{s,t}^{d'}$ with d' > d but as small as possible. If no such domain has been calculated before, then it is V_c . Obviously, if there is no simple path using a node within d', then there will be none within d, so $PN_{s,t}^d$ is a subset of $PN_{s,t}^{d'}$. Similarly, we try to find a lower bound $PN_{s,t}^{d''}$ with d'' < d but as large as possible. If no such domain has been calculated, then it is the empty set. If there is a simple path using a node within d'', then this path is of course still valid with delay bound d, so all nodes in $PN_{s,t}^{d''}$ also belong to $PN_{s,t}^d$. If the upper and lower bounds are equal, then we already know $PN_{s,t}^d$ and nothing more needs to be done. Otherwise, the nodes that are included in $PN_{s,t}^{d'}$, but not in $PN_{s,t}^{d''}$, are the ones we still need to test whether they belong to $PN_{s,t}^d$, which is stated in line 6. Note that we assume a bit-vector representation of the domains, so we can apply bit-wise logical operators. After determining which nodes need to be tested, we apply one of the pruning methods discussed previously. The pruning method will remove all nodes from the nodes to test that do not belong to $PN_{s,t}^d$. It is free to ignore the input about nodes to test. The Path Enumeration method for instance cannot use it, the All Pair Shortest Path method on the other hand can and does use it. The final $PN_{s,t}^d$ is then assembled from the lower bound and the nodes that have passed the pruning.

The second step of the algorithm determines $FN_{s,t}^d$. It works basically the same way as the first step. One thing to note here is that the term upper bound still refers to a domain that was derived by allowing more delay and lower bound refers to a domain where less delay was allowed. The result is that for fixed nodes, the upper bound will contain fewer nodes than the lower bound. In case no suitable upper or lower bounds have been calculated, the upper bound will be the empty set and the lower bound V_c . For calculating the nodes for which we have to test whether they belong to $FN_{s,t}^d$ (line 16), that means we have to test all nodes that belong to $PN_{s,t}^d$ and are present in lower bound, but not in the upper bound. The restriction to $PN_{s,t}^d$ is especially useful at the beginning when no suitable lower bound is known. One of the discussed fixing methods is then called with the nodes to test and removes all nodes that do not belong to $FN_{s,t}^d$. As an additional input, the fixing method gets the extended component graph, restricted by the possible nodes. This for instance increases the efficiency of path calculations, because parts of the extended component graph are removed.

The last step of the algorithm is storing the calculated domains in the domain store, followed by a simplification step. The main idea here is that it is not necessary to store all calculated domains, because a lot of them will be the same. It is sufficient to store the same domain only twice; for the lowest and the highest delay for which this domain occurred. As a consequence, the memory requirements for storing domains can be reduced. On the other hand, the probability of finding exactly the requested domain already present in the domain store is decreased, but we will still get the same upper and lower bounds and save execution time that way.

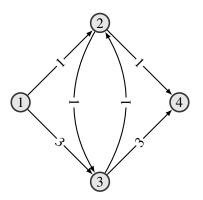


Figure 9.10: $FN_{s,t}^d$ can change without $PN_{s,t}^d$ changing.

As an implementation remark, note that $PN_{s,t}^d$ and $FN_{s,t}^d$ are stored completely independently from each other, because changing delays can add or remove nodes from one without affecting the other. It is easy to see that changing delay may change $PN_{s,t}^d$ without changing $FN_{s,t}^d$. Just think about a graph without fixed nodes where increasing the delay bound might add nodes to $PN_{s,t}^d$. The other direction is also possible, Figure 9.10 shows an example. For simplicity reasons we omitted the nodes for every arc. It holds that $PN_{1,4}^5 = PN_{1,4}^6 = \{1, 2, 3, 4\}$, but $FN_{1,4}^5 = \{1, 2, 4\}$ and $FN_{1,4}^6 = \{1, 4\}$. The main insight here is that even with perfect pruning, the resulting graph can contain delay-infeasible paths. This means that an incomplete fixing method that does not take delays into account applied to a pruned extended component graph derived by a complete pruning algorithm will still be incomplete. In this example, the fixing method cannot distinguish the situations for delay 5 and 6.

9.5 The Complete Preprocessing Algorithm

We have now finished the discussion of all components necessary for solving the VNMP preprocessing problem and are therefore able to show the algorithm for solving it. Algorithm 9.3 gives an high level overview of the procedure. It neglects the mentioned memoization steps for simplicity. In addition, there is a relevant implementation aspect to this algorithm. As presented, it solves the substrate domain problems one at a time for each virtual arc separately. We will refer to this as the default order. As a result, consecutive SDP instances will be very different in terms of starting node, end node, and delay constraint. It is possible to calculate the domains for the virtual arcs in another order. Instead of iterating through virtual arcs, we iterate through substrate nodes, and for each node solve the SDP for every virtual arc that may start at this node. We will refer to this order as substrate order. With this simple step, consecutive SDP instances will be more similar, which causes a performance increase up to 15%. See the results in Section 9.6.2 for more details.

Alg	Algorithm 9.3: Preprocessing for the VNMP							
Ir	Input : A VNMP Instance							
0	Output : Virtual arc domains $D_f, \forall f \in A'$							
1 C	1 Calculate simplified block tree of substrate;							
2 fo								
3	VirtualArcDomain D;							
4	forall the allowed locations ms of s(f) do							
5	forall the <i>allowed locations mt of t(f)</i> do							
	<pre>// solving the substrate domain problem</pre>							
6	SubstrateDomain D';							
7	Calculate path in block tree from ms to mt;							
8	Add domain parts known from block tree path to D';							
9	Calculate delay bounds for all crossed components;							
10	forall the crossed components do							
11	Execute algorithm 9.2;							
12	Add result to D';							
13	end							
14	D.add(D');							
15	end							
16	end							
17	17 Store D for f;							
18 ei	nd							

9.6 Results

In this section, we will analyze the different aspects of the preprocessing algorithm. Now follows a description of the default configuration of the preprocessing algorithm which is used unless specified otherwise. We also refer to following sections that offer further explanations and experimental evidence as to why a particular setting was chosen as default setting. The evaluation was done on instances with full load and a memory limit of 5 GB.

In the default setting, the preprocessing algorithm uses the block tree decomposition with all simplifications (see Section 9.6.1). The virtual arc domains are built in substrate order (see Section 9.6.2). Knowledge already gathered while calculating the domain of a virtual arc is not used when evaluating further mapping possibilities for the source and target node of the virtual arc (see Section 9.6.3). The following pruning methods are utilized:

None This method does not perform any pruning. All nodes and arcs within a component are possible.

Heuristic The heuristic pruning scheme described in Section 9.3.1.

APSP The pruning scheme based on shortest delays between nodes as described in 9.3.2.

- **PathEnumeration** The path enumeration scheme presented in 9.3.4. A node limit of 40 nodes is employed, meaning that components containing more than 40 nodes will be solved by APSP. See Section 9.6.7 for details.
- **TwoFlow** The ILP model TWOFLOW presented in 9.3.3 is used for pruning. If CPLEX has stored a solution from the last solving procedure, it is discarded. Section 9.6.6 contains details. The CPLEX solver object is not discarded when the model is modified (9.6.4).
- **FlowInFlow** The ILP model FLOWINFLOW presented in 9.3.3 is used for pruning. Again, a possibly stored solution is discarded (see Section 9.6.6) and the solver object is kept (see Section 9.6.4).
- Now follows a description of the employed fixing methods:
- **None** Does not perform any fixing, only the nodes by which a component is entered and left are fixed.
- **Dominators** Utilizes the idea of using dominators for fixing presented in 9.3.5.
- **Testing** The fixing by testing approach introduced in 9.3.5.
- **TestingSAP-0** A variant of testing that only tests strong articulation points. These strong articulation points are determined statically based on the complete extended component graph.
- **TestingSAP-1** This method recomputes the strong articulation points depending on the nodes of the extended component graph that were determined to be possible $(PN_{s,t}^d)$.
- **PathEnumeration** The path enumeration approach presented in 9.3.4, extended to calculate fixed nodes as well. A node limit of 40 nodes is employed. If this limit is exceeded, Testing is used. See Section 9.6.7 for details.
- **FixFlow** The ILP model FIXFLOW as presented in 9.3.7 is used for fixing. A possibly stored solution is discarded (see Section 9.6.6). The solver object is kept during modifications and inequality 9.17 is only applied for the testing node *k*. See Section 9.6.5 for details.

Unless stated otherwise, all combinations of pruning and fixing methods are evaluated. In the following sections, we will present the motivation for the choice of parameters and behaviors for the default configuration. Then we will analyze the characteristics of the preprocessing algorithm in more detail. The following definitions will apply during the discussion of the results. Symbol t will be used to denote the run-time of the preprocessing algorithm in CPU-seconds.

Definition 9.6.1 (Relative Node-Pruning Performance PN_{rel}). The relative node-pruning performance PN_{rel} is defined as $PN_{rel} = \frac{\sum_{f \in A'} |PN_f|}{|A'||V|}$. Note that smaller values mean better performance because more nodes could be excluded from PN_f .

Table 9.1: Influence of different block tree decomposition configurations on the average required CPU-time (t) in seconds and the pruning and fixing performance. TN is the average number of nodes of the tree decomposition.

Configuration	t[s]	PN _{rel} [%]	PA _{rel} [%]	FN _{rel} [%]	FA _{rel} [%]	TN
No Decomposition	204.2	67.9	62.6	4.0	0.9	1.0
With Decomposition	16.1	37.2	26.9	5.2	1.2	51.3
With Bridge Opt.	16.0	37.2	26.9	5.2	1.2	42.5
With Node Opt.	16.0	37.2	26.9	5.2	1.2	27.4
With Both Opt.	16.0	37.2	26.9	5.2	1.2	18.6

Definition 9.6.2 (Relative Arc-Pruning Performance PA_{rel}). *The relative arc-pruning performance* PA_{rel} *is defined as* $PA_{rel} = \frac{\sum_{f \in A'} |PA_f|}{|A'||A|}$.

The definitions of the relative node-fixing performance FN_{rel} and relative arc-fixing performance FA_{rel} are analogous, but for fixing, larger values are better. As an example, $PN_{rel} = 0.2$ means that on average, every virtual arc can only use 20% of the substrate's nodes.

9.6.1 Influence of Block Tree Decomposition

Calculating the simplified block tree decomposition is the first step when executing the preprocessing algorithm. In this section, we test the influence this step has on the overall preprocessing algorithm, especially with respect to run-time, pruning and fixing performance. Additionally, we test the influence of the suggested simplifications. Those simplifications were the removal of component and articulation point nodes of degree one (henceforth called node optimization) and the removal of component nodes containing only two nodes (bridge optimization). We tested on instances of size 50. Testing with larger instances was not possible, because the required run-time exploded when omitting the block tree decomposition, especially for the ILP methods. Table 9.1 shows the behavior of the preprocessing algorithm depending on the chosen configuration. It can be seen that there is a huge difference between the performance with the decomposition or without it. By using the tree decomposition, the algorithm requires only 8% of the original run-time. Additionally, the pruning and fixing performance is better. For instance, without the decomposition, the preprocessing tells us that every virtual arc may use about 68% of the substrate nodes on average, with the decomposition this is reduced to 37%. The difference is even larger for the possible arcs. For fixing, the additional benefit of the decomposition is not as pronounced, which is due to the fact that not a lot of nodes or arcs can be fixed to begin with. This difference occurs because of the use of incomplete pruning or fixing methods. Keep in mind that for all configurations all combinations of pruning and fixing methods have been tested, for instance pruning with method None and fixing with method None. With those methods and without decomposition, nothing can be pruned or fixed. By performing the decomposition, we can exclude nodes and arcs that are not touched by the path within the block tree. For a method selection that uses complete methods, like PathEnumeration, the decomposition does not give

(a	ı) without ILF	e methods		(b) with ILP methods					
	Evaluat	ion Order		Evaluation Order					
Size	Default	Substrate	Size	Default	Substrate				
20	0.01 =	0.01 >	20	0.19 =	0.18 =				
50	0.12 =	0.12 >	50	43.77 >	37.38 =				
200	2.99 >	2.80 =	200	1261.19 >	1253.36 =				

Table 9.2: Influence of the evaluation order of virtual arc domains on the average required runtime for preprocessing.

any additional advantages other than better run-time. The last column of Table 9.1 shows the average number of nodes of the block tree. Without decomposition, there is of course just one node, containing the complete substrate graph. More interesting is the situation with respect to the number of nodes with decomposition. Just using decomposition, the block tree contains 51 nodes on average, which is even more than the substrate size. By using bridge optimization, nine nodes can be removed, while node optimization can remove 24 nodes. By using both optimizations, the simplified block tree uses only 18 nodes. That means the preprocessing algorithm is able to find a structural representation of the input substrate graph that contains only 35% of the substrate's nodes. Unfortunately, this additional compactness does not lead to any performance gains.

Based on these results, we chose to use the block tree decomposition with both simplifications.

9.6.2 Influence of the Domain Evaluation Order

In Section 9.5, we have outlined two different ordering possibilities for calculating the domains of virtual arcs, the default order and the substrate order. To analyze the difference between these orderings, we have tested the preprocessing algorithm on instances of size 20, 50 and 200. The results are summarized in Table 9.2. Note that we have distinguished between preprocessing configurations that do not use ILP based pruning/fixing methods (9.2a) and configurations that use ILP based methods either for pruning, fixing, or both (9.2b), since the required run-times are very different.

We can see that for ILP based methods, using substrate order always requires less run-time than the default order and this difference is also statistically significant for all but the smallest instance size. The largest improvement is achieved for size 50, where using substrate order requires 15% less run-time. This advantage is reduced to 0.6% for size 200. Every reported value is the average of 540 runs.

As for the preprocessing configurations that do not use ILP methods, we can see that the overhead of having to determine for each substrate node the virtual arcs that may originate there is prohibitive for small instances. The default order performs better than the substrate order. Note that due to rounding the reported average run-times are the same, but still significantly different. For the largest instance size, using substrate order results in 6% lower run-times, so the overhead of using substrate order starts paying off. Every reported value is the average of 720 runs.

		Fixing								
Pruning	Cfg.	None	Dominators	Testing	TestingSAP-0	TestingSAP-1	PathEnum.	FixFlow		
None	Α	2.2 =	2.2 =	2.5 =	2.3 =	4.1 =	2.9 =	31.2 =		
	В	7.6 >	7.7 >	28.2 >	15.4 >	160.7 >	15.6 >	2658.3 >		
	C	8.2 >	8.3 >	27.7 >	16.0 >	161.2 >	15.9 >	2417.4 >		
Heuristic	Α	2.2 =	2.2 =	2.6 =	2.5 =	4.2 =	3.2 =	31.4 =		
	В	3.1 >	3.1 >	17.3 >	10.3 >	91.6 >	10.6 >	1493.3 >		
	C	8.4 >	8.4 >	28.5 >	16.7 >	166.4 >	16.6 >	2451.3 >		
APSP	Α	2.3 =	2.3 =	2.4 =	2.3 =	3.6 =	3.0 =	31.0 =		
	В	7.7 >	7.7 >	18.2 >	11.5 >	93.3 >	11.6 >	1387.6 >		
	C	10.4 >	10.5 >	29.8 >	18.0 >	175.7 >	17.2 >	2496.9 >		
PathEnum.	Α	2.9 =	2.9 =	3.0 =	2.9 =	4.1 =	3.0 =	32.5 =		
	В	8.3 >	8.3 >	17.1 >	11.4 >	80.7 >	11.5 >	1289.7 >		
	C	11.4 >	11.4 >	30.2 >	18.8 >	177.6 >	17.3 >	2447.9 >		
TwoFlow	Α	1465.5 >	1460.1 >	1460.1 >	1460.7 >	1456.3 >	1473.0 >	1489.1 =		
	В	822.4 =	824.0 =	835.1 =	825.8 =	889.3 =	830.4 =	2041.1 >		
	C	1543.1 >	1547.5 >	1564.8 >	1557.5 >	1707.8 >	1579.8 >	3954.0 >		
FlowInFlow	Α	1739.9 >	1740.5 >	1736.0 >	1739.0 >	1742.1 >	1758.6 >	1763.6 =		
	В	958.8 =	960.7 =	974.4 =	963.4 =	1029.0 =	974.4 =	2166.0 >		
	C	1775.7 >	1775.6 >	1797.5 >	1787.3 >	1942.5 >	1831.2 >	4200.6 >		

Table 9.3: Influence of utilizing partially known domains on the required preprocessing runtime.

Based on these results, we decided to use substrate order as default setting for the preprocessing algorithm.

9.6.3 Influence of Partially Known Domains

When calculating the domain of a virtual arc, we combine the results of all relevant substrate domains. At the end of Section 9.1, we have argued that this might seem wasteful, because we test for instance whether a node belongs to $PN_{s,t}^d$, even though we might already know from another substrate domain that it belongs to the domain of the virtual arc. In this section we show that the benefit of calculating valid substrate domains, which might be useful for the domains of other virtual arcs, outweighs the cost of possibly redundant testing in most cases.

We tested three different configurations. Configuration A is the standard configuration, which calculates every substrate domain, regardless of what is already known about the virtual arc domain. Configuration B skips the evaluation of a substrate domain (for one component), if it cannot give any new information. In algorithm 9.2, we calculate nodes that have to be tested for membership in $PN_{s,t}^d$. If we already know for all nodes that they will be present in PN_f , we skip the pruning step. The same holds for the fixing step: If we know for all nodes that would have to be tested, that they cannot belong to FN_f , then we skip the fixing step. If pruning or fixing have been skipped, the calculated substrate domain is invalid and cannot be stored for later use. Configuration C goes even further and just removes nodes for which it is already known if they either belong to PN_f or cannot belong to FN_f from the nodes that have to be tested during the pruning and fixing step respectively.

We have tested all three configurations on instances of size 20, 50, and 200. The results were consistent across all size classes, therefore Table 9.3 only shows the results for size 200.

Table 9.4: Influence of the method used for changing TwoFlow models on the required preprocessing time and the time required for changing the model (Setup Time).

		t[s]	Setup Time[s]			
	Size	Default	Tracking	Default	Tracking		
-	20	1.1 >	0.2 =	0.8	0.0		
	50	79.0 >	41.4 =	35.0	0.4		
	200	2026.4 >	1468.0 =	518.9	5.6		

For all pruning methods that are not ILP based, configuration A is the best choice by a very wide margin. For non-ILP-based fixing, using another configuration results in a 5 to 40 fold increase in run-time, when FixFlow is used we get an 70 fold increase. The situation is reversed for ILP based pruning methods. Here a 40% reduction in run-time can be achieved by using configuration B instead of A. Configuration C however still performs worst. Interestingly, the benefit of using configuration A for FixFlow outweighs the cost of using it for ILP based pruning. This demonstrates the tradeoff between possibly computing too much, but creating good domain bounds that can be utilized during future calculations (configuration A) and only calculating what is needed, but possibly repeatedly for different virtual arcs (configuration B). With good bounds, we can skip calling a pruning or fixing method altogether, which is beneficial if the bound creation did not take a lot of time. For slow pruning methods, the cost for creating good bounds is prohibitive, which is why configuration B performs better. The fraction of nodes that can be fixed is rather low (see results in Section 9.6.8), so good bounds are also essential when using FixFlow. Configuration C combines the worst of both worlds, the calculated domains cannot be reused and during domain calculation we still need to execute pruning and fixing methods (albeit with a reduced number of nodes).

9.6.4 Modification of TwoFlow

In this section, we will look at the performance differences caused by either destroying the solver object before the model is modified and then rebuilding it with the new model, or letting the CPLEX solver object track changes to the TwoFlow model. It is not clear beforehand which method is better, because both have overheads. For the first method, we have the cost of instantiating CPLEX solver objects, for the second we have the cost of the solver object having to change its internal model representation (e.g., the simplex tableau) to correspond to the modified model. As a side note, we do not offer an analysis of the alternative of destroying both solver and model and rebuilding from scratch, because we have seen in preliminary runs that preprocessing an instance with this method took more than two hours, while the other choices finished within seconds. Because the results for FlowInFlow are similar, we will only discuss TwoFlow.

We tested the preprocessing algorithm with TwoFlow as pruning method and None for fixing on instance sizes 20, 50, and 200. For TwoFlow, we applied the default changing method (change the model without an attached solver object) and the tracking method (change the model with an attached solver object). Table 9.4 shows the results.

It can be seen that the tracking method is vastly superior to the default changing method, which

Table 9.5: Influence of different configurations of FixFlow on the required preprocessing time.

	Configuration									
Size	Α	В	C	D						
20	0.3 >	0.3 >	0.0 =	0.1 >						
50	6.1 >	6.2 >	2.6 >	2.2 =						
200	76.1 >	74.8 >	40.3 >	31.0 =						

requires 38% more time for preprocessing for the largest instance size. Table 9.4 also shows the total setup time required for the two configurations. The setup time is the time needed for changing the model from solving $PN_{s,t}^d$ to $PN_{s',t'}^{d'}$. There is a factor of 90 difference between the setup times for the default and tracking method, which means that it is 90 times faster to modify a model that is tracked by a CPLEX solver object than to repeatedly create new solver objects. Based on these results we chose the tracking method for TwoFlow and FlowInFlow.

9.6.5 Modification of FixFlow

In this section we test for FixFlow what we tested for TwoFlow in Section 9.6.4: The tradeoff between either tracking model changes with the CPLEX solver object or rebuilding the solver object once the model has been modified. However, for FixFlow we had a look at an additional possibility for modification. Inequalities 9.17 of FIXFLOW limit the flow through every node in the component and forbid flow through the node for which we want to determine whether it belongs to $FN_{s,t}^d$ (the testnode). Our design goal for the FIXFLOW model was that it can be reconfigured by just changing coefficients. Since every node may be the testnode, we need inequalities 9.17 for every node. As we need them for every node anyway, we can use them to limit the flow. There is, however, another possibility. We could only forbid flow through the testnode, which means we only have to add one inequality instead of |V|, which makes the model smaller. On the other hand, extracting the actual path from a solution to FIXFLOW gets more complicated, as the flow from s to t will no longer be a simple path. To extract a simple path, we need to perform a shortest path computation in the graph induced by the flow values. The second downside of only using one inequality is that removing constraints from a model is usually an expensive operation, which is why we look at this tradeoff in more detail in this section.

We tested four different FixFlow configurations. Configuration A uses the default changing setup (rebuilding the CPLEX solver object) and the original inequalities 9.17. In configuration B, only one inequality is used. Configuration C uses the tracking method (the CPLEX solver object gets notified about changes to the model) and the original inequalities. Configuration D uses the tracking method and only one inequality. These four configurations were used for preprocessing (with pruning method None) on instances of size 20, 50, and 200. Table 9.5 shows the results.

It can be seen that when the CPLEX solver object does not have to track modifications to the model, it does not matter whether coefficients are modified (A) or constraints are removed (B). The preprocessing will be slow either way. The situation becomes more interesting when we

use the tracking method, since we can see a significant difference here between coefficient modification (C) and constraint removal (D). For the smallest instance size, coefficient modification is faster, while the other instances constraint removal offers a distinct speed advantage. For instances of size 200, constraint removal is 25% faster. This means that it is more important to save on the number of constraints in the model for larger instances than it is to have simple model modification methods.

Based on these results, we used configuration D for FixFlow.

9.6.6 Removal of ILP solutions

During the discussion of ILP based pruning and fixing methods, we stated that rapid reconfigurability of an ILP formulation is very important since we need to solve very closely related problems in succession. By reconfigurability we mean the ability to solve a related problem by just changing some coefficients of the model. The alternative of course would be to rebuild the model from scratch. While conceptually simpler, this has a huge performance penalty. So, in CPLEX terms, we always use the same model and solver object, cf. Section 9.6.4. That means when we modify a model and try to solve it, CPLEX might have stored the solution to a previous variant of the model. Usually, this is done to speed up the re-solving process of the model. For instance when adding additional cuts, a previous solution might be modified to satisfy the new constraints so that a valid solution to the new model is available from the beginning. In our case however, even though the models are closely related, a solution to one model is not related to the solution of the modified model at all. Just think about changing the source node of a flow. The previous flow from another source (i.e., the previous solution) is not helpful for finding a new solution. Therefore, it might be the case that CPLEX spends too much time trying to adapt a solution instead of solving the new model. Removing a present solution before resolving might speed up the process. In this section, we will test this hypothesis.

We tested all three ILP based pruning/fixing methods without clearing and with clearing solutions. As the fixing method for the ILP based pruning methods (TwoFlow and FlowInFlow) we used None, which we also used as pruning method for FixFlow. From the instance set, we used the sizes 20, 50, and 200. Table 9.6 shows the result.

It can be seen that for all three methods, clearing the solution is significantly better than keeping it. The margin by which it is better however becomes smaller with larger instance size. This is not surprising, as the time spent on adapting a solution is small in relation to the time required for solving the model for larger instances. For the largest instances, clearing the solution results in a run-time reduction of 3% for TwoFlow, 1.5% for FlowInFlow, and 12% for FixFlow. Based on these results, we chose to clear the solution for all three ILP based methods.

9.6.7 Cutoff Size for Path Enumeration

The time and memory requirements of the PathEnumeration method are exponential in the size of the component for which it is executed. Therefore, we only execute PathEnumeration if the number of nodes contained in a component is below a certain threshold. For larger components, we fall back to APSP if we perform pruning, or to Testing if we perform fixing. In this section, we analyze the influence of this threshold (referred to as node limit) on the performance of Table 9.6: Influence of clearing a (possibly) present solution before resolving an ILP model on the required preprocessing run-time.

		Clear Solution				
Method	Size	No	Yes			
TwoFlow	20	0.2 >	0.2 =			
	50	42.8 >	41.2 =			
	200	1489.5 >	1442.3 =			
FlowInFlow	20	0.3 >	0.2 =			
	50	55.0 >	53.1 =			
	200	1766.8 >	1738.7 =			
FixFlow	20	0.1 >	0.1 =			
	50	2.6 >	2.1 =			
	200	35.1 >	30.9 =			

PathEnumeration. As test set, we used all instances from the VNMP instance set and applied node limits from 10 to 100 nodes in increments of 10 nodes. We used PathEnumeration as pruning and fixing method. Table 9.7 shows the results.

It can be seen that for sizes up to 100, the run-time differences caused by different node limits are negligible and every configuration is able to preprocess all instances up to size 200. However, beginning with instance size 30, some calls to PathEnumeration have to fall back to APSP or Testing and with larger instance sizes ever larger node limits are required to avoid falling back. The tradeoff between node limit and fraction of fallbacks can be seen especially well for instance size 200 and node limits 50 and 60. We can see that for node limit 50, we fall back in 7% of all cases and require on average 3.4 seconds of run-time. By increasing the node limit to 60, we do not fall back any longer, at the cost of an average run-time of 14.8 seconds. That means we have more than a five fold increase of run-time when we try to solve 7% of calls to PathEnumeration exactly. This tradeoff gets even worse when we consider the increase of relative arc pruning performance PA_{rel} we get by spending so much more time. By never falling back to APSP, we are able to exclude an additional 0.2% of substrate arcs from PA_f on average for every $f \in A'$. By going beyond instance size 200, we can see the memory limit coming into effect as executions of the preprocessing algorithm start to fail. The algorithm can perform preprocessing for only six out of 30 instances when a node limit of 100 nodes is used, for instances of size 1000. Note that the reported run-times, fallbacks and arc pruning performances are averages of successful executions only, which means that the reported values cannot be directly compared if some instances could not be preprocessed. This is nicely demonstrated by the (apparent) increase in fallbacks when increasing the node limit from 90 to 100 for instance size 1000.

The choice of node limit generally has very little influence on PA_{rel} . The largest difference is caused by changing the node limit from 10 to 20 nodes, which results in about 1% more pruning for small to medium sized instances. Also note that we only show the results for PA_{rel} because for PN_{rel} , FN_{rel} and FA_{rel} , the differences are even smaller.

Based on these results, we chose to use a node limit of 40 nodes, since this is the largest value for the node limit so that all instances can be preprocessed. We could have reduced the node limit

Table 9.7: Influence of different node limits for PathEnumeration on the average required CPUtime (t) in seconds, the number of executions not terminated due to excessive memory requirements (# Successes), the fraction of APSP and Testing fallbacks and the relative arc pruning performance (PA_{rel}).

						No	de Limit	;			
	Size	10	20	30	40	50	60	70	80	90	100
t[s]	20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	50	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
	100	0.7	0.7	0.9	0.9	0.9	1.2	1.2	1.2	1.2	1.2
	200	2.4	2.5	2.7	3.1	3.4	14.8	14.3	16.1	16.1	16.0
	500	13.9	14.1	16.1	26.4	58.3	117.3	133.5	150.0	162.0	216.6
	1000	68.9	69.8	73.4	75.6	151.6	324.1	329.9	326.2	318.7	330.4
# Successes	20	30	30	30	30	30	30	30	30	30	30
	30	30	30	30	30	30	30	30	30	30	30
	50	30	30	30	30	30	30	30	30	30	30
	100	30	30	30	30	30	30	30	30	30	30
	200	30	30	30	30	30	30	30	30	30	30
	500	30	30	30	30	29	23	19	17	15	11
	1000	30	30	30	30	29	18	18	18	17	6
Fallbacks[%]	20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	30	12.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	50	35.5	9.6	3.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	100	53.6	19.6	3.1	3.1	3.1	0.0	0.0	0.0	0.0	0.0
	200	81.7	35.8	16.4	8.1	7.2	0.0	0.0	0.0	0.0	0.0
	500	91.8	85.2	72.3	65.8	55.1	43.3	37.2	32.1	24.5	0.0
	1000	95.5	90.6	83.3	80.6	75.0	68.9	68.9	68.9	69.0	70.5
PA _{rel} [%]	20	26.9	26.9	26.9	26.9	26.9	26.9	26.9	26.9	26.9	26.9
	30	29.3	28.4	28.4	28.4	28.4	28.4	28.4	28.4	28.4	28.4
	50	25.2	24.6	24.3	23.9	23.9	23.9	23.9	23.9	23.9	23.9
	100	20.9	19.8	19.3	19.3	19.3	19.1	19.1	19.1	19.1	19.1
	200	19.4	18.2	17.8	17.5	17.5	17.3	17.3	17.3	17.3	17.3
	500	17.9	17.8	17.7	17.6	17.4	17.4	16.9	16.4	15.7	13.5
	1000	15.6	15.6	15.6	15.6	15.5	15.7	15.7	15.7	15.7	16.1

further, but then we would have moved even farther away from the ideal of a complete pruning (fixing) method. Note that this choice of node limit means that employing PathEnumeration is only complete for instances up to size 50. For sizes 100 and 200, it is close to complete, since less than 10% of PathEnumeration calls are solved by APSP or Testing. The pruning efficiency cost is at most 0.2%. For the largest instances, the behavior of PathEnumeration is actually closer to APSP or Testing than to its own. Another possibility would have been to set a node limit of 60 up to size 200 and use a limit of 40 for sizes 500 and 1000, which would have kept PathEnumeration exact up to size 200 while still being able to preprocess all instances. We decided against that to keep the configuration simple.

9.6.8 Comparison of Pruning and Fixing Methods

Now that we have determined the best configuration for the preprocessing algorithm and its pruning and fixing methods, we can evaluate the performance of preprocessing depending on the selected pruning and fixing methods, the general behavior of the algorithm and the usefulness of memoization.

We tested all combinations of pruning and fixing methods on the complete instance set with full load, with one exception. The ILP-based pruning methods (TwoFlow and FlowInFlow) were not tested on instance sizes 500 and 1000 due to their run-time requirements.

Table 9.8 shows the required run-time of preprocessing with different pruning and fixing methods. It can be seen that up to size 50, all configurations that do not use ILP based methods require basically the same run-time. The differences only start showing with larger instance sizes. For sizes 100 and 200, using TestingSAP-1 as fixing method requires consistently more run-time than the rest. For the larger instances, it is possible to see the influence of interactions between pruning and fixing methods on the required run-time. For instance, at size 500 and using Testing as fixing method, preprocessing using None as pruning is slower than preprocessing with APSP as pruning method. This is surprising, as None does nothing and should be faster. However, by not performing any pruning, Testing has to test more nodes, so in total it is more beneficial to use APSP as pruning method. For sizes 500 and 1000, there starts being a distinct run-time difference between PathEnumeration and the faster methods. Note that PathEnumeration is not exact any longer for these sizes, as discussed in Section 9.6.7. TestingSAP-1, however, is still slower. Also observe that TestingSAP-0 is faster than Testing, because it restricts testing only to the strong articulation points in the extended component graph. The required run-time when using None as pruning and fixing method gives the time necessary to calculate the virtual arc domains by just using the simplified block tree. Generally, the additional run-time cost of using pruning or fixing methods that actually do something is rather small.

Until now we have neglected to discuss the ILP based pruning and fixing methods. From Table 9.8, we see that they are not competitive in terms of required run-time, even though we spent some effort to implement them efficiently. FixFlow can be applied up to size 1000 (taking about a factor of 10 longer than other methods), but for TwoFlow and FlowInFlow, the runtime is too high for the largest sizes. Interestingly, size 200 is also the largest size for which PathEnumeration can be applied in an exact configuration. Note that we do not use such a configuration, but in Section 9.6.7 we have shown that it would require 16 seconds on average for instances of size 200, which compares quite favorably to the 1450 seconds TwoFlow requires. To calculate exact domains for larger instances, either more memory (for PathEnumeration) or more run-time (for TwoFlow) than we were able to supply has to be invested.

Of course, analyzing the required run-time alone is insufficient, since different methods for pruning and fixing have different pruning and fixing performances. Table 9.9 shows the relative node pruning performance PN_{rel} for the presented pruning methods. As a general tendency, the larger the instances, the better the node pruning performance. For size 20, half of the substrate's nodes can be excluded from the domain of every virtual arc, which increases to nearly 85% for size 1000. Unsurprisingly, the exact methods achieve the best results and the difference to the other pruning methods is significant. However, by using exact methods we can only exclude about 2% of additional substrate nodes from PN_f , while the additional run-time cost, as shown

					Fixing			
Size	Pruning	None	Dominators	Testing	TestingSAP-0	TestingSAP-1	PathEnum.	FixFlow
20	None	0.0	0.0	0.0	0.0	0.0	0.0	0.1
	Heuristic	0.0	0.0	0.0	0.0	0.0	0.0	0.1
	APSP	0.0	0.0	0.0	0.0	0.0	0.0	0.1
	PathEnum.	0.0	0.0	0.0	0.0	0.0	0.0	0.1
	TwoFlow	0.2	0.2	0.2	0.2	0.2	0.2	0.2
	FlowInFlow	0.2	0.2	0.2	0.2	0.2	0.2	0.3
30	None	0.0	0.0	0.0	0.0	0.1	0.0	0.6
	Heuristic	0.0	0.0	0.0	0.0	0.1	0.0	0.6
	APSP	0.0	0.0	0.0	0.0	0.1	0.0	0.6
	PathEnum.	0.0	0.0	0.0	0.0	0.0	0.0	0.6
	TwoFlow	5.3	5.3	5.3	5.3	5.3	5.2	5.8
	FlowInFlow	6.5	6.5	6.6	6.5	6.5	6.5	7.1
50	None	0.1	0.1	0.1	0.1	0.2	0.1	2.1
	Heuristic	0.1	0.1	0.1	0.1	0.2	0.1	2.2
	APSP	0.1	0.1	0.1	0.1	0.2	0.1	2.1
	PathEnum.	0.1	0.1	0.1	0.1	0.1	0.1	2.0
	TwoFlow	41.8	42.2	42.2	41.5	42.5	41.5	43.8
	FlowInFlow	52.7	53.7	52.9	53.3	54.0	53.0	54.6
100	None	0.6	0.7	0.7	0.7	1.2	0.8	8.7
	Heuristic	0.7	0.7	0.8	0.7	1.2	0.9	8.4
	APSP	0.7	0.7	0.7	0.7	1.0	0.9	8.2
	PathEnum.	0.8	0.8	0.9	0.8	1.1	0.8	8.6
	TwoFlow	416.0	419.6	413.1	413.1	418.0	413.6	422.5
	FlowInFlow	490.2	491.5	496.3	493.4	491.3	495.5	503.0
200	None	2.1	2.2	2.4	2.3	4.1	2.9	30.8
	Heuristic	2.2	2.2	2.6	2.5	4.2	3.1	31.0
	APSP	2.2	2.2	2.4	2.3	3.6	3.0	30.5
	PathEnum.	2.8	2.9	3.0	2.9	4.0	2.9	31.3
	TwoFlow	1468.8	1472.8	1473.4	1467.3	1465.6	1479.7	1490.0
	FlowInFlow	1759.9	1763.9	1755.5	1747.6	1764.4	1777.4	1787.7
500	None	11.5	11.5	14.4	12.8	31.5	26.1	286.0
	Heuristic	12.0	11.9	15.8	14.1	32.6	27.1	287.1
	APSP	11.9	12.1	13.9	12.8	28.2	24.9	285.7
	PathEnum.	23.3	24.3	25.2	24.0	42.0	26.6	302.4
1000	None	59.3	59.6	70.3	64.7	128.5	76.1	988.5
	Heuristic	61.0	61.4	75.0	69.5	132.1	81.4	993.2
	APSP	61.7	61.6	67.8	64.3	118.1	74.3	983.9
	PathEnum.	67.8	68.0	74.3	70.9	125.0	74.1	988.0

Table 9.8: Mean required run-time in seconds of preprocessing with different pruning and fixing methods.

Table 9.9: Relative node pruning performance in percent for the presented pruning methods.

	Size									
Pruning	20	30	50	100	200	500	1000			
None	50.20 >	47.37 >	38.94 >	29.32 >	24.12 >	20.64 >	17.65 >			
Heuristic	50.20 >	47.37 >	38.94 >	29.32 >	24.12 >	20.64 >	17.65 >			
APSP	49.10 >	45.66 >	36.76 >	27.34 >	22.36 >	18.31 >	15.63 >			
PathEnum.	48.90 =	45.29 =	36.10 =	26.80 =	21.93 =	18.24 =	15.62 =			
TwoFlow	48.90 =	45.29 =	36.10 =	26.74 =	21.88 =	-	-			
FlowInFlow	48.90 =	45.29 =	36.10 =	26.74 =	21.88 =	-	-			

Table 9.10: Relative arc pruning performance in percent for the presented pruning methods.

	Size										
Pruning	20	30	50	100	200	500	1000				
None	33.28 >	37.39 >	34.03 >	27.50 >	24.51 >	22.29 >	18.85 >				
Heuristic	29.73 >	33.18 >	29.57 >	24.64 >	22.32 >	21.25 >	18.17 >				
APSP	27.58 >	30.22 >	25.99 >	21.32 >	19.52 >	17.90 >	15.61 >				
PathEnum.	26.88 =	28.42 =	23.94 =	19.28 =	17.49 =	17.57 =	15.56 =				
TwoFlow	26.88 =	28.42 =	23.94 =	19.15 =	17.34 =	-	-				
FlowInFlow	26.88 =	28.42 =	23.94 =	19.15 =	17.34 =	-	-				

in Table 9.8, may lie between 30% (size 100) or 200% (size 500). Whether the benefit is worth the additional run-time will have to be seen. Note that for sizes 100 and 200, PathEnumeration does not produce the same results as TwoFlow or FlowInFlow (but is not significantly different). This is due to the cutoff rule discussed in detail in Section 9.6.7. For the two largest size classes, the performance of PathEnumeration is quite close to its fallback methods APSP and Testing. Generally, APSP performs very similar to PathEnumeration. The difference is basically the fraction of nodes for which a path that is not simple and which fulfills the delay constraints exists. As a further observation, the heuristic pruning is not able to prune any nodes (in addition to None), which means that there never is a situation where a node in the extended component graph has no incoming or outgoing arcs so that it can be removed. This is also the reason why APSP does not include this step.

Table 9.10 shows the relative arc pruning performance PA_{rel} for the presented pruning methods. We can again observe that the pruning efficiency increases with the instance size. In contrast to the node pruning performance, now we can see the value of the heuristic pruning. The difference in pruning performance when compared to None is caused by removing the incoming arcs of *s* and the outgoing arcs of *t* while calculating $PA_{s,t}^d$ within an extended component. With this rule alone we can remove about 4% of arcs from PA_f , which is reduced to 0.5% for the largest instance sizes. Considering the delays of paths by using APSP allows us to remove another 3% of arcs. And finally, by calculating the exact domain, we can remove an additional 2%. As

				Size			
Fixing	20	30	50	100	200	500	1000
None	7.74 <	6.60 <	5.13 <	2.95 <	1.53 <	0.59 <	0.30 <
Dominators	7.74 <	6.60 <	5.13 <	2.95 <	1.53 <	0.59 <	0.30 <
Testing	7.78 =	6.69 =	5.22 =	3.03 =	1.61 =	0.62 =	0.31 =
TestingSAP-0	7.74 <	6.60 <	5.13 <	2.95 <	1.53 <	0.59 <	0.30 <
TestingSAP-1	7.78 =	6.69 =	5.22 =	3.03 =	1.61 =	0.62 =	0.31 =
PathEnum.	7.78 =	6.69 =	5.22 =	3.03 =	1.61 =	0.62 =	0.31 =
FixFlow	7.78 =	6.69 =	5.22 =	3.03 =	1.61 =	0.62 =	0.31 =

Table 9.11: Relative node fixing performance in percent for the presented fixing methods.

Table 9.12: Relative arc fixing performance in percent for the presented fixing methods.

				Size			
Fixing	20	30	50	100	200	500	1000
None	1.86 <	1.67 <	1.15 <	0.67 <	0.26 <	0.10 <	0.05 <
Dominators	1.86 <	1.67 <	1.15 <	0.67 <	0.26 <	0.10 <	0.05 <
Testing	1.94 =	1.77 =	1.26 =	0.73 =	0.30 =	0.11 =	0.06 =
TestingSAP-0	1.86 <	1.67 <	1.15 <	0.67 <	0.26 <	0.10 <	0.05 <
TestingSAP-1	1.86 <	1.67 <	1.15 <	0.67 <	0.26 <	0.10 <	0.05 <
PathEnum.	1.94 =	1.77 =	1.26 =	0.73 =	0.30 =	0.11 =	0.06 =
FixFlow	1.94 =	1.77 =	1.26 =	0.73 =	0.30 =	0.11 =	0.06 =

with the node pruning performance, there is nearly no benefit in using PathEnumeration for the largest instance size when comparing to APSP, as a lot of calls to PathEnumeration end up being answered by APSP.

As we are able to exclude about 75% of nodes from PN_f and 80% of arcs from PA_f , one could hope that we are also able to fix a significant number of nodes and arcs. Unfortunately, this is not the case, as Table 9.11 shows for the node fixing performance and Table 9.12 for the arc fixing performance. The presented results are based on using None as pruning method. For node fixing, we can see that for small instance sizes, we can fix about 1.5 nodes for every virtual arc (7.7% of 20 nodes) which increases to three nodes for every virtual arc for instances of size 1000. The advantage of exact methods, while statistically significant, is negligible. It can be seen that Dominators has exactly the same performance as None, so the extended component graph does not contain dominators. For our instances this is not surprising since for every arc in the substrate there also exists the reverse arc. Therefore, the extended component graphs are strongly node biconnected and no dominators exist. This method is only interesting in situations when reverse arcs are missing. Also TestingSAP-0 fails to find any nodes to fix.

For the arc fixing performance, the situation is basically the same, but the performance is even lower. The astute reader will have noticed that we only used None as pruning method for com-

Table 9.13: Average relative node pruning (PN), node fixing (FN), arc pruning (AP) and arc fixing (AF) performance of PathEnumeration for an individual substrate domain evaluation in percent.

Size	PN	FN	PA	FA
20	23.15	19.34	10.06	6.32
30	24.16	14.12	12.36	3.91
50	20.96	9.27	12.30	2.23
100	16.04	5.37	11.81	1.31
200	14.27	2.72	12.40	0.52
500	12.91	1.10	14.70	0.20
1000	11.47	0.56	13.07	0.09

paring the different fixing methods, but the fixing methods depend on the employed pruning method. We have already shown the influence on the run-time. As for the relative node fixing performance, there is no detectable difference depending on the employed pruning. The only observable difference is the arc fixing performance of TestingSAP-1. By using APSP or PathEnumeration as pruning method, it reaches the fixing performance of Testing. However, it requires much more run-time, which makes TestingSAP-1 uninteresting as fixing method and is the reason why we do not show the more detailed data.

A related question regarding the pruning and fixing capabilities of the different methods is the performance cost of having to combine the different substrate domains. Until now, we have only looked at the final virtual arc domains. However, they are of course weaker (i.e., less restrictive) than the substrate domains they are built upon. Table 9.13 shows the average domain pruning and fixing performance of PathEnumeration (used both for pruning and fixing) for individual substrate domain calculations, before they are combined into a virtual arc domain. For instance, it shows that for size 20, a delay restricted simple path in the substrate nodes. That means we are only uncertain about 4% of substrate nodes. This uncertainty increases with instance size. When we compare these values to PN_{rel} and FN_{rel} , we see that due to the combination of substrate domains, we loose roughly half of the pruning and fixing performance, i.e., PN_{rel} is doubled and FN_{rel} is halved. This, however, can also be interpreted in a positive way. Once we are able to fix the locations of the source and the target node of a virtual arc, we can expect to be able to remove half of the nodes that we considered usable and we can double the number of nodes that we know we have to use.

Until now we have concentrated on the final result of the preprocessing algorithm. We will now focus on its inner workings, for example how many domains have to be calculated and what the benefit of memoization is. The results presented are based on the preprocessing algorithm using PathEnumeration as pruning and fixing method. Table 9.14 shows the characteristics of the preprocessing algorithm 9.3. The average VNMP instance of size 1000 contains 1700 virtual arcs, which means that we have to calculate 1700 virtual arc domains (CDom). Because of the high average number of mapping locations for virtual nodes (see Chapter 5), this corresponds

Table 9.14: Properties of the preprocessing algorithm 9.3: Average number of virtual arcs |A'|, calculated virtual arc domains (CDom), substrate domain requests (Dom Req), domain requests that were not memoized (DR Miss) and domain requests within a component (DR Comp).

Size	A'	CDom	Dom Req	DR Miss	DR Comp
20	432	432	6291	803	358
30	629	629	16588	2062	1659
50	947	947	60044	5548	6417
100	1753	1753	420592	19280	26830
200	1695	1695	1086983	63321	121473
500	1732	1732	3197825	313422	661395
1000	1723	1723	7843384	1047682	2456685

Table 9.15: Properties of algorithm 9.2 for calculating $PN_{s,t}^d$ within a component: Average number of domain requests that are memoized (Pos Hit), that are not memoized (Pos Miss), that have equal upper and lower bounds (Pos EqB), that have to be calculated by executing the pruning method (Pruning), the stored $PN_{s,t}^d$ within the components (Sto. D.) and the domain simplification efficiency for possible nodes.

Size	Pos Hit	Pos Miss	Pos EqB	Pruning	Sto. D.	SEff[%]
20	46	312	206	106	59	33.2
30	166	1493	1017	476	306	45.9
50	563	5854	4411	1442	946	47.4
100	2170	24660	20265	4395	2783	47.3
200	9672	111801	96415	15386	9133	44.4
500	83308	578087	501803	76284	52939	32.8
1000	323052	2133633	1931410	202223	138851	31.5

to calculating 7.8 million substrate domains (Dom Req). However, only 1 million of those are unique (DR Miss). By storing the result of the substrate domain calculation, we can reduce the number of calculations to a seventh of what we would have to do without memoization. For calculating the unique substrate domains, we have to calculate 2.4 million substrate domains within the crossed components (DR Comp). It can be seen that the path of a substrate connection in the simplified block tree crosses about 2.4 components on average. For smaller instances, fewer components have to be crossed. This is the point where execution of algorithm 9.2 begins to calculate $PN_{s,t}^d$ and $FN_{s,t}^d$ within a component.

The properties of the $PN_{s,t}^d$ calculation are shown in Table 9.15. Out of the 2.4 million substrate domain requests within a component, only 300000 can be served directly from memory (PosHit). For the remaining 2.1 million requests (Pos Miss), we now calculate upper and lower bounds for $PN_{s,t}^d$. For 1.9 million requests, the upper and lower bounds are equal (Pos EqB), so we know $PN_{s,t}^d$ and do not need to call the pruning method. Note that the distribution between direct

Table 9.16: Properties of algorithm 9.2 for calculating $FN_{s,t}^d$ within a component: Average number of domain requests that are memoized (Fix Hit), that are not memoized (Fix Miss), that have equal upper and lower bounds (Fix EqB), that have to be calculated by executing the fixing method (Fixing), the stored $FN_{s,t}^d$ within the components (Sto. D.) and the domain simplification efficiency for fixed nodes.

Size	Fix Hit	Fix Miss	Fix EqB	Fixing	Sto. D.	SEff[%]
20	43	315	223	92	45	35.0
30	140	1519	1150	369	201	50.9
50	403	6014	5042	972	516	56.0
100	1136	25694	23252	2442	1142	58.5
200	2580	118893	111257	7636	3120	61.2
500	7361	654033	621246	32787	13097	60.9
1000	17699	2438987	2357299	81688	33148	59.5

domain hits and equal bounds is skewed because of the domain simplification that is performed at the end of algorithm 9.2. For the remaining 200000 requests, we have to actually calculate the domain by using the pruning method. Of those calculated domains, we have to store only 138000, which gives a simplification efficiency (SEff) of 30%, which means that we save 30% of memory.

The properties of the $\text{FN}_{s,t}^d$ calculation of algorithm 9.2 are shown in Table 9.16. For fixing nodes, even fewer requests can be served from memory (Fix Hit). However, the number of times we actually have to execute a fixing procedure is lower than for the pruning procedure, the rest of the domain requests can be answered due to equal bounds. In addition, the simplification efficiency is better than for the calculation of possible nodes. This is because generally, very few nodes can be fixed, so $\text{FN}_{s,t}^d$ stays the same for a wide range of delays, which makes simplification work very well.

To sum it all up, due to the design of the preprocessing algorithm, especially with respect to domain bounds and memoization techniques, the selected pruning method (PathEnumeration) is only called 200000 times and the fixing method 80000 times to calculate 7.8 million substrate domains. This is a reduction of the required computations of 97.4% for pruning and 99% for fixing. In addition, the substrate domain problems that have to be solved eventually are smaller than the original problem, since we only consider a component instead of the complete substrate graph, which also increases efficiency.

Based on the presented data regarding the node and arc pruning performances, one could ask if it is possible to prune the substrate graph of the input VNMP instance as a whole, i.e., remove nodes and arcs that do not occur in any PN_f or PA_f . Related to this is which nodes or arcs have to be used, i.e., occur in any FN_f or FA_f . To explore this, we executed the preprocessing algorithm using PathEnumeration as pruning and fixing method on the complete VNMP instance set with loads from 0.1 to 1. We tested with different loads because with fewer virtual networks the probability that some parts of the substrate cannot be used increases. The results are presented in Table 9.17. By PN_S we denote the fraction of substrate nodes that occur in any PN_f , PA_S

Table 9.17: Influence of the VNMP instance load on the preprocessing algorithm's run-time (t[s]), the time required to calculate the simplified block tree $(t_b[s])$, the fraction of substrate nodes (PN_S) and arcs (PA_S) which are contained within at least one PN_f or PA_f and the fraction of substrate nodes (FN_S) and arcs (FA_S) which are contained within at least one FN_f or FA_f.

		Load									
	Size	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
t[s]	20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	50	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1
	100	0.3	0.3	0.4	0.5	0.5	0.6	0.7	0.7	0.8	0.8
	200	1.0	1.3	1.5	1.7	1.9	2.1	2.2	2.5	2.7	2.9
	500	11.4	15.7	17.6	18.9	19.8	21.6	21.9	23.2	24.3	25.4
	1000	15.9	26.0	34.5	40.9	46.7	51.9	58.4	63.0	68.1	72.2
$t_{b}[s]$	20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	100	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	200	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	1000	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
$PN_S[\%]$	20	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	30	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	50	99.5	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	100	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	200	98.2	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	500	90.1	99.1	99.9	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	1000	86.9	97.0	99.4	100.0	100.0	100.0	100.0	100.0	100.0	100.0
$\mathbf{PA}_{S}[\%]$	20	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	30	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	50	99.5	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	100	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	200	97.8	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	500	89.9	99.1	99.9	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	1000	87.1	96.7	99.3	100.0	100.0	100.0	100.0	100.0	100.0	100.0
FN _S [%]	20	26.7	34.0	39.0	43.7	47.3	51.2	54.2	55.8	58.2	60.7
	30 50	28.9 24.1	34.6 28.6	38.4 31.4	43.4 33.7	46.4 35.8	48.1 37.5	50.4 39.6	52.2 41.5	54.7 43.9	56.6 45.3
	100	24.1	28.0	27.5	29.9	31.7	33.3	39.6	35.7	45.9 37.1	45.5 38.2
	200	16.9	19.3	27.5	29.9	23.9	25.1	26.1	27.4	28.3	29.3
	500	9.0	19.5	12.1	13.1	13.9	14.9	15.6	16.4	28.5 17.1	29.3 17.8
	1000	5.4	6.7	7.5	8.4	9.0	9.6	10.1	10.4	11.0	11.5
FA _S [%]	20	15.4	19.7	22.8	25.7	28.0	30.5	32.3	33.6	35.0	36.4
FAS[70]	30	17.0	20.0	22.8	25.7	28.0	28.4	29.9	30.8	32.2	33.5
	50	17.0	17.7	19.6	21.2	27.1	23.5	29.9	25.9	27.2	28.1
	100	14.0	17.7	19.0	18.5	19.8	20.9	24.8	23.9	27.2	24.3
	200	8.2	10.2	11.7	12.8	19.8	14.6	15.3	16.1	16.8	24.5 17.4
	500	3.8	5.3	6.0	6.7	7.2	7.8	8.3	8.7	9.2	9.5
	1000	2.1	2.8	0.0 3.4	3.9	4.3	4.6	6.5 4.9	6.7 5.1	9.2 5.4	9.3 5.7
	1000	2.1	2.0	3.4	5.9	4.3	4.0	4.9	3.1	3.4	5.7

Table 9.18: Properties of the simplified block tree (average number of nodes N, number of component nodes C and articulation point nodes A), average number of fixed components (FC) and different ways of crossing (CR) them per virtual arc.

Size	Ν	Α	С	FC	CR
20	6.8	6.1	0.7	0.5	2.5
30	11.2	9.7	1.5	0.9	4.3
50	18.6	15.8	2.8	1.5	4.2
100	35.0	30.8	4.2	1.7	3.5
200	71.8	62.6	9.2	2.3	4.6
500	154.9	134.9	20.0	2.4	7.9
1000	250.8	219.7	31.1	2.9	11.4

gives the same information about substrate arcs. The fraction of nodes that are fixed in any FN_f is labeled by FN_S , for arcs by FA_S .

Before we have a look if pruning the substrate graph is feasible, we should analyze the required run-times. Until now we have only looked at instances with load 1, now we can see the run-time increase depending on the load. Significant run-times of the preprocessing algorithm only occur at size 200 and above. The interesting thing to note here is the sub-linear growth in required run-time. For size 1000, going from load 0.1 to 0.2 adds 10 seconds to the run-time, from 0.2 to 0.3 8.5 seconds, to 0.4 adds 6.4 seconds, and going from load 0.9 to 1 adds only 4.2 seconds. This is caused by the employed memoization and bounding techniques which get more efficient when more substrate domains have to be calculated. This is doubly true for PathEnumeration, since for evaluating the domain within a component from one source to one target, we have to enumerate the paths to all nodes within the component. This is inefficient if the domains to the other target nodes are never requested. So the 16 seconds required for load 0.1 is basically the warmup-time to calculate paths which get mostly reused when more virtual networks are added. Table 9.17 also shows that the time to build the simplified block graph (labeled t_b) is insignificant, even for the largest instances. Note that this time is independent of the load.

Going back to the substrate pruning idea, we see that this is not feasible. Only for the largest instance sizes and lowest loads some substrate nodes and arcs could be removed. Only for load 0.1 and sizes 500 and 1000 the savings are of a meaningful magnitude, around 10% for nodes and arcs. Far more interesting is the fraction of nodes FN_S and and arcs FA_S for which we know that we have to use them. In the extreme case of size 20 and full load, 60% of the substrate nodes and 36% of the arcs have to be used. For the largest instances, this is reduced to 11% for nodes and 6% for arcs. This information can be used for instance to derive lower bounds for the substrate usage cost.

To finalize the discussion of the preprocessing algorithm, we will have a look at miscellaneous properties that did not fit into the discussion before. They are presented in Table 9.18. Previously (in Table 9.1), we have only presented the average size of the simplified block tree over a limited range of instance sizes, Table 9.18 shows the size for all instance sizes (N). It can be seen that

the simplified block tree only requires 25% to 35% of the number of nodes in the substrate graph to describe the main structure of the substrate. The following two columns in the table show the distribution of nodes in the block tree. Most of the nodes represent articulation points (A), only 10% to 15% of the nodes represent components (C). Note that this number slightly underrepresents the number of components that are found while calculating the block tree, since components that are only connected via one articulation point to the rest of the graph get removed by the simplification procedure.

We have shown that only a few nodes can be fixed for virtual arcs. There is however a related property that we have not touched upon: fixed components. These are components that have to be touched by a virtual arc. This includes just using one node of the component and stands in contrast to the definition of crossed component, which excludes components of which only one node is used. This difference is essential when we take the resources within components into account. Even if a virtual arc only uses one node of a component, it consumes resources. This will become relevant in Chapter 10 for determining satisfiability of a VNMP instance. Table 9.18 shows that a virtual arc has to use between 0.5 and 2.9 components (column FC). The last column (CR) shows by how many ways these forced components can be traversed on average.

9.7 Conclusion

In this chapter, we have presented a preprocessing algorithm for VNMP instances. Its main aim is to calculate tight domains of the virtual arcs, i.e., which substrate nodes or arcs can be used by the implementation of a virtual arc and which substrate nodes or arcs have to be used. This information is especially useful for removing unnecessary variables and constraints in VNMP models.

The preprocessing algorithm works in two phases. The first phase is independent of the virtual network configuration of the instance. It calculates the simplified block tree of the substrate, which contains two types of nodes: Articulation points, which directly represent a node of the substrate graph, and component nodes, which represent a subgraph of the substrate whose shadow is biconnected. The results have shown that the simplified block tree is rather compact, requiring about a quarter of the substrate nodes to represent the global structure of the substrate. The second phase of the preprocessing algorithm derives the domains for the virtual arcs by using the block tree as a guide. Just using the tree alone gives very good results, depending on the instance size 50% to 83% of substrate nodes and 67% to 81% of arcs can be excluded for each virtual arc as unusable. Note that these numbers are high, even though they are based on all mapping possibilities of the source and target node of the virtual arcs. We have demonstrated that when the locations are fixed, about half of the remaining nodes and arcs can be excluded. As for determining which nodes and arcs have to be fixed, only very few nodes and arcs can be identified per virtual arc.

To further improve this performance, we have introduced methods for determining possible nodes (the pruning methods) and fixed nodes (the fixing methods) within the graph represented by a component node of the block tree. We extend this graph by adding a node for every arc, which results in the extended component graph. This allows us to concentrate exclusively on nodes. The pruning problem is \mathcal{NP} -complete, while fixing is in \mathcal{P} . For both problems we presented heuristic and exact solution methods and evaluated them. Based on the results, three interesting method selections for preprocessing can be identified: just using the simplified block tree (None as pruning and fixing method), which gives the best performance for invested time, using APSP as pruning and Testing as fixing method, which is the middle ground between runtime and pruning/fixing performance and using PathEnumeration for pruning and fixing for the best performance at the cost of doubling run-time (in relation to doing nothing) in the worst case. The ILP based pruning and fixing methods were not competitive, even though some effort was expended to increase their efficiency.

A very interesting point to note is that PathEnumeration beats the ILP methods to a large extent. This is surprising, since usually it is not a good idea to solve a problem by enumerating all solutions and one would expect that the more sophisticated methods perform better. There is a number of factors that make PathEnumeration perform so well. First of all, by enumerating simple paths originating at a specific node and limited by a certain delay, we do not only solve one pruning or fixing problem, we solve all pruning and fixing problems that use the originating node as source node and a delay not larger than the current bound. Also the ILP based approaches can solve more than one problem per execution (remember that a path proves for all contained nodes that they are possible in the case of pruning and that only nodes contained in a found path can be fixed), but not as many as PathEnumeration. For PathEnumeration, it is also possible to build upon previous executions when the delay bound is increased. This is not possible for the ILP based methods. The third factor is the sparseness of the extended component graphs which makes enumerating all paths feasible. We expect that the ILP based methods would not fail with denser graphs, but they will definitely require more memory and even more run-time. On the topic of memory, path enumeration starts to fail for larger instances due to its memory requirements, which is why we needed to add a fallback to simpler methods if the extended component graphs are too large. We performed some optimizations to reduce the memory requirements (like storing only unions and intersections of paths), but there is still significant room for improvement.

During the evaluation, we have shown some pruning and fixing methods that seem to be useless. We have included them because the pruning and fixing methods are applicable to more than just preprocessing. Due to external factors (for instance a partial solution for the VNMP instance), we might be able to deduce that a substrate node cannot be used any longer. Based on this new information, we can execute the pruning and fixing methods within the changed component and derive new information. In these cases, a fixing method like Dominators might be useful, even though it is not so during preprocessing, due to the structure of the VNMP instances.

We have shown that for good performance while executing the pruning and fixing methods it is essential that domain bounds are calculated and used, since a very large fraction of domain requests can be answered because the upper and lower bounds are equal. Even if they are not equal, they are useful to reduce the number of nodes for which we have to test whether they are possible or fixed. Memoization is a central component of fast preprocessing.

9.8 Future Work

In this section, we discuss possibilities for future research on the topic of preprocessing. The preprocessing for VNMP instances produces a lot of useful information, especially when

using Path Enumeration. It might be possible to develop specialized VNMP problem formulations that make use of this information, for instance a model that works on the simplified block tree with additional variables that select a specific path within crossed biconnected component. One important caveat to the presented preprocessing method is that it works best with sparse graphs. A computational study is still necessary to determine the characteristics of this method for dense graphs, for applications outside of the telecommunication network domain. It is clear that the decomposition based on the block tree will not work any more, since we will most likely end up with a single biconnected component. Therefore well performing pruning and fixing algorithms will be key. We expect that methods that take the delay into account are going to perform far better in terms of pruning or fixing performance than the alternatives, but this also warrants further investigation.

We saw that the order in which the virtual arc domains are calculated can influence the execution speed. As a refinement, it could be tested if additional improvements can be achieved by looking at the target nodes of virtual arcs in addition of the source nodes. That means, when we know the virtual arcs that may start at a particular substrate node, we could group them according to their possible target nodes. For one particular pair of source and target nodes and the virtual arcs that may be implemented between them, it might be beneficial to impose an additional order based on the delay. Here different strategies (high delay first, low delay first, ...) could be tested. As an additional benefit, after we have solved the substrate domain problems for one particular source node in the substrate, it is possible to erase all memoized data associated with this node, since we will not need it again. This could make preprocessing applicable to even larger instances.

At the moment, the selection of pruning and fixing methods is static, with the exception of PathEnumeration, which falls back to other methods if the component graphs grow too large. It might be beneficial to add more fine-grained control here, and fall back in a staggered way, e.g., first try to apply PathEnumeration, if the problem is too complex fall back to Testing or APSP, if this still uses too much time, just use None. However, for the tested instance sizes, there was never a big run-time difference between applying Testing/APSP and None, so this might only be beneficial for even larger VNMP instances. Another possibility would be to apply PathEnumeration of calls to a pruning method to derive better bounds. It might also be possible to initialize the stored domains within a component with interesting values, for instance with the domains for all pairs of nodes while using the shortest possible delay as delay bound.

A refinement for the pruning methods could be using the All Pair Shortest Path pruning while keeping track of the shortest paths (instead of just their delays). If we accept a node, but the shortest paths from *s* to this node and from this node to *t* are not disjoint, then we use an exact method to determine if this node really belongs to $PN_{s,t}^d$. Since the pruning performance of APSP is very close to the exact methods, it might be feasible to use an ILP based approach also for the largest instance sizes without too much of a performance hit.

We have discarded two ILP formulation ideas for the node testing problem because they did not fit our requirements. However, it might be interesting to see if they offer advantages for larger, denser graphs and in situations were reconfigurability is not a huge concern. For solving the FIXFLOW model, it was beneficial to only have one constraint which has to be removed instead of having |V| constraints where coefficients have to be adapted. This modification might also be possible for the TWOFLOW and FLOWINFLOW models and speed up preprocessing.

A further idea to improve preprocessing efficiency while using ILP pruning or fixing would be collecting all required domains within components and then solving them in one go for the same s, t and k values, but with increasing d, because the possible nodes for low d values are still possible for higher d and do not need to be tested again. The same applies to fixed nodes, for increasing d, only nodes that have been fixed for lower d need to be tested again. Depending on the number of different d values that have to be tested, it might even be beneficial to solve another problem: what is the lowest d such that a simple path from s to t over k exists or what is the highest d such that no simple path from s to t not containing k exists respectively. With this, we would have to solve two optimization problems for each k, instead of k satisfaction problems for each delay bound. This idea directly leads to a more space efficient method of storing domains for memoization. Currently, we store complete domains within components for one source-target node pair sorted by delay. Another possibility would be to store for each node (and source and target node) the minimum allowed delay required to make it possible and the maximum allowed delay to keep it fixed. Depending on the number of different values for the delay bound that occur this could lead to significant memory savings, but of course causes the domain extraction time to be linear in the number of nodes since we need to assemble the domain based on delay values.

We have shown that the combination of substrate domains to build the domains of virtual arcs doubles the possible nodes and arcs and halves the fixed ones. However, we usually combine a lot of different substrate domains (for size 1000 2500 for the expected case) to achieve this doubling. That means that each additional substrate domain only adds very little additional information. It might be possible to rank substrate domain calculation according to their potential of adding new information. For instance, if a substrate domain calculation is done for a connection that starts and ends at nodes for which no such calculation has been executed until now, then it has a high potential for adding new possible nodes/arcs and removing fixed nodes and arcs. It might be possible to calculate substrate domains exactly for the ones with the highest potential and then use the knowledge already gathered about the domain of the virtual arc to speed up the calculation of the complete domain. If we use the information about the already gathered domain only in this limited way, it might be possible to avoid the run-time penalty that we have observed.

A surprising number of substrate nodes or arcs can be fixed, i.e., they have to be used. This information might be useful for defining more intelligent neighborhoods, for instance the clearing neighborhood as discussed in Chapter 6 can skip evaluating nodes which have to be used anyway. Such nodes and arcs are also interesting in their own right. If they are removed from the substrate, the flexibility afforded by being able to chose mapping locations for virtual nodes is not enough to ensure a feasible solution to the VNMP instance, so they are very critical. Of course, when capacity constraints are considered also other nodes might be critical in this sense.

CHAPTER 10

Constraint Programming

10.1 Introduction

In this chapter, we investigate Constraint Programming (CP) approaches for solving the VNMP. For a treatment of the basic working principles of CP, see Section 2.2.11. Section 10.2 introduces CP formulations for the VNMP. In Section 10.3, we apply the lessons learned while designing Construction Heuristics (Chapter 6) to devise heuristic branching rules that guide CP towards feasible solutions. Methods for strengthening propagation are discussed in Section 10.4. The analysis of the different improvements for the CP approach can be found in Section 10.5. Section 10.6 summarizes the results and Section 10.7 shows possible directions for future work.

10.2 Models

We will present two different CP models for solving the VNMP. Their main difference is the type of variables used for modeling. The presented model in Section 10.2.1 utilizes binary variables. For instance, that means for representing an implementing path for a virtual arc, we need variables for each substrate arc telling us whether this substrate arc is used to implement this particular virtual arc. Another possibility is using set variables as presented in Section 10.2.2. With set variables, we only need one variable to define the implementing path of a virtual arc. Usually, CP models based on set variables perform better, both in terms of run-time and memory requirements [46]. We will evaluate whether this is true for the VNMP models in Section 10.5. The following CP models assume that preprocessing as defined in Chapter 9 has been executed. As a short summary, $PN_f \subseteq V$ is the set of substrate arcs that a virtual arc can use (the possible nodes). $PA_f \subseteq A$ is the set of substrate arcs that a virtual arc has to use (the fixed nodes), and $FA_f \subseteq A$ is the set of substrate arcs that a virtual arc has to use (the fixed nodes), and $FA_f \subseteq A$ is the set of substrate arcs that a virtual arc has to use (the fixed nodes). If no preprocessing has taken place, $PN_f = V$, $\forall f \in A'$, $PA_f = A$, $\forall f \in A'$ and $FN_f = FA_f = \emptyset$, $\forall f \in A'$.

Variab	ble	Domain	Description
x_i^k	$\forall (k,i) \in M$	[0, 1]	Mapping for virtual nodes
y_e^f	$\forall f \in A', \ \forall e \in \mathbf{PA}_f$	[0,1]	Substrate arcs for virtual arcs
$egin{array}{c} z^f_i \ u^V_i \ u^V_i \end{array}$	$\forall f \in A', \ \forall i \in \mathbf{PN}_f$	[0,1]	Substrate nodes for virtual arcs
u_i^V	$\forall i \in V$	[0,1]	Used substrate nodes
u_e^A	$\forall e \in A$	[0,1]	Used substrate arcs
$u_e^A a_i^{CPU}$	$\forall i \in V$	$[0, \sum_{k \in V'} c_k]$	Additional CPU resources
$a_e^{\mathbf{BW}}$	$\forall e \in A$	$\left[0, \sum_{f \in A'}^{n \in Y} b_f\right]$	Additional bandwidth

Table 10.1: Variables of the CP model based on binary variables

10.2.1 Binary Model

To formulate a CP model for the VNMP based on binary variables, we need two sets of main variables. The first set of variables x_i^k , $\forall (k,i) \in M$, is used to specify the mapping of each virtual node. If x_i^k is true, virtual node k is mapped to substrate node i. We define $x_i^k = 0$, $\forall (k,i) \in (V' \times V) \setminus M$, to simplify the model definition. To define the paths used to implement the virtual arcs, we use variables y_e^f , $\forall f \in A'$, $\forall e \in PA_f$. If y_e^f is true, substrate arc e is used to implement virtual arc f. These variables are sufficient to define a solution to the VNMP. However, to be able to express the constraints we need further auxiliary variables. First, there are variables z_i^f , $\forall f \in A'$, $\forall i \in PN_f$. If z_i^f is true, substrate node i is crossed by the implementing path of virtual arc f. This information is required to define the CPU constraints. Variables u_e^A , $\forall e \in A$, are true for all substrate arcs which are used to implement virtual arcs. They are required to define the objective. To be able to buy additional CPU resources if necessary, we use integer variables a_i^{CPU} , $\forall i \in V$. They specify for each substrate node how much additional CPU resources have been bought. Integer variables a_e^{BW} , $\forall e \in A$, fulfill the same role for additional bandwidth for each substrate arc. A summary of the used variables and their domains is shown in Table 10.1.

The complete CP model for the VNMP based on binary variables (CPBIN) is defined by equations (10.1)–(10.11). We begin the discussion of the model with the objective function as shown in (10.1). The aim is to minimize the cost incurred by having to buy additional resources. If it is possible to realize the virtual network load with the available resources, we want to reduce the cost incurred by using the substrate network as much as possible. K denotes a sufficiently large constant, for instance the sum of all p_i^V and p_e^A .

$$\min \sum_{i \in V} K p^{\text{CPU}} a_i^{\text{CPU}} + \sum_{e \in A} K p^{\text{BW}} a_e^{\text{BW}} + \sum_{i \in V} p_i^V u_i^V + \sum_{e \in A} p_e^A u_e^A$$
(10.1)

For a valid description of a VNMP solution, we must first enforce that every virtual node is mapped exactly to one substrate node.

$$\sum_{(k,i)\in M} x_i^k = 1 \qquad \forall k \in V' \tag{10.2}$$

The implementations of the virtual arcs have to be simple paths from the mapping location of the source of the virtual arc to the mapping location of the target of the virtual arc within the substrate. We use the idea of network flows to formulate this. Each virtual arc sends one unit of flow across the substrate network, which enters at the mapping location of the source of the virtual arc and leaves at the mapping location of the target. At the nodes we have flow conservation, which means that all incoming flow has to leave again. The flow defines the implementing path for each virtual arc. This is enforced by the following two constraints, which also link the flow to the z_i^f variables. Remember that we have defined $x_i^k = 0$ for $(k, i) \notin M$.

$$\sum_{e \in \mathbf{PA}_f \mid t(e)=i} y_e^f + x_i^{s(f)} = z_i^f \qquad \forall f \in A', \ \forall i \in \mathbf{PN}_f$$
(10.3)

$$\sum_{e \in \mathsf{PA}_f | s(e) = i} y_e^f + x_i^{t(f)} = z_i^f \qquad \forall f \in A', \ \forall i \in \mathsf{PN}_f$$
(10.4)

The CPU load caused by hosting virtual nodes and routing data may not exceed the available resources. If necessary, more resources have to be bought. The amount of CPU resources that have to be bought for a particular substrate node are the resources that are missing after the mapped virtual nodes and all traversing virtual arcs have consumed the available resources, but at least zero. We have stated the CPU constraint in this way so that a_i^{CPU} is always as small as possible. If we had chosen for example an inequality to formulate the CPU constraint, a complete assignment to the main variables (x_i^k and y_e^f) would not cause a_i^{CPU} to become assigned, it would just force a lower bound. Additional branching would be needed to find a valid assignment to a_i^{CPU} . Suffice it to say, this would be very inefficient, since we already know that the smallest possible value will be valid and also the best assignment according to the employed objective function.

$$\max(0, \sum_{(k,i)\in M} c_k x_i^k + \sum_{f\in A'|i\in \mathsf{PN}_f} b_f z_i^f - c_i) = a_i^{\mathsf{CPU}} \qquad \forall i \in V$$
(10.5)

In a similar fashion, the bandwidth required by virtual arcs crossing a substrate arc may not exceed the available resources. If necessary, more resources have to be bought.

$$\max(0, \sum_{f \in A' | e \in \mathbf{PA}_f} b_f y_e^f - b_e) = a_e^{\mathbf{BW}} \qquad \forall e \in A \qquad (10.6)$$

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The third resource we need to take care of is the delay. The implementing path for the virtual arcs may not exceed the respective allowed delays.

$$\sum_{e \in \mathsf{PA}_f} d_e y_e^f \le d_f \qquad \qquad \forall f \in A' \tag{10.7}$$

To make the objective function work as intended, we need to set u_i^V if at least one virtual node is mapped to substrate node *i*.

$$\max(x_i^k \mid (k,i) \in M) = u_i^V \qquad \forall i \in V \qquad (10.8)$$

In addition, we need to set u_e^A if at least one virtual arc uses substrate arc e.

$$\max(y_e^f \mid e \in \mathbf{PA}_f) = u_e^A \qquad \qquad \forall e \in A \qquad (10.9)$$

As the last part of the model, we need to set variables which we know to be set based on the results of preprocessing.

$$z_i^f = 1 \qquad \qquad \forall f \in A', \ \forall i \in FN_f \tag{10.10}$$

$$y_e^f = 1 \qquad \qquad \forall f \in A', \ \forall e \in \mathsf{FA}_f \tag{10.11}$$

10.2.2 Set Model

As with the discussion of CPBIN, before we can state the constraints we need to define the used variables. To define a VNMP solution, we again need two sets of main variables. The set variables P_f , $\forall f \in A'$, define for each virtual arc the set of substrate arcs that are used to implement the virtual arc. The set variables X_i , $\forall i \in V$, define for each substrate node which virtual nodes it hosts. Note that this is an indirect way of specifying the locations of the virtual nodes. Defining it this way will become useful later when stating the CPU constraint for substrate nodes.

In addition to those main variables, we need auxiliary variables that allow us to formulate the resource constraints. To define the set of virtual arcs that utilize a substrate arc, we use variables Y_e , $\forall e \in A$. These variables are in some sense the duals of P_f , so they cannot express any additional information. However, we need both variable types to specify (and implement) constraints, P_f to define the delay constraints and Y_e for the bandwidth constraints. Variables Z_i , $\forall i \in V$, contain for each substrate node the virtual arcs that are crossing it. The remaining variables are the same as for CPBIN. Binary variables u_i^V , $\forall i \in V$, to specify which substrate nodes are used and u_e^A , $\forall e \in A$, for the used substrate arcs. Integer variables a_i^{CPU} , $\forall i \in V$, denote bought CPU resources for each substrate node, variables a_e^{BW} , $\forall e \in A$, bought bandwidth capacities for each substrate arc. To simplify stating the model we define the following four (constant) sets: δ_k^+ and δ_k^- , $\forall k \in V'$, the sets of virtual arcs leaving and entering a virtual

Variab	ole	Domain	Description		
P_f	$\forall f \in A'$	$[\emptyset PA_f]$	Substrate arcs for virtual arcs		
X_i	$\forall i \in V$	$[\emptyset \{k \mid (k,i) \in M\}]$	Virtual nodes mapped to a substrate node		
Y_e	$\forall e \in A$	$[\{f \in A' \mid e \in FA_f\} \dots \{f \in A' \mid e \in PA_f\}]$	Virtual arcs using a substrate arc		
Z_i	$\forall i \in V$	$[\{f \in A' \mid i \in FN_f\} \dots \{f \in A' \mid i \in PN_f\}]$	Virtual arcs using a substrate node		
u_i^V	$\forall i \in V$	[0, 1]	Used substrate nodes		
u_e^A	$\forall e \in A$	[0, 1]	Used substrate arcs		
a_i^{CPU}	$\forall i \in V$	$[0, \sum_{k \in V'} c_k]$	Additional CPU resources		
$a_e^{\mathbf{BW}}$		$[0, \sum_{f \in A'}^{n \in I} b_f]$	Additional bandwidth		
δ_k^+		$\{f \in A' \mid s(f) = k\}$	Outgoing arcs of a virtual node		
$\begin{array}{c} Z_i \\ u_i^V \\ u_e^A \\ a_i^{\text{CPU}} \\ a_e^{\text{BW}} \\ \delta_{k}^+ \\ \delta_{k}^- \\ \delta_{i}^- \\ \delta_{i}^- \end{array}$	$\forall k \in V'$	$\{f \in A' \mid t(f) = k\}$	Incoming arcs of a virtual node		
δ_i^{+}	$\forall i \in V$	$\{e \in A \mid s(e) = i\}$	Outgoing arcs of a substrate node		
δ_i^-		$\{e \in A \mid t(e) = i\}$	Incoming arcs of a substrate node		

Table 10.2: Variables and auxiliary constants of the CP model based on set variables

node and δ_i^+ and δ_i^- , $\forall i \in V$, the sets of substrate arcs leaving and entering a substrate node. A summary of the used variables, constants and their domains is shown in Table 10.2. The complete CP model for the VNMP based on set variables (CPSET) is defined by equations (10.12)–(10.20). We begin the discussion of CPSET with the objective function as shown in (10.12). As before, the aim is to minimize the cost incurred by having to buy additional resources. If it is possible to realize the virtual network load with the available resources, we want

$$\min \sum_{i \in V} K p^{\text{CPU}} a_i^{\text{CPU}} + \sum_{e \in A} K p^{\text{BW}} a_e^{\text{BW}} + \sum_{i \in V} p_i^V u_i^V + \sum_{e \in A} p_e^A u_e^A$$
(10.12)

One type of constraint that we will make heavy use of during the definition of the constraints for CPSET is the disjoint union, which we denote by the symbol \sqcup . Its semantics are the same as the regular union, with the additional constraint that the sets being combined are disjoint. As an example, the constraint $A \sqcup B = C$ states that the set C contains all elements from sets A and B and each element within C has to have a unique source, i.e., either A or B but not both. This allows us to state the mapping constraint as shown in equation (10.13) in an elegant way. All virtual nodes have to be hosted on exactly one substrate node.

$$\bigsqcup_{i \in V} X_i = V' \tag{10.13}$$

The CPSET model is based on the idea of network flows. The flow conservation constraints are shown in equations (10.14) and (10.15). They state that all virtual arcs that use one of the incoming arcs of a substrate node together with all virtual arcs whose source is mapped to the substrate node give the virtual arcs that are traversing the substrate node. All traversing virtual arcs need to leave either by using an outgoing arc or the mapping of the target of the virtual arc. By using the disjoint union we enforce that the implementing path for every virtual arc is simple.

$$\bigsqcup_{e \in \delta_i^-} Y_e \sqcup \bigsqcup_{k \in X_i} \delta_k^+ = Z_i \qquad \forall i \in V \qquad (10.14)$$

$$\bigsqcup_{e \in \delta_i^+} Y_e \sqcup \bigsqcup_{k \in X_i} \delta_k^- = Z_i \qquad \forall i \in V \qquad (10.15)$$

The following equalities take care of the resource constraints. The amount of CPU resources that have to be bought for a particular substrate node are the resources that are missing after the mapped virtual nodes and all traversing virtual arcs have consumed the available resources, but at least zero.

$$\max(0, \sum_{k \in X_i} c_k + \sum_{f \in Z_i} b_f - c_i) = a_i^{\text{CPU}} \qquad \forall i \in V$$
(10.16)

In a similar fashion, the bandwidth constraints state that the amount of extra bandwidth that has to be bought for a substrate arc is the bandwidth that is missing after the virtual arcs using that arc have used up the available resources, but at least zero.

$$\max(0, \sum_{f \in Y_e} b_f - b_e) = a_e^{\mathsf{BW}} \qquad \forall e \in A$$
(10.17)

The delay constraints can be stated in a straight forward manner, the delay of all substrate arcs used to implement a virtual arc may not exceed the maximum allowed delay for the virtual arc.

$$\sum_{e \in P_f} d_e \le d_f \qquad \qquad \forall f \in A' \tag{10.18}$$

To make the objective function work as intended, we need to connect the mapping decisions with the variable that is used to track whether a substrate node is used to host a virtual node. If the set of hosted virtual nodes is not empty, then the substrate node is used.

$$\min(1, |X_i|) = u_i^V \qquad \qquad \forall i \in V \qquad (10.19)$$

In the same way, we connect the substrate arc usage to the virtual arcs using the substrate arc. If the set of virtual arcs using a substrate arc is not empty, then the substrate arc is used.

$$\min(1, |Y_e|) = u_e^A \qquad \qquad \forall e \in A \qquad (10.20)$$

As a last step, we need to add the channeling between the substrate arcs used to implement the virtual arcs and the virtual arcs crossing substrate arcs.

$$e \in P_f \Leftrightarrow f \in Y_e \qquad \qquad \forall e \in A, \ \forall f \in A' \tag{10.21}$$

This completes the CPSET model. Note that in comparison to CPBIN, we do not need to add additional constraints to include information about fixed nodes or arcs. The domains calculated for the virtual arcs during preprocessing can be used directly to define the domains of the CPSET variables (see Table 10.2); no further constraints or considerations are necessary.

An additional characteristic of CPSET when compared to CPBIN is its greater compactness. For instance, CPBIN requires one constraint per virtual node to ensure valid mappings, for CPSET a single constraint is sufficient. Flow conservation only requires two constraints for every substrate node, while CPBIN needs 2|A'|. On the other hand, we need an additional type of variables (P_f) to formulate a valid model of the VNMP. In Section 10.5 we will analyze the practical difference between CPBIN and CPSET.

10.3 Heuristic Branching

When trying to find solutions to the VNMP by using the CPBIN or CPSET formulations, we need to define a branching strategy. After propagation has been performed and the domains of the variables cannot be reduced any longer (and not all variables have been assigned a value), branching needs to be performed to basically try out some assignments to a variable and see if this leads to a valid solution.

As default strategy for CPBIN and CPSET we use the following. We first try to assign the mapping variables $(x_i^k \text{ and } X_i)$. During branching, we select the unassigned mapping variable with the highest degree (i.e., occurs in the most constraints), and assign it the value one (for CPBIN) or one virtual node from its domain (for CPSET). In effect, with this branching decision we fix the mapping of one virtual node.

Once the mapping has been fixed, we branch on the variables concerned with the implementing path of a virtual arc $(y_e^f \text{ and } P_f)$. We select an arbitrary variable and set its value to zero (CPBIN), or remove a substrate arc (CPSET). In effect, we forbid the usage of a substrate arc for a virtual arc. This might seem counter-intuitive, as we would like to directly fix an implementing path instead of removing substrate arcs until only one path remains. However, it is not possible to specify a branching strategy in GECODE that builds implementing paths in a logical way (e.g., for a virtual arc select the substrate arc going away from the mapping location of its source node, then the following arc and so on). Instead, arcs are basically randomly selected across the substrate arcs cannot be used to form a simple implementing path. Preliminary runs showed a very bad performance when using this kind of strategy.

However, also the strategy of forbidding the use of a single substrate arc for a virtual arc has its weakness. First of all, there are a lot of decisions required to reach a complete assignment for all variables, and secondly, the implementing paths are still not built in a coherent fashion. To alleviate those problems, we implemented a custom branching strategy, making use of the results presented in Chapter 6. A branching decision is either the mapping of a virtual node or a complete assignment of an implementing path for a virtual arc. We used CH-O to calculate the branching decision. In the context of CP that means the following. As long as there is a virtual arc which is implementable (source and target nodes have been mapped and does not have a (complete) implementing path), implement a virtual arc, otherwise map a virtual node (arc emphasis). If we implement a virtual arc, we select the most delay constrained virtual arc from the virtual network that in total has the most stringent delay constraints. For this virtual arc, we need to find an implementing path. However, just generating one path as possible implementation would mean that the search is not complete (i.e., not all solutions can be found). Therefore, we create multiple paths by following all outgoing arcs from the mapping location of the source of the virtual arc (denoted as continuation arcs) and then finding a path that causes the least increase to the substrate usage cost. During branching, these alternatives are tried in order of the increase in substrate usage cost they cause. Still, this is not complete as we do not try all possible paths, so in case we cannot find a solution by using one of the calculated paths, we just assign one of the continuation arcs and complete the implementation of the virtual arc with a path in a subsequent branching decision.

When we have to map a virtual node, we select the node with the DLHeavyVN strategy, i.e., from the virtual network that is most delay constrained we select the node that has the highest CPU requirements (see Section 6.2). CH-O uses MostFree to determine the mapping location, i.e., the virtual node is mapped to the substrate node which still has the most resources left. In context of CP that means that we first try to map the virtual node to the substrate node with the most free resources. If we fail to find a valid solution using this decision, we try the substrate node with the second most free resources and so on.

By using this branching strategy, the performance of the CP models is at least as good as CH-O. We expect it to be better since we can make use of the results of propagation and for instance exclude mapping possibilities that would have been chosen by CH-O.

10.4 Strengthening Propagation

The task of propagation is to reduce the domains of variables as much as possible, based on the imposed constraints. If the domain of a variable becomes empty, we know that the current assignment of values to variables is inconsistent and it is not possible to find a solution that satisfies all constraints with it. If the current assignment is inconsistent, we want to detect that as early as possible. A way to facilitate this would be using the DomReachability propagator [140, 141], which is used to find constrained paths in a network. However, this propagator has a high memory overhead. We would need to store three times a graph of the size of the substrate network for each of the virtual arcs. In addition, it is based on dominator trees, which cannot be used when we want to consider the delay constraints. These problems were actually one of the main reasons for developing the preprocessing methods as presented in Chapter 9, as we needed a method which is less sensitive to a high number of virtual arcs and can also consider the delay constraints. So as a first step to strengthen propagation, we remove useless values from the domains of the variables by using the preprocessing techniques.

This is something we can do once at the beginning when we start searching for a solution, but does not help during the search. Based on mapping decisions or chosen implementing paths, some nodes or arcs might become forced for virtual arcs or might not be possible any longer. This is detected only in the simplest cases. To improve on this situation, it would of course be possible to re-apply the preprocessing methods on the residual substrate network (i.e., the substrate network with the capacities that are still available) and the virtual arcs that still need to be implemented. This would mean either a very high overhead or a complex implementation making use of upper and lower domain bounds for virtual arcs not only according to the allowed delay, but also the available parts of the substrate network. Therefore, we chose a simpler approach. As outlined in Chapter 9, the preprocessing does not directly calculate the domains for the virtual arcs. Instead, it calculates domains for delay constrained paths in the substrate network and combines the relevant domains (depending on the possible mapping locations of the source and target node of a virtual arc) to derive the final domain of the virtual arc. Every time the mapping targets change, there is a possibility that the domain of the virtual arc changes. We chose to update the domain only when a mapping has been fixed, because then we have the highest probability of actually reducing the domain for the virtual arc.

In total, we update the domain of a virtual arc f three times during the search for a solution. The first time at the beginning, based on the results of the preprocessing procedure. Then, when either s(f) or t(f) has been mapped and the last time when both virtual nodes have been mapped. These domain updates just reuse information already generated during preprocessing, so they are very fast and do not require any additional memory (aside from keeping the preprocessing information during search). As a downside, these domains are based on the initial resources available in the substrate network and not on the current resource levels.

10.5 Results

In this section, we evaluate the performance of the two proposed formulations for the VNMP and the various improvements described in the previous section. We will mainly focus on solving the VNMP-S. We do that by removing the objective we presented for the CPBIN and CPSET models and instead add a constraint forcing C_a to be zero. Another approach would have been to minimize C_a . Preliminary runs have shown that this reduces the number of VNMP instances for which a valid solution can be found since the search gets stuck at finding an improvement to a bad solution (in terms of C_a) instead of finding a valid one. We test the following configurations:

we test the following configurations.

- **CPB-B** The CPBIN model with standard branching and no improvements.
- **CPS-B** The CPSET model with standard branching and no improvements.
- **CPS-PE** The CPSET model using PathEnumeration as pruning and fixing strategy for preprocessing.
- **CPS-BR** The CPS-PE configuration, but in addition using the heuristic branching introduced in Section 10.3.

	Size	CPB-B	CPS-B	CPS-PE	CPS-BR	CPS-VA	CPS-NE	CPS-OP
# Valid	20	41	38	80	110	111	108	112
	30	15	12	48	104	105	99	105
	50	0	0	23	98	102	100	102
	100	0	0	6	77	81	78	81
Mem.	20	0	0	0	0	0	0	0
	30	43	0	0	0	0	0	0
	50	69	57	2	0	0	0	0
	100	112	60	38	23	21	10	21
$\mathbf{C}_{\mathbf{u}}$	20	1182.5	981.1	1011.2	1062.8	1067.5	1051.5	914.6
	30	1813.1	1238.2	1399.7	1543.4	1550.5	1504.7	1336.1
	50	-	-	2004.2	2332.3	2368.9	2324.4	2163.4
	100	-	-	3487.7	4203.7	4310.1	4222.6	4097.9
t[s]	20	6299.9	6810.1	3339.5	884.1	828.4	1066.0	6996.9
	30	8523.2	8890.8	5954.2	1332.0	1248.5	1751.5	8579.6
	50	9801.8	6300.8	7756.6	1831.3	1497.2	1664.8	9830.0
	100	9984.1	5659.1	6856.9	1822.2	1998.0	2668.1	8724.1
t _n [ms]	20	1.1	1.1	0.9	0.6	0.6	0.7	0.8
	30	3.8	1.9	1.4	1.5	1.4	1.4	1.8
	50	13.1	4.2	2.7	2.8	2.5	2.5	3.3
	100	78.0	24.7	9.1	6.4	5.7	6.9	7.3
M. Peak [MB]	20	570	105	23	38	42	28	95
	30	1127	653	120	89	96	70	216
	50	1587	210	446	231	249	183	414
	100	3124	462	440	769	835	654	1271
# Opt	20	0	0	0	0	0	0	37
	30	0	0	0	0	0	0	18
	50	0	0	0	0	0	0	2
	100	0	0	0	0	0	0	0

Table 10.3: Performance of different CP configurations depending on instance size.

- **CPS-VA** The CPS-BR configuration enhanced by using the propagation strengthening technique for virtual arcs outlined in Section 10.4.
- **CPS-NE** The same configuration as CPS-VA, but the heuristic branching uses node emphasis (i.e., maps all virtual nodes first), instead of arc emphasis.
- **CPS-OP** The CPS-VA configuration used for solving VNMP-O instead of VNMP-S.

The motivation behind this choice of configurations will become clear when the computational results are being discussed. We use VNMP instances of size 20, 30, 50, and 100, at loads 0.1, 0.5, 0.8, and 1 as test instances. A time-limit of 10000 seconds and a memory limit of 5 GB was employed.

Table 10.3 shows the performance of the different CP configurations depending on instance size. First, we will have a look at the basic models and their characteristics, i.e., configuration CPB-B using the CPBIN model and CPS-B using CPSET. When considering the number of valid solutions (out of 120 instances) found by the CPBIN and CPSET models (labeled # Valid), it can be seen that those numbers are very low, and that CPBIN has a slight advantage. Not a single valid solution can be found for instances of size 50 and 100.

In terms of the number of times the solution procedure had to be aborted due to the memory limit (labeled Mem.), we can see that CPSET is far better than CPBIN, for instances of size 100 nearly every execution of CPBIN has to be aborted, while CPSET succeeds at least for half of the instances. With respect to the average substrate usage cost C_u , we can see that the solutions found by CPSET are cheaper. However, these numbers are only based on valid solutions and as CPBIN produces more of those we cannot conclude that solutions by CPSET are cheaper.

The average required run-time is labeled by t[s] in the table. If the execution had to be aborted, we assume a run-time of 10000 seconds. We can observe that CPSET is faster than CPBIN. This statement is also supported by the average time required to perform propagation before another branching decision is necessary (labeled as node-time t_n). Note that the reported values of t_n are only based on instances where the solution process was not aborted due to the memory limit.

With respect to the peak memory consumption (labeled M. Peak), we can again observe an advantage of CPSET. These values are based on instances where CP did not fail due to memory reasons. The last shown property, the number of instances that could be solved to optimality (labeled Opt.), is of course zero for CPB-B and CPS-B, as they solve VNMP-S and not VNMP-O.

To sum it all up, while CPBIN is able to solve a bit more instances, CPSET requires far less memory and also has faster propagation. Therefore, we considered CPSET to be more promising and continued with this model as basis for further improvements.

When we activate preprocessing (CPS-PE), we can observe a huge improvement in terms of performance. Now, CP is also able to find valid solutions for instances of size 50 and 100, and fails due to memory reasons only for a third of instances of size 100. Also the propagation time is decreased, since preprocessing can remove a lot of superfluous variables.

With heuristic branching (CPS-BR), we get another boost in performance. CP fails due to memory reasons only for the instances of size 100. It is interesting to see that in some cases the peak memory requirements have increased. This is due to two effects. First of all, these numbers are based on more, and more challenging, instances (since fewer executions fail due to the memory limit). Secondly, due to the employed heuristic branching scheme, we get far deeper in the search tree than with the standard branching, which gets stuck early on due to undetected inconsistencies. However, most of the performance increase can be explained by the capability of CH-O for finding valid solutions. CH-O on its own finds 99 valid solutions for size 20, 96 for size 30, 91 for size 50 and 77 for size 100. Only the remaining instances that could be solved benefit from CP, for size 100, there is no benefit at all.

The performance of CP can be increased a little bit by using improved propagation (CPS-VA). Two instances do not fail any more due to memory limits and four more valid solutions can be found for the largest instance size. It is remarkable that the average propagation time does not increase, even though we perform more propagation. The peak memory requirements on the

	Size	CPB-B	CPS-B	CPS-PE	CPS-BR	CPS-VA	CPS-NE	CPS-OP
# Valid	0.10	19	19	66	120	120	120	120
	0.50	16	15	46	119	119	118	119
	0.80	13	11	31	101	106	99	106
	1.00	8	5	14	49	54	48	55
Mem.	0.10	30	24	0	0	0	0	0
	0.50	69	58	22	0	0	0	0
	0.80	72	29	16	3	2	1	2
	1.00	53	6	2	20	19	9	19
$\mathbf{C}_{\mathbf{u}}$	0.10	922.6	705.0	1166.3	1381.9	1382.0	1376.7	1236.7
	0.50	1471.4	1142.0	1489.4	2441.9	2440.8	2409.5	2217.2
	0.80	1607.7	1358.5	1579.3	2667.5	2740.4	2670.6	2516.3
	1.00	1713.4	1334.0	1475.5	2119.4	2319.8	2299.9	2115.4
t[s]	0.10	8230.2	7128.2	4455.1	0.1	0.1	0.0	6087.5
	0.50	8501.4	4707.4	4572.4	84.6	84.7	168.0	9407.4
	0.80	8753.7	6652.7	6287.2	1333.7	1000.6	1675.4	9733.1
	1.00	9123.6	9172.5	8592.5	4451.3	4486.7	5307.1	8902.6
t _n [ms]	0.10	0.7	0.4	0.4	0.5	0.6	0.6	0.9
	0.50	7.0	1.1	1.2	1.8	1.9	1.9	2.9
	0.80	8.7	7.9	3.4	3.6	3.5	3.4	4.5
	1.00	13.1	11.2	7.0	5.1	3.9	5.4	4.4
M. Peak [MB]	0.10	616	252	89	12	13	10	30
	0.50	1461	434	289	199	209	156	513
	0.80	1273	540	325	450	490	354	778
	1.00	1046	278	281	395	443	395	552
# Opt	0.10	0	0	0	0	0	0	49
	0.50	0	0	0	0	0	0	7
	0.80	0	0	0	0	0	0	1
	1.00	0	0	0	0	0	0	0

Table 10.4: Performance of different CP configurations depending on instance load

other hand increase due to the space required for preprocessing data. Now, CP is also able to solve more instances than CH-O for size 100.

The next tested configuration (CPS-NE) uses node emphasis instead of arc emphasis for the heuristic brancher. We tested this configuration because we suspected that there could be a possible benefit in combination with the better propagation. We propagate only if a virtual node has been mapped. If we map the virtual nodes early, then we get better domains for the virtual arcs faster, which might help with the search for valid solutions. However, the results show that this was not the case, fewer valid solutions could be found.

As the last experiment, we tried to tackle VNMP-O with the CPS-OP configuration. We can see that some instances can be solved to optimality, but these are not a significant fraction of the total VNMP instances. Based on the reported C_u values, we can see an improvement of about 10% when compared to CPS-VA, at the expense of a six fold increase in required run-time.

Table 10.4 shows the performance of the different CP configurations depending on the load of the VNMP instances. Again we can observe the advantage of CPSET compared to CPBIN with respect to the number of memory aborts, especially for instances of high load. CPS-PE and CPS-BR give huge benefits across the board. It is interesting to see that CPS-BR fails more often for the instances of highest load than CPS-PE. Because of the heuristic branching, the search gets far deeper into the search tree until inconsistencies are encountered. That also means that the memory requirements are higher, which leads to more failed instances. With CPS-BR, CP is able to solve all instances of the lowest load. Improved propagation (CPS-VA) is only useful for instances of the highest load and using node emphasis causes performance regressions for those instances. CPS-OP can basically only prove optimality for instances of the lowest load levels.

10.6 Conclusion

In this chapter, we have introduced two Constraint Programming models for the VNMP, one based on binary variables and one based on sets. In addition, we presented methods for improving the performance of those approaches, such as utilizing preprocessing to reduce the number of variables, heuristic branching or additional propagation during search.

Our computational results showed that the formulation based on set variables is far better in terms of required memory than the formulation using binary variables, even though it requires more variable types. By using preprocessing, the memory requirements can be reduced further. The main component necessary for finding valid VNMP solutions turned out to be the use of the heuristic brancher. This brancher however was the sole determining factor if a valid solution could be found. Especially for instances of the largest size a solution was either found by following the first suggestion of the brancher or not at all. This situation could be slightly improved by using the advanced propagation. Only the smallest instances of lowest load could be solved to optimality.

Based on these results, the main problem of the CP approaches is weak propagation, i.e., inconsistent assignments are detected too late. This was also confirmed by inspection of the search tree. We saw cases where the branching decision that caused the partial assignment to be inconsistent was 50 levels above the location in the search tree where inconsistencies were detected. Even if the tree was binary, the search could never (in a practical amount of time) back-track to the wrong decision and revert it. In the following section, we will give some ideas on how to improve that situation.

The question that remains now is of course whether the CP approach is promising, since we have not presented another exact method for solving the VNMP. However, anticipating the results we will show in the following chapter, which is concerned with Integer Linear Programming approaches, the performance of CP is very bad. We were not even able to solve VNMP instances larger than 100 nodes. Nevertheless, the CP approach is interesting, since it does not require the constraints to be linear, which might make it the only promising exact solution method for extensions to the VNMP.

10.7 Future Work

We could show that the main problem of the CP method is the weak propagation. One possibility to alleviate this problem is to reapply the preprocessing methods during search, but as we have already pointed out, such an approach would have either a high overhead or be very intricate. Even if such an approach was used, it would still be focused only on one virtual arc. It is not possible to detect for instance that a group of nodes or arcs has not enough resources left to implement some virtual arcs. Just imagine three virtual arcs, each requiring one unit of bandwidth. The sources of the virtual arcs are mapped to the same substrate node, and all targets are mapped to another node. If there are two paths between those two substrate nodes, with a capacity of one unit of bandwidth each, it is immediately apparent that no solution can exist. The CP approach, even with the preprocessing techniques, is not able to detect this situation, so some techniques with a more global view are needed.

One possible approach would be to consider the subgraphs of the substrate network corresponding to one component of the block tree calculated during preprocessing. For each of the subgraphs, we keep track of the virtual arcs that need to cross it. Note that even in case the source and target node of a virtual arc have not been fixed, its implementation may be forced to cross a particular subgraph.

Based on this, we can define the following problem. We are given a subgraph of the substrate (with the currently remaining resources) and a set of virtual arcs that have to cross the subgraph. For each virtual arc, we have a set of configurations of how it may cross the subgraph, which are ultimately depending on the different mapping configurations for its source and target node. These configurations are pairs of nodes by which the virtual arc enters the subgraph and by which it leaves. A solution to this problem is a selection of one configuration for each virtual arc and an implementing path through the subgraph, such that the resource constraints are satisfied. In addition, the selection of configurations needs to be consistent. All virtual arcs with the same source node have to enter from the same substrate node and all virtual arcs with the same target node have to leave by the same node.

This problem can be solved at different levels of relaxation. We might employ single commodity flow algorithms or formulate this problem as an integer linear program. This approach might be strengthened further by also incorporating the mapping decisions, that means, when a virtual arc enters a subgraph due to mapping, then we have at the entering node not only the CPU cost caused by the transferred data, but also the cost for hosting the virtual node. We are convinced that this method can lead to further improvements, but in first prototype implementations inconsistencies were still not detected early enough.

As a further technique to improve performance, we could activate the advanced propagation techniques outlined in this section only when needed (i.e., when we start to find inconsistencies). The best thing to to in this case would be to track back in the search tree, and check every partial assignment (once again, but now with better propagation) for consistency, until we find a consistent assignment (or at least cannot prove inconsistency) and then continue with the search without the advanced propagation until it is needed again. However, this behaviour is not natively supported by GECODE and would require a custom search engine.

CHAPTER 11

Mixed Integer Linear Programming

11.1 Introduction

In this chapter, we will introduce (Mixed) Integer Linear Programming (ILP) formulations for the Virtual Network Mapping Problem. Section 11.2 presents a multi-commodity flow based ILP formulation for the VNMP, while a Column Generation approach based on paths is shown in Section 11.3. Computational results can be found in Section 11.4. We conclude in Section 11.5 and give promising directions for future work in Section 11.6. A precursor to the work presented in this chapter was published in [88].

11.2 Multi-Commodity Flow Model

This section presents a multi-commodity flow ILP formulation for the VNMP. Its objective is to find the cheapest possible implementation of all virtual networks within the substrate (solving VNMP-O). Later on, we will present variants of this formulation with the aim of finding a feasible solution, and if this is not possible, the cheapest possible way of adding resources to the substrate so that all virtual networks fit into the substrate (VNMP-S).

For the following ILP model, we assume that preprocessing as defined in Chapter 9 has taken place. As a short summary, $PN_f \subseteq V$ is the set of substrate nodes that a virtual arc $f \in A'$ can use (the possible nodes). $PA_f \subseteq A$ is the set of substrate arcs that a virtual arc can use (the possible arcs). Correspondingly, $FN_f \subseteq V$ is the set of substrate nodes that a virtual arc has to use (the fixed nodes), and $FA_f \subseteq A$ is the set of substrate arcs that a virtual arc has to use (the fixed arcs). If no preprocessing has taken place, $PN_f = V$, $\forall f \in A'$, $PA_f = A$, $\forall f \in A'$ and $FN_f = FA_f = \emptyset$, $\forall f \in A'$.

The model utilizes decision variables $x_i^k \in \{0, 1\}$, $\forall (k, i) \in M$, to indicate where the virtual nodes are located in the substrate graph and $y_e^f \in \{0, 1\}$, $\forall f \in A'$, $\forall e \in \mathsf{PA}_f$, to indicate if a virtual connection is implemented by using a substrate connection. To simplify the model, we define $x_i^k = 0$, $\forall (k, i) \in (V' \times V) \setminus M$. The decision variable $z_i^f \in \{0, 1\}$, $\forall f \in A'$, $\forall i \in \mathsf{PN}_f$,

indicates that a substrate node is used to route a virtual connection. Further auxiliary decision variables are $u_i^V \in \{0, 1\}$, $\forall i \in V$, to indicate that a substrate node hosts at least one virtual node and $u_e^A \in \{0, 1\}$, $\forall e \in A$, to indicate that a substrate arc is used to implement at least one virtual connection.

The complete model is defined by inequalities (11.1)-(11.16).

(FLOW) min
$$\sum_{i \in V} p_i^V u_i^V + \sum_{e \in A} p_e^A u_e^A$$
(11.1)

$$\sum_{(k,i)\in M} x_i^k = 1 \qquad \forall k \in V'$$
(11.2)

$$\sum_{e \in \mathsf{PA}_f | t(e) = i} y_e^f + x_i^{s(f)} - \sum_{e \in \mathsf{PA}_f | s(e) = i} y_e^f - x_i^{t(f)} = 0 \qquad \forall f \in A', \ \forall i \in \mathsf{PN}_f$$
(11.3)

$$\sum_{e \in \mathsf{PA}_f | t(e) = i} y_e^f + x_i^{s(f)} \le z_i^f \qquad \forall f \in A', \ \forall i \in \mathsf{PN}_f \qquad (11.4)$$

$$\sum_{(k,i)\in M} c_k x_i^k + \sum_{f\in A' \mid i\in \mathsf{PN}_f} b_f z_i^f \le c_i \qquad \forall i\in V$$
(11.5)

$$\sum_{f \in A' \mid e \in \mathsf{PA}_f} b_f y_e^f \le b_e \qquad \forall e \in A \tag{11.6}$$

$$\sum_{e \in \mathsf{PA}_f} d_e y_e^f \le d_f \qquad \forall f \in A' \tag{11.7}$$

$$y_e^f = 1 \qquad \forall f \in A', \ \forall e \in \mathsf{FA}_f \qquad (11.8)$$
$$z_i^f = 1 \qquad \forall f \in A', \ \forall i \in \mathsf{FN}_f \qquad (11.9)$$

$$z_i^f = 1 \qquad \forall f \in A', \ \forall i \in FN_f \quad (11.9)$$
$$x_i^k \le u_i^V \qquad \forall (k,i) \in M \quad (11.10)$$

$$\begin{aligned} x_i^{\kappa} &\leq u_i^{\kappa} & \forall (k,i) \in M & (11.10) \\ y_e^{f} &\leq u_e^{A} & \forall f \in A', \ \forall e \in \mathsf{PA}_f & (11.11) \\ u_i^{V} &\geq 0 & \forall i \in V & (11.12) \end{aligned}$$

$$\begin{aligned} u_i &\geq 0 & \forall i \in V \\ u_e^A &\geq 0 & \forall e \in A \end{aligned} \tag{11.12}$$

$$x_i^k \in \{0, 1\}$$
 $\forall (k, i) \in M$ (11.14)

$$y_e^f \in \{0, 1\} \qquad \forall e \in A, \ \forall f \in A' \qquad (11.15)$$

 $z_i^f \in \{0,1\} \qquad \forall i \in V, \, \forall f \in A'$ (11.16)

Table 11.1 gives a short summary of the used variables, constants, and functions.

The objective of FLOW (11.1) is to minimize the total cost incurred due to hosting virtual nodes on substrate nodes and using substrate arcs to implement virtual arcs. Equalities (11.2) ensure that each virtual node is mapped to exactly one substrate node, subject to the mapping constraints. The flow conservation constraints (11.3) make sure that for each virtual connection there is a connected path in the substrate network going from the location of the source of the virtual connection to the location of the target of the virtual connection. Linking constraints (11.4) make certain that variables z_i^f are equal to one when the corresponding node is used to

Table 11.1: Summary of the used variables, constants and functions to define FLOW ($i \in V$, $e \in A, k \in V', f \in A', l \in A \cup A'$)

Symbol	Meaning	Symbol	Meaning	Symbol	Meaning
G(V, A)	Substrate graph	G'(V', A')	Virtual graph	x_i^k	Map node k to i
c_i	Available CPU	c_k	Required CPU	y_e^f	Use arc e for f
d_e	Delay	d_f	Max. allowed delay	z_i^f	Use node i for f
b_e	Available bandwidth	b_f	Required bandwidth	u_i^V	Use node <i>i</i>
M	Set of allowed mappings	s(l)	Source node of arc l	u_e^A	Use arc e
p_i^V	Node price	t(l)	Target node of arc l		
p_e^A	Arc price				

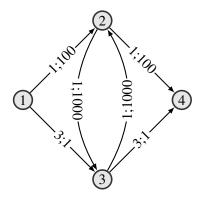


Figure 11.1: Substrate configuration showing that fixing nodes can strengthen FLOW. The arcs are labeled by their delay and costs.

route the traffic of a particular virtual connection. Since z_i^f is binary, this also ensures that the implementing path of the virtual connection is simple. Inequalities (11.5)–(11.7) ensure that the solutions are valid with regard to CPU, bandwidth, and delay constraints. The constraints (11.8) and (11.9) incorporate the knowledge about fixed nodes and arcs into the model. To keep the exposition easy to follow, we chose to include superfluous variables that are explicitly set in the model instead of defining for instance z_i^f only for arcs in PN_f but not in FN_f. In addition, we know that only few nodes and arcs can be set, which limits the overhead caused by additional constraints. A discussion on the usefulness of those two constraint classes fill follow shortly. Linking constraints (11.10) and (11.11) force variables u_i^V and u_e^A to be (at least) one when the corresponding substrate node or arc is used by any virtual node or arc. Inequalities (11.12) and (11.13) ensure that the model is bounded even if some substrate nodes or arcs are not used by any virtual connection. The results presented in Chapter 9 show that this can occur for large instances from the VNMP instance set with low loads. Note that while the model only includes integrality constraints for x_i^k , y_e^f and z_i^f (11.14)–(11.16), constraints (11.12) and (11.13) also cause variables u_i^V and u_e^A to be integral (and binary).

The presented model makes good use of the information provided by preprocessing regarding the substrate nodes and arcs that might be used, by reducing the number of variables (see y_e^{f}) and constraints (see (11.3) and (11.4)) required to formulate the VNMP. By adding equations (11.8) and (11.9), we incorporate information about nodes and arcs that have to be used. Now the question is whether adding those constraints is actually beneficial. The first instinctive reaction would say that yes, adding these is beneficial because we constrain more and should be able to improve the formulation, which means a better (i.e., higher) lower bound on the solution cost derived via calculating the LP relaxation of FLOW. On second thought, however, we recall what it means for a node to be fixed: The implementation of the virtual arc has to use this node, i.e., there is no possible way of not using this node. Therefore, also the LP relaxation will have to use this node, even without adding an explicit constraint, so adding it does not improve the model. This argumentation is incorrect, because it neglects the interaction of the delay bound with the LP relaxation. Due to the nature of the LP relaxation, we can get multiple fractional flows as implementation of a virtual arc, instead of one unit flow. Referring to inequalities (11.7), we can see that crossing a substrate arc only fractionally also incurs the delay only fractionally. As an example, a flow of 0.5 may cross an arc with a delay of 20 even if the delay bound is 10 (neglecting for now the other half of the flow which may incur additional delays). Therefore, we need to identify cases where a fractional flow can go around a fixed node or arc, because in these cases using (11.8) and (11.9) will be useful. One possible way of achieving this would be to use None as pruning and Testing as fixing method (as defined in Chapter 9), so arcs that are not actually possible remain within PN_f which may then carry flow avoiding fixed nodes or arcs. But even with complete pruning, it is possible to avoid fixed nodes. We present an example for this in the following.

Figure 11.1 shows a substrate graph labeled with delay values and costs for using the arcs. We disregard bandwidths, CPU resources and node usage costs because they are not relevant for this example. Assume that a virtual arc needs to be implemented from node 1 to node 4 with a delay limit of 5. As a result, we know that node 2 is fixed. Note that this graph is perfectly pruned. It is easy to see that the optimal result of FLOW will be implementing the virtual arc via the path 1-2-4 for a total solution cost of 200. Now lets consider the LP relaxation of FLOW in this scenario, if we do not fix node 2. It will try to put as much flow as possible across the path 1-3-4, because this is the cheapest possibility. In this case, $\frac{3}{4}$ units of flow, i.e., $y_{(1,3)}^f = y_{(3,4)}^f = \frac{3}{4}$. The rest of the flow uses the path 1-2-4, i.e., $y_{(1,2)}^f = y_{(2,4)}^f = \frac{1}{4}$. In total, this gives a delay of 5 for the implementation of the virtual arc and a substrate usage cost of 51.5, which is very far away from the integer optimal solution cost of 200. If we fix node 2, we require one unit of flow crossing node 2. With this additional constraint, the optimal solution of the LP relaxation will also be 200. This shows that equations (11.9) strengthen FLOW. A similar example can be constructed for equations (11.8). Note that for node fixing to work, inequalities (11.4) have to be equalities. In the presented form, we would only limit the incoming flow to one, but not force it to be one. We will show in Section 11.4 whether fixing nodes and arcs has any effect if inequalities (11.4) are used in their presented form.

As with previous network-flow based ILP formulations, we have the problem of disconnected flow circulations, i.e., there is the main flow from the appropriate source to the appropriate target node, but there may be also flow circulations disconnected from this main flow. Two

factors help reduce the occurrences of these circulations for FLOW. First of all, we have an objective function that tries to minimize substrate usage costs. Therefore, circulations can only occur within the part of the substrate network that is used by some part of the calculated VNMP solution. Note that if we had an objective that directly depends on the utilization of substrate nodes and arcs, circulations would not be possible for an optimal solution. The second factor is the delay constraint. Since any disconnected flow circulation also counts towards the delay limit, it puts a boundary on the length of the circulation. However, even though we have these mitigating factors, we still can have disconnected circulations. As a consequence, this requires a bit more thought when extracting a solution from the solved FLOW model, but this is not problematic. A bigger problem is that the circulations can circumvent the strengthening by fixed nodes or arcs. If a node or arc is fixed, it could simply cause a flow circulation, without having an influence on the main flow. The substrate shown in Figure 11.1 is too small to show this effect. Subtour elimination constraints or directed connection cuts could be used to solve this problem. Note that we do not need to remove all circulations, only those that involve fixed nodes. We will leave the inclusion of cuts as shown in (11.17) to FLOW as future work and concentrate on determining if fixing nodes and arcs has a measurable effect on the performance of FLOW. We expect that circulations involving fixed nodes only become a problem for the largest instances.

$$\sum_{e \in \{(i,j) \in \mathsf{PA}_f | i,j \in S\}} y_e^f \le |S| - 1 \qquad \forall f \in A', \ \forall S \subset \mathsf{PN}_f : |S \cap \mathsf{FN}_f| > 0 \tag{11.17}$$

The presented formulation of the VNMP has one weakness: it fails if a VNMP instance is unsolvable. By failing we mean it detects unsolvability, but does not give any more information, such as where additional resources might be bought so that all virtual networks can be implemented. We already used this aproach for the heuristic methods presented in Chapters 6, 7, and 8. We will now present a modification of FLOW that gives the cheapest possible way to make a VNMP instance feasible if it is not feasible and otherwise minimizes the substrate usage costs. We will call the adapted model FLOW-A.

FLOW-A requires additional information in the form of the cost of one unit of additional bandwidth p^{BW} and the cost of one unit of additional CPU resources p^{CPU} . We also need two additional types of variables. Variables a_i^{CPU} , $\forall i \in V$, to denote the amount of added CPU resources and a_e^{BW} , $\forall e \in A$, to denote the amount of added bandwidth resources. The objective of FLOW-A is given in (11.18). K denotes a sufficiently large constant, for instance the sum of all p_i^V and p_e^A .

$$\min \quad \sum_{i \in V} K p^{\text{CPU}} a_i^{\text{CPU}} + \sum_{e \in A} K p^{\text{BW}} a_e^{\text{BW}} + \sum_{i \in V} p_i^V u_i^V + \sum_{e \in A} p_e^A u_e^A$$
(11.18)

To allow for added CPU capacity, inequalities (11.5) are replaced by inequalities (11.19).

$$\sum_{(k,i)\in M} c_k x_i^k + \sum_{f\in A'|i\in \mathsf{PN}_f} b_f z_i^f \le c_i + a_i^{\mathsf{CPU}} \qquad \forall i\in V$$
(11.19)

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For added bandwidth capacity, inequalities (11.6) are replaced by inequalities (11.20).

$$\sum_{e \in A' \mid e \in \mathsf{PA}_f} b_f y_e^f \le b_e + a_e^{\mathsf{BW}} \qquad \forall e \in A \tag{11.20}$$

To forbid selling of resources, inequalities (11.21) and (11.22) are added to FLOW-A.

$$a_i^{\text{CPU}} \ge 0 \qquad \qquad \forall i \in V \tag{11.21}$$

$$a_e^{\text{BW}} \ge 0 \qquad \qquad \forall e \in A \qquad (11.22)$$

In Section 11.4, we will show how the additional variables influence the behaviour of FLOW-A compared to FLOW.

11.3 Path-based Model

f

In this section, we present an ILP model for the VNMP based on paths. The main difference between this model and the one presented in Section 11.2 is that we now use decision variables to decide which path in the substrate implements a virtual connection, instead of deciding which substrate arc is part of the implementing arc. Since the set of paths may be exponential in size, so is this model (in principle). To get around this problem, we use delayed column generation (see Section 2.2.12).

Before we can show the complete model, some definitions are required. First, we need the extended graph $G_P(V_P, A_P)$ based on the substrate graph G(V, A): $V_P = V' \cup V \cup V''$, where V'' contains a copy \overline{k} of each node $k \in V'$, and $A_P = A \cup \{(k, i), (i, \overline{k}) \mid (k, i) \in M\}$. The delays of all additional arcs are zero, the available bandwidth larger than $\sum_{f \in A'} b_f$. By using this construction, the implementation of a virtual arc f always has to connect the same nodes $(s(f) \text{ and } \overline{t(f)} \text{ in } V_P)$, regardless of the mapping decision, which simplifies the model definition. Furthermore, since the virtual nodes contained in G_P only have outgoing edges and the copies of the virtual nodes only have incoming edges, every path implementing a virtual arc is valid. Without the copies of the virtual nodes (i.e., incoming and outgoing arcs for virtual nodes) we could have paths that cross other virtual nodes before reaching the target node. On the topic of paths, for all virtual arcs $f \in A'$ let P_f be the set of all simple paths from s(f) to $\overline{t(f)}$ in G_P not exceeding the delay bound d_f .

The model utilizes decision variables $w_{p_f} \in \{0,1\}$, $\forall p_f \in P_f$ to indicate which path is chosen to implement virtual arc f. As for FLOW, we have variables x_{ki} specifying if virtual node kis mapped to substrate node i. The presented model is capable of adding resources if required (akin to FLOW-A), so we need variables u_i^V , $\forall i \in V$, to indicate if a substrate node is used to host virtual nodes, u_e^A , $\forall e \in A$, to indicate if a substrate arc is used to implement virtual arcs, a_i^{CPU} , $\forall i \in V$, to indicate the amount of added CPU resources and a_e^{BW} , $\forall e \in A$, to indicate the amount of added bandwidth resources. Like FLOW-A, p^{CPU} denotes the cost of one additional unit of CPU resources, p^{BW} denotes the cost of one additional unit of bandwidth and K a sufficiently large constant. The complete model is defined by inequalities (11.23)–(11.34). The variables in parenthesis denote the dual variables associated with the constraint.

(PATH) min
$$\sum_{i \in V} K p^{\text{CPU}} a_i^{\text{CPU}} + \sum_{e \in A} K p^{\text{BW}} a_e^{\text{BW}} + \sum_{i \in V} p_i^V u_i^V + \sum_{e \in A} p_e^A u_e^A$$
 (11.23)

$$(\mu_f) \qquad \qquad \sum_{p_f \in P_f} w_{p_f} = 1 \qquad \qquad \forall f \in A' \tag{11.24}$$

$$(\pi_{fki}) \qquad \sum_{p_f \in P_f: (k,i) \in p_f \lor (i,\overline{k}) \in p_f} -w_{p_f} + x_{ki} \ge 0 \qquad \qquad \forall (k,i) \in M, \ \forall f \in A'$$

$$(11.25)$$

$$\sum_{(k,i)\in M} x_i^k = 1 \qquad \forall k \in V' \tag{11.26}$$

$$(\lambda_i) \quad -\sum_{f \in A'} \sum_{p_f \in P_f: i \in p_f} b_f w_{p_f} - \sum_{(k,i) \in M} c_k x_i^k \ge -c_i - a_i^{\text{CPU}} \quad \forall i \in V$$

$$(11.27)$$

$$(\epsilon_e) \qquad \qquad \sum_{f \in A'} \sum_{p_f \in P_f: e \in p_f} -b_f w_{p_f} \ge -b_e - a_e^{\mathrm{BW}} \quad \forall e \in A \tag{11.28}$$

$$(\gamma_{ef}) \qquad \sum_{p_f \in P_f: e \in p_f} -w_{p_f} + u_e^A \ge 0 \qquad \forall e \in A, \ \forall f \in A' \quad (11.29)$$

$$-x_i^k + u_i^V \ge 0 \qquad \qquad \forall (k,i) \in M \qquad (11.30)$$

$$a_i^{\text{CPU}} \ge 0 \qquad \qquad \forall i \in V \qquad (11.31)$$

$$a_e^{\text{BW}} \ge 0 \qquad \qquad \forall e \in A \qquad (11.32)$$

$$w_{p_f} \in \{0, 1\} \qquad \qquad \forall f \in A', \ \forall p_f \in P_f \qquad (11.33)$$

$$x_{ki} \in \{0, 1\}$$
 $\forall (k, i) \in M$ (11.34)

The objective function (11.23) states that first the cost of adding additional resources has to be minimized. If those costs cannot be reduced any more, then the cost of using substrate components has to be reduced as much as possible. Equalities (11.24) ensure that for every virtual arc exactly one implementing path is chosen. The linking constraints (11.25) state that if a specific path is used, the corresponding mapping variables have to be set too. Equalities (11.26) state that exactly one mapping target for every virtual node has to be chosen. Inequalities (11.27) implement the CPU constraint, the bandwidth constraint is implemented by inequalities (11.28). The linking constraints (11.29) ensure that variables u_e^A are set if paths are selected that use those arcs. Constraints (11.30) fulfill the same role for variables u_i^V , which are set if a virtual node is mapped to them. Inequalities (11.31) forbid selling CPU resources, inequalities (11.32) forbid it for bandwidth capacities. Note that while the model only includes constraints to restrict w_{p_f} and x_{ki} to binary values, due to the employed constraints and the objective function, u_i^V

and u_e^A are binary as well. Since the CPU and bandwidth capacities are integral, $a_i^{\rm CPU}$ and $a_e^{\rm BW}$ are too. We do not need to add delay constraints since they are covered by the definition of P_f . The restricted master problem is derived from PATH by using only a small, nonempty subset P'_{f} of P_{f} to construct the model. For the pricing subproblem, we need to find a path p_{f} with negative reduced costs. The reduced costs $\overline{c_{w_{p_f}}}$ of a path p_f are calculated as follows:

$$\overline{c_{w_{p_f}}} = 0 - (\mu_f - \sum_{(k,i)\in M|(k,i)\in p_f\vee(i,\overline{k})\in p_f} \pi_{fki} - \sum_{e\in p_f\cap A} b_f\epsilon_e - \sum_{e\in p_f\cap A} \gamma_{ef} - \sum_{i\in p_f\cap V} b_f\lambda_i) =$$
$$= -\mu_f + \sum_{(k,i)\in M|(k,i)\in p_f\vee(i,\overline{k})\in p_f} \pi_{fki} + \sum_{e\in p_f\cap A} (b_f\epsilon_e + \gamma_{ef}) + \sum_{i\in p_f\cap V} b_f\lambda_i$$
(11.35)

Note that μ_f is unrestricted, while variables π_{fki} , ϵ_e , γ_{ef} and λ_i have to be greater or equal to zero. Looking at the structure of equation (11.35), we can see that the costs are determined by two parts: one part (μ_f) is constant for the virtual arc for which we are trying to find an improving path. The second part actually depends on the nodes and arcs used by the path. Since we add the values of the dual variables to the cost and the dual variables are always positive due to the way the corresponding constraints are stated, we need to minimize those costs. In effect, we are searching for a cheapest path within G_P that satisfies the delay restrictions. The costs of the arcs c are set as follows, the costs of nodes are added to all incoming arcs:

$$\mathcal{L}_{e}^{f} = b_{f}\epsilon_{e} + \gamma_{ef} + b_{f}\lambda_{t(e)} \qquad \forall e \in A$$
(11.36)

$$c_{ki}^{f} = \pi_{fki} + b_{f}\lambda_{i} \qquad \forall (k,i) \in M \qquad (11.37)$$
$$c_{i\overline{k}}^{f} = \pi_{fki} \qquad \forall (k,i) \in M \qquad (11.38)$$

$$=\pi_{fki} \qquad \qquad \forall (k,i) \in M \qquad (11.38)$$

Costs c_e^f apply to arcs within A_P that come from the substrate graph, costs c_{ki}^f apply to arcs which connect a virtual node to the substrate and costs $c_{i\bar{k}}^f$ apply to arcs which connect the substrate to the copy of a virtual node. It is important to note that the costs can never be negative. Summing up, to create a path variable for a virtual arc f that can potentially reduce the cost of the LP relaxation of PATH, we need to find a resource constrained shortest path in G_P from s(f)to t(f), with arc costs defined by equations (11.36)–(11.38). Note that in this case, the delay bound is the constrained resource. Since we are dealing with non-negative arc costs, we can use the Dynamic Program for the Resource Constrained Shortest Path Problem from [69] to identify such a path. If the sum of $-\mu_f$ and the cost of the path is negative, we have identified a path that we have to add to the model. If no such path exists for all $f \in A'$, then we have successfully solved the LP relaxation of PATH.

A central problem when applying delayed column generation is determining the initial set of variables, which has to be sufficient to allow a solution to the problem. For the VNMP, this condition is problematic, since just finding a valid solution (which could be used to initialize P'_{f}) is \mathcal{NP} -complete. One possible way around this problem would be the introduction of additional arcs between all pairs of substrate nodes with infinite bandwidth, zero delay, and very high costs. Paths using these arcs could be used to initialize P'_{f} .

By using the extension that we are allowed to purchase additional resources as needed it is possible to avoid that problem. A simple method for constructing initial paths could be to just randomly map virtual nodes to arbitrary substrate nodes (allowed by the mapping constraints) and implement every virtual arc with its delay shortest path. This path, together with the mapping decision, defines a path p_f in G_P , which is the sole member of P'_f . This procedure assumes that for every virtual arc f and all possible mappings of s(f) and t(f) a path within the delay bound d_f exists. This condition holds for the VNMP instance set, otherwise selecting the mapping targets requires more thought or penalties for exceeding the delay. For a better selection of initial paths, the heuristics presented in the previous chapters can be used. These of course can also be used to speed up solving FLOW by supplying good upper bounds on the optimal solution. In Section 11.4.3 we will analyze the influence of an initial solution on the performance of FLOW, in Section 11.4.4 we will utilize CH-O (see Section 6.5.1) to derive an initial set of paths for PATH.

11.4 Results

In this section, we present the evaluation of the different ILP models for the VNMP. We executed the different models with a time-limit of 10000 seconds and a memory limit of 5 GB. The reported times include also the time necessary for preprocessing. We use the complete VNMP instance set with loads of 0.1, 0.5, 0.8 and 1. As constants we use $p^{\text{CPU}} = 1$, $p^{\text{BW}} = 5$, and K = 1000000. The motivation for the difference in cost between additional CPU and bandwidth resources is that it is easier to add another server than to add more bandwidth, which might mean constructing a new physical connection if other options like renting additional bandwidth are not available.

As for the different models, we will now compare the following FLOW configurations. A discussion of the influence of an initial solution on FLOW and the performance of PATH follows later in this section.

FLOW-B The basic FLOW model, without using any preprocessing.

- **FLOW-P1** The FLOW model with activated preprocessing, but using None as pruning method. Preprocessing is executed with the same settings as outlined in Chapter 9. Fixing of nodes or arcs is not performed within FLOW. Nevertheless, the preprocessing algorithm requires a method specification for the fixing method. We use None, unless otherwise specified.
- FLOW-P2 FLOW-P1 with better preprocessing. APSP is used as pruning method.
- FLOW-P3 FLOW-P1 with the best preprocessing. PathEnumeration is used as pruning method.
- **FLOW-F** FLOW-P3, but in addition, nodes and arcs are being fixed. PathEnumeration is used as fixing method.
- FLOW-FE The FLOW-F configuration, but with equalities instead of the inequalities (11.4).

- **FLOW-A** The FLOW-A model. Otherwise, the FLOW-FE configuration is used, i.e., preprocessing with PathEnumeration, nodes and arcs are fixed and node usage is linked to incoming flow by using equalities.
- **FLOW-MA** A variant of FLOW-A which only minimizes the cost of additional resources to buy, solving VNMP-S. After this variant is solved, we either get a valid solution to the original VNMP instance (but not an optimal one) or the cheapest possible solution that requires additional resources if the instance is infeasible.
- **FLOW-S** Another configuration that focuses on satisfiability instead of optimality. It is a variant of the FLOW-FE configuration, but removes the objective function. As with FLOW-MA, we get a valid solution to the original VNMP instance if possible. If the instance is infeasible, we get no hints about recourse actions to make it feasible.

11.4.1 Solving Characteristics of FLOW Configurations

In this section, we will compare the main solving characteristics of the different FLOW configurations. By solving characteristics we mean how far the different configurations get when trying to solve VNMP instances, for instance, do they find optimal solutions or do they run out of time before they are able to solve the LP relaxation in the root node. We also consider the standard characteristics like the required run-time or the gap between the lower bound and the best found integer feasible solution after all of the available run-time has been used. We will now discuss the different characteristics that we are going to report on in more detail.

When trying to solve a VNMP instance with one of the configurations of FLOW, we can either succeed (i.e., find an optimal solution) or we can fail to varying degrees. The most serious failure is exceeding the memory limit. Due to technical limitations, we do not get any information about the solving process in this case, even though useful results might have been created before the memory limit was hit. We will denote this type of failure by Mem. The next type of failure is failing to solve the LP relaxation of the root node in the Branch-and-Bound tree, which means we do not get any lower bounds. We will label this type of failure LP. If we succeed in solving the root node and start branching, we can fail to find an integer feasible solution to the VNMP instance that we are trying to solve (denoted by NS). Depending on the configuration of FLOW, it might be possible to buy additional resources. Even if we found an integer feasible solution, it might not be valid for the original VNMP instance because additional resources need to be bought. This type of failure we will denote with AR. If none of these failure reasons apply, then we succeeded in finding an integer feasible solution to the VNMP instance that we are trying to solve. However, we may fail to prove the optimality of this solution (if it is the optimal solution). We will call this condition Feas. If none of the previous failures apply, then we have found an optimal solution to the VNMP instance we are trying to solve, denoted by Opt. Since FLOW-S does not have an objective to optimize, it counts as Opt if a feasible solution has been found. In addition to the state that the solving process ended in, we will also report on other characteristics, the first of which is the achieved gap between the final lower bound (lb) on the solution

cost and cost of the best found integer feasible solution (ub). The gap (in %) is calculated as $100\frac{ub-lb}{ub}$. A gap of 1% means that the best found solution can be improved by at most 1%. If

we do not have access to lower or upper bounds (i.e., we failed before Feas), we assume a gap of 100%. We also report on the achieved objective values (Obj). However, for the objective we cannot define a useful default value if we fail before Feas, so the reported objective values are only based on instances where at least a feasible solution not requiring additional resources was found. The required run-time t in seconds is not as problematic, if we fail due to Mem we assume a run-time of 10000 seconds. For gap, objective, and run-time, we present average values. Last but not least, we are going to report on the number of variables (Vars) used to model the VNMP instances. This allows us to judge the effectiveness of preprocessing in terms of model size. Note that we report the size of the model after CPLEX has performed its own reductions. That has two consequences. First of all, this allows us to really see the benefit of preprocessing. If we would report on the size of the model before the reductions, any reduction in the number of necessary variables caused by preprocessing could in principle be matched by the reductions performed by CPLEX. Preprocessing only adds real value if it reduces the number of variables in addition to the reductions of CPLEX. The second consequence and downside of using the number of variables after the reductions is that the results are not necessarily consistent. We have seen instances where the final number of variables is smaller if weaker preprocessing is used. Of course, the number of variables in a model is only an indicator for the "hardness" of a model but by no means the sole determining factor. We could observe instances where a smaller model for the same instance took far longer to solve to optimality. The reported numbers of variables are based on instances that did not fail due to Mem.

Based on these definitions, we can present the main characteristics of different configurations of FLOW depending on the instance size in Table 11.2.

For the smallest instance sizes, we can see that every configuration is able to solve all of the 120 instances (30 instances with four load levels each). The average time required to to so however is very different. Unsurprisingly, FLOW-B requires the most run-time with more than 100 seconds on average. By adding preprocessing without any pruning and fixing, the run-time requirements can be reduced to one third of the original run-time.

Interestingly, adding additional pruning capabilities increases the required run-time in some cases. The reason is not the run-time required by the enhanced preprocessing methods, as we know from Chapter 9 that they are negligible compared to the total run-time. The reason is the performance variability of ILP [36, 109]. Simple changes to a model which should be performance neutral (or improve performance) like changing the order of constraints may sometimes cause unexpected degradations. At least the number of variables present in the ILP model is reduced by using enhanced preprocessing methods. Starting to fix nodes gives another boost to the preprocessing performance. FLOW-F only requires less than 17% of the run-time of FLOW-B, but linking flow with node usage by equality does not give an additional advantage. By adding the possibility of buying additional resources, we slow the solving process down significantly. It also causes a noticeable increase in variables necessary for modeling the VNMP instances. The two configurations that focus on finding a valid solution, FLOW-MA and FLOW-S, unsurprisingly are much faster than the optimizing configurations. The solutions that are found by them are about 50% more expensive than the optimal solutions with respect to substrate usage costs.

Starting with instance size 30, the configurations begin to fail. Most noticeable of them is the FLOW-B configuration, which is not able to find an integer feasible solution for one instance.

Table 11.2: Main characteristics of different configurations of FLOW depending on the instance size.

Size	Method	Mem	LP	NS	AR	Feas	Opt	Gap[%]	Obj	t[s]	Vars
20	FLOW-B	0	0	0	0	0	120	0.0	803.2	102.2	6269.7
	FLOW-P1	0	0	0	0	0	120	0.0	803.2	28.6	2437.8
	FLOW-P2	0	0	0	0	0	120	0.0	803.2	34.6	2039.0
	FLOW-P3	0	0	0	0	0	120	0.0	803.2	29.6	1962.2
	FLOW-F	0	0	0	0	0	120	0.0	803.2	16.9	1871.2
	FLOW-FE	0	0	0	0	0	120	0.0	803.2	17.1	1884.4
	FLOW-A	0	0	0	0	0	120	0.0	803.2	44.5	2070.4
	FLOW-MA	0	0	0	0	0	120	0.0	1237.5	0.3	2077.4
	FLOW-S	0	0	0	0	0	120	0.0	1271.0	0.2	2091.3
30	FLOW-B	0	0	1	0	9	110	0.1	1139.9	1155.9	18271.4
	FLOW-P1	0	0	0	0	8	112	0.2	1141.3	905.8	8661.9
	FLOW-P2	0	0	0	0	6	114	0.1	1141.3	690.6	7116.6
	FLOW-P3	0	0	0	0	5	115	0.1	1140.9	690.9	6408.7
	FLOW-F	0	0	0	0	5	115	0.1	1140.8	653.6	6258.7
	FLOW-FE	0	0	0	0	4	116	0.1	1140.6	638.8	6301.2
	FLOW-A	0	0	0	0	6	114	0.1	1140.9	635.2	6473.0
	FLOW-MA	0	0	0	0	0	120	0.0	1901.8	1.0	6670.3
	FLOW-S	0	0	0	0	0	120	0.0	1934.1	0.9	6625.9
50	FLOW-B	1	1	4	0	23	91	0.6	1728.1	2895.2	51474.8
	FLOW-P1	0	0	3	0	16	101	0.4	1742.4	2006.5	21382.1
	FLOW-P2	0	0	1	0	14	105	0.6	1753.8	1718.0	17825.8
	FLOW-P3	0	0	0	0	16	104	0.6	1760.0	1676.9	15193.8
	FLOW-F	0	0	1	0	15	104	0.5	1752.8	1639.7	15033.7
	FLOW-FE	0	0	2	0	12	106	0.3	1746.5	1601.0	15050.5
	FLOW-A	0	0	0	1	17	102	0.5	1751.9	1879.9	15971.0
	FLOW-MA	0	0	0	0	0	120	0.0	3131.5	2.9	16106.5
	FLOW-S	0	0	0	0	0	120	0.0	3179.0	2.7	15288.9
100	FLOW-B	1	45	6	0	27	41	1.4	2939.1	6780.5	212914.1
	FLOW-P1	0	8	14	0	48	50	1.7	3291.9	6054.2	71727.4
	FLOW-P2	0	3	6	0	53	58	1.6	3385.3	5534.4	57764.0
	FLOW-P3	1	3	2	0	56	58	1.6	3415.5	5484.2	49632.6
	FLOW-F	0	3	2	0	59	56	1.5	3420.7	5648.9	49346.6
	FLOW-FE	0	3	3	0	55	59	1.4	3410.1	5432.7	49079.0
	FLOW-A	2	3	3	0	56	56	1.3	3403.4	5601.8	51844.1
	FLOW-MA	0	0	0	0	0	120	0.0	6207.9	10.9	50784.4
	FLOW-S	0	0	0	0	0	120	0.0	6189.7	10.6	48418.3
200	FLOW-B	0	69	4	0	13	34	0.4	3325.1	7410.6	457143.5
	FLOW-P1	0	39	9	0	31	41	1.6	3973.1	6856.6	133156.1
	FLOW-P2	0	24	13	0	37	46	1.7	4218.3	6514.9	109582.3
	FLOW-P3	0	18	17	0	41	44	1.7	4253.3	6455.4	93957.8
	FLOW-F	0	17	18	0	38	47	1.5	4264.8	6343.9	92999.5
	FLOW-FE	0	15	15	0	41	49	1.5	4342.4	6275.2	92228.6
	FLOW-A	0	20	16	1	36	47	1.4	4191.6	6372.2	102392.0
	FLOW-MA	0	0	0	0	0	120	0.0	8849.6	28.5	102075.4
	FLOW-S	0	0	0	0	0	120	0.0	8872.3	27.1	91624.3
500	FLOW-B	73	6	10	0	5	26	0.2	3171.8	8131.2	433322.5
	FLOW-P1	0	84	2	0	7	27	0.2	3448.6	7879.0	316870.9
	FLOW-P2	0	68	3	0	15	34	0.7	4423.6	7384.5	250480.3
	FLOW-P3	0	67	2	0	18	33	0.7	4503.6	7440.8	243814.4
	FLOW-F	0	64	6	0	17	33	0.6	4488.7	7385.6	241183.3
	FLOW-FE	0	63	2	0	22	33	0.7	4706.8	7334.0	240455.4
	FLOW-A	0	67	6	0	14	33	0.4	4293.6	7452.2	279763.4
	FLOW-MA	0	0	0	0	0	120	0.0	13381.6	146.0	278027.4
	FLOW-S	0	0	0	0	0	120	0.0	13774.0	174.6	241346.1
1000	FLOW-B	90	3	4	0	15	8	1.5	3677.5	9707.1	416302.4
	FLOW-P1	19	61	13	0	11	16	0.7	3721.4	9104.5	468323.0
	FLOW-P2	3	66	14	0	13	24	0.6	4657.2	8209.1	420043.5
	FLOW-P3	3	65	15	0	12	25	0.6	4617.8	8146.5	418783.2
	FLOW-F	2	64	14	0	13	27	0.6	4834.0	8058.6	420225.3
	FLOW-FE	2	59	14	0	18	27	0.7	5277.2	7978.8	417890.5
	FLOW-A	3	68	8	0	15	26	0.8	4948.1	8062.7	509628.8
	FLOW-MA	15	0	0	0	0	105	0.0	16698.6	1630.9	463200.4
	FLOW-S	1	0	0	0	0	119	0.0	21265.4	673.8	422138.7

Other than that, the solving performance of all configurations is similar, between 110 and 115 instances can be solved to optimality, for the rest we get feasible solutions. The average gap is also very small. Notice how FLOW-B seems to produce better results based on the reported objective value than the other configurations. This is because the result for the instance that failed with NS is missing from the average objective value. Again, we can see significant runtime improvements by activating preprocessing and this time using improved pruning (FLOW-P2) gives an additional advantage. Notice how preprocessing is able to reduce the number of required variables to a third of the original value. FLOW-FE shows a distinct advantage when compared to FLOW-F in terms of required run-time and is even able to prove optimality for one more instance. For size 30, FLOW-A is the fastest solution method, the ability to add additional resources no longer seems to be a disadvantage. The last two configurations are still very fast and able to find valid solutions to all instances. Note however that the gap to the optimal solution values increases, the found solutions now being 60% more expensive than the optimal solutions. By increasing the substrate size to 50 nodes, we can observe the first failures of FLOW-B due to the memory limit. None of the other methods have problematic memory consumption, but they start failing due to NS. Also, the number of instances where only a valid solution could be found increases. Still, all methods employing preprocessing can solve more than 100 instances to optimality. FLOW-A is again slower than the other methods and also keeps being slower for larger instance sizes. Also notice how the run-time advantage of methods using preprocessing starts to shrink, but the difference in variables is still huge.

At size 100 we have reached the point where only half of the instances can be solved to optimality by the optimization configurations using preprocessing (FLOW-P1 – FLOW-A). For the remaining instances we mostly find feasible solutions, but for some we fail earlier at LP or NS. The most common result for FLOW-B is to fail at LP, which means gaps and objective values cannot be compared meaningfully with the other algorithms.

By doubling the instance size again to 200, we get a significant number of failures for all optimization configurations, but still a surprising number of instances can be solved to optimality by the configurations using preprocessing. This is the last size for which we can compare the number of variables in a meaningful way, as FLOW-B starts failing at Mem for the larger sizes. FLOW-B almost requires five times more variables than FLOW-FE. For this size class we also have the second and last occurrence of a solution found by FLOW-A that requires additional resources.

For size 500, FLOW-B requires too much memory in most cases. All other optimization configurations predominately fail at LP. Observe that if a valid solution can be found, in most cases also its optimality can be proven or the remaining gap is very small.

Applying the different FLOW configurations to the last instance size, we can see that also FLOW-P1 starts to fail a significant number of times due to Mem. The configurations using more preprocessing fail predominantly at LP. Again, if we find a valid solution, we either achieve low gaps or can prove optimality, but this happens less often than for the previous size class. Also the configurations concentrating on just finding valid solutions start to fail, especially FLOW-MA, which needs too much memory. FLOW-S just fails once due to Mem and manages to find valid solutions for all other instances. Here we can clearly see the run-time and memory cost of not only finding valid solutions, but also being able to derive cheap recourse actions if the VNMP

Table 11.3: Main characteristics of different configurations of FLOW depending on the instance load.

Load	Method	Mem	LP	NS	AR	Feas	Opt	Gap[%]	Obj	t[s]	Vars
0.10	FLOW-B	0	3	4	0	23	180	0.2	1866.3	1812.7	97877.5
	FLOW-P1	0	0	3	0	16	191	0.1	1906.7	1237.4	24635.7
	FLOW-P2	0	0	0	0	7	203	0.0	1937.8	428.8	16877.0
	FLOW-P3	0	0	0	0	7	203	0.0	1938.0	440.0	16362.7
	FLOW-F	0	0	0	0	5	205	0.0	1937.8	378.5	16209.0
	FLOW-FE	0	0	0	0	5	205	0.0	1937.8	330.3	15320.2
	FLOW-A	0	0	0	0	6	204	0.0	1938.3	395.0	20833.0
	FLOW-MA	0	0	0	0	0	210	0.0	3504.0	11.2	23754.1
	FLOW-S	0	0	0	0	0	210	0.0	3508.9	10.3	19896.2
0.50	FLOW-B	44	25	13	0	21	107	0.2	1985.6	5213.3	201956.9
	FLOW-P1	0	58	3	0	32	117	0.5	2348.0	4614.2	130523.2
	FLOW-P2	0	31	9	0	39	131	0.5	2891.2	4135.5	104773.1
	FLOW-P3	0	27	11	0	42	130	0.5	2924.8	4061.7	100501.8
	FLOW-F	0	26	11	0	43	130	0.4	2959.5	4065.7	99785.8
	FLOW-FE	0	22	6	0	49	133	0.5	3201.5	3955.1	99474.9
	FLOW-A	0	27	10	0	42	131	0.4	2979.1	4084.8	117538.6
	FLOW-MA	0	0	0	0	0	210	0.0	6697.3	45.6	115672.5
	FLOW-S	0	0	0	0	0	210	0.0	6888.1	44.8	98781.1
0.80	FLOW-B	60	45	3	0	23	79	0.7	1766.8	6442.6	201773.8
	FLOW-P1	0	75	11	0	38	86	1.0	2306.7	6128.9	209788.7
	FLOW-P2	0	66	7	0	44	93	1.1	2586.2	5973.0	168153.4
	FLOW-P3	1	64	6	0	49	90	1.1	2623.4	5923.0	161912.1
	FLOW-F	0	61	8	0	49	92	1.1	2686.2	5927.3	160086.1
	FLOW-FE	0	61	6	0	48	95	0.9	2734.8	5831.7	159641.8
	FLOW-A	2	63	8	0	48	89	1.0	2603.3	6004.9	188727.4
	FLOW-MA	0	0	0	0	0	210	0.0	8959.1	111.9	185858.2
	FLOW-S	0	0	0	0	0	210	0.0	9879.9	113.1	158224.1
1.00	FLOW-B	61	51	9	0	25	64	1.0	1739.9	7207.3	253519.9
	FLOW-P1	19	59	24	0	35	73	1.4	2157.8	6782.6	194607.1
	FLOW-P2	3	64	21	0	48	74	1.8	2493.8	6654.7	201272.1
	FLOW-P3	3	62	19	0	50	76	1.8	2549.5	6674.9	192672.3
	FLOW-F	2	61	22	0	50	75	1.5	2542.8	6626.9	194292.1
	FLOW-FE	2	57	24	0	50	77	1.5	2634.3	6612.9	193650.0
	FLOW-A	3	68	15	2	48	74	1.3	2430.0	6686.0	223339.1
	FLOW-MA	15	0	0	0	0	195	0.0	9717.4	871.7	179565.3
	FLOW-S	1	0	0	0	0	209	0.0	11956.2	340.3	194892.7

instance does not have a valid solution. For the smaller sizes, there was no clear difference between FLOW-MA and FLOW-S.

We have discussed the characteristics of different FLOW configurations based on the instance size. Table 11.3 shows the same characteristics, but now based on the instance load. For the lowest load of 0.1, finding at least feasible solutions is not a problem, just FLOW-B fails at LP or NS for some instances. All other configurations manage to find optimal solutions for most instances. Note that for low loads the run-time requirements are very different for the compared

configurations, higher loads will cause the run-time requirements to be more similar. The same holds true for the number of variables. Also observe that the solutions found by FLOW-F and FLOW-FE are the same, but FLOW-FE is faster. Both configurations are better than FLOW-P3. By increasing the load to 0.5, we see that FLOW-B already starts to fail due to Mem, all other configurations start failing due to LP. Most instances can still be solved to optimality. Going to load 0.8 increases the number of failures due to LP further, we also start seeing failures due to Mem for configurations other than FLOW-B. The average gap reaches 1%. The main difference when using VNMP instances with full load is that for some instances the different configurations fail to find valid solutions more often. It can also be seen that the two instances for which FLOW-A produces a solution which requires additional resources are those with highest load. There is not a lot of observable difference between the characteristics of the configurations FLOW-P2 to FLOW-A, only FLOW-P1 and FLOW-B are worse.

Based on these results, one thing is clear: if we want to tackle instances with more than 50 substrate nodes and with more than 50% load with FLOW, then preprocessing in some form is essential. For the largest instance sizes and highest loads, using more advanced preprocessing techniques becomes important. Valid solutions can be found to nearly all instances in surprisingly short time. Especially for large instances with high loads, solving the LP relaxation in the root node is very time consuming. The differences between the configurations from FLOW-P1 to FLOW-A are not very visible and require further analysis. Preprocessing has a very pronounced effect on the number of variables, which also translates to shorter run-times, albeit to a lesser extent. This is not surprising since in some sense preprocessing just removes unnecessary ballast but does not make the core problem easier to solve.

11.4.2 Comparison of FLOW Configurations

In the previous section, we have analyzed the properties of different configurations of FLOW. It could be seen that adding preprocessing improves the performance, but a more detailed analysis (for instance if fixing nodes reduces run-times in a statistically significant way or if going from FLOW-P2 to FLOW-P3 is beneficial) could not be presented. This is the main goal for this section. We will present a comparison of the configurations FLOW-P1 to FLOW-A and determine which one actually performs best. We excluded FLOW-B, because the results presented in the previous section showed it to be very clearly worse than the alternatives. FLOW-MA and FLOW-S are not considered since they do not perform optimization. To be able to perform a meaningful comparison, we restrict ourselves to instances for which all compared configurations at least managed to find a valid solution that does not require additional resources.

Table 11.4 shows the comparison of the different FLOW configurations based on instance size. The values labeled # Inst. give the count of instances used as basis for the other presented properties. For instance the data for size 20 is based on all 120 instances, but for size 1000 the different configurations produced comparable results only for 27 instances. The other properties have the same meaning as before. The presented values are geometric means, except for the gap values, which are arithmetic means (since values of zero can occur).

Based on the presented results, we can see that FLOW-P1 is outclassed for all but the smallest instance sizes in terms of the achieved solution and always significantly slower than the best configurations. For the largest instance size, FLOW-P1 is nearly ten times slower than the best

	Size	FLOW-P1	FLOW-P2	FLOW-P3	FLOW-F	FLOW-FE	FLOW-A
# Inst.	20	120	120	120	120	120	120
	30	120	120	120	120	120	120
	50	117	117	117	117	117	117
	100	97	97	97	97	97	97
	200	71	71	71	71	71	71
	500	34	34	34	34	34	34
	1000	27	27	27	27	27	27
Gap[%]	20	0.00 =	0.00 =	0.00 =	0.00 =	0.00 =	0.00 =
	30	0.16 >	0.11 =	0.10 =	0.08 =	0.08 =	0.10 =
	50	0.41 >	0.35 >	0.32 =	0.26 =	0.26 =	0.31 =
	100	1.58 >	1.10 >	0.97 >	0.96 =	0.84 =	0.94 >
	200	1.51 >	0.93 >	0.80 >	0.73 =	0.63 =	0.72 =
	500	0.23 >	0.06 =	0.05 =	0.04 =	0.06 =	0.06 =
	1000	0.73 >	0.10 =	0.11 =	0.08 =	0.07 =	0.12 =
Obj	20	758.9 =	758.9 =	758.9 =	758.9 =	758.9 =	758.9 =
	30	1086.2 >	1086.2 =	1085.9 =	1085.8 =	1085.7 =	1085.9 =
	50	1659.2 >	1658.6 >	1658.2 =	1657.7 =	1657.6 =	1657.8 =
	100	3104.7 >	3095.2 >	3092.9 =	3093.5 =	3091.4 =	3092.9 =
	200	3673.6 >	3660.6 >	3656.2 >	3655.7 =	3653.8 =	3654.4 =
	500	3326.3 =	3325.4 =	3325.4 =	3325.1 =	3325.5 =	3325.1 =
	1000	3677.2 >	3665.4 =	3665.7 =	3665.8 =	3665.5 =	3666.2 =
t[s]	20	1.0 >	0.8 >	0.8 =	0.8 =	0.7 =	0.8 >
	30	7.8 >	5.6 >	5.2 >	4.9 =	4.8 =	4.7 =
	50	41.1 >	27.2 >	24.3 =	22.0 =	22.0 =	25.0 =
	100	437.5 >	283.4 >	275.2 >	284.4 >	250.7 >	237.3 =
	200	511.2 >	270.4 >	247.3 >	225.5 =	215.9 =	225.3 =
	500	417.0 >	91.8 >	105.9 >	90.3 >	81.9 =	84.2 =
	1000	3373.9 >	507.1 >	469.5 >	404.9 =	352.9 =	399.8 >
Vars	20	1350.5 >	1240.7 >	1213.0 >	1152.6 =	1160.2 =	1230.4 >
	30	4478.5 >	3669.4 >	3455.1 >	3464.5 >	3366.5 =	3538.5 >
	50	11442.5 >	9838.5 >	8480.6 >	8275.8 >	8432.2 =	9446.0 >
	100	32832.4 >	27831.2 >	24132.6 >	23648.2 >	23223.7 =	25757.1 >
	200	46666.0 >	36512.6 >	30493.7 >	30487.8 >	28645.3 =	36273.1 >
	500	44079.5 >	28478.0 >	27718.1 >	22596.9 =	25789.7 =	32490.7 >
	1000	87962.1 >	49807.1 >	50308.2 >	50537.9 >	44325.1 =	69993.1 >

Table 11.4: Comparison of FLOW configurations utilizing preprocessing, depending on the instance size.

configuration. Adding additional preprocessing capabilities by using pruning by APSP improves the situation a bit. For instance sizes 20, 30, 500 and 1000 FLOW-P2 achieves solutions not significantly different from the best, both in terms of gap and objective. For the instances of medium size, PathEnumeration manages better pruning and FLOW-P2 cannot keep up. For the largest instances, the distance between APSP and PathEnumeration shrinks, due to the fallback behaviour of PathEnumeration, and FLOW-P2 manages to create comparable results. It also speeds up the solving process itself, giving a factor of 4 to 6 improvement over FLOW-P1. By using PathEnumeration as pruning method (FLOW-P3), it is possible to achieve the best known objectives for all but one size class and the best gaps except for medium sized instances. FLOW-P3 also starts getting competitive in terms of run-time, but is still more than 30% slower than the best for the largest instances.

By switching on fixing (FLOW-F), we are now able to achieve the best possible results in terms of gap and objective function. The difference to FLOW-P3 shows that utilizing the information about nodes that have to be used results in a meaningful improvement and is not only a theoretical advantage. It is moderate in terms of gap and objective value, but very noticeable in terms of required run-time. To achieve the very best performance, we need to link the use of substrate nodes to the incoming flow via equalities (FLOW-FE). This configuration achieves the best values for gap, objective, required run-time and variables in nearly all cases.

The case of FLOW-A is interesting to analyze. On one hand, the model is larger in principle, because we have to add variables that allow buying additional resources. These additional variables even increase the number of variables in the simplified model in a disproportionate way, i.e., the difference in variables between FLOW-FE and FLOW-A is larger than the number of added variables. Therefore, we could expect worse performance. On the other hand, allowing to buy additional resources makes finding an integer feasible solution easier, which potentially speeds up the solving process. Based on the results shown in Table 11.4, it seems that these influences balance each other. FLOW-A requires far more variables, but is still competitive with respect to solving time, especially for instances of medium size.

Table 11.5 shows the same data, but this time depending on instance load. Regarding the quality of the solutions, we get basically the same results as previously. One difference to note is that FLOW-P1 is not even competitive for the lowest loads, both in terms of average gap and objective. By increasing the level of preprocessing and utilizing fixing information, we get the best solutions. With respect to the time required to achieve a solution, we can see that for loads 0.1 and 0.5, FLOW-A is the fastest configuration. For higher loads, FLOW-FE is fastest.

11.4.3 Starting with a Valid Solution

The aim of this section is to analyze the influence of using an initial solution created by one of the heuristics presented in the previous chapters on FLOW. As base configurations we use FLOW-A and FLOW-MA, to solve VNMP-O and VNMP-S respectively. Note that we chose FLOW-A instead of FLOW-FE and FLOW-MA instead of FLOW-S because they can use an initial solution which requires additional resources. Such solutions are likely to be found by heuristics, especially for the larger instance sizes and could not be used otherwise (e.g., when using FLOW-FE), so all the effort expended on creating them would have been wasted.

	Size	FLOW-P1	FLOW-P2	FLOW-P3	FLOW-F	FLOW-FE	FLOW-A
# Inst.	0.10	207	207	207	207	207	207
	0.50	149	149	149	149	149	149
	0.80	123	123	123	123	123	123
	1.00	107	107	107	107	107	107
Gap[%]	0.10	0.12 >	0.01 =	0.02 =	0.01 =	0.01 =	0.02 >
	0.50	0.50 >	0.28 >	0.26 >	0.23 =	0.22 =	0.21 =
	0.80	0.98 >	0.73 >	0.58 >	0.54 =	0.47 =	0.55 =
	1.00	1.26 >	0.91 >	0.86 =	0.79 =	0.73 =	0.86 >
Obj	0.10	1476.4 >	1475.7 =	1475.7 =	1475.7 =	1475.7 =	1475.7 =
	0.50	1868.9 >	1866.6 >	1866.5 >	1866.1 =	1866.0 =	1866.0 =
	0.80	1863.5 >	1860.7 >	1858.3 =	1858.2 =	1857.5 =	1858.0 =
	1.00	1818.2 >	1814.3 >	1813.5 =	1813.6 =	1812.4 =	1813.6 >
t[s]	0.10	6.6 >	2.8 >	2.7 >	2.5 >	2.5 >	2.2 =
	0.50	43.3 >	26.6 >	24.4 >	23.6 >	21.8 >	21.5 =
	0.80	127.4 >	103.1 >	94.4 >	91.2 >	84.1 =	96.3 =
	1.00	244.5 >	189.6 >	186.6 =	174.9 =	168.4 =	228.6 >
Vars	0.10	6039.1 >	4596.8 >	4218.4 >	4044.8 >	3965.8 =	4891.2 >
	0.50	12578.3 >	10699.3 >	9623.5 >	9380.9 >	9410.1 =	10205.0 >
	0.80	14696.6 >	12567.9 >	11381.9 >	11101.7 >	10958.3 =	11741.8 >
	1.00	13974.3 >	12081.4 >	11126.3 >	10930.4 >	10881.9 =	11872.2 >

Table 11.5: Comparison of FLOW configurations utilizing preprocessing, depending on the instance load.

Table 11.6: Main solving characteristics of FLOW-A (FA) and FLOW-MA (FMA) when an initial solution is created by CH-O (CH), LS-O (LS), or VNS-O (VNS).

Method	Mem	LP	NS	AR	Feas	Opt
FA	3	165	36	3	138	495
FA-CH	29	0	0	116	198	497
FA-LS	28	0	0	74	239	499
FA-VNS	28	0	0	58	255	499
FMA	15	0	0	0	0	825
FMA-CH	28	0	0	0	0	812
FMA-LS	27	0	0	0	0	813
FMA-VNS	27	0	0	0	0	813

As heuristics to create initial solutions we use CH-O, LS-O, and VNS-O with a time-limit of 500 seconds. The time required to create the initial solution of course counts towards the total time-limit of 10000 seconds.

The solving characteristics of the different configurations of FLOW-A and FLOW-MA are shown in Table 11.6. We use the same definitions as in Section 11.4.1. When solving VNMP-O with FLOW-A, using a heuristic to create an initial solution has a very pronounced influence on the solving characteristics. While the basic FLOW-A configuration (FA) fails to solve the LP relaxation in the root node 165 times or fails to find an integer feasible solution 36 times (out of 840 instances), it never fails due to those reasons when an initial solution is supplied. Interestingly, an initial solution increases the number of instances that fail due to Mem. This is most likely caused by FLOW-A being able to solve the LP relaxation when an initial solution is supplied, but later running out of memory during Branch & Bound. For most instances that failed due to LP, a valid solution, or at least a solution requiring additional resources can be found. Supplying an initial solution has barely any influence on the number of instances that can be solved to optimality. Based on the number of instances for which a valid solution could be found, VNS-O is the best heuristic to use to create the initial solution. When solving VNMP-S with FLOW-MA (FMA), the only difference caused by supplying an initial solution is that more instances fail due to Mem.

Table 11.7 shows a comparison of the different FLOW-A configurations. As in the previous section, we restrict the comparison to those instances for which all configurations where able to find at least a valid solution. The number of instances used for comparison is labeled by # Inst. It can be seen that beginning with size 200 a significant fraction of instances is rejected. The influence of an initial solution on the achieved gap and objective values is negligible. Only with respect to the required run-time is it advantageous to supply an initial solution, which has to be created by CH-O. Using CH-O has the most benefit for the largest instance sizes. The run-time overhead of LS-O is too high to make it worthwhile. Note that we use VNS-O exactly in the configuration as introduced in Chapter 8. That means that the only termination criterion is the elapsed time and as a consequence FA-VNS requires at least 500 seconds to solve an instance. However, even if we subtract 500 seconds from the reported run-times, it still is far from competitive. Using VNS-O does not show any advantage with respect to gap or objective value, so an evaluation of more practical termination criteria does not seem warranted. Also with respect to the required Branch & Bound nodes (# Nodes) VNS-O is only as good as the other heuristics are.

The same data, but now according to instance load is presented in Table 11.8. Again we can observe that the presence of an initial solution only has a significant influence on the required run-time and CH-O seems to be the right choice. A solution created by LS-O is only beneficial for instances of the highest load.

The performance of the different FLOW-MA configurations when solving VNMP-S is shown in Table 11.9. Note that we do not present the average objective and gap values in this case, because we have already shown that the different configurations of FLOW-MA either find a valid solution or fail due to the memory limit. For solving VNMP-S it is very clear that using an initial solution created by CH is an advantage in terms of required run-time. In contrast to VNMP-O, better initial solutions consistently lead to fewer required nodes in the Branch & Bound tree, with the

# Inst. 20 120 30 120 50 120 100 113 200 79 500 46 1000 35 Gap[%] 20 0.0 = 30 0.1 = 50 50 0.5 = 50	$\begin{array}{c} 120 \\ 120 \\ 113 \\ 79 \\ 46 \\ 35 \\ 0.0 = \\ 0.1 = \\ 0.5 = \\ 1.3 = \end{array}$	$ \begin{array}{r} 120\\ 120\\ 120\\ 113\\ 79\\ 46\\ 35\\ 0.0 = \\ 0.1 = \\ 0.5 = \\ 1.4\\ \end{array} $	$ \begin{array}{r} 120\\ 120\\ 120\\ 113\\ 79\\ 46\\ 35\\ 0.0 = \\ 0.1 = \\ 0.5 = \\ \end{array} $
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{r} 120\\ 113\\ 79\\ 46\\ 35\\ 0.0 = \\ 0.1 = \\ 0.5 = \\ \end{array} $	$ \begin{array}{r} 120\\ 113\\ 79\\ 46\\ 35\\ 0.0 =\\ 0.1 = \end{array} $
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c} 113 \\ 79 \\ 46 \\ 35 \\ 0.0 = \\ 0.1 = \\ 0.5 = \\ 1.3 = \\ \end{array} $	$ \begin{array}{r} 113 \\ 79 \\ 46 \\ 35 \\ 0.0 = \\ 0.1 = \\ 0.5 = \\ \end{array} $	$ \begin{array}{r} 113 \\ 79 \\ 46 \\ 35 \\ 0.0 = \\ 0.1 = \end{array} $
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 79 \\ 46 \\ 35 \\ 0.0 = \\ 0.1 = \\ 0.5 = \\ 1.3 = \end{array}$	$79 \\ 46 \\ 35 \\ 0.0 = \\ 0.1 = \\ 0.5 =$	79 46 35 0.0 = 0.1 =
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c} 46 \\ 35 \\ 0.0 = \\ 0.1 = \\ 0.5 = \\ 1.3 = \\ \end{array} $	46 35 0.0 = 0.1 = 0.5 =	$46 \\ 35 \\ 0.0 = \\ 0.1 =$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c c} 35 \\ 0.0 = \\ 0.1 = \\ 0.5 = \\ 1.3 = \end{array}$	35 0.0 = 0.1 = 0.5 =	35 0.0 = 0.1 =
Gap[%] 20 0.0 = 30 0.1 = 50 0.5 =	0.0 = 0.1 = 0.5 = 1.3 =	0.0 = 0.1 = 0.5 =	0.0 = 0.1 =
$\begin{array}{c} 30\\50 \end{array} \qquad \begin{array}{c} 0.1 = \\ 0.5 = \end{array}$	0.1 = 0.5 = 1.3 =	0.1 = 0.5 =	0.1 =
50 0.5 =	0.5 = 1.3 =	0.5 =	
	1.3 =		0.5 =
		1.4	
100 1.4 =		1.4 =	1.4 =
200 1.4 >	1.2 =	1.2 =	1.2 =
500 0.4 =	0.4 =	0.4 =	0.3 =
1000 0.6 >	0.4 =	0.4 =	0.4 =
Obj 20 803.2 =	803.2 =	803.2 =	803.2 =
30 1140.8 =	1141.3 =	1140.7 =	1140.9 =
50 1758.0 =	1758.1 =	1757.9 =	1758.2 =
100 3398.2 =	3397.1 =	3398.2 =	3395.6 =
200 4140.7 >	4134.4 =	4134.6 =	4134.6 =
500 4248.8 =	4249.6 =	4250.2 =	4248.5 =
1000 4353.7 =	4346.1 =	4347.4 =	4346.4 =
t[s] 20 0.9 =	0.8 =	1.0 >	516.5 >
30 4.9 =	4.9 =	5.7 >	650.2 >
50 30.3 >	28.7 =	32.4 >	963.9 >
100 421.7 >	410.5 =	488.3 >	2742.7 >
200 341.6 >	341.3 =	400.8 >	2336.1 >
500 306.6 >	277.2 =	330.4 >	1499.3 >
1000 730.1 >	645.5 =	674.4 =	1545.7 >
# Nodes 20 921.7 =	583.2 =	500.4 =	698.5 =
30 1452.4 =	1532.6 =	1993.7 =	2047.2 =
50 2756.6 >	2873.8 >	2219.0 =	2245.4 =
100 5657.1 =	4409.3 =	4840.8 =	5728.0 =
200 1528.4 =	1513.4 =	1599.3 =	1574.5 =
500 558.0 =	503.4 =	539.8 =	625.7 =
1000 715.1 =	656.3 =	732.7 =	636.7 =

Table 11.7: Comparison of FLOW-A (FA) configurations depending on the instance size.

	Load	FA	FA-CH	FA-LS	FA-VNS
# Inst.	0.10	210	210	210	210
	0.50	165	165	165	165
	0.80	135	135	135	135
	1.00	123	123	123	123
Gap[%]	0.10	0.0 =	0.0 =	0.0 =	0.0 =
	0.50	0.4 =	0.3 =	0.3 >	0.3 =
	0.80	1.0 >	0.9 =	0.8 =	0.9 =
	1.00	1.4 =	1.4 =	1.4 =	1.4 =
Obj	0.10	1938.3 =	1937.9 =	1937.9 =	1937.9 =
	0.50	2749.6 >	2748.4 =	2749.4 =	2748.2 =
	0.80	2528.9 >	2525.5 =	2524.9 =	2525.0 =
	1.00	2447.6 =	2447.2 =	2447.6 =	2446.3 =
t[s]	0.10	2.5 >	2.3 =	3.1 >	597.0 >
	0.50	43.5 =	43.2 =	50.3 >	1206.5 >
	0.80	155.6 =	147.9 =	154.2 =	1669.0 >
	1.00	357.0 >	360.1 =	352.2 =	2041.1 >
# Nodes	0.10	170.9 >	158.4 =	177.4 =	162.9 =
	0.50	1280.1 =	937.4 =	1365.4 =	1632.8 =
	0.80	4501.3 =	3847.3 =	3597.9 =	3937.8 =
	1.00	4647.0 >	4515.3 =	4399.6 =	4767.5 =

Table 11.8: Comparison of FLOW-A (FA) configurations depending on the instance load.

fewest nodes required by FMA-VNS. This method might even be competitive with respect to run-time with a better termination criterion. The data presented in Table 11.10 shows the same tendencies. When considering run-time, FMA-CH performs best. The higher the instance load, the more important good initial solutions are, and again FMA-VNS performs best. It is notable that solving VNMP-S requires barely any branching.

In conclusion, we have shown that using an initial solution (preferably one created by CH-O) can reduce the time required to solve an instance. However, it does not increase the number of instances that can be solved to optimality or decrease the achieved gap. The influence of the employed heuristic is much more pronounced when solving VNMP-S. We will denote FA-CH by ILP-O and FLOW-S by ILP-S.

11.4.4 Feasibility of PATH

In this section, we are mainly concerned with PATH and whether this formulation is substantially better than FLOW. Using PATH, we only implemented a Column Generation approach, instead of full Branch & Price. Therefore, we will compare lower bounds on the objective instead of found feasible solutions. PATH should be able to achieve significantly better (i.e., higher) lower bounds than FLOW. For FLOW, we only showed the influence of utilizing knowledge about

	Size	FMA	FMA-CH	FMA-LS	FMA-VNS
# Inst.	20	120	120	120	120
	30	120	120	120	120
	50	120	120	120	120
	100	120	120	120	120
	200	120	120	120	120
	500	120	120	120	120
	1000	92	92	92	92
t[s]	20	0.2 >	0.2 =	0.3 >	500.2 >
	30	0.8 >	0.6 =	0.9 >	500.4 >
	50	2.5 >	1.8 =	2.4 >	501.1 >
	100	9.1 >	7.3 =	15.1 >	504.6 >
	200	23.9 >	19.5 =	52.1 >	510.3 >
	500	119.0 >	93.9 =	310.8 >	572.4 >
	1000	244.0 >	189.4 =	502.5 >	652.9 >
# Nodes	20	0.0 =	0.0 =	0.0 =	0.0 =
	30	0.1 =	0.0 =	0.0 =	0.0 =
	50	0.2 =	0.0 =	0.0 =	0.0 =
	100	0.1 =	0.0 =	0.0 =	0.0 =
	200	4.2 >	0.6 =	0.0 =	0.0 =
	500	46.4 >	1.7 =	2.4 =	0.2 =
	1000	98.5 >	14.4 >	12.8 =	3.7 =

Table 11.9: Comparison of FLOW-MA (FMA) configurations depending on the instance size.

Table 11.10: Comparison of FLOW-MA (FMA) configurations depending on the instance load.

	Load	FMA	FMA-CH	FMA-LS	FMA-VNS
# Inst.	0.10	210	210	210	210
	0.50	210	210	210	210
	0.80	209	209	209	209
	1.00	183	183	183	183
t[s]	0.10	9.7 >	8.9 =	16.2 >	508.5 >
	0.50	37.8 >	32.3 =	116.4 >	528.9 >
	0.80	86.1 >	67.9 =	177.7 >	549.1 >
	1.00	71.8 >	51.3 =	147.7 >	536.3 >
# Nodes	0.10	0.0 =	0.0 =	0.0 =	0.0 =
	0.50	1.6 >	0.2 =	0.0 =	0.0 =
	0.80	34.3 >	5.7 >	0.0 =	0.7 =
	1.00	41.9 >	2.1 =	8.0 >	1.3 =

Table 11.11: Number of instances for which all configurations created a result (# Inst.), average lower bounds (LB.) and average required run-time (t[s]) for FLOW and PATH configurations.

	Size	PATH	F-P3N	F-FEN	F-P3	F-FE
# Inst.	20	120	120	120	120	120
	30	120	120	120	120	120
	50	116	116	116	116	116
	100	45	45	45	45	45
	200	31	31	31	31	31
	500	28	28	28	28	28
	1000	13	13	13	13	13
LB.	20	737.6 <	737.0 <	737.1 <	785.5 =	785.0 =
	30	1073.8 <	1072.4 <	1072.4 <	1117.8 =	1118.5 =
	50	1653.1 <	1650.8 <	1650.8 <	1711.2 =	1712.3 =
	100	2391.8 <	2386.3 <	2386.4 <	2440.4 =	2440.2 =
	200	2534.3 <	2523.4 <	2523.5 <	2566.0 =	2566.2 =
	500	3082.9 <	3071.4 <	3072.2 <	3110.6 =	3111.6 =
	1000	3996.8 <	3990.8 <	3992.8 <	4052.8 =	4053.2 =
t[s]	20	22.7 >	1.6 >	0.9 =	1.3 >	1.2 >
	30	243.5 >	12.3 >	7.5 =	15.8 >	12.9 >
	50	1476.0 >	34.5 >	20.8 =	47.0 >	45.2 >
	100	1648.2 >	24.2 >	6.6 =	15.6 >	13.3 >
	200	884.3 >	24.8 >	5.4 =	8.6 >	6.6 >
	500	1557.5 >	79.5 >	31.7 =	46.1 >	38.7 >
	1000	4321.2 >	324.2 >	62.9 =	96.6 >	64.5 =

fixed nodes on the final result and not on the achieved lower bound in the root node. We will also consider this aspect in this section.

The following PATH and FLOW configurations are going to be compared:

PATH The PATH model. The initial set of paths is derived from the solution found by CH-O.

F-P3 The FLOW-P3 configuration, all cuts CPLEX uses in its default setting are used.

F-P3N The same as F-P3, but all automatic cuts of CPLEX are deactivated.

F-FE The FLOW-FE configuration making use of all cuts. This configuration utilizes information about fixed nodes or arcs, which F-P3 does not.

F-FEN The same as F-FE, but all cuts are deactivated.

Note that in addition to the influence of fixing nodes and arcs, we also compare the performance of the FLOW models with and without automatic cuts. For a fair comparison of model strength, PATH has to be compared to FLOW without cuts, and is expected to achieve better bounds. For practical purposes, it should also create better bounds than FLOW with cuts to be considered useful. Table 11.11 shows a comparison of the different configurations. We only consider instances for which all methods were able to derive a valid lower bound. It can be seen that beginning with size 100, only for a small fraction of instances a bound could be obtained. This low number is due to PATH, the FLOW based configurations were far more successful. For size 1000, PATH fails mostly because of its memory requirements, for smaller sizes too many pricing iterations are required.

When comparing the achieved lower bounds, it can be observed that PATH indeed creates better bounds than both FLOW configurations without automatic cuts, and the difference is statistically significant (not shown in the table). However, the difference is small, especially compared to the bound increase achieved by using the automatic cuts. As for the difference in bounds when fixing nodes and arcs, we see that fixing results in slightly higher bounds, but the difference is not significant. This is surprising since we have already shown in Section 11.4.2 that using information about fixed nodes or arcs has a significant positive influence on the final result.

When considering the required run-time, we see that PATH is not competitive at all. This alone however should not be discouraging, since it is well known [174] that Column Generation requires careful tuning to achieve lower run-times, which we did not perform in this case. Automatic cuts and fixing nodes have an interesting influence on the required run-time. Without the overhead of the automatic cuts, fixing nodes allows solving the LP much faster than without, especially for larger instances. With the automatic cuts, fixing is still faster, but only barely. In addition, activating the cuts decreases the required run-time when not fixing nodes, but increases it when fixing. We do not show the results depending on instance load since they show essentially the same.

11.5 Conclusion

In this chapter, we could show that the preprocessing techniques as presented in Chapter 9 are essential for solving the VNMP with FLOW. The largest improvement is achieved by activating preprocessing. Using more powerful preprocessing techniques gives another boost to performance. The achieved objective values are usually very similar, the influence of preprocessing mostly concerns the required run-time. In total, FLOW-FE performed best. This configuration uses FLOW with the highest levels of preprocessing, utilizing also information about fixed nodes and links incoming flow and node usage with equality. FLOW-A, which allows buying additional resources in the substrate, could have been better than FLOW-FE since integer feasible solutions can be found more easily, but this advantage did not materialize. Nevertheless, FLOW-A has a performance very similar to FLOW-FE, and for instances of medium size or low loads offers a small advantage with respect to run-time. The run-time can be further improved by starting from a solution created by CH-O.

The difference between methods for solving VNMP-S and VNMP-O is huge, both in number of solved instances and required run-time. FLOW-S fails only to produce a valid solution for a single instance of the largest size due to memory reasons, which also exceeds the capabilities of the best heuristics presented in the previous chapters. The FLOW variants solving VNMP-O fail earlier and more often. Additionally, there is a run-time difference of a factor between 10 and 500.

Concerning PATH, we have shown that it is not a promising approach for solving the VNMP. While it is able to derive better bounds than FLOW without automatic cuts used by CPLEX, it does not even come close to the bounds achieved by using automatic cuts. In addition, implementing a full Branch & Price scheme based on PATH would require significant effort, since this is not supported by CPLEX and additional work would be required to make it competitive with respect to run-time. The performance of PATH depends critically on the delays, since this constraint is hidden within the pricing problem. In preliminary work with another instance set, where most virtual arcs just used a large value for the allowed delay if it was not meant to be constraining, PATH achieved even worse results. It stands to reason that for instances where delays are more critical, PATH might offer a useful advantage over FLOW.

11.6 Future Work

The incorporation of knowledge about fixed nodes or arcs could be refined. We have shown that the information about fixed nodes can strengthen the model, because it helps forbidding fractional flows that would exceed the delay limit if they were integer. However, this is only true if the fixed node is not an articulation point in the shadow of the substrate. If it is an articulation point, then there is by definition no other way than to use this node, regardless of delay limits. The same holds true for fixed arcs, if they are bridges in the shadow of the substrate. Therefore, the corresponding constraints could be omitted in those cases.

During the definition of FLOW-A, we have forbidden the possibility of selling resources that we do not use. However, there are some applications where this would be an interesting possibility. For instance, if we own contended resources that we do not need to use ourselves, it might be profitable to rent them out. This of course raises the question of how to determine which resources to keep so that future virtual networks can still be accommodated.

If a VNMP instance has no valid solution, we only considered changes to the substrate network. It might beneficial to study the possibility of changing the virtual network requests. For instance, we could allocate less bandwidth to a virtual arc than it requires, which would incur a penalty. Depending on the costs for adding additional resources, this approach might make more sense. For full flexibility, one could consider both options at the same time. Some care would be required to keep the model linear.

The presented ILP models (and the VNMP in general) use a very simple delay model. One possible enhancement would be to assume the delay on a substrate connection to be normally distributed instead of constant. In addition to the maximum allowed delay, a virtual arc also specifies the probability of exceeding this delay. Since the means and variances of normal distributions are additive (assuming independent distributions), it is possible to modify FLOW by adding another variable type that tracks the sum of the variances for each virtual arc. The only challenge is that to check whether we exceed the delay bounds, we need the standard deviation, which is the square root of the variance. To calculate this, linear approximations of the square root function have to be employed, as presented for instance in [136].

Based on the presented results of FLOW-S and previous heuristics, it might make sense to combine both, since FLOW-S is rather fast when trying to find valid solutions for an VNMP instance. These valid solutions could be further optimized by one of the heuristics. PATH does not work for the VNMP, but that does not mean that the Column Generation idea itself is without merit for the VNMP. It may be possible to devise other formulations, for instance focusing on finding good groups of virtual nodes for a substrate node instead of finding good paths for virtual arcs.

CHAPTER 12

Application Study

12.1 Introduction

In the previous chapters, we have presented different methods for solving the VNMP. In this chapter, we will give an example of how these methods can be used to solve problems occurring in practice. In particular, we use one of the presented methods to evaluate at which locations in the substrate we have to invest (i.e., increase the amount of available resources) to be able to host more virtual networks.

Note the difference between this scenario and what we did previously for solving the VNMP where we allow to buy additional resources. In the VNMP scenario, we know which virtual networks we want to embed into the substrate and are able to determine, if the substrate does not have enough resources, the cheapest way of buying additional resources to fit all virtual networks into the substrate. Now we try to determine beforehand where we need to buy more resources, so that future virtual networks can be successfully embedded and do not need to be rejected. Of course, we do not know how future virtual networks will look like, so our objective is to increase the probability that a new virtual network can be embedded into the substrate, the embedding probability $p_{\rm em}$.

The remainder of this chapter is structured as follows. We give an overview on related work in Section 12.2. In Section 12.3, we present the Virtual Network Mapping model used in this chapter. It is slightly different from the VNMP, since the work presented here actually partly preceded the results from previous chapters. This section also outlines the differences in the used VNMP instances, as precursors of the VNMP instances as presented in Chapter 5 have been used. The method of determining the embedding probabilities is outlined in Section 12.4. The results of the evaluation can be found in Section 12.5. We conclude in Section 12.6. The work presented in this chapter has been published in [92].

12.2 Related Work

A physical network has to be able to transfer the occurring traffic and avoid packet loss and delay. The amount of data to be transferred during normal operation is not known beforehand and has to be estimated. There are different mechanisms available to ensure adequate quality of service by avoiding or minimizing link overload in the network.

To a certain extent, Traffic Engineering (TE) can be used to optimize the resource-usage of flows in the network [58]. By using TE, the physical network can carry more traffic, but the optimization is difficult and requires a careful selection of objective functions [78]. In addition to TE, the mechanism of Admission Control (AC) can be used. With AC, the load in the network is analyzed before new traffic is allowed to enter the network to avoid overload situations. This mechanism was proposed for the Internet in [157].

Both mechanisms are rather complex and require constant monitoring of the network. A much simpler approach is to use Capacity Overprovisioning (CO). With CO, the network is overdimensioned, which makes overload in unexpected scenarios very unlikely. It is often stated that CO has high initial costs in comparison to other methods, but as the network usually must also provide backup capacity for unexpected failure scenarios, the bandwidth requirements for CO are similar to those of AC [128]. To determine the best capacities for links and routers is not easy, even when accurate information about traffic patterns in a network are provided. A method to find network elements which might have insufficient capacity is implemented in the Resilyzer framework [79, 127]. It can analyze all network failure scenarios with given probability and determine for example the link overload probability, which can be used for CO.

In this chapter we present a new method to determine such bottlenecks that can be alleviated by CO, which is not based on failure probabilities but on the probability that a new virtual network can be embedded. To the best of our knowledge, no other previous work examined the use of a virtual network embedding algorithm for finding bottlenecks in the substrate.

12.3 Network Traffic Model

The network traffic model employed in this chapter is the same model as for the VNMP, with one exception. For the VNMP, we have only one type of resource available on the substrate nodes, the CPU capacity. It is used for hosting virtual nodes and to transfer data across the node. Here, the CPU capacity does not have this dual role and is only required for hosting virtual networks. For routing, there is a second type of resource at the substrate nodes, the routing capacity. For one unit of transferred data, one unit of routing capacity is used at the substrate node. Within this chapter, we will denote this problem by VNMP.

The second major difference in the employed methodology in contrast to the previous chapters concerns the used instance set [87]. Here we use substrate networks derived from the rock-etfuel [163], scan [70] and lucient [25] projects. The instances were created similarly to the method presented in Chapter 5. They still contain virtual networks designed to represent Stream, Web, P2P and VoIP applications, albeit not the same number of each of those networks. Further differences will be mentioned as relevant, for much more detail on the design of the instances and this variant of the VNMP, see [88].

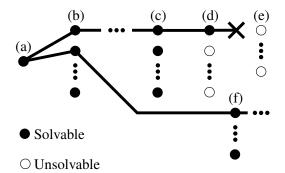


Figure 12.1: Example of an extension procedure execution.

12.4 Methodology

The basic idea of the conducted experiments is to show that by solving VNMP instances that do not have a valid solution we can extract valuable information in order to improve the substrate network. By improvement we mean that the probability that an additional virtual network (VN) can be embedded into the substrate (the embedding probability $p_{\rm em}$) is increased, and that more virtual networks can be embedded into the substrate before $p_{\rm em}$ reaches zero.

To show this, we perform two experiments. We start with the VNMP instances available at [87]. For those instances, we determine $p_{\rm em}$ separately for each VN type because we want to show the influence of different VN types on $p_{\rm em}$. The embedding probability of an instance is determined by adding a VN of a specific type and checking whether this new problem is solvable or not. This is repeated $n_{\rm S}$ times and $p_{\rm em}$ is calculated as the fraction of solvable cases in relation to the number of tested ones. In this work, we use $n_{\rm S} = 50$ as tradeoff between required run-time and confidence in the calculated $p_{\rm em}$. We continue this process with the first found solvable problem (i.e., the added VN could be successfully embedded in the substrate) in a depth-first fashion until $p_{\rm em}$ reaches zero or we have added ten additional VNs. Then the procedure tracks back to an instance with the least amount of added VNs that has not had its $p_{\rm em}$ determined. The motivation for this traversal order is that we want to see how $p_{\rm em}$ develops while adding additional VNs but also cover a reasonable fraction of the search space of instances reachable from the initial problem. We chose the upper limit of ten additional VNs because as more VNs are added, the resource bottlenecks in the substrate depend more and more on the exact configuration of the added VNs and not on the initial situation in the substrate and therefore become less relevant when we want to determine where we want to add additional resources to the substrate. Additionally, we used a limit of 101 $p_{\rm em}$ evaluations, which allows the cutoff point of ten added VNs to be reached ten times. For the whole procedure (which we will call extension procedure from now on) we set a time limit of one day and we executed it for each of the four VN types used for the VNMP.

Figure 12.1 shows an example execution of the extension procedure. It starts with problem instance (a), an instance from the instance set. It then determines p_{em} for this instance by repeatedly extending (a) with one random VN (generated as in [88]) and checking whether the

new instances (b) are solvable or not. In the illustrated case, every newly generated instance was solvable. We continue with the first found instance and repeat this process until we reach c. For c, we determine a $p_{\rm em}$ of 2%, because only one of 50 instances is solvable. We continue with the only found solvable instance (d). Node (d) has a $p_{\rm em}$ of 0%, so there are no instances with which to continue; we track back to an instance with the least amount of added VNs that has not had its $p_{\rm em}$ determined, in this case the second solvable instance created from (a). The extension procedure then continues with (f) and so on until one of the mentioned limits is reached or no more instances remain.

During the extension procedure, a lot of unsolvable instances (i.e., VN configurations that cannot be embedded into the substrate) are created. For the second experiment, we add resources to the substrates of the starting instances based on the reasons why those instances from the first run of the extension procedure were unsolvable and then execute the extension procedure again.

The following sections describe in detail how we determined the solvability or unsolvability of a problem instance during the calculation of $p_{\rm em}$, how the reasons for unsolvability were extracted and how they were translated into substrate changes.

12.4.1 Proving Unsolvability and Extracting Reasons

During the execution of the extension procedure, we want to collect reasons (i.e., location and amount of missing resources) why some VN configurations could not be mapped into the substrate. From these reasons, we want to derive changes to the substrate network. To get the unsolvability reasons, we solve a modified version of the Integer Linear Program presented in [88]. It calculates the cheapest changes to the substrate (i.e., added resources), so that a specific VN configuration can be embedded. We have looked at this problem also in previous chapters. Here we consider an extended version, because we do not only allow changing bandwidth and CPU capacities, but also changing the delay on substrate arcs and the routing capacity on substrate nodes. We will now present the used ILP in detail.

The directed graph G = (V, A) with node set V and arc set A represents the substrate network. The available CPU power of a substrate node is denoted with $c_i \in \mathbb{N}^+$, $\forall i \in V$, its routing capacity with $r_i \in \mathbb{N}^+$, $\forall i \in V$. The delay of a substrate arc is denoted by $d_e \in \mathbb{N}^+$, $\forall e \in A$, its available bandwidth with $b_e \in \mathbb{N}^+$, $\forall e \in A$. The components of the VN graph G' = (V', A') are the VNs that have to be embedded into the substrate. The constant $c_k \in \mathbb{N}^+$, $\forall k \in V'$ determines the required CPU power of a virtual node. The required bandwidth by a virtual connection is defined by $b_f \in \mathbb{N}^+$, $\forall f \in A'$ and the maximum allowed delay by $d_f \in \mathbb{N}^+, \forall f \in A'$. The set $M \subseteq V' \times V$ defines the allowed mappings between virtual and substrate nodes. The functions $s: A \cup A' \to V \cup V'$ and $t: A \cup A' \to V \cup V'$ associate each arc of G and G' with their source and target nodes, respectively. The coefficients p^{CPU} , p^{RC} , p^{BW} and p^{DL} define the cost of adding one unit of CPU or routing capacity to a substrate node, a unit of bandwidth to a substrate arc or removing a unit of delay from a substrate arc. We use $p^{\text{CPU}} = p^{\text{RC}} = 1$, $p^{\text{BW}} = 5$ and $p^{\text{DL}} = 20$ to reflect the fact that it is easy to increase the resources at the substrate nodes, but adding bandwidth at substrate arcs may be difficult and expensive because new cables may need to be run. Most expensive is a change to delays, which basically means a switch of data transmission technology.

The ILP utilizes the decision variables $x_i^k \in \{0,1\}, \ \forall k \in V', \ \forall i \in V$ to indicate where the virtual nodes are located in the substrate graph and $y^f_e \in \{0,1\}, \ \forall f \in A', \ \forall e \in A$ to indicate if a virtual connection is implemented by using a substrate connection. The decision variable $z_i^f \in \{0, 1\}, \forall f \in A', \forall i \in V \text{ indicates that a substrate node is used to route a virtual connection. The variables <math>a_i^{\text{CPU}}, \forall i \in V$ represent the added CPU capacity to a substrate node, $a_i^{\text{RC}}, \forall i \in V$ the added routing capacity. Variables $a_e^{\text{BW}}, \forall e \in A$ represent the added bandwidth to a substrate arc and $a_e^{\text{DL}}, \forall e \in A$ the removed delay. Note that a_e^{DL} is always non negative. The auxiliary variable d_e^f , $\forall e \in A$, $\forall f \in A'$ represents the delay a substrate arc has when used to implement a virtual arc and is required due to technical reasons pertaining to the implementation of delay constraints when delay changes on substrate arcs are possible. The complete ILP is defined by inequalities (12.1)–(12.18).

$$\min \quad \sum_{i \in V} (p^{\text{CPU}} a_i^{\text{CPU}} + p^{\text{RC}} a_i^{\text{RC}}) + \sum_{e \in A} (p^{\text{BW}} a_e^{\text{BW}} + p^{\text{DL}} a_e^{\text{DL}})$$
(12.1)

 $\sum_{(h,i) \in \mathbb{N}}$

$$x_i^k = 1 \qquad \forall k \in V' \tag{12.2}$$

$$\sum_{e \in A \mid t(e) = i} y_e^f + x_i^{s(f)} - \sum_{e \in A \mid s(e) = i} y_e^{f} - x_i^{t(f)} = 0 \qquad \forall i \in V, \, \forall f \in A'$$
(12.3)

$$\sum_{e \in A|t(e)=i}^{i} y_e^f + x_i^{s(f)} \le z_i^f \qquad \forall i \in V, \ \forall f \in A'$$
(12.4)

$$\sum_{(k,i)\in M} c_k x_i^k \le c_i + a_i^{\text{CPU}} \qquad \forall i \in V$$
(12.5)

$$\sum_{f \in A'} b_f z_i^f \le r_i + a_i^{\text{RC}} \qquad \forall i \in V$$
(12.6)

$$\sum_{f \in A'} b_f y_e^f \le b_e + a_e^{\mathsf{BW}} \qquad \forall e \in A$$

$$d_e y_e^f - a_e^{\mathsf{DL}} \le d_e^f \qquad \forall e \in A, \, \forall f \in A'$$
(12.8)

$$a_e^{DL} \le d_e^{-1} \qquad \forall e \in A, \ \forall f \in A' \qquad (12.8)$$
$$a_e^{DL} \le d_e - 1 \qquad \forall e \in A, \ \forall f \in A' \qquad (12.9)$$

$$\sum_{e \in A} d_e^f \leq d_f \qquad \forall f \in A' \qquad (12.10)$$
$$x_i^k \in \{0,1\} \qquad \forall (k,i) \in M \qquad (12.11)$$
$$y_e^f \in \{0,1\} \qquad \forall e \in A, \forall f \in A' \qquad (12.12)$$
$$z_i^f \in \{0,1\} \qquad \forall i \in V, \forall f \in A' \qquad (12.13)$$

$$a_i^{\text{CPU}} \in \mathbb{R}_0^+ \qquad \forall i \in V, \quad (12.15)$$

$$a_i^{\text{CPU}} \in \mathbb{R}_0^+ \qquad \forall i \in V \qquad (12.14)$$

$$a_i^{\text{RC}} \in \mathbb{R}_0^+ \qquad \forall i \in V \qquad (12.15)$$

$$a_e^{\text{BW}} \in \mathbb{R}_0^+ \qquad \forall e \in A \qquad (12.16)$$

$$a_e^{\text{DL}} \in \mathbb{R}_0^+ \qquad \forall e \in A, \quad \forall f \in A' \qquad (12.18)$$

(12.18)

The objective is given by (12.1), the total cost of added resources is to be minimized. Equalities (12.2) ensure that each virtual node is mapped to exactly one substrate node, subject to the mapping constraints. The flow conservation constraints (12.3) make sure that for each virtual connection there is a connected path in the substrate. Linking constraints (12.4) ensure that variables z_i^f are equal to one when the corresponding node is used to route the traffic of a particular virtual connection. Inequalities (12.5)-(12.7) ensure that the solution is valid with regard to CPU, routing capacity and bandwidth constraints while also considering added resources. Incorporating changes to the substrate delay is not as straight forward. Inequalities (12.8) together with the domain of d_e^f ensure that d_e^f is either zero if substrate arc e is not used by virtual arc f (i.e., y_e^f is zero) or has the correct delay value (i.e., defined delay minus delay reduction) if the substrate arc is used. Inequalities (12.9) make certain that even after reduction, the delay of a substrate connection is at least 1. Inequalities (12.10) ensure that the solution is valid with regards to the delay constraints. Equations (12.11)–(12.18) define the domains of the used variables. Note that the model only includes integrality constraints for x_i^k , y_e^f and z_i^f (12.11)– (12.13), the already covered constraints together with the objective function cause a_i^{CPU} , a_i^{RC} , a_e^{BW} and a_e^{DL} to be integral as well. Every non-zero *a* variable in an optimal solution counts as one failure reason because a resource had to be added in order to embed all VNs.

12.4.2 Reacting to Failure Reasons

Executing the extension procedure for one instance typically creates ≈ 3000 unsolvable VN configurations. With the help of the ILP formulation from the previous section, ≈ 8000 reasons for unsolvability can be extracted. These have to be condensed into concrete changes for the substrate. The following lists the four parameters we used for calculating the changes to the substrate:

- *f* Function that calculates a descriptive value from a set of missing resources, e.g., mean or max
- s Scaling factor for the result of f to calculate the added amount of resources
- r How often resources (of one type) have to be missing at a specific location in the substrate in relation to the maximum number of reported missing resources in order for this location to receive additional resources
- *n* Maximum number of resource changes in the substrate (per resource type)

Our aim is to add more resources to the most critical parts of the substrate network. Critical parts are those that are frequently reported as having too few resources of a specific type available. Let this report count be called m, and the highest count m_{max} . Note that the amount of missing resources that is reported is not yet relevant. So, for routing capacity, bandwidth, CPU capacity and delay separately, we regard substrate nodes (in case of routing and CPU capacity) and arcs (for bandwidth and delay) in descending order of m. All locations with $m \ge r \cdot m_{\text{max}}$, but at most n, will receive additional resources. These cutoff rules ensure that we do not add resources to too many locations in the substrate (which would not be economical) and also not to locations which only rarely miss resources. The amount of additional resources is determined by applying f to the reported amounts of missing resources at the selected locations and multiplying the result by s. In this work, we used f = mean, s = 3, r = 0.3, n = 5. That means that for each resource type separately, at most five locations receive additional resources. If, for example, bandwidth was reported to be missing on a substrate arc ten times, and bandwidth was not required on another arc more often, then every substrate arc that required more bandwidth at least three times gets additional bandwidth. If these are more than five arcs, the five arcs that reported missing bandwidth most often are selected to receive additional resources. For one particular substrate arc selected to receive additional bandwidth, we calculate the mean reported missing bandwidth and add three times as much. For consistency reasons, we apply s also to delay changes (ensuring that the resulting delay is still at least one), even though this is not strictly necessary, since delay is not consumed in the same way as for instance bandwidth is by additional virtual networks.

12.5 Results

From the instance set, we chose 80 instances with a load of 0.8. Due to the computational demand of the extension procedure, we could not use the complete instance set. We set a timelimit of 300 seconds to solve the presented ILP model, for substrates of size 100 (the largest size for this instance set) a time-limit of 500 seconds was employed. If the optimal solution to the ILP was not found within the time-limit, we used the best found feasible solution if the gap to the optimal solution was smaller than 95%. Preliminary runs showed that for larger substrates the feasible solution reported by CPLEX sometimes was the result of heuristics CPLEX runs before it actually starts to solve the ILP. For this problem, the solutions generated in this way add a lot of resources to almost all nodes and arcs in the substrate and are therefore not helpful for finding the real bottlenecks in the substrate. The gap limit ensures that we do not use those solutions.

12.5.1 VNMP Instance Properties

In this section we show the properties of the 80 VNMP instances used as base for the performed experiments. Table 12.1 shows the number of nodes and arcs and the number of virtual networks of each type contained in the instances. The first thing to note is that we only tested 10 instances of size 100, instead of the 14 for all other instance sizes. The reason for this is that for size 100, the instance set does not contain as many instances as for the other sizes since some of the rocketfuel networks used as basis for the substrate networks are smaller than 100 nodes. The development of the number of substrate arcs, virtual nodes and virtual arcs is similar to the instances presented in Chapter 5. The remaining part of the table shows the average number of virtual networks for each type. These instances however have been created by completely specifying a substrate network and then adding virtual networks of a random type and checking if a valid solution exists until no more virtual networks could be added. As a result, the instances contain more Stream and Web slices, since they require fewer connections and also not as many

Table 12.1: Number of used VNMP instances per size (#) and the average number of substrate arcs (A), virtual nodes (V'), virtual arcs (A') and contained Stream, Web, P2P, and VoIP virtual networks for those instances.

Size	#	A	V '	A'	Stream	Web	P2P	VoIP
20	14	53.7	118.2	146.0	6.6	8.5	4.4	4.1
30	14	95.4	215.0	264.0	13.1	14.6	7.6	7.8
40	14	126.1	275.4	382.0	16.4	18.5	6.9	7.1
50	14	172.1	308.0	523.9	14.4	16.5	7.5	6.4
70	14	259.4	478.7	835.6	18.1	17.9	6.1	7.6
100	10	399.0	648.0	1276.9	14.0	15.8	6.8	9.1

Table 12.2: Fraction of failure reasons of a specific type (missing CPU- (C) or routing- (R) capacity, missing bandwidth (B) or too much delay (D)) during the first run of the extension procedure for each VN type in relation to the total number of found failure reasons in percent. (May not add up to exactly 100% due to rounding.)

	Stream			Web				P2P				VoIP				
Size	C	R	B	D	С	R	B	D	C	R	B	D	C	R	В	D
20	39.9	56.0	4.1	0.0	12.5	66.2	0.0	21.3	11.9	80.3	7.8	0.0	11.5	88.0	0.2	0.2
30	35.5	43.7	20.7	0.0	10.6	40.8	12.3	36.3	5.1	53.3	41.6	0.0	4.3	48.1	40.3	7.3
40	42.9	46.1	11.0	0.0	8.9	37.5	9.1	44.4	13.3	58.8	27.9	0.0	16.2	42.2	12.9	28.7
50	39.5	38.6	21.9	0.0	6.2	38.0	12.7	43.1	1.8	78.7	19.0	0.5	4.6	45.5	31.6	18.4
70	53.7	33.4	12.8	0.0	9.2	28.0	13.3	49.4	6.0	61.7	31.7	0.6	5.6	58.7	34.0	1.7
100	48.9	36.5	14.6	0.0	4.0	42.6	3.3	50.1	5.2	88.8	6.1	0.0	19.1	74.1	5.1	1.6

resources as the P2P and VoIP networks and therefore have a higher probability to fit into the substrate network. The total number of virtual networks contained in the instances does not rise because their size grows with the substrate network size.

12.5.2 Extension Procedure

After we executed the extension procedure for all 80 instances and for each of the four virtual network (VN) types, the extracted failure reasons were distributed as shown in Table 12.2. It can be seen that in general missing routing capacities are prevalent, no matter which type the additionally added VNs are. For Stream VNs, the fraction of missing routing capacities is reduced while CPU capacities are missing more often with increasing substrate size. Normally, one would expect the development to be the other way around, because routing capacities are needed at multiple nodes in the substrate network to implement a virtual connection, while CPU capacities are only needed at the start and the end of a virtual connections and the total required routing capacity increases while the CPU capacity stays the same. This would cause the probability of missing routing capacity to increase. The reason why missing CPU resources become more prominent is that the nodes of stream VNs require a lot of CPU capacity (because they have to split and distribute video streams). The VN sizes grow proportional to the substrate sizes, therefore the Stream VNs for larger substrates can contain more nodes which perform

video stream splitting which in turn requires more CPU resources. In relation to the missing CPU and routing capacities, the substrate link bandwidths only play a subsidiary role and no link was found to have too much delay, which is not surprising since the virtual connections in Stream VNs are not delay constrained.

This is not true for Web VNs which are heavily delay constrained. For substrates of size 100, more than half of all found failure reasons are too high delay on some substrate arc. This fraction rises with increasing substrate size because the average length of virtual arc implementations and therefore the total accumulated delay grows.

When embedding additional P2P VNs into the substrate, missing bandwidths on substrate links become a significant issue for the first time, even though missing routing capacities still dominate. Again, this is because the (already high) bandwidth requirements of P2P VNs require a lot of routing capacity in the substrate nodes and depending on the particular substrate configuration one resource is more limiting than the other. Also note that a bias against bandwidth changes exists, since adding one unit of bandwidth costs five times more than a unit of routing capacity. That means that if an additional VN can be added either by adding four units of routing capacity or adding one unit of bandwidth, four units of routing capacity are reported missing because this is the cheaper solution. However, this effect can also be witnessed the other way around with this VN type. Even though P2P VNs are not delay constrained, for sizes 50 and 70 delays were reported missing. This can happen because the initial instances contain all four VN types, i.e., also some with are delay constrained. By reducing the delay somewhere, the virtual connections with stringent delay requirements can be implemented by using a previously impossible substrate path which frees up resources for new VNs where they are needed. This can be the cheaper solution, even though decreasing link delay by one unit costs twenty times more than adding a unit of routing capacity for instance.

For VoIP VNs, missing bandwidths are again prominent (after missing routing capacities). Also link delays play some role, but since VoIP VNs are not as delay constrained as Web VNs it is not surprising to see that delays are less often a failure reason when compared to Web VNs. Unfortunately we can offer no conclusive reason as to why the fraction of delay reasons fluctuates so much for VoIP VNs. Larger instances reduce the probability that the presented ILP can be solved to optimality, so one possible explanation could be that feasible solutions which only change other resources are predominantly found. We will not go into detail regarding the performance of the ILP, but in a nutshell, it works well up to substrate sizes of 50, i.e., more than 99% of executions yield useful results (a valid solution within the gap limit). For sizes 70 and 100, there is a split between Stream and Web VNs on the one hand and P2P and VoIP VNs on the other. For Stream and Web VNs, we still get useful results more than 90% of the time. For P2P and VoIP VNs, this is reduced to 85% for instances of size 70 and even 45% for size 100. Note that at the time these experiments were performed, the preprocessing techniques outlined in Chapter 9 had not been developed.

Based on the reported missing resources for each instance, we added resources to each VNMP instance as outlined in Section 12.4.2. Table 12.3 shows the change to the available resources in the substrate because of the added resources.

It can be seen that only a very small amount of resources was added to the substrate, as was our goal. Generally, the total available amount of resources was increased by less than one percent.

		Stream			Web				P2P				VoIP				
Si	ize	С	R	B	D	С	R	B	D	C	R	В	D	C	R	B	D
	20	1.4	0.7	0.2	0.0	0.1	0.3	0.0	-8.4	0.1	2.1	0.0	0.0	0.2	2.2	0.0	-1.2
	30	0.6	0.4	0.2	0.0	0.0	0.1	0.0	-5.9	0.1	1.0	0.1	0.0	0.1	0.9	0.1	-3.8
	40	0.4	0.3	0.1	0.0	0.0	0.1	0.0	-7.5	0.1	1.2	0.1	0.0	0.2	1.2	0.1	-3.5
	50	0.2	0.1	0.2	-0.2	0.0	0.1	0.1	-6.5	0.0	0.8	0.3	-0.9	0.1	0.9	0.3	-5.7
	70	0.2	0.1	0.0	-0.2	0.0	0.0	0.0	-2.3	0.0	0.5	0.1	-0.3	0.1	0.5	0.1	-0.4
1	00	0.1	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	0.3	0.0	0.0	0.0	0.3	0.0	-0.1

Table 12.3: Relative change of the available resources in the substrate due to the found failure reasons in percent.

Table 12.4: Average $p_{\rm em}$ in percent for the first run of the extension procedure and for the second run with additional substrate resources.

	Stre	eam	W	eb	P2	2P	VoIP		
Size	1st	2nd	1st	2nd	1st	2nd	1st	2nd	
20	13.6	38.8	18.4	59.7	7.4	20.2	7.9	20.4	
30	13.1	39.6	14.7	60.8	3.3	18.2	3.7	15.5	
40	13.5	42.7	11.7	60.6	4.3	15.4	3.1	14.3	
50	15.7	40.7	9.0	51.7	11.6	13.6	7.2	14.7	
70	12.7	33.5	11.3	31.5	2.6	11.7	2.9	15.4	
100	8.0	31.7	12.2	30.9	0.0	0.9	0.0	0.6	

The most notable exception to this are the delays when adding Web or VoIP VNs. There are three factors that work together to cause this. First of all, Web and VoIP VNs are delay constrained, so even though delays are not very often reported to be too high, the reported magnitude *is* high. Secondly, as we have already pointed out, *s* is also applied when calculating delay changes for uniformity reasons, even though it is not strictly necessary. If the delay of a substrate connection is reduced all future VNs will benefit, at least in our simplified model where substrate link delays are independent of link load. The same is not true for the other resource types, since they are used up by additional VNs, which is the reason why we add more resources than are reported missing. The third contributing factor to the large delay changes is the fact that the sum of all delays in the substrate is less than the sum of the other resource types, so changing the delay by some fixed amount will be a larger relative change than for the other resources.

The following section will answer the question whether the resources added to the substrate could influence $p_{\rm em}$ in a meaningful way.

12.5.3 Change to the Embedding Probability

The complete development of $p_{\rm em}$ during both runs of the extension procedure can be seen in Figures 12.2 and 12.3. They show how often a specific $p_{\rm em}$ occurred depending on the number of added VNs. Stream and Web have been combined into one graph, as have P2P and VoIP, because they show very similar behaviour. Clearly visible is the shift of the clusters to the right, i.e., more VNs can be added before the substrate is full. Figure 12.2b shows that for

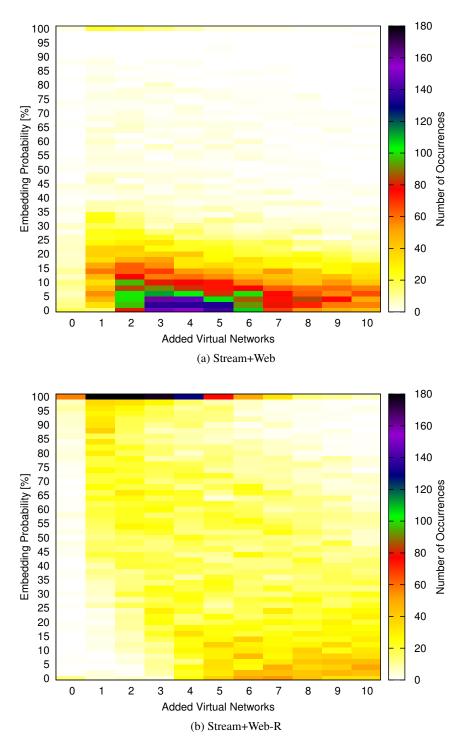


Figure 12.2: $p_{\rm em}$ development for both runs of the extension procedure for Stream and Web VNs. Suffix R denotes the second run with additional resources for the substrates.

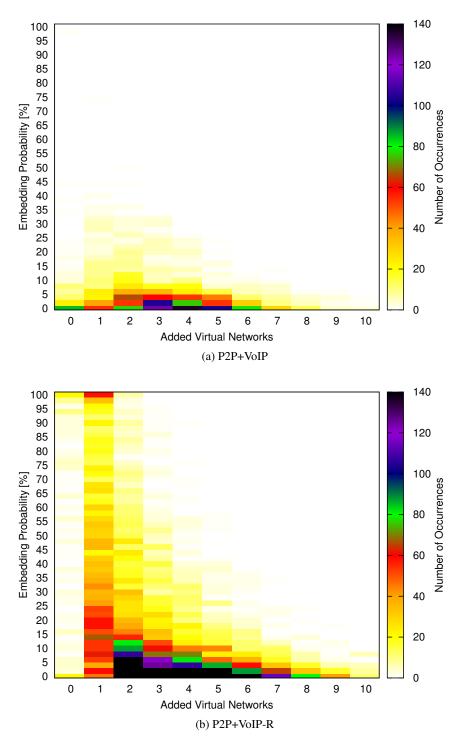


Figure 12.3: $p_{\rm em}$ development for both runs of the extension procedure for P2P and VoIP VNs. Suffix R denotes the second run with additional resources for the substrates.

Stream and Web VNs, the added resources cause a substantial fraction of instances to stay at $p_{\rm em} = 1$ during the first five added VNs. This is in contrast to the situation before additional resources were added, as seen in Figure 12.2a. After five added VNs, most instances show a $p_{\rm em}$ smaller than 0.2. A similar, although not as pronounced, development can be seen for P2P and VoIP VNs in Figure 12.3. It can be seen that during the first run of the extension procedure (Figure 12.3a) it happens very often that the initial VNMP instance cannot be extended with an additional VN. Nearly every initial instance has $p_{\rm em} \leq 0.12$. After adding additional resources, $p_{\rm em}$ of the initial instance covers the whole range of possibilities. Adding further VNs reduces $p_{\rm em}$ far more rapidly than for Stream and Web VNs. On the whole the added resources are more effective for influencing $p_{\rm em}$ for Stream and Web VNs than they are for P2P and VoIP VNs. There are two possible explanations for this behaviour. First of all, since the first run of the extension procedure for P2P and VoIP VNs often could not extend the initial instance, the collected failure reasons only contain reasons why adding one VN might fail. Therefore, after adding resources, only adding the first VN works well and $p_{\rm em}$ rapidly approaches zero afterwards. Another possible explanation is that the used settings for determining where and how much resources are added might not cause enough resources to be added so that multiple VNs can be embedded in the substrate. The chosen settings work well for Stream and Web VNs, but might be too conservative for P2P and VoIP VNs, which require more resources per VN. In Table 12.4, we show the average embedding probability for all instances generated by both runs of the extension procedure. It can be seen that the greatest gains have been achieved for Web VNs (which also had the largest resource change). A close second are Stream VNs, where less

than 1% change in resources increased $p_{\rm em}$ by 20% and more on average. The improvements for P2P and VoIP VNs were not as great, especially for instances of size 100 where the ILP fails to extract useful failure reasons due to time and memory constraints.

12.6 Conclusion

In this chapter, we presented a method how an ILP as presented in Chapter 11 can be adapted to successfully identify bottlenecks in a substrate network. Our results show that less than one percent of additional resources in the substrate network can increase the probability that additional VNs can be embedded by 20% or more. We also showed that different use cases (i.e., different VNs) lead to different resources being added to the substrate. Therefore, Virtual Network Operators (VNOs) are able to optimize their networks to cater best to the VN types they encounter the most, or are most profitable.

With the presented method, a VNO could monitor its current network situation and decide whether leasing additional resources at a specific location is necessary to provide a certain service availability. Knowing these bottlenecks in advance offers the VNO the possibility to add additional resources before it is too late.

We have shown in this chapter the flexibility of the VNMP solution approaches. They can be easily adapted to solve related practical problems occurring when operating VNs.

CHAPTER **13**

Comparison and Conclusions

13.1 Introduction

In the final chapter of this work, we present a comparison of the main algorithms introduced previously, both for solving VNMP-S and VNMP-O. The algorithms chosen for comparison are CH-S and CH-O as defined in Section 6.5.1, LS-S and LS-O as defined in Section 6.5.2, VND-S and VND-O as defined in Section 6.5.3, MA-O as defined in Section 7.5, GRASP-O as defined in Section 8.4.1, VNS-O as defined in Section 8.4.2, and ILP-S and ILP-O as defined in Section 11.4.3. For an even more extensive presentation of the results achieved by those algorithms, we refer to Appendix A.

The presented data is based on the full VNMP instance set as defined in Chapter 5 at loads 0.1, 0.5, 0.8 and 1, giving a total of 840 VNMP instances. Comparing the exact algorithms (ILP-S and ILP-O) to the others is problematic, since they may fail to produce any solution at all, for instance when they are aborted due to the memory limit. To offer a meaningful comparison, we chose the following approach: For every VNMP instance, we keep track of the highest additional resource cost C_a and usage cost C_u produced by any compared algorithm. If one of the exact methods failed to generate a result, we assume it generated a result with the worst C_u increased by one, the worst C_a increased by one, and required 10000 seconds (the time-limit used for the exact methods).

In Section 13.2, we analyze the number of valid solutions found by each algorithm. Section 13.3 is concerned with the C_a of the found solutions, while the achieved relative ranks R_{rel} are covered in Section 13.4. The gap of the substrate usage cost (and the definition of this metric) is discussed in Section 13.5. Required run-times are presented in Section 13.6. We conclude in Section 13.7 and outline future work in Section 13.8.

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			1000				500				200				100				50				30				20	Size
1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	Load
0	0 <	> 6	27 =	0 <	$^{3}_{\wedge}$	21 <	30 =	0 <	6 <	22 <	30 =	$^{3}_{\wedge}$	18 < 18	26 <	30 =	7 <	25 <	29 =	30 =	13 <	23 <	30 =	30 =	13 <	27 =	29 =	30 =	CH-O
0 <	8	25 <	30 =	0 <	20 <	27 =	30 =	12 <	28 =	30 =	30 =	26 <	29 =	30 =	30 =	27 =	30 =	30 =	30 =	28 =	30 =	30 =	30 =	24 <	29 =	30 =	30 =	LS-O
2 <	17 <	28 =	30 =	18 < 18	30 =	30 =	30 =	27 =	30 =	30 =	30 =	27 =	29 =	30 =	30 =	28 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	27 =	29 =	30 =	30 =	LS-0 VND-0
2 <	12 <	26 < 100	30 =	8	26 <	28 =	30 =	27 =	28 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	MA-0
0 <	16 < 16	26 <	30 =	10 < 10	24 <	28 =	30 =	29 =	29 =	30 =	30 =	28 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	GRASP-O
2 <	12 <	27 =	30 =	9 <	24 <	29 =	30 =	27 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	VNS-0
0 ^	0 <	20 < 10	30 =	0 <	7 <	27 =	30 =	21 <	25 <	29 =	30 =	28 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	ILP-O
0 ~	$\frac{1}{\wedge}$		26 <	0 <	9 <	24 <	30 =	7 <	21 <	26 < 100	30 =	27 =	28 =	30 =	30 =	27 =	30 =	30 =	30 =	28 =	30 =	30 =	30 =	23 <	29 =	30 =	30 =	CH-S
1 ^	10 < 0	24 <	30 =	4 ^	18 <	26 <	30 =	19 <	27 =	29 =	30 =	24 <	29 =	29 =	30 =	28 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	27 =	29 =	30 =	30 =	LS-S
2 <	17 <	28 =	30 =	16 <	30 =	30 =	30 =	28 =	30 =	30 =	30 =	28 =	29 =	29 =	30 =	29 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	28 =	29 =	30 =	30 =	VND-S
29 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	30 =	ILP-S

13.2 Number of Valid VNMP Solutions

In this section, we focus on the capabilities of the main algorithms with respect to VNMP-S and present the number of valid solutions found in Table 13.1. The very best performance is achieved by ILP-S, which only fails to find a valid solution for one single instance. However, most of the other compared algorithms achieve nearly the same performance for instances up to size 100. VND-S is competitive up to size 200, but then fails often for instances with load 0.8 and 1. The same is true for MA-O, GRASP-O, and VNS-O. The hardest VNMP instances with respect to VNMP-S are those of size 500 and 1000 with full load. For those, ILP-S is able to find valid solutions for nearly all instances. The next best algorithm (VND-O) finds 18 valid solutions for size 500 and 2 for size 1000. Therefore, there is still room for improvement for heuristics when solving VNMP-S for the largest instances.

13.3 Additional Resource Cost

Table 13.2 presents the average achieved C_a of the compared algorithms. This allows us to judge how far the found solutions are away from validity ($C_a = 0$). The most interesting comparison can be performed for instances of size 1000 and load 1. We know from the previous section that all algorithms (with the exception of ILP-S) fail to find valid solutions for basically all instances. Now we can see, that while those algorithms are the same with respect to the number of solved instances, there are huge differences as to their distance to optimality. The best algorithm is VND-S, with an average C_a of 303. The construction heuristic geared towards solving VNMP-S is more than ten times worse, with a cost of 3200. When we use CH-O, the cost is 23300, nearly a 77-fold increase compared to the best value. ILP-O achieves basically the same cost. This is the result of how we set the costs if no solution is found. Note how the average cost of 1352 for ILP-S is marked as the best value. From the previous section, we know that this average is the result of 29 solved instances (i.e., 0 for C_u) and one instance for which no result was produced, which is punished by using the C_a value from the worst algorithm (CH-O). This is better than failing for nearly every instance with small values of C_a .

13.4 Relative Rank

We now start considering the performance of the different algorithms with respect to VNMP-O. In Table 13.3, we show their average relative ranks. Unsurprisingly, ILP-O dominates according to this metric, only starting with size 100 other methods achieve better results. This is mostly due to ILP-O failing because of excessive memory requirements. For larger instances, ILP-O cannot be beaten for a load of 0.1. When we consider the performance of the heuristic methods, we see that MA-O and VNS-O are close to ILP-O up to size 100. For larger sizes VND-O is best and also able to outperform ILP-O in some instances. The algorithms focusing on VNMP-S generally have the worst ranks. Considering that CH-O often fails to find valid solutions, it is surprising that ILP-S is even worse. For size 500 and load 0.1, this method consistently finds the most expensive solutions. However, this is reversed for the highest loads and largest instance sizes, where ILP-S performs best since all other methods struggle to find valid solutions.

			1000				500				200				100				50				30				20	Size
1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	_	1.00	0.80	0.50	_	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	e Load
23278.3 >	3993.5 >	237.9 >	1.1 =	13765.3 >	2075.8 >	25.4 >	0.0 =	5480.0 >	648.9 >	86.2 >	0.0 =	1470.6 >	301.3 >	31.5 =	0.0 =	1905.5 >	27.5 >	1.5 =	0.0 =	786.9 >	62.5 >	0.0 =	0.0 =	386.0 >	55.3 =	2.3 =	0.0 =	CH-O
2527.5 >	290.0 >	2.5 >	0.0 =	1963.2 >	339.3 >	1.9 =	0.0 =	229.8 >	2.2 =	0.0 =	0.0 =	30.7 =	2.3 =	0.0 =	0.0 =	124.8 =	0.0 =	0.0 =	0.0 =	50.7 =	0.0 =	0.0 =	0.0 =	101.3 >	6.0 =	0.0 =	0.0 =	LS-O
659.1 >	45.5 >	0.4 =	0.0 =	57.9 >	0.0 =	0.0 =	0.0 =	4.8 =	0.0 =	0.0 =	0.0 =	14.7 =	2.3 =	0.0 =	0.0 =	8.4 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	29.5 =	6.0 =	0.0 =	0.0 =	VND-O
748.5 >	112.9 >	2.1 =	0.0 =	271.7 >	35.8 =	1.8 =	0.0 =	11.0 =	1.2 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	VND-O MA-O
881.8 >	98.0 >	2.1 =	0.0 =	158.1 >	29.1 >	1.3 =	0.0 =	0.2 =	1.3 =	0.0 =	0.0 =	10.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	GRASP-O
1024.0 >	173.4 >	47.3 =	0.0 =	238.8 >	34.1 >	1.1 =	0.0 =	24.3 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	VNS-O
23279.2 >	3993.5 >	158.4 >	0.0 =	13765.3 >	1902.3 >	20.0 =	0.0 =	1742.9 >	74.0 >	8.3 =	0.0 =	119.5 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	ILP-0
3211.0 >	510.1 >	34.8 >	1.9 =	2214.2 >	381.6 >	16.7 >	0.0 =	585.0 >	132.6 >	69.1 =	0.0 =	144.0 =	12.5 =	0.0 =	0.0 =	21.6 =	0.0 =	0.0 =	0.0 =	2.8 =	0.0 =	0.0 =	0.0 =	36.0 >	1.7 =	0.0 =	0.0 =	CH-S
811.4 >	151.0 >	3.1 >	0.0 =	496.1 >	106.2 >	2.3 =	0.0 =	99.4 >	3.0 =	0.8 =	0.0 =	53.7 >	2.3 =	2.3 =	0.0 =	22.7 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	46.3 =	6.0 =	0.0 =	0.0 =	LS-S
	49.3 >	0.6 =	0.0 =	47.6 >	0.0 =	0.0 =	0.0 =	4.0 =	0.0 =	0.0 =	0.0 =	5.5 =	2.3 =	2.3 =	0.0 =	5.3 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	11.8 =	6.0 =	0.0 =	0.0 =	VND-S
1352.6 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	ILP-S

Table 13.2: Average C_a achieved by the main algorithms presented in this work.

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Size	Load	CH-0	LS-O	O-UND-O	MA-O	GRASP-O	O-SNV	ILP-O	CH-S	LS-S	S-DND-S	ILP-S
20	0.10	0.619 >	0.209 >	0.186 >	0.013 =	0.086 >	0.031 >	0.000 =	0.749 >	0.506 >	0.657 >	0.971 >
	0.50	0.698 >	0.382 >	0.321 >	0.096 >	0.220 >	0.103 >	0.000 =	0.844 >	0.571 >	0.703 >	0.969 >
	0.80	0.726 >	0.460 >	0.411 >	0.124 >	0.266 >	0.130 >	0.000 =	0.863 >	0.648 >	0.764 >	0.910 >
	1.00	0.883 >	0.523 >	0.421 >	0.132 >	0.278 >	0.150 >	0.000 =	0.837 >	0.613 >	0.688 >	0.765 >
30	0.10	0.560 >	0.199 >	0.167 >	0.014 =	< 660.0	0.028 >	0.000 =	0.771 >	0.490 >	0.715 >	< 0600 >
	0.50	0.626 >	0.384 >	0.304 >	0.092 >	0.231 >	0.107 >	0.000 =	0.852 >	0.587 >	0.760 >	0.993 >
	0.80	0.745 >	0.449 >	0.359 >	0.150 >	0.277 >	0.123 >	0.000 =	0.854 >	0.604 >	0.758 >	0.960 >
	1.00	0.859 >	0.482 >	0.403 >	0.183 >	0.286 >	0.136 >	0.000 =	0.846 >	0.621 >	0.715 >	0.889 >
50	0.10	0.541 >	0.276 >	0.165 >	0.019 =	0.179 >	0.048 >	0.000 =	0.831 >	0.551 >	0.775 >	1.000 >
	0.50	0.680 >	0.454 >	0.366 >	0.155 >	0.277 >	0.111 >	0.000 =	0.884 >	0.618 >	0.785 >	0.988 >
	0.80	0.729 >	0.484 >	0.371 >	0.183 >	0.319 >	0.129 >	0.000 =	0.868 >	0.630 >	0.779 >	0.983 >
	1.00	0.937 >	0.514 >	0.421 >	0.188 >	0.331 >	0.119 >	0.000 =	0.794 >	0.636 >	0.706 >	0.874 >
100	0.10	0.630 >	0.411 >	0.319 >	0.151 >	0.270 >	0.092 >	0.000 =	0.876 >	0.644 >	0.818 >	1.000 >
	0.50	0.697 >	0.448 >	0.309 >	0.270 >	0.325 >	0.142 >	0.000 =	0.881 >	0.659 >	0.784 >	0.975 >
	0.80	0.797 >	0.450 >	0.323 >	0.231 >	0.362 >	0.140 >	0.019 =	0.853 >	0.644 >	0.749 >	0.940 >
	1.00	0.967 >	0.465 >	0.339 >	0.240 >	0.393 >	0.137 >	0.067 =	0.762 >	0.647 >	0.667 >	0.838 >
200	0.10	0.596 >	0.464 >	0.324 >	0.219 >	0.254 >	0.121 >	0.000 =	0.892 >	0.694 >	0.809 >	0.978 >
	0.50	0.710 >	0.461 >	0.240 >	0.272 >	0.364 >	0.125 >	0.070 =	0.892 >	0.673 >	0.757 >	0.956 >
	0.80	0.915 >	0.450 >	0.174 =	0.289 >	0.397 >	0.155 =	0.185 =	0.859 >	0.647 >	0.669 >	0.850 >
	1.00	< 7997 >	0.680 >	0.190 =	0.265 >	0.324 >	0.201 =	0.300 =	0.808 >	0.647 >	0.562 >	0.677 >
500	0.10	0.598 >	0.445 >	0.254 >	0.341 >	0.282 >	0.169 >	0.000 =	0.878 >		0.828 >	1.000 >
	0.50	0.694 >	0.399 >	0.077 =	0.301 >	0.404 >	0.230 >	0.244 =	0.909 >	0.691 >	0.721 >	0.891 >
	0.80	0.964 >	0.474 >	0.017 =	0.261 >	0.361 >	0.320 >	0.804 >	0.817 >	0.642 >	0.528 >	0.671 >
	1.00	< 960.0	0.794 >	0.096 =	0.366 >	0.342 >	0.407 >	0.993 >	0.830 >		0.332 >	0.239 >
1000	0.10	0.630 >	0.423 >	0.128 >	0.378 >	0.373 >	0.213 >	0.000 =	0.889 >	0.696 >	0.813 >	0.975 >
	0.50	0.868 >	0.348 >	0.070 =	0.316 >	0.365 >	0.331 >	0.411 >	0.917 >	0.683 >	0.641 >	0.716 >
	0.80	< 7997 >	0.555 >	0.111 =	0.388 >	0.377 >	0.432 >	1.000 >	0.764 >	0.598 >	0.420 >	0.342 >
	1.00	0.913 >	0.702 >	0.269 >	0.346 >	0.425 >	0.478 >	1.000 >	0.769 >	0.446 >	0.179 >	0.057 =

13.5 Substrate Usage Cost Gap

The previous section allowed a small insight into the relative performance of the compared algorithms when solving VNMP-O. However, as we have outlined when defining the ranking procedure, by ranking we lose the information about the distance between solutions (in terms of cost). In this section, we will focus on the difference of the created solutions with respect to C_u . As a measure, we chose the substrate usage cost gap C_u -gap. For an algorithm A, it is calculated by dividing the difference between the C_u of the result created by A and the best C_u by the C_u of A. If A's result is not valid, we assume a C_u -gap of 100%. A C_u -gap of 1% means that the best result is 1% cheaper than the result of the algorithm in question. Table 13.4 presents the average C_u -gap values achieved by the algorithms compared in this chapter.

It can be seen, that for instances below size 200, VNS-O is very close to the results achieved by ILP-O. For load 0.1, the gap is about 0.5%, for load 0.5 it is 2%, for 0.8 4%, and 7% for load 1. Also MA-O is close to the performance of ILP-O. It can be observed that the results of ILP-S are very often the most expensive. They are only exceeded for some instances by CH-O due to the 100% gap assumed when the final result is not valid. Even the results of CH-S, LS-S, and VND-S are in most cases not as expensive. Note that the presented gaps are in relation to the best found solution, which is usually created by ILP-O. For a significant fraction of instances, ILP-O cannot prove the optimality of the found solution and a gap of about 1% remains, as shown in Chapter 11. Therefore, to get an approximation of the gap between the heuristic solutions and the best solution value that might be obtainable, 1% can be added to the reported C_u -gaps.

13.6 Required Run-time

The last property of the presented algorithms we compare is their required run-time, which is presented in Table 13.5. For MA-O, GRASP-O, and VNS-O, the required run-time is constant since the algorithms were executed until the time-limit was reached. The applied time-limit of 1000 seconds for VND-based methods can be observed for the largest instances. Also note how the run-time, especially for full load and a size of 1000, exceeds the time-limit by a large margin. This is due to the fact that the time-limit is checked after a neighborhood has been searched. For the very largest instances, even LS-O had to be aborted due to the time-limit. That may be an indication that the employed neighborhoods grow too large when the instance size rises and some alternatives need to be developed. It is remarkable how close the run-time of ILP-S is to the heuristic alternatives.

13.7 Conclusion

The Internet has ossified. It cannot react to changing demands placed upon it by different applications. A way to get around this problem is to use virtual networks, which can be adapted in terms of structure, employed protocols, and available resources to perfectly fit to specific applications or application classes. These virtual networks need to be hosted in the available physical network. In this work, we have considered an abstraction of this problem, the Virtual Network Mapping Problem (VNMP), which is \mathcal{NP} -hard. More precisely, we considered two

Table 13.4: Average C_u -gap in percent achieved by the main algorithms presented in this work.

Size	Load	CH-O	LS-O	O-UND-O	MA-O	GRASP-O	O-SNV	ILP-O	CH-S	LS-S	S-UN-S	ILP-S
20	0.10	19.4 >	5.7 >	5.0 >	0.0 =	2.0 >	0.5 >	0.0 =	23.0 >	17.9 >	20.3 >	48.2 >
	0.50	29.5 >	8.7 >	< T.T >	1.7 >	4.8 >	2.1 >	0.0 =	31.3 >	24.2 >	28.0 >	38.2 >
	0.80	32.5 >	15.2 >	14.1 >	2.8 >	6.3 >	3.3 >	0.0 =	32.8 >	27.5 >	29.4 >	34.4 >
	1.00	68.3 >	29.9 >	20.9 >	4.0 >	8.1 >	4.4 >	0.0 =	44.9 >	30.8 >	30.3 >	29.3 >
30	0.10	22.6 >	3.8 >	2.7 >	0.2 =	1.7 >	0.4 >	0.0 =	26.9 >	20.5 >	25.5 >	47.0 >
	0.50	25.5 >	7.4 >	6.1 >	1.6 >	4.3 >	1.9 >	0.0 =	31.9 >	25.1 >	28.5 >	40.6 >
	0.80	44.5 >	11.9 >	10.1 >	4.0 >	7.2 >	3.5 >	0.0 =	34.5 >	27.8 >	30.8 >	40.8 >
	1.00	67.9 >	19.6 >	13.5 >	6.7 >	9.1 >	6.0 >	0.0 =	38.3 >	28.8 >	30.6 >	36.5 >
50	0.10	19.8 >	4.7 >	2.7 >	0.2 =	2.4 >	0.4 >	0.0 =	25.7 >	20.3 >	24.6 >	45.2 >
	0.50	30.3 >	8.4 >	6.3 >	2.5 >	4.7 >	1.8 >	0.0 =	32.4 >	27.2 >	29.6 >	45.4 >
	0.80	42.0 >	12.2 >	9.4 >	5.0 >	7.6 >	4.3 >	0.0 =	34.3 >	29.7 >	32.0 >	44.4 >
	1.00	83.8 >	24.7 >	20.3 >	> 0.0	12.7 >	7.2 >	0.0 =	41.9 >	36.0 >	35.0 >	42.1 >
100	0.10	21.7 >	6.0 >	4.3 >	1.2 >	2.7 >	0.8 >	0.0 =	28.9 >	24.1 >	26.7 >	40.0 >
	0.50	38.0 >	8.4 >	6.6 >	5.9 >	6.7 >	4.5 >	0.0 =	34.5 >	32.1 >	33.4 >	46.3 >
	0.80	57.6 >	14.1 >	12.7 >	8.6 >	10.0 >	7.4 >	= 0.0	40.3 >	32.6 >	33.8 >	44.6 >
	1.00	93.1 >	25.5 >	21.0 >	11.9 >	19.1 >	10.6 >	6.7 =	42.3 >	44.3 >	36.1 >	42.1 >
200	0.10	19.9 >	6.5 >	3.7 >	2.2 >	2.9 >	1.1 >	0.0 =	30.0 >	26.0 >	28.4 >	41.1 >
	0.50	49.1 >	8.8 >	5.6 >	5.9 >	7.2 >	4.2 >	5.0 =	44.4 >	34.1 >	33.0 >	45.8 >
	0.80	86.7 >	17.2 >	7.8 =	14.8 >	13.3 >	7.5 =	17.7 =	57.1 >	39.8 >	34.3 >	45.2 >
	1.00	100.0 >	65.1 >	19.0 =	19.2 >	15.5 >	18.8 =	30.0 =	86.0 >	57.3 >	37.5 >	43.6 >
500	0.10	16.8 >	5.8 >	2.9 >	4.4 >	3.7 >	2.5 >	0.0 =	29.5 >	26.0 >	28.2 >	44.5 >
	0.50	51.0 >	16.4 >	3.3 =	12.3 >	13.2 >	8.8 >	17.9 =	50.9 >	41.9 >	34.2 >	43.6 >
	0.80	93.3 >	38.0 >	1.6 =	18.4 >	25.8 >	25.1 >	78.7 >	81.6 >	58.8 >	33.3 >	42.7 >
	1.00	100.0 >	100.0 >	40.1 =	74.4 >	68.3 >	71.4 >	100.0 >	100.0 >	> 89.8	61.0 >	27.2 =
1000	0.10	28.2 >	7.7 >	3.3 >	7.1 >	7.1 >	4.7 >	0.0 =	41.9 >	28.7 >	30.7 >	46.9 >
	0.50	80.3 >	23.2 >	9.7 =	20.1 >	20.6 >	17.5 >	37.8 =	73.3 >	49.3 >	40.9 >	44.8 >
	0.80	100.0 >	73.9 >	43.3 =	61.5 >	49.9 >	61.8 >	100.0 >	98.1 >	> 76.9 >	59.7 =	36.7 =
	1.00	100.0 >	100.0 >	93.3 >	93.3 >	100.0 >	93.6 >	100.0 >	100.0 >	97.4 >	94.7 >	9.1 =

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			1000				500				200				100				50				30				20	Size
1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	1.00	0.80	0.50	0.10	Load
2.1 >	1.5 >	0.8 >	0.3 >	0.8 >	0.6 >	0.3 >	0.1 >	0.2 =	0.2 =	0.1 =	0.0 >	0.1 =	0.1 =	0.1 =	0.0 >	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	CH-O
1009.4 >	860.3 >	328.6 >	25.1 >	427.3 >	245.4 >	96.1 >	7.3 >	77.6 >	55.0 >	25.0 >	2.3 >	24.3 >	18.2 >	9.1 >	1.0 >	3.3 >	2.2 >	1.1 >	0.1 >	0.9 >	0.6 >	0.3 >	0.0 >	0.3 >	0.2 >	0.1 >	0.0 >	LS-O
1013.9 >	1064.9 >	1011.3 >	216.6 >	1013.2 >	909.8 >	366.0 >	41.7 >	209.5 >	153.1 >	63.6 >	7.7 >	47.9 >	37.0 >	19.3 >	2.5 >	6.7 >	5.3 >	2.7 >	0.3 >	1.9 >	1.6 >	0.8 >	0.1 >	0.6 >	0.5 >	0.3 >	0.0 >	VND-O
503.9 >	502.7 >	502.3 >	500.5 >	501.5 >	501.2 >	500.7 >	500.2 >	500.5 >	500.4 >	500.2 >	500.1 >	200.3 >	200.2 >	200.1 >	200.0 >	200.1 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	MA-0
504.8 >	502.8 >	501.6 >	500.3 >	501.3 >	501.0 >	500.5 >	500.2 >	500.6 >	500.5 >	500.2 >	500.0 >	200.2 >	200.2 >	200.1 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	GRASP-O
506.3 >	503.8 >	501.8 >	500.4 >	502.1 >	501.4 >	500.7 >	500.2 >	500.7 >	500.3 >	500.3 >	500.1 >	200.3 >	200.2 >	200.1 >	200.0 >	200.1 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	200.0 >	VNS-0
10019.6 >	10124.3 >	10075.2 >	2264.7 >	10038.8 >	10030.9 >	8988.5 >	536.9 >	10006.3 >	9768.4 >	6470.8 >	48.5 >	9912.9 >	8933.8 >	3108.9 >	4.1 >	4565.6 >	2576.0 >	372.9 >	0.4 >	1899.2 >	466.1 >	96.6 >	0.2 >	85.8 >	10.7 >	0.8 >	0.1 >	ILP-O
1.9 =	1.3 =	0.7 =	0.1 =	0.8 =	0.5 =	0.3 =	0.0 =	0.3 >	0.2 >	0.1 =	0.0 =	0.1 >	0.1 >	0.1 =	0.0 >	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 =	0.0 >	0.0 =	0.0 =	0.0 =	0.0 =	CH-S
356.4 >	109.3 >	12.2 >	0.5 >	80.8 >	19.3 >	2.3 >	0.2 >	6.2 >	1.6 >	0.5 >	0.1 >	1.0 >	0.3 >	0.2 >	0.0 >	0.3 >	0.1 >	0.1 >	0.0 >	0.1 >		0.0 >	0.0 >	0.0 >	0.0 >	0.0 >	< 0.0	LS-S
1076.5 >	717.5 >	129.5 >	1.8 >	579.8 >	80.0 >	5.7 =	0.0 =	51.5 >	6.6 >	1.8 =	0.0 =	4.3 >	1.9 =	0.9 =	0.0 =	0.6 >	0.0 =	0.0 =	0.0 =	0.0 >	0.0 =	0.0 =	0.0 =	0.0 >	0.0 =	0.0 =	0.0 =	VND-S
1865.5 >	579.0 >	208.2 >	42.4 >	432.9 >	161.9 >	79.3 >	24.3 >	55.5 >	32.8 >	16.8 >	3.4 >	20.8 >	13.3 >	7.1 >	1.3 >	5.3 >	3.7 >	1.8 >	0.2 >	1.8 >	1.0 >	0.5 >	0.1 >	0.4 >	0.3 >	0.1 >	0.0 >	ILP-S

Table 13.5: Average run-time in seconds required by the main algorithms presented in this work.

variants, VNMP-S, where just a valid solution (i.e., an embedding of the virtual networks into the physical network) needs to be found, and VNMP-O, where we want to find a valid and cheap embedding.

As a first approach for solving the VNMP, we presented Construction Heuristics. Then we defined neighborhood structures to improve solutions found by the Construction Heuristics to create Local Search algorithms. We studied how the different neighborhood structures can be used to form Variable Neighborhood Descent algorithms. Based on the experience of solving the VNMP with those methods, we developed Greedy Randomized Adaptive Search Procedures, Memetic Algorithms and Variable Neighborhood Search approaches to further increase performance. Having covered some of the most important meta-heuristics for combinatorial optimization problems, we focused on exact methods for solving the VNMP. As preparation for that, we developed a preprocessing algorithm which makes use of the properties of telecommunication networks to extract useful knowledge from a VNMP instance, such as which nodes have to be used in any case. The preprocessing methods proved to be essential when we applied Constraint Programming and Integer Linear Programming to the VNMP.

As for the performance of the different developed algorithms, some of course proved to be better suited than others. The Constraint Programming and Column Generation approaches we had to reject out of hand. Constraint Programming showed poor performance when trying to find valid VNMP solutions and very poor performance when solving VNMP-O. The main problems where high memory consumption and insufficient detection of inconsistent partial assignments. The Column Generation approach was dismissed, since it offered no advantages over a compact ILP formulation for the VNMP while substantially increasing the required run-time.

The other algorithms were more successful. Due to the performance of ILP-S, VNMP-S can be considered solved, at least for the instance types and sizes used in this work. ILP-S was also surprising because of its extremely low run-time requirements for an exact method. Just for one single instance from the VNMP instance set ILP-S failed to find a valid solution due to memory requirements. In fact, without the memory limit, ILP-S is able to solve this instance within the time-limit. Among the tested heuristic methods, VND-S showed the best performance for solving VNMP-S. This algorithm is able to solve all but the instances from the largest two size classes. In addition, the results it creates are very close to validity.

When considering the VNMP-O, the overall best performing algorithm in terms of solution quality is ILP-O. We could demonstrate the importance of our developed preprocessing algorithm, which reduces the required run-time and memory for solving VNMP instances significantly. However, the dominance of ILP-O is not as complete as ILP-S's on VNMP-S. For the largest instances, VND-O produces the best results. For the most challenging instances, ILP-S is the best algorithms since it is the only one capable of finding valid solutions. Even though ILP-O produces the best results, the heuristic methods, especially MA-O and VNS-O are very close. On average, the solutions found by ILP-O cost about 5% less than those of the best heuristic methods.

13.8 Future Work

As the previous chapters already contained remarks on promising research avenues for each of the investigated subtopics, we will focus here on those ideas which are supported by data from this chapter. At the end, we present directions for future work on Virtual Network Mapping. One major area for future improvement is the performance of the heuristic approaches for the larger instance sizes and especially in cases of high load. We have seen that the load of an instance has a far more pronounced influence on the results achieved by a heuristic than the instance size. The first step for doing this is to incorporate the information derived by preprocessing, similar to what we did for the exact methods. Especially the defined neighborhood structures can be refined. For instance, it does not make sense to remove the implementation of a virtual arc from a substrate node if we know that the virtual arc has to use that substrate node. We have also seen an indication that the neighborhoods become too large when the instance size larger than 100 nodes, the problem lies in the much increased mapping possibilities. Methods that do not slow to a crawl when confronted with a high number of mapping possibilities need to be developed.

We have shown that ILP-S is excellent for solving the VNMP-S, but the created solutions leave something to be desired in terms of C_u . Combined with one of the presented heuristic methods, we could create a method that can basically guarantee that it will find a valid solution to a VNMP instance (if it exists) and that the solution will be close to optimal.

If the utilization of an ILP approach is not desirable, another approach to increase the capability of the heuristics to find valid solutions could be to solve VNMP instances incrementally. That means, we first solve the instance with load 0.1 and once a valid solution has been found, we add further virtual networks until the target load is reached.

In this work, we have considered the offline variant of the VNMP and built a solid foundation for future work on this topic. The following lists what we believe are core research directions for the VNMP:

Online VNMP: As we have outlined in Chapter 4, most work on the VNMP considers the online variant of VNMP in the sense that virtual networks arrive and depart. The performance of the algorithms presented in this work needs to be evaluated in such a setting. The required run-time will be of prime importance as it determines how fast virtual networks can be accepted. However, arriving and departing virtual networks are only a part of the possible dynamic behaviours of the VNMP. Virtual networks may change their requirements, for instance if more people connect to them. At the very least, bandwidth requirements will increase and the behaviour of the presented algorithms needs to be studied when such small scale changes occur. The substrate network is also dynamic and can change in structure, for example if leasing resources somewhere else in the physical network gets cheaper, or if hardware fails. One interesting question would be how long it takes to modify an existing solution, such that it does not use a particular substrate node or arc any longer. Models for the cost of changing a virtual network configuration need to be studied, together with their implications on the performance and complexity of algorithms solving the VNMP.

- **Distributed VNMP:** Current models of the VNMP assume global knowledge of the substrate network. However, this is not possible for networks on the scale of planets, so algorithms that work in a distributed fashion need to be studied further. The methods presented in this work could be used as efficient solvers for parts of the network where global knowledge is feasible.
- **Stochastic VNMP:** We have already stated that the whole system of network virtualization is very dynamic. This can be handled in two ways, we could solve the problem again when something changes, or we could add the possibility of change to the model of the VNMP. The first approach is the online VNMP. Here, we deal with the second one. As an example, instead of assuming that a virtual network has a constant requirement for bandwidth, we could model it using a suitable statistical distribution depending on the application. Instead of a solution that does not exceed the available resource capacities, we would search for a solution with a probability of capacity violations below a certain threshold.
- **Nonlinear VNMP:** Models of the VNMP usually do not consider non-linear behaviour in the network. As an example, the observed delay of a link in the physical network heavily depends on the amount of data that is transferred across this link. This dependency is highly non-linear, especially when a link's capacity limit is approached. Furthermore, virtual nodes are not free of overhead and mapping multiple virtual nodes on a substrate node should at least cause some additional CPU load. Algorithms able to cope with non-linear behaviour, or which are at least able to approximate it, need to be studied.
- **Split-Flow VNMP:** A major assumption of the definition of VNMP in this work is, that a virtual connection can only be implemented by using a single path in the substrate network. Doing otherwise would cause erratic behaviour with respect to transmission delays on the virtual link. However, a lot of applications are not delay sensitive or can deal with high delay variances. In addition, allowing multiple paths simplifies the VNMP. There are some interesting possibilities that arise when multiple paths are allowed. For instance, virtual links can be implemented that have more capacity than any single connection in the substrate network. Depending on the application, it might be possible to split up the workload of a single virtual node and spread it across multiple substrate nodes.
- **Physical VNMP:** At the moment, the behaviour of algorithms for solving the VNMP is only evaluated theoretically. There are no experiments in this work where the developed algorithms are used to perform Virtual Network Mapping in a real network, or at least a simulated network. Such experiments would be highly informative, as they can give hints which aspects of the VNMP model should be refined to get it close enough to real world behaviour. For instance, it could turn out that the behaviour of a physical network with non-linearities is not so far removed from the assumed linear behaviour as to make the investigation of algorithms capable of dealing with non-linearities absolutely essential.

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APPENDIX A

Solutions in Detail

Here, we present the detailed results achieved by CH-O, CH-R, CH-S, LS-O, LS-S, VND-O, VND-S, GRASP-O, MA-O, VNS-O, CP-O, CP-S, ILP-O, and ILP-S for each individual instance of the VNMP instance set, which is the basis for the analysis carried out in the previous chapters. This data allows for comparisons with future algorithms solving the VNMP and is also available online [87]. We report the additional resource cost C_a and substrate usage cost C_u of the final VNMP solution. The required run-time is labeled by t[s]. For the ILP methods, we also report the final gaps in percent. The gap is 100% if the LP relaxation of the root node could not be solved. For the ILP and CP methods, column Opt shows if the optimality of the solution could be proven. Note that for ILP-S, any valid solution is optimal. Values may be missing due to early termination because of the applied memory limit. Table A.1 gives an overview where the results can be found.

Methods	Size	Nr. 0–14	Nr. 15–29	Size	Nr. 0–14	Nr. 15–29
	20	A.2	A.3	30	A.4	A.5
CH-O, CH-R, CH-S,	50	A.6	A.7	100	A.8	A.9
LS-O, LS-S	200	A.10	A.11	500	A.12	A.13
	1000	A.14	A.15			
VND-O, VND-S,	20	A.16	A.17	30	A.18	A.19
GRASP-O, MA-O,	50	A.20	A.21	100	A.22	A.23
VNS-O	200	A.24	A.25	500	A.26	A.27
VIN3-0	1000	A.28	A.29			
CP-O, CP-S	20	A.30	A.31	30	A.32	A.33
CF-0, CF-3	50	A.34	A.35	100	A.36	A.37
	20	A.38	A.39	30	A.40	A.41
ILP-O. ILP-S	50	A.42	A.43	100	A.44	A.45
ILF-0, ILF-5	200	A.46	A.47	500	A.48	A.49
	1000	A.50	A.51			

Table A.1: Tables showing the achieved results for the VNMP instance set, depending on employed methods, the instance size, and the instance number (Nr.)

Table A.2: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 20, instance numbers (Nr.) 0–14.

			1	сн-о			CH-R			CH-S			LS-O			LS-S	
Sz.	Nr.	Load	C_a	C_u	t[s]	C_{a}	Cu	t[s]	Ca	C_u	t[s]	C_{a}	C_u	t[s]	C_{a}	C_u	t[s]
20	0	0.10	0	572	0.0	0	607	0.0	0	595	0.0	0	513	0.0	0	560	0.0
		0.50	0	1063	0.0	0	921	0.0	0	1048	0.0	0	829	0.2	0	931	0.0
		0.80	0	1223	0.0	0	1161	0.0	0	1284	0.0	0	1107	0.2	0	1223	0.0
		1.00	625	1382	0.0	520	1224	0.0	0	1382	0.0	0	1184	0.3	0	1336	0.0
	1	0.10	0	377	0.0	0	474	0.0	0	377	0.0	0	361	0.0	0	377	0.0
		0.50	0	791	0.0	545	1017	0.0	0	910	0.0	0	737	0.1	0	791	0.0
		0.80 1.00	0	1040 1168	0.0	695 1626	980 1003	0.0	0	1030 1153	0.0	0 113	804 994	0.2	0	958 1139	0.0
	2	0.10	0	511	0.0	1020	604	0.0	0	511	0.0	0	484	0.5	0	511	0.0
	2	0.10	0	960	0.0	0	1017	0.0	0	1032	0.0	0	791	0.0	0	960	0.0
		0.80	0	1297	0.0	0	1106	0.0	0	1388	0.0	0	972	0.1	0	1249	0.0
		1.00	0	1470	0.0	430	1431	0.0	0	1623	0.0	0	1201	0.2	0	1360	0.0
	3	0.10	0	739	0.0	0	553	0.0	0	798	0.0	0	535	0.0	0	698	0.0
	0	0.50	0	1147	0.0	0	1018	0.0	0	1147	0.0	0	960	0.1	0	1147	0.0
		0.80	0	1309	0.0	535	1235	0.0	0	1397	0.0	0	1107	0.1	0	1271	0.0
		1.00	0	1582	0.0	947	1437	0.0	0	1550	0.0	0	1380	0.2	0	1474	0.0
	4	0.10	0	543	0.0	0	589	0.0	0	543	0.0	0	506	0.0	0	511	0.0
		0.50	0	1048	0.0	0	914	0.0	0	1048	0.0	0	818	0.1	0	1011	0.0
		0.80	0	1248	0.0	0	1090	0.0	0	1350	0.0	0	989	0.2	0	1248	0.0
		1.00	658	1399	0.0	385	1236	0.0	146	1488	0.0	0	1124	0.4	0	1399	0.1
	5	0.10	0	464	0.0	0	486	0.0	0	530	0.0	0	401	0.0	0	464	0.0
		0.50	0	978	0.0	105	1020	0.0	0	1116	0.0	0	832	0.1	0	978	0.0
		0.80	0	1166	0.0	0	1018	0.0	0	1245	0.0	0	888	0.2	0	1131	0.0
		1.00	0	1177	0.0	0	1134	0.0	0	1311	0.0	0	1057	0.1	0	1142	0.0
	6	0.10	0	493	0.0	0	389	0.0	0	493	0.0	0	389	0.0	0	479	0.0
		0.50	0	1143	0.0	0	963	0.0	0	1280	0.0	0	860	0.1	0	1071	0.0
		0.80	0	1406	0.0	0	1267	0.0	0	1422	0.0	0	1229	0.1	0	1406	0.0
	7	1.00	425	1422	0.0	0	1422	0.0	0	1446	0.0	0	1422	0.1	0	1422	0.0
	7	0.10	0	831	0.0	0	573	0.0	0	819	0.0	0	551	0.0	0	725	0.0
		0.50 0.80	0	1289 1418	0.0	0 225	1069 1285	0.0	0	1334 1506	0.0	0	982 1237	0.1 0.2	0	1207 1418	0.0
		1.00	0	1590	0.0	965	1285	0.0	155	1630	0.0	0	1461	0.2	0	1506	0.0
	8	0.10	0	656	0.0	0	418	0.0	0	716	0.0	0	351	0.0	0	559	0.0
	0	0.50	0	1092	0.0	0	884	0.0	0	1219	0.0	0	841	0.1	0	1049	0.0
		0.80	0	1171	0.0	0	1162	0.0	0	1378	0.0	0	1070	0.2	0	1214	0.0
		1.00	0	1442	0.0	0	1326	0.0	55	1442	0.0	0	1273	0.1	0	1353	0.0
	9	0.10	0	782	0.0	0	672	0.0	0	754	0.0	0	643	0.0	0	753	0.0
		0.50	0	1094	0.0	0	1129	0.0	0	1121	0.0	0	1011	0.1	0	1059	0.0
		0.80	0	1163	0.0	0	1392	0.0	0	1254	0.0	0	1106	0.1	0	1163	0.0
		1.00	305	1455	0.0	572	1445	0.0	0	1560	0.0	0	1287	0.3	0	1355	0.0
	10	0.10	0	660	0.0	0	722	0.0	0	717	0.0	0	598	0.0	0	660	0.0
		0.50	0	1093	0.0	0	1124	0.0	0	1236	0.0	0	923	0.1	0	1027	0.0
		0.80	0	1374	0.0	0	1459	0.0	50	1454	0.0	0	1252	0.1	0	1302	0.0
		1.00	1035	1538	0.0	825	1502	0.0	65	1538	0.0	1035	1502	0.1	1035	1538	0.0
	11	0.10	0	308	0.0	0	272	0.0	0	362	0.0	0	217	0.0	0	308	0.0
		0.50	0	933	0.0	0	793	0.0	0	997	0.0	0	638	0.1	0	829	0.0
		0.80 1.00	0	1077 1077	0.0	0	839 1023	0.0	0 145	1077 1224	0.0	0	977 942	0.1	0	1077 997	0.0
	12	0.10	0	573	0.0	0	650	0.0	145	573	0.0	0	557	0.1	0	573	0.0
	12	0.10	70	1153	0.0	235	1004	0.0	0	1236	0.0	0	1022	0.0	0	1121	0.0
		0.80	625		0.0	700	1173	0.0	0		0.0	0	1137	0.4	0	1407	0.0
		1.00	790	1607	0.0	3302	1343	0.0	0	1613	0.0	0	1238	0.4	0	1476	0.0
	13	0.10	0	638	0.0	0	802	0.0	0	644	0.0	0	480	0.0	0	621	0.0
		0.50	0	900	0.0	0	1004	0.0	0	972	0.0	0	733	0.1	0	888	0.0
		0.80	0	1267	0.0	530	1279	0.0	0	1294	0.0	0	1109	0.2	0	1277	0.0
		1.00	50	1293	0.0	1320	1338	0.0	0	1338	0.0	0	1224	0.2	0	1330	0.0
	14	0.10	0	636	0.0	0	681	0.0	0	636	0.0	0	535	0.0	0	576	0.0
		0.50	0	1195	0.0	0	977	0.0	0	1203	0.0	0	828	0.1	0	1195	0.0
		0.80	0	1310	0.0	0	1231	0.0	0	1310	0.0	0	1214	0.2	0	1310	0.0
		1.00	250	1344	0.0	2405	1301	0.0	0	1456	0.0	250	1301	0.1	0	1344	0.0

Table A.3: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 20, instance numbers (Nr.) 15–29.

				сн-о			CH-R		l	CH-S			LS-O		l	LS-S	
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	Cu	t[s]	C_{a}	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	C_{a}	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]
20	15	0.10	0	430	0.0	0	539	0.0	0	524	0.0	0	424	0.0	0	524	0.0
		0.50	0	905	0.0	0	1057	0.0	0	980	0.0	0	722	0.1	0	863	0.0
		0.80	0	1094	0.0	0	964	0.0	0	1140	0.0	0	964	0.2	0	1140	0.0
		1.00	0	1299	0.0	354	1169	0.0	0	1474	0.0	0	1029	0.3	0	1224	0.0
	16	0.10	0	931	0.0	0	837	0.0	0	1092	0.0	0	736	0.0	0	961	0.0
		0.50	0	1373	0.0	0	1204	0.0	0	1398	0.0	0	895	0.3	0	1273	0.0
		0.80	0 2575	1542 1564	0.0	0 2363	1415 1543	0.0	0	1564 1584	0.0 0.0	0	1227 1285	0.4	0	1462 1564	0.0 0.0
	17	1.00 0.10	2373	563	0.0	2303	583	0.0	0	563	0.0	0	453	0.4	0	549	0.0
	17	0.10	0	1061	0.0	0	1132	0.0	0	1136	0.0	0	982	0.0	0	1061	0.0
		0.80	0	1356	0.0	0	1311	0.0	0	1489	0.0	0	1169	0.1	0	1299	0.0
		1.00	758	1561	0.0	1048	1560	0.0	129	1623	0.0	0	1297	0.5	0	1561	0.1
	18	0.10	0	528	0.0	0	507	0.0	0	528	0.0	0	439	0.0	0	517	0.0
		0.50	0	1112	0.0	0	920	0.0	0	1179	0.0	0	891	0.1	0	1074	0.0
		0.80	1015	1271	0.0	0	1197	0.0	0	1271	0.0	180	1093	0.2	180	1271	0.1
		1.00	1050	1328	0.0	1734	1262	0.0	0	1374	0.0	940	1157	0.3	325	1328	0.1
	19	0.10	0	643	0.0	0	627	0.0	0	662	0.0	0	584	0.0	0	643	0.0
		0.50	0	1401	0.0	0	1030	0.0	0	1493	0.0	0	1008	0.1	0	1309	0.0
		0.80	0	1500	0.0	845	1405	0.0	0	1535	0.0	0	1226	0.3	0	1491	0.0
		1.00	905	1535	0.0	1118	1473	0.0	0	1573	0.0	0	1318	0.3	0	1535	0.1
	20	0.10	0	396	0.0	0	546	0.0	0	434	0.0	0	396	0.0	0	403	0.0
		0.50	0	1182	0.0	0	739	0.0	0	1182	0.0	0	640	0.1	0	1083	0.0
		0.80	20	1322	0.0	280	892	0.0	0	1322	0.0	0	835	0.2	0	1307	0.0
	~ .	1.00	0	1366	0.0	2568	1102	0.0	0	1391	0.0	0	1008	0.3	0	1366	0.0
	21	0.10	0	484	0.0	0	507	0.0	0	535	0.0	0	475	0.0	0	484	0.0
		0.50	0	898	0.0	0	893	0.0	0	902	0.0	0	758	0.1	0	898	0.0
		0.80	0 421	1052 1300	0.0	0	1050	0.0	0	1221 1360	0.0 0.0	0	952 1097	0.1	0	1052 1268	0.0
	22	1.00 0.10	421	647	0.0	646 0	1158 662	0.0	0	647	0.0	0	646	0.2	0	647	0.1 0.0
	22	0.10	0	748	0.0	0	801	0.0	0	794	0.0	0	747	0.0	0	748	0.0
		0.80	0	1120	0.0	0	898	0.0	0	1110	0.0	0	898	0.1	0	1120	0.0
		1.00	0	1120	0.0	0	905	0.0	0	1171	0.0	0	905	0.5	0	1127	0.0
	23	0.10	0	612	0.0	0	593	0.0	0	754	0.0	0	535	0.0	0	591	0.0
		0.50	0	1180	0.0	0	919	0.0	0	1256	0.0	0	779	0.1	0	1039	0.0
		0.80	0	1490	0.0	0	1066	0.0	0	1500	0.0	0	990	0.2	0	1390	0.0
		1.00	0	1515	0.0	0	1338	0.0	0	1564	0.0	0	1131	0.2	0	1415	0.0
	24	0.10	0	716	0.0	0	767	0.0	0	716	0.0	0	683	0.0	0	716	0.0
		0.50	0	1044	0.0	0	910	0.0	0	1128	0.0	0	910	0.1	0	930	0.0
		0.80	0	1184	0.0	0	1153	0.0	0	1247	0.0	0	1153	0.1	0	1184	0.0
		1.00	0	1258	0.0	283	1226	0.0	0	1395	0.0	0	1257	0.2	0	1257	0.0
	25	0.10	0	525	0.0	0	533	0.0	0	568	0.0	0	432	0.0	0	525	0.0
		0.50	0	955	0.0	0	1058	0.0	0	990	0.0	0	827	0.1	0	902	0.0
		0.80	0	1195	0.0	345	1164	0.0	0	1394	0.0	0	1043	0.1	0	1131	0.0
	26	1.00	0	1481	0.0	30	1214	0.0	0	1522	0.0	0	1043	0.3	0	1357	0.0
	26	0.10 0.50	0	559 1024	0.0 0.0	0	508 946	0.0	0	559 1058	0.0 0.0	0	420 937	0.0	0	527 1022	0.0 0.0
		0.80	0	1024	0.0	0	1069	0.0	0	1101	0.0	0	1024	0.1	0	1022	0.0
		1.00	730	1240	0.0	0	1112	0.0	0	1213	0.0	0	1112	0.1	0	1240	0.0
	27	0.10	0	687	0.0	0	699	0.0	0	687	0.0	0	607	0.0	0	654	0.0
	27	0.50	0	1086	0.0	0	1271	0.0	0	1347	0.0	0	901	0.1	0	1086	0.0
		0.80	0	1284	0.0	0	1461	0.0	0	1538	0.0	0	1068	0.3	0	1286	0.0
		1.00	590	1446	0.0	385	1511	0.0	0	1538	0.0	385	1233	0.4	0	1538	0.0
	28	0.10	0	568	0.0	0	531	0.0	0	630	0.0	0	552	0.0	0	568	0.0
	-	0.50	0	1249	0.0	0	1225	0.0	0	1247	0.0	0	1034	0.1	0	1187	0.0
		0.80	0	1367	0.0	0	1347	0.0	0	1576	0.0	0	1243	0.2	0	1368	0.0
		1.00	315	1581	0.0	0	1409	0.0	385	1581	0.0	315	1390	0.2	30	1436	0.1
	29	0.10	0	460	0.0	0	431	0.0	0	460	0.0	0	387	0.0	0	460	0.0
		0.50	0	1029	0.0	175	866	0.0	0	1050	0.0	0	593	0.1	0	1040	0.0
		0.80	0	1249	0.0	40	1079	0.0	0	1285	0.0	0	886	0.2	0	1192	0.0
		1.00	97	1377	0.0	333	1326	0.0	0	1433	0.0	0	1334	0.1	0	1377	0.0

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Table A.4: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 30, instance numbers (Nr.) 0–14.

				сн-о			CH-R			CH-S			LS-O			LS-S	
Sz.	Nr.	Load	C_a	C_u	t[s]	C_a	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]
30	0	0.10	0	633	0.0	0	641	0.0	0	633	0.0	0	611	0.0	0	619	0.0
		0.50	0	1332	0.0	0	1289	0.0	0	1327	0.0	0	1036	0.2	0	1287	0.0
		0.80	0	1509	0.0	0	1560	0.0	0	1714	0.0	0	1454	0.3	0	1553	0.0
	1	1.00	0	1711 793	0.0	905	1882	0.0	0	2091 932	0.0	0	1586	0.3	0 0	1716	0.0
	1	0.10 0.50	0	1470	0.0	0	887 1539	0.0	0	952 1789	0.0 0.0	0	713 1149	0.1 0.5	0	876 1649	0.0 0.0
		0.30	0	1951	0.0	2808	1927	0.0	0	2382	0.0	0	1510	0.5	0	2077	0.0
		1.00	4041	2330	0.0	4782	2067	0.0	0	2474	0.0	0	1689	1.3	0	2457	0.1
	2	0.10	0	1288	0.0	0	1028	0.0	0	1467	0.0	0	860	0.1	0	1250	0.0
		0.50	0	1818	0.0	0	1634	0.0	0	2135	0.0	0	1628	0.3	0	1925	0.0
		0.80	0	2176	0.0	0	1866	0.0	0	2384	0.0	0	1810	0.6	0	2210	0.1
		1.00	233	2288	0.0	3102	2084	0.0	0	2546	0.0	0	2216	0.4	0	2432	0.1
	3	0.10	0	1000	0.0	0	701	0.0	0	1000	0.0	0	679	0.0	0	844	0.0
		0.50	0	1347	0.0	0	1177	0.0	0	1558	0.0	0	1095	0.2	0	1365	0.0
		0.80	565	1879	0.0	0	1234	0.0	0	1871	0.0	0	1203	0.7	0	1815	0.0
		1.00	1636	2096	0.0	1467	1869	0.0	0	2162	0.0	0	1745	1.3	0	2110	0.1
	4	0.10	0	1036	0.0	0	979	0.0	0	1024	0.0	0	701	0.1	0	994	0.0
		0.50	0	1610	0.0	0	1563	0.0	0	1733	0.0	0	1376	0.4	0	1602	0.0
		0.80	0	1696	0.0	80	1595	0.0	0	1838	0.0	0	1436	0.6	0	1688	0.0
	Ę	1.00	0	1788	0.0	385	1938	0.0	0	2219	0.0	0	1647	0.5	0	1807	0.1
	5	0.10	0	737 1429	0.0	0	641 1163	0.0	0	801 1480	0.0 0.0	0	612 1105	0.0 0.4	0 0	712 1426	0.0 0.0
		0.50 0.80	0	1649	0.0	0	1588	0.0	0	1969	0.0	0	1402	0.4	0	1614	0.0
		1.00	0	1906	0.0	657	1618	0.0	0	2002	0.0	0	1462	0.0	0	1774	0.0
	6	0.10	0	853	0.0	0.57	786	0.0	0	971	0.0	0	624	0.1	0	899	0.0
	0	0.50	0	1467	0.0	0	1414	0.0	0	1637	0.0	0	1206	0.4	0	1515	0.0
		0.80	330	1796	0.0	145	1642	0.0	Ő	1824	0.0	0	1333	0.7	0	1755	0.0
		1.00	5	1779	0.0	5376	1769	0.0	0	2051	0.0	0	1544	0.6	0	1864	0.1
	7	0.10	0	897	0.0	0	769	0.0	0	1002	0.0	0	684	0.0	0	768	0.0
		0.50	0	1589	0.0	0	1355	0.0	0	1661	0.0	0	1226	0.5	0	1596	0.0
		0.80	0	1834	0.0	0	1686	0.0	0	2031	0.0	0	1510	0.8	0	1965	0.1
		1.00	415	2296	0.0	3097	1948	0.0	0	2362	0.0	0	1752	1.3	0	2285	0.1
	8	0.10	0	1489	0.0	0	1275	0.0	0	1561	0.0	0	1192	0.1	0	1375	0.0
		0.50	0	1659	0.0	0	1457	0.0	0	1746	0.0	0	1269	0.6	0	1541	0.0
		0.80	0	1758	0.0	0	1761	0.0	0	1773	0.0	0	1629	0.6	0	1746	0.1
	9	1.00	0	1758 743	0.0	200 0	1915 654	0.0	0	1838 822	0.0 0.0	0	1625 571	1.0 0.0	0 0	1773 762	0.1 0.0
	9	0.10 0.50	0	1638	0.0	0	1423	0.0	0	1924	0.0	0	1164	0.0	0	1784	0.0
		0.80	0	2065	0.0	0	1939	0.0	0	2234	0.0	0	1486	0.5	0	2145	0.0
		1.00	0	2242	0.0	678	2179	0.0	0	2382	0.0	0	1788	0.9	0	2280	0.1
	10	0.10	0	610	0.0	0	811	0.0	0	610	0.0	0	593	0.0	0	610	0.0
		0.50	0	1375	0.0	0	1292	0.0	0	1375	0.0	0	1122	0.3	0	1289	0.0
		0.80	0	1662	0.0	0	1436	0.0	0	1717	0.0	0	1259	0.8	0	1538	0.0
		1.00	0	1930	0.0	494	1539	0.0	0	1977	0.0	0	1349	1.3	0	1817	0.1
	11	0.10	0	908	0.0	0	625	0.0	0	1064	0.0	0	597	0.1	0	914	0.0
		0.50	0	1430	0.0	405	1360	0.0	0	1627	0.0	0	1188	0.2	0	1459	0.0
		0.80	140	2088	0.0	3507	2094	0.0	0	2155	0.0	0	1663	0.4	0	1894	0.0
		1.00	3089	2184	0.0	5516	2278	0.0	10	2327	0.0	1481	2043	0.3	0	2184	0.2
	12	0.10	0	754	0.0	0	867	0.0	0	871	0.0	0	754	0.0	0	827	0.0
		0.50	0	1439	0.0	0	1369	0.0	0	1539	0.0	0	1102	0.2	0	1392	0.0
		0.80	0	1923	0.0	125		0.0	0	2081	0.0	0	1434	0.6	0	1821	0.0
	12	1.00	0	2160	0.0	262	1972	0.0	0	2216	0.0	0	1742	0.7	0	2150	0.0
	13	0.10 0.50	0	674 1424	0.0	0	655 1476	0.0	0	698 1523	0.0 0.0	0	542 1102	0.0 0.3	0	617 1386	0.0 0.0
		0.50	0	1424	0.0	422	1476	0.0	0	1921	0.0	0	1102 1517	0.3	0 0	1380	0.0
		1.00	0	1929	0.0	422	1791	0.0	0	2132	0.0	0	1517	0.5	0	1926	0.0
	14	0.10	0	902	0.0	430	940	0.0	0	935	0.0	0	809	0.0	0	884	0.1
	17	0.10	0	1771	0.0	0	1593	0.0	0	1909	0.0	0	1413	0.1	0	1802	0.0
		0.80	0	1825	0.0	0	1631	0.0	0	2065	0.0	0	1490	1.1	0	1923	0.1
		1.00	1875	2019	0.0	3089	1799	0.0	0	2347	0.0	0	1812	1.2	0	2109	0.1
															-		

Table A.5: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 30, instance numbers (Nr.) 15–29.

				сн-о		I	CH-R			CH-S			LS-O			LS-S	
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Ca	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	C_{a}	C_u	t[s]
30	15	0.10	0	1245	0.0	0	1160	0.0	0	1226	0.0	0	1019	0.1	0	1163	0.0
		0.50	0	1633	0.0	0	1582	0.0	0	1633	0.0	0	1298	0.4	0	1623	0.0
		0.80	0	1867	0.0	0	1650	0.0	0	2084	0.0	0	1531	1.1	0	1779	0.1
	16	1.00	0	2304	0.0	623	1901	0.0	0	2304	0.0	0	1773	2.7	0	2206	0.1
	16	0.10 0.50	0	633 1382	0.0	0	534 1196	0.0	0	649 1556	0.0	0	461 997	0.0 0.2	0	589 1279	0.0 0.0
		0.30	0	1777	0.0	1070	1505	0.0	0	1797	0.0	0	1229	0.2	0	1704	0.0
		1.00	289	1924	0.0	1814	1796	0.0	0	1931	0.0	0	1521	1.2	0	1900	0.0
	17	0.10	0	976	0.0	0	999	0.0	0	1065	0.0	0	904	0.1	0	992	0.0
		0.50	0	1392	0.0	0	1412	0.0	0	1427	0.0	0	1132	0.5	0	1370	0.0
		0.80	305	1779	0.0	1725	1654	0.0	0	1972	0.0	0	1463	1.0	0	1772	0.1
		1.00	3630	1901	0.0	933	1711	0.0	0	2053	0.0	0	1604	1.2	0	1922	0.1
	18	0.10	0	579	0.0	0	658	0.0	0	629	0.0	0	561	0.0	0	561	0.0
		0.50	0	1458	0.0	25	1530	0.0	0	1612	0.0	0	1200	0.3	0	1381	0.0
		0.80	0	1831	0.0	727	2061	0.0	0	1928	0.0	0	1534	0.4	0	1724	0.0
	10	1.00	0	2181	0.0	597	2292	0.0	0	2304	0.0	0	1750	0.5	0	2066	0.1
	19	0.10	0	955	0.0	0	810	0.0	0	1005	0.0	0	548	0.0	0	984	0.0
		0.50 0.80	0	1643 1928	0.0	0 270	1614 1820	0.0	0	1833 2136	0.0	0	1432 1570	0.4 0.9	0	1706 1932	0.0 0.0
		1.00	315	2248	0.0	3234	2168	0.0	0	2150	0.0	0	1818	1.2	0	2210	0.0
	20	0.10	0	1195	0.0	0	1165	0.0	0	1222	0.0	0	941	0.1	0	1050	0.0
	20	0.50	0	1771	0.0	0	1513	0.0	0	1807	0.0	0	1490	0.4	0	1761	0.0
		0.80	0	2011	0.0	0	1710	0.0	0	1985	0.0	0	1686	1.0	0	1973	0.0
		1.00	0	2129	0.0	38	1885	0.0	0	2200	0.0	0	1842	1.4	0	2129	0.1
	21	0.10	0	897	0.0	0	684	0.0	0	878	0.0	0	640	0.0	0	812	0.0
		0.50	0	1283	0.0	0	1243	0.0	0	1332	0.0	0	1010	0.1	0	1283	0.0
		0.80	0	1636	0.0	0	1629	0.0	0	1769	0.0	0	1460	0.3	0	1565	0.0
		1.00	46	1875	0.0	1314	1887	0.0	75	2012	0.0	41	1743	0.3	0	1875	0.1
	22	0.10	0	968	0.0	0	1013	0.0	0	1003	0.0	0	929	0.0	0	968	0.0
		0.50	0	1554	0.0	0	1442	0.0	0	1706	0.0	0	1400	0.3	0	1479	0.0
		0.80	75 529	1813 2249	0.0	0	1693 1757	0.0	0	2048 2292	0.0	0	1491 1631	0.6 1.1	0	1727 2110	0.1 0.1
	23	1.00 0.10	0	766	0.0	0	1213	0.0	0	876	0.0	0	738	0.0	0	841	0.1
	25	0.10	0	1589	0.0	0	1582	0.0	0	1949	0.0	0	1214	0.5	0	1643	0.0
		0.80	115	2240	0.0	40	1974	0.0	0	2300	0.0	0	1492	1.0	0	2135	0.1
		1.00	270	2410	0.0	785	2360	0.0	0	2380	0.0	0	1745	1.0	0	2184	0.1
	24	0.10	0	626	0.0	0	571	0.0	0	630	0.0	0	564	0.0	0	626	0.0
		0.50	0	1398	0.0	10	1498	0.0	0	1799	0.0	0	1278	0.3	0	1479	0.0
		0.80	0	1936	0.0	0	1862	0.0	0	2187	0.0	0	1700	0.6	0	2155	0.0
		1.00	309	2234	0.0	5062	2445	0.0	0	2514	0.0	0	2139	0.6	0	2349	0.1
	25	0.10	0	1123	0.0	0	1381	0.0	0	1222	0.0	0	972	0.1	0	1187	0.0
		0.50	0	1645	0.0	0	1709	0.0	0	1886	0.0	0	1533	0.5	0	1694	0.0
		0.80 1.00	0 1370	2029 2095	0.0	615 6659	1872 2057	0.0	0	2147 2310	0.0	0	1587 1722	0.9 1.5	0	1925 2125	0.1 0.1
	26	0.10	1370	750	0.0	0039	528	0.0	0	763	0.0	0	518	0.0		763	0.1
	20	0.10	0	1479	0.0	0	1068	0.0	0	1528	0.0	0	1009	0.0	0	1410	0.0
		0.80	345	1941	0.0	590	1736	0.0	0	1938	0.0	0	1364	0.5	0	1825	0.0
		1.00	4841	2047	0.0	7421	2034	0.0	0	2143	0.0	Ő	1554	0.5	0	2024	0.1
	27	0.10	0	773	0.0	0	651	0.0	0	751	0.0	0	561	0.0	0	638	0.0
		0.50	0	1396	0.0	0	1174	0.0	0	1504	0.0	0	1106	0.2	0	1267	0.0
		0.80	0	1699	0.0	0	1665	0.0	0	1958	0.0	0	1407	0.6	0	1659	0.0
		1.00	0	1968	0.0	686	1910	0.0	0	2057	0.0	0	1517	0.9	0	1919	0.1
	28	0.10	0	1031	0.0	0	1131	0.0	0	1073	0.0	0	962	0.0	0	1006	0.0
		0.50	0	1422	0.0	0	1457	0.0	0	1619	0.0	0	1238	0.1	0	1422	0.0
		0.80	0	1769	0.0	0	1555	0.0	0	2033	0.0	0	1553	0.3	0	1671	0.1
	20	1.00	0	2017	0.0	20	1800	0.0	0	2229	0.0	0	1608	0.7		1914	0.1
	29	0.10 0.50	0	872 1524	0.0	0	495 1508	0.0	0	935 1668	0.0	0	415 1090	0.1 0.2	0	798 1445	0.0 0.0
		0.30	0	1858	0.0	0	1771	0.0	0	1944	0.0	0	1437	0.2	0	1761	0.0
		1.00	715	1920	0.0	0	2067	0.0	0	1944	0.0	0	1743	0.4	0	1920	0.0
				. =												. = .	

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Table A.6: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 50, instance numbers (Nr.) 0–14.

Sz. N 50	Nr. 0	Load	C_{a}	C													
50	0		\sim_a	C_u	t[s]	C_a	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	C_{a}	C_u	t[s]
		0.10	0	1313	0.0	0	1291	0.0	0	1311	0.0	0	1020	0.1	0	1289	0.0
		0.50	0	2410	0.0	0	2522	0.0	0	2571	0.0	0	1872	1.5	0	2411	0.1
		0.80	0	2967	0.0	4241	2977	0.0	0	3275	0.0	0	2445	2.8	0	2991	0.1
		1.00	7008	3614	0.1	18804	3663	0.0	0	3859	0.0	2640	3240	4.0	0	3731	1.3
	1	0.10	0	1472	0.0	0	1449	0.0	0	1638	0.0	0	1340	0.1	0	1470	0.0
		0.50	0	2262	0.0	0	2246	0.0	0	2395	0.0	0	1826	0.7	0	2208	0.0
		0.80	0	2720	0.0	13	2643	0.0	0	3180	0.0	0	2088	2.2	0	2937	0.1
		1.00	757	3669	0.0	5574	3489	0.0	0	3756	0.0	0	2634	2.8	0	3678	0.1
	2	0.10	0	1126	0.0	0	1071	0.0	0	1175	0.0	0	749	0.1	0	1069	0.0
		0.50	0	2453	0.0	0	2474	0.0	0	2610	0.0	0	2043	0.5	0	2372	0.1
		0.80	0	3002	0.0	0	2862	0.0	0	3116	0.0	0	2373	1.1	0	2915	0.1
	2	1.00	1148	3097	0.0	1287	3273	0.0	0	3386	0.0	520	2668	1.6	95	3049	0.3
	3	0.10	0	1410	0.0	0	1444	0.0	0	1588	0.0	0	1211	0.1	0	1512	0.0
		0.50	0	2602	0.0	0 0	2315	0.0	0	2957	0.0	0	1905	1.4 2.4	0	2575	0.1
		0.80 1.00	958	3282 3633	0.0 0.0	5673	2536 3013	0.0	0	3416 3776	0.0 0.0	0	2370 2502	4.0	0	3223 3573	0.1
	4	0.10	938	1313	0.0	0	1300	0.0	0	1360	0.0	0	1194	0.1	0	1317	0.1
	4	0.10	0	2232	0.0	0	2275	0.0	0	2303	0.0	0	1904	0.1	0	2175	0.0
		0.80	0	2604	0.0	0	2746	0.0	0	2750	0.0	0	2188	1.8	0	2560	0.1
		1.00	218	3106	0.0	2794	3124	0.0	0	3219	0.0	0	2723	2.2	0	3124	0.1
5	5	0.10	0	1397	0.0	0	1376	0.0	0	1470	0.0	0	1280	0.1	0	1431	0.0
	5	0.50	Ő	2143	0.0	0	1888	0.0	0	2290	0.0	0	1664	1.2	0	2111	0.0
		0.80	0	2676	0.0	0	1989	0.0	0	2837	0.0	0	2031	2.7	0	2720	0.1
		1.00	0	2907	0.0	616	2406	0.0	Õ	3172	0.0	0	2221	4.0	0	2831	0.1
	6	0.10	0	900	0.0	145	1205	0.0	0	947	0.0	0	825	0.1	0	911	0.0
		0.50	0	2096	0.0	30	2500	0.0	0	2431	0.0	0	1732	0.8	0	2064	0.0
		0.80	100	2727	0.0	385	2655	0.0	0	2988	0.0	0	2085	2.3	0	2640	0.1
		1.00	370	3012	0.0	4080	3132	0.0	0	3264	0.0	0	2460	3.0	0	2954	0.1
	7	0.10	0	1152	0.0	0	1617	0.0	0	1227	0.0	0	1129	0.1	0	1198	0.0
		0.50	0	2351	0.0	1387	2558	0.0	0	2513	0.0	0	2010	1.2	0	2268	0.1
		0.80	40	2746	0.0	905	2823	0.0	0	2731	0.0	0	2464	1.4	0	2653	0.1
		1.00	360	3060	0.0	1674	3192	0.1	0	3412	0.0	0	2643	3.3	0	3086	0.1
	8	0.10	0	1053	0.0	130	1224	0.0	0	1194	0.0	0	946	0.1	0	1097	0.0
		0.50	0	2202	0.0	190	2219	0.0	0	2337	0.0	0	1619	0.8	0	2159	0.1
		0.80	0	2732	0.0	200	2609	0.0	0	3180	0.0	0	2127	2.2	0	2824	0.1
		1.00	7306	3537	0.0	6770	3608	0.0	0	3873	0.0	0	2956	3.2	0	3648	0.8
	9	0.10	0	1623	0.0	0	1566	0.0	0	1736	0.0	0	1392	0.2	0	1623	0.0
		0.50	0	2369	0.0	0	2263	0.0	0	2370	0.0	0	1818	1.1	0	2229	0.1
		0.80	0	2683	0.0	0	2542	0.0	0	2803	0.0	0	2076	2.1	0	2596	0.1
	10	1.00 0.10	0	3144 1079	0.0 0.0	1251 0	3038 1256	0.0	0	3438 1094	0.0 0.0	0	2404 988	4.7 0.1	0	3122 1046	0.1
	10	0.10	0	2677	0.0	0	2346	0.0	0	2894	0.0	0	1862	1.4	0	2637	0.0
		0.80	0	3001	0.0	20	2780	0.0	0	3284	0.0	0	2165	2.1	0	2908	0.1
		1.00	1657	3380	0.0	3819	3073	0.0	0	3840	0.0	0	2726	3.6	0	3330	0.1
1	11	0.10	0	1380	0.0	0	1249	0.0	0	1402	0.0	0	978	0.2	0	1336	0.0
		0.50	Ő	2498	0.0	45	2110	0.0	0	2507	0.0	0	1703	0.9	0	2375	0.0
		0.80	400	3011	0.0	17	2928	0.0	0	3261	0.0	0	2353	1.6	0	2941	0.1
		1.00	3373	3428	0.0	4288	3412	0.0	Õ	3671	0.0	585	3129	2.2	585	3204	0.8
	12	0.10	0	1042	0.0	0	981	0.0	0	1098	0.0	0	889	0.1	0	1026	0.0
		0.50	0	2326	0.0	0	2049	0.0	0	2493	0.0	0	1837	1.1	0	2343	0.1
		0.80	0	2600	0.0	0	2647	0.0	0	2827	0.0	0	2364	1.1	0	2598	
		1.00	2609	2810	0.0	4501	2865	0.0	25	3174	0.0	0	2682	1.4	0	2810	0.5
	13	0.10	0	1375	0.0	0	1312	0.0	0	1438	0.0	0	1086	0.1	0	1358	0.0
		0.50	0	2247	0.0	0	2132	0.0	0	2303	0.0	0	1897	0.9	0	2222	0.0
		0.80	0	3116	0.0	105	2728	0.0	0	3175	0.0	0	2389	2.1	0	2995	0.1
		1.00	5020	3454	0.0	4965	3490	0.0	80	3495	0.0	0	3190	3.1	0	3428	0.8
	14	0.10	0	1278	0.0	0	1360	0.0	0	1286	0.0	0	1096	0.1	0	1250	0.0
		0.50	0	2144	0.0	0	2098	0.0	0	2202	0.0	0	1859	0.8	0	2149	0.0
		0.80	0	2606	0.0	38	2458	0.0	0	2673	0.0	0	2096	2.7	0	2562	0.1
		1.00	5	2724	0.0	544	2575	0.0	0	2918	0.0	0	2283	3.0	0	2665	0.1

Table A.7: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 50, instance numbers (Nr.) 15–29.

			(СН-О			CH-R			CH-S			LS-O			LS-S	
Sz.	Nr.	Load	C_a	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]
50	15	0.10	0	1396	0.0	0	1297	0.0	0	1449	0.0	0	1161	0.1	0	1398	0.0
		0.50	0	2618	0.0	55	2688	0.0		2775	0.0	0	2091	1.2	0	2577	0.1
		0.80	275	3074	0.1	480	2997	0.0	0	3137	0.0	0	2278	3.1	0	3029	0.1
	16	1.00 0.10	4494	3581 1305	0.0 0.0	2336 0	3397 1549	0.1	0	3770 1369	0.0	0 0	3227 1180	3.6 0.1	0	3689 1295	0.2
	10	0.10	0	2234	0.0	0	2237	0.0	0	2326	0.0	0	1509	1.9	0	2151	0.0
		0.80	0	2682	0.0	1529	2754	0.0	0	2711	0.0	0	2142	3.1	0	2627	0.1
		1.00	25	3083	0.0	3606	3047	0.0	0	3409	0.0	0	2390	4.5	0	3062	0.1
	17	0.10	0	1276	0.0	0	1475	0.0	0	1422	0.0	0	1103	0.1	0	1314	0.0
		0.50	0	2502	0.0	0	2445	0.0	0	2576	0.0	0	2191	0.9	0	2555	0.0
		0.80	0	2945	0.0	0	2670	0.0	0	3249	0.0	0	2388	2.1	0	2991	0.1
		1.00	430	3665	0.0	1667	3124	0.0	0	4059	0.0	0	2755	4.1	0	3763	0.1
	18	0.10	0	1034	0.0	0	995	0.0	0	1093	0.0	0	863	0.1	0	1025	0.0
		0.50	45	2144	0.0	0	2250	0.0	0	2367	0.0	0	1945	0.7	0	2148	0.1
		0.80	0	2937	0.0	0	2545	0.0	0	3089	0.0	0	2433	2.3	0	2994	0.1
		1.00	2401	3438	0.0	7945	3049	0.0	0	3680	0.0	0	2894	3.9	0	3542	0.4
	19	0.10	0	1187	0.0	0	1090	0.0	0	1433	0.0	0	1065	0.1	0	1311	0.0
		0.50	0	2133	0.0	0	1753	0.0	0	2331	0.0	0	1628	1.3	0	2234	0.1
		0.80	0 5028	2769	0.0 0.0	0	2298 2932	0.0 0.0	0	2737 3128	0.0	0	2042	1.9 2.6	0	2605 3012	0.1
	20	1.00 0.10	5028 0	3012 1837	0.0	6512 0	1831	0.0	0	2150	0.0	0	2585 1659	2.0 0.2	0	2027	0.4
	20	0.10	0	2353	0.0	295	2504	0.0	0	2590	0.0	0	2042	0.2	0	2462	0.0
		0.80	0	2838	0.0	1210	3096	0.0	0	2946	0.0	0	2513	1.4	0	2870	0.1
		1.00	2774	3256	0.1	5274	3320	0.1	0	3234	0.0	0	2841	4.3	0	3306	0.1
	21	0.10	0	1482	0.0	0	1523	0.0	0	1489	0.0	0	1210	0.1	Ő	1322	0.0
		0.50	0	2187	0.0	0	2416	0.0	0	2410	0.0	0	1814	1.0	0	2144	0.1
		0.80	10	2817	0.0	230	2906	0.0	0	3225	0.0	0	2431	3.0	0	2949	0.1
		1.00	590	3403	0.0	10	3341	0.0	0	3553	0.0	0	2801	5.1	0	3352	0.1
	22	0.10	0	1532	0.0	0	1602	0.0	0	1853	0.0	0	1377	0.1	0	1684	0.0
		0.50	0	2661	0.0	0	2707	0.0	0	3092	0.0	0	2093	1.5	0	2778	0.1
		0.80	0	3072	0.0	0	3120	0.0	0	3264	0.0	0	2425	2.6	0	3075	0.1
		1.00	79	3334	0.0	3984	3673	0.0	0	3766	0.0	0	2841	3.3	0	3385	0.1
	23	0.10	0	1209	0.0	0	1151	0.0	0	1318	0.0	0	1035	0.1	0	1165	0.0
		0.50	0	2700	0.0	0	2718	0.0	0	2861	0.0	0	2184	1.2	0	2655	0.1
		0.80	0	3100	0.0	450	3071	0.0	0	3272	0.0	0	2484	2.6	0	3060	0.1
	24	1.00 0.10	0	3148 1047	0.0 0.0	557 0	3159 1013	0.0 0.0	0	3378 1137	0.0	0	2680 803	2.5 0.1	0	3185 1057	0.1
	24	0.10	0	2259	0.0	80	2092	0.0	0	2883	0.0	0	1645	1.9	0	2461	0.0
		0.30	0	3193	0.0	449	2092	0.0	0	3510	0.0	0	2276	2.8	0	3189	0.1
		1.00	0	3415	0.0	3401	3503	0.0	0	3732	0.0	0	2925	3.7	0	3462	0.1
	25	0.10	0	1243	0.0	0	1139	0.0	0	1343	0.0	0	962	0.1	Ő	1214	0.0
		0.50	0	2420	0.0	0	1856	0.0	0	2438	0.0	0	1737	1.4	0	2347	0.0
		0.80	0	2637	0.0	0	2201	0.0	0	2716	0.0	0	2115	1.6	0	2532	0.1
		1.00	0	2844	0.0	2202	2473	0.0	0	3084	0.0	0	2280	3.0	0	2820	0.1
	26	0.10	0	1071	0.0	0	1222	0.0	0	1193	0.0	0	929	0.1	0	1085	0.0
		0.50	0	2357	0.0	0	2070	0.0	0	2507	0.0	0	1709	1.3	0	2354	0.0
		0.80	0	2824	0.0	115	2656	0.0	0	3048	0.0	0	2308	2.1	0	2840	0.1
		1.00	4	2980	0.0	2238	2957	0.0	0	3266	0.0	0	2579	2.5	0	3055	0.1
	27	0.10	0	1497	0.0	0	1631	0.0	0	1609	0.0	0	1397	0.0	0	1502	0.0
		0.50	0	2469	0.0	0	2376	0.0	0	2563	0.0	0	2019	1.1	0	2398	0.1
		0.80	0	2842	0.0	0	2553	0.0	0	2877	0.0	0	2179	1.9	0	2733	0.1
	28	1.00 0.10	0	3099 1787	0.0	1411	2884	0.0	0	3066	0.0	0	2370	3.4	0	3012	0.1
	20	0.10	0	2941	0.0 0.0	0 0	1857 2875	0.0 0.0	0	2044 3192	0.0	0	1154 2279	0.3 1.1	0	1807 2990	0.0
		0.30	0	3703	0.0	1977	3599	0.0	0	4059	0.0	0	2926	2.8	0	3832	0.1
		1.00	10551	4483	0.0	17697	4325	0.0	543	4039	0.0	0	3643	2.8	0	4232	0.1
	29	0.10	0	953	0.0	0	844	0.0	0	1207	0.0	0	620	0.1	0	42 <i>32</i> 974	0.0
	_/	0.10	0	2583	0.0	0	2321	0.0	0	2705	0.0	0	1977	0.9	0	2660	0.1
		0.80	0	3125	0.0	0	2880	0.0	0	3332	0.0	0	2463	1.9	0	3082	0.1
		0.00															

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Table A.8: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 100, instance numbers (Nr.) 0–14.

				СН-О		(CH-R			CH-S			LS-O			LS-S	
Sz.	Nr.	Load	Ca	C_u	t[s]	C_a	Cu	t[s]	Ca	C_u	t[s]	C_a	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	Cu	t[s]
100	0	0.10	0	2527	0.0	0	2959	0.0	0	2943	0.0	0	2286	0.8	0	2854	0.0
		0.50	0	4209	0.1	0	4028	0.1	0	4557	0.1	0	3235	9.0	0	4145	0.1
		0.80	580	5556 5991	0.1	1355	5123	0.1	0	6047	0.1	0	4014	26.9	0	5627	0.3
	1	1.00 0.10	4264	3310	0.1	7294 0	5728 3714	0.1		6831 3533	0.2	0	5042 2788	26.9 1.5	0	6128 3264	0.4 0.0
	1	0.10	0	4966	0.0	1915	5018	0.0		5855	0.0	0	4279	7.6	0	5300	0.0
		0.80	1305	5744	0.1	4514	5758	0.1	0	6503	0.1	0	4867	11.2	0	5911	0.2
		1.00	7855	6617	0.1	7721	6583	0.1	0	7135	0.1	0	5664	17.3	0	6742	0.3
	2	0.10	0	1522	0.0	0	1687	0.0	0	1531	0.0	0	1401	0.3	0	1504	0.0
	_	0.50	0	3898	0.1	0	3984	0.1	0	4366	0.0	0	2965	9.1	0	3945	0.2
		0.80	0	4832	0.1	1019	4445	0.1	0	5550	0.1	0	3769	18.6	0	4851	0.3
		1.00	416	5649	0.1	3743	5494	0.1	0	6227	0.1	0	4668	21.6	0	5772	0.6
	3	0.10	0	2913	0.0	0	3436	0.0	0	3497	0.0	0	2553	2.1	0	3169	0.0
		0.50	0	5305	0.1	0	5126	0.1	0	6172	0.1	0	4096	12.8	0	5849	0.2
		0.80	0	6301	0.1	60	5767	0.1	0	7232	0.1	0	4747	25.5	0	6590	0.3
		1.00	1626	7346	0.1	5479	6792	0.1	0	7713	0.1	0	5666	31.0	0	7346	0.4
	4	0.10	0	2414	0.0	0	2356	0.0	0	2699	0.0	0	1980	1.0	0	2628	0.0
		0.50	0	4938	0.1	905	4998	0.1	0	5361	0.1	0	4099	9.3	0	5150	0.2
		0.80	355	5885	0.1	2385	5591	0.1	0	6278	0.1	0	4651	18.1	0	5789	0.3
		1.00	539	6312	0.1	2175	6035	0.1	0	6733	0.1	410	5141	25.1	0	6354	0.9
	5	0.10	0	2139	0.0	0	2231	0.0	0	2557	0.0	0	1927	0.8	0	2475	0.0
		0.50	0	4643	0.1	0	5026	0.1	0	5040	0.1	0	3692	11.6	0	4733	0.2
		0.80	0	5307	0.1	735	5714	0.1	0	6049	0.1	0	4375	18.3	0	5362	0.3
		1.00	0	6272	0.1	8113	6426	0.2	0	6701	0.2	0	4863	24.8	0	5950	0.5
	6	0.10	0	2577	0.0	0	2648	0.0	0	3249	0.0	0	2297	1.3	0	3088	0.0
		0.50	0	4578	0.1	30	4213	0.1	0	5276	0.1	0	3750	10.8	0	5029	0.2
		0.80	250	5590	0.1	312	5056	0.1	0	6365	0.1	0	4527	21.0	0	5943	0.3
	7	1.00	1705	6983 3981	0.1	4566	6170	0.1		7251 4044	0.1	0	5502	28.7	0	6992 3856	0.4
	/	0.10	0		0.0	0	3216	0.0			0.0	0	2761	1.8 8.8	0		0.0
		0.50 0.80	65 75	5187 6144	0.1	65 305	4727 5835	0.1	6	5304 6591	0.0	0	3930 4791	8.8 19.0	0	5244 6171	0.2
		1.00	310	6624	0.1	3041	6385	0.1	6	7189	0.1	0	5381	27.2	0	6819	0.3
	8	0.10	0	2380	0.1	0	2359	0.0	0	2785	0.0	0	1831	0.7	0	2378	0.4
	0	0.10	0	4620	0.0	0	4514	0.0	0	5077	0.0	0	3661	5.7	0	4684	0.0
		0.80	675	5400	0.1	68	5822	0.1	0	5985	0.1	0	4236	14.4	0	5461	0.2
		1.00	1557	6541	0.1	6343	6973	0.1	0	6916	0.1	160	5202	22.6	160	6177	3.7
	9	0.10	0	2531	0.0	0	2755	0.0	0	2796	0.0	0	2153	1.1	0	2530	0.0
		0.50	70	4521	0.1	186	4276	0.1	0	4860	0.1	0	3365	14.3	70	4156	1.3
		0.80	70	5275	0.1	354	4982	0.1	0	5701	0.1	70	4055	25.0	70	4946	2.5
		1.00	70	5984	0.1	4406	5788	0.1	0	6638	0.1	70	4959	25.5	70	5728	2.7
	10	0.10	0	2937	0.0	0	2998	0.0	0	3291	0.0	0	2179	1.1	0	2844	0.0
		0.50	0	4004	0.0	0	3624	0.1	0	4449	0.0	0	2910	6.0	0	3979	0.1
		0.80	0	4969	0.1	661	4914	0.1	0	5438	0.1	0	4141	12.4	0	4898	0.2
		1.00	448	5824	0.1	4094	5500	0.1	0	6114	0.1	0	4865	21.7	0	5791	0.3
	11	0.10	0	3882	0.0	0	3646	0.0	0	4325	0.0	0	2778	1.9	0	3875	0.0
		0.50	0	5648	0.1	70	4845	0.1	0	5704	0.1	0	4362	8.2	0	5579	0.2
		0.80	0	6424	0.1	1764	5955	0.1	0	7006	0.1	0	5164	16.8	0	6538	0.3
		1.00	1960	7043	0.1	18262	7313	0.2	1840	7701	0.2	0	5803	26.2	925	6370	4.9
	12	0.10	0	2896	0.0	0	2570	0.0	0	3265	0.0	0	2273	1.1	0	2850	0.0
		0.50	0	5184	0.1	0	4324	0.1	0	5444	0.1	0	3441	9.3	0	5178	0.2
		0.80	205		0.1	85	5355		0	6736		0	4553	16.2	0	6299	0.3
		1.00	550	6967	0.1	2149	6430	0.1	0	7395	0.1	0	5590	20.5	85	6114	3.0
	13	0.10	0	2705	0.0	0	2774	0.0	0	2820	0.0	0	2192	0.8	0	2710	0.0
		0.50	0	4967	0.1	185	5199	0.1	0	5252	0.1	0	3966	11.3	0	5108	0.2
		0.80	0	5739	0.1	90	5845	0.1	0	6332	0.1	0	4898	20.0	0	5977	0.3
		1.00	635	6670	0.1	6815	6628	0.1	0	7339	0.1	0	5792	22.3	0	6608	0.4
	14	0.10	0	3196	0.0	0	3339	0.0	0	3409	0.0	0	2783	1.8	0	3273	0.0
		0.50	0	5138	0.1	0	5175	0.1		6012	0.1	0	4181	10.1	0	5216	0.2
		0.80	0 1811	6414 7048	0.1	0 2304	6161 6773	0.1	0	6810 7323	0.1	0	4986 5591	22.1 31.8	0	6264 6816	0.3
		1.00															

Table A.9: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 100, instance numbers (Nr.) 15–29.

		1	1	сн-о		(CH-R		I	CH-S			LS-0		I	LS-S	
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	Cu	t[s]	Ca	C_u	t[s]	C_{a}	Cu	t[s]	C_{a}	C_u	t[s]	Ca	C_u	t[s]
100	15	0.10	0	2541	0.0	0	2304	0.0	0	2591	0.0	0	1940	0.9	0	2495	0.0
		0.50	0	4275	0.0	0	3881	0.0	0	4338	0.0	0	3359	5.1	0	4191	0.1
		0.80	0	5350	0.1	405	4894	0.1	0	5798	0.1	0	4445	12.2	0	5439	0.2
		1.00	550	6054	0.1	1760	6019	0.1	0	6912	0.1	0	5148	15.1	0	6108	0.3
	16	0.10	0	3185	0.0	0	3039	0.0	0	3553	0.0	0	2547	2.3	0	3299	0.0
		0.50	0	4801	0.0	301	4722	0.1	0	5151	0.0	0	3934	10.3	0	4822	0.2
		0.80	0	5656	0.1	1746	5895	0.1	0	5930	0.1	0	4610	21.1	0	5695	0.3
	17	1.00	909	6199	0.1	3343	6242	0.1	0	6667	0.1	0	5474	33.6	0	6382	0.7
	17	0.10	0	2529	0.0	0	2583	0.0	0	2793	0.0	0	2157	0.6	0	2595	0.0
		0.50	0	5283	0.0	90	5111	0.0	0	5884	0.0	0	4300 5143	9.7 14.2	0	5393	0.1
		0.80 1.00	0	5953 6842	0.1	1258 8986	6088 7118	0.1	0	6681 7128	0.1	0	5794	14.2	0	6005 6776	0.2
	18	0.10	0	2399	0.0	0	2585	0.1	0	2419	0.0	0	2048	0.7	0	2304	0.0
	10	0.10	0	4357	0.0	0	3984	0.0	0	4669	0.0	0	3284	7.5	0	4256	0.0
		0.80	0	5185	0.1	1543	5233	0.1	370	5719	0.1	0	3966	14.1	0	5037	0.1
		1.00	3130	6252	0.1	2798	6450	0.1	2475	6579	0.1	0	4881	21.9	115	5526	3.6
	19	0.10	0	2624	0.0	0	2703	0.0	0	2996	0.0	0	2305	1.1	0	2884	0.0
		0.50	0	4377	0.1	0	4511	0.1	0	4561	0.1	0	3547	9.5	0	4265	0.2
		0.80	0	5009	0.1	390	5315	0.1	0	5608	0.1	0	4201	15.2	0	4953	0.3
		1.00	3461	5766	0.1	5447	5605	0.1	0	6120	0.1	0	4626	22.8	0	5579	0.4
	20	0.10	0	1946	0.0	0	2009	0.0	0	2091	0.0	0	1637	0.5	0	2025	0.0
		0.50	0	4777	0.0	0	4573	0.0	0	5357	0.0	0	3733	6.5	0	4983	0.1
		0.80	0	5752	0.1	890	4950	0.1	0	6236	0.1	0	4322	17.6	0	5939	0.2
		1.00	509	6347	0.1	1460	5811	0.1	0	6614	0.1	0	5136	24.6	0	6610	0.3
	21	0.10	0	2596	0.0	0	2660	0.0	0	3029	0.0	0	2051	1.4	0	2777	0.0
		0.50	470	4481	0.1	0	4618	0.1	0	4937	0.1	0	3492	8.2	0	4709	0.2
		0.80	3895	5448	0.1	4520	5802	0.1	0	6070	0.1	0	4415	18.5	0	5694	0.3
		1.00	7014	5840	0.1	9414	6263	0.1	0	6942	0.1	0	4765	21.8	0	6027	0.4
	22	0.10	0	1209	0.0	0	1413	0.0	0	1238	0.0	0	1097	0.2	0	1218	0.0
		0.50	0	4152	0.0	0	3809	0.1	0	4379	0.0	0	3013	7.1	0	4139	0.1
		0.80	0	5441	0.1	152	4887	0.1	0	6014	0.1	0	3957	17.0	0	5316	0.3
		1.00	0	5848	0.1	790	5522	0.1	0	6649	0.1	0	4450	24.0	0	5947	0.3
	23	0.10	0	2007 4380	0.0	0 520	2916	0.0	0	2254	0.0	0	1681	0.3	0	2108	0.0
		0.50 0.80	340 790	5419	0.1 0.1	520 1045	4408 5413	0.1	0	5235 6075	0.1 0.1	0	3182 4153	9.6 19.4	0	4525 5612	0.1
		1.00	1640	6321	0.1	14984	6145	0.1	0	7157	0.1	0	5311	19.4	0	6316	0.3
	24	0.10	0	3153	0.0	0	3063	0.0	0	3233	0.0	0	2678	0.7	0	3150	0.0
	21	0.50	0	4961	0.0	0	4568	0.1	0	5563	0.1	0	4043	6.9	0	5154	0.1
		0.80	Ő	6337	0.1	1023	5696	0.1	0	6753	0.1	0	4801	16.2	0	6337	0.3
		1.00	273	6595	0.1	2731	6275	0.1	0	7196	0.1	0	5162	21.6	0	6648	0.3
	25	0.10	0	2520	0.0	65	2736	0.0	0	2747	0.0	0	2088	1.4	0	2467	0.0
		0.50	0	4727	0.0	0	4290	0.1	0	4831	0.1	0	3662	9.8	0	4433	0.2
		0.80	0	5600	0.1	2986	5690	0.1	0	6464	0.1	0	4651	18.8	0	5461	0.3
		1.00	181	6649	0.1	5600	6907	0.1	0	7156	0.1	0	5271	33.5	0	6653	0.3
	26	0.10	0	2868	0.0	0	3052	0.0	0	3335	0.0	0	2450	0.6	0	2924	0.0
		0.50	0	4437	0.0	60	4484	0.0	0	4707	0.0	0	3749	3.7	0	4513	0.1
		0.80	55	5751	0.1	325	5625	0.1	0	6244	0.1	0	4585	17.9	0	5989	0.2
		1.00	615	6491	0.1	2856	6141	0.1	0	6909	0.1	0	5214	21.8	0	6468	0.3
	27	0.10	0	2113	0.0	0	2321	0.0	0	2153	0.0	0	1745	0.3	0	2073	0.0
		0.50	0	4514	0.0	0	4595	0.0	0	4961	0.0	0	3318	6.3	0	4433	0.1
		0.80	0		0.1	510	5624	0.1	0	6194	0.1	0	4417	14.1	0	5551	0.3
	20	1.00	385	5960	0.1	952	5930	0.1	0	6580	0.1	280	4950	13.3	255	5641	2.1
	28	0.10 0.50	0	2790 5380	0.0	0 703	2675 5108	0.0	0	2976 5829	0.0	0	2297 4358	1.1 15.7		2902 5321	0.0
		0.50	0 785	6060	0.1	902	5108	0.1	0	6634	0.1 0.1	0	4358 4793	23.5		6058	0.2
		1.00	1465	6874	0.1	3094	6528	0.1	0	7206	0.1	0	5312	38.4	0	6869	0.5
	29	0.10	0	2807	0.0	0	3021	0.0	0	3363	0.0	0	2526	1.1	0	3014	0.4
		0.10	0	5076	0.0	0	4545	0.0	0	5566	0.0	0	3661	14.3	0	5198	0.0
		0.80	0	5955	0.1	1923	5699	0.1	0	6959	0.1	0	4680	22.1	0	6167	0.3
		1.00	240	6266	0.1	3814	6090	0.1	0	7390	0.1	0	5100	24.4	0	6537	0.4
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Table A.10: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 200, instance numbers (Nr.) 0–14.

				сн-о			CH-R			CH-S			LS-O			LS-S	
Sz.	Nr.	Load	Ca	C_{u}	t[s]	C_{a}	Cu	t[s]	C_a	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	C_a	Cu	t[s]
200	0	0.10	0	2781	0.0	0	3139	0.0	0	3071	0.0	0	2415	1.3	0	2948	0.0
		0.50	15	6499	0.1	738	7015	0.1	0	7063	0.1	0	4804	16.6	0	6683	0.2
		0.80	70	7771	0.1	4537	8931	0.2	0	8984	0.1	0	6100	31.5	0	8033	0.4
		1.00	1911	9075	0.2	9313	10248	0.2	0	10329	0.2	16	7008	46.3	0	9453	0.5
	1	0.10	0	2229	0.0	0	2804	0.0	0	2809	0.0	0	1915	1.2	0	2428	0.1
		0.50	0	5705	0.1	1558	6056	0.1	152	6682	0.1	0	4276	25.7	0	5847	0.4
		0.80	1782	7610	0.2	5283	7825	0.2	214	8383	0.2	0	6010	45.7	0	7443	0.9
	2	1.00	6584	9578	0.2	11628	9646	0.3	2009	10351	0.2	291	7798	62.8	0	9357	2.8
	2	0.10 0.50	0	3032 5777	0.0	0 385	3293 5646	0.0 0.1	0	3685 6131	0.0	0	2690 4326	1.9 16.4	0	3351 5720	0.1 0.3
		0.30	0	7261	0.1	585 1901	7329	0.1	0	8438	0.1	0	4526 5715	32.3	0	7375	0.5
		1.00	2647	9158	0.2	8931	8881	0.2	0	10935	0.2	0	7109	75.6	0	9390	2.7
	3	0.10	0	4387	0.0	527	4505	0.2	0	5014	0.0	0	3779	4.6	0	4788	0.1
	5	0.50	100	7619	0.1	2832	7640	0.1	0	8340	0.1	0	5942	26.8	0	7922	0.7
		0.80	270	9055	0.1	2138	8633	0.2	0	9769	0.2	0	7173	48.6	0	9354	1.0
		1.00	393	10138	0.2	3829	9908	0.2	86	10612	0.2	140	8207	63.2	100	9155	10.9
	4	0.10	0	3356	0.0	0	3973	0.1	0	3930	0.0	0	3097	2.8	0	3740	0.1
		0.50	0	6441	0.1	1110	7178	0.1	0	7071	0.1	0	5140	18.3	0	6675	0.2
		0.80	0	8874	0.2	1618	8692	0.2	0	9774	0.2	0	6926	51.8	0	9445	0.5
		1.00	2099	10999	0.2	4810	9913	0.2	0	11794	0.2	899	8622	78.5	0	11302	1.7
	5	0.10	0	3740	0.0	0	3779	0.0	0	4182	0.0	0	3230	2.7	0	4004	0.1
		0.50	120	6923	0.1	1	7222	0.2	120	7687	0.1	0	5402	37.0	0	7163	0.3
		0.80	1240	8773	0.2	3573	8914	0.3	140	9388	0.2	0	6881	76.2	40	7853	11.0
		1.00	3029	10007	0.3	4080	9909	0.3	120	10895	0.3	382	8747	92.7	205	9135	15.9
	6	0.10	0	4577	0.0	90	4822	0.0	0	5297	0.0	0	3517	5.6	0	4919	0.1
		0.50	0	7598	0.1	3860	7780	0.1	0	8231	0.1	0	5613	31.5	0	7765	0.4
		0.80	739	9499	0.2	17089	10385	0.2	0	10569	0.2	0	7191	56.5	0	9553	0.6
	7	1.00	4553	11680	0.2	32203	10890	0.3	169	11936	0.3	0	9246	59.7	0	10880	1.9
	7	0.10	0	3558	0.0	1250	3927	0.0	0	4547	0.0	0	3108	3.0	0	4346	0.1
		0.50 0.80	0	7712 10352	0.1 0.2	1259 3076	8374 10421	0.1 0.2	0	8742 11280	0.1	0	5698 7586	27.7 63.8	0	8236 10421	0.3 0.9
		1.00	4007	11758	0.2	7036	11238	0.2	2278	12069	0.2	541	9019	92.9	1035	10421	15.2
	8	0.10	0	1873	0.0	0000	2176	0.0	0	2046	0.0	0	1546	0.5	0	1971	0.0
	0	0.50	0	6417	0.1	760	7049	0.1	0	7658	0.1	0	4899	24.1	0	6760	0.3
		0.80	430	8930	0.2	2615	9901	0.2	261	10236	0.2	0	6726	49.2	0	9014	0.8
		1.00	4614	11950	0.2	19800	11899	0.3	509	12330	0.3	0	8402	69.7	0	10764	1.1
	9	0.10	0	2428	0.0	0	2824	0.0	0	2765	0.0	0	2265	1.3	0	2661	0.1
		0.50	25	6693	0.1	296	6868	0.1	25	7444	0.1	0	5216	33.3	25	6044	5.4
		0.80	25	8198	0.2	3110	9068	0.3	25	8792	0.2	0	6332	69.7	25	7293	10.5
		1.00	8678	9961	0.3	12752	9616	0.3	25	11106	0.3	135	8434	103.5	25	9157	15.0
	10	0.10	0	2140	0.0	0	2247	0.0	0	2388	0.0	0	1840	0.4	0	2324	0.0
		0.50	0	6611	0.1	295	6635	0.1	0	7034	0.1	0	5045	17.2	0	6704	0.2
		0.80	130	9180	0.1	1333	8967	0.2	0	9593	0.2	0	6911	47.8	0	8978	0.4
	11	1.00	9394	10797	0.2	16382	10805	0.2	0	11413	0.2	0	8375	76.2	0	10563	1.9
	11	0.10	0	2449	0.0	0	2475	0.0	0	2893	0.0	0	2150	1.4	0	2653	0.1
		0.50	250	6800	0.1	380	6825	0.1	0	7755	0.1	0	5101	35.9	0	7130	0.3
		0.80 1.00	350 17462	10157 11850	0.2	11667 33827	10463 11930	0.3 0.3	0 260	11318 12920	0.2	0 140	7943 8926	79.5 96.5	0	10591 12127	0.6 0.8
	12	0.10	0	3200	0.0	45	3811	0.0	0	3806	0.0	0	2757	2.3	0	3631	0.8
	12	0.50	0	6058	0.1	206	5988	0.1	0	6587	0.1	0	4839	19.8	0	6264	0.3
		0.80		7670		4650	7444	0.1	0	8298		0	5813	45.2	0	7778	0.5
		1.00	1374	9706	0.2	27009	9256	0.3	4768	10229	0.3	0	7488	71.4	0	9518	0.7
	13	0.10	0	2419	0.0	0	3275	0.0	0	2634	0.0	0	2231	0.9	0	2634	0.1
		0.50	0	4928	0.1	615	5719	0.1	0	5392	0.1	0	4006	10.1	0	5128	0.2
		0.80	0	7293	0.2	6790	8075	0.2	0	8728	0.2	0	5430	45.2	0	7677	0.5
		1.00	11084	9999	0.2	18983	9255	0.3	0	10634	0.2	0	6829	73.2	0	9795	1.7
	14	0.10	0	3791	0.0	0	5439	0.1	0	4452	0.0	0	3487	4.5	0	4222	0.1
		0.50	0	6231	0.1	40	7724	0.1	0	6908	0.1	0	4994	26.0	0	6363	0.3
		0.80	600	8728	0.2	3419	10365	0.3	0	9801	0.2	0	7279	48.6	0	8650	0.6
		1.00	3409	10386	0.3	5986	10436	0.3	0	10929	0.3	0	8046	65.2	0	9506	1.3

Table A.11: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 200, instance numbers (Nr.) 15–29.

				СН-О			CH-R			CH-S			LS-0			LS-S	
Sz.	Nr.	Load	C_a	C_u	t[s]	C_a	C_u	t[s]	C_a	C_u	t[s]	C_a	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]
200	15	0.10	0	3101	0.0	0	3784	0.0	0	3096	0.0	0	2645	1.5	0	2966	0.1
		0.50	0	5915	0.1	210	6156	0.1	0	6495	0.1	0	4545	22.7	0	6023	0.3
		0.80	0	7723	0.2	1723	8040	0.2	0	8426	0.2	0	5770	46.7	0	7766	0.5
	16	1.00	2295	9120	0.2	5349	9458	0.2	10	9858	0.2	45	7388	72.2	10	8008	13.0
	16	0.10	0	3021	0.0	0	3560	0.0	0	3647	0.0	0	2731	1.9	0	3477	0.1
		0.50 0.80	0	5508 8416	0.1	225 3502	5593 7661	0.1 0.2	0	6363 9510	0.1	0	4313 5656	16.3 67.9	0	5721 8702	0.3
		1.00	3588	10403	0.2	9093	9531	0.2	583	11989	0.2	395	7710	89.8	535	9241	16.0
	17	0.10	0	2381	0.0	0	2682	0.0	0	2962	0.0	0	1917	2.3	0	2684	0.1
	17	0.10	0	7493	0.0	603	7155	0.0	0	8358	0.0	0	5491	39.8	0	7566	0.1
		0.80	297	9681	0.2	3361	10053	0.3	0	11151	0.2	0	7390	68.0	0	9415	0.6
		1.00	3826	11663	0.3	22415	11530	0.3	85	12826	0.3	0	8733	110.0	0	11426	3.4
	18	0.10	0	2763	0.0	0	2992	0.0	0	3070	0.0	0	2254	2.9	0	2865	0.1
		0.50	0	6426	0.1	640	6827	0.1	0	6747	0.1	0	5074	25.1	0	6532	0.3
		0.80	110	8632	0.2	4640	9351	0.3	0	9354	0.2	0	6773	65.0	0	8799	0.6
		1.00	5073	9603	0.3	20267	10228	0.3	100	10157	0.3	0	7720	101.8	0	10024	2.4
	19	0.10	0	2649	0.0	0	3726	0.0	0	2926	0.0	0	2581	1.0	0	2824	0.1
		0.50	0	5477	0.1	772	6578	0.1	0	5970	0.1	0	4464	11.7	0	5358	0.2
		0.80	1245	8678	0.1	6243	8702	0.2	110	9785	0.2	0	6585	46.5	0	8825	0.8
		1.00	2143	9615	0.2	8616	9980	0.2	148	10907	0.2	12	8061	59.4	0	9825	1.3
	20	0.10	0	3617	0.0	0	4096	0.1	0	4567	0.0	0	2716	3.7	0	3917	0.1
		0.50	0	7468	0.1	1852	6701	0.1	0	7807	0.1	0	5109	39.0	0	7431	0.3
		0.80	3975	9478	0.2	7573	8477	0.2	45	10479	0.2	25	6711	68.3	25	8575	8.4
		1.00	11918	11079	0.3	17564	9841	0.3	470	11374	0.2	613	8528	78.3	623	9875	12.8
	21	0.10	0	3217	0.0	0	3500	0.0	0	3454	0.0	0	2617	2.8	0	3406	0.1
		0.50	0	6330	0.1	1941	6682	0.1	0	6794	0.1	0	4919	29.2	0	6468	0.3
		0.80	470	8652	0.2	975	8355	0.2	0	9463	0.2	0	6666	66.2	0	8760	0.6
		1.00	6928	10102	0.2	11431	9306	0.3	315	10952	0.3	0	8050	78.9	0	9869	2.0
	22	0.10	0	3572	0.0	170	3636	0.0	0	4263	0.0	0	3144	2.5	0	4108	0.1
		0.50	50	6925	0.1	1896	7194	0.2	0	7340	0.1	0	5413	32.1	0	7092	0.4
		0.80	355	8576	0.2	2435	8970	0.2	570	9241	0.2	0	6783	58.9	0	8501	0.6
	22	1.00	2747 0	10086	0.3	11568 0	9796	0.3	579 0	11023 4242	0.3	0	7872	104.7 2.5	165 0	9116 3964	15.8
	23	0.10 0.50	0	3933 7767	0.0	0	4262 8269	0.0	0	8534	0.0	0	5625	2.5	0	8145	0.1 0.3
		0.80	155	9409	0.1	3118	10009	0.1	0	10373	0.1	0	7196	63.0	0	9617	0.5
		1.00	2579	11244	0.2	5574	11922	0.2	457	12530	0.2	175	9771	74.5	175	10804	10.1
	24	0.10	0	2643	0.0	0	2680	0.0	0	3063	0.0	0	2231	1.7	0	2875	0.1
		0.50	0	6723	0.1	1323	6526	0.1	0	7494	0.1	0	5252	23.3	0	6912	0.3
		0.80	130	8396	0.2	6897	8262	0.2	0	9253	0.2	0	6101	56.7	0	8395	0.5
		1.00	7567	9878	0.2	15560	10260	0.3	10	11128	0.3	85	8506	68.7	0	10469	2.5
	25	0.10	0	3635	0.0	0	3623	0.0	0	4103	0.0	0	2976	3.2	0	3857	0.1
		0.50	1751	7172	0.1	1958	7917	0.1	1775	8002	0.1	0	5192	34.0	0	7196	0.3
		0.80	2721	9022	0.2	15645	9305	0.2	1781	9882	0.2	40	6483	56.6	0	8591	1.6
		1.00	16072	9962	0.2	41028	10234	0.2	2372	10797	0.2	130	7333	65.7	0	9379	1.9
	26	0.10	0	3283	0.0	0	3916	0.0	0	3637	0.0	0	3027	2.2	0	3547	0.1
		0.50	275	5500	0.1	550	5864	0.1	0	6108	0.1	0	4742	17.1	0	5853	0.3
		0.80	2102	8641	0.2	5852	8351	0.2	1134	8842	0.2	0	6516	48.8	0	8607	0.6
		1.00	5909	9427	0.2	19286	9415	0.3	1594	10481	0.3	1611	7839	60.8	0	9382	3.0
	27	0.10	0	2898	0.0	0	2986	0.0	0	3166	0.0	0	2561	1.1	0	3080	0.0
		0.50	0	6054	0.1	80	6208	0.1	0	6162	0.1	0	4740	22.9	0	6129	0.3
		0.80	0	7820		926	7950	0.2	0	8564	0.2	0	6095	51.4	0	7973	0.5
	20	1.00	2153	9057	0.2	9719	8701	0.3	108	9768	0.2	0	7005	85.1	0	9116	2.0
	28	0.10	0	3278	0.0		3501	0.0	0	3676	0.0	0	2678	1.8	0	3454	0.1
		0.50	0	5730	0.1	62	5972	0.1		6506	0.1	0	4476	11.9	0	5819	0.2
		0.80	1456	8479	0.1	3647	8300	0.2	269	9061	0.2	1255	5884	38.0	0	8382	0.4
	20	1.00	9279	10623	0.2	21759	10169	0.2	496	11073	0.2	1255	8245	65.6	80	9000	13.8
	29	0.10	0	3421	0.0	0	3647	0.1		3865	0.0	0	2452	4.1	0	3571	0.1
		0.50 0.80	0 194	7206 9262	0.1	688 2715	7998 9810	0.2	0	7437 10150	0.1	0	4874 6823	30.4 56.2	0	7091 9158	0.3
		1.00	1084	10443	0.2	9158	10802	0.3		10150	0.2	30	8133	83.5	30	9138	13.4
										10210	. 0.2						1 12.4

Table A.12: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 500, instance numbers (Nr.) 0–14.

a	••			CH-O			CH-R	ا مع	a	CH-S			LS-O			LS-S	
Sz.	Nr.	Load	Ca	Cu	t[s]	Ca	Cu	t[s]	C_a	Cu	t[s]	Ca	C_u	t[s]	Ca	Cu	t[s]
500	0	0.10	0	4639	0.1	897	6702	0.2	0	5318	0.1	0	3991	14.4	0	4869	0.3
		0.50	0	9322	0.3	2696	11370	0.4	0	10397	0.3	0	7148	106.0	0	9593	0.
		0.80	3395	12835	0.5	7780	15754	0.7	0	14895	0.5	0	9912	221.6	0	13114	1.
		1.00	16483	16274	0.8	34288	19718	1.0	3538	18925	0.8	1767	13745	385.7	26	14452	101.
	1	0.10	0	4382	0.1	2235	5523	0.1	0	5021	0.0	0	4060	4.1	0	4799	0.
		0.50	145	9123	0.2	2974	10614	0.4	0	10409	0.2	0	7231	63.8	0	9645	0.
		0.80	1264	13307	0.4	7211	13727	0.5	270	13953	0.4	0	9135	198.9	0	10688	31.0
		1.00	9320	15518	0.6	15351	16821	0.7	1139	17511	0.6	1797	12164	312.4	37	13361	75.
	2	0.10	0	4213	0.1	2295	5164	0.1	0	4771	0.1	0	3563	9.0	0	4644	0.
		0.50	0	8279	0.2	1694	11026	0.4	0	9983	0.2	0	6749	63.7	0	9007	0.
		0.80	2459	13825	0.5	6911	16454	0.7	0	15069	0.5	0	9808	230.3	0	14273	1.
		1.00	16322	17531	0.8	26042	19283	1.0	1987	19586	0.7	718	13580	418.7	0	16982	29.
	3	0.10	0	3569	0.1	170	3703	0.1	0	4012	0.0	0	3153	5.7	0	3822	0.
		0.50	0	7746	0.2	994	9867	0.3	0	8426	0.2	0	6142	52.4	0	7861	0.
		0.80	0	13022	0.5	7621	14865	0.6	0	13986	0.5	0	9798	193.1	0	12658	1.
		1.00	4795	16388	0.7	25696	17495	0.9	255	18825	0.7	390	12850	386.0	0	16322	10.
	4	0.10	0	2728	0.1	0	4108	0.1	0	3405	0.0	0	2729	2.5	0	3262	0.
		0.50	0	8967	0.3	5901	10319	0.5	0	9980	0.3	0	6848	87.9	0	9231	0.
		0.80	45	12775	0.6	3284	14316	0.6	0	13575	0.5	0	9475	193.7	0	12261	1.
	_	1.00	5868	16026	0.8	10654	16537	0.9	5	16855	0.7	92	12075	348.9	5	12859	61.
	5	0.10	0	3420	0.1	160	4547	0.1	0	3982	0.1	0	2971	5.6	0	3753	0.
		0.50	0	10418	0.4	3766	12711	0.5	0	11563	0.3	0	7368	168.4	0	11015	0.
		0.80	4851	14490	0.6	21752	15667	0.8	20	16297	0.6	2530	10558	335.8	0	14213	8.
	,	1.00	16673	17427	0.8	28631	17898	0.9	1079	18720	0.7	3472	13089	537.1	715	14721	89.
	6	0.10	0	3757	0.1	541	4389	0.1	0	4367	0.0	0	3412	6.1	0	4201	0.
		0.50	5	9320	0.3	1458	11341	0.4	5	10598	0.2	0	7308	94.4	0	10059	0.
		0.80	886	12920	0.5	11505	14845	0.8	5	14779	0.5	0	9742	233.8	40	11393	40.
	_	1.00	8250	16495	0.8	28714	18642	1.0	544	18350	0.8	1006	13171	395.9	220	14095	97.
	7	0.10	0	3763	0.1	520	4604	0.1	0	4290	0.0	0	3414	4.7	0	4197	0.
		0.50	12	9619	0.3	8211	12100	0.4	419	10680	0.3	0	8008	80.7	12	8978	11.
		0.80	5255	15456	0.7	14600	17041	0.8	3561	17611	0.6	0	11983	338.6	12	13023	56.
		1.00	12441	18699	0.9	19709	19006	1.0	3909	20477	0.8	321	15659	511.3	262	16255	103.
	8	0.10	0	3607	0.1	0	5400	0.1	0	4893	0.1	0	3219	10.3	0	4505	0.
		0.50	0	9553	0.4	192	12537	0.6	0	10879	0.4	0	7036	122.8	0	9856	0.
		0.80	2349	15746	0.8	19266	18420	1.1	0	17910	0.7	0	11398	363.0	0	15925	1.
		1.00	19908	19533	1.0	30225	20327	1.2	906	22797	1.0	456	15974	559.5	20	16041	109.
	9	0.10	0	2923	0.1	1147	4028	0.1	0	3622	0.0	0	2541	4.5	0	3279	0.
		0.50	20	8164	0.3	3747	9209	0.3	20	9104	0.2	0	5836	88.4	0	8489	0.
		0.80	3052	12907	0.5	15420	14233	0.6	739	13599	0.4	0	8655	209.6	0	12698	2.
		1.00	9392	15606	0.6	21374	16239	0.8	4967	17645	0.6	1215	12149	323.3	2220	13142	65.
	10	0.10	0	3623	0.1	150	5686	0.1	0	4438	0.0	0	3192	6.7	0	4007	0.
		0.50	0	8586	0.3	2533	11985	0.5	0	10208	0.3	0	6117	99.1	0	9055	0.
		0.80	344	13430	0.6	9302	16689	0.7	214	15785	0.5	0	10272	239.0	0	13936	1.
		1.00	20951	17789	0.8	25223	18480	0.9	1982	20284	0.8	825	13128	463.8	385	14340	107.
	11	0.10	0	4104	0.1	140	4510	0.1	0	4616	0.0	0	3567	9.8	0	4482	0.
		0.50	0	10219	0.3	1206	12049	0.4	0	10925	0.3	0	8037	90.3	0	10256	0.
		0.80	778	12513	0.5	3404	13959	0.6	75	14083	0.4	0	9849	174.5	0	12549	1.
		1.00	5612	15057	0.6	16226	16281	0.7	7489	17062	0.6	364	12588	249.7	3090	13441	53.
	12	0.10	0	3760	0.1	95	4123	0.1	0	3948	0.0	0	3279	3.2	0	3876	0.
		0.50	435	10761	0.4	3352	12110	0.5	0	12073	0.3	0	7699	118.4	0	10947	0.
		0.80	7555	14944	0.6		15788	0.7	3001	16505	0.5		10995	284.8	30	13543	43
		1.00	19224	17711	0.8	45729	17485	0.8	7480	19377	0.7	5911	14612	389.0	170	15902	90.
	13	0.10	0	3197	0.1	3435	4570	0.1	0	3681	0.0	0	2901	2.4	0	3387	0.
		0.50	0	8910	0.3	3301	9558	0.4	0	10247	0.3	0	6097	105.2	0	9202	0.
		0.80	491	12995	0.5	11789	12357	0.6	170	13764	0.5	0	9074	210.5	0	13040	1
		1.00	6984	17009	0.7	27600	16633	0.9	501	18842	0.7	1500	12886	389.8	224	14647	80.
	14	0.10	0	4163	0.1	539	5325	0.1	0	4545	0.0	0	3661	6.9	0	4378	0.
		0.50	0	10190	0.3	5114	12951	0.5	0	11211	0.3	0	7528	115.8	0	10400	0.
		0.80	3412	15201	0.6	17179	17470	0.8	17	17351	0.6	770	11480	255.7	620	12310	50.
		1.00	21324	19392	0.9	39018	20434	1.1	1611	22932	0.9	1820	16084	561.5	645	16859	116.

Table A.13: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 500, instance numbers (Nr.) 15–29.

				СН-О			CH-R		I	CH-S			LS-O			LS-S	
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	C_a	C_u	t[s]	C_{a}	C_u	t[s]	C_a	C_{u}	t[s]	C_a	C_u	t[s]
500	15	0.10	0	4337	0.1	0	5984	0.1	0	5471	0.1	0	3833	11.3	0	5204	0.2
		0.50	0	8564	0.3	1701	10815	0.5	0	10023	0.3	0	6338	98.9	0	8986	0.7
		0.80	822	13489	0.5	9295	15193	0.8	308	15349	0.5	415	10391	207.0	100	12308	33.9
	16	1.00 0.10	18419 0	18031 3922	0.8	34280 190	18153 4793	1.0 0.1	3129 0	19613 4532	0.8	2238 0	14331 3078	419.2 5.9	403 0	15751 4402	85.5 0.2
	10	0.10	0	9070	0.1	1328	11054	0.1	0	10079	0.0	0	6445	77.0	0	9295	0.2
		0.80	0	12612	0.5	4375	15005	0.4	0	14224	0.5	0	9235	214.8	0	12815	1.4
		1.00	6602	16230	0.7	13534	18495	0.9	1345	17828	0.7	2520	13264	389.1	10	14113	91.6
	17	0.10	0	2567	0.1	0	3382	0.1	0	2955	0.0	0	2269	3.2	0	2890	0.1
		0.50	17	7829	0.2	5112	9467	0.3	5	8516	0.2	5	5683	54.6	5	6540	12.1
		0.80	17	11091	0.4	12934	12166	0.5	5	12464	0.4	5	7910	128.8	5	9627	28.2
		1.00	3856	13976	0.5	33832	14764	0.7	938	16571	0.5	148	11585	203.7	143	12254	48.5
	18	0.10	0	3925	0.1	0	5084	0.1	0	4581	0.1	0	3588	6.6	0	4357	0.2
		0.50 0.80	0 783	9346 14895	0.3	424 3583	11412 16682	0.5	0 98	10631 17355	0.3	0	6776 10124	113.6 340.5	0	9683 15150	0.9 1.8
		1.00	8189	19675	1.0	25385	19737	1.2	1342	21090	1.0	989	14257	704.4	650	16383	119.4
	19	0.10	0109	3746	0.1	445	4754	0.1	0	4561	0.0	0	3249	8.5	0.50	4401	0.2
	.,	0.50	0	9705	0.4	1283	11328	0.5	0	11099	0.3	0	7442	141.8	0	9819	1.0
		0.80	2328	15313	0.8	13788	15842	0.9	0	17020	0.7	0	10744	383.9	0	15392	4.5
		1.00	22383	19932	1.1	24383	20815	1.2	1454	21649	1.0	406	16481	659.6	10	17407	128.3
	20	0.10	0	3720	0.1	872	4680	0.1	0	4587	0.0	0	3304	9.7	0	4393	0.2
		0.50	0	9524	0.3	2766	11041	0.5	0	10252	0.3	0	7394	100.6	0	9819	0.8
		0.80	140	13534	0.6	7417	15145	0.8	5	15010	0.5	0	9603	252.7	0	13754	1.4
	21	1.00	10625	17590	0.9	29717	18913	1.0	4471	19531	0.8	1188	13498	492.1	140	14724	99.8
	21	0.10	0	4110	0.1	0	4754	0.1		4601 9953	0.0		3245	6.6	0	4170	0.2
		0.50 0.80	20 1719	8759 13366	0.3	9236 16118	11794 14810	0.4	20 558	16133	0.2	20 1055	6527 10042	65.8 162.5	20 981	8207 11649	10.6 35.7
		1.00	10421	17057	0.4	38611	18385	0.8	2376	19933	0.5	3188	13273	345.3	2450	14595	67.4
	22	0.10	0	2848	0.1	105	4504	0.1	0	3519	0.0	0	2547	4.9	0	3409	0.1
		0.50	33	8694	0.3	3434	10729	0.4	33	10256	0.3	33	6045	78.1	33	7219	14.0
		0.80	2914	12548	0.5	6969	13449	0.6	1007	13975	0.5	53	9059	171.6	193	10618	34.7
		1.00	9597	14935	0.7	23708	16508	0.8	1795	17428	0.6	993	12239	275.4	33	13119	66.4
	23	0.10	0	4083	0.1	0	4985	0.1	0	4694	0.1	0	3483	15.0	0	4561	0.2
		0.50	0	9658	0.4	1269	10183	0.5	0	10429	0.3	0	6930	110.2	0	9562	0.9
		0.80	444	13460	0.6	10164	13810	0.8	290	14859	0.6	194	10391	243.8	0	13262	7.4
	24	1.00 0.10	29457 0	17271 3263	0.9	39710 250	17403 4395	1.1	3871 0	20132 4001	0.9	14050 0	14235 3044	539.6 5.4	315 0	15032 3890	122.1 0.2
	24	0.10	0	8513	0.1	1363	9432	0.1	0	9538	0.1	0	6397	109.4	0	9006	0.2
		0.80	75	12952	0.4	5554	12786	0.7	15	14740	0.6	0	8971	291.1	15	10842	45.8
		1.00	25367	17170	0.9	36228	16999	1.0	267	19446	0.8	104	13225	500.7	40	15004	86.8
	25	0.10	0	4055	0.1	859	5614	0.2	0	5008	0.1	0	3630	12.4	0	4822	0.2
		0.50	0	10040	0.4	2762	12273	0.5	0	11636	0.3	0	7668	101.0	0	10443	0.9
		0.80	2290	15194	0.6	10050	17291	0.8	390	18089	0.6	5	11519	277.9	485	13489	47.3
		1.00	8822	18437	0.8	18950	20014	1.0	1477	21844	0.9	135	14532	508.8	520	16206	98.0
	26	0.10	0	3366	0.1	2326	5271	0.1	0	4184	0.0	0	2964	6.6	0	4081	0.2
		0.50	75	10152	0.4	3026	10883	0.5	0	11443	0.3	0	7263	125.7	0	10593 14108	1.0
		0.80 1.00	5587 23692	14590 17156	0.6 0.8	18790 39414	15660 15962	0.8	230 1069	16986 19577	0.6	0 1612	10760 13051	297.0 445.0	0 500	14108	5.0 79.7
	27	0.10	23092	3876	0.8	1549	6053	0.9	0	4903	0.0	0	3619	10.1	0	4554	0.2
	2.	0.50	0	8821	0.2	4299	11196	0.4	0	10770	0.2	0	6712	71.5	0	9535	0.6
		0.80	1	14665	0.5		15164		0	16322	0.5	0	10745	201.3		11893	40.7
		1.00	18469	16927	0.7	29088	18139	0.8	1209	19218	0.7	1297	14397	293.8	0	17175	18.2
	28	0.10	0	3731	0.1	75	4690	0.1	0	4331	0.0	0	3099	7.2	0	4132	0.1
		0.50	0	9044	0.3	7452	11418	0.5	0	11149	0.3	0	6582	103.9	0	9516	0.8
		0.80	4672	14631	0.6	14427	15100	0.7	470	16131	0.5	2791	10204	282.7	165	12498	44.4
		1.00	14827	17823	0.9	31728	17750	1.0	2965	18709	0.8	5457	13473	469.4	1650	15089	80.5
	29	0.10	0	4423	0.1	0	5809	0.2	0	5261	0.1	0	4019	10.5	0	5032	0.2
		0.50 0.80	0	8437 12656	0.3	990 8556	10250 14821	0.5	0	9194 13500	0.3	0	6672 9740	73.8 222.2	0	8731 13124	0.7 1.5
		1.00	8686	12030	0.0	24144	14821	1.1	1325	18572	0.0	2916	13776	340.1	0	16853	39.8
		1.00	0000	10010			10200			10072	0.7		10/10	0.0.1		100000	27.0

Table A.14: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 1000, instance numbers (Nr.) 0–14.

				СН-О			CH-R			CH-S			LS-O			LS-S	
Sz.	Nr.	Load	C_a	C_u	t[s]	Ca	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	C_a	$\mathbf{C}_{\mathbf{u}}$	t[s]	C_a	C_u	t[s]
1000	0	0.10	0	5196	0.3	0	6776	0.4	0	6322	0.1	0	4110	40.7	0	5788	0.5
		0.50	41	15071	0.8	7216	18404	1.2	0	17968	0.7	0	10740	402.4	0	15908	2.1
		0.80	3663	22502	1.5	17413	24043	2.0	251	25885	1.3	200	17226	1003.8	165	18760	183.1
		1.00	20216	29398	2.1	29304	29570	2.5	2177	33235	2.0	1651	26417	1016.0	727	26087	473.6
	1	0.10	0	4094	0.2	466	6048	0.4	0	5319	0.1	0	3547	13.1	0	4717	0.4
		0.50	15	12987	0.7	2033	16765	1.1	130	14191	0.5	0	9121	237.5	0	12703	5.0
		0.80	2433	21052	1.3	8171	23812	1.7	493	22946	1.1	240	15189	821.6	180	16934	140.1
		1.00	16846	26477	1.9	51031	29050	2.5	2159	30554	1.6	1637	23646	1007.4	888	23011	300.8
	2	0.10	0	5091	0.2	0	6681	0.3	0	6139	0.1	0	4284	27.9	0	5782	0.5
		0.50	216	12957	0.8	3055	13865	1.0	0	14232	0.6	0	9831	320.1	0	13830	1.7
		0.80	7705	20612	1.5	20259	20468	1.7	1114	23987	1.2	0	15050	849.1	105	16449	156.6
		1.00	16323	25039	1.9	36308	25547	2.1	4966	28409	1.7	1661	21440	1015.6	500	21211	301.2
	3	0.10	0	3813	0.2	770	5177	0.2	0	4170	0.0	0	3234	6.8	0	4049	0.3
		0.50	78	12973	0.7	5971	17540	1.1	15	14622	0.5	0	9527	283.6	0	13367	2.5
		0.80	620	20989	1.5	28508	25024	2.0	126	23903	1.2	0	14518	867.3	0	20749	15.6
		1.00	20795	25665	2.0	42092	27458	2.4	2106	29794	1.7	804	22216	1011.0	702	20716	320.7
	4	0.10	10	4879	0.2	2102	7188	0.4	10	5804	0.1	0	4497	26.1	0	5616	0.5
		0.50	411	13991	1.0	8431	19291	1.5	50	15668	0.7	50	9719	313.1	50	10835	57.9
		0.80	5337	18736	1.4	15900	23388	2.0	245	21161	1.2	301	13637	942.3	50	15212	165.4
		1.00	21328	25619	2.1	36127	29322	2.6	3267	30343	1.8	5032	21518	1001.7	687	23039	426.8
	5	0.10	17	5767	0.3	255	8344	0.5	17	6978	0.1	0	5123	33.3	0	6841	0.0
		0.50	66	15091	0.9	8281	18482	1.2	17	16693	0.7	0	10688	336.5	0	15907	3.
		0.80	6039	21893	1.4	14785	23403	1.8	417	24847	1.3	547	16205	984.3	0	22382	29.9
		1.00	12657	26901	1.8	29003	27717	2.2	1451	29828	1.8	1172	23564	1005.2	716	22591	306.0
	6	0.10	0	5927	0.3	170	7551	0.4	0	6664	0.1	0	4994	36.2	0	6661	0.5
		0.50	1247	12758	0.8	3821	17146	1.2	0	14312	0.6	0	9971	308.8	0	13228	1.
		0.80	4198	19513	1.6	26573	24380	2.2	20	23080	1.4	409	15486	750.3	209	16778	145.
	_	1.00	28518	25317	2.2	52866	29555	2.8	3164	29314	1.9	7573	22447	1000.5	965	22248	407.
	7	0.10	0	3686	0.2	0	4993	0.3	0	4066	0.1	0	2963	12.5	0	3823	0.4
		0.50	0	10625	0.7	8620	15125	1.2	0	12967	0.6	0	7477	247.4	0	11059	1.'
		0.80	4890	19680	1.4	32328	23706	2.1	136	22255	1.2	951	15856	796.1	28	16496	154.5
		1.00	25492	24462	2.0	40601	27330	2.5	2977	29990	1.8	5400	22799	1011.2	670	22894	428.2
	8	0.10	0	4595	0.3	860	7078	0.4	0	5596	0.1	0	3941	35.9	0	5039	0.5
		0.50	0	11685	0.7	3519	15387	1.1	0	12875	0.6	0	7422	282.6	0	12322	1.
		0.80	8167	20085	1.7	25673	24750	2.3	363	22444	1.4	467	14977	1005.3	241	17441	189.5
		1.00	20002	24877	2.2	31579	28807	2.6	2515	27968	2.0	2912	22309	1019.1	789	22448	393.0
	9	0.10	0	4041	0.3	95	6490	0.4	0	5029	0.1	0	3653	16.0	0	4539	0.5
		0.50	0	12248	0.9	4475	16862	1.3	0	14389	0.7	0	8974	336.4	0	12258	1.9
		0.80	5838	19365	1.4	14139	23388	1.9	145	21958	1.2	73	14346	913.4	140	16037	154.
		1.00	23001	25099	2.0	28037	26838	2.6	1439	29700	1.9	2231	21493	1000.7	450	21273	380.
	10	0.10	0	3646	0.2	180	5128	0.3	0	4237	0.1	0	3328	13.0	0	4176	0.4
		0.50	5	12952	0.8	6521	14687	1.1	5	14187	0.6	0	9219	255.5	0	12910	1.8
		0.80	6926	21602	1.4	20906	21365	1.7	331	24471	1.3	993	15839	908.3	544	17259	192.
		1.00	13299	25286	1.8	30759	24897	2.2	3384	30429	1.8	4352	22692	1013.0	3806	21980	378.2
	11	0.10	0	4686	0.3	3574	6800	0.4	0	5715	0.1	0	4176	22.5	0	5394	0.5
		0.50	542	12145	0.7	11967	16322	1.1	0	14044	0.6	0	8509	262.0	0	12242	2.
		0.80	2403	19234	1.3	13307	22450	1.8	169	22770	1.2	181	14173	707.2	181	16983	139.3
		1.00	17012	25742	1.9	33457	28041	2.3	1993	29217	1.8	1754	21984	1000.3	1085	23480	282.
	12	0.10	0	4532	0.2	432	6597	0.4	0	5091	0.1	0	3829	22.3	0	4970	0.
		0.50	1735	14863	1.0	11285	19083	1.8	190	18036	0.9	0	10308	409.1	0	14729	2.
		0.80	5453	21292	1.6		25065	2.3	2103	25600	1.5	329	15930	804.4	8	17364	168.
		1.00	27342	27460	2.1	30033	28396	2.7	5144	31464	2.0	1377	22319	1017.0	756	22552	305.
	13	0.10	0	5221	0.2	296	6592	0.4	0	5841	0.1	0	4616	26.8	0	5805	0.
		0.50	0	13085	0.9	7155	19246	1.4	0	14707	0.7	0	8252	375.0	0	12694	1.
		0.80	1964	17975	1.4	9505	23430	2.1	0	20666	1.3	0	12354	660.6	0	17397	8.
		1.00	41279	25331	2.1	38544	27805	2.6	2743	28391	2.0	3643	22878	1006.4	1155	23725	423.
	14	0.10	0	4056	0.3	479	7611	0.5	0	4972	0.1	0	3734	23.0	0	4721	0.:
		0.50	84	11605	0.8	5315	15639	1.2	10	13875	0.7	0	7936	267.2	0	11951	2.0
		0.80	2927	17229	1.5	10246	21418	1.9	75	19858	1.2	0	11664	751.6	5	13994	113.
		1.00	33164	25902	2.3	40044	28222	2.7	4513	30282	2.0	1217	22897	1020.9	886	22179	333.2

Table A.15: Performance of CH-O, CH-R, CH-S, LS-O, and LS-S for instance size (Sz.) 1000, instance numbers (Nr.) 15–29.

		1	1	СН-О			CH-R			CH-S		1	LS-O		1	LS-S	
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	C _u	t[s]	Ca	C_u	t[s]	C_{a}	$\mathbf{C}_{\mathbf{u}}$	t[s]	Ca	C _u	t[s]	C_a	Cu	t[s]
1000	15	0.10	0	5059	0.3	1675	6601	0.4	0	5843	0.1	0	4567	26.0	0	5690	0.5
		0.50	0	13171	0.8	2381	16368	1.1	0	15489	0.6	0	10002	290.7	0	13783	1.8
		0.80 1.00	755	20466	1.3 2.1	10115 28888	22057	1.6 2.4	35 3188	22961	1.2	415 2758	15160	911.2 1007.4	440 281	16561 22455	157.8
	16	0.10	11479 0	25693 4785	0.3	20000	26727 5519	0.3	5188	30088 5813	1.9 0.1	2738	22403 3772	1007.4	281	5400	353.8 0.4
	10	0.50	0	12126	0.6	5998	14833	1.0	0	13759	0.5	0	9026	290.3	0	13007	1.5
		0.80	1402	17668	1.2	15555	21353	1.6	155	21499	1.1	0	13390	618.1	0	18085	5.1
		1.00	23767	25688	1.8	28961	26463	2.2	1577	29450	1.8	903	20220	1013.3	15	21646	314.4
	17	0.10	0	4004	0.3	4300	5514	0.3	0	4984	0.1	0	3389	22.4	0	4661	0.5
		0.50	31	13453	0.8	4769	16688	1.1	1	16223	0.7	1	9936	405.5	1	11590	59.7
		0.80	9575	22070	1.5	17569	24406	2.0	1889	26220	1.4	226	16870	850.7	156	17902	159.3
	18	1.00 0.10	18155 0	24743 4181	1.8 0.2	40331	27112 5835	2.1	3879 0	30238 5054	1.6	1012	21500 3779	1014.9 23.8	420 0	22546 4801	275.1 0.4
	10	0.10	470	12661	0.2	13274	18247	1.3	35	15322	0.1	0	8971	378.0	0	13450	2.0
		0.80	3164	20325	1.6	28276	24765	2.1	483	22891	1.4	0	13489	928.1	0	20619	6.0
		1.00	40578	26496	2.3	71771	29686	3.0	2094	30215	2.2	389	21162	1001.7	490	21208	310.0
	19	0.10	0	4393	0.2	959	6433	0.4	0	5149	0.1	0	3739	26.8	0	4837	0.4
		0.50	21	13148	0.9	4783	19214	1.4	21	14948	0.7	11	9297	404.5	21	10868	50.2
		0.80	981	19866	1.5	15869	25079	2.1	21	22115	1.2	11	14222	883.4	36	15861	132.4
	20	1.00 0.10	26342 0	27032 6019	2.1 0.3	54438 782	29111 8487	2.8 0.5	1846 0	29894 6842	1.9 0.2	644 0	22904 5320	1016.7 42.6	506 0	21920 6492	317.2 0.7
	20	0.10	201	12484	0.3	1994	14251	1.2	0	14119	0.2	0	9306	311.6	0	12735	1.8
		0.80	809	20242	1.4	9509	22372	1.9	635	22541	1.2	625	14714	884.2	0	20371	10.5
		1.00	16688	26072	2.0	38667	27244	2.4	5921	28323	1.7	4495	22454	1019.1	370	22364	373.7
	21	0.10	0	4786	0.3	373	7937	0.5	0	5932	0.2	0	4247	39.5	0	5313	0.7
		0.50	0	12449	1.1	6377	16635	1.5	0	14753	0.9	0	8916	497.6	0	12943	2.5
		0.80	1007	17922	1.5	17534	22065	1.9	138	20535	1.3	273	13164	1007.3	89	14567	166.2
	22	1.00	21296	23839	2.0	43268	26489	2.7	2013	29574	1.9	1951	23347	1011.5	797	21333	452.4
	22	0.10 0.50	0 1661	4254 13169	0.3	1187 10071	6232 18168	0.4	0 477	5252 15233	0.1	0	3867 8246	26.2 458.7	0	4828 12930	0.6 2.4
		0.30	3284	20370	1.2	29163	23258	2.6	494	22936	1.6	9	13864	1007.6	0	12930	20.1
		1.00	20535	25283	2.5	68871	27012	3.1	3914	29400	2.3	4303	22148	1012.7	2298	21630	462.5
	23	0.10	5	4924	0.2	1250	6196	0.3	5	5429	0.1	0	4277	24.3	0	5379	0.5
		0.50	36	13865	0.8	6538	16844	1.2	20	15186	0.6	0	9885	387.1	10	10944	56.8
		0.80	3892	19010	1.4	17020	23117	1.8	50	22561	1.2	169	13448	779.1	30	15052	143.7
	24	1.00	41686	27139	2.1	61185	29942	2.5	4382	31180	1.9	1978	22948	1005.2	784	22996	389.8
	24	0.10 0.50	0 27	3960 13225	0.2	0 2777	4600 14779	0.3	02	4724 15007	0.1	02	3505 10199	13.4 341.4	02	4522 11877	0.4 43.5
		0.30	6348	21149	1.6	15229	22423	1.1	1239	23469	1.4	873	16355	990.4	1128	17690	197.4
		1.00	14117	24242	2.0	26530	26305	2.5	4185	28953	2.0	2682	21888	1004.8	976	21728	358.4
	25	0.10	0	4824	0.3	793	7973	0.5	0	6523	0.1	0	4317	36.3	0	6159	0.6
		0.50	0	12830	1.1	7232	19970	1.7	0	14713	0.8	0	8892	384.4	0	13129	2.2
		0.80	6483	21993	1.8	20834	26704	2.4	599	24887	1.7	0	15233	1007.1	0	21138	20.7
		1.00	26302	27529	2.5	55995	30587	3.1	2180	31566	2.4	1761	22097	1006.7	5	21479	417.0
	26	0.10	0	5847	0.3	356	7035	0.4	0	6429	0.1	0	4823	26.9	0	6229	0.5
		0.50 0.80	20 2238	13267 20437	0.8 1.4	4944 12939	15823 22721	1.1 1.8	30 524	14646 23118	0.6	0	9548 15514	312.5 950.1	0	13624 21218	1.7 19.1
		1.00	20349	25838	2.0	35679	26706	2.2	2452	28831	1.2	1534	21892	1007.1	313	21218	361.5
	27	0.10	0	3375	0.2	0	5355	0.4	25	4175	0.1	0	3144	13.9	0	4125	0.4
		0.50	0	9773	0.6	4461	13440	1.1	25	11563	0.4	0	7396	190.8	0	10657	1.2
		0.80	907	17975	1.6	10136	21828	2.3	25	19861	1.2	0	12740	758.8	0	17960	5.6
		1.00	12904	25403	2.3	35436	28267	3.2	1510	28785	2.1	315	22238	1003.0	0	24187	97.5
	28	0.10	0	5476	0.3	1696	8154	0.5	0	6467	0.1	0	4821	25.9	0	6203	0.5
		0.50	15	13716	0.7	3905	18345	1.3	15	14871	0.6	10	10416	262.9	10	11352	43.3
		0.80 1.00	1613 23215	20802 27435	1.5 2.3	11516 49253	26213 32750	2.1	335 2796	23182 31563	1.4	10 2137	15178 23282	832.4 1007.2	220 1569	16584 24318	140.0 389.7
	29	0.10	0	5329	0.2	89	7895	0.4	2790	5978	0.1	0	4442	30.0	1309	5801	0.5
	_/	0.50	215	12357	0.2	6702	16262	1.4	0	13893	0.7	0	8549	305.7	0	13195	4.5
		0.80	8784	19037	1.5	28753	23123	2.1	2694	22283	1.2	1388	14329	634.6	575	15769	138.0
		1.00	43662	24147	2.0	51276	26103	2.5	10396	28020	1.7	6548	21979	1005.4	735	21436	356.8

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Table A.16: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 20, instance numbers (Nr.) 0–14.

		_		VND-O			VND-S			GRASE			MA-0			VNS-	
Sz.	Nr.	Load	C_a	C_u	t[s]	Ca	C_u	t[s]	C_a	C_u	t[s]	C_a	C_u	t[s]	Ca	C_u	t[
20	0	0.10	0	469	0.1	0	595	0.0	0	469	200.0	0	469	200.0	0	469	20
		0.50	0	829	0.3	0	1023	0.0	0	813	200.0	0	778	200.0	0	778	20
		0.80	0	1102	0.4	0	1269	0.0	0	1073	200.0	0	1001	200.0	0	977	20
		1.00	0	1164	0.9	0	1382	0.0	0	1166	200.0	0	1101	200.0	0	1084	20
	1	0.10	0	361	0.0	0	377	0.0	0	351	200.0	0	351	200.0	0	351	20
		0.50	0	719	0.3	0	910	0.0	0	702	200.0	0	686	200.0	0	668	20
		0.80	0	820	0.2	0	989	0.0	0	737	200.0	0	730	200.0	0	703	20
		1.00	0	976	0.6	0	1139	0.0	0	871	200.0	0	813	200.0	0	871	20
	2	0.10	0	484	0.0	0	511	0.0	0	484	200.0	0	484	200.0	0	484	20
		0.50	0	753	0.3	0	1026	0.0	0	753	200.0	0	722	200.0	0	717	20
		0.80	0	954	0.6	0	1340	0.0	0	913	200.0	0	893	200.0	0	893	20
		1.00	0	1139	1.1	0	1451	0.0	0	1086	200.0	0	1051	200.0	0	1061	20
	3	0.10	0	535	0.0	0	739	0.0	0	535	200.0	0	467	200.0	0	467	20
		0.50	0	954	0.2	0	1147	0.0	0	953	200.0	0	869	200.0	0	869	20
		0.80	0	1067	0.4	0	1309	0.0	0	1107	200.0	0	1060	200.0	0	1060	20
		1.00	0	1318	0.6	0	1582	0.0	0	1291	200.0	0	1239	200.0	0	1239	20
	4	0.10	0	497	0.0	0	543	0.0	0	506	200.0	0	497	200.0	0	497	20
		0.50	0	818	0.2	0	1048	0.0	0	840	200.0	0	772	200.0	0	772	20
		0.80	0	989	0.4	0	1248	0.0	0	963	200.0	0	918	200.0	0	964	20
		1.00	0	1217	0.5	0	1399	0.0	0	1090	200.0	0	1075	200.0	0	1075	20
	5	0.10	0	401	0.0	0	464	0.0	0	401	200.0	0	401	200.0	0	401	20
		0.50	0	832	0.2	0	978	0.0	0	764	200.0	0	733	200.0	0	733	20
		0.80	0	888	0.4	0	1166	0.0	0	888	200.0	0	854	200.0	0	854	20
		1.00	0	1046	0.5	0	1177	0.0	0	1046	200.0	0	935	200.0	0	966	20
	6	0.10	0	382	0.0	0	493	0.0	0	374	200.0	0	374	200.0	0	374	20
		0.50	0	832	0.2	0	1143	0.0	0	840	200.0	0	791	200.0	0	785	20
		0.80	0	1154	0.5	0	1406	0.0	0	1075	200.0	0	1075	200.0	0	1041	20
		1.00	0	1422	0.2	0	1446	0.0	0	1287	200.0	0	1231	200.0	0	1256	20
	7	0.10	0	543	0.1	0	819	0.0	0	521	200.0	0	521	200.0	0	521	20
		0.50	0	982	0.3	0	1289	0.0	0	966	200.0	0	931	200.0	0	979	20
		0.80	0	1237	0.4	0	1418	0.0	0	1079	200.0	0	1116	200.0	0	1076	20
		1.00	0	1461	0.5	0	1593	0.0	0	1322	200.0	0	1263	200.0	0	1263	20
	8	0.10	0	351	0.0	0	656	0.0	0	347	200.0	0	347	200.0	0	347	20
	0	0.50	0	826	0.3	0	1135	0.0	0	767	200.0	0	752	200.0	0	752	20
		0.80	0	1070	0.4	0	1246	0.0	0	1039	200.0	0	1033	200.0	0	1022	20
		1.00	0	1246	0.4	0	1442	0.0	0	1163	200.0	0	1109	200.0	0	1119	20
	9	0.10	0	649	0.1	0	754	0.0	0	643	200.0	0	643	200.0	0	643	20
		0.50	0	984	0.4	0	1094	0.0	0	984	200.0	0	984	200.0	0	984	20
		0.80	0	1071	0.6	0	1163	0.0	0	1071	200.0	0	1042	200.0	0	1042	20
		1.00	0	1197	1.3	0	1466	0.0	0	1210	200.0	0	1109	200.0	0	1106	20
	10	0.10	0	598	0.0	0	660	0.0	0	598	200.0	0	559	200.0	0	559	20
	10	0.10	0	944	0.0	0	1093	0.0	0	905	200.0	0	905	200.0	0	923	20
		0.80	0	1252	0.2	0	1374	0.0	0	1143	200.0	0	1098	200.0	0	1098	20
		1.00	530	1465	0.5	0	1538	0.3	0	1297	200.0	0	1220	200.0	0	1209	20
	11	0.10	0	217	0.0	0	308	0.0	0	217	200.0	0	217	200.0	0	217	20
		0.50	0	703	0.1	0	933	0.0	0	579	200.0	0	579	200.0	0	579	20
		0.80	0	873	0.5	0	1077	0.0	0	713	200.0	0	633	200.0	0	633	20
		1.00	0	942	0.3	0	1077	0.0	0	919	200.0	0	919	200.0	0	854	20
	12	0.10	0	557	0.0	0	573	0.0	0	557	200.0	0	499	200.0	0	499	20
	12	0.50	0	981	0.3	0	1153	0.0	0	887	200.0	0	848	200.0	0	801	20
		0.80	0	1137	0.8	0	1488	0.0	0		200.0	0	999	200.0	0	999	
		1.00	0	1227	1.1	0	1611	0.0	0	1184	200.0	0	1091	200.0	0	1136	20
	13	0.10	0	480	0.1	0	644	0.0	0	480	200.0	0	480	200.0		476	20
	15	0.10	0	708	0.1	0	940	0.0	0	705	200.0	0	696	200.0		696	20
						0									1		20
		0.80	0	1109	0.5	0	1330	0.0		987	200.0	0	938	200.0		950	
	14	1.00	0	1224	0.5		1338	0.0		1122	200.0	0	1046	200.0	0	1020	20
	14	0.10	0	539	0.0	0	636	0.0		539	200.0	0	535	200.0		535	20
		0.50	0	818	0.4	0	1195	0.0		828	200.0	0	818	200.0	0	818	20
		0.80	0	1115	0.9	0	1310	0.0	0	1026	200.0	0	950	200.0	0	967	20
		1.00	0	1267	0.6	0	1344	0.0	0	1220	200.0	0	1164	200.0	0	1164	20

Table A.17: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 20, instance numbers (Nr.) 15–29.

				VND-O			VND-S			GRASP			MA-C			VNS-	
Sz.	Nr.	Load	C_a	C_u	t[s]	C_a	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	C_a	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]
20	15	0.10	0	424	0.0	0	524	0.0	0	318	200.0	0	318	200.0	0	318	200.0
		0.50	0	678	0.2	0	951	0.0	0	678	200.0	0	570	200.0	0	597	200.0
		0.80	0	930	0.5	0	1140	0.0	0	865	200.0	0	837	200.0	0	822	200.0
		1.00	0	1077	0.5	0	1299	0.0	0	1012	200.0	0	967	200.0	0	967	200.0
	16	0.10	0	734	0.1	0	987	0.0	0	611	200.0	0	611	200.0	0	649	200.0
		0.50	0	895	0.5	0	1373	0.0	0	895	200.0	0	895	200.0	0	893	200.0
		0.80	0	1227	0.8	0	1564	0.0	0	1186	200.0	0	1137	200.0	0	1137	200.0
		1.00	0	1285	0.8	0	1564	0.0	0	1263	200.0	0	1186	200.0	0	1263	200.0
	17	0.10	0	453	0.0	0	563	0.0	0	453	200.0	0	453	200.0	0	453	200.0
		0.50	0	982	0.2	0	1061	0.0	0	935	200.0	0	935	200.0	0	950	200.0
		0.80	0	1169	0.3	0	1356	0.0	0	1131	200.0	0	1093	200.0	0	1093	200.0
	10	1.00	0	1382	0.5	0	1561	0.0	0	1280	200.0	0	1181	200.0	0	1215	200.0
	18	0.10	0	467	0.0	0	528	0.0	0	439	200.0	0	439	200.0	0	439	200.0
		0.50	0	891	0.3	0	1112	0.0	0	858	200.0	0	786	200.0	0	783	200.0
		0.80	180	1093	0.5	180	1271	0.2	0	964	200.0	0	906	200.0	0	956	200.0
	10	1.00	325	1216	0.5	325	1328	0.2	0	1080	200.0	0	1030	200.0	0	1080	200.0
	19	0.10	0	603	0.0	0	662	0.0	0	579	200.0	0	579	200.0	0	584	200.0
		0.50	0	981	0.4	0	1449	0.0	0	957	200.0	0	929	200.0	0	929	200.0
		0.80	0	1226	0.7	0	1535	0.0	0	1225	200.0	0	1199	200.0	0	1199	200.0
	•	1.00	0	1318	0.6	0	1535	0.0	0	1318	200.0	0	1318	200.0	0	1318	200.0
	20	0.10	0	371	0.0	0	434	0.0	0	371	200.0	0	371	200.0	0	381	200.0
		0.50	0	640	0.3	0	1182	0.0	0	640	200.0	0	626	200.0	0	625	200.0
		0.80	0	835	0.5	0	1322	0.0	0	773	200.0	0	742	200.0	0	773	200.0
	21	1.00	0	1008	0.6	0	1366	0.0	0	1008	200.0	0	897	200.0	0	913	200.0
	21	0.10	0	475	0.0	0	484	0.0	0	475	200.0	0	421	200.0	0	421	200.0
		0.50	0	758	0.1	0	898	0.0	0	711	200.0	0	675	200.0	0	675	200.0
		0.80	0	952	0.3	0	1052	0.0	0	905	200.0	0	882	200.0	0	882	200.0
		1.00	0	1148	0.4	0	1268	0.0	0	1024	200.0	0	1024	200.0	0	1024	200.0
	22	0.10	0	646	0.0	0	647	0.0	0	524	200.0	0	524	200.0	0	524	200.0
		0.50	0	747	0.2	0	748	0.0	0	676	200.0	0	670	200.0	0	709	200.0
		0.80	0	898	0.8	0	1120	0.0	0	849	200.0	0	802	200.0	0	841	200.0
	22	1.00	0	905	1.1	0	1127	0.0	0	905	200.0	0	887	200.0	0	894	200.0
	23	0.10	0	535	0.0	0	591	0.0	0	535	200.0	0	499	200.0	0	535	200.0
		0.50	0	779	0.3	0	1180	0.0	0	779	200.0	0	727	200.0	0	766	200.0
		0.80	0	972	0.4	0	1490	0.0	0	923	200.0	0	908	200.0	0	908	200.0
	24	1.00	0	1131	0.5	0	1515	0.0	0	1037	200.0	0	1062	200.0	0	1039	200.0
	24	0.10	0	683	0.1	0	716	0.0	0	643	200.0	0	620	200.0	0	620	200.0
		0.50	0	910	0.2	0	1044	0.0	0	910	200.0	0	910	200.0	0	910	200.0
		0.80	0	1153	0.3	0	1184	0.0	0	1044	200.0	0	965	200.0	0	1018	200.0
	25	1.00	0	1247	0.6	0	1258	0.0	0	1153	200.0	0	1131	200.0	0	1131	200.0
	25	0.10	0	432	0.0	0	525	0.0	0	432	200.0	0	432	200.0	0	432	200.0
		0.50	0	827	0.1	0	955	0.0	0	827 993	200.0	0	809	200.0	0	809	200.0
		0.80	0	1043 1043	0.2	0	1195 1481	0.0	0		200.0		993 1043	200.0 200.0	0	993	200.0
	24	1.00	0		0.5			0.0	0	1043		0			0	1043	200.0
	26	0.10	0	369	0.0	0	559	0.0	0	369	200.0		369	200.0	0	369	200.0
		0.50	0	926	0.3		1024	0.0		819	200.0	0	866	200.0	0	866	200.0
		0.80	0	1024	0.2	0	1047	0.0	0	939	200.0		939	200.0	0	939	200.0
	27	1.00	0	1112	0.5	0	1240	0.0 0.0	0	1076 607	200.0	0	1065 607	200.0	0	1034	200.0
	27	0.10	0	607	0.0		687				200.0			200.0	0	607	
		0.50	0	872	0.4	0	1086	0.0	0	872	200.0	0	807	200.0	0	807	200.0
		0.80	0	1068	0.8	0	1376			1068	200.0	0	1068	200.0	0	1068	
	20	1.00	0	1202	0.9	0	1538	0.0	0	1147	200.0		1147	200.0	0	1109	200.0
	28	0.10	0	552	0.0	0	584	0.0	0	531	200.0		531	200.0	0	531	200.0
		0.50	0	953	0.5	0	1247	0.0	0	946	200.0		946	200.0	0	946	
		0.80	0	1201	0.6	0	1370	0.0	0	1179	200.0	0	1076	200.0	0	1141	200.0
	20	1.00	30	1341	0.7	30	1436	0.6	0	1270	200.0	0	1196	200.0	0	1152	200.0
	29	0.10	0	387	0.0	0	460	0.0	0	383	200.0	0	369	200.0	0	369	200.0
		0.50	0	593	0.2	0	1040	0.0	0	574	200.0		574	200.0	0	574	200.0
		0.80 1.00	0	878 1162	0.5	0	1285	0.0	0	876	200.0	0	876	200.0	0	855 1085	200.0
				1162	0.6	0	1377	0.0	0	1121	200.0	0	1064	200.0	0	11185	1 20010

Table A.18: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 30, instance numbers (Nr.) 0–14.

				VND-O)		VND-S		(GRASP	-0		MA-0)		VNS-0	Э
Sz.	Nr.	Load	Ca	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	C_{a}	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]
30	0	0.10	0	611	0.0	0	633	0.0	0	606	200.0	0	606	200.0	0	606	200.0
		0.50	0	1020	0.5	0	1371	0.0	0	1036	200.0	0	1016	200.0	0	1016	200.0
		0.80 1.00	0	1425 1545	0.9 1.2	0	1598 1750	0.0 0.0	0	1270 1482	200.0 200.0	0	1254 1425	200.0 200.0	0	1270 1389	200.0 200.0
	1	0.10	0	711	0.1	0	932	0.0	0	694	200.0	0	677	200.0	0	682	200.0
		0.50	0	1211	0.9	0	1759	0.0	0	1126	200.0	0	1079	200.0	0	1082	200.0
		0.80	0	1510	1.8	0	2183	0.0	0	1392	200.0	Ő	1359	200.1	Õ	1351	200.0
		1.00	0	1820	2.6	0	2457	0.1	0	1641	200.0	0	1547	200.0	0	1547	200.0
	2	0.10	0	898	0.1	0	1366	0.0	0	826	200.0	0	826	200.0	0	826	200.0
		0.50	0	1527	1.3	0	2027	0.0	0	1508	200.0	0	1378	200.0	0	1409	200.0
		0.80	0	1734	1.3	0	2353	0.0	0	1629	200.0	0	1610	200.0	0	1539	200.0
		1.00	0	2151	1.1	0	2432	0.0	0	1879	200.0	0	1835	200.1	0	1822	200.1
	3	0.10	0	679	0.1	0	1000	0.0	0	679	200.0	0	679	200.0	0	679	200.0
		0.50 0.80	0	1083 1258	0.7 2.2	0	1399 1879	0.0 0.0	0	1095 1166	200.0 200.0	0	1083 1118	200.0 200.0	0	1083 1118	200.0 200.0
		1.00	0	1700	2.2	0	2146	0.0	0	1641	200.0	0	1612	200.0	0	1593	200.0
	4	0.10	0	701	0.1	0	994	0.0	0	712	200.0	0	701	200.0	0	701	200.0
		0.50	0	1265	0.9	0	1629	0.0	0	1251	200.0	0	1217	200.0	Õ	1251	200.0
		0.80	0	1436	1.0	0	1715	0.0	0	1384	200.0	0	1365	200.0	0	1290	200.0
		1.00	0	1628	1.2	0	1807	0.0	0	1628	200.0	0	1577	200.0	0	1539	200.0
	5	0.10	0	606	0.1	0	737	0.0	0	612	200.0	0	606	200.0	0	606	200.0
		0.50	0	1164	0.8	0	1480	0.0	0	1076	200.0	0	1043	200.0	0	1043	200.0
		0.80	0	1384	0.9	0	1700	0.0	0	1246	200.0	0	1263	200.0	0	1243	200.1
	6	1.00	0	1459 624	1.4		1899	0.0	0	1387 624	200.0	0	1405	200.0	0	1405	200.0
	6	0.10 0.50	0	624 1199	0.1 0.6	0	971 1587	0.0 0.0	0	624 1127	200.0 200.0	0	623 1131	200.0 200.0	0	623 1136	200.0 200.0
		0.30	0	1344	1.3	0	1786	0.0	0	1323	200.0	0	1326	200.0	0	1299	200.0
		1.00	0	1456	1.9	0	1880	0.0	0	1473	200.0	0	1388	200.0	0	1384	200.0
	7	0.10	0	671	0.1	0	910	0.0	0	671	200.0	0	671	200.0	0	671	200.0
		0.50	0	1226	1.2	0	1633	0.0	0	1222	200.0	0	1175	200.0	0	1175	200.0
		0.80	0	1469	2.8	0	2000	0.0	0	1485	200.0	0	1408	200.1	0	1409	200.0
		1.00	0	1774	3.3	0	2355	0.0	0	1543	200.0	0	1567	200.0	0	1529	200.0
	8	0.10	0	1192	0.2	0	1541	0.0	0	1111	200.0	0	1111	200.0	0	1111	200.0
		0.50	0	1316	1.0	0	1707	0.0	0	1269	200.0	0	1269	200.0	0	1269	200.0
		0.80 1.00	0	1569 1606	1.6 2.7	00	1773 1773	0.0 0.0	0	1521 1578	200.0 200.0	0	1490 1578	200.0 200.0	0	1504 1578	200.0 200.0
	9	0.10	0	571	0.1	0	822	0.0	0	571	200.0	0	571	200.0	0	571	200.0
		0.50	0	1157	0.8	0	1924	0.0	0	1129	200.0	0	1124	200.0	0	1129	200.0
		0.80	0	1468	1.7	0	2205	0.0	0	1441	200.0	Ő	1401	200.0	Õ	1401	200.0
		1.00	0	1788	1.7	0	2336	0.0	0	1690	200.0	0	1626	200.0	0	1626	200.0
	10	0.10	0	593	0.0	0	610	0.0	0	593	200.0	0	593	200.0	0	593	200.0
		0.50	0	1112	0.7	0	1375	0.0	0	1082	200.0	0	997	200.0	0	1039	200.0
		0.80	0	1228	1.7	0	1662	0.0	0	1228	200.0	0	1153	200.0	0	1200	200.0
	1.1	1.00	0	1349	2.5	0	1930	0.0	0	1345 597	200.0	0	1335	200.0	0	1307	200.0
	11	0.10 0.50	0	608 1155	0.1 0.7		985 1507	0.0 0.0	0	1144	200.0 200.0		597 1122	200.0 200.0	0	597 1109	200.0 200.0
		0.80	0	1634	1.3	0	1990	0.0	0	1590	200.0	0	1447	200.0	0	1438	200.0
		1.00	0	2071	1.5	0	2184	0.1	0	1868	200.0	0	1811	200.0	0	1722	200.0
	12	0.10	0	759	0.0	0	871	0.0	0	754	200.0	0	754	200.0	0	754	200.0
		0.50	0	1071	0.5	0	1467	0.0	0	1071	200.0	0	1071	200.0	0	1071	200.0
		0.80	0	1311	2.1	0	1923	0.0	0	1311	200.0	0	1311	200.1	0	1311	200.0
		1.00	0	1719	1.8	0	2160	0.0	0	1694	200.0	0	1650	200.1	0	1634	200.0
	13	0.10	0	542	0.0	0	698	0.0	0	542	200.0	0	542	200.0	0	542	200.0
		0.50	0	1081	0.7	0	1462	0.0	0	1081	200.0	0	1061	200.0	0	1061	200.0
		0.80 1.00	0	1448 1505	1.3 1.4	0	1717 2014	0.0 0.0	0	1498 1505	200.0 200.0	0	1407 1455	200.0 200.0	0	1386 1482	200.0 200.0
	14	0.10	0	777	0.1		935	0.0	0	759	200.0		749	200.0	0	749	200.0
	14	0.10	0	1346	0.1	0	1911	0.0	0	1318	200.0	0	1285	200.0	0	1285	200.0
		0.80	0	1451	1.9	0	1974	0.0	0	1419	200.0	0	1376	200.0	0	1403	200.0
		1.00	0	1652	2.0	0	2109	0.0	0	1595	200.0	0	1583	200.1	0	1583	200.1

Table A.19: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 30, instance numbers (Nr.) 15–29.

				VND-O			VND-S			GRASP			MA-0			VNS-	
Sz.	Nr.	Load	C_a	C_u	t[s]	C_a	C_u	t[s]	C_a	C_u	t[s]	C_a	C_u	t[s]	C_a	C_u	t[s]
30	15	0.10	0	993	0.1	0	1245	0.0	0	977	200.0	0	977	200.0	0	993	200.0
		0.50	0	1298	0.9	0	1633	0.0	0	1298	200.0	0	1270	200.0	0	1270	200.0
		0.80	0	1520	2.7	0	1911	0.0		1510	200.0	0	1512	200.1	0	1499	200.0
	17	1.00	0	1726	3.0	0	2304	0.0	0	1729	200.0	0	1693	200.1	0	1689	200.0
	16	0.10	0	461	0.1	0	633	0.0		461	200.0	0	458	200.0	0	458	200.0
		0.50	0	957	0.6	0	1382	0.0	0	969	200.0	0	916	200.0	0	916	200.0
		0.80	0	1159	1.4		1778	0.0		1045	200.0	0	971	200.0		996	200.
	17	1.00	0	1547	1.0	0	1900 1027	0.0		1371 795	200.0	0	1272	200.0 200.0	0	1249	200. 200.
	17	0.10 0.50	0	780 1132	0.3		1427	0.0 0.0		1096	200.0 200.0	0	776 1066	200.0	0	776 1043	200.
		0.30	0	1463	1.2	0	1849	0.0	0	1388	200.0	0	1319	200.0	0	1313	200.
		1.00	0	1648	1.5	0	1999	0.0	0	1588	200.0	0	1519	200.0	0	1515	200.
	18	0.10	0	561	0.0	0	579	0.0	0	561	200.0	0	561	200.0	0	561	200.
	10	0.10	0	1235	0.0	0	1458	0.0	0	1200	200.0	0	1192	200.0	0	1192	200.
		0.30	0	1534	0.7	0	1438	0.0	0	1532	200.0	0	1532	200.0	0	1532	200.
		1.00	0	1746	1.2	0	2181	0.0	0	1746	200.0	0	1714	200.0	0	1714	200.
	19	0.10	0	516	0.1	0	1005	0.0	0	502	200.0	0	502	200.0	0	502	200.
	1)	0.10	0	1270	1.3	0	1774	0.0	0	1263	200.0	0	1251	200.0	0	1205	200.
		0.80	0	1518	2.3	0	2065	0.0	0	1532	200.0	0	1420	200.0	0	1420	200.
		1.00	0	1760	2.5	0	2294	0.0	0	1695	200.0	0	1593	200.0	0	1624	200
	20	0.10	0	817	0.2	0	1222	0.0	0	896	200.0	0	803	200.0	0	803	200
	20	0.10	0	1490	0.2	0	1824	0.0	0	1465	200.0	0	1450	200.0	0	1423	200.
		0.80	0	1686	2.3	0	2011	0.0	0	1686	200.0	0	1672	200.0	0	1646	200.
		1.00	0	1842	3.2	0	2170	0.0	0	1794	200.0	0	1792	200.0	0	1792	200.
	21	0.10	0	640	0.1	0	897	0.0	0	640	200.0	0	581	200.0	0	581	200.
		0.50	0	993	0.4	0	1283	0.0	0	1003	200.0	0	928	200.0	0	928	200
		0.80	Ő	1420	0.8	0	1636	0.0	0	1242	200.0	0	1239	200.0	0	1239	200
		1.00	0	1647	1.1	0	1875	0.0	0	1422	200.0	0	1430	200.0	0	1399	200.
	22	0.10	0	905	0.1	0	1003	0.0	0	929	200.0	0	881	200.0	0	881	200.
		0.50	0	1380	0.6	0	1554	0.0	0	1333	200.0	0	1312	200.0	0	1338	200.
		0.80	0	1418	1.6	0	1813	0.0	0	1430	200.0	0	1395	200.0	0	1395	200.
		1.00	0	1631	1.4	0	2249	0.0	0	1565	200.0	0	1531	200.0	0	1490	200.
	23	0.10	0	738	0.1	0	876	0.0	0	738	200.0	0	738	200.0	0	738	200.
		0.50	0	1199	0.8	0	1724	0.0	0	1201	200.0	0	1187	200.0	0	1187	200
		0.80	0	1413	2.2	0	2263	0.0	0	1440	200.0	0	1376	200.0	0	1346	200.
		1.00	0	1667	3.3	0	2282	0.0	0	1721	200.0	0	1598	200.0	0	1531	200
	24	0.10	0	547	0.1	0	630	0.0	0	525	200.0	0	525	200.0	0	525	200.
		0.50	0	1278	0.5	0	1547	0.0	0	1278	200.0	0	1205	200.0	0	1205	200.
		0.80	0	1627	1.9	0	2203	0.0	0	1575	200.0	0	1427	200.0	0	1352	200
		1.00	0	2046	2.1	0	2375	0.0	0	1753	200.0	0	1701	200.0	0	1670	200
	25	0.10	0	972	0.3	0	1222	0.0	0	952	200.0	0	849	200.0	0	869	200.
		0.50	0	1514	1.5	0	1772	0.0	0	1389	200.0	0	1286	200.0	0	1355	200
		0.80	0	1560	2.2	0	1970	0.0	0	1550	200.0	0	1532	200.1	0	1520	200
		1.00	0	1765	2.2	0	2190	0.0	0	1679	200.0	0	1630	200.0	0	1652	200
	26	0.10	0	518	0.1	0	763	0.0	0	518	200.0	0	518	200.0	0	518	200
		0.50	0	1009	0.6	0	1479	0.0	0	1009	200.0	0	1009	200.0	0	1009	200
		0.80	0	1364	0.9	0	1941	0.0	0	1270	200.0	0	1258	200.0	0	1258	200
		1.00	0	1517	1.6	0	2047	0.0	0	1517	200.0	0	1477	200.0	0	1505	200
	27	0.10	0	583	0.1	0	751	0.0	0	549	200.0	0	549	200.0	0	549	200
		0.50	0	1044	0.3	0	1390	0.0	0	1029	200.0	0	1017	200.0	0	1013	200
		0.80	0	1407	1.0	0	1740		0	1378	200.0	0	1279	200.0	0	1284	
		1.00	0	1540	1.6	0	2006	0.0	0	1434	200.0	0	1434	200.0	0	1434	200
	28	0.10	0	1002	0.1	0	1031	0.0	0	939	200.0	0	939	200.0	0	939	200
		0.50	0	1232	0.5	0	1422	0.0	0	1232	200.0	0	1232	200.0	0	1232	200
		0.80	0	1547	1.0	0	1769	0.0	0	1501	200.0	0	1413	200.0	0	1413	200
		1.00	0	1608	1.5	0	2013	0.0	0	1568	200.0	0	1473	200.0	0	1473	200
	29	0.10	0	415	0.1	0	872	0.0	0	411	200.0	0	411	200.0	0	411	200
		0.50	0	1065	0.6	0	1524	0.0	0	1024	200.0	0	963	200.0	0	963	200
		0.80	0	1430	1.0	0	1858	0.0	0	1397	200.0	0	1328	200.0	0	1363	200
		1.00	0	1686	0.8	0	1944	0.1	0	1494	200.0	0	1458	200.0	0	1503	200

Table A.20: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 50, instance numbers (Nr.) 0–14.

		1		VND-O	•		VND-S			GRASP	- 0		MA-0)		VNS-0	0
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	Cu	t[s]	Ca	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	C_{a}	Cu	t[s]
50	0	0.10	0	1022	0.3	0	1311	0.0	0	1022	200.0	0	974	200.0	0	988	200.0
		0.50 0.80	0	1840 2281	3.7	0	2499	0.0	0	1832 2224	200.0	0	1796 2242	200.1		1763	200.0
		1.00	0	3116	8.8 6.4	0	3030 3793	0.0	0	2828	200.0 200.0	0	2682	200.0 200.1	0	2173 2501	200.0 200.1
	1	0.10	0	1340	0.4	0	1494	0.0	0	1340	200.0	0	1322	200.0	0	1322	200.0
		0.50	0	1764	1.9	0	2302	0.0	0	1703	200.0	0	1673	200.0	0	1668	200.0
		0.80	0	1956	5.1	0	3096	0.0	0	1960	200.0	0	1906	200.1	0	1937	200.0
		1.00	0	2659	8.2	0	3678	0.0	0	2611	200.0	0	2454	200.1	0	2365	200.0
	2	0.10	0	749	0.1	0	1175	0.0	0	747	200.0	0	719	200.0	0	719	200.0
		0.50	0	1968	2.5	0	2496	0.0	0	1949	200.0	0	1782	200.0	0	1844	200.0
		0.80 1.00	0 95	2326 2636	3.1 5.6	0	3045 3335	0.0	0	2322 2597	200.0 200.0	0	2222 2468	200.1 200.1		2223 2534	200.0 200.1
	3	0.10	0	1118	0.4	0	1588	0.0	0	1140	200.0	0	1118	200.1	0	1131	200.1
	5	0.50	0	1927	2.1	0	2740	0.0	0	1882	200.0	0	1841	200.0	0	1803	200.0
		0.80	0	2355	5.3	0	3323	0.0	0	2245	200.0	0	2108	200.0	0	2082	200.1
		1.00	0	2390	8.6	0	3573	0.1	0	2418	200.0	0	2268	200.1	0	2260	200.1
	4	0.10	0	1182	0.3	0	1360	0.0	0	1193	200.0	0	1182	200.0	0	1182	200.0
		0.50	0	1881	1.5	0	2261	0.0	0	1878	200.0	0	1870	200.1	0	1870	200.0
		0.80	0	2092	5.9	0	2646	0.0	0	2084	200.1	0	2018	200.0		2009	200.0
	5	1.00 0.10	0	2598 1295	9.0 0.3	0	3160 1470	0.0	0	2494 1240	200.1 200.0	0	2387 1238	200.2 200.0		2357 1238	200.1 200.0
	5	0.10	0	1664	2.0	0	2272	0.0	0	1644	200.0	0	1631	200.0	0	1592	200.0
		0.80	0	1920	7.1	0	2824	0.0	0	1924	200.1	0	1905	200.0	0	1905	200.1
		1.00	0	2245	7.1	0	2935	0.0	0	2243	200.1	0	2145	200.1	0	2096	200.1
	6	0.10	0	825	0.2	0	947	0.0	0	801	200.0	0	787	200.0	0	787	200.0
		0.50	0	1749	1.8	0	2096	0.0	0	1698	200.1	0	1698	200.0	0	1698	200.0
		0.80	0	2052	4.5	0	2727	0.0	0	2023	200.0	0	1968	200.0	0	1968	200.1
	7	1.00	0	2450	4.2	0	3087	0.0	0	2389	200.1	0	2320	200.1	0	2303	200.0
	7	0.10 0.50	0	1116 1994	0.1 2.1	0	1227 2324	0.0	0	1116 1980	200.0 200.0	0	1116 1964	200.0 200.0		1116 1964	200.0 200.0
		0.30	0	2300	3.0	0	2703	0.0	0	2243	200.0	0	2207	200.0	0	2191	200.0
		1.00	0	2524	7.0	0	3120	0.0	0	2481	200.0	0	2370	200.0	0	2414	200.0
	8	0.10	0	875	0.3	0	1119	0.0	0	841	200.0	0	798	200.0	0	807	200.0
		0.50	0	1535	3.1	0	2214	0.0	0	1507	200.0	0	1432	200.1	0	1432	200.0
		0.80	0	2002	6.0	0	2899	0.0	0	1993	200.0	0	1920	200.0	0	1865	200.0
		1.00	0	2712	6.6	0	3786	2.8	0	2647	200.1	0	2553	200.0	0	2493	200.0
	9	0.10	0	1392 1793	0.4 2.5	0	1683 2268	0.0	0	1392	200.0	0	1371	200.0	0	1392	200.0
		0.50 0.80	0	2044	4.5	0	2684	0.0	0	1815 2002	200.0	0	1733 1959	200.0 200.1		1733 1933	200.0 200.1
		1.00	0	2437	4.9	0	3260	0.0	0	2422	200.1	0	2289	200.1	0	2293	200.1
	10	0.10	0	973	0.2	0	1094	0.0	0	973	200.0	0	973	200.0	0	973	200.0
		0.50	0	1791	2.4	0	2798	0.0	0	1875	200.0	0	1780	200.1	0	1780	200.1
		0.80	0	2132	3.8	0	3026	0.0	0	2145	200.0	0	2103	200.1	0	2040	200.0
		1.00	0	2635	7.5	0	3419	0.0	0	2501	200.0	0	2433	200.0	0	2450	200.1
	11	0.10	0	916	0.4	0	1402	0.0	0	974	200.0	0	886	200.0		886	200.0
		0.50 0.80	0	1701 2380	2.0 2.8	0	2507 3020	0.0	0	1583 2133	200.0 200.0	0	1534 2097	200.0 200.1		1575 2089	200.0 200.1
		1.00	158	2992	5.4	158	3366	6.7	0	2888	200.0	0	2688	200.1	0	2555	200.1
	12	0.10	0	880	0.4	0	1098	0.0	0	881	200.0	0	872	200.0	0	872	200.0
		0.50	0	1770	2.9	0	2396	0.0	0	1751	200.0	0	1768	200.0	0	1729	200.0
		0.80	0	2305	2.9	0	2670	0.0	0	2183	200.0	0	2099	200.1	0	2130	200.1
		1.00	0	2630	2.3	0	2988	0.5	0	2381	200.1	0	2364	200.0	0	2347	200.0
	13	0.10	0	1086	0.3	0	1438	0.0	0	1086	200.0	0	1081	200.0	0	1081	200.0
		0.50	0	1833	2.6	0	2286	0.0	0	1791	200.0	0	1748	200.1	0	1706	200.1
		0.80 1.00	0	2329	4.5	0	3094	0.0	0	2217 2794	200.1		2108	200.0	0	2092	200.0 200.0
	14	0.10	0	2897 1061	7.2 0.3		3432 1286	0.5	0	1074	200.0 200.0	0	2518 1057	200.1 200.0		2514 1057	200.0
	14	0.10	0	1760	2.3	0	2202	0.0	0	1829	200.0	0	1736	200.0	0	1736	200.0
		0.80	0	2037	5.5	0	2670	0.0	0	2096	200.0	0	1948	200.1	0	1988	200.1
		1.00	0	2179	5.8	0	2773	0.0	0	2207	200.0	0	2157	200.0	0	2015	200.1

Table A.21: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 50, instance numbers (Nr.) 15–29.

-		_		VND-0			VND-S			GRASP			MA-C			VNS-	
Sz.	Nr.	Load	Ca	Cu	t[s]	Ca	Cu	t[s]	Ca	Cu	t[s]	Ca	C_u	t[s]	Ca	Cu	t[s]
50	15	0.10	0	1132	0.3	0	1446	0.0	0	1132	200.0	0	1132	200.0	0	1132	200.0
		0.50	0	2025	2.3		2648	0.0	0	1949	200.0	0	1880	200.1		1907	200.0
		0.80 1.00	0	2205 2898	9.8 5.7		3079 3713	0.0 0.6	0	2191 2883	200.1 200.0	0	2157	200.0 200.1	0	2128 2582	200.0
	16	0.10	0	1162	0.2		1315	0.0	0	1113	200.0	0	2672 953	200.1	0	953	200.1
	10	0.10	0	1635	4.1	0	2250	0.0	0	1472	200.0	0	1493	200.0	0	1456	200.0
		0.80	0	2072	5.5	0	2719	0.0	0	1996	200.0	0	1955	200.0	0	1911	200.0
		1.00	Ő	2319	8.7	0	3149	0.1	Õ	2310	200.2	0	2251	200.1	0	2244	200.1
	17	0.10	0	1103	0.2	0	1376	0.0	0	1110	200.0	0	1103	200.0	0	1110	200.0
		0.50	0	2071	3.0	0	2603	0.0	0	2064	200.0	0	2039	200.0	0	2008	200.0
		0.80	0	2290	4.9	0	3164	0.0	0	2294	200.1	0	2250	200.0	0	2216	200.0
		1.00	0	2732	6.8	0	3886	0.0	0	2697	200.0	0	2726	200.0	0	2553	200.0
	18	0.10	0	839	0.1	0	1093	0.0	0	864	200.0	0	839	200.0	0	839	200.0
		0.50	0	1945	1.4	0	2195	0.0	0	1796	200.0	0	1785	200.0	0	1740	200.
		0.80	0	2395	5.9	0	3104	0.0	0	2347	200.1	0	2281	200.0	0	2248	200.
		1.00	0	2959	7.7	0	3542	1.8	0	2789	200.0	0	2608	200.0	0	2547	200.
	19	0.10	0	961	0.4	0	1361	0.0	0	973	200.0	0	949	200.0	0	949	200.0
		0.50	0	1499	2.8	0	2322	0.0	0	1498	200.0	0	1494	200.0	0	1481	200.
		0.80	0	1963	5.2	0	2743	0.0	0	1865	200.0	0	1777	200.0	0	1736	200.
	20	1.00	0	2613	4.7	0	3012	1.9	0	2392	200.1	0	2351	200.0	0	2243	200.
	20	0.10 0.50	00	1626 1998	0.6 2.6		2150 2566	0.0 0.0	0	1637 1968	200.0 200.0	0	1626 1966	200.0 200.0	0	1626 1966	200. 200.
		0.30	0	2473	4.1	0	2901	0.0	0	2420	200.0	0	2360	200.0	0	2368	200.
		1.00	0	2891	7.2	0	3306	0.0	0	2420	200.1	0	2791	200.1	0	2718	200.
	21	0.10	0	1210	0.2	0	1482	0.0	0	1210	200.1	0	1213	200.0	0	1213	200.
		0.50	Ő	1763	1.9	0	2237	0.0	0	1781	200.0	0	1735	200.0	0	1727	200.
		0.80	Ő	2352	6.2	0	3016	0.0	0	2257	200.0	0	2194	200.0	0	2228	200.
		1.00	0	2681	9.4	0	3399	0.0	0	2687	200.0	0	2599	200.1	0	2617	200.
	22	0.10	0	1355	0.4	0	1808	0.0	0	1305	200.0	0	1305	200.0	0	1338	200.
		0.50	0	2113	2.9	0	2869	0.0	0	2047	200.0	0	2072	200.0	0	2073	200.
		0.80	0	2322	7.7	0	3219	0.0	0	2362	200.1	0	2275	200.1	0	2234	200.
		1.00	0	2833	7.2	0	3488	0.0	0	2618	200.0	0	2593	200.1	0	2583	200.
	23	0.10	0	1035	0.1	0	1279	0.0	0	997	200.0	0	986	200.0	0	986	200.
		0.50	0	2117	3.6	0	2719	0.0	0	2144	200.0	0	2068	200.0	0	2065	200.
		0.80	0	2521	5.1	0	3142	0.0	0	2364	200.1	0	2347	200.1	0	2308	200.
	~ 1	1.00	0	2616	4.4	0	3267	0.0	0	2551	200.0	0	2601	200.0	0	2516	200.
	24	0.10	0	803	0.3	0	1137	0.0	0	784	200.0	0	784	200.0	0	784	200.
		0.50 0.80	0	1570 2159	3.5 6.2		2570 3314	0.0 0.0	0	1553 2181	200.0	0	1444 2181	200.0 200.1	0	1444 2144	200. 200.
		I	0	2139		0			0		200.0	0	2699				200.
	25	1.00 0.10	0	856	7.3 0.6		3533 1343	0.0 0.0	0	2830 856	200.0 200.0	0	823	200.0 200.0	0	2646 823	200.
	25	0.10	0	1720	3.6	0	2413	0.0	0	1556	200.0	0	1537	200.0	0	1537	200.
		0.80	0	2126	3.0	0	2620	0.0	0	2014	200.0	0	1931	200.0	0	1929	200.
		1.00	Ő	2217	3.7	0	2908	0.0	Õ	2206	200.1	0	2087	200.1	0	2073	200.
	26	0.10	0	929	0.1	0	1193	0.0	0	937	200.0	0	920	200.0	0	920	200.
		0.50	0	1680	2.7	0	2406	0.0	0	1709	200.0	0	1646	200.0	0	1625	200.
		0.80	0	2216	6.2	0	2926	0.0	0	2189	200.0	0	2186	200.0	0	2142	200.
		1.00	0	2493	7.5	0	3055	0.0	0	2569	200.0	0	2376	200.0	0	2311	200.
	27	0.10	0	1340	0.3	0	1545	0.0	0	1397	200.0	0	1340	200.0	0	1340	200.
		0.50	0	2008	2.5	0	2469	0.0	0	1999	200.0	0	1998	200.1	0	1998	200.
		0.80	0	2123	5.6	0	2842	0.0	0	2145	200.0	0	2140	200.0	0	2130	200.
		1.00	0	2357	7.0	0	3079	0.0	0	2376	200.1	0	2330	200.0	0	2328	200.
	28	0.10	0	1154	0.6	0	2066	0.0	0	1099	200.0	0	1082	200.0	0	1082	200
		0.50	0	2198	4.5	0	3064	0.0	0	2170	200.0	0	2142	200.0	0	2028	200
		0.80	0	2865	6.1	0	3978	0.0	0	2787	200.1	0	2659	200.1	0	2648	200
	-	1.00	0	3271	8.2	0	4279	0.5	0	3311	200.0	0	3065	200.2	0	2913	200
	29	0.10	0	620	0.2	0	1139	0.0	0	620	200.0	0	620	200.0	0	620	200
		0.50	0	1844	3.1		2721	0.0	0	1808	200.0	0	1754	200.0		1685	200.
		0.80	0	2365 2630	3.8 8.6		3171 3419	0.0 0.0	0	2339 2635	200.0 200.1	0	2295 2483	200.0 200.0	0	2307 2515	200. 200.
		1.00															

Table A.22: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 100, instance numbers (Nr.) 0–14.

se. load C.s. C.s. C.s. C.s. C.s. L.s. L.s. <thl.s.< th=""> <thl.s.< th=""> <thl.s.< th=""> L.s</thl.s.<></thl.s.<></thl.s.<>					VND-0)		VND-S	5	(GRASP	-0		MA-0)		VNS-0	Э
0.50 0 202 1 0 21.01 0 31.49 200.1 0 41.49 200.4 40.40 40.00 1 0.10 0 5566 47.7 0.1 0 3772 0.21 0 41.49 200.1 0 41.49 200.0 0 2575 200.0 0 2575 200.0 0 2575 200.0 0 2575 200.1 0 41.59 200.1 0 4551 200.1 0 4551 200.1 0 4551 200.1 0 2575 200.1 0 2571 200.1 0 2571 200.1 0 2571 200.1 0 2572 200.1 0 2572 200.1 0 2572 200.1 0 2572 200.1 0 2572 200.1 0 2572 200.1 0 2572 200.1 0 2572 200.1 0 2572 200.1 0 2572 <																		
0.80 0 0.00 0.90 0.723 0.1 0.0 0.90 0.0 0.4054 2002 0.10 0.0 0.4054 2002 1 0.10 0 2785 2.30 0 3398 0.1 0.0 2755 200.0 0.0 2757 200.0 0 4759 201.1 0.0 4759 200.1 0.0 4739 200.2 1.00 0 5435 36.0 0.751 0.0 0 2132 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0 1219 200.0 0	100	0																
100 0 906 40.7 0 6178 0.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 2757 200.0 0 200.0 0 200.0 0 200.0 0 200.0 0 200.0 0 200.0 0 200.0 0 200.0 0 200.0 0 200.0 0 200.0 0 200.0 0 200.0 0 200.0 0 200.0 0 200.0																		
1 0.10 0 278 20.0 0 2757 20.00 0 2757 20.01 0 2757 20.01 0 2757 20.01 0 2757 20.01 0 2757 20.01 0 4769 20.01 0 4769 20.01 0 4739 20.01 0.00 0 2757 18.3 0 4052 0.01 0 2352 20.01 0 2121 20.00 0 1219 20.01 0 2352 20.01 0.00 0 400 400 4052 40.3 0 3528 20.01 0 2354 20.01 0 3552 20.01 0 3553 20.01 0 2354 20.01 0 2355 20.01 0 2354 20.01 0 2355 20.01 0 2354 20.01 0 2355 20.01 0 2354 20.01 0 2355 20.01 0 <											1				1			
0.50 0 218 17.4 0 5398 0.1 0 4764 200.1 0 4759 200.2 0 4139 200.1 1.00 0 5335 38.0 0 6742 0.6 0 4335 200.3 0 5417 200.4 0 2219 200.0 0 2132 200.0 0 2133 200.0 0 2133 200.0 0 2139 200.0 0 2339 200.1 0 2430 200.1 0 2430 200.1 0 2430 200.1 0 2439 200.1 0 2439 200.1 0 2439 200.1 0 4430 200.1 0 4430 200.1 0 3435 200.1 0 3435 200.1 0 3435 200.1 0 3435 200.1 0 3435 200.1 0 3435 200.1 0 3435 200.1 0 3435 200.1		1																
0.80 0 0.4778 2.80 0.6 6742 0.6 0.543 2.00.3 0 1.71 20.04 0 1.25 200.3 2 0.10 0 1.351 1.8 0 1.51 0.0 1.51 20.01 0.0 2.725 20.01 0.0 1.212 20.00 0 1.714 20.01 0.0 2.956 20.01 0 2.956 20.01 0 2.956 20.01 0 2.956 2.000 0 2.951 2.001 0 2.956 2.001 0 4.950 2.001 0 4.950 2.001 0 4.950 2.001 0 3.951 2.001 0 3.951 2.001 0 3.951 2.001 0 3.951 2.001 0 3.951 2.001 0 5.45 2.000 0 1.914 2.001 0 3.951 2.001 0 3.951 2.001 0 3.951 2.001 0 3.951		•									1				1			
2 0.10 0 1301 1.8 0 1402 0.1 0 2001 0 2002 000 0 2003 0.00 0 2003 0.00 0 2003 0.00 0 2004 0 4550 200.0 0 4780 2001 0 4550 200.0 0 2164 2001 0 4500 0 4402 2001 10 0 0 4564 3.9 0 550 0 5660 0.01 0 4500 20.03 0 450 20.01 0 554 20.00 0 154 20.00 0 554 20.00 0 1545 20.00 0 1545 20.00 0 1545 20.00 0 1545 20.00 0 1543 20.00 0 1545 20.00 0 150 0 1570 20.00 10 1505 10 1443 20.01 0 150																		
0.50 0 0.2 75 1.8 0 4956 0.1 0 2846 200.0 0 7.84 200.1 0 4957 200.3 1.00 0 2456 3.55 0.0 3298 0.0 0 2564 20.0 0 212 200.0 0 2142 200.1 0 200.1 200.1 200.1 200.1			1.00	0	5435	38.0	0	6742	0.6	0	5435	200.3	0	5417	200.4	0	5215	200.3
0.80 0.0 6456 8.5 0.0 8564 0.0.0 0 3.748 20.0.1 0 3.742 20.0.1 3 0.10 0 2.50 3.9 0.1 0.0 2.502 0.1 0.0 2.502 0.0 0 2.562 2.00.1 0 3.350 2.00.3 0 2.500 3.00 3.850 2.00.3 0.0 2.550 0.0.3 0.0 3.550 2.00.3 0.0 3.550 2.00.3 0.0 3.550 2.00.3 0.0 0.554 2.00.1 0.0 5.541 2.00.1 0.0 5.541 2.00.1 0.0 5.541 2.00.1 0.0 3.571 2.00.0 0.0 3.571 2.00.1 0.0 3.571 2.00.1 0.0 3.571 2.00.1 0.0 3.571 0.0 0.0 1.01 0.0 3.571 0.0 0.0 2.582 2.00.1 0.0 3.571 0.01 0.0 2.572 0.01 0.0 2.572		2	0.10	0		1.8	0		0.0	0			0			0		
1.00 0 4.405 4.20 0 5.80 0.0 2.54 2.00.1 0 4.53 2.00.1 0 4.53 2.00.1 0 3.54 2.00.3 0.0 3.54 2.00.1 3.54 2.00.3 3.00 3.55 2.00.3 3.55 2.00.3 3.55 2.00.3 3.55 2.00.3 3.55 2.00.3 3.55 2.00.3 3.55 2.00.3 3.55 2.00.3 3.55 2.00.3 3.55 2.00.3 3.55 2.00.3 3.55 2.00.1 3.55 3								1			1				1			
3 0.10 0 2504 309 0 2364 00.0 0 217 2000 0 2003 0.50 0 4655 49.3 0 6680 0.1 0 4990 20.3 0 4550 20.3 0 4550 20.3 0 4550 20.3 0 4550 20.3 0 4550 20.0 0 5541 20.0 0 5541 20.0 0 5541 20.0 0 3541 20.0 0 3541 20.0 0 3541 20.0 0 3541 20.0 0 3571 20.0 0 3977 20.0 0 3541 20.0 0 3551 20.0 0 3571 20.0 0 3971 20.0 0 3541 20.0 0 3541 20.0 0 3542 20.0 0 3542 20.0 0 3542 20.0 0 3542 20.0 0																		
0.50 0 9.50 1.6 9.99 20.1 0 9.93 20.3 0 9.93 20.3 0 9.93 20.3 0 9.93 20.3 0 9.93 20.3 0 9.95 20.3 0 9.95 20.3 0 9.95 20.3 0 9.95 20.3 0 9.95 20.3 0 9.95 20.3 0 5.55 20.0 0 5.56 20.0 0 5.54 20.0 0 5.54 20.0 0 5.54 20.0 0 5.57 20.0 0 9.01 0 3.65 20.0 0 4.53 20.0 0 4.35 20.0 0 4.35 20.0 0 4.35 20.0 0 4.35 20.0 0 4.35 20.0 0 4.35 20.0 0 4.35 20.0 0 4.35 20.0 0 4.35 20.0 0 4.35 20.0 20.0 1.35<		2													1			
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			1.00	0	3319	01.5	0	0530	0.1	0	3492	200.2	0	3330	200.0	0	3310	200.0

Table A.23: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 100, instance numbers (Nr.) 15–29.

				VND-O)		VND-S	5	•	GRASP	-0		MA-0)		VNS-0	0
Sz.	Nr.	Load	C_a	$\mathbf{C}_{\mathbf{u}}$	t[s]	C_a	$\mathbf{C}_{\mathbf{u}}$	t[s]	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	C_a	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]
100	15	0.10	0	1861	3.2	0	2591	0.0	0	1887	200.0	0	1850	200.0	0	1822	200.0
		0.50	0	3293	11.6	0	4283	0.0	0	3183	200.0	0	3103	200.1	0	3064	200.1
		0.80	0	4413	25.9	0	5504	0.1	0	4323	200.0	0	4317	200.1	0	4243	200.1
	16	1.00 0.10	0	5251 2531	25.5 4.7	0	6118 3454	0.1 0.0	0	5347 2530	200.4 200.1	0	5254 2519	200.1 200.1	0	5221 2499	200.0 200.1
	10	0.10	0	3894	18.8	0	4885	0.0	0	3976	200.1	0	3957	200.1	0	3850	200.1
		0.80	0	4493	47.9	0	5796	0.1	0	4466	200.0	0	4567	200.2	0	4527	200.4
		1.00	0	5217	61.2	0	6609	0.3	Õ	5167	200.2	0	5198	200.3	0	5157	200.7
	17	0.10	0	2003	2.2	0	2744	0.0	0	2045	200.0	0	2003	200.0	0	2003	200.0
		0.50	0	4339	16.7	0	5453	0.0	0	4226	200.1	0	4217	200.2	0	4151	200.0
		0.80	0	4986	27.4	0	6123	0.1	0	4872	200.1	0	4693	200.4	0	4859	200.2
		1.00	0	5791	31.0	0	6888	0.1	0	5905	200.1	0	5717	200.2	0	5702	200.2
	18	0.10	0	2002	1.5	0	2419	0.0	0	2013	200.0	0	1956	200.0	0	1945	200.0
		0.50	0	3143	17.1	0	4376	0.1	0	3214	200.0	0	3158	200.0	0	3023	200.1
		0.80	0	3914	28.1	0	5150	0.1	0	3863	200.1	0	3706	200.0	0	3641	200.2
	10	1.00	115	4763	44.8	0	6356	0.7	0	4778	200.0	0	4699	200.2	0	4527	200.0
	19	0.10	0	2210	3.8	0	2947	0.0	0	2220	200.0	0	2219	200.0	0	2204	200.1
		0.50	0	3514	21.8	0	4375	0.1	0	3451	200.0	0	3352	200.4	0	3346	200.2
		0.80	00	4048 4543	27.2 43.5	0	5060 5686	0.1	0	4001 4374	200.2 200.4	0	3977 4530	200.4 200.2	0	3990 4359	200.0 200.0
	20	1.00 0.10	0	1630	0.8	0	2091	0.1 0.0	0	1609	200.4		1548	200.2	0	1548	200.0
	20	0.10	0	3648	15.5	0	5106	0.0	0	3607	200.0	0	3600	200.0	0	3544	200.0
		0.80	0	4338	37.8	0	6065	0.1	0	4366	200.0	0	4262	200.0	0	4170	200.2
		1.00	0	4963	52.1	0	6644	0.2	0	5001	200.0	0	4853	200.4	0	4711	200.0
	21	0.10	0	1973	4.3	0	2866	0.0	Õ	2015	200.0	0	1950	200.0	0	1923	200.0
		0.50	0	3414	19.7	0	4798	0.1	0	3440	200.1	0	3434	200.0	0	3434	200.2
		0.80	0	4249	41.9	0	5769	0.1	0	4158	200.2	0	4154	200.5	0	4107	200.1
		1.00	0	4708	46.6	0	6111	0.1	0	4815	200.0	0	4570	200.2	0	4570	200.1
	22	0.10	0	1097	0.6	0	1238	0.0	0	1065	200.0	0	1065	200.0	0	1065	200.0
		0.50	0	2882	17.4	0	4200	0.0	0	2961	200.1	0	2855	200.0	0	2863	200.0
		0.80	0	3881	35.2	0	5445	0.1	0	3958	200.3	0	3869	200.3	0	3901	200.1
		1.00	0	4395	38.6	0	6033	0.1	0	4510	200.2	0	4365	200.2	0	4282	200.3
	23	0.10	0	1572	1.6	0	2254	0.0	0	1603	200.0	0	1518	200.0	0	1519	200.0
		0.50	0	3084	17.9	0	4672	0.1	0	3023	200.0	0	2959	200.2	0	3018	200.0
		0.80	00	4064	40.3	0	5685	0.1	0	4282 5420	200.2	00	4160 5312	200.3	0	4109	200.0
	24	1.00 0.10	0	5315 2584	39.7 2.1		6413 3233	0.1 0.0	0	2503	200.4 200.0		2490	200.2 200.0	0	5134 2482	200.1 200.1
	24	0.10	0	3991	18.3	0	5235	0.0	0	3992	200.0	0	4020	200.0	0	3886	200.1
		0.80	0	4810	31.2	0	6442	0.0	0	4707	200.2	0	4606	200.2	0	4554	200.0
		1.00	0	4948	62.0	0	6741	0.1	0	5252	200.2	0	4988	200.1	0	4853	200.0
	25	0.10	0	2088	2.2	0	2535	0.0	Õ	2088	200.0	0	1940	200.0	0	1976	200.1
		0.50	0	3587	23.4	0	4569	0.1	0	3539	200.0	0	3440	200.1	0	3518	200.3
		0.80	0	4648	34.0	0	5558	0.1	0	4479	200.2	0	4555	200.2	0	4453	200.4
		1.00	0	5150	51.6	0	6758	0.1	0	5145	200.4	0	5020	200.0	0	4860	200.4
	26	0.10	0	2384	1.6	0	3194	0.0	0	2357	200.0	0	2353	200.0	0	2360	200.0
		0.50	0	3753	9.1	0	4552	0.0	0	3735	200.1	0	3751	200.1	0	3590	200.0
		0.80	0	4535	33.5	0	6142	0.1	0	4652	200.2	0	4458	200.0	0	4355	200.3
		1.00	0	5091	36.2	0	6596	0.1	0	5103	200.1	0	4954	200.2	0	5081	200.5
	27	0.10	0	1690	1.1	0	2153	0.0	0	1465	200.0	0	1480	200.0	0	1445	200.0
		0.50	0	3175	13.2	0	4531	0.0	0	3221	200.1	0	3122	200.1	0	3013	200.0
		0.80		4371	21.6		5636	0.1		4378			4197				200.3
	28	1.00	255 0	5031 2265	27.2	95 0	5567 2976	59.4	0	4777	200.2		4789	200.1 200.0	0	4688	200.4 200.0
	20	0.10 0.50	0	4364	2.8 17.6	0	2976 5456	0.0	0	2178 4292	200.0 200.0		2219 4277	200.0	0	2219 4202	200.0
		0.30	0	4693	38.7	0	6154	0.1	0	4292	200.0	0	4658	200.2	0	4629	200.0
		1.00	0	5401	47.2	0	7019	0.1	0	5306	200.2	0	5519	200.4	0	5115	200.4
						0	3087	0.0	0	2403	200.1	0	2421	200.1	0	2416	200.0
	29	0.10	0	2525													
	29	0.10 0.50	0	2523 3622	3.0 24.4				0	3616	200.2	0	3580	200.0	0		200.0
	29	0.10 0.50 0.80		2523 3622 4549	24.4 48.3	0	5318 6258	0.1 0.1	0 0	3616 4663	200.2 200.1	00	3580 4577	200.0 200.1	0	3501 4454	200.0 200.3

Table A.24: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 200, instance numbers (Nr.) 0–14.

			1	VND-0	Э		VND-S	5		GRASP	-0		MA-C)	l	VNS-0)
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	Cu	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Ca	$\mathbf{C}_{\mathbf{u}}$	t[s]	C_{a}	C_u	t[s]
200	0	0.10	0	2277	6.7	0	3035	0.0	0	2247	500.0	0	2265	500.0	0	2222	500.1
		0.50	0	4616	65.1	0	6799	0.1	0	4760	500.2	0	4631	500.5	0	4612	500.4
		0.80	0	6050	112.2	0	8179	0.1	0	6173	500.0	0	6047	500.1	0	5811	500.2
		1.00	0	7003	121.9	0	9502	0.2	0	6885	500.1	0	7104	500.1	0	6857	500.3
	1	0.10	0	1915	2.7	0	2535	0.0	0	1879	500.0	0	1891	500.1	0	1858	500.1
		0.50	0	4132	70.4	0	5974	0.1	0	4334	500.1	0	4302	500.3	0	4212	500.6
		0.80	0	5706	159.1	0	7391	11.1	0	5932	500.8	0	5834	500.6	0	5904	500.1
		1.00	0	7601	137.1	0	9234	20.0	0	7613	500.0	0	7394	500.6	0	7189	500.9
	2	0.10	0	2669	6.4	0	3478	0.0	0	2673	500.1	0	2650	500.1	0	2677	500.0
		0.50	0	4261	31.6	0	5910	0.1	0	4389	500.2	0	4203	500.1	0	4135 5278	500.3
		0.80	0	5398	153.0 178.2	0	7485 9725	0.2	0	5649	500.6 500.2	0	5516 7051	500.0 500.9	0	6722	500.4
	3	1.00 0.10	0	6676 3699	178.2	0	4918	0.7 0.0	0	7036 3612	500.2	0	3650	500.9	0	3579	500.5 500.0
	3	0.10	0	5850	85.5	0	7894	30.9	0	5968	500.1	0	6144	500.2	0	5815	500.0
		0.80	0	6965	139.6	0	9359	72.9	0	7104	500.4	0	7017	500.2	0	6921	500.0
		1.00	100	8081	159.8	100	8951	363.9	0	8190	500.2	0	7952	500.3	100	7946	501.0
	4	0.10	0	2921	10.0	0	3805	0.0	0	2896	500.0	0	2882	500.0	0	2854	500.1
		0.50	0	4870	41.4	0	6757	0.1	0	5080	500.4	0	4905	500.4	0	4819	500.3
		0.80	0	6398	174.8	0	9562	0.1	0	6845	500.2	0	6524	500.1	0	6320	500.2
		1.00	0	8240	218.1	0	11288	0.7	0	8569	500.1	0	8111	500.9	0	7991	501.3
	5	0.10	0	3012	12.4	0	4086	0.0	0	3089	500.1	0	2994	500.0	0	2994	500.0
		0.50	0	5150	100.4	0	7151	6.0	0	5217	500.0	0	5298	500.4	0	5182	500.8
		0.80	0	6435	224.4	0	8654	22.6	40	6840	500.1	10	6681	500.4	0	6516	500.1
		1.00	20	7388	353.2	20	8429	523.5	0	7601	500.4	130	7742	500.0	40	7289	500.6
	6	0.10	0	3334	20.6	0	5047	0.0	0	3411	500.2	0	3262	500.2	0	3310	500.0
		0.50	0	5307	92.3	0	7907	0.1	0	5546	500.5	0	5280	500.8	0	5272	500.0
		0.80	0	6829	147.3	0	9469	2.7	0	7064	501.0	0	6929	501.0	0	6818	500.6
		1.00	0	8076	191.4	0	10897	6.3	0	8231	501.4	0	7962	500.5	0	8030	500.3
	7	0.10	0	2883	10.0	0	4504	0.0	0	3025	500.0	0	2932	500.0	0	2802	500.1
		0.50	0	5657	72.4	0	8369	0.1	0	5808	500.0	0	5640	500.2	0	5562	500.5
		0.80	0	7221	172.1	0	10621	0.2	0	7499	500.3	0	7371	500.6	0	7131	500.0
		1.00	0	8518	188.2	0	11595	15.8	0	8917	501.2	0	8705	500.2	0	8524	500.9
	8	0.10	0	1530	1.9	0	2046	0.0	0	1511	500.0	0	1520	500.0	0	1511	500.0
		0.50	0	4746	56.4	0	7117	0.1	0	4851	500.1	0	4818	500.2	0	4779	500.1
		0.80	0	6262	144.8	0	9732	13.2	0	6468	501.0	0	6449	500.3	0	6305	500.1
	9	1.00 0.10	0	8083 2264	159.1 3.5	0	10777 2765	23.3 0.0	0	8337 2241	500.2 500.0	0	8067 2264	500.3 500.0	0	8133 2262	500.0 500.0
	9	0.10	0	5070	83.8	0	7299	7.3	0	5249	500.0	0	5184	500.0	0	5108	500.0
		0.80	0	5890	254.8	0	8466	13.7	0	6078	500.2	0	6141	500.4	0	5924	500.1
		1.00	0	7854	312.9	0	10055	20.0	0	8003	500.4	0	7813	500.1	0	7952	500.2
	10	0.10	0	1792	1.6	0	2375	0.0	0	1810	500.0	0	1734	500.0	0	1734	500.0
		0.50	0	5040	34.6	0	6830	0.1	0	4944	500.2	0	4919	500.2	0	4812	500.0
		0.80	0	6718	149.6	0	9066	0.1	0	6947	500.2	0	6722	500.3	0	6720	500.5
		1.00	0	7897	154.0	0	10630	1.4	0	8236	501.1	0	7828	500.1	0	7894	500.8
	11	0.10	0	2070	2.6	0	2747	0.0	0	1999	500.0	0	1889	500.0	0	1869	500.0
		0.50	0	4961	73.8	0	7254	0.1	0	5067	500.4	0	5032	500.8	0	5005	500.2
		0.80	0	7695	184.8	0	10777	0.2	0	7474	500.3	0	7669	500.0	0	7343	500.4
		1.00	0	8669	304.8	0	12127	0.3	0	9074	501.4	0	8704	501.2	0	8696	500.3
	12	0.10	0	2733	7.6	0	3761	0.0	0	2719	500.0	0	2718	500.1	0	2704	500.0
		0.50	0	4635	57.0	0	6317	0.1	0	4781	500.1	0	4749	500.2	0	4630	500.0
		0.80	0	5735	92.8	0	7884	0.2	0	5758	500.2	0	5745		0		500.3
		1.00	0	6990	225.8	0	9638	0.2	0	7257	500.8	0	7278	500.7	0	7070	500.0
	13	0.10	0	2209	2.9	0	2706	0.0	0	2201	500.0	0	2206	500.0	0	2201	500.0
		0.50	0	3819	63.7	0	5247	0.1	0	3953	500.1	0	3873	500.4	0	3853	500.0
		0.80	0	5031	149.3	0	7815	0.2	0	5204	500.2	0	5173	500.1	0	5055	500.3
	14	1.00	0	6416	207.2	0	9871	0.5	0	6744	500.4	0	6442	500.8	0	6119	500.4
	14	0.10	0	3292	18.4	0	4318	0.0	0	3339	500.1	0	3412	500.3	0	3279	500.1 500.4
		0.50 0.80	0	4803 6857	63.0 160.9	0	6509 8785	0.1 0.2	0	5023 7093	500.0 501.5	0	4818 7032	500.0 500.9	0	4932 6918	500.4
		1.00	0	7482	186.6	0	9518	0.2	0	7678	501.5	0	7696	500.9	0	7639	500.2
		1.00	0	1702	100.0	0	7510	0.4	0	1010	501.7	0	7070	500.5	0	1059	501.0

Table A.25: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size
(Sz.) 200, instance numbers (Nr.) 15–29.

		1	1	VND-	0		VND-S			GRASE	-0		MA-C)		VNS-0)
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	C_{a}	C _u	t[s]	$\mathbf{C}_{\mathbf{a}}$	C _u	t[s]	$\mathbf{C}_{\mathbf{a}}$	Cu	t[s]	$\mathbf{C}_{\mathbf{a}}$	Cu	t[s]
200	15	0.10	0	2631	5.3	0	3045	0.0	0	2562	500.1	0	2546	500.0	0	2554	500.1
		0.50	0	4419	58.1	0	6125	0.1	0	4485	500.2	0	4496	500.1	0	4355	500.2
		0.80	0	5506	164.4	0	7859	0.1	0	5832	500.4	0	5706	500.9	0	5676	500.0
		1.00	0	6968	134.0	0	9258	15.9	5	7264	500.9	0	6944	500.7	0	6745	500.0
	16	0.10	0	2700	6.3	0	3571	0.0	0	2694	500.0	0	2689	500.1	0	2652	500.0
		0.50	0	4103	46.9	0	5862	0.1	0	4171	500.5	0	4124	500.1	0	4009	500.4
		0.80	0	5450	158.5	0	8900	0.2	0	5735	500.1	0	5682	500.7	0	5660	500.2
		1.00	0	7128	214.3	0	10720	22.1	0	7504	501.0	0	7558	500.1	0	7261	501.6
	17	0.10	0	1943	4.4	0	2837	0.0	0	1851	500.0	0	1851	500.1	0	1851	500.1
	17	0.50	0	5287	82.7	0	7698	0.1	0	5286	500.5	0	5300	500.2	0	5094	500.5
		0.80	0	7095	141.2	0	9643	0.2	0	7310	500.5	0	7183	500.2	0	7132	500.5
		1.00	0	8576	251.3	0	11573	1.1	0	8770	500.5	0	8654	500.5	0	8365	500.0
	18	0.10	0	2159	12.4	0	2980	0.0	0	2065	500.2	0	2060	500.8	0	2042	500.9
	10	0.10	0	4635	80.4	0	6669	0.0	0	4624	500.1	0	4568	500.0	0	4521	500.1
			0			0			0		500.8	0			0		
		0.80		6442	172.7		8854	0.3		6364 7434			6427	500.4		6265	500.8
	10	1.00	0	7457	249.5	0	9800	28.6	0		500.5	0	7272	501.4	0	7615	501.7
	19	0.10	0	2414	4.1	0	2889	0.0	0	2394	500.0	0	2415	500.1	0	2402	500.1
		0.50	0	4347	35.7	0	5502	0.1	0	4297	500.1	0	4301	500.2	0	4182	500.1
		0.80	0	6480	116.2	0	9378	0.3	0	6653	500.1	0	6710	500.9	0	6438	500.7
	•	1.00	0	7581	119.4	0	10351	14.3	0	7742	500.1	0	7662	500.3	0	7374	500.3
	20	0.10	0	2627	8.2	0	4118	0.0	0	2619	500.1	0	2640	500.1	0	2604	500.1
		0.50	0	5076	62.1	0	7554	0.1	0	5013	500.1	0	4971	500.2	0	4895	500.6
		0.80	0	6384	168.6	0	9645	14.3	0	6743	500.7	25	6682	500.8	0	6561	500.0
		1.00	25	8128	198.2	0	10652	69.4	0	8345	500.8	25	8162	501.3	588	8009	500.4
	21	0.10	0	2512	7.9	0	3518	0.0	0	2516	500.1	0	2545	500.1	0	2481	500.0
		0.50	0	4811	52.4	0	6612	0.1	0	4878	500.2	0	4812	500.1	0	4758	500.4
		0.80	0	6539	151.4	0	8874	0.2	0	6880	501.2	0	6510	500.1	0	6788	500.3
		1.00	0	7445	199.1	0	9870	15.0	0	8055	501.1	0	7453	500.5	0	7908	501.0
	22	0.10	0	3046	8.5	0	4213	0.0	0	3084	500.0	0	3031	500.0	0	2997	500.0
		0.50	0	5436	66.5	0	7159	0.1	0	5342	500.7	0	5350	500.1	0	5321	500.1
		0.80	0	6731	140.1	0	8708	0.2	0	6809	501.0	0	6732	500.0	0	6598	500.2
		1.00	0	7742	222.7	0	10890	6.0	0	7871	500.1	0	7853	500.5	0	7851	501.2
	23	0.10	0	2571	11.7	0	4196	0.0	0	2699	500.0	0	2797	500.0	0	2571	500.1
		0.50	0	5191	58.2	0	8257	0.1	0	5352	500.5	0	5318	500.0	0	5025	500.0
		0.80	0	6835	120.3	0	9738	0.2	0	6793	500.4	0	6532	500.2	0	6343	500.5
		1.00	0	9031	256.4	0	11393	16.0	0	9102	500.0	175	8972	500.0	0	9171	501.1
	24	0.10	0	2278	2.7	0	2970	0.0	0	2209	500.0	0	2208	500.0	0	2207	500.0
		0.50	0	5032	63.5	0	7111	0.1	0	5175	500.0	0	5009	500.2	0	4771	500.1
		0.80	0	5899	134.6	0	8662	0.2	0	6176	500.3	0	5994	500.0	0	6009	500.1
		1.00	0	8010	216.9	0	10681	3.5	0	7771	500.2	0	7780	500.0	0	7778	500.4
	25	0.10	0	2919	8.8	0	3957	0.0	0	2866	500.0	0	2829	500.0	0	2805	500.1
	20	0.50	0	5100	69.8	0	7152	6.5	0	4992	500.0	0	5001	500.2	0	4913	500.4
		0.80	0	6154	150.8	0	8577	20.3	0	6203	500.0	0	6108	500.2	0	6136	500.1
		1.00	0	7074	218.7	0	9729	23.3	0	7364	500.6	0	7330	500.2	0	7286	500.1
	26	0.10	0	2992	12.6	0	3590	0.0	0	2997	500.0	0	2943	500.0	0	2920	500.2
	20	0.10	0	4498	87.6	0	5922	0.0	0	4598	500.0	0	4608	500.1	0	4469	500.2
		0.30	0	6348	152.8	0	8519	12.4	0	6408	501.0	0	6430	500.0	0	6247	500.7
		1.00	0	7398	219.2	0	9393	12.4	0	7635	501.0	0	7476	500.5 501.5	0	7371	501.3
	27	0.10	0	2555	1.9	0	3166	0.0	0	2437	500.0	0	2478	500.1	0	2478	500.0
	21																
		0.50	0	4632	48.3	0	6317	0.1	0	4643	500.1	0	4611	500.2	0	4620	500.2
		0.80	0	5823		0	8099	0.2	0	6041	500.3	0	6013		0	5923	500.3
		1.00	0	6655	202.7	0	9045	14.2	0	6889	500.3	0	6753	501.1	0	6815	500.2
	28	0.10	0	2630	7.1	0	3615	0.0	0	2545	500.0	0	2389	500.0	0	2397	500.0
		0.50	0	4272	56.3	0	5952	0.1	0	4189	500.2	0	4151	500.1	0	4052	500.2
		0.80	0	5713	114.1	0	8237	10.8	0	6128	500.2	0	5796	500.8	0	5845	500.7
		1.00	0	7744	194.4	0	9275	187.7	0	8211	500.7	0	7808	500.6	0	7655	500.0
	29	0.10	0	2531	7.0	0	3760	0.0	0	2455	500.0	0	2391	500.2	0	2368	500.2
		0.50	0	4826	47.9	0	7198	0.1	0	4894	500.4	0	4634	500.0	0	4676	500.2
		0.80	0	7064	138.6	0	9158	0.3	0	6956	500.6	0	6770	500.8	0	6697	501.0
		1.00	0	7805	288.2	0	9998	135.4	0	7977	500.5	0	7833	500.3	0	7595	500.4

Table A.26: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 500, instance numbers (Nr.) 0–14.

				VND-O			VND-S			GRASP-			MA-O		_	VNS-0	
Sz.	Nr.	Load	Ca	C_u	t[s]	Ca	C_u	t[s]	Ca	C_u	t[s]	C_a	C_u	t[s]	C_a	C_u	t[s]
500	0	0.10	0	3876	42.0	0	4988	0.1	0	3838	500.2	0	3846	500.0	0	3793	500.0
		0.50	0	6952	229.7	0	9727	0.2	0	7167	500.1	0	7172	500.6	0	7118	500.0
		0.80	0	9165	887.1	0	13114	0.5	0	9530	501.8	0	9787	500.3	0	9582	502.
		1.00	26	12890	1009.9	26	15806	1001.1	194	13827	500.2	231	13924	501.4	231	13062	501.
	1	0.10	0	3749	51.1	0	4910	0.0	0	3828	500.1	0	3934	500.3	0	3817	500.
		0.50	0	6510	309.9	0	9735	0.2	0	6953	500.0	0	6813	500.3	0	6818	501.
		0.80	0	8452	643.1	0	13259	36.6	0	9321	500.4	0	9019	500.4	0	8982	502.
		1.00	35	11716	896.5	35	14438	1001.4	55	12580	500.4	13	13087	502.3	37	11838	503.
	2	0.10	0	3475	53.3	0	4820	0.1	0	3520	500.1	0	3588	500.4	0	3384	500.
		0.50	0	6501	215.9	0	9112	0.2	0	6829	500.4	0	6722	500.3	0	6762	500.
		0.80	0	9319	994.7	0	14273	0.6	0	9768	500.7	0	9643	500.4	0	9743	501
		1.00	0	13096	1031.7	0	18406	80.8	240	14091	500.6	0	13636	500.5	0	13921	500.
	3	0.10	0	3082	30.8	0	3907	0.0	0	3108	500.0	0	3124	500.0	0	3046	500.
		0.50	0	6098	187.2	0	7963	0.2	0	6202	500.2	0	6121	500.6	0	6106	500.
		0.80	0	8799	1028.9	Ő	12861	0.4	0	9538	500.2	0	9651	502.0	0	9413	500.
		1.00	0	11894	1037.1	0	16369	4.4	0	12682	501.0	0	12177	500.1	0	12201	500.
	4	0.10	0	2633	9.3	0	3323	0.0	0	2631	500.0	0	2630	500.0	0	2625	500
	4	0.10	0	6540		0	9406	0.0	0	6843	501.2	0	1	500.0	0		500
			0		333.1 698.1		12390					0	6733 9300	500.9	0	6783	503
		0.80		9127		0		0.5	0	9643	502.0					9508	
	_	1.00	0	11498	1042.1	0	15213	96.6	0	12659	503.0	5	11987	500.7	5	12208	501
	5	0.10	0	2998	25.5	0	3862	0.0	0	2951	500.1	0	2944	500.0	0	2896	500
		0.50	0	6967	610.1	0	11169	0.3	0	7409	501.4	0	7326	501.0	0	7399	500
		0.80	0	9970	1000.6	0	14375	4.0	0	10697	501.3	0	10421	500.6	0	10295	503
		1.00	308	12753	1001.8	93	15957	1037.3	468	14832	502.3	530	13490	503.9	615	13444	500
	6	0.10	0	3270	46.5	0	4386	0.0	0	3305	500.3	0	3223	500.1	0	3217	500
		0.50	0	6890	405.4	0	10049	31.4	0	7350	500.1	0	7227	500.2	0	6984	501
		0.80	0	8984	1030.9	0	13276	60.5	0	9592	500.3	0	9927	501.6	0	9851	500
		1.00	185	12434	1039.0	120	15583	1002.3	120	13315	502.5	120	12932	500.5	120	12996	504
	7	0.10	0	3256	40.6	0	4290	0.0	0	3381	500.1	0	3272	500.1	0	3247	500
		0.50	0	7664	513.1	0	10053	25.1	0	7783	501.2	0	7964	501.1	0	7725	501
		0.80	0	11302	1000.1	0	15800	275.8	0	12010	501.5	0	11733	503.2	0	11877	500
		1.00	115	14322	1001.6	115	17843	1012.9	12	16823	501.5	250	14718	501.4	250	15023	501
	8	0.10	0	3059	73.4	0	4606	0.1	0	3165	500.1	0	3207	500.3	0	3015	500
		0.50	0	6557	657.0	Ő	9999	0.3	0	7000	500.1	0	6861	500.3	0	7104	500
		0.80	0	10742	1045.4	0	16243	0.6	0	11073	500.5	0	10918	501.6	0	11105	500
		1.00	0	13979	1045.4	0	19822	189.5	0	15110	500.9	0	14482	500.6	0	15607	505
	9	0.10	0	2480	30.9	0	3470	0.0	0	2490	500.9	0	2623	500.0	0	2434	500
		0.50	0	5588	333.5	0	8472	26.0	0	5904	500.4	0	5991	500.1	0	5881	500
		0.80	0	8510	784.1	0	12676	43.7	0	8724	500.4	0	8626	500.1	0	9067	500
	10	1.00	0	11692	1029.5	0	16230	306.9	0	12509	501.1	0	12556	500.6	0	12235	500
	10	0.10	0	3145	41.8	0	4117	0.1	0	3062	500.0	0	3246	500.1	0	3128	500
		0.50	0	6003	375.3	0	9254	0.3	0	6160	500.4	0	6073	500.9	0	6023	500
		0.80	0	9655	676.5	0	14007	0.6	0	9659	501.0	0	10003	501.4	0	9840	500
		1.00	0	13097	1037.9	46	16492	1007.1	0	13281	502.0	385	13164	500.1	0	13839	500
	11	0.10	0	3513	19.1	0	4610	0.0	0	3471	500.4	0	3513	500.5	0	3513	500
		0.50	0	7540	310.8	0	10411	0.3	0	8085	500.5	0	7891	501.5	0	7953	500
		0.80	0	9417	567.8	0	12731	0.4	10	9744	500.0	0	9692	500.2	0	9881	501
		1.00	0	11712	1003.2	0	15413	244.1	0	12423	500.3	0	12330	501.8	280	12371	501
	12	0.10	0	3098	9.8	0	4014	0.0	0	3098	500.0	0	3096	500.1	0	3096	500
		0.50	0	7209	453.8	0	11058	0.3	0	7716	501.8	0	7485	500.6	0	7357	500
		0.80	0	10124	993.7	0	16037	125.9	0	11356	500.3	0	10830	500.3	5	11000	503
		1.00	0	12797	1037.7	0	18317	440.7	440	14389	500.0	25	14220	501.2	5	13617	501
	13	0.10	0	2899	16.1	Ő	3434	0.0	0	2895	500.0	0	2858	500.0	0	2860	500
		0.50	0	5966	203.4	0	9346	0.2	0	6087	500.1	0	5934	500.0	0	5894	500
		0.80	0	8972	529.9	0	13215	0.4	0	9283	501.0	0	9095	502.4	0	9119	501
		1.00	49	12696	1021.8	49	15625	1037.6	0	13146	500.7	49	12963	501.0	145	13196	501
	14	0.10	49	3549	44.7		4501	0.0	0	3535	500.7	49	3647	500.1	145	3451	
	14					0							1				500
		0.50	0	7067	461.0	0	10534	0.2	0	7245	501.8	0	7186	500.7	0	7197	501
		0.80 1.00	0	10278	1042.0	0	15870	71.4	750	10618	502.6	0	10630	500.2	750	10781	500 505
			275	15289	1053.1	15	20098	1015.5	1248	17547	501.7	645	15412	502.9	800	15954	505

Table A.27: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 500, instance numbers (Nr.) 15–29.

				VND-0	Э		VND-	S	1	GRASP	0		MA-O			VNS-O	
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	$\mathbf{C}_{\mathbf{a}}$	C _u	t[s]	$\mathbf{C}_{\mathbf{a}}$	C _u	t[s]	$\mathbf{C}_{\mathbf{a}}$	C _u	t[s]
500	15	0.10	0	3641	73.2	0	5289	0.1	0	3673	500.3	0	3671	500.2	0	3647	500.0
		0.50	0	6151	361.6	0	9280	0.2	0	6408	501.5	0	6357	501.4	0	6183	501.4
		0.80	0	9656	1029.1	0	14306	115.3	70	10602	500.9	35	10365	500.8	100	10537	500.5
		1.00	10	13722	1000.1	75	16961	1000.7	0	14463	501.4	35	14140	501.2	260	14365	504.8
	16	0.10	0	2960	38.8	0	4581	0.0	0	3030	500.3	0	3098	500.1	0	3007	500.0
		0.50	0	6258	313.0	0	9436	0.2	0	6346	500.2	0	6377	500.3	0	6281	501.0
		0.80	0	8758	1022.2	0	12919	0.4	0	9102	500.4	0	9180	501.9	0	9087	501.0
	17	1.00	5 0	12286	1046.9	30	15699 2955	1014.1	10	13036	500.2 500.0	10	12989	500.1 500.2	10 0	13006	500.2
	17	0.10 0.50	0	2215 5638	11.7 182.0	0	2955 8191	0.0 20.5	05	2183 5655	500.0	0	2188 5713	500.2	0	2160 5797	500.0 500.1
		0.30	0	7552	687.8	0	11509	36.9	5	7914	500.2	0	7889	500.0	5	7706	500.1
		1.00	0	10869	884.7	0	14079	109.4	23	11465	501.5	30	11242	500.1	5	10975	501.2
	18	0.10	0	3512	35.5	0	4432	0.0	0	3595	500.6	0	3594	500.2	0	3556	500.2
	10	0.10	0	6626	301.7	0	9815	0.0	0	6877	500.5	0	6806	500.5	0	6728	501.7
		0.80	0	9344	1018.7	0	15415	0.5	0	9999	501.0	0	9999	503.2	3	10055	501.1
		1.00	0	13930	1002.3	0	19481	341.1	236	15025	500.1	514	14178	500.2	1045	14379	506.0
	19	0.10	0	3123	62.3	0	4511	0.1	0	3234	500.4	0	3193	500.3	0	3211	500.5
		0.50	0	7380	404.2	0	9965	0.3	0	7612	500.4	0	7535	501.6	0	7488	501.6
		0.80	0	10284	1002.8	0	15392	1.2	0	11730	502.4	0	10543	505.2	0	10611	503.4
		1.00	0	15385	1042.9	0	20147	40.2	95	16302	506.6	10	15447	500.2	0	15708	502.5
	20	0.10	0	3336	21.9	0	4522	0.0	0	3284	500.1	0	3313	500.1	0	3275	500.0
		0.50	0	6942	479.0	0	9923	0.3	0	7265	500.2	0	7205	500.1	0	7104	501.2
		0.80	0	9157	832.0	0	13713	53.3	0	9771	501.4	0	9568	500.0	0	9538	502.4
		1.00	140	13358	1003.5	140	18061	1011.9	95	14276	501.3	140	13641	503.2	210	14028	501.6
	21	0.10	0	3101	40.9	0	4578	0.0	0	3195	500.0	0	3110	500.3	0	3067	500.0
		0.50	0	5902	406.9	0	9383	29.8	0	6404	500.2	20	6389	501.3	0	6633	500.1
		0.80	0	8931	991.5	0	14010	136.4	5	10101	502.0	981	10173	501.6	0	9439	500.8
		1.00	0	12602	1001.8	0	16905	462.5	852	13017	501.2	2450	13483	500.9	1394	13814	501.2
	22	0.10	0	2527	72.6	0	3519	0.0	0	2497	500.0	0	2497	500.0	0	2477	500.1
		0.50	0	5853	423.4	0	9410	32.9	33	6291	500.5	33	6118	501.4	33	5943	500.1
		0.80	0	8389	1012.2	0	13008	152.5	33	9284	501.2	53	9105	502.2	160	9408	500.2
		1.00	0	11750	1017.4	0	16226	324.5	33	12819	500.9	53	12357	504.0	213	12213	500.4
	23	0.10	0	3336	63.6	0	4694	0.1	0	3481	500.2	0	3528	500.6	0	3500	500.6
		0.50	0	6540	428.1	0	9842	0.3	0	6725	500.3	0	6731	500.4	0	6803	500.5
		0.80	0	9106	1008.9	0	13632	1.8	0	9505	502.8	0	9553	500.6	0	9710	502.0
	24	1.00 0.10	0	12634 2747	1033.1 89.0	0	18518 3948	237.2 0.0	1 0	14416 2899	501.4 500.3	480 0	13006 2922	505.2 500.0	315 0	14041 2908	503.1 500.1
	24	0.10	0	6230	410.6	0	9173	0.0	0	6399	500.5	0	6392	500.0	0	6356	500.1
		0.30	0	8669	1001.9	0	12918	66.0	0	9133	500.2	0	8989	502.2	0	8996	500.2
		1.00	0	12018	1006.0	0	16876	238.3	0	13248	500.0	0	12448	501.7	0	13204	503.6
	25	0.10	0	3631	35.4	0	4956	0.1	0	3507	500.3	0	3606	500.1	0	3596	500.2
	25	0.50	0	7411	428.2	0	10623	0.3	0	7771	500.2	0	7674	500.8	0	7519	502.0
		0.80	0	11048	1000.1	0	14185	999.4	0	11903	501.2	5	11496	500.9	0	11712	500.8
		1.00	305	13828	1042.4	315	17860	1028.2	390	14827	500.1	480	14547	501.1	480	14849	501.1
	26	0.10	0	3033	42.0	0	4184	0.0	0	2972	500.0	0	3113	500.2	0	2928	500.2
		0.50	0	6924	342.3	0	10625	0.5	0	7105	500.4	0	7184	500.9	0	6903	501.3
		0.80	0	10142	1002.5	0	14119	103.8	0	10897	500.1	0	10931	501.0	0	11189	500.6
		1.00	0	12362	1047.6	20	15650	1040.4	50	13326	500.3	405	13284	501.3	405	12926	504.8
	27	0.10	0	3503	31.7	0	4626	0.1	0	3576	500.0	0	3601	500.3	0	3531	500.0
		0.50	0	6415	292.1	0	9717	0.2	0	6533	500.2	0	6740	500.9	0	6554	501.0
		0.80	0	9944	927.5	0	14386	43.1	0	10540	501.0	0	10514	500.3	0	10513	500.9
		1.00	0	13425	1003.6	0	17330	13.1	167	14332	500.3	0	13721	504.7	0	13177	502.9
	28	0.10	0	3032	35.7	0	4465	0.0	0	3035	500.1	0	3154	500.1	0	3050	500.3
		0.50	0	6513	274.0	0	9716	0.3	0	6723	500.6	0	6562	500.0	0	6563	500.0
		0.80	0	9860	800.9	0	14309	65.6	0	10478	500.8	0	9889	500.6	0	9862	501.0
		1.00	285	12589	1001.2	350	15724	1034.8	15	14008	502.9	1290	13091	501.3	340	13464	500.1
	29	0.10	0	3814	62.8	0	5124	0.1	0	3997	500.1	0	3789	500.1	0	3815	500.5
		0.50	0	6518	334.3	0	8895	0.2	0	6613	500.3	0	6560	501.1	0	6412	500.2
		0.00															
		0.80 1.00	0	9009 13274	1032.5 1009.6	0	13180 17571	0.7 18.8	0	10127 13867	500.1 503.5	0	9738 13503	500.6 501.0	0	9510 13633	502.5 500.1

Table A.28: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 1000, instance numbers (Nr.) 0–14.

				VND-O)		VND-S	5		GRASP-	0	I	MA-O		I	VNS-O	
Sz.	Nr.	Load	Ca	$\mathbf{C}_{\mathbf{u}}$	t[s]	C_{a}	Cu	t[s]	$\mathbf{C}_{\mathbf{a}}$	Cu	t[s]	$\mathbf{C}_{\mathbf{a}}$	Cu	t[s]	$\mathbf{C}_{\mathbf{a}}$	Cu	t[s]
1000	0	0.10	0	4002	169.5	0	6023	0.1	0	4066	500.1	0	4161	500.3	0	4105	501.3
		0.50	0	10173	1001.8	0	15908	0.7	0	10479	500.9	0	11017	502.8	0	10435	504.7
		0.80	165	16396	1004.9	0	23962	723.0	1120	17394	500.9	165	16757	501.3	15	17332	509.0
	1	1.00	617	25140	1004.6	191	31778	1062.6	716	26359	503.7	523	25113	511.5	1291	25589	502.0
	1	0.10 0.50	0	3352	160.5 804.7	0	4841	0.1 1042.5	0	3500 9230	500.2 500.9	0	3564 9417	500.0	0	3386 9261	500.1
		0.50	93	8718 14461	804.7	15 0	11474 20847	380.6	0	9230 14984	500.9	0 110	15082	500.2 503.3	0 110	15513	501.0 500.9
		1.00	848	21414	1041.3	645	27135	1124.7	441	23130	502.1	779	22406	503.9	1059	22265	508.4
	2	0.10	0	4024	303.7	045	5945	0.1	0	4224	500.4	0	4266	501.1	0	4148	500.1
	2	0.50	0	9526	1034.5	0	13551	115.2	0	9929	500.1	0	9907	500.7	0	9846	500.3
		0.80	0	14721	1172.8	0	21191	1001.6	60	14826	500.0	105	14821	500.7	105	14778	502.4
		1.00	240	19995	1003.3	0	25236	978.1	3476	21794	501.8	85	20292	502.7	0	21828	511.4
	3	0.10	0	3183	81.7	0	4225	0.0	0	3240	500.1	0	3296	500.1	0	3135	500.1
		0.50	0	9127	1058.5	0	13362	83.2	0	9682	500.1	0	9472	500.6	0	9586	501.7
		0.80	0	13998	1046.4	0	21102	349.0	0	14715	500.7	0	14091	500.1	0	14211	500.6
		1.00	2	18835	1004.3	242	26054	1089.6	245	22159	505.3	0	19758	505.8	700	20647	505.3
	4	0.10	0	4415	216.7	0	5630	17.9	0	4616	500.2	0	4486	501.3	0	4511	500.2
		0.50	0	9361	1044.0	0	14012	139.5	50	9970	502.4	50	9676	505.3	50	9913	502.1
		0.80	0	13536	1006.5	0	19171	934.8	50	14336	501.6	50	13828	501.4	50	14242	508.7
		1.00	583	21503	1016.7	289	29230	1195.9	1358	23893	511.8	607	22448	500.4	824	23125	510.0
	5	0.10	0	4875	198.9	0	6841	19.4	0	5189	500.2	0	5065	500.1	0	4952	500.2
		0.50	0	9889	1000.8	0	15660	256.0	0	10776	503.5	0	10623	500.1	0	10714	500.2
		0.80	0	15818	1039.9	0	22306	405.0 1001.5	370	17524	501.8	0	16406	501.8	0	16274	505.3
	6	1.00 0.10	586 0	21117 4957	1010.3 223.4	70 0	28299 6789	0.1	1067 0	21961 4900	502.9 500.7	361 0	21982 5032	502.7 500.9	470 0	22525 4947	505.6 500.4
	0	0.10		9438	1075.6	0	13399	0.1	0	10120	500.7	0	9860	500.9	0	9940	502.6
		0.80	209	15055	1075.0	209	19397	1135.0	0	15613	505.1	209	15360	502.6	0	15664	502.0
		1.00	840	21214	1012.6	312	27707	1160.3	117	22356	501.1	870	21125	502.5	1589	23091	500.0
	7	0.10	0	2747	251.9	0	4115	0.1	0	2837	500.3	0	3086	500.0	0	2825	500.2
		0.50	0	7407	726.3	0	11289	0.5	0	7640	501.4	0	7644	501.5	0	7782	501.7
		0.80	0	14343	1055.3	15	19408	1083.0	0	15271	505.8	17	15605	501.2	17	14608	501.3
		1.00	606	21510	1001.2	300	27212	1068.0	1250	22715	502.7	1152	22695	503.0	1576	22714	501.5
	8	0.10	0	3685	155.6	0	5300	0.1	0	4043	500.4	0	3760	500.2	0	3676	500.8
		0.50	0	7209	1023.4	0	12741	0.5	0	7482	501.1	0	7924	501.0	0	7939	501.4
		0.80	241	15233	1001.5	427	20567	1003.9	95	17283	507.3	168	15584	503.8	635	16356	506.6
		1.00	664	21941	1010.3	293	25790	1152.5	1639	22590	511.2	787	21958	500.2	872	21856	510.2
	9	0.10	0	3490	202.5	0	4687	0.1	0	3566	500.3	0	3604	500.1	0	3537	500.2
		0.50	0	8437	1003.9	0	12546	0.6	0	8920	500.3	0	8748	500.6	0	9253	500.1
		0.80	0	14113	1046.4	54	19688	1103.9	0	14505	502.6	140	14227	502.2	140	14295	500.1
	10	1.00	445	19812	1004.5	576	26125	1084.9	775	20566	503.5	635	19365	500.1	372	20820	508.8
	10	0.10	0	3114 8881	566.1	0	4256	0.1	0	3383	500.0	0	3272 9021	500.0 500.8	0	3200 9135	500.4
		0.50 0.80	195	14986	1104.5 1077.6	180	12850 21157	122.2 1160.4	240	9197 16103	501.0 500.6	205	15345	500.8	485	15387	500.9 500.9
		1.00	3571	20931	1077.0	964	29324	1180.4	240	21356	500.0	3494	21739	502.0	4061	21781	511.9
	11	0.10	0	3910	226.5	0	5517	0.1	270	4026	500.1	0	4152	500.8	4001	4099	500.5
		0.50	0	8113	1051.6	0	12241	83.1	0	9026	501.6	0	8501	501.7	0	8443	500.0
		0.80	181	13879	1025.7	181	19710	1051.7	0	14538	501.4	181	14248	500.1	102	13947	503.9
		1.00	895	21569	1004.4	444	28019	1010.0	237	21877	500.4	769	22065	501.4	854	22061	500.8
	12	0.10	0	3634	189.6	0	5372	0.1	0	3706	500.9	0	3863	500.4	0	3660	500.8
		0.50	0	9669	1075.4	0	14584	154.0	0	10548	506.8	0	10792	507.1	0	10960	504.7
		0.80	3	15518	1073.6	13	20770	1172.7	68	16812	500.7	78	15829	508.9	78	15979	507.0
		1.00	756	21013	1010.0	420	26258	1171.0	1534	23337	506.8	796	22100	502.3	2650	23860	506.0
	13	0.10	0	4415	217.9	0	5895	0.1	0	4488	500.0	0	4453	500.0	0	4346	500.6
		0.50	0	7777	1012.5	0	12976	0.6	0	8543	503.2	0	8002	504.5	0	8241	502.6
		0.80	0	11924	1074.7	0	17614	217.1	0	14021	500.7	0	12399	501.3	0	12553	508.2
		1.00	492	21687	1002.6	234	30050	1222.0	667	21642	500.6	927	21426	503.9	1349	23440	501.5
	14	0.10	0	3675	105.1	0	4816	0.1	0	3842	500.0	0	3765	500.3	0	3729	500.0
		0.50	0	7609	990.7	0	12272	0.6	0	7609	503.1	0	7644	501.3	0	8179	504.0
		0.80	0	11535	1030.9	0	17609	188.4	0	12810	500.7	0	11962	500.1	0	12439 22796	507.5
		1.00	300	20934	1008.4	171	26818	1048.9	866	23107	513.8	462	20678	501.3	601	22/90	503.5

Table A.29: Performance of VND-O, VND-S, GRASP-O, MA-O, and VNS-O for instance size (Sz.) 1000, instance numbers (Nr.) 15–29.

		_	_	VND-O			VND-			GRASP-			MA-O			VNS-O	
Sz.	Nr.	Load	C_a	C_u	t[s]	Ca	C_u	t[s]	C_a	C_u	t[s]	C_a	C_u	t[s]	C_a	C_u	t[s]
000	15	0.10	0	4379	208.0	0	5923	0.1	0	4732	500.1	0	4523	501.1	0	4445	500.8
		0.50	0	9442	1024.9	0	13953	0.6	0	9975	501.8	0	10023	500.0	0	10017	500.5
		0.80	25	15244	1057.0	25	19378	1075.9	202	18378	500.8	415	15068	500.4	407	15623	505.5
		1.00	271	21226	1019.0	129	26157	1123.0	75	22238	509.9	356	21376	506.0	341	22488	502.2
	16	0.10	0	3407	346.0	0	5535	0.1	0	3722	500.1	0	3722	500.2	0	3445	500.3
		0.50	0	8562	1039.1	0	13259	0.5	0	9132	501.5	0	8977	500.2	0	8794	501.7
		0.80	0	12980	1077.0	0	17842	186.6	0	13968	503.2	0	13296	501.0	0	13738	500.6
		1.00	15	19855	1004.9	92	25731	1229.2	89	21632	500.4	0	19838	505.7	0	21329	505.6
	17	0.10	0	3277	140.2	0	4789	0.1	0	3335	501.0	0	3360	500.1	0	3318	500.3
		0.50	0	9638	1000.8	0	14267	95.8	1	10148	503.0	1	9848	500.1	0	10037	500.1
		0.80	131	16398	1040.0	130	20956	1139.3	202	17271	504.7	247	16226	504.6	202	16822	504.9
		1.00	420	20852	1011.4	173	26541	1003.7	2079	22834	509.7	323	20633	503.2	513	21909	502.2
	18	0.10	0	3609	163.3	0	4996	0.1	0	3797	500.1	0	3808	501.1	0	3694	500.0
	10	0.50	0	8722	967.1	0	14021	0.7	0	9226	501.4	0	8974	505.8	0	9003	502.0
		0.80	0	13556	1107.8	0	20666	1.9	0	14504	503.2	0	14620	501.1	0	14519	502.4
		1.00	0	19996	1008.2	0	27385	833.5	330	22020	503.6	145	20219	505.1	1092	22447	509.3
	19	0.10	0	3441	209.8	0	5071	0.1	0	3746	500.1	0	3548	500.2	0	3397	500.8
	19																
		0.50	0	8623	1006.1	0	13490	196.9	0	9312	500.3	11	9274	500.2	0	9393	504.9
		0.80	0	14182	1042.0	0	19998	396.4	0	14865	502.6	11	14463	502.1	471	15046	500.9
	•	1.00	496	20822	1013.5	10	26340	1022.3	669	22114	500.4	506	20657	509.8	657	22706	508.2
	20	0.10	0	5129	234.3	0	6581	0.2	0	5239	500.1	0	5344	500.1	0	5278	500.1
		0.50	0	8886	1041.4	0	13071	0.6	0	9170	500.9	0	9224	503.1	0	9297	500.6
		0.80	0	14090	1020.5	0	20926	22.8	0	15476	505.4	0	15009	503.1	10	15026	502.9
		1.00	350	20310	1013.2	70	27316	1104.1	113	22324	503.4	1306	21099	501.8	640	21640	512.0
	21	0.10	0	4076	286.3	0	5436	0.2	0	4215	501.5	0	4230	500.3	0	4140	501.4
		0.50	0	8891	1002.1	0	13172	0.8	0	8998	500.8	0	9178	503.4	0	9299	500.8
		0.80	89	13080	1009.4	89	17547	1001.2	0	14081	503.9	89	13284	500.4	89	13539	501.6
		1.00	490	20092	1014.8	550	25878	1016.7	499	21015	505.4	642	20671	501.3	1159	20412	503.2
	22	0.10	0	3846	106.9	0	5040	0.1	0	3896	500.1	0	3830	501.0	0	3811	500.4
		0.50	0	7637	1002.8	0	12930	115.0	0	8498	501.2	0	7736	505.2	1366	8050	503.7
		0.80	0	13609	1167.7	0	19922	428.6	0	14097	503.2	0	13833	504.9	1366	14049	502.4
		1.00	2293	20607	1005.1	758	28152	1012.5	860	20302	510.0	2268	20076	501.0	2138	21291	507.7
	23	0.10	0	4208	106.8	0	5379	13.9	0	4293	500.5	0	4224	500.0	0	4251	500.1
		0.50	0	9397	1046.3	0	14073	94.4	0	9806	501.6	0	9689	501.8	0	9633	500.9
		0.80	20	12808	1119.4	20	17995	1128.8	185	13746	502.2	20	13188	501.2	0	13434	504.3
		1.00	774	21556	1001.2	540	27914	1091.1	1795	23157	506.0	950	21029	510.1	1418	22339	512.2
	24	0.10	0	3384	68.5	0	4662	0.1	0	3517	500.1	0	3459	500.8	0	3422	500.5
	2.	0.50	2	9596	1075.2	2	12818	1081.3	2	10189	500.8	2	10178	504.7	2	9859	503.7
		0.80	2	16223	1107.3	2	20791	1045.3	102	16224	502.8	1028	16382	506.1	911	16547	508.1
		1.00	971	20759	1013.9	528	24957	1068.8	551	22007	503.5	981	21229	503.8	1034	21980	512.6
	25	0.10	0	4114	379.7	0	6284	0.2	0	4250	500.1	0	4285	501.9	0	4209	500.8
	20	0.10	0	8360	1112.3	0	13407	0.2	0	8705	503.9		8889	506.9	0	8783	500.8
		0.30	0	14777	1112.3		21769	840.6	0	15925	505.6		15506	506.9		16295	501.5
		1.00	5	20428	100.2		21769	840.6 1081.9	1147	23330	505.6	270	20241	506.2	691	22011	501.2
	21					338											
	26	0.10	0	4570	166.6	0	6407	0.1	0	4766	500.0		4699	500.9	0	4807	500.1
		0.50	0	9124	1029.4	0	13540	88.5	0	9672	502.0	0	9702	501.0	0	9394	505.1
		0.80	0	15371	1137.7	0	20996	369.8	30	15910	502.6	0	15967	507.6	0	15970	502.2
	27	1.00	308	21298	1000.9	194	26020	1033.0	1552	22214	509.1	308	21712	514.3	242	22463	505.1
	27	0.10	0	2977	231.3	0	4175	0.1	0	3216	500.4	0	3141	500.2	0	3023	500.0
		0.50	0	7174	870.6	0	10707	0.4	0	7425	502.5	0	7298	500.6	0	7257	500.1
		0.80	0	12510	1112.6	0	18046	1.2	205	14122	501.6	0	13030	502.6	0	13011	501.3
		1.00	0	19228	1013.2	28	25302	1016.2	370	21183	502.4	240	19644	501.9	30	21289	507.5
	28	0.10	0	4536	260.1	0	6309	0.1	0	4679	500.1	0	4846	500.1	0	4666	500.9
		0.50	10	9881	1070.3	0	13777	204.2	10	10552	500.2	0	10323	504.7	0	10287	500.1
		0.80	10	14949	1023.9	135	19721	1079.9	10	15670	506.9	150	15129	500.6	10	15469	501.6
		1.00	1569	23433	1013.9	451	29311	1006.6	125	25106	505.3	1753	23220	502.9	1882	25455	504.3
	29	0.10	0	4111	322.0	0	5978	0.1	0	4303	500.2	0	4417	500.4	0	4257	500.0
		0.50	0	8080	1043.7	0	13549	4.5	0	8428	500.2	0	8648	502.3	0	8640	500.1
						0	19509	896.3	0	17023	501.4	0	14389	500.0	0	14809	503.0
		0.80	0	13828	1047.6		1/50/	090.5	0			0	14309	500.0	0	14009	

Table A.30: Performance of CP-O and CP-S for instance size (Sz.) 20, instance numbers (Nr.) 0–14.

			1	(СР-О	0.2 1 0 595 0.0				
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Opt	$\mathbf{C}_{\mathbf{a}}$			Opt
20	0	0.10	0	469			0			0
		0.50	0	778	6.1	1	0	1042	0.2	0
		0.80	0	1223	9991.3	0	0	1238	0.5	0
	1	1.00	0	1288	9995.6	0	0	1382	0.8	0
	1	0.10 0.50	0	351 668	0.0 0.5	1	0	377 791	0.0	0
		0.30	0	870	10000.3	0	0	1007	0.1	0
		1.00	0	921	9995.6	0	0	1007	0.2	0
	2	0.10	0	484	2146.7	1	0	511	0.0	0
		0.50	0	900	10000.5	0	0	990	0.1	0
		0.80	0	983	9995.5	0	0	1298	0.4	0
		1.00	0	1538	10000.4	0	0	1540	0.6	0
	3	0.10	0	467	31.6	1	0	739	0.0	0
		0.50	0	1006	9994.5	0	0	1147	0.1	0
		0.80	0	1175	9998.3	0	0	1305	0.2	0
	4	1.00	0	1438	10000.5	0	0	1578	0.4	0
	4	0.10 0.50	0	497 981	3.7	1	0	543	0.0	0
		0.30	0	1037	10000.5 10000.5	0	0	1048 1352	0.1	0
		1.00	0	1295	10000.3	0	0	1488	0.2	0
	5	0.10	0	401	0.7	1	0	464	0.0	0
		0.50	Ő	879	9970.9	0	0	978	0.1	0
		0.80	0	990	10000.3	0	0	1166	0.3	0
		1.00	0	1081	9997.3	0	0	1222	0.4	0
	6	0.10	0	374	1.7	1	0	493	0.0	0
		0.50	0	841	9983.3	0	0	1143	0.1	0
		0.80	0	1215	9979.4	0	0	1406	0.2	0
	-	1.00	0	1422	10000.3	0	0	1446	0.4	0
	7	0.10	0	521	9.1	1	0	819	0.0	0
		0.50 0.80	0	1034 1216	9997.0 9994.3	0	0	1289 1460	0.2 0.4	0
		1.00	0	1503	9994.3 9995.5	0	0	1590	0.4	0
	8	0.10	0	347	309.9	1	0	662	0.0	0
	0	0.50	0	849	9995.5	0	Ő	1092	0.2	0
		0.80	0	1122	9978.6	0	0	1171	0.5	0
		1.00	0	1273	9999.2	0	0	1442	0.6	0
	9	0.10	0	643	37.2	1	0	754	0.0	0
		0.50	0	1089	9995.6	0	0	1121	0.2	0
		0.80	0	1142	10000.4	0	0	1230	0.4	0
		1.00	0	1271	9995.5	0	0	1549	0.5	0
	10	0.10	0	559	44.3	1	0	660	0.0	0
		0.50 0.80	0	979 1268	10000.4 10000.5	0	0	1093 1374	0.1 0.3	0
		1.00	0	0	9994.5	0	0	0	9961.6	0
	11	0.10	0	217	0.5	1	0	308	0.0	0
		0.50	Ő	711	9981.7	0	0	933	0.1	0
		0.80	0	890	9993.7	0	0	1077	0.3	0
		1.00	0	973	9995.5	0	0	1077	0.4	0
	12	0.10	0	499	4.0	1	0	573	0.0	0
		0.50	0	882	10000.5	0	0	1153	0.2	0
		0.80	0	1257	9995.5	0	0	1440	0.4	0
	12	1.00	0	1382	10000.5	0	0	1586	0.6	0
	13	0.10	0	476	2733.9	1	0	689	0.0	0
		0.50	0	800 992	9995.5 10000.5	0	0	880 1310	0.4	0
		0.80 1.00	0	992 1315	10000.5	0	0	1310	0.8	0
	14	0.10	0	535	2.5	1	0	636	0.0	0
	11	0.10	0	947	9997.0	0	0	1195	0.0	0
		0.80	0	1219	10000.4	0	0	1310	0.4	0
		1.00	0	0	9989.3	0	0	0	10000.4	0

Table A.31: Performance of CP-O and CP-S for instance size (Sz.) 20, instance numbers (Nr.) 15–29.

~					СР-О				CP-S	
Sz.	Nr.	Load	Ca	Cu	t[s]	Opt	Ca	Cu	t[s]	Opt
20	15	0.10	0	318	3.0	1	0	474	0.0	0
		0.50	0	525	2.8	1	0	949	0.2	
		0.80	0	880	10000.5	0	0	1138	0.5	
		1.00	0	0	10000.5	0	0	0	9997.9	
	16	0.10	0	611	9.7	1	0	931	0.0	
		0.50	0	1047	9982.7	0	0	1373	0.2	
		0.80		1328	9995.6	0	0	1584	0.4	
		1.00	0	1482	9995.6	0	0	1584	0.5	
	17	0.10	0	453	0.1	1	0	563	0.0	
		0.50	0	923	2.7	1	0	1061	0.1	
		0.80	0	1323	10000.4	0	0	1447	0.3	
	10	1.00	0	1489	10000.4	0	0	1589	0.5	
	18	0.10	0	439	1.3	1	0	528	0.0	
		0.50	0	985	9994.7	0	0	1112	0.1	
		0.80	0	1245	10000.5	0	0	1271	95.1	
	10	1.00	0	0	9995.5	0	0	0	9893.5	
	19	0.10	0	579	2575.0	1	0	662	0.0	
		0.50	0	987	10000.5	0	0	1416	0.2	
		0.80	0	1353	9993.8	0	0	1500	0.5	
	•	1.00	0	0	9995.5	0	0	0	9981.1	
	20	0.10	0	371	0.2	1	0	434	0.0	
		0.50	0	958	9995.6	0	0	1172	0.1	
		0.80	0	1153	9949.1	0	0	1322	0.3	
	21	1.00	0	1366	9995.6	0	0	1391	0.4	
	21	0.10	0	421	1.9	1	0	484	0.0	
		0.50	0	697	10000.4	0	0	898	0.2	
		0.80	0	909	9979.4	0	0	1103	0.5	
	22	1.00	0	1211	9996.3	0	0	0	9989.3	
	22	0.10	0	524	0.1	1	0	647	0.0	
		0.50	0	669	0.3	1	0	748	0.1	
		0.80	0	802	2.0	1	0	1120	0.3	
		1.00	0	887	10000.4	0	0	1127	0.4	
	23	0.10	0	499	2102.5	1	0	591	0.0	
		0.50	0	936	9995.5	0	0	1180	0.2	
		0.80	0	1066	9995.1	0	0	1461	0.4	
	24	1.00	0	1361	10000.5	0	0	1559	0.7	
	24	0.10	0	620	14.3	1	0	716	0.0	
		0.50	0	930	9997.9	0	0	1075	0.2	
		0.80	0	1162	9995.0	0	0	1184	0.5	
	~~	1.00	0	1247	10000.0		0	1307	0.7	
	25	0.10	0	432	1.0	1	0	525	0.0	
		0.50	0	894	9995.5	0	0	955	0.1	
		0.80		1148	9999.0	0	0	1249	0.4	
	~	1.00	0	1357	9994.5	0	0	1481	9525.2	
	26	0.10	0	369	9.1	1	0	559	0.0	
		0.50	0	1022	9995.5	0	0	1022	0.1	
		0.80	0	962	9992.6		0	1054	0.3	
	27	1.00	0	0	9995.4	0	0	0	9995.5	
	27	0.10	0	607	1.9	1	0	687	0.0	
		0.50		807	0.8	1	0	1086	0.2	
		0.80	0	1115	9994.3	0	0	1446	0.4	
	20	1.00	0	0	9995.2		0	0	9975.3	
	28	0.10		531	12.0	1	0	617	0.0	
		0.50	0	1128	9995.4	0	0	1247	0.1	
		0.80	0	1286	9995.4	0	0	1502	0.4	
		1.00	0	0	9999.8	0	0	0	10000.4	
	29	0.10	0	368	10.3	1	0	460	0.0	
		0.50	0	681	9995.3	0	0	1029	0.1	
		0.80	0	1135	9995.5	0	0	1258	0.4	0
		1.00	0	1354	9995.5	0	0	1418	0.6	0

Table A.32: Performance of CP-O and CP-S for instance size (Sz.) 30, instance numbers (Nr.) 0–14.

			1	(СР-О				CP-S	
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	C_{u}	t[s]	Opt	$\mathbf{C}_{\mathbf{a}}$	Cu	t[s]	Opt
30	0	0.10	0	606	368.4	1	0	633	0.0	0
		0.50	0	1139	9994.0	0	0	1327	0.3	0
		0.80	0	1430	9995.5	0	0	1509	0.7	0
		1.00	0	1561	10000.3	0	0	1808	0.9	0
	1	0.10	0	836	9999.9	0	0	866	0.1	0
		0.50	0	1383	10000.2	0	0	1579	0.8	0
		0.80	0	0	99999.6	0	0	0	9989.1	0
	2	1.00	0	0	9991.5	0	0 0	0	10000.2	0
	2	0.10 0.50	0	959 1700	9998.8 9995.3	0	0	1237 1861	0.1 0.6	0
		0.80	0	0	9995.4	0	0	0	9971.4	0
		1.00	0	0	10000.4	0	0	0	9981.4	0
	3	0.10	0	789	9995.4	0	0	1031	0.0	0
		0.50	0	1348	10000.4	0	0	1380	0.3	0
		0.80	0	1796	9995.4	0	0	1939	0.8	0
		1.00	0	0	9999.7	0	0	0	10000.4	0
	4	0.10	0	701	16.4	1	0	994	0.0	0
		0.50	0	1578	10000.3	0	0	1629	0.4	0
		0.80	0	1710	9993.9	0	0	1745	0.8	0
		1.00	0	1767	10000.2	0	0	2114	1.2	0
	5	0.10	0	606	283.0	1	0	737	0.0	0
		0.50	0	1287	10000.4	0	0	1422	0.3	0
		0.80	0	1533	9998.8	0	0	1736	0.7	0
		1.00	0	0	9993.9	0	0	0	9999.5	0
	6	0.10	0	823	9995.6	0	0	875	0.1	0
		0.50 0.80	0	1359 1689	9999.5 10000.4	0	0 0	1479 1731	0.6 1.6	0
		1.00	0	1089	9994.8	0	0	0	9940.7	0
	7	0.10	0	759	10000.0	0	0	910	0.0	0
	,	0.50	0	1390	10000.4	0	0	1627	0.5	0
		0.80	0	1780	10000.1	0	0	1939	1.4	0
		1.00	0	2048	9995.4	0	0	2408	2.1	0
	8	0.10	0	1208	10000.5	0	0	1491	0.0	0
		0.50	0	1395	10000.2	0	0	1688	0.5	0
		0.80	0	1534	9998.2	0	0	1739	1.0	0
		1.00	0	0	9994.3	0	0	0	10000.4	0
	9	0.10	0	571	1232.1	1	0	834	0.0	0
		0.50	0	1409	9995.2	0	0	1813	0.5	0
		0.80	0	1966	9999.1	0	0	2170	1.4	0
		1.00	0	0	9995.5	0	0	0	10000.4	0
	10	0.10	0	593	3.9	1	0	610	0.0	0
		0.50	0	1054	10000.5	0	0	1375	0.2	0
		0.80 1.00	0	1393 1560	10000.4 9997.7	0	0 0	1643 1930	0.5 0.9	0
	11	0.10	0	646	9997.7	0	0	937	0.9	0
	11	0.10	0	1297	9991.3	0	0	1459	0.0	0
		0.80	0	1682	9994.6	0	0	2088	0.7	0
		1.00	0	2151	9993.7	0	0	2327	1.7	0
	12	0.10	Ő	754	0.1	1	0	869	0.0	0
		0.50	0	1300	9995.4	0	0	1439	0.3	0
		0.80	0	1703	10000.5	0	0	2015	0.7	0
		1.00	0	0	9998.1	0	0	0	9998.9	0
	13	0.10	0	542	3.7	1	0	686	0.0	0
		0.50	0	1184	10000.3	0	0	1464	0.3	0
		0.80	0	1535	10000.3	0	0	1787	0.8	0
		1.00	0	1719	9996.9	0	0	1967	1.1	0
	14	0.10	0	749	2316.7	1	0	891	0.0	0
		0.50	0	1391	10000.4	0	0	1813	0.4	0
		0.80	0	1695	10000.4	0	0	1865	1.0	0
		1.00	0	2162	10000.2	0	0	2162	1.7	0

Table A.33: Performance of CP-O and CP-S for instance size (Sz.) 30, instance numbers (Nr.) 15–29.

<u>Sz.</u> 30	Nr. 15 16	Load 0.10 0.50 0.80 1.00 0.10 0.50	Ca 0 0 0 0	Cu 977 1270 1691	t[s] 70.0 143.2	Opt	C _a	Cu 1209	t[s]	Opt 0
30	16	0.50 0.80 1.00 0.10 0.50	0 0 0	1270					0.0	0
		0.80 1.00 0.10 0.50	00		143.2					
		1.00 0.10 0.50	0	1691		1	0	1633	0.3	0
		0.10 0.50		1071	9995.1	0	0	1903	0.9	0
		0.50		1853	9993.5	0	0	2304	1.4	0
	17		0	458	288.9	1	0	633	0.0	0
	17		0	1186	9995.5	0	0	1556	0.3	0
	17	0.80	0	1700	9995.6	0	0	1778	0.6	0
	17	1.00	0	1810	9999.8	0	0	1931	0.9	0
		0.10	0	784	9994.2	0	0	992	0.1	0
		0.50	0	1392	10000.4	0	0	1392	0.5	0
		0.80	0	1588	9896.3	0	0	1934	1.2	0
		1.00	0	0	9995.5	0	0	0	10000.1	0
	18	0.10	0	561	0.1	1	0	646	0.0	0
		0.50	0	1271	10000.4	0	0	1481	0.3	0
		0.80	0	1596	9994.5	0	0	1831	0.6	0
	10	1.00	0	1879	9995.6	0	0	2292	1.0	0
	19	0.10	0	498	4880.8	1	0	975	0.0	0
		0.50	0	1394	10000.3	0	0	1640	0.7	0
		0.80	0	1669	99999.0	0	0	1962	1.8	0
		1.00	0	0	9995.2	0	0	0	9995.5	0
	20	0.10	0	910	9994.9	0	0	1171	0.0	0
		0.50	0	1525	9995.2	0	0	1803	0.5	0
		0.80	0	1791	9995.3	0	0	2023	1.2	0
		1.00	0	1999	10000.1	0	0	2157	2.0	0
	21	0.10	0	640	10000.2	0	0	897	0.0	0
		0.50	0	994	10000.4	0	0	1283	0.2	0
		0.80	0	1613	9995.4	0	0	1636	0.6	0
	22	1.00	0	1844	9995.5	0	0	1968	1.9	0
	22	0.10	0	881	116.0	1	0	1003	0.0	0
		0.50	0	1437	10000.4	0	0	1554	0.3	0
		0.80	0	1481	10000.2	0	0	1958	0.7	0
	22	1.00	0	1881	9919.3	0	0	2268	1.1	0
	23	0.10	0	738	3.0	1	0	755	0.0	0
		0.50	0	1753	9994.7	0	0	1764	0.5	
		0.80		2003	9995.6	0	0	2254	1.0	0
	24	1.00	0	525	9962.3	0	0	0	9946.7 0.1	
	24	0.10		525	52.9		0	630		
		0.50 0.80	0	1459 1707	10000.3 9995.1	0	0	1543 2123	0.6 1.8	
						0	0		9995.5	0
	25	1.00 0.10		0 930	10000.1 9995.5	0	0	0 1198	0.0	0
	23	0.10		1592	9993.5	0	0	1756	0.0	0
		0.80	0	1728	9903.3	0	0	2019	1.2	0
		1.00	0	2154	99999.9	0	0	2019	1.2	0
	26	0.10	0	653	9999.6	0	0	763	0.0	0
	20	0.10	0	1304	10000.3	0	0	1509	0.0	0
		0.80	0	1598	9995.1	0	0	1977	1.0	0
		1.00	0	0	9999.4	0	0	0	9979.9	0
	27	0.10	0	549	1.2	1	0	751	0.0	0
	21	0.10	0	1198	10000.3	0	0	1432	0.0	0
		0.80	0	1574	9995.5	0	0	1693	0.5	0
		1.00	0	1843	10000.0	0	0	2124	1.3	0
	28	0.10		962	10000.0	0	0	1031	0.0	0
	20	0.10		1317	9995.2	0	0	1422	0.0	0
		0.30	0	1455	10000.4	0	0	2013	0.5	0
		1.00	0	2013	9995.0	0	0	2013	1.3	0
	29	0.10	0	411	521.5	1	0	915	0.0	0
		0.10	0	1094	10000.3	0	0	1524	0.0	0
		0.80	0	1639	99999.6	0	0	1858	0.5	0
		1.00	0	1751	9903.7	0	0	1920	0.9	0
		1.00	L Ŭ			. <u> </u>	Ŭ			

Table A.34: Performance of CP-O and CP-S for instance size (Sz.) 50, instance numbers (Nr.) 0–14.

					СР-О				CP-S	
Sz.	Nr.	Load	C_{a}	C_u	t[s]	Opt	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	Opt
50	0	0.10	0	1085	9908.4	0	0	1271	0.1	0
		0.50	0	2186	9992.4	0	0	2410	2.0	0
		0.80	0	2686	9988.8	0	0	3047	4.8	0
		1.00	0	0	10000.3	0	0	0	9932.4	0
	1	0.10	0	1425	10000.4	0	0	1492	0.2	0
		0.50	0	1955	9997.5	0	0	2277	1.0	0
		0.80	0	2376	10000.1	0	0	2744	2.8	0
		1.00	0	0	99999.9	0	0	0	9974.1	0
	2	0.10	0	830	9960.1	0	0	1175	0.0	0
		0.50	0	2058	9995.4	0	0	2453	1.0	0
		0.80	0	2794	10000.2	0	0	2988	2.3	0
		1.00	0	0	9995.2	0	0	0	9995.4	0
	3	0.10	0	1171	9995.3	0	0	1425	0.1	0
		0.50	0	2208	9995.4	0	0	2645	1.1	0
		0.80	0	2864	10000.0	0	0	3430	2.7	0
		1.00	0	0	10000.1	0	0	0	10000.1	0
	4	0.10	0	1213	9995.3	0	0	1317	0.1	0
		0.50	0	2182	10000.1	0	0	2237	1.0	0
		0.80	0	2443	9989.1	0	0	2625	2.2	0
		1.00	0	3191	10000.3	0	0	3255	3.4	0
	5	0.10	0	1344	9901.0	0	0	1429	0.1	0
		0.50	0	2000	9995.5	0	0	2065	0.8	0
		0.80	0	2429	10000.2	0	0	2742	2.6	0
		1.00	0	2555	9923.1	0	0	3188	58.9	0
	6	0.10	0	852	9921.5	0	0	861	0.1	0
		0.50	0	2062	10000.4	0	0	2342	1.2	0
		0.80	0	2519	9980.4	0	0	2708	2.8	0
		1.00	0	2990	10000.5	0	0	3088	4.1	0
	7	0.10	0	1116	10000.3	0	0	1169	0.0	0
		0.50	0	2449	9995.6	0	0	2480	0.9	0
		0.80	0	2702	9998.0	0	0	2795	2.2	0
		1.00	0	0	10000.5	0	0	0	9995.5	0
	8	0.10	0	1009	9995.6	0	0	1084	0.1	0
		0.50	0	1960	9995.4	0	0	2181	2.2	0
		0.80	0	2333	9968.5	0	0	2843	5.5	0
		1.00	0	0	10000.2	0	0	0	10000.2	0
	9	0.10	0	1483	9999.4	0	0	1617	0.1	0
		0.50	0	1992	9945.4	0	0	2289	0.6	0
		0.80	0	2258	10000.2	0	0	2587	1.7	0
		1.00	0	0	9987.6	0	0	0	9993.3	0
	10	0.10	0	1006	9990.2	0	0	1046	0.1	0
		0.50	0	2307	9996.0	0	0	2674	1.3	0
		0.80	0	2839	9933.2	0	0	2942	2.7	0
		1.00	0	3473	9995.4	0	0	3782	4.7	0
	11	0.10	0	1201	10000.4	0	0	1402	0.0	0
		0.50	0	2183	9995.6	0	0	2507	0.7	0
		0.80	0	2591	9995.4	0	0	2893	2.0	0
		1.00	0	0	9945.9	0	0	0	9997.3	0
	12	0.10	0	956	9999.6	0	0	1098	0.1	0
		0.50	0	2202	10000.3	0	0	2326	1.1	0
		0.80	0	2418	9995.5	0	Õ	2600	2.7	0
		1.00	0	0	9999.4	0	0	0	10000.4	0
	13	0.10	0	1136	9876.1	0	0	1375	0.1	0
	10	0.10	0	2058	9995.6	0	0	2291	1.0	0
		0.80	0	2710	9995.5	0	0	3143	2.1	0
		1.00	0	0	9998.7	0	0	0	10000.4	0
	14	0.10	0	1105	10000.5	0	0	1314	0.1	0
	14	0.10	0	2038	9995.6	0	0	2144	0.1	0
		0.30	0	2593	9995.5	0	0	2756	2.3	0
		1.00	0	2564	10000.5	0	0	2937	3.2	0
		1.00		2007	10000.5		0		5.2	

Table A.35: Performance of CP-O and CP-S for instance size (Sz.) 50, instance numbers (Nr.) 15–29.

					СР-О				CP-S	
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	Opt	C_a	C_u	t[s]	Opt
50	15	0.10	0	1314	10000.4	0	0	1350	0.1	0
		0.50	0	2353	9967.5	0	0	2582	1.4	0
		0.80	0	2965	10000.5	0	0	3149	3.7	0
		1.00	0	3793	9911.8	0	0	3793	5.5	0
	16	0.10	0	1232	9999.2	0	0	1306	0.1	0
		0.50	0	1894	9995.5	0	0	2243	1.4	0
		0.80	0	2443	10000.4	0	0	2739	3.1	0
		1.00	0	2829	9937.9	0	0	3041	5.2	0
	17	0.10	0	1321	9999.6	0	0	1400	0.1	0
		0.50	0	2518	9995.5	0	0	2633	1.0	0
		0.80	0	3089	9926.8	0	0	3278	2.5	0
		1.00	0	0	9745.9	0	0	0	9691.6	0
	18	0.10	0	839	408.5	1	0	1093	0.1	0
		0.50	0	2134	9999.0	0	0	2239	1.1	0
		0.80	0	2812	10000.3	0	0	2972	2.7	0
		1.00	0	3142	10000.3	0	0	3494	4.6	0
	19	0.10	0	949	10000.4	0	0	1458	0.1	0
		0.50	0	1955	10000.4	0	0	2236	1.2	0
		0.80	0	2601	10000.3	0	0	2788	2.8	0
		1.00	0	0	9995.5	0	0	0	9976.4	0
	20	0.10	0	1819	9908.4	0	0	1939	0.1	0
		0.50	0	2402	9982.7	0	0	2485	0.9	0
		0.80	0	2821	9995.5	0	0	2844	2.4	0
		1.00	0	3282	9989.9	0	0	3290	3.2	0
	21	0.10	0	1254	10000.4	0	0	1482	0.1	0
		0.50	0	1944	9980.3	0	0	2275	1.0	0
		0.80	0	3023	9992.6	0	0	3065	2.7	0
		1.00	0	3426	9915.0	0	0	3484	3.9	0
	22	0.10	0	1495	9991.1	0	0	1607	0.1	0
		0.50	0	2666	10000.5	0	0	2778	1.3	0
		0.80	0	2919	9991.9	0	0	3176	2.5	0
		1.00	0	0	9994.6	0	Ő	0	9981.2	0
	23	0.10	0	1060	10000.5	0	0	1228	0.1	0
	20	0.50	0	2444	9948.0	0	0	2700	0.9	0
		0.80	0	2976	10000.5	0	0	3151	1.9	0
		1.00	0	3061	10000.3	0	0	3201	3.0	0
	24	0.10	0	984	9994.1	0	Ő	1138	0.1	0
		0.50	0	2203	10000.5	0	0	2480	1.3	0
		0.80	0	3281	9995.4	0	0	3350	3.5	0
		1.00	0	3572	9995.4	0	0	3705	5.3	0
	25	0.10	0	982	9867.5	0	0	1293	0.1	0
	20	0.50	0	2223	9996.9	0	0	2420	1.1	0
		0.80	0	0	9921.9	0	0	0	9995.5	0
		1.00	0	0	9885.6	0	0	0	10000.4	0
	26	0.10	0	941	9995.5	0	0	1146	0.0	0
	20	0.50	0	2345	10000.3	0	0	2405	1.0	0
		0.80	0	2737	10000.3	0	0	2933	2.3	0
		1.00	0	2867	9961.0	0	0	3106	3.7	0
	27	0.10	0	1397	9997.6	0	0	1497	0.1	0
	21	0.10	0	2356	9997.0 9995.6	0	0	2507		0
		0.30	0	2530	99931.7	0	0	2856	0.9	
	20	1.00		2778	9995.2 10000.3	0	0	3160	3.1	
	28	0.10		1505	9995.4	0	0	1772	0.3	
		0.50	0	2672		0	0	2978	3.5	0
		0.80		0	9995.5	0	0	0	9998.1	0
	-	1.00	0	0	10000.4	0	0	0	9995.4	0
	29	0.10	0	620	1433.3	1	0	865	0.1	
		0.50	0	2459	9995.5		0	2650	1.1	
		0.80	0	3332	9995.1	0	0	3383	4.0	0
		1.00	0	0	10000.4	0	0	0	9950.3	0

Table A.36: Performance of CP-O and CP-S for instance size (Sz.) 100, instance numbers (Nr.) 0-14.

Sz. Nr. Load $\begin{vmatrix} CP-O \\ C_a \end{vmatrix} \begin{vmatrix} C_u \\ C_u \end{vmatrix} t[s] Opt C_a $									CP-S	
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Opt	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	Opt
100	0	0.10	0	2554	9995.8	0	0	2637	0.3	0
		0.50	0	3969	9985.8	0	0	4261	6.1	0
		0.80	0	0	9960.6	0	0	0	9995.5	0
		1.00	-	-	-	-	-	-	-	-
	1	0.10	0	3372	9929.4	0	0	3372	0.6	0
		0.50	0	5134	10000.4	0	0	5345	6.0	0
		0.80	0	0	9768.7	0	0	0	9760.6	0
	2	1.00	0	0	9692.8	0	0	0	9685.4	0
	2	0.10 0.50	0	1360	9970.4	0	0	1522 3992	0.3	0
		0.30	0	3562 4797	10000.1 9421.4	0	0	4797	4.1	0
		1.00	-	4/9/	9421.4	-	0	4/9/	11.0	0
	3	0.10	0	2891	9924.9	0	0	2961	1.0	0
	5	0.10	0	5407	10000.5	0	0	5541	8.2	0
		0.80	0	0	9997.4	0	0	0	9998.0	0
		1.00	-	-	-	-	-	-	-	-
	4	0.10	0	2519	9985.2	0	0	2572	0.8	0
	-	0.50	0	4973	9995.0	0	0	5055	9.9	0
		0.80	0	0	9999.4	0	0	0	9999.6	0
		1.00	_	-	-	_	_	-	-	-
	5	0.10	0	2060	10000.5	0	0	2381	0.5	0
		0.50	0	4506	10000.2	0	0	4618	6.6	0
		0.80	0	5223	9995.1	0	0	5500	18.1	0
		1.00	-	-	-	-	-	-	-	-
	6	0.10	0	2706	9981.8	0	0	2759	1.9	0
		0.50	0	4679	9729.9	0	0	4679	7.9	0
		0.80	0	5946	9998.9	0	0	6019	15.8	0
		1.00	0	0	9997.4	0	0	0	9988.3	0
	7	0.10	0	3959	9995.6	0	0	3981	0.5	0
		0.50	0	5139	9779.4	0	0	5213	5.6	0
		0.80	0	5960	9939.0	0	0	6165	14.8	0
		1.00	-	-	-	-	-	-	-	-
	8	0.10	0	2139	9989.1	0	0	2362	0.4	0
		0.50	0	4236	10000.4	0	0	4602	3.6	0
		0.80	0	0	9961.5	0	0	0	9995.4 10000.3	0
	9	1.00 0.10	0	0 2465	9997.7 9934.6	0	0	0 2622	0.6	0
	,	0.10	0	2405	10000.1	0	0	0	10000.1	0
		0.80	0	0	10000.1	0	0	0	10000.1	0
		1.00	-	-	- 10000.5	-	-	-	- 10000.5	-
	10	0.10	0	2809	9995.4	0	0	2947	0.6	0
		0.50	0	3331	9997.6	0	Ő	4073	3.2	0
		0.80	0	4761	9994.5	0	0	5114	8.3	0
		1.00	-	-	-	-	-	-	-	-
	11	0.10	0	3759	9907.6	0	0	3965	4.9	0
		0.50	0	5520	10000.2	0	0	5603	9.4	0
		0.80	0	6380	9993.9	0	0	6535	21.5	0
		1.00	-	-	-	-	-	-	-	-
	12	0.10	0	2713	99999.0	0	0	2865	0.4	0
		0.50	0	4408	9995.1	0	0	5229	5.3	0
		0.80	-	-	-	-	-	-	-	-
		1.00	-	-	-	-	-	-	-	-
	13	0.10	0	2761	9995.4	0	0	2777	0.3	0
		0.50	0	4672	10000.0	0	0	5001	7.7	0
		0.80	0	5621	9998.7	0	0	5996	16.2	0
	1.4	1.00	-	-		-	-	-	-	-
	14	0.10	0	3103	9992.2	0	0	3263	0.4	0
		0.50	0	4858	9932.5	0	0	5117	6.0	0
		0.80 1.00	0	6114	9999.1	0	0	6355	14.8	0
		1.00	-	-	-	-	-	-	-	

Table A.37: Performance of CP-O and CP-S for instance size (Sz.) 100, instance numbers (Nr.) 15–29.

~			-		СР-О		~		CP-S	
Sz.	Nr.	Load	Ca	Cu	t[s]	Opt	Ca	Cu	t[s]	Op
100	15	0.10	0	2421	9995.0	0	0	2548	0.4	(
		0.50	0	4060	9964.9	0	0	4256	3.9	
		0.80	0	5273	9992.0	0	0	5415	8.4	
	16	1.00	0	0	9994.9	0	0	0	9994.9	
	16	0.10	0	3285	9886.6	0	0	3320	0.3	
		0.50	0	4641	9995.3	0	0	4772	3.5	
		0.80	0	5309	9995.1	0	0	5814	10.2	
	17	1.00	-			-	-	-	-	
	17	0.10	0	2492	9992.1	0	0	2548	0.4	
		0.50	0	5346	10000.5	0	0	5412	3.9	
		0.80 1.00	0	6218 0	9993.8 9949.1	0	0	6342 0	8.3 9995.4	
	18	0.10	0	2326	9949.1	0	0	2414	0.3	
	10	0.10	0	4012	9900.0	0	0	4356	4.4	
		0.30	0	4865	9992.9	0	0	4330 5144	11.8	
		1.00	-	4005	3332.3	-	-	5144		`
	19	0.10	0	2762	9927.5	0	0	2762	0.4	
	19			3701	9927.3	0	0	4485		
		0.50 0.80	0	3701	9994.9	-	-	4485	8.5	
		1.00	-	_	-	-	-	-	-	
	20	0.10	0	1923	9999.7	0	0	1964	0.3	
	20	0.10	0	4539	99995.4	0	0	4730	5.0	
		0.30	0	5503	9993.4	0	0	5925	10.9	
		1.00	0	6091	9999.6	0	0	6401	15.7	
	21	0.10	0	2722	9999.0	0	0	2740	0.7	
	21	0.10	0	3871	9992.4	0	0	4494	7.7	
		0.30	0	0	9995.2	0	0	4494	9963.1	
		1.00						0		`
	22	0.10	- 0	- 1172	- 10000.5	- 0	- 0	1246	0.2	
	22	0.10	0	3893	9997.2	0	0	4201	3.5	
		0.30	0	5498	99997.2	0	0	5715	10.1	
		1.00	0	0	9953.2	0	0	0	9995.0	
	23	0.10	0	2056	10000.3	0	0	2066	0.4	
	23	0.10	0	4217	10000.3	0	0	4468	5.0	
		0.80	0	5638	9999.5	0	0	5799	13.9	
		1.00	-	- 5058	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	-	-	5199	- 15.9	`
	24	0.10	0	2953	9992.8	0	0	3214	0.3	
	24	0.10	0	5056	10000.2	0	0	5091	4.8	
		0.80	0	6131	10000.2	0	0	6309	12.2	
		1.00	-	0151	10000.1	-	-	0507	-	`
	25	0.10	0	2473	9904.0	0	0	2549	0.3	
	25	0.10	0	4048	9997.9	0	0	4582	4.6	
		0.80	0	4960	9999.6	0	0	5609	13.2	
		1.00	-		-	-	-		- 15.2	
	26	0.10	0	2833	9982.1	0	0	3021	0.2	(
	20	0.50	0	4380	10000.1	0	0	4437	3.4	
		0.80	0	5641	9998.6	0	0	5831	12.8	
		1.00	0	6283	9994.0	0	0	6437	17.8	
	27	0.10	0	2128	9979.0	0	0	2142	0.4	
		0.50	0	4448	9975.2	0	0	4599	3.5	(
		0.80	0	5608	9988.1	0	0	5924	9.8	
		1.00	0	0	99999.4	0	0	0	9976.3	
	28	0.10	0	2603	10000.4	0	0	2908	0.4	
	20	0.50	0	4883	9977.1	0	0	5320	5.9	
		0.80	0	5445	9954.1	0	0	5993	14.4	
		1.00	0	0	9980.7	0	0	0	9987.2	
	29	0.10	0	2842	10000.0	0	0	2891	0.7	
		0.10	0	5015	9965.9	0	0	5130	7.1	
		0.80	0	0	99999.6	0	0	0	10000.0	
			Ĭ	0	99999.7	0	0	0	10000.0	I '

Table A.38: Performance of ILP-O and ILP-S for instance size (Sz.) 20, instance numbers (Nr.) 0–14.

			1		ILP-O	ILP-S						
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	Gap	Opt	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Gap	Opt
20	0	0.10	0	469	0.0	0.0	1	0	835	0.0	0.0	1
		0.50	0	778	1.7	0.0	1	0	1255	0.2	0.0	1
		0.80	0	977	0.9	0.0	1	0	1284	0.4	0.0	1
		1.00	0	1057	2.7	0.0	1	0	1475	0.6	0.0	1
	1	0.10	0	351	0.0	0.0	1	0	736	0.0	0.0	1
		0.50	0	668	0.2	0.0	1	0	1048	0.1	0.0	1
		0.80	0	668	0.4	0.0	1	0	1096	0.2	0.0	1
		1.00	0	786	3.2	0.0	1	0	1269	0.2	0.0	1
	2	0.10	0	484	0.4	0.0	1	0	1010	0.0	0.0	1
		0.50	0	706	0.5	0.0	1	0	1353	0.2	0.0	1
		0.80	0	862	3.6	0.0	1	0	1606	0.5	0.0	1
		1.00	0	1028	646.5	0.0	1	0	1586	0.7	0.0	1
	3	0.10	0	467	0.0	0.0	1	0	1122	0.0	0.0	1
		0.50	0	826	0.5	0.0	1	0	1445	0.1	0.0	1
		0.80	0	1055	18.1	0.0	1	0	1493	0.1	0.0	1
	4	1.00	0	1185	5.3	0.0	1	0	1493	0.2	0.0	1
	4	0.10	0	497	0.0	0.0	1	0	675	0.0	0.0	1
		0.50	0	772 915	0.1	0.0	1	0	1043 1304	0.1	0.0	1
		0.80 1.00	0	1043	0.4 0.7	0.0 0.0	1	0	1488	0.1 0.2	0.0 0.0	1
	5	0.10	0	401	0.7	0.0	1	0	771	0.2	0.0	1
	5	0.10	0	733	0.0	0.0	1	0	1160	0.0	0.0	1
		0.80	0	816	0.9	0.0	1	0	1310	0.1	0.0	1
		1.00	0	935	2.8	0.0	1	0	1310	0.1	0.0	1
	6	0.10	0	374	0.0	0.0	1	0	757	0.2	0.0	1
	0	0.50	0	768	1.4	0.0	1	0	1326	0.1	0.0	1
		0.80	0	941	2.4	0.0	1	0	1369	0.1	0.0	1
		1.00	0	1159	2.7	0.0	1	0	1446	0.2	0.0	1
	7	0.10	0	521	0.0	0.0	1	0	802	0.0	0.0	1
		0.50	0	931	0.8	0.0	1	0	1508	0.2	0.0	1
		0.80	0	1076	2.0	0.0	1	0	1494	0.3	0.0	1
		1.00	0	1158	56.4	0.0	1	0	1596	0.4	0.0	1
	8	0.10	0	347	0.1	0.0	1	0	618	0.0	0.0	1
		0.50	0	752	3.7	0.0	1	0	1305	0.2	0.0	1
		0.80	0	1004	28.6	0.0	1	0	1405	0.4	0.0	1
		1.00	0	1066	32.7	0.0	1	0	1442	0.5	0.0	1
	9	0.10	0	643	0.1	0.0	1	0	1056	0.0	0.0	1
		0.50	0	984	0.5	0.0	1	0	1317	0.1	0.0	1
		0.80	0	1042	0.8	0.0	1	0	1591	0.2	0.0	1
		1.00	0	1106	1.7	0.0	1	0	1585	0.3	0.0	1
	10	0.10	0	559	0.0	0.0	1	0	998	0.0	0.0	1
		0.50	0	905	0.2	0.0	1	0	1306	0.1	0.0	1
		0.80	0	1095	0.4	0.0	1	0	1502	0.1	0.0	1
		1.00	0	1172	5.5	0.0	1	0	1502	0.2	0.0	1
	11	0.10	0	217	0.0	0.0	1	0	639	0.0	0.0	1
		0.50	0	576	0.3	0.0	1	0	1036	0.1	0.0	1
		0.80	0	633	0.5	0.0	1	0	1217	0.1	0.0	1
	12	1.00	0	854 499	3.9 0.0	0.0	1	0	1217 939	0.2 0.0	0.0 0.0	1
	12	0.10		499 801		0.0	1	0				
		0.50 0.80	0	944	0.4	0.0	1	0	1246 1558	0.1	0.0	1
		1.00	0	1050	2.9 8.9	0.0 0.0	1	0	1611	0.2 0.5	0.0 0.0	1
	13	0.10	0	476	0.2	0.0	1	0	1329	0.5	0.0	1
	15	0.10	0	671	2.5	0.0	1	0	1653	0.1	0.0	1
		0.30	0	878	53.0	0.0	1	0	1739	1.3	0.0	1
		1.00	0	954	918.8	0.0	1	0	1739	1.5	0.0	1
	14	0.10	0	535	0.0	0.0	1	0	776	0.0	0.0	1
	11	0.10	0	818	0.3	0.0	1	0	1288	0.0	0.0	1
		0.80	0	899	1.7	0.0	1	0	1456	0.1	0.0	1
		1.00	0	1107	22.9	0.0	1	0	1409	0.3	0.0	1
			L Ŭ								5.0	

Table A.39: Performance of ILP-O and ILP-S for instance size (Sz.) 20, instance numbers (Nr.) 15–29.

		1	1		ILP-O				ILP-S			
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Gap	Opt	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Gap	Opt
20	15	0.10	0	318	0.1	0.0	1	0	847	0.0	0.0	1
		0.50	0	525	0.3	0.0	1	0	1212	0.2	0.0	1
		0.80	0	800	0.8	0.0	1	0	1521	0.4	0.0	1
		1.00	0	919	1.0	0.0	1	0	1551	0.6	0.0	1
	16	0.10	0	611	0.0	0.0	1	0	1062	0.0	0.0	1
		0.50	0	866	0.5	0.0	1	0	1499	0.1	0.0	1
		0.80	0	1094	1.2 2.4	0.0	1	0	1499	0.2	0.0	1
	17	1.00 0.10	0	1186 453	0.0	0.0 0.0	1	0	1559 1113	0.3	0.0	1
	17	0.10	0	923	0.0	0.0	1	0	1235	0.0	0.0	1
		0.80	0	1076	0.7	0.0	1	0	1546	0.2	0.0	1
		1.00	0	1164	1.9	0.0	1	0	1656	0.2	0.0	1
	18	0.10	0	439	0.0	0.0	1	0	820	0.0	0.0	1
		0.50	0	783	0.6	0.0	1	0	934	0.1	0.0	1
		0.80	0	906	3.7	0.0	1	0	1125	0.1	0.0	1
		1.00	0	1030	5.9	0.0	1	0	1339	0.2	0.0	1
	19	0.10	0	579	0.1	0.0	1	0	1030	0.0	0.0	1
		0.50	0	915	3.9	0.0	1	0	1520	0.2	0.0	1
		0.80	0	1174	103.8	0.0	1	0	1595	0.8	0.0	1
		1.00	0	1256	644.4	0.0	1	0	1557	1.1	0.0	1
	20	0.10	0	371	0.0	0.0	1	0	565	0.0	0.0	1
		0.50	0	598	0.3	0.0	1	0	1233	0.1	0.0	1
		0.80	0	715	1.1	0.0	1	0	1293	0.2	0.0	1
	21	1.00	0	860	26.4	0.0	1	0	1481	0.2	0.0	1
	21	0.10 0.50	0	421 675	0.0	0.0	1	0	813	0.0	0.0	1
		0.30	0	882	1.8 14.8	0.0 0.0	1	0	1257 1420	0.2 0.4	0.0	1
		1.00	0	950	94.9	0.0	1	0	1420	0.4	0.0	1
	22	0.10	0	524	0.0	0.0	1	0	995	0.0	0.0	1
		0.50	0	669	0.2	0.0	1	0	912	0.1	0.0	1
		0.80	0	802	0.4	0.0	1	0	1333	0.2	0.0	1
		1.00	0	887	1.0	0.0	1	0	1168	0.2	0.0	1
	23	0.10	0	499	0.1	0.0	1	0	1078	0.0	0.0	1
		0.50	0	727	0.8	0.0	1	0	1530	0.2	0.0	1
		0.80	0	892	39.4	0.0	1	0	1477	0.5	0.0	1
		1.00	0	937	8.7	0.0	1	0	1549	0.5	0.0	1
	24	0.10	0	620	0.1	0.0	1	0	1214	0.0	0.0	1
		0.50	0	910	0.3	0.0	1	0	1497	0.1	0.0	1
		0.80	0	965	0.6	0.0	1	0	1505	0.3	0.0	1
	25	1.00	0	1131	2.2	0.0	1	0	1505	0.3	0.0	1
	25	0.10 0.50	0	432 809	0.0	0.0 0.0	1	0	1101 1076	0.0 0.1	0.0	1
		0.30	0	993	1.3	0.0	1	0	1371	0.1	0.0	1
		1.00	0	1043	1.7	0.0	1	0	1452	0.2	0.0	1
	26	0.10	0	369	0.0	0.0	1	0	813	0.0	0.0	1
	20	0.50	0	811	0.3	0.0	1	0	1083	0.1	0.0	1
		0.80	0	939	0.6	0.0	1	0	1129	0.2	0.0	1
		1.00	0	1000	0.7	0.0	1	0	1389	0.2	0.0	1
	27	0.10	0	607	0.1	0.0	1	0	1018	0.0	0.0	1
		0.50	0	807	0.3	0.0	1	0	1279	0.1	0.0	1
		0.80	0	1028	1.0	0.0	1	0	1608	0.2	0.0	1
		1.00	0	1109	1.2	0.0	1	0	1497	0.5	0.0	1
	28	0.10	0	531	0.1	0.0	1	0	941	0.0	0.0	1
		0.50	0	928	1.4	0.0	1	0	1308	0.1	0.0	1
		0.80	0	1042	2.0	0.0	1	0	1524	0.2	0.0	1
		1.00	0	1084	22.4	0.0	1	0	1524	0.3	0.0	1
	29	0.10	0	368	0.0	0.0	1	0	908	0.0	0.0	1
		0.50	0	574	0.4	0.0	1	0	1210	0.1	0.0	1
		0.80	0	836	32.2	0.0	1	0	1395	0.3	0.0	1
		1.00	0	1028	44.1	0.0	1	0	1486	0.6	0.0	1

Table A.40: Performance of ILP-O and ILP-S for instance size (Sz.) 30, instance numbers (Nr.) 0–14.

								ILP-S					
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	Gap	Opt	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	Gap	Opt	
30	0	0.10	0	606	0.1	0.0	1	0	1127	0.1	0.0	1	
		0.50	0	1016	1.0	0.0	1	0	1497	0.2	0.0	1	
		0.80 1.00	0	1208 1300	5.0 97.4	0.0	1	0	2022 2059	0.4 0.7	0.0 0.0	1	
	1	0.10	0	649	0.5	0.0 0.0	1	0	1684	0.7	0.0	1	
		0.50	0	1047	2714.2	0.0	1	0	2330	1.4	0.0	1	
		0.80	0	1298	2165.2	0.0	1	0	2416	3.5	0.0	1	
		1.00	0	1401	9998.5	2.5	0	0	2570	5.4	0.0	1	
	2	0.10	0	826	0.2	0.0	1	0	1752	0.1	0.0	1	
		0.50	0	1365	18.7	0.0	1	0	2459	1.1	0.0	1	
		0.80	0	1464	11.7	0.0	1	0	2706	2.0	0.0	1	
	2	1.00	0	1635	4737.1	0.0	1	0	2730	3.5	0.0	1	
	3	0.10 0.50	0	679 1083	0.1 2.9	0.0 0.0	1	0	1165 1756	0.0	0.0 0.0	1	
		0.30	0	1118	4.8	0.0	1	0	2278	0.3	0.0	1	
		1.00	0	1526	1119.9	0.0	1	0	2448	0.9	0.0	1	
	4	0.10	0	701	0.1	0.0	1	0	1516	0.1	0.0	1	
		0.50	0	1213	4.7	0.0	1	0	1770	0.3	0.0	1	
		0.80	0	1267	6.5	0.0	1	0	2185	0.5	0.0	1	
		1.00	0	1486	18.1	0.0	1	0	2359	0.8	0.0	1	
	5	0.10	0	606	0.1	0.0	1	0	1176	0.0	0.0	1	
		0.50	0	1043	4.1	0.0	1	0	1812	0.3	0.0	1	
		0.80	0	1209	510.5	0.0	1	0	1908	0.4	0.0	1	
	6	1.00 0.10	0	1309	50.2 1.3	0.0	1	0	2001	0.8 0.2	0.0	1	
	0	0.10	0	617 1079	5.1	0.0 0.0	1	0	1306 2138	0.2	0.0 0.0	1	
		0.80	0	1207	9998.3	0.0	0	0	2296	2.5	0.0	1	
		1.00	0	1339	9998.4	7.0	0	0	2296	4.8	0.0	1	
	7	0.10	0	671	0.1	0.0	1	0	1138	0.1	0.0	1	
		0.50	0	1175	1.1	0.0	1	0	1814	0.7	0.0	1	
		0.80	0	1339	9.6	0.0	1	0	2015	0.9	0.0	1	
		1.00	0	1395	36.2	0.0	1	0	2013	1.7	0.0	1	
	8	0.10	0	1111	0.1	0.0	1	0	1620	0.1	0.0	1	
		0.50	0	1267	4.6	0.0	1	0	1978	0.5	0.0	1	
		0.80	0	1475	2.9	0.0	1	0	2214	0.6	0.0	1	
	9	1.00 0.10	0	1549 571	5.3 0.1	0.0 0.0	1	0	2257 1503	0.9 0.1	0.0 0.0	1	
		0.10	0	1094	4.9	0.0	1	0	2308	0.1	0.0	1	
		0.80	0	1374	243.8	0.0	1	0	2554	1.6	0.0	1	
		1.00	0	1530	9998.3	1.4	0	0	2474	2.6	0.0	1	
	10	0.10	0	593	0.1	0.0	1	0	1068	0.0	0.0	1	
		0.50	0	997	0.3	0.0	1	0	1663	0.2	0.0	1	
		0.80	0	1139	1.3	0.0	1	0	1987	0.4	0.0	1	
		1.00	0	1307	10.5	0.0	1	0	2168	0.4	0.0	1	
	11	0.10	0	597	0.1	0.0	1	0	1606	0.1	0.0	1	
		0.50	0	1109	0.6	0.0	1	0	2043	0.2	0.0	1	
		0.80 1.00	0	1396 1626	48.8 836.2	0.0 0.0	1	0	2200 2278	0.5 0.9	0.0 0.0	1	
	12	0.10	0	754	0.1	0.0	1	0	1147	0.0	0.0	1	
		0.50	0	1049	0.3	0.0	1	0	1551	0.2	0.0	1	
		0.80	0	1311	1.0	0.0	1	0	2216	0.3	0.0	1	
		1.00	0	1546	1.3	0.0	1	0	2243	0.7	0.0	1	
	13	0.10	0	542	0.1	0.0	1	0	1464	0.1	0.0	1	
		0.50	0	1061	1.1	0.0	1	0	1972	0.5	0.0	1	
		0.80	0	1380	5.7	0.0	1	0	2209	0.5	0.0	1	
	1.4	1.00	0	1405	9.7	0.0	1	0	2205	0.9	0.0	1	
	14	0.10	0	749	0.1	0.0	1	0	1660	0.1	0.0	1	
		0.50 0.80	0	1282 1323	3.4 1.6	0.0 0.0	1	0	1958 2356	0.5	0.0 0.0	1	
		1.00	0	1488	85.6	0.0	1	0	2527	1.1	0.0	1	
		1.00		1 100	55.0	0.0			2521	1.5	0.0	1	

Table A.41: Performance of ILP-O and ILP-S for instance size (Sz.) 30, instance numbers (Nr.) 15–29.

					ILP-O				ILP-S				
Sz.	Nr.	Load	C_a	C_u	t[s]	Gap	Opt	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Gap	Opt	
30	15	0.10	0	977	0.1	0.0	1	0	1581	0.1	0.0	1	
		0.50	0	1270	0.6	0.0	1	0	2033	0.3	0.0	1	
		0.80	0	1406	23.3	0.0	1	0	1971	0.8	0.0	1	
	16	1.00	0	1590	51.7	0.0	1	0	2148	1.3	0.0	1	
	16	0.10	0	458	0.1	0.0	1	0	1208	0.0	0.0	1	
		0.50 0.80	0	896 971	1.8 3.2	0.0 0.0	1	0	1842 1894	0.2	0.0	1	
		1.00	0	1202	36.2	0.0	1	0	1959	0.4	0.0	1	
	17	0.10	0	776	0.2	0.0	1	0	1326	0.0	0.0	1	
	17	0.50	0	1043	2.9	0.0	1	0	1797	0.6	0.0	1	
		0.80	0	1313	11.7	0.0	1	0	2036	0.7	0.0	1	
		1.00	0	1366	127.2	0.0	1	Ő	2195	1.4	0.0	1	
	18	0.10	0	561	0.1	0.0	1	0	1035	0.1	0.0	1	
		0.50	0	1192	0.5	0.0	1	0	1969	0.3	0.0	1	
		0.80	0	1473	1.2	0.0	1	0	2308	0.4	0.0	1	
		1.00	0	1669	13.5	0.0	1	0	2498	0.8	0.0	1	
	19	0.10	0	498	2.4	0.0	1	0	1284	0.1	0.0	1	
		0.50	0	1118	111.5	0.0	1	0	2514	1.5	0.0	1	
		0.80	0	1287	586.1	0.0	1	0	2443	3.7	0.0	1	
		1.00	0	1465	9998.5	4.8	0	0	2483	8.6	0.0	1	
	20	0.10	0	803	0.1	0.0	1	0	1184	0.1	0.0	1	
		0.50	0	1317	2.9	0.0	1	0	2015	0.8	0.0	1	
		0.80	0	1539	182.7	0.0	1	0	2275	2.2	0.0	1	
		1.00	0	1658	8166.4	0.0	1	0	2301	2.7	0.0	1	
	21	0.10	0	581	0.1	0.0	1	0	1246	0.0	0.0	1	
		0.50	0	916	1.5	0.0	1	0	1616	0.2	0.0	1	
		0.80	0	1169	2.9	0.0	1	0	1926	0.3	0.0	1	
	22	1.00	0	1273	30.7	0.0	1	0	1926	0.4	0.0	1	
	22	0.10	0	881	0.1	0.0	1	0	1152	0.0	0.0	1	
		0.50 0.80	0	1312 1395	0.4 0.7	0.0	1	0	1753 2428	0.2	0.0	1	
		1.00	0	1417	1.1	0.0 0.0	1	0	2428	0.3	0.0	1	
	23	0.10	0	738	0.1	0.0	1	0	940	0.1	0.0	1	
	25	0.50	0	1178	0.7	0.0	1	0	1731	0.5	0.0	1	
		0.80	0	1332	23.0	0.0	1	0	2303	0.8	0.0	1	
		1.00	0	1497	135.7	0.0	1	Ő	2182	1.0	0.0	1	
	24	0.10	0	525	0.1	0.0	1	0	991	0.2	0.0	1	
		0.50	0	1202	3.1	0.0	1	0	2294	1.7	0.0	1	
		0.80	0	1339	32.9	0.0	1	0	2709	2.3	0.0	1	
		1.00	0	1371	866.2	0.0	1	0	2810	4.3	0.0	1	
	25	0.10	0	849	0.1	0.0	1	0	1714	0.1	0.0	1	
		0.50	0	1283	2.6	0.0	1	0	1850	0.6	0.0	1	
		0.80	0	1462	8.0	0.0	1	0	2132	0.8	0.0	1	
		1.00	0	1620	116.7	0.0	1	0	2328	2.7	0.0	1	
	26	0.10	0	518	0.1	0.0	1	0	835	0.1	0.0	1	
		0.50	0	981	1.3	0.0	1	0	2028	0.3	0.0	1	
		0.80	0	1165	6.6	0.0	1	0	2288	0.5	0.0	1	
	27	1.00	0	1365	147.0	0.0	1	0	2293	0.8	0.0	1	
	27	0.10	0	549	0.1	0.0	1	0	980	0.0	0.0	1	
		0.50 0.80	0	1013 1262	0.5 35.4	0.0 0.0	1	0	1642 2137	0.2	0.0	1	
		1.00		1422					2157			1	
	28	0.10	0	939	187.4 0.1	0.0	1	0	1505	0.9	0.0	1	
	20	0.10	0	1232	0.1	0.0	1	0	1736	0.1	0.0	1	
		0.80	0	1385	4.5	0.0	1	0	2308	0.3	0.0	1	
		1.00	0	1448	8.2	0.0	1	0	2338	0.4	0.0	1	
	29	0.10	0	411	0.1	0.0	1	0	1069	0.0	0.0	1	
		0.50	0	940	0.9	0.0	1	0	1718	0.2	0.0	1	
		0.80	0	1278	45.2	0.0	1	0	1947	0.4	0.0	1	
		1.00	0	1354	86.3	0.0	1	0	2067	0.5	0.0	1	
					1								

Table A.42: Performance of ILP-O and ILP-S for instance size (Sz.) 50, instance numbers (Nr.) 0–14.

			ILP-O					ILP-S				
Sz.	Nr.	Load	C_{a}	C_u	t[s]	Gap	Opt	$\mathbf{C}_{\mathbf{a}}$	C_{u}	t[s]	Gap	Opt
50	0	0.10	0	974	0.6	0.0	1	0	1731	0.4	0.0	1
		0.50	0	1732	97.7	0.0	1	0	3859	2.5	0.0	1
		0.80	0	1991	5787.4	0.0	1	0	4019	6.3	0.0	1
		1.00	0	2131	9999.0	3.2	0	0	4378	7.0	0.0	1
	1	0.10	0	1322	0.5	0.0	1	0	2103	0.5	0.0	1
		0.50	0	1612	25.2	0.0	1	0	3087	2.2	0.0	1
		0.80 1.00	0	1837 2162	9998.8 9999.0	0.7 4.8	0	0 0	3754 3868	4.1 11.0	0.0 0.0	1
	2	0.10	0	719	0.2	4.0 0.0	1	0	1537	0.1	0.0	1
	2	0.50	0	1776	2.5	0.0	1	0	3032	1.2	0.0	1
		0.80	0	2124	49.4	0.0	1	0	3686	1.1	0.0	1
		1.00	0	2370	398.5	0.0	1	0	3873	2.2	0.0	1
	3	0.10	0	1118	0.2	0.0	1	0	2028	0.1	0.0	1
		0.50	0	1779	3.0	0.0	1	0	3193	1.0	0.0	1
		0.80	0	1972	11.4	0.0	1	0	3666	1.7	0.0	1
		1.00	0	2022	16.4	0.0	1	0	4193	2.8	0.0	1
	4	0.10	0	1182	0.3	0.0	1	0	1763	0.2	0.0	1
		0.50	0	1870	7.9	0.0	1	0	3033	1.4	0.0	1
		0.80	0	1969	66.4	0.0	1	0	3606	2.6	0.0	1
	_	1.00	0	2203	290.0	0.0	1	0	3678	3.2	0.0	1
	5	0.10	0	1238	0.3	0.0	1	0	2190	0.2	0.0	1
		0.50	0	1571	11.7	0.0	1	0	2892	1.4	0.0	1
		0.80 1.00	0	1866 1905	20.5 931.3	0.0	1	0 0	3632 3602	2.6 3.9	0.0	1
	6	0.10	0	787	0.2	0.0 0.0	1	0	1213	0.2	0.0	1
	0	0.10	0	1657	2.1	0.0	1	0	2726	1.1	0.0	1
		0.80	0	1884	6.9	0.0	1	0	3425	2.1	0.0	1
		1.00	0	2214	258.2	0.0	1	0	3463	2.6	0.0	1
	7	0.10	0	1116	0.2	0.0	1	0	1939	0.1	0.0	1
		0.50	0	1948	2.1	0.0	1	0	3577	1.2	0.0	1
		0.80	0	2161	12.2	0.0	1	0	3485	2.2	0.0	1
		1.00	0	2370	185.7	0.0	1	0	3987	3.2	0.0	1
	8	0.10	0	780	2.5	0.0	1	0	2015	0.5	0.0	1
		0.50	0	1420	506.3	0.0	1	0	4054	4.1	0.0	1
		0.80	0	1720	9998.9	1.6	0	0	4377	10.9	0.0	1
		1.00	0	2201	99999.1	8.1	0	0	4605	20.5	0.0	1
	9	0.10	0	1371	0.2	0.0	1	0	2305	0.1	0.0	1
		0.50	0	1709	1.3	0.0	1	0	2268	0.8	0.0	1
		0.80	0	1873	116.3	0.0	1	0	2866	2.6	0.0	1
	10	1.00 0.10	0	2117 973	10.2 0.2	0.0 0.0	1	0 0	3380 1277	2.3 0.3	0.0 0.0	1
	10	0.10	0	1765	129.0	0.0	1	0	3811	2.1	0.0	1
		0.80	0	1981	2813.1	0.0	1	0	4165	3.6	0.0	1
		1.00	0	2141	9998.9	1.7	0	0	4325	4.5	0.0	1
	11	0.10	0	886	0.1	0.0	1	0	2184	0.1	0.0	1
		0.50	0	1534	2.0	0.0	1	0	3384	1.0	0.0	1
		0.80	0	1935	9998.6	0.3	0	0	3685	1.9	0.0	1
		1.00	0	2388	9998.7	2.0	0	0	3809	4.7	0.0	1
	12	0.10	0	872	0.3	0.0	1	0	1560	0.1	0.0	1
		0.50	0	1706	70.0	0.0	1	0	3246	1.5	0.0	1
		0.80	0	1954	9998.6	0.4	0	0	3491	1.7	0.0	1
	12	1.00	0	2089	9998.8	1.2	0	0	3640	3.5	0.0	1
	13	0.10	0	1081	0.2	0.0	1	0	1987	0.1	0.0	1
		0.50	0	1657	1.5	0.0	1	0	2962	1.1	0.0	1
		0.80	0	1982	51.8	0.0	1	0	3303	1.8	0.0	1
	14	1.00 0.10	0	2304 1057	217.6 0.2	0.0 0.0	1	0 0	3778 1461	2.8 0.2	0.0	1
	14	0.10	0	1057	1.8	0.0	1	0	2402	0.2	0.0	1
		0.30	0	1925	37.4	0.0	1	0	3201	1.7	0.0	1
		1.00	0	1962	8.2	0.0	1	0	3313	2.4	0.0	1
		1.00		1702	0.2	5.0		0	0010		0.0	

Table A.43: Performance of ILP-O and ILP-S for instance size (Sz.) 50, instance numbers (Nr.) 15–29.

		1	1		ILP-O					ILP-S		
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Gap	Opt	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Gap	Opt
50	15	0.10	0	1132	0.3	0.0	1	0	1767	0.2	0.0	1
		0.50	0	1865	56.2	0.0	1	0	3401	1.6	0.0	1
		0.80	0	2010	129.1	0.0	1	0	3627	2.9	0.0	1
	16	1.00	0	2358	6428.8	0.0	1	0	3987	3.3	0.0	1
	16	0.10	0	953	0.7	0.0	1	0	1899	0.1	0.0	1
		0.50 0.80	0	1456 1860	2.2	0.0	1	0	3596 3795	1.8 3.2	0.0 0.0	1
		1.00	0	2038	27.7 2273.3	0.0	1	0	3939	3.5	0.0	1
	17	0.10	0	1103	0.2	0.0	1	0	1546	0.2	0.0	1
	17	0.50	0	1963	2.6	0.0	1	0	3164	1.3	0.0	1
		0.80	Õ	2185	120.4	0.0	1	0	3541	2.2	0.0	1
		1.00	0	2436	3794.3	0.0	1	0	3828	3.8	0.0	1
	18	0.10	0	839	0.2	0.0	1	0	1773	0.1	0.0	1
		0.50	0	1725	2.6	0.0	1	0	2818	1.2	0.0	1
		0.80	0	2222	4.3	0.0	1	0	3736	2.1	0.0	1
		1.00	0	2418	818.4	0.0	1	0	4052	3.2	0.0	1
	19	0.10	0	949	0.2	0.0	1	0	1720	0.1	0.0	1
		0.50	0	1462	46.1	0.0	1	0	3128	1.3	0.0	1
		0.80	0	1653	95.5	0.0	1	0	3376	2.6	0.0	1
		1.00	0	1995	9998.8	0.6	0	0	3563	2.7	0.0	1
	20	0.10	0	1626	0.3	0.0	1	0	2865	0.1	0.0	1
		0.50	0	1965	3.1	0.0	1	0	3258	0.6	0.0	1
		0.80	0	2249	53.5	0.0	1	0	3567	2.4	0.0	1
	21	1.00	0	2647	6524.8	0.0	1	0	3878	3.1	0.0	1
	21	0.10 0.50	0	1208 1703	0.7	0.0 0.0	1	0	2479 3157	0.1 1.4	0.0 0.0	1
		0.30	0	2137	7210.6	0.0	1	0	4026	2.6	0.0	1
		1.00	0	2467	9998.7	2.0	0	0	4115	4.0	0.0	1
	22	0.10	0	1305	0.9	0.0	1	0	2192	0.4	0.0	1
		0.50	0	2020	11.5	0.0	1	0	3755	1.5	0.0	1
		0.80	Ő	2207	116.5	0.0	1	Õ	3949	3.8	0.0	1
		1.00	0	2445	9998.9	0.7	0	0	4300	6.2	0.0	1
	23	0.10	0	986	0.1	0.0	1	0	1960	0.1	0.0	1
		0.50	0	1985	11.5	0.0	1	0	2993	1.0	0.0	1
		0.80	0	2239	162.5	0.0	1	0	3623	1.6	0.0	1
		1.00	0	2361	591.2	0.0	1	0	3849	1.8	0.0	1
	24	0.10	0	784	0.3	0.0	1	0	2076	0.3	0.0	1
		0.50	0	1412	31.8	0.0	1	0	3394	2.5	0.0	1
		0.80	0	2072	9998.7	3.8	0	0	4222	6.6	0.0	1
		1.00	0	2430	9998.8	5.5	0	0	4319	6.7	0.0	1
	25	0.10	0	823	0.2	0.0	1	0	1652	0.1	0.0	1
		0.50	0	1463	2.4	0.0	1	0	2674	1.2	0.0	1
		0.80 1.00	0	1859 2011	6.9 114.2	0.0	1	0	3197 3336	1.9 2.2	0.0 0.0	1
	26	0.10	0	920	0.2	0.0 0.0	1	0	1553	0.1	0.0	1
	20	0.50	0	1575	2.5	0.0	1	0	2857	0.7	0.0	1
		0.80	0	1994	314.1	0.0	1	0	3641	1.9	0.0	1
		1.00	0	2147	998.5	0.0	1	0	3778	3.0	0.0	1
	27	0.10	0	1309	0.1	0.0	1	0	2185	0.1	0.0	1
		0.50	0	1984	1.7	0.0	1	0	3296	0.6	0.0	1
		0.80	0	2094	3.1	0.0	1	0	3020	1.3	0.0	1
		1.00	0	2282	8.8	0.0	1	0	3464	2.0	0.0	1
	28	0.10	0	1082	1.7	0.0	1	0	3313	0.8	0.0	1
		0.50	0	1976	9998.7	3.9	0	0	4406	10.9	0.0	1
		0.80	0	2449	9999.1	8.1	0	0	4736	22.7	0.0	1
		1.00	0	2676	9999.3	11.6	0	0	4716	28.0	0.0	1
	29	0.10	0	620	0.2	0.0	1	0	1741	0.2	0.0	1
		0.50	0	1658	6.5	0.0	1	0	3000	1.9	0.0	1
		0.80	0	2168	71.3	0.0	1	0	3441	5.0	0.0	1
		1.00	0	2306	3110.4	0.0	1	0	4170	8.2	0.0	1

Table A.44: Performance of ILP-O and ILP-S for instance size (Sz.) 100, instance numbers (Nr.) 0–14.

					ILP-O					ILP-S		
Sz.	Nr.	Load	Ca	C_u	t[s]	Gap	Opt	Ca	Cu	t[s]	Gap	Opt
100	0	0.10	0	2226	0.8	0.0	1	0	3211	0.6	0.0	1
		0.50	0	3015	10.4	0.0	1	0	5501	6.2	0.0	1
		0.80	0	3691	10001.4	1.5	0		6764	13.5	0.0	1
	1	1.00	0	4444	10001.7	5.2	0		7466	20.7	0.0	1
	1	0.10 0.50	0	2757 3910	1.2 9999.8	0.0	1	0	4302 6785	1.1	0.0	1
		0.30		4361	10000.9	0.2	0	0	7133	6.9 10.7	0.0	1
		1.00	0	4770	10000.9	3.5	0	0	7764	16.0	0.0	1
	2	0.10	0	1219	0.7	0.0	1	0	2049	0.9	0.0	1
	-	0.50	0	2723	378.9	0.0	1	0	5891	4.9	0.0	1
		0.80	0	3281	10000.7	2.0	0	0	6833	9.0	0.0	1
		1.00	0	3884	10001.4	6.6	0	0	7351	15.1	0.0	1
	3	0.10	0	2185	71.3	0.0	1	0	4786	3.3	0.0	1
		0.50	0	3595	10000.8	1.7	0	0	7480	29.3	0.0	1
		0.80	0	6301	10002.2	39.4	0	0	7918	44.9	0.0	1
		1.00	1626	7346	10003.5	100.0	0	0	8279	113.7	0.0	1
	4	0.10	0	1776	7.2	0.0	1	0	3257	1.9	0.0	1
		0.50	0	3663	10000.5	2.4	0	0	6941	13.1	0.0	1
		0.80	0	4119	10001.0	4.9	0	0	7706	20.9	0.0	1
		1.00	0	4398	10001.5	6.2	0	0	7839	29.4	0.0	1
	5	0.10	0	1904	1.2	0.0	1	0	3067	1.4	0.0	1
		0.50	0	3432	31.0	0.0	1	0	6101	6.3	0.0	1
		0.80	0	3838	2059.4	0.0	1	0	6736	13.8	0.0	1
		1.00	0	4199	10001.8	2.6	0	0	7150	18.1	0.0	1
	6	0.10	0	2250	5.5	0.0	1	0	3883	3.8	0.0	1
		0.50	0	3497	944.7	0.0	1		5462	8.6	0.0	1
		0.80	0	4133	10001.2	0.9	0	0	6727	21.9	0.0	1
	7	1.00	0	4637	10001.9	3.4	0	0	7980	21.7	0.0	1
	/	0.10		2552 3614	1.2 30.5	0.0	1	0	4132 6073	1.1 6.0	0.0	1
		0.50 0.80		4455	10001.3	0.0 2.6	0	0	7091	13.7	0.0 0.0	1
		1.00	0	4782	10001.3	4.4	0	0	8264	17.2	0.0	1
	8	0.10	0	1765	1.0	0.0	1	0	3603	0.9	0.0	1
	0	0.50	0	3167	257.5	0.0	1	0	6530	4.0	0.0	1
		0.80	0	3772	10000.7	1.8	0	0	7266	7.2	0.0	1
		1.00	0	4508	10001.6	5.0	0	0	7582	9.6	0.0	1
	9	0.10	0	1990	1.3	0.0	1	0	2937	1.3	0.0	1
		0.50	0	3278	1616.1	0.0	1	0	6032	4.9	0.0	1
		0.80	0	3829	3107.8	0.0	1	0	6647	8.6	0.0	1
		1.00	0	4267	10001.5	0.8	0	0	7162	11.4	0.0	1
	10	0.10	0	2143	1.2	0.0	1	0	3741	1.1	0.0	1
		0.50	0	2726	30.7	0.0	1	0	5431	4.5	0.0	1
		0.80	0	3751	10001.0	3.1	0	0	6381	6.5	0.0	1
		1.00	0	4018	10001.8	3.6	0	0	7016	10.9	0.0	1
	11	0.10	0	2724	5.4	0.0	1		4904	6.1	0.0	1
		0.50	0	3984	6036.7	0.0	1	0	7065	15.5	0.0	1
		0.80	1060	4675	10001.7	2.4	0		8027	30.6	0.0	1
	12	1.00 0.10	1960	7043	10003.0	100.0	0		8443	44.6	0.0	1
	12			2192	0.6 580.7	0.0	1		4178	0.4	0.0	1
		0.50 0.80	0	3276 3841	580.7	0.0 0.9	1	0	6776 7554	3.4 7.3	0.0	1
		1.00		4563	10001.1	3.9	0	0	7780	10.0	0.0	1
	13	0.10		2145	1.3	0.0	1	0	3207	0.6	0.0	1
	15	0.10		3732	1.0000.0	1.3	0	0	6353	7.6	0.0	1
		0.80	0	4440	10000.0	4.1	0	0	7432	13.1	0.0	1
		1.00	0	4802	10001.1	4.0	0	0	7880	21.9	0.0	1
	14	0.10	0	2693	3.2	0.0	1	0	4245	0.6	0.0	1
	11	0.10	0	3802	668.9	0.0	1	0	6572	6.1	0.0	1
		0.80	0	4357	10000.9	2.6	0	0	7184	13.0	0.0	1

Table A.45: Performance of ILP-O and ILP-S for instance size (Sz.) 100, instance numbers (Nr.) 15–29.

			1		ILP-O					ILP-S		
Sz.	Nr.	Load	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Gap	Opt	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	Gap	Opt
100	15	0.10	0	1822	0.7	0.0	1	0	3592	1.1	0.0	1
		0.50	0	2955	191.0	0.0	1	0	5939	5.1	0.0	1
		0.80	0	3830	10000.4	0.3	0	0	6803	7.7	0.0	1
	16	1.00	0	4503	10001.2	1.9	0	0	7505	11.4	0.0	1
	16	0.10	0	2496	0.9	0.0	1	0	3639	0.6	0.0	1
		0.50	0	3702	9.8	0.0	1	0	4941	3.2	0.0	1
		0.80 1.00	0	4113 4622	5490.9 10000.9	0.0	1 0	0	6278 6902	6.9 10.2	0.0	1
	17	0.10	0	1973	2.4	0.0	1	0	3401	1.2	0.0	1
	17	0.50	0	3861	2176.9	0.0	1	0	7456	6.5	0.0	1
		0.80	0	4308	10000.5	2.6	0	0	8000	13.3	0.0	1
		1.00	0	4877	10000.9	5.0	0	0	7986	16.6	0.0	1
	18	0.10	0	1889	0.7	0.0	1	0	2603	0.8	0.0	1
		0.50	0	2980	99999.9	1.3	0	0	6363	6.3	0.0	1
		0.80	0	3383	10001.2	2.7	0	0	7573	11.0	0.0	1
		1.00	0	4004	10002.3	7.3	0	0	7890	16.5	0.0	1
	19	0.10	0	2148	0.8	0.0	1	0	3214	0.7	0.0	1
		0.50	0	3182	1058.8	0.0	1	0	5594	7.8	0.0	1
		0.80	0	3680	10001.8	3.7	0	0	6359	11.2	0.0	1
		1.00	0	4039	10002.3	2.7	0	0	6769	15.4	0.0	1
	20	0.10	0	1548	0.9	0.0	1	0	2586	0.7	0.0	1
		0.50	0	3468	99999.9	1.1	0	0	6784	7.4	0.0	1
		0.80	0	3924	10001.3	1.4	0	0	7405	12.9	0.0	1
	21	1.00	0	4336	10001.8	3.0	0	0	8055	19.4	0.0	1
	21	0.10	0	1923	2.3 4776.0	0.0	1	0	3799	1.6 7.7	0.0	1
		0.50 0.80	0	3283 3893	10002.0	0.0	0	0	6795 7724	16.0	0.0	1
		1.00	0	4168	10002.0	5.3	0	0	7971	19.8	0.0	1
	22	0.10	0	1065	0.5	0.0	1	0	1454	0.5	0.0	1
	22	0.50	0	2776	12.1	0.0	1	0	5583	3.9	0.0	1
		0.80	0	3686	9470.6	2.2	0	0	6795	8.3	0.0	1
		1.00	0	3979	10001.4	1.7	0	Ő	7620	13.8	0.0	1
	23	0.10	0	1518	1.1	0.0	1	0	2845	1.2	0.0	1
		0.50	0	2922	3318.1	0.0	1	0	5319	4.7	0.0	1
		0.80	0	3857	10001.4	1.3	0	0	6785	10.2	0.0	1
		1.00	0	4506	10002.1	2.2	0	0	7491	18.1	0.0	1
	24	0.10	0	2458	1.2	0.0	1	0	3910	1.0	0.0	1
		0.50	0	3762	19.0	0.0	1	0	6516	6.1	0.0	1
		0.80	0	4311	10000.9	1.3	0	0	7833	10.9	0.0	1
		1.00	0	4505	10002.2	2.5	0	0	7773	14.5	0.0	1
	25	0.10	0	1940	2.0	0.0	1	0	4142	0.9	0.0	1
		0.50	0	3291	1048.4	0.0	1	0	6984	5.2	0.0	1
		0.80 1.00	0	3925 4505	10001.3 10001.7	2.4 6.2	0 0	0	7912 7932	12.9 19.3	0.0	1
	26	0.10	0	2340	1.7	0.2	1	0	3340	0.5	0.0	1
	20	0.10	0	3534	39.2	0.0	1	0	6147	4.9	0.0	1
		0.80	0	3997	10001.3	1.3	0	0	7801	12.7	0.0	1
		1.00	0	4351	10001.8	3.4	0	0	8173	20.0	0.0	1
	27	0.10	0	1445	1.0	0.0	1	0	2193	0.9	0.0	1
		0.50	0	2868	5.5	0.0	1	0	6244	3.6	0.0	1
		0.80	0	3712	5991.7	0.0	1	0	7219	6.7	0.0	1
		1.00	0	3985	7333.2	1.5	0	0	7465	9.2	0.0	1
	28	0.10	0	2137	0.8	0.0	1	0	3765	1.3	0.0	1
		0.50	0	3872	24.8	0.0	1	0	7234	6.4	0.0	1
		0.80	0	4183	1864.8	0.0	1	0	7817	12.0	0.0	1
		1.00	0	4589	10001.5	3.1	0	0	8307	16.2	0.0	1
	29	0.10	0	2377	4.1	0.0	1	0	4260	1.5	0.0	1
		0.50	0	3387	9999.8	1.9	0	0	6603	7.2	0.0	1
		0.80	0	4213	10000.9	2.8	0	0	7541	11.5	0.0	1
		1.00	0	4405	10001.3	2.4	0	0	7990	16.6	0.0	1

Table A.46: Performance of ILP-O and ILP-S for instance size (Sz.) 200, instance numbers (Nr.) 0–14.

		1	1		ILP-O					ILP-S		
Sz.	Nr.	Load	C_{a}	C_u	t[s]	Gap	Opt	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Gap	Opt
200	0	0.10	0	2215	6.3	0.0	1	0	3643	1.3	0.0	1
		0.50	0	4452	10001.7	0.6	0	0	8102	14.0	0.0	1
		0.80	0	5370	10004.0	2.3	0	0	10515	30.3	0.0	1
		1.00	0	6080	10004.9	4.9	0	0	11662	41.3	0.0	1
	1	0.10	0	1841	5.4	0.0	1	0	2688	3.4	0.0	1
		0.50	0	3922	10003.5	0.6	0	0	7578	17.6	0.0	1
		0.80	0	5201	10005.2	3.8	0	0	9675	26.2	0.0	1
	2	1.00 0.10	0	6407 2650	10005.2	6.0 0.0	0	0	11754 4520	32.2 4.5	0.0	1
	2	0.10	0	3998	5.6 126.6	0.0	1	0	4320 7427	13.8	0.0	1
		0.80	0	4828	120.0	3.1	0	0	9940	22.7	0.0	1
		1.00	0	5877	10004.1	7.5	0	0	11183	41.8	0.0	1
	3	0.10	0	3513	2.1	0.0	1	0	5700	1.8	0.0	1
	5	0.50	0	5553	6799.3	0.0	1	0	8456	12.6	0.0	1
		0.80	0	6422	10003.7	0.2	0	0	10759	21.2	0.0	1
		1.00	0	7172	10005.6	1.7	0	0	11644	29.8	0.0	1
	4	0.10	0	2807	4.5	0.0	1	0	5494	2.7	0.0	1
		0.50	0	4674	83.8	0.0	1	0	9154	11.9	0.0	1
		0.80	0	5783	10003.9	1.4	0	0	11533	22.6	0.0	1
		1.00	0	6715	10004.2	5.9	0	0	12783	38.8	0.0	1
	5	0.10	0	2980	1.4	0.0	1	0	3968	0.8	0.0	1
		0.50	0	4816	110.4	0.0	1	0	8105	9.2	0.0	1
		0.80	0	5797	10003.1	1.4	0	0	9541	17.0	0.0	1
		1.00	0	6432	10005.6	6.6	0	0	11174	28.6	0.0	1
	6	0.10	0	3202	86.4	0.0	1	0	5947	4.1	0.0	1
		0.50	0	4826	10002.0	2.2	0	0	8797	24.8	0.0	1
		0.80	739	9499	10004.8	100.0	0	0	11354	48.0	0.0	1
	-	1.00	4553	11680	10005.2	100.0	0	0	12118	56.5	0.0	1
	7	0.10	0	2776	3.4	0.0	1	0	5156	4.0	0.0	1
		0.50	0	5243	10002.1	1.9 100.0	0	0	9020	16.3	0.0	1
		0.80 1.00	100	10352 7157	10004.1 10005.8	7.7	0	0	10904 12432	24.3 41.4	0.0	1
	8	0.10	0	1511	4.7	0.0	1	0	3097	41.4	0.0	1
	0	0.10	0	6417	10002.9	33.8	0	0	9390	24.8	0.0	1
		0.80	430	8930	10006.1	100.0	0	0	11296	52.8	0.0	1
		1.00	4614	11950	10008.0	100.0	0	0	12491	116.7	0.0	1
	9	0.10	0	2238	1.8	0.0	1	0	3222	1.1	0.0	1
		0.50	0	4888	10002.6	0.5	0	0	9978	14.7	0.0	1
		0.80	0	5541	10006.3	1.8	0	0	11119	27.8	0.0	1
		1.00	8678	9961	10008.0	100.0	0	0	13220	47.2	0.0	1
	10	0.10	0	1734	1.2	0.0	1	0	2487	1.3	0.0	1
		0.50	0	4733	10002.5	1.4	0	0	8743	13.2	0.0	1
		0.80	0	6237	10004.5	1.6	0	0	11916	24.7	0.0	1
		1.00	0	6978	10006.1	3.6	0	0	12960	38.4	0.0	1
	11	0.10	0	1849	17.3	0.0	1	0	3583	16.0	0.0	1
		0.50	250	6800	10005.8	100.0	0	0	10490	49.9	0.0	1
		0.80	350	10157	10008.9	100.0	0	0	13103	123.1	0.0	1
	12	1.00	17462	11850	10012.1	100.0	0	0	14207	355.1	0.0	1
	12	0.10 0.50	0	2696 4467	5.8 10002.4	0.0 0.1	1	0	4656 7421	6.1 15.6	0.0	1
		0.30	0	5195	9094.6	0.1	1	0	9823	25.4	0.0	1
		1.00	0	6237	10006.3	5.4	0	0	12088	40.3	0.0	1
	13	0.10	0	2169	5.9	0.0	1	0	3486	3.0	0.0	1
		0.10	0	3792	334.0	0.0	1	0	6706	17.8	0.0	1
		0.80	0	7293	10006.8	37.6	0	0	9397	29.8	0.0	1
		1.00	0	5577	10007.0	6.6	0	0	11255	44.0	0.0	1
	14	0.10	0	3194	1172.5	0.0	1	0	6366	7.5	0.0	1
		0.50	0	6231	10002.5	31.7	0	0	8345	20.2	0.0	1
		0.80	600	8728	10005.9	100.0	0	0	11720	60.7	0.0	1
		1.00	3409	10386	10007.5	100.0	0	0	12888	94.8	0.0	1

Table A.47: Performance of ILP-O and ILP-S for instance size (Sz.) 200, instance numbers (Nr.) 15–29.

		1	1		ILP-O					ILP-S		
Sz.	Nr.	Load	C_{a}	C_u	t[s]	Gap	Opt	C_{a}	C_u	t[s]	Gap	Opt
200	15	0.10	0	2489	5.8	0.0	1	0	4206	1.8	0.0	1
		0.50	0	4193	71.4	0.0	1	0	8536	11.2	0.0	1
		0.80	0	5067	10003.7	1.1	0	0	9553	22.7	0.0	1
	16	1.00	0	5756	10004.2	3.8	0	0	11220	35.2	0.0	1
	16	0.10	0	2649 3941	9.0	0.0	1	0	4321	3.2 17.3	0.0	1
		0.50 0.80	0	5201	7659.6 10004.4	0.0 2.4	1	0	7721 10027	27.6	0.0	1
		1.00	3588	10403	10004.4	100.0	0	0	12455	47.6	0.0	1
	17	0.10	0	1838	3.5	0.0	1	0	3533	3.4	0.0	1
	17	0.50	0	4627	10002.0	1.6	0	0	8508	16.6	0.0	1
		0.80	0	6196	10003.9	8.4	0	0	11135	28.8	0.0	1
		1.00	3826	11663	10006.6	100.0	0	0	12952	49.9	0.0	1
	18	0.10	0	2037	4.2	0.0	1	0	3959	3.2	0.0	1
		0.50	0	4313	10003.6	0.2	0	0	8541	19.6	0.0	1
		0.80	0	5763	10005.7	4.9	0	0	10523	33.8	0.0	1
		1.00	5073	9603	10009.1	100.0	0	0	11741	53.0	0.0	1
	19	0.10	0	2351	2.2	0.0	1	0	3668	1.9	0.0	1
		0.50	0	3989	6174.1	0.0	1	0	7039	11.1	0.0	1
		0.80	0	5797	10003.7	6.5	0	0	10318	23.0	0.0	1
		1.00	0	6712	10004.5	9.5	0	0	11954	31.2	0.0	1
	20	0.10	0	2598	23.1	0.0	1	0	5224	3.9	0.0	1
		0.50	0	4652	10001.7	0.4	0	0	8840	13.6	0.0	1
		0.80	0	5735	10003.7	5.4	0	0	10642	24.6	0.0	1
	21	1.00	0	6756	10004.8	9.0	0	0	11786	30.3	0.0	1
	21	0.10	0	2481	1.2	0.0	1	0	3710	1.1	0.0	1
		0.50	0	4566	25.7	0.0	1	0	7960	8.4	0.0	1
		0.80	0	5921	3776.0	0.0	1	0	10463	15.8	0.0	1
	22	1.00	0	6341 2997	10003.6	1.8	0	0	12437	19.3	0.0	1
	22	0.10	0	4950	3.9 169.8	0.0 0.0	1		5071 9109	2.0 18.7	0.0	1
		0.50 0.80	0	5970	10004.4	0.0	0	0	10918	26.9	0.0	1
		1.00	0	6855	10004.4	4.5	0	0	12466	53.0	0.0	1
	23	0.10	0	2537	3.8	0.0	1	0	3988	2.9	0.0	1
	25	0.50	0	4893	10003.1	1.5	0	0	8621	19.8	0.0	1
		0.80	0	5711	10004.5	3.7	0	0	9939	32.2	0.0	1
		1.00	0	7260	10006.1	9.2	0	0	12632	40.8	0.0	1
	24	0.10	0	2193	3.2	0.0	1	0	3424	1.8	0.0	1
		0.50	0	4617	65.5	0.0	1	0	8088	9.5	0.0	1
		0.80	0	5517	10005.6	2.9	0	0	9712	24.0	0.0	1
		1.00	0	6342	10006.0	3.4	0	0	11662	31.8	0.0	1
	25	0.10	0	2790	21.4	0.0	1	0	5007	3.1	0.0	1
		0.50	0	4645	10003.1	5.4	0	0	8918	21.2	0.0	1
		0.80	0	5549	10006.1	8.3	0	0	11289	39.4	0.0	1
		1.00	0	6347	10006.0	8.1	0	0	12373	36.9	0.0	1
	26	0.10	0	2895	17.9	0.0	1	0	4772	6.7	0.0	1
		0.50	0	4330	2205.0	0.0	1	0	7295	16.8	0.0	1
		0.80	0	5744	10005.9	2.0	0	0	10030	28.0	0.0	1
	27	1.00	0	6257 2416	10007.3 1.1	4.4 0.0	0	0	11055 3547	45.8 1.2	0.0	1
	21	0.10 0.50	0	4503	253.7	0.0	1	0	7710	9.0	0.0	1
		0.30	0	5544	10003.3	0.0	0	0	9842	9.0	0.0	1
		1.00	0	5947	10005.3	3.1	0	0	10985	25.2	0.0	1
	28	0.10	0	2331	2.1	0.0	1	0	4481	1.4	0.0	1
	20	0.50	0	3864	10001.4	2.3	0	0	8410	14.6	0.0	1
		0.80	0	5286	10004.7	6.5	0	0	10270	27.4	0.0	1
		1.00	0	6481	10007.1	8.3	0	0	12288	44.9	0.0	1
	29	0.10	0	2318	28.0	0.0	1	0	4985	2.6	0.0	1
	••	0.50	0	4320	10002.2	3.4	0	0	9784	19.7	0.0	1
		0.80	0	6018	10048.8	7.6	0	0	11948	55.3	0.0	1
		1.00	1084	10443	10009.3	100.0	0	0	12903	74.3	0.0	1
					I	1						

Table A.48: Performance of ILP-O and ILP-S for instance size (Sz.) 500, instance numbers (Nr.) 0–14.

		1			ILP-O					ILP-S		
Sz.	Nr.	Load	Ca	C_u	t[s]	Gap	Opt	Ca	C_u	t[s]	Gap	Opt
500	0	0.10	0	3722	250.6	0.0	1	0	6596	13.5	0.0	1
		0.50	0	6726	736.2	0.0	1	0	12177	46.1	0.0	1
		0.80	0	8286	10027.2	2.9	0	0	15490	96.7	0.0	1
		1.00	16483	16274	10025.7	100.0	0	0	20935	193.9	0.0	1
	1	0.10	0	3633	1985.5	0.0	1	0	6325	7.6	0.0	1
		0.50	145	9123	10015.4	100.0	0	0	11265	60.7	0.0	1
		0.80	1264	13307	10025.8	100.0	0	0	14666	136.4	0.0	1
	2	1.00 0.10	9320 0	15518 3329	10029.5 36.4	100.0 0.0	0	0	21198 6433	1064.4 7.2	0.0 0.0	1
	2	0.10	0	6243	10010.4	1.2	0	0	10252	37.8	0.0	1
		0.80	2459	13800	10010.4	100.0	0	0	15297	88.4	0.0	1
		1.00	16322	17450	10021.0	100.0	0	0	20207	160.2	0.0	1
	3	0.10	0	2987	14.3	0.0	1	0	5033	11.4	0.0	1
	2	0.50	0	5782	10015.3	1.1	0	0	10002	56.0	0.0	1
		0.80	0	13022	10043.3	76.8	0	0	15190	187.8	0.0	1
		1.00	4795	16388	10047.5	100.0	0	0	25129	408.4	0.0	1
	4	0.10	0	2555	31.7	0.0	1	0	5284	11.7	0.0	1
		0.50	0	8967	10019.6	32.8	0	0	11402	57.5	0.0	1
		0.80	45	12775	10025.6	100.0	0	0	15281	157.1	0.0	1
		1.00	5868	16026	10042.8	100.0	0	0	22320	505.1	0.0	1
	5	0.10	0	2858	38.5	0.0	1	0	5262	7.1	0.0	1
		0.50	0	6648	10014.6	3.0	0	0	12007	56.7	0.0	1
		0.80	4851	14490	10023.0	100.0	0	0	16711	151.2	0.0	1
		1.00	16673	17427	10033.8	100.0	0	0	19699	204.0	0.0	1
	6	0.10	0	3167	237.8	0.0	1	0	5495	327.7	0.0	1
		0.50	0	6556	10039.3	0.1	0	0	12300	332.3	0.0	1
		0.80	886	12920	10053.6	100.0	0	0	15354	368.3	0.0	1
	7	1.00	8250	16495	10049.3	100.0	0	0	20361	412.6	0.0	1
	7	0.10	0	3220	15.3	0.0	1	0	4535	19.1	0.0	1
		0.50 0.80	0 5255	7170 15456	10017.0 10031.1	0.3 100.0	0	0	12733 17862	101.7 165.2	0.0 0.0	1
		1.00	12441	18699	10031.1	100.0	0	0	21870	210.3	0.0	1
	8	0.10	0	2973	2610.9	0.0	1	0	6539	17.9	0.0	1
	0	0.50	0	9553	10027.4	76.5	0	0	12954	90.3	0.0	1
		0.80	2349	15746	10032.4	100.0	0	0	22026	297.6	0.0	1
		1.00	19908	19533	10035.8	100.0	0	0	25113	649.3	0.0	1
	9	0.10	0	2420	35.8	0.0	1	0	4565	7.5	0.0	1
		0.50	20	8164	10021.5	100.0	0	0	10223	76.5	0.0	1
		0.80	3052	12907	10029.1	100.0	0	0	14751	163.4	0.0	1
		1.00	9392	15606	10035.2	100.0	0	0	23490	587.3	0.0	1
	10	0.10	0	2962	29.2	0.0	1	0	5691	13.0	0.0	1
		0.50	0	8586	10017.5	36.4	0	0	11010	65.3	0.0	1
		0.80	344	13336	10024.4	100.0	0	0	17645	136.4	0.0	1
		1.00	20951	17789	10035.6	100.0	0	0	22149	207.8	0.0	1
	11	0.10	0	3446	11.1	0.0	1	0	5288	5.9	0.0	1
		0.50	0	7218	10019.1	0.9	0	0	11641	67.4	0.0	1
		0.80	778	12513	10028.2	100.0	0		13469	91.0 171.7	0.0	1
	12	1.00 0.10	5612 0	15057 3061	10036.1 9.1	100.0 0.0	0	0	17559 5015	6.7	0.0 0.0	1
	12	0.10	435	10761	9.1	100.0	0	0	10685	73.4	0.0	1
		0.30	7555	14944	10020.8	100.0	0	0	15064	133.3	0.0	1
		1.00	19224	17711	10025.5	100.0	0	0	18521	245.7	0.0	1
	13	0.10	0	2780	20.6	0.0	1	0	4777	9.3	0.0	1
		0.50	0	8910	10019.0	36.4	0	0	12142	73.9	0.0	1
		0.80	491	12995	10027.8	100.0	0	0	14288	115.6	0.0	1
		1.00	6984	17009	10034.8	100.0	0	0	23228	352.7	0.0	1
	14	0.10	0	3451	12.7	0.0	1	0	5398	7.0	0.0	1
		0.50	0	10190	10019.7	37.7	0	0	11941	65.2	0.0	1
		0.80	3412	15201	10031.7	100.0	0	0	17504	120.1	0.0	1
		1.00	21324	19392	10044.4	100.0	0	0	24708	689.4	0.0	1

Table A.49: Performance of ILP-O and ILP-S for instance size (Sz.) 500, instance numbers (Nr.) 15–29.

Sz. Nr. Load Ca Ca Ca Ca Val Cap Opt 500 15 0.10 0 3565 29.1 0.0 1 0 7629 12.0 0.0 1 0.80 822 13489 10029.4 1000 0 15173 141.8 0.0 1 16 0.10 1531 1044.8 1000 0 2512 686.2 0.0 1 0.50 0 6033 7791.4 0.0 1 0 1452.6 680.0 0 11 1.00 6602 16230 10035.2 100.0 0 23400 339.8 0.0 1 0.50 0 7944 103.5 2.5 0 0 1443.8 0.0 1 0.50 0 7956 10037.1 100.0 0 18.3 0.0 1 1.00 3856 13756 100.1 1						ILP-O					ILP-S		
	Sz.	Nr.	Load	C_a	C_u	t[s]	Gap	Opt	$\mathbf{C}_{\mathbf{a}}$	$\mathbf{C}_{\mathbf{u}}$	t[s]	Gap	Opt
0.80 822 1489 10029.4 100.0 0 15173 141.8 0.0 1 16 0.10 0 2891 104.6 0.0 1 0 15172 105.0 0.0 1 0.50 0 6033 7791.4 0.0 1 0 11562 69.8 0.0 1 0.50 0 7942 10031.5 2.5 0.0 114536 130.8 0.0 1 0.50 0 5440 189.4 0.0 1 0 9449 43.8 0.0 1 0.80 0 7656 10027.8 1.4 0 0 14538 1758.4 0.0 1 1.00 3545 10702.0 1.4 0 0 11116 64.7 0.0 1 0.80 783 14895 10303.8 100.0 0 0 12375 311.2 0.0 1 0.80 783 <	500	15	0.10		3565	29.1	0.0	1	0	7629	12.0	0.0	1
1.00 18419 18031 10044.8 100.0 0 25124 686.2 0.0 1 0.50 0 6033 7791.4 0.0 1 0 1152 698.0 1 0.80 0 7942 10031.5 2.5 0 0 14536 130.8 0.0 1 0.80 0 7942 10031.5 2.5 0 0 14536 130.8 0.0 1 0.00 0 24400 339.8 0.0 1 0 3660 5.4 0.0 1 0.80 0 7056 10027.8 1.4 0 0 14341 86.7 0.0 1 0.80 783 14895 1003.4 100.0 0 1 0 140.4 136.9 100 1 0 110.0 10.0 110.0 10.0 10.0 1 0 156.9 100.1 1 0.0 10.0 110.0													
16 0.10 0 233 7791.4 0.0 1 0 6172 10.05 60.0 1 0.80 0 7742 10031.5 2.5 0 0 14536 130.8 0.0 1 1.00 6602 1203 295 0.0 1 0 9449 43.8 0.0 1 0.50 0 5440 189.4 0.0 1 0 9449 43.8 0.0 1 0.00 0 5450 10037.1 100.0 0 0 14568 175.8 10.0 1 0.00 0 3455 10037.1 100.0 0 0 1116 64.7 0.0 1 0.00 0 345 10037.1 100.0 0 0 1116 64.7 0.0 1 0.50 0 9346 1002.0 0 1234 84.1 0.0 1 1 0 554 <													
0.50 0 693 7942 10031.5 2.5 0 0 1456 130.8 0.0 1 1.00 6602 16230 10035.2 100.0 0 23400 339.8 0.0 1 1.7 0.10 0 21630 189.4 0.0 1 0 3660 5.4 0.0 1 0.80 0 7056 10027.8 1.4 0 0 14341 86.7 0.0 1 1.00 3856 13976 10021.0 3.1 0 0 14341 86.7 0.0 1 0.50 0 3445 1202.0 1.1 0 14341 10.0 1 0 14441 1.0 1 1.0 1.													
0.80 00 7942 10031.5 2.5 00 0 14736 130.8 0.0 1 170 0.10 0 2108 29.5 0.0 1 0 3660 5.4 0.0 1 0.50 0 5440 189.4 0.0 1 0 9449 43.8 0.0 1 1.00 3856 13976 10037.1 100.0 0 0 18638 175.8 0.0 1 0.50 0 9346 10022.0 3.1 0 0 11116 64.7 0.0 1 1.00 8189 1957<10043.9		16											
1.00 6602 16230 1003.52 100. 0 2400 339.8 0.0 1 1.7 0.10 0 2108 29.5 0.0 1 0 3660 5.4.8 0.0 1 0.50 0 5544 1807.4 180.0 0 14341 86.7 0.0 1 1.00 3856 13976 10037.1 100.0 0 18638 175.8 0.0 1 0.80 783 14895 1003.6 100.0 0 0 15484 12.97 0.0 1 0.80 783 14895 1003.6 100.0 0 22375 311.2 0.0 1 0.50 0 6978 1002.10 3.3 0 0 12364 84.1 0.0 1 0.50 0 6677 1003.3 1000.0 0 2188 10352.0 0.0 1 0.50 0 5619 10													
17 0.0 0 5440 189.4 0.0 1 0 3660 5.4 0.0 1 0.80 0 7056 10027.8 1.4 0 0 18538 175.8 0.0 1 1.00 3856 13976 10027.8 1.4 0 0 18538 175.8 0.0 1 0.50 0 9346 10022.0 34.1 0 0 11116 64.7 0.0 1 0.50 0 9346 10022.0 34.1 0 0 15484 129.7 0.0 1 0.50 0 6978 10021.0 3.3 0 0 12364 84.1 0.0 1 0.50 0 6600 10014.9 2.4 0 0 12364 84.1 0.0 1 0.00 0 2235 1313 1002.7 6.3 0.0 1 10 5563 6.7 0.0													
0.50 0 5440 189.4 0.0 1 0 94.38 0.0 1 1.00 3856 13976 10037.1 100.0 0 0 18638 175.8 0.0 1 18 0.10 0 3445 22.9 0.0 1 0 5716 12.9 0.0 1 0.50 0 9346 10022.0 34.1 0 0 15444 12.9 0.0 1 0.50 0 6978 1002.1 3.3 0 0 12364 84.1 0.0 1 0.50 0 6978 1002.1 3.3 0 0 12364 84.1 0.0 1 0.00 2238 15313 10032.1 3.3 0 0 14839 86.3 0.0 1 0.00 0 3267 6.3 0.0 1 0 5563 6.7 0.0 1 0.50 <		17											
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$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			0.50	0	6978	10021.0	3.3	0	0	12364	84.1	0.0	1
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				2328		10032.8	100.0	0	0	20738	340.4	0.0	
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0.80 4672 14631 10036.5 100.0 0 0 20703 261.2 0.0 1 1.00 14827 17823 10042.2 100.0 0 0 23266 314.9 0.0 1 29 0.10 0 3777 55.9 0.0 1 0 5939 23.4 0.0 1 0.50 0 6028 412.1 0.0 1 0 9413 55.9 0.0 1 0.80 0 12656 10038.0 40.6 0 0 15136 96.1 0.0 1		28	0.10	0	3004	67.7	0.0	1	0	5919	14.4	0.0	1
1.00 14827 17823 10042.2 100.0 0 0 23266 314.9 0.0 1 29 0.10 0 3777 55.9 0.0 1 0 5939 23.4 0.0 1 0.50 0 6028 412.1 0.0 1 0 9413 55.9 0.0 1 0.80 0 12656 10038.0 40.6 0 0 15136 96.1 0.0 1			0.50	0	9044	10026.3	73.9	0	0	12079	101.6	0.0	1
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0.80 0 12656 10038.0 40.6 0 0 15136 96.1 0.0 1		29											
1.00 8686 16546 10054.0 100.0 0 0 20333 146.9 0.0 1			1.00	8686	16546	10054.0	100.0	0	0	20333	146.9	0.0	1

Table A.50: Performance of ILP-O and ILP-S for instance size (Sz.) 1000, instance numbers (Nr.) 0–14.

					ILP-O					ILP-S		
Sz.	Nr.	Load	C_a	C_u	t[s]	Gap	Opt	$\mathbf{C}_{\mathbf{a}}$	C_u	t[s]	Gap	Opt
1000	0	0.10	0	3912	705.0	0.0	1	0	7977	62.4	0.0	1
		0.50	41	15071	10069.3	100.0	0	0	16389	194.1	0.0	1
		0.80	3663	22502	10211.6	100.0	0	0	30527	721.7	0.0	1
		1.00	-	-	-	-	-	0	37444	1640.8	0.0	1
	1	0.10	0	3262	10003.7	0.3	0	0	5873	32.2	0.0	1
		0.50	0	8308	10041.0	3.0	0	0	14212	161.8	0.0	1
		0.80	2433	21052	10087.1	100.0	0	0	27025	481.2	0.0	1
		1.00	16846	26477	10147.8	100.0	0	0	35046	1097.6	0.0	1
	2	0.10	0	3966	83.2	0.0	1	0	7125	41.0	0.0	1
		0.50	0	8850	10085.0	2.9	0	0	14804	246.5	0.0	1
		0.80	7705	20612	10160.9	100.0	0	0	28623	586.4	0.0	1
		1.00	-	-	-	-	-	0	35462	1546.1	0.0	1
	3	0.10	0	3056	65.7	0.0	1	0	5323	46.3	0.0	1
		0.50	78	12973	10074.6	100.0	0	0	13739	194.4	0.0	1
		0.80	620	20989	10120.9	100.0	0	0	26914	692.8	0.0	1
		1.00	-	-	-	-	-	0	35649	1954.9	0.0	1
	4	0.10	0	4378	80.0	0.0	1	0	8169	37.8	0.0	1
		0.50	411	13991	10102.2	100.0	0	0	21674	258.2	0.0	1
		0.80	5337	18736	10149.8	100.0	0	0	27998	794.1	0.0	1
		1.00	-	-	-	-	-	0	35095	1845.7	0.0	1
	5	0.10	0	4801	298.4	0.0	1	0	8539	34.8	0.0	1
		0.50	0	9183	10104.3	1.6	0	0	21531	266.0	0.0	1
		0.80	6039	21893	10102.0	100.0	0	0	29258	532.4	0.0	1
		1.00	-	-	-	-	-	0	35228	1694.8	0.0	1
	6	0.10	0	4770	143.2	0.0	1	0	8831	61.9	0.0	1
		0.50	0	8751	10102.6	2.3	0	0	14137	234.4	0.0	1
		0.80	4198	19513	10140.7	100.0	0	0	28161	563.6	0.0	1
		1.00	-	-	-	-	-	0	35958	1269.1	0.0	1
	7	0.10	0	2642	174.9	0.0	1	0	5834	49.8	0.0	1
		0.50	0	6971	10075.1	1.5	0	0	12615	168.6	0.0	1
		0.80	4890	19680	10146.1	100.0	0	0	28173	516.8	0.0	1
		1.00	-	-	-	-	-	0	34359	2069.2	0.0	1
	8	0.10	0	3484	131.3	0.0	1	0	7233	21.1	0.0	1
		0.50	0	6724	10055.6	0.5	0	0	10636	173.9	0.0	1
		0.80	8167	20085	10129.6	100.0	0	0	27346	514.0	0.0	1
	0	1.00	-	-	-	-	-	0	33449	2172.6	0.0	1
	9	0.10	0	3376	10009.6	0.1		0	6783	46.7	0.0	1
		0.50	0	12248	10075.6	75.3		0	18876	222.8	0.0	1
		0.80	5838	19365	10090.7	100.0	0	0	27061	569.4	0.0	1
	10	1.00	-		540.5	-	-	0	33438	1344.9	0.0	1
	10	0.10	0	3032	540.5 10099.9	0.0	1	0	6460	87.6	0.0	1
		0.50	0	8311		2.7	0	0	14394	253.1	0.0	1
		0.80	6926 13299	21602 25286	10155.7 10153.0	100.0 100.0	0	0	28290 35415	523.6 1169.9	0.0	1
	11	1.00						0				
	11	0.10 0.50	0	3754 7722	2621.8 10039.5	0.0 2.1	1	0	7068 12766	28.9 155.6	0.0	1
			2403	19234	10039.3	100.0		0	28316	396.4	0.0	1
		0.80 1.00	2405	19234	10084.3	100.0	-	0	28516 34903	1328.4	0.0	1
	12	0.10	0	3454	299.3	0.0	- 1	0	7075	24.3	0.0	1
	12		1735	14863	10089.3	100.0		0	17087	24.5		1
		0.50 0.80	5453	21292	10089.3	100.0		0	27696	420.5	0.0	1
				21292	10120.5				35966			
	13	1.00 0.10	- 0	4204	- 90.9	- 0.0	- 1	0	7622	1552.8 54.2	0.0	1
	13	0.10	0	13085	10090.5	80.8	0	0	19931	239.4	0.0	1
		0.80 1.00	1964	17975	10137.9	100.0	0	0	26705	577.3 5516.8	0.0	1
	14		0	- 3457	129.8	0.0	- 1	0	35871 6745	33.0	0.0	1
	14	0.10 0.50	84	11605	129.8	100.0	0	0	6745 11500	33.0 175.6	0.0	1
		0.30	2927	17229	10036.4	100.0	0	0	25899	602.7	0.0	1
		1.00	2721	11229	- 10090.8	100.0		0	35056	3160.0	0.0	1
		1.00					-	0	55050	5100.0	0.0	·

Table A.51: Performance of ILP-O and ILP-S for instance size (Sz.) 1000, instance numbers (Nr.) 15–29.

					ILP-O					ILP-S		
Sz.	Nr.	Load	C_a	C_u	t[s]	Gap	Opt	C_{a}	C_u	t[s]	Gap	Opt
1000	15	0.10	0	4310	155.7	0.0	1	0	8216	40.3	0.0	1
		0.50	0	13171	10055.9	32.9	0	0	15171	160.3	0.0	1
		0.80	755	20466	10088.0	100.0	0	0	26156	378.3	0.0	1
		1.00	-	-	-	-	-	0	33574	769.7	0.0	1
	16	0.10	0	3379	56.9	0.0	1	0	6218	38.2	0.0	1
		0.50	0	8248	10106.8	0.8	0	0	14510	250.1	0.0	1
		0.80	1402	17668	10167.9	100.0	0	0	27824	604.6	0.0	1
		1.00	-	-	-	-	-	0	35507	1360.2	0.0	1
	17	0.10	0	3147	10008.0	0.5	0	0	6173	47.1	0.0	1
		0.50	0	9120	10081.2	1.5	0	0	15934	243.1	0.0	1
		0.80	9575	22070	10115.8	100.0	0	0	30242	600.5	0.0	1
		1.00	18155	24743	10156.9	100.0	0	0	34411	889.2	0.0	1
	18	0.10	0	3561	221.5	0.0	1	0	6544	21.1	0.0	1
		0.50	470	12661	10069.1	100.0	0	0	18935	177.4	0.0	1
		0.80	3164	20325	10113.0	100.0	0	0	27722	643.3	0.0	1
		1.00	-	-	-	-	-	-	-	-	-	-
	19	0.10	0	3228	10003.9	2.1	0	0	6740	25.6	0.0	1
		0.50	21	13148	10060.5	100.0	0	0	18448	177.5	0.0	1
		0.80	981	19866	10103.3	100.0	0	0	27358	377.6	0.0	1
		1.00	-	-	-	-	-	0	34735	1285.8	0.0	1
	20	0.10	0	4874	216.4	0.0	1	0	8248	31.1	0.0	1
		0.50	0	8140	10064.8	1.8	0	0	13761	178.1	0.0	1
		0.80	809	20242	10117.8	100.0	0	0	28112	388.9	0.0	1
		1.00	-	- 20212	-	-	-	0	33578	727.3	0.0	1
	21	0.10	0	3826	713.0	0.0	1	0	7456	68.9	0.0	1
		0.50	0	8017	10122.7	2.0	0	0	20455	289.2	0.0	1
		0.80	1007	17922	10163.7	100.0	0	0	28622	602.9	0.0	1
		1.00	-	-	10105.7	100.0	-	0	35560	1055.6	0.0	1
	22	0.10	0	3666	10006.4	0.5	0	0	6880	35.7	0.0	1
	22	0.10	1661	13169		100.0	0	0	17223	193.9	0.0	1
		0.30	3284	20370	10059.0 10145.5	100.0	0	0	26152	1260.9	0.0	1
		1.00	- 3204	20370	10145.5	100.0	-	0	35924	1200.9	0.0	1
	22			4160	100.6	-						
	23	0.10	0	4162	109.6	0.0	1	0	6915	21.3	0.0	1
		0.50	36	13865	10053.6	100.0	0	0	14934	206.0	0.0	1
		0.80	3892	19010	10097.3	100.0	0	0	26239	386.9	0.0	1
		1.00	-	-	-	-	-	0	36037	1443.3	0.0	1
	24	0.10	0	3361	37.2	0.0	1	0	6189	10.2	0.0	1
		0.50	0	8857	10068.5	2.5	0	0	14901	215.8	0.0	1
		0.80	6348	21149	10162.6	100.0	0	0	27857	591.4	0.0	1
		1.00	14117	24242	10130.1	100.0	0	0	34531	915.9	0.0	1
	25	0.10	0	3887	10015.3	0.3	0	0	7466	60.7	0.0	1
		0.50	0	12830	10083.4	76.8	0	0	19888	230.0	0.0	1
		0.80	-	-	-	-	-	0	29531	587.1	0.0	1
		1.00	-	-	-	-	-	0	35631	1872.4	0.0	1
	26	0.10	0	4450	44.3	0.0	1	0	7523	21.8	0.0	1
		0.50	0	8277	10050.9	2.9	0	0	13145	155.1	0.0	1
		0.80	2238	20437	10113.4	100.0	0	0	27649	1115.4	0.0	1
		1.00	-	-	-	-	-	0	32847	1291.7	0.0	1
	27	0.10	0	2881	228.2	0.0	1	0	5581	69.0	0.0	1
		0.50	0	6943	10052.2	0.9	0	0	11883	180.1	0.0	1
		0.80	907	17975	10135.7	100.0	0	0	24847	406.0	0.0	1
		1.00	-	-	-	-	-	0	35397	1219.2	0.0	1
	28	0.10	0	4438	535.1	0.0	1	0	8098	54.6	0.0	1
		0.50	0	9527	10087.5	2.5	0	0	16811	196.1	0.0	1
		0.80	1613	20802	10158.6	100.0	0	0	27594	401.0	0.0	1
		1.00	-	- 20002	-	-	-	0	36884	1349.2	0.0	1
	29	0.10	0	4053	212.3	0.0	1	0	6991	63.7	0.0	1
		0.50	215	12357	10078.9	100.0	0	0	12804	222.4	0.0	1
		0.80	8784	19037	10112.0	100.0	0	0	26014	533.3	0.0	1
		1.00	-	-	-		-	0	32722	895.0	0.0	1
		1.00	1	· · ·						070.0	0.0	



Curriculum Vitae

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Education

since 2010	PhD studies in Computer Science , <i>Vienna University of Technology</i> . Dissertation on the topic of "Optimization Challenges of the Future Federated Internet"
2008–2010	Master studies in <i>Computational Intelligence</i> , <i>Vienna University of Technology</i> . Master thesis on the topic of "Automatic Generation of 2-AntWars Players with Genetic Pro- gramming Master examination passed with distinction
2006–2008	Bachelor studies in Software & Information Engineering , Vienna University of Technology. Bachelor thesis on the topic of "Scatter Search" Bachelor examination passed with distinction
2000–2005	Upper Secondary School , <i>School for higher technical education Donaustadt</i> , Vienna. Department of computer engineering Diploma thesis on the topic of 2-D motion capture General qualification for university entrance passed with distinction
1996-2000	Lower Secondary School, Polgarstraße, Vienna.

1992–1996 Elementary School, Essling, Vienna.

Work Experience

since 2011	University Assistant , Algorithm and Data Structures Group, Institute of Computer Graphics and Algorithms, Vienna University of Technology.
2010–2011	Research Assistant , Algorithm and Data Structures Group, Institute of Computer Graphics and Algorithms, Vienna University of Technology.
2005–2006	Military Duty, Department of Performance Medicine, Van-Swieten-Kaserne.

2001–2003 **Internship**, Department of Information-management, Generali Holding Vienna AG (one month each year).

Languages

German native tongue English fluent

Research Interests

My current research interests are telecommunication problems, with the focus on virtual network mapping. I studied various heuristic and exact methods for solving the offline variant of this problem and plan to extend those methods to solve online and dynamic variants, which are closer to the real world. Generally, I am interested in any method for automatic problem solving, like meta-heuristics or algorithms from the field of artificial intelligence. I find the emergent self-organization and evolutionary dynamics that can be observed in genetic programming fascinating.

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