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# Consumption-based capital asset pricing models: Empirical evidence from the Austrian capital market

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*To my parents,  
Julia, Thomas and Martin*

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# Abstract

Using Austrian data, two consumption-based capital asset pricing models were empirically tested and estimated in this study, namely the simple power utility model first introduced by Mehra and Prescott and Abel's more general "Catching up with the Joneses". First of all, the performance of these models using historical averages and reasonable values of the relative risk aversion coefficient was examined. In addition, the model parameters were formally estimated using the generalized method of moments approach as suggested by Hansen and the overall fit of the models was tested by Hansen's  $J_T$  test. Additionally, this empirical study examines whether the used time series satisfy the necessary model assumptions like a lognormal distribution of returns on a risky asset and consumption growth. In order to be able to adopt the generalized method of moments the underlying stationarity assumptions of the time series was tested.

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# Chapter 1

## Introduction

From the very beginning of financial markets and stock exchanges, investors as well as analysts and economists have been fascinated by the movement of stock prices. Great efforts have been made in order to understand and, as a desirable result, be able to forecast the prices of these financial instruments. The fast rise of consumption-based capital asset pricing models (CCAPM) started with Lucas' paper on asset pricing in an exchange economy in 1978 [11], where he first analysed the stochastic behaviour of equilibrium asset prices in a one-good, pure exchange economy. Lucas linked the pricing of assets to levels of consumptions, or more precisely, to the utility an individual receives from a certain consumption level. The general setup of a representative individual maximizing its utility depending on consumption levels has been the starting point for a growing number of CCAPM getting more and more complex.

Numerous empirical studies testing CCAPM have been conducted, in the beginning mostly using data from the US. In most cases, simple CCAPM are not able to match the historical stock and bond returns simultaneously when using a realistic and therefore low relative risk aversion parameter. Asset pricing puzzles are the consequence and the main goal of researchers is to solve these puzzles. Recently, CCAPM were exposed to empirical tests in other countries than the US and even at an international level more often, but the results remain virtually equal to the evidence from the US.

Ever since Hansen has published his heavily cited article on the large sample properties of generalized method of moments (GMM) estimators [9], this approach has become common practice when estimating the parameters of these non-linear CCAPM. As GMM does not demand strong distributional assumptions it is suitable for financial data where the variables are often serially correlated and heteroscedastic and hence, a conventional estimation would not result in satisfying estimators. Furthermore, the overall fit of a CCAPM can be tested using the set of moment conditions implied by GMM.

In this thesis Austrian time series data including returns on a risky asset and a bond as well as aggregate private consumption was used in order to test and estimate two CCAPM. Starting with Mehra and Prescott's well-known simple power utility model described in [13] the attempt was made to provide evidence for the validity of this model, which links asset prices to the covariance of returns on a risky asset and consumption growth. Moreover, a generalization of the simple model was taken into account. Abel's "Catching up with the Joneses" [1] introduces external habit formation, which means that individuals care not only about their own consumption, but also about the others' consumption levels which are regarded as a benchmark.

Using Austrian data both models were examined whether they can match the historical averages of returns on stocks and bonds when considering reasonable values for the relative risk aversion parameter. Furthermore, the models were estimated using Hansen's GMM approach and the fit of the models was tested by using Hansen's  $J_T$  test.

The thesis is organized into the following chapters. Chapter 2 shortly summarizes the basic principles which are needed to develop the necessary understanding of CCAPM. Chapter 3 introduces the simple power utility model and Abel's "Catching up with the Joneses". What is more, chapter 3 contains a discussion of two popular asset pricing puzzles, namely the "equity premium puzzle" and the "risk-free rate puzzle". Chapter 4 gives a description of the Austrian data and the time series obtained from this data. Furthermore, this chapter explains the tests on lognormality and stationarity applied to the time series and depicts the results. Chapter 5 introduces GMM and how GMM is implemented in the statistic software R. Moreover, the application of GMM to the simple model and Abel's model is described. Chapter 6 finally shows the results of this empirical study. First, both models are examined whether they can match historical averages when using a reasonable relative risk aversion coefficient, second, the results of the GMM procedure are stated and third, a comparison of the results of the models is given.

## Chapter 2

# Basic Principles

Consumption-based capital asset pricing models (CCAPM) are meant to link asset prices to consumption. More precisely these models link expected returns to the covariance of returns and (aggregate) consumption. In order to develop these models it is important to understand the principles used in the models. If not stated otherwise, this chapter is based on [14] and will shortly introduce the basics that are used.

### 2.1 Probability and stochastic processes

Any model with uncertainty refers to a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  which is given by

- $\Omega$ , which is the state space of possible outcomes.  $\omega \in \Omega$  represents a possible realization of all uncertain objects of the model.
- $\mathcal{F}$ , which is a  $\sigma$ -algebra in  $\Omega$  and therefore has the following properties
  1.  $\Omega \in \mathcal{F}$
  2.  $\forall F \in \mathcal{F} \Rightarrow F^C \equiv \Omega \setminus F \in \mathcal{F}$
  3. for  $F_1, F_2, \dots \in \mathcal{F} \Rightarrow \bigcup_{n=1}^{\infty} F_n \in \mathcal{F}$ .
- $\mathbb{P}$ , which is a probability measure, i.e., a function  $\mathbb{P} : \mathcal{F} \rightarrow [0, 1]$  with  $\mathbb{P}(\Omega) = 1$  and for any sequence  $A_1, A_2, \dots$  of disjoint events  $\mathbb{P}(\bigcup_{m=1}^{\infty} A_m) = \sum_{m=1}^{\infty} \mathbb{P}(A_m)$ .

In order to model information and information revelation the concept of an information filtration is used. A filtration is given by a sequence of  $\sigma$ -algebras  $(\mathcal{F}_t)_{t \in \mathcal{T}}$  on  $\Omega$ , where the time horizon is given by  $\mathcal{T} = \{1, 2, \dots, T\}$  and for all  $s, t \in \mathcal{T}$  with  $s < t$  it follows that  $\mathcal{F}_s \subseteq \mathcal{F}_t \subseteq \mathcal{F}$ . The interpretation of this filtration is that the increasing  $\sigma$ -algebras describe the information available at time  $t$ . We assume that information at time 0 is trivial and therefore



$\mathcal{F}_0 = \{\emptyset, \Omega\}$  which indicates that no information is available at all. We additionally assume that all information is revealed at the final date  $T$  and therefore  $\mathcal{F}_T = \mathcal{F}$ . Using this concept a filtered probability space can be written as  $(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F})_{t \in \mathcal{T}})$ .

In order to take the information available into account when considering expectations of random variables like stock prices or dividends conditional expectations are needed and the following important theorem will be used:

**Theorem 2.1** (*The Law of Iterated Expectations*) *If  $\mathcal{F}$  and  $\mathcal{G}$  are two  $\sigma$ -algebras with  $\mathcal{F} \subseteq \mathcal{G}$  and  $X$  is a random variable, then  $E[E[X|\mathcal{G}]|\mathcal{F}] = E[X|\mathcal{F}]$ . In particular, if  $(\mathcal{F})_{t \in \mathcal{T}}$  is an information filtration representing the information available at time  $t$  and  $s < t$ , we have*

$$E_s[E_t[X]] = E_s[X], \quad (2.1)$$

where  $E_s[X] = E[X|\mathcal{F}_s]$  and  $E_t[X] = E[X|\mathcal{F}_t]$ .

Using this conditional expectation it is possible to define conditional variances, conditional covariances and conditional correlations in the same way as it is done for their unconditional equivalent.

$$\text{Var}_t[X] = E_t[(X - E_t[X])^2]$$

$$\text{Cov}_t[X, Y] = E_t[(X - E_t[X])(Y - E_t[Y])]$$

$$\text{Corr}_t = \rho_t = \frac{\text{Cov}_t[X, Y]}{\sqrt{\text{Var}_t[X] \text{Var}_t[Y]}}.$$

When considering multi-period models uncertainty is represented by stochastic processes. Given a filtered probability space  $(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t \in \mathcal{T}})$  a stochastic process is a collection  $(X_t)_{t \in \mathcal{T}}$  where each  $X_t$  is a random variable. A stochastic process  $(X_t)_{t \in \mathcal{T}}$  is said to be adapted to the filtration  $(\mathcal{F}_t)_{t \in \mathcal{T}}$  if every random variable  $X_t, t \in \mathcal{T}$  is  $\mathcal{F}_t$ -measurable. The interpretation of an adapted stochastic process is the following: at each time  $t$  the value  $X_t$  of the stochastic process can be derived using only information from  $\mathcal{F}_t$ , therefore the information available at time  $t$  is sufficient to know  $X_t$ .

## 2.2 State-price deflators

### 2.2.1 The discrete-time multi-period framework

Following [14], in a discrete-time multi-period framework with time set  $\mathcal{T} = \{0, 1, 2, \dots, T\}$  a state price deflator (or often referred to as stochastic discount factor) is defined as an adapted stochastic process  $\zeta = (\zeta_t)_{t \in \mathcal{T}}$  with

1.  $\zeta_0 = 1$
2.  $\zeta_t > 0$  for all  $t = 1, 2, \dots, T$
3.  $\text{Var}_t[\zeta_t] < \infty$  for any  $t \in \mathcal{T}$
4. for any basic asset  $i = 1, \dots, I$  and any  $t \in \mathcal{T}$ , the price satisfies

$$P_{it} = E_t \left[ \sum_{s=t+1}^T D_{is} \frac{\zeta_s}{\zeta_t} \right], \quad (2.2)$$

where  $P_{it}$  denotes the price of asset  $i$  at time  $t$  and  $D_{it}$  denotes the dividend paid for asset  $i$  from time  $t$  to  $t+1$ . Condition 1 is just a normalization, condition 3 is technically needed to ensure that some relevant conditional expectations exist. The pricing rule in condition 4 shows, that the price at time  $t$  is given in terms of future dividends and the state-price deflator.

When considering  $t' > t$  we get  $P_{it'} = E_{t'} \left[ \sum_{s=t'+1}^T D_{is} \frac{\zeta_s}{\zeta_{t'}} \right]$  and by rewriting the price at time  $t$  it follows that

$$\begin{aligned} P_{it} &= E_t \left[ \sum_{s=t+1}^T D_{is} \frac{\zeta_s}{\zeta_t} \right] \\ &= E_t \left[ \sum_{s=t+1}^{t'} D_{is} \frac{\zeta_s}{\zeta_t} + \sum_{s=t'+1}^T D_{is} \frac{\zeta_s}{\zeta_t} \right] \\ &= E_t \left[ \sum_{s=t+1}^{t'} D_{is} \frac{\zeta_s}{\zeta_t} + \frac{\zeta_{t'}}{\zeta_t} \sum_{s=t'+1}^T D_{is} \frac{\zeta_s}{\zeta_{t'}} \right] \\ &= E_t \left[ \sum_{s=t+1}^{t'} D_{is} \frac{\zeta_s}{\zeta_t} + \frac{\zeta_{t'}}{\zeta_t} E_{t'} \left[ \sum_{s=t'+1}^T D_{is} \frac{\zeta_s}{\zeta_{t'}} \right] \right]. \end{aligned}$$

The last part of the equation is the price of asset  $i$  at time  $t'$  and therefore we have

$$P_{it} = E_t \left[ \sum_{s=t+1}^{t'} D_{is} \frac{\zeta_s}{\zeta_t} + P_{it'} \frac{\zeta_{t'}}{\zeta_t} \right]. \quad (2.3)$$

Hence, state-price deflators make it possible to link the price of an asset to the price of the same asset at another point in time  $t'$ .

Now consider the important special case of  $t' = t + 1$  and the expression for the price simplifies to

$$P_{it} = E_t \left[ \frac{\zeta_{t+1}}{\zeta_t} (P_{i,t+1} + D_{i,t+1}) \right]. \quad (2.4)$$

As a result the price of an asset can be expressed in terms of the next period price and the next period dividend.

Defining the gross rate of return of asset  $i$  at time  $t + 1$  as

$$R_{i,t+1} = (P_{i,t+1} + D_{i,t+1}) / P_{it}$$

it follows that

$$1 = E_t \left[ \frac{\zeta_{t+1}}{\zeta_t} R_{i,t+1} \right]. \quad (2.5)$$

Rewriting these terms leads to an expression for the expected returns on the risky assets

$$1 = E_t \left[ \frac{\zeta_{t+1}}{\zeta_t} \right] E_t [R_{i,t+1}] + \text{Cov}_t \left[ \frac{\zeta_{t+1}}{\zeta_t}, R_{i,t+1} \right]$$

and thus

$$E_t [R_{i,t+1}] = \frac{1}{E_t \left[ \frac{\zeta_{t+1}}{\zeta_t} \right]} - \frac{\text{Cov}_t \left[ \frac{\zeta_{t+1}}{\zeta_t}, R_{i,t+1} \right]}{E_t \left[ \frac{\zeta_{t+1}}{\zeta_t} \right]}. \quad (2.6)$$

Introducing a risk-free asset we define the risk-free gross rate of return  $R_t^f$  as the return over the period between  $t$  and  $t + 1$ . This return will be known at time  $t$  and therefore the following equation holds

$$1 = E_t \left[ \frac{\zeta_{t+1}}{\zeta_t} R_t^f \right] = R_t^f E_t \left[ \frac{\zeta_{t+1}}{\zeta_t} \right]$$

and thus

$$R_t^f = \left( E_t \left[ \frac{\zeta_{t+1}}{\zeta_t} \right] \right)^{-1}. \quad (2.7)$$

Substituting (2.7) into (2.6) gives an expression for the excess return of asset  $i$  over the period  $t$  to  $t + 1$

$$E_t [R_{i,t+1}] - R_t^f = - \frac{\text{Cov}_t \left[ \frac{\zeta_{t+1}}{\zeta_t}, R_{i,t+1} \right]}{E_t \left[ \frac{\zeta_{t+1}}{\zeta_t} \right]}. \quad (2.8)$$

Later on we will use the following expression, which is only another rewriting of (2.8), as the covariance of the state-price deflator and the return can be written as the product of the correlation and the standard deviations.

$$\text{EER}_{i,t+1} := \text{E}_t[R_{i,t+1}] - R_t^f = -\rho_t \left[ R_{i,t+1}, \frac{\zeta_{t+1}}{\zeta_t} \right] \sigma_t[R_{i,t+1}] \frac{\sigma_t \left[ \frac{\zeta_{t+1}}{\zeta_t} \right]}{\text{E}_t \left[ \frac{\zeta_{t+1}}{\zeta_t} \right]} \quad (2.9)$$

### 2.2.2 Properties of state-price deflators

The two most important questions concerning state-price deflators are:

1. When do they exist?
2. When are they unique?

The answers to these questions are given in the following theorems<sup>1</sup>.

**Theorem 2.2** *A state-price deflator exists if and only if prices admit no arbitrage.*

As most models used in asset pricing theory assume that markets admit no arbitrage, it is possible to assume the existence of a state-price deflator.

**Theorem 2.3** *Assume prices admit no arbitrage. Then there is a unique state-price deflator if and only if the market is complete. If the market is incomplete, several state-price deflators exist.*

### 2.2.3 Lognormal state-price deflators and returns

Assuming that a state-price deflator  $\zeta$  is lognormally distributed, it follows that<sup>2</sup>

$$\text{E}_t \left[ \frac{\zeta_{t+1}}{\zeta_t} \right] = \text{E}_t \left[ e^{\ln \frac{\zeta_{t+1}}{\zeta_t}} \right] = \exp \left\{ \text{E}_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t} \right] + \frac{1}{2} \text{Var}_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t} \right] \right\}.$$

If a risk-free asset exists, the continuously compounded risk-free rate of return is given by

$$r_t^f = \ln R_t^f = -\ln \text{E}_t \left[ \frac{\zeta_{t+1}}{\zeta_t} \right] = -\text{E}_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t} \right] - \frac{1}{2} \text{Var}_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t} \right]. \quad (2.10)$$

If the state-price deflator and the gross rate of return on a risky asset  $R_{i,t+1}$  are jointly lognormally distributed, the product  $\frac{\zeta_{t+1}}{\zeta_t} R_{i,t+1}$  will also be lognormally

---

<sup>1</sup>Proofs of the theorems in discrete time can be found in [14].

<sup>2</sup>See Appendix A for details.

distributed and therefore it follows from (2.5) and (2.10) that

$$\begin{aligned}
1 &= E_t \left[ \frac{\zeta_{t+1}}{\zeta_t} R_{i,t+1} \right] \\
&= E_t \left[ e^{\ln \frac{\zeta_{t+1}}{\zeta_t} + \ln R_{i,t+1}} \right] \\
&= \exp \left\{ E_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t} + \ln R_{i,t+1} \right] + \frac{1}{2} \text{Var}_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t} + \ln R_{i,t+1} \right] \right\}.
\end{aligned}$$

Using the logarithm on both sides of the equation we get

$$\begin{aligned}
0 &= E_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t} + \ln R_{i,t+1} \right] + \frac{1}{2} \text{Var}_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t} + \ln R_{i,t+1} \right] \\
&= E_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t} \right] + \frac{1}{2} \text{Var}_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t} \right] + E_t[\ln R_{i,t+1}] + \\
&\quad + \frac{1}{2} \text{Var}_t[\ln R_{i,t+1}] + \text{Cov}_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t}, \ln R_{i,t+1} \right]
\end{aligned}$$

and substituting into (2.10) the expected log-excess rate of return is given by

$$\text{eer}_{i,t+1} = E_t[r_{i,t+1}] - r_t^f = -\text{Cov}_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t}, r_{i,t+1} \right] - \frac{1}{2} \text{Var}_t[r_{i,t+1}], \quad (2.11)$$

where  $r_{i,t+1} = \ln R_{i,t+1}$  denotes the continuously compounded rate of return on a risky asset.

## 2.3 Optimizing consumption

We assume that individuals have preferences for consumption and therefore have to plan how much they want to consume at different points in time and different states of the world. By participating in financial markets individuals are able to reallocate consumption over time and states.

In a discrete-time framework each individual has to choose a consumption process  $c = (c_t)_{t \in \mathcal{T}}$ , with  $\mathcal{T} = \{0, 1, \dots, T\}$  and  $c_t$  denoting the random consumption at time  $t$ . Participating in the financial market takes place by choosing a trading strategy  $\theta = (\theta_t)_{t=0,1,\dots,T-1}$ , with  $\theta_t$  representing the portfolio held from time  $t$  to time  $t+1$ . As  $\theta_t$  may depend on the information available at time  $t$ ,  $\theta$  is an adapted stochastic process. The individual has an endowment process  $e = (e_t)_{t \in \mathcal{T}}$ , where  $e_0$  is the initial endowment at time 0 and  $e_t$  is the possible state-dependent income received at time  $t$ .

### 2.3.1 Utility functions

Different levels of consumption are ranked by a utility function  $u : \mathbb{R}^+ \rightarrow \mathbb{R}$  and the following common properties are assumed:

- $u$  is continuous and twice continuously differentiable.
- $u$  is increasing, i.e.,  $u(x) > u(y)$  if  $x > y$ .  
This assumption implicates that individuals are "greedy", they prefer high consumption to low consumption. It follows that  $u' > 0$ .
- $u$  is concave, i.e.,  $u(\alpha x + (1 - \alpha)y) \geq \alpha u(x) + (1 - \alpha)u(y)$  for  $\alpha \in [0, 1]$ .  
This assumption models that individuals are risk-averse and prefer a sure consumption level to an uncertain one. Therefore,  $u'' \leq 0$  is needed which says that  $u$  is assumed to be a concave function.
- $u'(\infty) := \lim_{c \rightarrow \infty} u'(c) = 0$ , this means that marginal utility tends to zero when wealth tends to infinity.

The degree of risk aversion is associated with  $u''$ . One possibility to measure risk aversion are the Arrow-Pratt measures which are defined as follows:

**Definition 2.1** *The absolute risk aversion is given by*

$$ARA(c) = -\frac{u''(c)}{u'(c)}. \quad (2.12)$$

**Definition 2.2** *The relative risk aversion is given by*

$$RRA(c) = -\frac{cu''(c)}{u'(c)} = cARA(c). \quad (2.13)$$

In a discrete-time multi-period setting individuals can consume at each point in time in  $\mathcal{T} = \{0, 1, 2, \dots, T\}$ , where consumption is stochastic and therefore modelled as a stochastic process  $c = (c_t)_{t \in \mathcal{T}}$ . Hence we need to generalize the setting. We do that by defining a multi-date utility function  $U(c_0, c_1, \dots, c_T)$ .

CCAPM often assume time-additivity, which means that the utility an individual gets from consumption in one period does not directly depend on what was consumed in earlier periods or what is planned to be consumed in later periods.

**Definition 2.3** *A utility function in a discrete-time model is said to be time-additive if*

$$U(c_0, c_1, \dots, c_T) = \sum_{t=0}^T u_t(c_t), \quad (2.14)$$

where each  $u_t$  is a 'single-dated' utility function.

Additionally, it is typically assumed that  $u_t(c_t) = e^{-\delta t} u(c_t)$  for all  $t$ . In economical terms this means that the utility an individual gets from a given consumption level is basically the same for all dates, but the individual prefers to consume any given number of goods sooner than later. This is modelled by the subjective time preference rate  $\delta$ , which is assumed to be constant over time and independent of the consumption level. More impatient individuals therefore have higher  $\delta$ 's.

For a discrete-time multi-period framework we get

$$U(c_0, c_1, \dots, c_T) = \sum_{t=0}^T e^{-\delta t} u(c_t). \quad (2.15)$$

An important class of utility functions used in CCAPM are the CRRA (Constant Relative Risk Aversion) or power utility functions. This class is defined as

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad (2.16)$$

where  $c > 0$ ,  $\gamma > 0$  and  $\gamma \neq 1$ .

From  $u'(c) = c^{-\gamma}$  and  $u''(c) = -\gamma c^{-\gamma-1}$  it follows that the absolute and relative risk aversion are given by

$$ARA(c) = -\frac{u''(c)}{u'(c)} = \frac{\gamma}{c}, \quad RRA(c) = cARA(c) = \gamma. \quad (2.17)$$

The relative risk aversion parameter is therefore constant across consumption levels  $c$ . In [13] Mehra and Prescott argue, that there is evidence that most investors have a relative risk aversion above 1 and below 10.

### 2.3.2 The optimal consumption plan

An individual wants to optimize its consumption over time at time 0. Assuming time-additive utility the problem is a maximization problem of the sum of the utility of time 0 consumption and the expected utility of future consumption:

$$\max_{\boldsymbol{\theta}} u(c_0) + \sum_{t=1}^T e^{-\delta t} \mathbb{E}[u(c_t)] \quad (2.18)$$

$$\begin{aligned} \text{s.t.} \quad c_0 &\leq e_0 - \boldsymbol{\theta}_0 \mathbf{P}_0, \\ c_t &\leq e_t + D_t^\theta, \quad t = 1, \dots, T, \\ c_0, c_1, \dots, c_T &\geq 0. \end{aligned}$$

where  $D_t^\theta = \boldsymbol{\theta}_{t-1}(\mathbf{P}_t + \mathbf{D}_t) - \boldsymbol{\theta}_t \mathbf{P}_t$ . At time  $t-1$  we invested in the portfolio  $\boldsymbol{\theta}_{t-1}$ , at time  $t$  we sell the portfolio at price  $\mathbf{P}_t$  and get dividends  $\mathbf{D}_t$  for it. From time  $t$  to  $t+1$  we invest in the portfolio  $\boldsymbol{\theta}_t$  and we pay prices  $\mathbf{P}_t$  for this investment. Since we assume that individuals prefer more consumption to less, it is clear, that the budget constraints will hold as equalities. As we assumed infinite marginal utility  $u'(c)$  at  $c = 0$  the non-negativity constraints are automatically satisfied. Furthermore we assume that individuals act as price takers, hence prices are unaffected by their portfolio choices. The constraint on time  $t$  consumption is therefore given as

$$c_t = e_t + \boldsymbol{\theta}_{t-1}(\mathbf{P}_t + \mathbf{D}_t) - \boldsymbol{\theta}_t \mathbf{P}_t \quad (2.19)$$

and using (2.19) the optimization problem can be written as

$$\max_{\boldsymbol{\theta}} u(e_0 - \boldsymbol{\theta}_0 \mathbf{P}_0) + \sum_{t=1}^T e^{-\delta t} \mathbb{E}[u(e_t + \boldsymbol{\theta}_{t-1}(\mathbf{P}_t + \mathbf{D}_t) - \boldsymbol{\theta}_t \mathbf{P}_t)]. \quad (2.20)$$

The only terms involving the initial portfolio  $\boldsymbol{\theta}_0 = (\theta_{10}, \theta_{20}, \dots, \theta_{I0})^T$  is

$$u(e_0 - \boldsymbol{\theta}_0 \mathbf{P}_0) + e^{-\delta} \mathbb{E}[u(e_1 + \boldsymbol{\theta}_0(\mathbf{P}_1 + \mathbf{D}_1) - \boldsymbol{\theta}_1 \mathbf{P}_1)]. \quad (2.21)$$

The first order condition with respect to  $\theta_{i0}$  implies

$$P_{i0} = \mathbb{E} \left[ e^{-\delta} \frac{u'(c_1)}{u'(c_0)} (P_{i1} + D_{i1}) \right]. \quad (2.22)$$

More generally the first order condition with respect to  $\theta_{it}$  implies

$$P_{it} = \mathbb{E}_t \left[ e^{-\delta} \frac{u'(c_{t+1})}{u'(c_t)} (P_{i,t+1} + D_{i,t+1}) \right], \quad (2.23)$$

where  $c_t$  and  $c_{t+1}$  are the optimal consumption rates of the individual. When comparing (2.23) with (2.4) it follows that

$$\frac{\zeta_{t+1}}{\zeta_t} = e^{-\delta} \frac{u'(c_{t+1})}{u'(c_t)}. \quad (2.24)$$



When rewriting the general expression of the state-price deflator we get an expression for  $\zeta_t$

$$\zeta_t = \frac{\zeta_t}{\zeta_{t-1}} \frac{\zeta_{t-1}}{\zeta_{t-2}} \dots \frac{\zeta_1}{\zeta_0} = e^{-\delta} \frac{u'(c_t)}{u'(c_{t-1})} e^{-\delta} \frac{u'(c_{t-1})}{u'(c_{t-2})} \dots e^{-\delta} \frac{u'(c_1)}{u'(c_0)}, \quad (2.25)$$

which leads to an expression for the state-price deflator assuming time-additive utility

$$\zeta_t = e^{-\delta t} \frac{u'(c_t)}{u'(c_0)}. \quad (2.26)$$

As a result the state-price deflator can be interpreted as the individual's marginal rate of substitution between consumption at time 0 and consumption at time  $t$ , which means the willingness of the individual to substitute consumption at time 0 for consumption at time  $t$ .

When again considering power utility as in (2.16) the state-price deflator is given by

$$\zeta_t = e^{-\delta t} \left( \frac{c_t}{c_0} \right)^{-\gamma}. \quad (2.27)$$

### 2.3.3 A representative individual

Since it is much easier to analyze models with only one individual rather than many, asset pricing models often assume the existence of a representative individual. Assuming that  $L$  individuals in the economy have time-additive utility as given in (2.15) with  $u_l$  denoting the utility function of individual  $l$  the utility function  $u_{\eta,t}$  of the representative individual is defined as

$$u_{\eta,t}(C) = \sup \left\{ \sum_{l=1}^L e^{-\delta_l t} \eta_l u_l(c_l) \mid \sum_{l=1}^L c_l \leq C \right\}, \quad (2.28)$$

where  $C$  denotes aggregate consumption which is defined as  $\sum_{l=1}^L c_l$  and  $\eta$  is a vector of strictly positive numbers giving weight to the utility functions of different individuals. As individuals are assumed to be greedy the constraint holds as equality and the first order condition with respect to  $c_l$  is given by

$$\frac{\partial}{\partial c_l} \left[ \sum_{l=1}^L e^{-\delta_l t} \eta_l u_l(c_l) + \lambda_t (C - \sum_{l=1}^L c_l) \right] \stackrel{!}{=} 0$$

and it follows that

$$\begin{aligned} e^{-\delta_l t} \eta_l u'_l(c_l) - \lambda_t &= 0 \\ e^{-\delta_l t} \eta_l u'_l(c_l) &= \lambda_t. \end{aligned} \quad (2.29)$$

As  $\lambda_t$  is a constant, in an optimum the marginal utility weighted by  $\eta_l$  is the same for all  $L$  individuals in the economy.

When expressing individual consumption as a function of optimal aggregate consumption  $c_l = c_l(C)$  and using  $e^{-\delta_l t} \eta_l u'_l(c_l(C)) = \lambda$  and  $C = \sum_{l=1}^L c_l(C)$ , which implies  $1 = \sum_{l=1}^L c'_l(C)$  when differentiated with respect to  $C$ , it follows that

$$u'_{\eta,t}(C) = \sum_{l=1}^L e^{-\delta_l t} \eta_l u'_l(c_l(C)) c'_l(C) = \lambda_t \sum_{l=1}^L c_l(C)' = \lambda_t. \quad (2.30)$$

$\lambda_t$  denotes the shadow price, i.e., the marginal change of the objective function.

Supposing that there existed a vector  $\boldsymbol{\eta}$  so that individual consumption  $c_{l,t}(\omega)$  optimizes  $u_{\eta,t}(C_t)$  for all points in time  $t = \{0, 1, \dots, T\}$  and all states  $\omega \in \Omega$  with  $C_t = \sum_{l=1}^L c_{l,t}$  it follows that

$$\frac{u'_{\eta,t+1}(C_{t+1})}{u'_{\eta,t}(C_t)} = \frac{\lambda_{t+1}}{\lambda_t} = \frac{e^{-\delta_l(t+1)} \eta_l u'_l(c_{l,t+1})}{e^{-\delta_l t} \eta_l u'_l(c_{l,t})} = e^{-\delta_l} \frac{u'_l(c_{l,t+1})}{u'_l(c_{l,t})}, \quad (2.31)$$

which is the marginal rate of substitution of individual  $l$  and according to (2.24) denotes a valid state-price deflator. Therefore, the state-price deflator of individual  $l$  can be expressed in terms of the utility function of a representative individual and aggregate consumption  $C$ .

It can be shown, that a weighting vector  $\boldsymbol{\eta}$  exists if the financial market is complete, hence a representative individual exists. In this case the state-price deflator is the same for every individual  $l$  and the state price deflator of the representative individual is unique.

## Chapter 3

# The Models

### 3.1 The simple power utility model

With the instruments developed in the previous chapter it is possible to link asset pricing to consumption and to derive a first approximation for the excess return. To do so, we link expected returns to the covariance of returns and consumption as it is shown in [14] and obtain two versions of the simple power utility model.

As discussed before the marginal rate of substitution of an individual defines a state-price deflator. We assume time-additive utility with a time preference rate  $\delta$  and a utility function  $u$ .

#### 3.1.1 Version 1

Additionally to the assumptions stated above we assume that

- i) the economy has a representative individual with constant relative risk aversion. Therefore we define utility as in (2.16), i.e.

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \quad (3.1)$$

where  $C$  denotes aggregate consumption.

- ii) Future aggregate consumption growth is lognormally distributed, i.e.

$$\ln(C_{t+1}/C_t) \sim N(\bar{g}, \sigma_C^2). \quad (3.2)$$

The first assumption leads to a marginal rate of substitution given by

$$\frac{u'(C_{t+1})}{u'(C_t)} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} = \exp\left\{-\gamma \ln\left(\frac{C_{t+1}}{C_t}\right)\right\}. \quad (3.3)$$

From the second assumption it follows (see Appendix A) that

$$\mathbb{E}_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} \right] = \mathbb{E}_t \left[ \exp \left\{ -\gamma \ln \left( \frac{C_{t+1}}{C_t} \right) \right\} \right] = \exp \left\{ -\gamma \bar{g} + \frac{1}{2} \gamma^2 \sigma_C^2 \right\} \quad (3.4)$$

and

$$\frac{\sigma_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} \right]}{\mathbb{E}_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} \right]} = \sqrt{e^{\gamma^2 \sigma_C^2} - 1} \approx \gamma \sigma_C, \quad (3.5)$$

where the approximation is based on  $e^x \approx 1 + x$  for  $x \approx 0$  and  $\sigma_C$  denotes the standard deviation of consumption growth. Another approximation is needed to reach the result that is used in most literature and also for empirical analysis. Considering the first-order Taylor approximation of the function  $f(x) = x^{-\gamma}$  around 1 gives  $f(x) \approx f(1) + f'(1)(x - 1) = 1 - \gamma(x - 1)$ . Using  $x = C_{t+1}/C_t$  it follows that

$$\left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \approx 1 - \gamma \left( \frac{C_{t+1}}{C_t} - 1 \right). \quad (3.6)$$

Substituting (3.5) and (3.6) into (2.9) we get an expected excess return on a risky asset as follows

$$\text{EER}_{i,t+1} \approx \gamma \sigma_C \rho_t \left[ R_{i,t+1}, \frac{C_{t+1}}{C_t} \right] \sigma_t [R_{i,t+1}]. \quad (3.7)$$

Substituting (3.4) into (2.7) the gross risk-free rate of return over the period  $t$  to  $t + 1$  is explicitly given by

$$R_t^f = e^\delta \left( \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] \right)^{-1} = \exp \left\{ \delta + \gamma \bar{g} - \frac{1}{2} \gamma^2 \sigma_C^2 \right\}. \quad (3.8)$$

The continuously compounded risk-free rate of return is then given as

$$r_t^f = \ln R_t^f = \delta + \gamma \bar{g} - \frac{1}{2} \gamma^2 \sigma_C^2. \quad (3.9)$$

### 3.1.2 Version 2

To get the relation, that is used in empirical studies in addition to assumptions i) and ii) we assume that

- iii) the gross rate of return on the risky assets and consumption growth are simultaneously lognormally distributed.

As consumption growth is lognormally distributed the one-period state-price deflator  $\zeta_{t+1}/\zeta_t$  is also lognormally distributed

$$\frac{\zeta_{t+1}}{\zeta_t} = e^{-\delta} \frac{u'(C_{t+1})}{u'(C_t)} = e^{-\delta} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} = \exp \left\{ -\delta - \gamma \ln \left( \frac{C_{t+1}}{C_t} \right) \right\}$$

with an expectation of  $-\delta - \gamma \bar{g}$  and a variance of  $\gamma^2 \sigma_C^2$ , and therefore we can use the results of section 2.2.3.

The continuously compounded risk-free rate of return is the same as in version 1 and we get an explicit continuously compounded excess rate of return as follows:

$$\begin{aligned} \text{eer}_{i,t+1} + \frac{1}{2} \text{Var}_t[r_{i,t+1}] &= -\text{Cov}_t \left[ \ln \frac{\zeta_{t+1}}{\zeta_t}, r_{i,t+1} \right] \\ &= \gamma \sigma_C \rho_t \left[ \ln \frac{C_{t+1}}{C_t}, r_{i,t+1} \right] \sigma_t[r_{i,t+1}]. \end{aligned} \quad (3.10)$$

### 3.1.3 Asset pricing puzzles

The simple model has been exposed to numerous empirical tests. Using data from different countries Campbell shows the results of such empirical tests in [5]. These tests focus on the question whether relation (3.10) holds for stock-indices and for reasonable values of the relative risk aversion coefficient  $\gamma$ .

Table 3.1 below shows empirical results for the US and several European countries. The first column of the table headed by  $\widetilde{\text{eer}}$  gives the annualized sample estimate of the left hand side of (3.10), which is the expected excess return of the risky asset over the risk-free asset, adjusted for half of the sample variance of the risky asset. The second column shows the annualized average risk-free rate  $\tilde{r}^f$ . The third column reports the annualized standard deviation of the return on the risky asset  $\tilde{\sigma}_R$ . The fourth column headed by  $\tilde{\sigma}_C$  shows the annualized standard deviation of consumption growth and the fifth column the correlation between the log stock return and consumption growth denoted by  $\tilde{\rho}$ . The last two columns show the implied relative risk aversion coefficient  $\gamma$ . The column headed  $\gamma$  uses (3.10) to get the estimated risk aversion. The other column shows  $\gamma$  if the correlation of stock returns and consumption growth is set to the counterfactual value of 1. Campbell argues, that this correlation is hard to measure accurately as it is easily affected by measurement errors in consumption and it is very sensitive to the measurement horizon. Therefore, it could be interesting to just ignore the correlation and setting it equal to 1.

Country	Period	$\widetilde{eer}$	$\widetilde{r}^f$	$\widetilde{\sigma}_R$	$\widetilde{\sigma}_C$	$\widetilde{\rho}$	$\gamma$	$\hat{\gamma}$
USA	74Q2-98Q3	8.071	0.896	15.271	1.071	0.205	240.6	49.3
FRA	73Q2-98Q3	8.308	2.715	23.175	2.922	-0.093	neg	12.3
GER	78Q4-97Q3	8.669	3.219	20.196	2.447	0.029	599.5	17.5
ITA	71Q2-98Q1	4.687	2.371	27.068	1.665	-0.006	neg	10.4
CH	82Q2-98Q4	14.898	1.393	21.878	2.123	-0.112	neg	32.1
NL	97Q2-98Q3	11.421	3.377	16.901	2.510	0.032	850.0	26.9
SWE	70Q1-99Q2	11.539	1.995	23.518	1.851	0.015	1713.2	26.5
UK	70Q1-99Q1	9.169	1.255	21.198	2.511	0.093	186.0	17.2

Table 3.1: The equity premium puzzle in selected countries

Several facts hold true over all countries:

1. The average real excess return on stock is high. Except for Italy this number is at least 8.1 or even higher. Furthermore, stock returns are volatile with a standard deviation between 15.3% and 27.1%.
2. The risk-free rate is low. In the considered countries it is mostly below 3%. The standard deviation of the risk-free asset is smaller than 3% for quaterly data<sup>1</sup>, therefore the real interest rate is much less volatile than the returns on a risky asset.
3. Real consumption growth has a low volatility, the standard deviation lies between 1.071% and 2.922%.
4. The relative risk aversion is either unrealistically high or negative due to a negative correlation of returns on the risky asset and consumption growth. Even when using a hypothetical correlation  $\rho$  equal to one,  $\hat{\gamma}$  is still too high when thinking of a realistic value between 1 and 10.

The figures derived by Campbell lead to different "asset pricing puzzles" as the results of the model are often far from being realistic when compared to historical averages. The two best know puzzles are the "equity premium puzzle" and the "risk-free rate puzzle", which will now be discussed briefly.

### The equity premium puzzle

The question, why the average real stock return is so high compared to the average short-term interest rate, is called the "equity premium puzzle" and was first analysed by Mehra and Prescott in [13]. As the average stock return is high and the interest rate is low, the expected excess return on stock is high. This high equity premium can only be explained by a very high relative

<sup>1</sup>See [5], Table 1 for details.

risk aversion coefficient as shown in Table 3.1. For reasonable values of risk aversion ranging from 1 to 10 the expected excess return is far too low when compared with the historical average.

### The risk-free rate puzzle

Even when allowing investors to have a high relative risk aversion coefficient the simple model is not realistic as a high  $\gamma$  can lead to the "risk-free rate puzzle". Very risk-averse investors have a strong desire to transfer wealth from periods with high consumption to periods with low consumption. As consumption has been growing steadily over time, a high risk aversion makes investors want to borrow in order to reduce the discrepancy between future and present consumption, and this demand increases the risk-free rate. Considering again (3.9) given by

$$r_t^f = \delta + \gamma \bar{g} - \frac{1}{2} \gamma^2 \sigma_C^2$$

it follows, that a low risk-free rate can only be obtained with a low or even negative time preference rate  $\delta$  that reduces the desire to borrow and therefore lowers the risk-free rate. It can be noticed that with a high  $\gamma$  and a positive mean growth rate  $\bar{g}$  the linear term on the right side of (3.9) is high. Of course,  $\gamma$  also appears in the quadric term of (3.9) and it can happen, that this term dominates the linear term if  $\gamma$  becomes high enough. The quadric term can be interpreted as modelling precautionary savings. Risk-averse investors have a desire to hold precautionary savings, as future consumption is uncertain. This could lower the risk free rate, as savings work against the desire to borrow. However, unrealistically high values of  $\gamma$  would be necessary to achieve a reasonable value of the quadric term as the low standard deviation of consumption growth  $\sigma_C$  depreciates the value of the term.

### 3.2 Abel's relative consumption model or "Catching up with the Joneses"

Abel's relative consumption model is an extension of version 2 of the simple model. It differs from the simple model in two ways. First we need an additional assumption to assumptions i), ii) and iii) of the simple model, which says that

- iv) consumption growth is independent and identically distributed.

Second, the representative individual maximizes a utility function now not only depending on the consumption level  $c_t$  but also on a benchmark  $X_t$ . This benchmark is an "external habit" depending on past aggregate consumption which is supposed to be unaffected by an individual's decision. The interpretation of such an external habit is that of  $X_t$  being the standard of living in the economy where individuals care about the lagged value of aggregate consumption. Therefore this model is often called the "Catching up with the Joneses". The general setup of Abel's model is described in [1] and [2], [5] introduces an external habit as the special case of  $X_t$  depending on one lag of aggregate consumption which is not influenced by a person's own past consumption.

The utility function of the representative individual is now given by

$$u(C_t, X_t) = \frac{1}{1-\gamma} \left( \frac{C_t}{X_t} \right)^{1-\gamma}, \quad \gamma > 0 \quad (3.11)$$

and the external habit is defined as

$$X_t = C_{t-1}^\kappa, \quad (3.12)$$

where  $\kappa$  measures the importance of the the consumption-benchmark ratio and is assumed to be a positive number in [1]. In [2] Abel restricts  $\kappa$  to  $0 < \kappa \leq 1$ . For  $\kappa = 0$  we are back in the simple model without a benchmark level of consumption.

Since  $u'(C_t, X_t) = (C_t/X_t)^{-\gamma} (1/X_t)$  and  $u''(C_t, X_t) = \gamma (C_t/X_t)^{-1}$  the relative risk aversion is still the constant  $\gamma$ .

It follows that the marginal rate of substitution can be stated as

$$\frac{u'(C_{t+1}, X_{t+1})}{u'(C_t, X_t)} = \frac{\left( \frac{C_{t+1}}{X_{t+1}} \right)^{-\gamma} \frac{1}{X_{t+1}}}{\left( \frac{C_t}{X_t} \right)^{-\gamma} \frac{1}{X_t}} = \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{X_t}{X_{t+1}} \right)^{1-\gamma}.$$

Using lagged aggregate consumption as a habit like in (3.12) we get



$$\frac{u'(C_{t+1}, X_{t+1})}{u'(C_t, X_t)} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{C_t}{C_{t-1}}\right)^{\kappa(\gamma-1)} \quad (3.13)$$

and as consumption growth is lognormally distributed the one-period state-price deflator is still lognormally distributed as well and is given by

$$\begin{aligned} \frac{\zeta_{t+1}}{\zeta_t} &= e^{-\delta} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{C_t}{C_{t-1}}\right)^{\kappa(\gamma-1)} \\ &= \exp \left\{ \delta - \gamma \ln \left(\frac{C_{t+1}}{C_t}\right) + \kappa(\gamma-1) \ln \left(\frac{C_t}{C_{t-1}}\right) \right\}. \end{aligned} \quad (3.14)$$

Using again relation (2.10) it follows that

$$\begin{aligned} E_t \left[ \frac{\zeta_{t+1}}{\zeta_t} \right] &= E_t \left[ \exp \left\{ -\delta - \gamma \ln \left(\frac{C_{t+1}}{C_t}\right) + \kappa(\gamma-1) \ln \left(\frac{C_t}{C_{t-1}}\right) \right\} \right] \\ &= \exp \left\{ -\delta - \gamma \bar{g} + \frac{\gamma^2 \sigma_C^2}{2} \right\} \exp \left\{ \kappa(\gamma-1) \ln \left(\frac{C_t}{C_{t-1}}\right) \right\} \end{aligned} \quad (3.15)$$

as the ratio  $\ln(C_t/C_{t-1})$  is already known at time  $t$  and the average risk-free rate is therefore given by

$$r_t^f = \delta + \gamma \bar{g} - \frac{1}{2} \gamma^2 \sigma_C^2 - \kappa(\gamma-1) \bar{g}. \quad (3.16)$$

The expected excess return on the risky asset follows as

$$\text{eer}_{i,t+1} + \frac{1}{2} \text{Var}_t[r_{i,t+1}] = \gamma \sigma_C \rho_t \left[ \ln \frac{C_{t+1}}{C_t}, r_{i,t+1} \right] \sigma_t[r_{i,t+1}], \quad (3.17)$$

which is the same as in (3.10), hence the expected excess return is not affected by this external habit.

Nevertheless, an external habit can help to explain asset pricing puzzles. Comparing (3.9) and (3.16) a positive  $\kappa$  reduces the risk-free rate because of the term  $-\kappa(\gamma-1)\bar{g}$ . As a result, higher values of the relative risk aversion  $\gamma$  can supposably be used to solve the equity premium puzzle without encountering the risk-free rate puzzle.

In chapter 6 the results for Austrian data and both models are described. For the simple model, a row for Austria was added to Table 3.1 and the figures were then compared to the other countries. This gave a first approximation for the parameter  $\gamma$  before it was formally estimated using the generalized method of moments (GMM) described in chapter 5. It was checked whether the equity premium puzzle and the risk-free rate puzzle exist in Austrian data and whether the Abel model can help to resolve the puzzles.

## Chapter 4

# Data and Tests

This chapter contains a description of the Austrian data used in this empirical study, the tests the data was exposed to and the generalized method of moments (GMM), which was used to estimate the parameters of the CCAPM.

### 4.1 Description of Austrian data

As it is crucial for the outcome of the analysis to have "good" data plenty of time was invested in order to obtain the most appropriate time series. Nevertheless, the used time series, especially the data for private consumption, are not perfect in several ways:

- It was not possible to obtain quarterly private consumption data for durable goods. As these goods offer consumption "services" beyond the period of purchase, it would have been more appropriate to take them not into account and thus to deduct their value from total consumption.
- Private consumption data is reported infrequently (quarterly) relative to financial data (daily). Furthermore, the quarterly time series for private consumption were available only from the first quarter of 1988 (1Q1988) onwards, which limits the observation period considerably.
- The time series for the population growth in Austria was only available at annual terms.

The Austrian data used to carry out this empirical study consists of 7 basic time series which can be found in Table 4.1. The sample period was chosen from 1Q1989 to 4Q2009 as only for this period of time all the necessary data was available. The time series were obtained from Thomson Datastream database, the database of the WIFO<sup>1</sup> and the webpage of Statistik Austria<sup>2</sup>.

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<sup>1</sup> *Österreichisches Institut für Wirtschaftsforschung*; Austrian Institute of Economic Research; [www.wifo.ac.at](http://www.wifo.ac.at)

<sup>2</sup> Statistics Austria - The Information Manager; [www.statistik.at](http://www.statistik.at)

Name	Basis <sup>3</sup>	Source	Description
ATX	D	Datastream	Austrian Trade Index
MV	D	Datastream	The market value is the sum of the share price multiplied by the number of ordinary shares in issue for each constituent and is given in Millions of EUR.
DIV	D	Datastream	The dividends are expressed as a percentage of the market value.
VIB	D	Datastream	For 1Q1989 - 4Q1998 the Vienna Interbank Offered Rate (Vibor) is used; as it was replaced by the Euro Interbank Offered Rate (Euribor) in 1999 the Euribor is used for 1Q1999 - 4Q2009.
POP	Y	Statistik Austria	The population is given as the population at the begin of the year.
CPI	M	Statistik Austria	The consumer price index is based on 1986.
CSP	Q	WIFO	The total real valued private consumption with services is given in Millions of EUR and based on prices in 2005.

Table 4.1: Basic time series used

Most financial data such as the stock price, market value, dividends or the risk-free rate was available as daily data. In order to obtain quarterly data the instructions described in the Data Appendix of [5] were followed. Private consumption was already given in quarterly, real terms based on the year 2005, however, the data was not seasonally adjusted. Most financial and consumption data needed to be adjusted from inflation by using the consumer price index based on 2005 and 1990. Population was given in annual terms and was transformed to quarterly data.

The adjustments were implemented in R and *"gretl"*, the source codes conducting the necessary calculations can be found in Appendix B. Figures 4.1 to 4.11 illustrate the time series used.

#### Austrian Traded Index - ATX

In order to derive quarterly data from daily data the monthly averages were calculated and the averages of the months March, June, September and December were used as end-of-quarter values. Figure 4.1 illustrates the history of this Austrian price index.

<sup>3</sup>D = daily, M = monthly, Y = yearly

**Market Value - MV**

The market value is only necessary to calculate quarterly dividends. To do so the end-of-quarter values are derived like the ATX end-of-quarter values by using the average of the corresponding months. The movement of the market value is depicted in Figure 4.2.

**Dividends on the ATX - DIV**

The dividends were given in percentage of the market value and were calculated by Datastream as a moving average of the dividends of the previous 11 months and the current month. In order to obtain quarterly data the percentage was multiplied with the corresponding market value. It was assumed that the dividends had been approximately constant during the last 12 months. Then, monthly dividends were obtained by using the corresponding end-of-month value. Finally, quarterly dividends were computed by adding up monthly dividends of the corresponding months. The dividends on the ATX are illustrated in Figure 4.3.

**Vibor/Euribor - VIB**

The quarterly risk-free rate was derived by computing the averages of the daily quotations of the Vienna Interbank Offered Rate from 1Q1989 to 4Q1998 and the averages of the daily quotations of the Euro Interbank Offered Rate from 1Q1999 to 4Q2009. The average annual rates were transformed in quarterly rates and are shown in Figure 4.4.

**Population - POP**

Data concerning population was given as annual data and was turned into quarterly data by assuming that growth had been constant over the year. The growth in population is depicted in Figure 4.5.

**Consumer Price Index - CPI**

In order to adjust the ATX, Vibor as well as consumption from inflation, the consumer price index was needed to be based on the year 2005 as real consumption is based on 2005. Furthermore, the given reference values of the results of the simple model are based on 1990. Therefore, the time series based on 1986 had been transformed so that it was based on the year 2005 as well as 1990. The time series based on 1990 is shown in Figure 4.6.

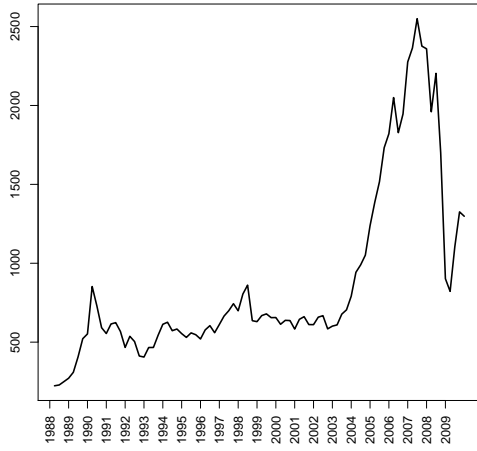


Figure 4.1: ATX-1Q88-4Q09

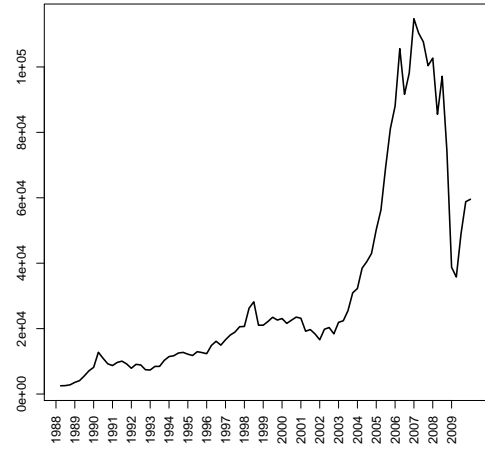


Figure 4.2: Market value-1Q88-4Q09

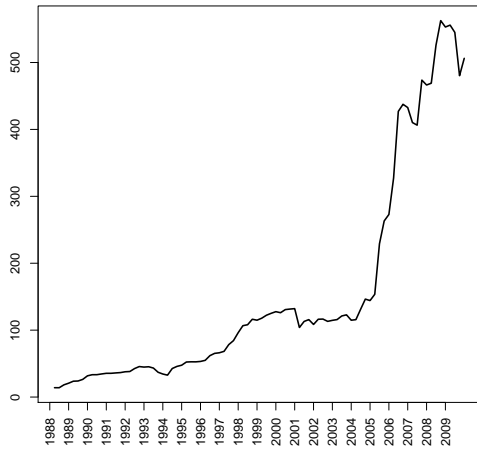


Figure 4.3: Dividends-1Q88-4Q09



Figure 4.4: Vibor/Euribor-1Q89-4Q09

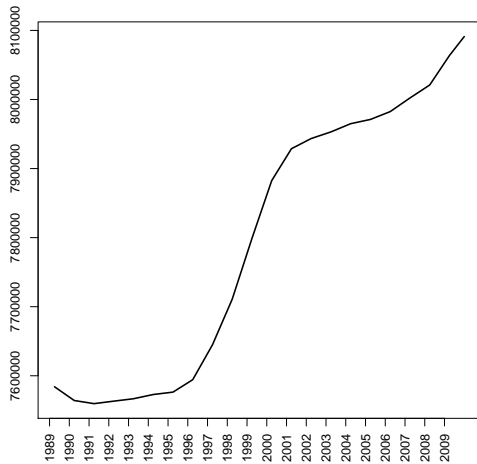


Figure 4.5: Population-1Q89-4Q09

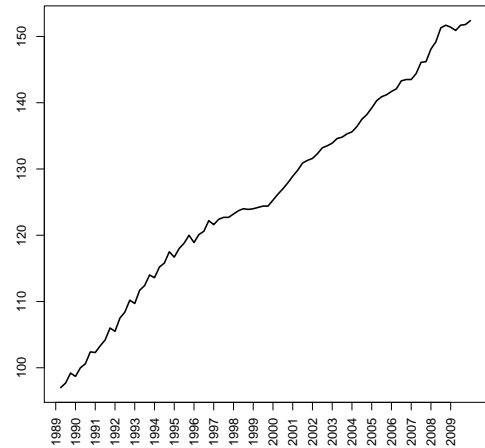


Figure 4.6: CPI-1Q89-4Q09

### Consumption - CPS

To obtain a seasonally adjusted time series for consumption, the data was adapted by using the Census X-12-ARIMA method<sup>4</sup>, which is included in the software package "*gretl*". It can be noted from Figure 4.8, which shows the original per capita consumption, that the mode of calculation of private consumption was changed in 1996 due to the introduction of the European System of Accounts (ESA 95) in 1995, which has since then been compulsory for all European membership countries. The seasonal adjustment of aggregate private consumption derived by "*gretl*" is depicted in Figure 4.7.

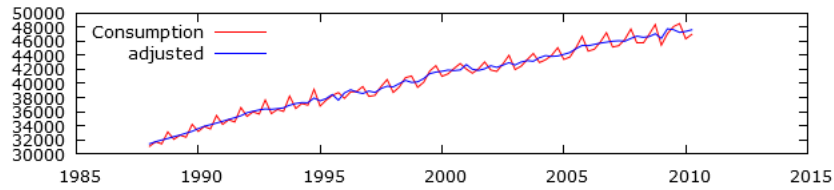


Figure 4.7: Seasonal adjustment of aggregate private consumption

Per capita consumption was then computed by dividing quarterly real consumption by population numbers. The original time series for per capita consumption, this means before seasonal adjustment, is shown in Figure 4.8. Using this data the original time series for consumption growth based on the preceding quarter ( $C_{t+1}/C_t$ ) was derived and is shown in Figure 4.10, where once again the change in the calculation system for private consumption can be noted. Figures 4.9 and 4.11, however, depict the time series finally used in this empirical study, where oscillation is much smaller when compared to the original consumption data.

Finally, Table 4.2 reports summary statistics for the basic time series. For the observations from 1Q1989 to 4Q2009, mean, standard deviation (Std.Dev), minimum, maximum and the first-order autocorrelation are stated<sup>5</sup>.

Name	Mean	Std.Dev	Min	Max	Autocorr.
ATX	874.863	564.378	223.350	2549.799	0.953
MV	31445.200	30530.920	2519.870	114746.200	0.965
DIV	154.691	160.234	13.946	562.584	0.960
VIB	1.093	0.590	0.088	2.322	0.962
CSP	500.785	43.406	414.780	570.825	0.960

Table 4.2: Summary statistics for basic time series

<sup>4</sup>See <http://www.census.gov/srd/www/x12a> for further details.

<sup>5</sup>MV is given in Mio EUR, VIB as a quarterly rate and all other positions in EUR.

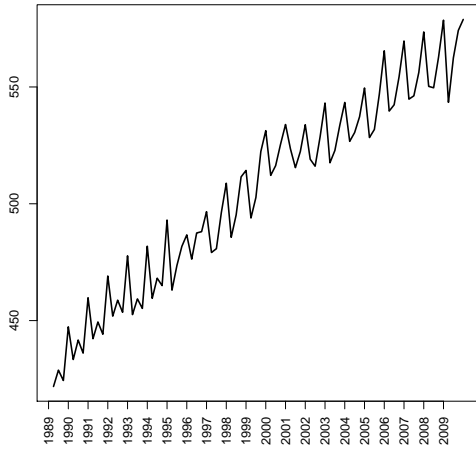


Figure 4.8: Per capita consumption - 1Q89-4Q09

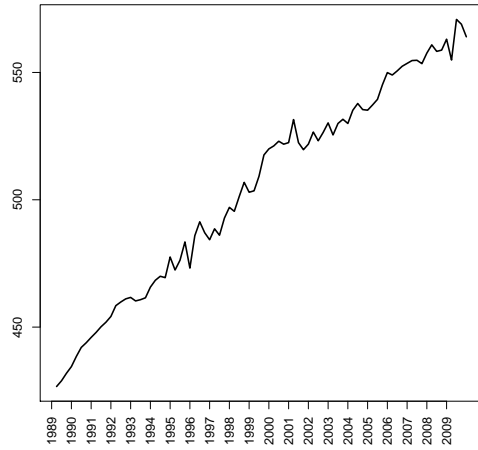


Figure 4.9: Per capita consumption seasonally adjusted - 1Q89-4Q09

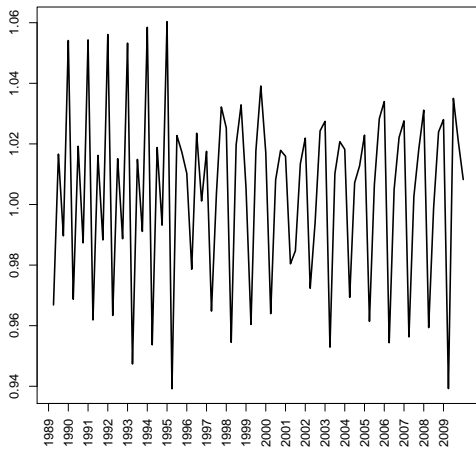


Figure 4.10: Consumption growth - 1Q89 - 4Q09

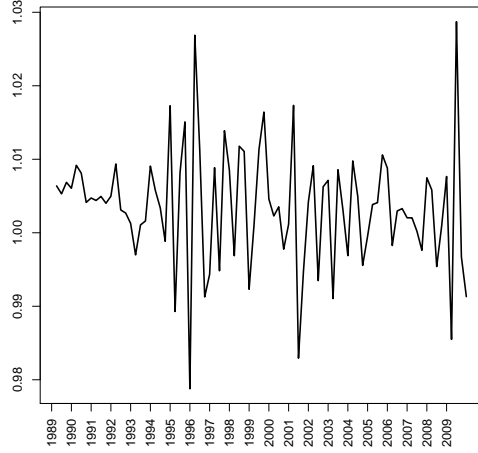


Figure 4.11: Consumption growth seasonally adjusted - 1Q89-4Q09

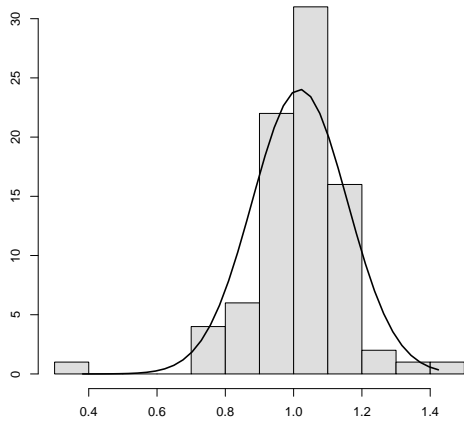


Figure 4.12: Histogram and density function for returns on the ATX

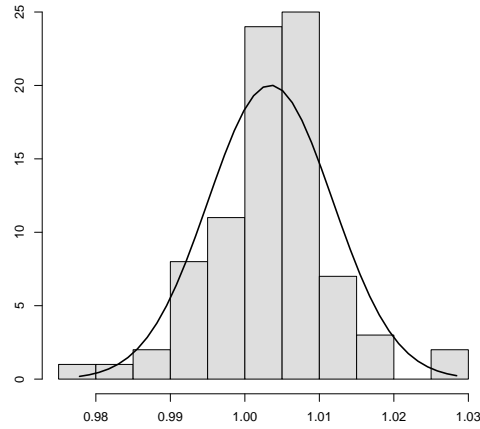


Figure 4.13: Histogram and density function for consumption growth

## 4.2 Testing the data

The data used in this empirical study was exposed to several tests. The different models and methods used demand various assumptions to deliver significant and reliable results. The testing in this section was meant to show which assumptions hold true and, on the other hand, which time series were not ideal when testing CCAPM. The tests were done using R and "gretl".

### 4.2.1 Tests on a lognormal distribution

The simple model assumes that consumption growth is lognormally distributed, i.e.

$$\ln(C_{t+1}/C_t) \sim N(\bar{g}, \sigma_C^2).$$

In version 2 of the simple model, it is additionally assumed that returns on risky assets and consumption growth are jointly lognormally distributed. If a vector of two random variables is bivariate normally distributed then it follows that each random variable itself is univariate normally distributed. Hence, log-returns on the ATX and log-consumption growth were tested. If they do not come from a univariate normal distribution, they can not be jointly lognormally distributed.

As a first step graphical techniques to evaluate the goodness of fit were used. Histograms provide some insights on the shape of a distribution, therefore the probability density function of a normal distribution with the empirical mean and standard deviation and the histogram of the data were drawn together in one plot, which is depicted in Figures 4.12 and 4.13.



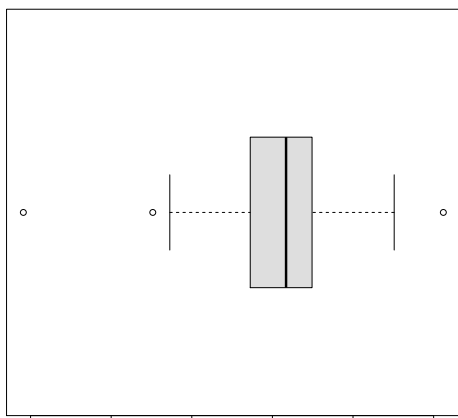


Figure 4.14: Boxplot for returns on the ATX

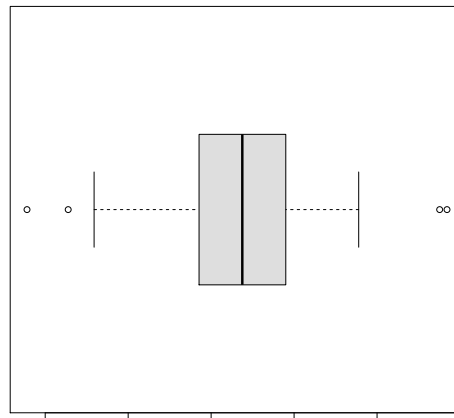


Figure 4.15: Boxplot for consumption growth

Another graphical way is to draw boxplots of the data which are shown in Figures 4.14 and 4.15.

Using these first impressions of the data depicted in the histograms and boxplots, a normal distribution of log-returns on the ATX and log-consumption growth is not very likely. In the histograms too many observations are located in the range near the empirical mean. Additionally, the data on returns on the ATX shows discrepancies at the tails. The consumption growth rates do not appear to be symmetrically distributed. The boxplots lead to a similar conclusion. The median of a normal distribution is supposed to be in the middle of the box, the whiskers should be of the same length and the outliers in Figure 4.14 indicate a deviation from the assumed normal distribution.

In a next step goodness of fit tests were used in order to test if it is reasonable that the data comes from a specific distribution. The null and alternative hypotheses are given as:

$H_0$ : Sample data comes from the stated distribution

$H_1$ : Sample data does not come from the stated distribution

The R package "nortest"<sup>6</sup> was used for testing whether the data comes from a normal population. This package includes 5 different tests for normality, namely the Anderson-Darling test (AD), Cramer-von Mises test (CvM), Lilliefors (or Kolmogorov-Smirnov) test (L), Pearson chi-square test (P) and the Sharpiro-Francia test (SF).

<sup>6</sup>Further details on the package "nortest" can be found on <http://cran.r-project.org/web/packages/nortest/nortest.pdf>

Return on ATX			Consumption Growth		
Test	Test Statistic	p-Value	Test	Test Statistic	p-Value
AD	1.3122	1.96e-03	AD	0.7862	3.98e-02
CvM	0.212	3.67e-03	CvM	0.1314	4.11e-02
L	0.0994	3.94e-02	L	0.0848	1.43e-01
P	12.286	3.34e-01	P	14.2857	2.18e-01
SF	0.9122	7.78e-05	SF	0.9646	2.25e-02

Table 4.3: Results of tests on lognormality

Considering the returns on the ATX the null hypothesis is rejected by all tests except the Pearson chi-square test on all conventional significance levels and the Lilliefors test at a level of significance of 0.01. The true p-value for the Pearson chi-square test lies somewhere between 0.2660 and 0.4314<sup>7</sup>, however it is recommended to prefer the results of the other tests as these are more reliable. For the consumption data the Pearson chi-square test and the Lilliefors test do again not reject the null hypothesis on conventional significance levels. For the other tests the null hypothesis is not rejected on the 0.01 significance level, but is rejected on the 0.1 and 0.05 significance levels. Hence, the tests provide evidence for the impression gained by the histograms and boxplots that the data does not come from a lognormal distribution. Furthermore, the tests fail to provide support for the assumption, that returns on the ATX and consumption growth are jointly lognormally distributed, as it would be needed in version 2 of the simple model.

#### 4.2.2 Tests on stationarity

In chapter 5 the generalized method of moments was used to estimate the parameters  $\delta$  and  $\gamma$ . The most important assumption in order to be able to apply GMM is the stationarity of the random variables used<sup>8</sup>. A stochastic process  $(x_t)_{t \in \mathcal{T}}$  is said to be weakly stationary if  $E[x_t^2] < \infty$  and

$$E[x_t] = \mu \quad (4.1)$$

$$\text{Cov}[x_t, x_{t+k}] = \gamma_k. \quad (4.2)$$

In order to test the respective time series whether they are stationary or not, the Augmented Dicky-Fuller test (ADF) and the KPSS test for stationarity (KPSS) were used. Both tests are included in the software package *"gretl"*, the definitions and descriptions are taken from [10] and [8].

<sup>7</sup>See <http://cran.r-project.org/web/packages/nortest/nortest.pdf>, pages 6-7 for details.

<sup>8</sup>All assumptions are covered in depth in [9].

In chapter 5 relation (2.5) will be used to set up the necessary moment conditions, therefore consumption growth as well as the returns on the ATX and the risk-free rate need to satisfy the stationarity assumption. Considering Figures 4.4, 4.11 and 4.16, returns on the ATX and consumption growth are likely to be stationary, the risk-free rate is supposedly not stationary.

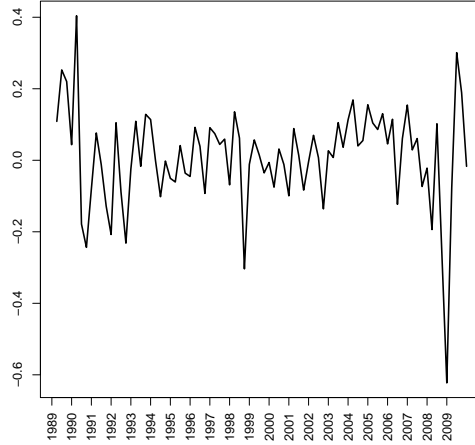


Figure 4.16: Log-returns on the ATX - 1Q88-4Q09

### ADF test

The ADF test is a test for the null hypothesis that a time series sample has a unit root. The model to which this test was applied to uses an autoregressive process of order  $p$  which is defined as follows:

**Definition 4.1** A  $p$ -th order autoregressive process ( $AR(p)$ ) satisfies the  $p$ -th order stochastic difference equation

$$y_t = c + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \varepsilon_t, \quad (4.3)$$

where  $c$  is a constant,  $\phi_p \neq 0$  and  $(\varepsilon_t)$  is a white noise error term, i.e.  $E[\varepsilon_t] = 0$ ,  $E[\varepsilon_t^2] = \sigma^2 > 0$  and  $E[\varepsilon_t \varepsilon_{t-j}] = 0$  for  $j \neq 0$ .

If the stability condition

$$\phi(z) = (1 - \phi_1 z^1 - \phi_2 z^2 - \dots - \phi_p z^p) \neq 0 \quad \forall |z| \leq 1 \quad (4.4)$$

holds, i.e., there are no roots on the unit circle, the process  $y_t$  is the unique weak-stationary solution to these stochastic difference equations given in (4.3). In contrast, if there is a unit roots ( $z = 1$ ), the equation has no weak-stationary solution, the solutions is then a so-called "unit-root processes" where the first

differences of the process rather than the process itself is stationary.

The ADF test now uses this methodology and tests whether  $\phi(z)$  has a unit root ( $z = 1$ ), which leads to non-stationarity, or not, which means that the process is stationary. Therefore, the null and alternative hypothesis are given as:

$H_0$ :  $\phi(1) = 0$ , i.e., the process has a unit root ( $z = 1$ )

$H_1$ :  $\phi(1) > 0$ , i.e., the process has no unit root and is therefore stationary

The following model is used in "gretl" to compute the ADF test, this is the  $t$ -statistic on  $\beta$  in the following expression

$$\Delta y_t = \mu_t + \beta y_{t-1} + \sum_{i=1}^{p-1} \gamma_i \Delta y_{t-i} + \varepsilon_t. \quad (4.5)$$

$(p - 1) := \bar{p}$  is the lag order,  $\gamma_i$  are the coefficients and  $(\varepsilon_t)$  is the error term as defined above. There are several forms for  $\mu_t$  namely  $\mu_t = 0$  (F1),  $\mu_t = \mu_0$  (F2) resulting in  $y_t$  being a AR(p) process with mean not equal to zero if the null hypothesis is rejected, and  $\mu = \mu_0 + \mu_1 t$  (F3) allowing for a linear trend.

As a matter of fact the condition  $\beta = (<) 0$  is equivalent to the stationarity condition  $\phi(1) = (>) 0$ , hence the null hypothesis  $H_0 : \beta = 0$  meaning non-stationarity was tested versus the alternative hypothesis  $H_1 : \beta < 0$  indicating a stationary AR( $\bar{p}$ ) process.

The number of lags  $\bar{p}$  was automatically determined in "gretl" by setting the lag order to a high number, then a sequential procedure was started which decremented  $\bar{p}$  until the  $t$ -statistic for the estimated parameter  $\gamma_{\bar{p}}$  was high enough.

As the alternative hypothesis is given by  $H_1 : \beta < 0$ , large negative values of the test statistic lead to rejection of the null hypothesis. The critical values for different levels of significance are given in the following table<sup>9</sup>.

Quantile	CV for F1	CV for F2	CV for F3
1%	-2.59056	-3.50906	-4.06996
5%	-1.94404	-2.89588	-3.46321
10%	-1.61775	-2.58485	-3.15776

Table 4.4: Critical values for the ADF test

<sup>9</sup>The critical values are calculated as recommended in [12] via the formula  $\beta_\infty + \beta_1/T + \beta_2/T^2$  with  $T = 84$  using the  $\beta$ -values stated in Table 1.

Using the return on the ATX data 3 lags were used in order to derive the results of the ADF test. For the risk-free rate 4 lags were used and for the consumption growth time series 6 lags were necessary.

ADF test - return on ATX				ADF test - return on VIB			
	Statistic	p-Value	$\hat{\beta}$		Statistic	p-Value	$\hat{\beta}$
F1	-0.379	5.48e-01	-0.006	F1	-1.128	2.36e-01	-8.06e-005
F2	-7.095	1.82e-01	-0.954	F2	-1.454	5.57e-01	-1.72e-02
F3	-7.041	1.89e-009	-0.956	F3	-2.629	2.67e-01	-5.70e-02

ADF test - con. growth			
	Statistic	p-Value	$\hat{\beta}$
F1	-0.404	5.38e-01	-0.001
F2	-10.732	1.43e-021	-1.879
F3	-11.193	8.33e-025	-1.942

Table 4.5: Results of the Augmented Dickey-Fuller test

Table 4.5 shows that all test statistics are negative, however, the test statistics for F1 are greater than the critical values in Table 4.4, hence the null hypothesis is accepted for all three time series on the conventional significance levels. For F2 and F3 the null hypothesis is rejected for returns on the ATX and consumption growth, i.e.  $\phi(z)$  has no unit roots. This leads to the conclusion, that these time series are assumed to be stationary and seem to have a mean not equal to zero. For consumption growth the statistic of F3 shows the smallest value which is an indicator for a linear trend in the data. The null hypothesis is accepted for the risk-free rate in both cases at conventional significance levels. The same conclusions follow when considering the estimate value  $\hat{\beta}$  of  $\beta$ . For F1 the values are pretty close to zero, the same can be noted for the risk-free rate when looking at F2 and F3. Considering the returns on the ATX and consumption growth  $\hat{\beta}$ , differs significantly from zero for F2 and F3 and therefore the null hypothesis is rejected for these cases.

Hence, the consumption growth process as well as the the risk-free rate process fail to satisfy the stationarity assumption needed for GMM.

### KPSS test

The KPSS test differs from the ADF test in that way, that the null hypothesis is the case of stationarity and the alternative hypothesis concerns the unit root option. The basic model (BM) is simple

$$\Delta y_t = \varepsilon_t - \nu \varepsilon_{t-1}, \quad (4.6)$$

where  $\nu$  is a constant and  $\varepsilon_t$  is a stationary process with mean zero and a variance  $\sigma$ . The null and alternative hypothesis are given by

$H_0 : \nu = 1$ , i.e., the process  $y_t = \nu_0 + \varepsilon_t$  is stationary

$H_1 : \nu < 1$ , i.e., the process  $y_t$  is not stationary

and the corresponding test statistic can be stated as

$$\eta = \frac{1}{T^2} \sum_{t=1}^T \frac{\hat{S}_t^2}{\hat{\sigma}_S^2} \quad (4.7)$$

with

$$\begin{aligned} e_t &= y_t - \bar{y}_T \\ \hat{S}_t &= \sum_{i=1}^t e_i \\ \hat{\gamma}_k &= \frac{1}{T} \sum_{i=1}^{T-k} e_{i+k} e_i \\ \hat{\sigma}_S^2 &= \hat{\gamma}_0 + 2 \sum_{j=1}^{I-1} w_j(I) \hat{\gamma}_j. \end{aligned}$$

$I = 4(T/100)^{1/4}$  was used and the weights for the estimation of  $\hat{\sigma}_S^2$  are given as  $w_j(I) = 1 - j/I$ . Under the null hypothesis the test statistic  $\eta$  has a well-defined but non-standard asymptotic distribution which has been tabulated by simulation and is stated in Table 4.6 for selected quantiles.

$$\eta \longrightarrow \int_0^1 V^2(s) ds,$$

where the process  $(V(s))_{s \in [0,1]}$  denotes a standard Brownian Bridge.

Generalizing this concept to the case where  $y_t$  is stationary around a deterministic trend the model (TM) is given by

$$\Delta y_t = \nu_1 + \varepsilon_t - \nu \varepsilon_{t-1}. \quad (4.8)$$

The null and alternative hypothesis are then given by:

$H_0 : \nu = 1$ , i.e., the process  $y_t = \nu_0 + \nu_1 t + \varepsilon_t$  is trendstationary

$H_1 : \nu < 1$ , i.e., the process  $y_t$  is not stationary.

The test statistic and the parameter estimates are the same as in (4.7), only  $e_t$  is defined as the residual from an ordinary least squares (OLS) regression of  $y_t$  on a constant and a linear trend:

$$e_t = y_t - \hat{\nu}_0 - \hat{\nu}_1 t.$$

Similarly to the first case the asymptotic distribution is given by an integral

$$\eta \longrightarrow \int_0^1 V_2^2(s) ds,$$

where the process  $(V(s))_{s \in [0,1]}$  is the so called second level Brownian Bridge.

For both cases  $H_0$  is rejected if the test statistic  $\eta$  is bigger than the corresponding critical values (CV), which are provided for the 90%, 95% and 99% quantiles by "*gretl*"<sup>10</sup>:

Sign.Level	CV for BM	CV for TM
1%	0.732	0.215
5%	0.467	0.148
10%	0.350	0.120

Table 4.6: Critical values for the KPSS test

Using the same number of lags as used for the ADF test the following results were obtained for the basic model and the trend model:

Time series	$\eta$ for BM	$\eta$ for TM
Return on the ATX	0.075	0.070
Risk-free rate	1.346	0.285
Consumption growth	0.392	0.044

Table 4.7: Results of the KPSS test

Comparing the test statistic  $\eta$  to the critical values for returns on the ATX and consumption growth, it is smaller for both models and both time series, hence the null hypothesis is accepted. Again it can be noted that consumption growth is likely to have a deterministic trend as the test statistic for the trend model is significantly smaller than the one for the basic model. The null hypothesis is rejected for the risk free rate in both models. Again consumption growth and the risk-free rate fail to satisfy the stationarity assumption for GMM.

In conclusion, both tests support the assumption that returns on the ATX follow a stationary process, hence the necessary assumption for GMM used in the next chapter is satisfied for this time series. Consumption growth seems to follow a trendstationary process and there is no evidence for a stationary risk-free rate. Therefore, both tests fail to provide evidence for stationarity of consumption growth and the risk-free rate as it would be needed for GMM.

<sup>10</sup>The critical values are computed via the method presented by Sephton in [15].

## Chapter 5

# Parameter Estimation

### 5.1 The Generalized Method of Moments

The generalized method of moments (GMM) as described by Hansen in [9] is used to formally estimate the parameters and evaluate the models considered in this analysis. The parameters are picked in a way that minimizes a weighted sum of squared pricing errors. The fit of the model can then be tested with Hansen's  $J_T$  test by looking at the sum of squared pricing errors. This short summary on GMM for asset pricing models is taken from [7].

#### General setup

Denoting the one-period state-price deflator  $\zeta_{t+1}/\zeta_t$  as a function of several parameters given in a  $1 \times q$  parameter vector  $\theta$ , we have  $\zeta_{t+1}/\zeta_t = m_{t+1}(\theta)$ . A general notation for any asset pricing model is given by

$$\begin{aligned} E[p_t] &= E[m_{t+1}(\theta)x_{t+1}] \text{ or} \\ 0 &= E[m_{t+1}(\theta)x_{t+1} - p_t], \end{aligned} \tag{5.1}$$

where  $x_{t+1}$  denotes the pay-off vector of an asset given by  $p_{t+1} + d_{t+1}$ .

Defining  $R_{t+1}$  as the vector of returns on different assets and using equation (2.5) the moment conditions can be expressed in terms of the state-price deflator and returns on the assets, which implies that

$$\begin{aligned} 1 &= E[m_{t+1}(\theta)R_{t+1}] \text{ or} \\ 0 &= E[m_{t+1}(\theta)R_{t+1} - 1]. \end{aligned} \tag{5.2}$$

We further define an error term  $v_t(\theta) = [m_{t+1}(\theta)R_{t+1} - 1]$  and  $g_T(\theta)$  as the sample mean of the  $v_t$ 's, when the parameter vector is  $\theta$  and the sample size



is  $T$ , and it follows that

$$g_T(\theta) = \frac{1}{T} \sum_{t=1}^T v_t(\theta) = E_T[v_t(\theta)] = E_T[m_{t+1}(\theta)R_{t+1} - 1], \quad (5.3)$$

where  $E_T[\cdot] = \frac{1}{T} \sum_{t=1}^T [\cdot]$  denotes the sample mean.

It is common to add instruments when testing asset pricing models. Using again equation (2.5) both sides can be multiplied by any variable  $z_t$  observed at time  $t$  before taking unconditional expectations which results in the following orthogonality conditions

$$\begin{aligned} E[z_t] &= E[m_{t+1}(\theta)R_{t+1}z_t] \text{ or} \\ 0 &= E\{[m_{t+1}(\theta)R_{t+1} - 1]z_t\}. \end{aligned}$$

This can be done for a whole vector of returns and instruments, multiplying each return by each instrument which can be denoted by

$$E\{[m_{t+1}(\theta)R_{t+1} - 1] \otimes z_t\} = 0, \quad (5.4)$$

where  $\otimes$  means "multiply every element by every other element" and the first element of the instrument vector is 1.

Assuming for example two returns  $R = [R^a, R^b]'$  (5.4) looks like

$$E \left\{ \begin{bmatrix} m_{t+1}(\theta)R_{t+1}^a \\ m_{t+1}(\theta)R_{t+1}^b \\ m_{t+1}(\theta)R_{t+1}^a z_t \\ m_{t+1}(\theta)R_{t+1}^b z_t \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ z_t \\ z_t \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (5.5)$$

### Two-step GMM estimation

To estimate the model parameters two steps are done:

1. Usually there are more moment conditions than parameters, hence  $\theta$  is chosen to make the pricing errors  $g_T(\theta)$  as small as possible. The first-stage estimate of  $\theta$  therefore minimizes a quadratic form of the sample mean of the errors,

$$\hat{\theta}_1 = \operatorname{argmin}_{\{\theta\}} g_T(\theta)' W g_T(\theta) \quad (5.6)$$

for a weighting matrix  $W$  that is often the identity matrix  $I$ . The weighting matrix shows how much attention is paid to each moment, using the identity matrix as weighting matrix  $W = I$  means that all assets are treated symmetrically. The derived estimate  $\hat{\theta}_1$  of the parameter vector

$\theta$  is consistent and asymptotically normal.

As some asset returns may have much more variance than others which implies that the sample mean will vary more from sample to sample, in a second step another weighting matrix is used for statistical reasons.  $T \text{Var}[g_T] \rightarrow \sum_{j=-\infty}^{\infty} E[v_t v'_{t-j}] = S$  suggests that the inverse of  $S$  might be a good weighting matrix and [9] shows, that it is the statistically optimal weighting matrix in the sense that it produces estimates with the lowest asymptotic variance. Hence in the second step less attention is paid to pricing errors from assets with high variances. Using  $\hat{\theta}_1$  an estimate  $\hat{S}$  of

$$S = \sum_{j=-\infty}^{\infty} E(v_t(\theta) v'_{t-j}(\theta)) \quad (5.7)$$

need to be formed to be able to continue with the second step. The estimation of  $S$  is described in the next section.

2. The second-stage estimate  $\hat{\theta}$  uses the estimated matrix  $\hat{S}$  as a weighting matrix in the quadratic form

$$\hat{\theta} = \underset{\{\theta\}}{\text{argmin}} g_T(\theta)' \hat{S}^{-1} g_T(\theta). \quad (5.8)$$

$\hat{\theta}$  is a consistent, asymptotically normal and asymptotically efficient estimate for the parameter vector  $\theta$ . Due to the choice of the weighting matrix  $W = \hat{S}^{-1}$  it has the smallest asymptotic variance-covariance matrix among all possible estimators corresponding to the moment conditions  $g_T(\theta)$ .

### **$J_T$ test**

The overall fit of an asset pricing model can finally be tested with the  $J_T$  test of overidentifying restrictions, which is given by

$$T J_T = T \min_{\{\theta\}} [g_T(\theta)' S^{-1} g_T(\theta)] \sim \chi^2(\# \text{moments} - \# \text{parameters}). \quad (5.9)$$

It states that assuming the model is specified correctly,  $T$  times the minimized value of the second-stage objective is distributed  $\chi^2$  with degrees of freedom equal to the number of moment conditions less the number of estimated parameters.

## 5.2 GMM in R

### 5.2.1 The Algorithm

To estimate the parameter vector  $\theta$  the R-package "gmm"<sup>1</sup> was used. This package includes the function *gmm*, which estimates  $\theta$  using the original two step version of GMM as proposed by [9]. The algorithm follows the two steps described in section 5.1 and is given below.

---

#### Two-step GMM algorithm

---

1. Compute  $\hat{\theta}_1 = \operatorname{argmin}_{\{\theta\}} g_T(\theta)^T g_T(\theta)$ .
  2. Compute the matrix  $\hat{S}$ .
  3. Compute  $\hat{\theta} = \operatorname{argmin}_{\{\theta\}} g_T(\theta)^T \hat{S}^{-1} g_T(\theta)$ .
- 

Additionally a refined version of the original GMM is available. This iterative version is implemented as follows.

---

#### Iterative GMM algorithm

---

1. Compute  $\hat{\theta}_1 = \operatorname{argmin}_{\{\theta\}} g_T(\theta)^T g_T(\theta)$ .
  2. Compute the matrix  $\hat{S}$ .
  3. Compute  $\hat{\theta}_2 = \operatorname{argmin}_{\{\theta\}} g_T(\theta)^T \hat{S}^{-1} g_T(\theta)$ .
  4. If  $\|\hat{\theta}_1 - \hat{\theta}_2\| < \textit{crit}$  stop, else  $\hat{\theta}_1 = \hat{\theta}_2$  and go to step 2.
  5. Define  $\hat{\theta}$  as  $\hat{\theta}_2$ .
- 

where *crit* can be set arbitrarily small .

---

<sup>1</sup>Further details on the package "gmm" can be found in [6] and on <http://cran.r-project.org/web/packages/gmm/gmm.pdf>

### 5.2.2 Estimation of the optimal weighting matrix $S$ and the asymptotic variance-covariance matrix $V(\theta)$

#### Estimation of $S$

For the second step of both algorithms it is needed to estimate the optimal weighting matrix  $S$  which is given in (5.7) by

$$S = \sum_{j=-\infty}^{\infty} E[v_t(\theta)v_{t-j}(\theta)'].$$

Using [3], [7] and [16] an estimate  $\hat{S}$  of  $S$  is obtained as follows. Supposing we have given a consistent (first step) estimate  $\hat{\theta}_1$  of  $\theta$ , the estimator for  $S$  derived in R is based on the empirical values of the error terms  $v_t(\theta)$ 's. More precisely, the estimator  $\hat{S}$  is based on the weighted empirical autocorrelations of the empirical error terms  $\hat{v}_t = v_t(\hat{\theta}_1) = [m_{t+1}(\hat{\theta}_1)R_{t+1} - 1] \otimes z_t$  as given below<sup>2</sup>.

$$\hat{S} = \frac{T}{T-q} \sum_{j=-T+1}^{T-1} k\left(\frac{j}{b}\right) \hat{\Gamma}(j), \quad (5.10)$$

with

$$\hat{\Gamma}(j) = \begin{cases} \frac{1}{T} \sum_{t=j+1}^T \hat{v}_t \hat{v}_{t-j}' & \text{for } j \geq 0 \\ \frac{1}{T} \sum_{t=-j+1}^T \hat{v}_{t+j} \hat{v}_t' & \text{for } j < 0 \end{cases} \quad (5.11)$$

and

$$k(x) = \frac{25}{12\pi^2 x^2} \left( \frac{\sin(6\pi x/5)}{6\pi x/5} - \cos(6\pi x/5) \right). \quad (5.12)$$

The factor  $T/(T-q)$  is a small sample degrees of freedom adjustment, where  $q$  is the dimension of the parameter vector  $\theta$ .  $b$  is the bandwidth parameter used, which is calculated as a function of the data and is described in more detail below.  $k(\cdot)$  describes the weights used to downweight higher-order autocorrelations, since they are based on only a few observations.  $k(\cdot)$  given in (5.12) is the so called "quadratic spectral kernel" recommended by Andrews in [3].

The bandwidth parameter  $b$  for a kernel  $k(\cdot)$  is estimated from the data as follows.

1. Specify  $p$  univariate approximating parametric models for  $\{\hat{v}_{at}\}$ , with  $a = 1, \dots, p$ , where  $\hat{v}_t = (\hat{v}_{1t}, \dots, \hat{v}_{pt})$ . The default procedure in R uses first order autoregressive (AR(1)) models for  $\{\hat{v}_{at}\}$ , with different parameters

<sup>2</sup>See the [3] for a detailed description and the appendix of [3] for the derivation.

for each  $a$ :  $(\rho_a, \sigma_a^2)$  denote the autoregressive and innovation variance parameters, respectively, for  $a = 1, \dots, p$ .

2. Estimate the parameters of the approximating parametric models by standard methods. To do so R uses the OLS method to get the estimates  $\{(\hat{\rho}_a, \hat{\sigma}_a^2) : a = 1, \dots, p\}$ .
3. Substitute these estimates into the formula

$$\hat{\alpha}(2) = \sum_{a=1}^p w_a \frac{4\hat{\rho}_a^2 \hat{\sigma}_a^4}{(1 - \hat{\rho}_a)^8} \bigg/ \sum_{a=1}^p w_a \frac{\hat{\sigma}_a^4}{(1 - \hat{\rho}_a)^4}$$

to get an estimate  $\hat{\alpha}(2)$  of  $\alpha(2)$ , where  $w_a$  is the diagonal element of the weighting matrix. Finally, use  $\hat{\alpha}(2)$  and

$$k_2 = -\frac{1}{2} \frac{d^2 k(x)}{dx^2} \bigg|_{x=0}$$

to calculate the estimate  $\hat{b}$  of the bandwidth parameter  $b$  as

$$\hat{b} = \left( \frac{2k_2^2 \hat{\alpha}(2)T}{\int k^2(x)dx} \right)^{1/5}.$$

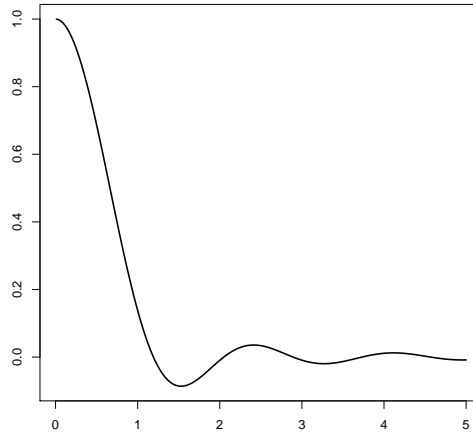


Figure 5.1: The quadratic spectral kernel

For the quadratic spectral kernel depicted in Figure 5.1 this leads to  $\hat{b} = 1.3221(\hat{\alpha}(2)T)^{1/5}$ .

Estimating  $S$  in this way necessarily generates a positive semi-definite estimator  $\hat{S}$  which can be inverted and then used as the weighting matrix for the second step of GMM.

### Estimation of $V(\theta)$

Relying on the central limit theorem, the asymptotic distribution of the GMM estimate  $\hat{\theta}$  of the parameter vector  $\theta$  is given by

$$\sqrt{T}(\hat{\theta} - \theta) \longrightarrow N(0, V(\theta)), \quad (5.13)$$

where  $\longrightarrow$  means convergence in distribution.  $V(\theta)$  denotes the asymptotic variance-covariance matrix of  $\hat{\theta}$ , with  $\hat{\theta}$  based on the estimated optimal weighting matrix  $\hat{S}^{-1}$ .  $V(\theta)$  can be expressed in a "sandwich" form<sup>3</sup>

$$V(\theta) = (B(\theta)'S(\theta)B(\theta))^{-1},$$

where  $S(\theta)$  denotes the optimal weighting matrix and  $B(\theta)$  is given as

$$B(\theta) = E \left[ \frac{\partial v_t(\theta)}{\partial \theta} \right].$$

An estimator of  $B(\theta)$  is given by

$$\hat{B} = \left[ \frac{\partial g_T(\theta)}{\partial \theta} \right]_{\theta=\hat{\theta}} = E_T \left[ \frac{\partial v_t(\theta)}{\partial \theta} \right]_{\theta=\hat{\theta}}, \quad (5.14)$$

where  $\hat{\theta}$  is the estimate of  $\theta$  and  $v_t(\theta) = [m_{t+1}(\theta)R_{t+1} - 1] \otimes z_t$  is again the error term. The derivatives of  $v_t(\theta)$  are obtained with the R-function *numericDeriv* which numerically evaluates the gradient of an expression.

Using the estimates  $\hat{S}$  and  $\hat{B}$  and estimate of  $V(\theta)$  is given as

$$\hat{V} = (\hat{B}'\hat{S}\hat{B})^{-1}. \quad (5.15)$$

---

<sup>3</sup>A formal proof is given in [4], Chapter 5.

### 5.3 GMM and the simple model

The state-price deflator of the simple model given by

$$\frac{\zeta_{t+1}}{\zeta_t} = e^{-\delta} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

uses two parameters  $\delta$  and  $\gamma$ . Writing this state-price deflator as a function of these parameters gives

$$m_{t+1}(\theta) = e^{-\delta} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}. \quad (5.16)$$

With  $\beta = e^{-\delta}$  the parameter vector is given by  $\theta = [\beta, \gamma]'$ .

By substituting (5.16) in (5.4) the following orthogonality conditions are obtained

$$E \left\{ \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} - 1 \right] \otimes z_t \right\} = 0, \quad (5.17)$$

where  $R_{t+1}$  denotes the vector of returns consisting of the returns on the ATX and the returns on the Vibor/Euribor, i.e.  $R_{t+1} = [R_{t+1}^{Atx}, R_{t+1}^{Vib}]'$ .

Using (5.3) the sample orthogonality conditions based on  $T$  observations are given as

$$g_T(\theta) = \frac{1}{T} \sum_{t=1}^T \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} - 1 \right] \otimes z_t \quad (5.18)$$

and the parameter vector  $\theta$  is estimated by minimizing  $g_T(\theta)' W g_T(\theta)$ .

The instrument sets (I-Set) used for the moment conditions given in (5.17) are the following.

1.  $z_t = [1, R_t^{Atx}]'$ , i.e. a constant and lagged return on the ATX
2.  $z_t = [1, R_t^{Vib}]'$ , i.e. a constant and lagged return on the Vibor/Euribor
3.  $z_t = [1, C_t/C_{t-1}]'$ , i.e. a constant and lagged consumption growth
4.  $z_t = [1, R_t^{Atx}, R_t^{Vib}]'$ , i.e. a constant, lagged return on the ATX and lagged return on the Vibor/Euribor
5.  $z_t = [1, R_t^{Atx}, C_t/C_{t-1}]'$ , i.e. a constant, lagged return on the ATX and lagged consumption growth
6.  $z_t = [1, R_t^{Vib}, C_t/C_{t-1}]'$ , i.e. a constant, lagged return on the Vibor/Euribor and lagged consumption growth
7.  $z_t = [1, R_t^{Atx}, R_t^{Vib}, C_t/C_{t-1}]'$ , i.e. a constant, lagged return on the ATX, lagged return on the Vibor/Euribor and lagged consumption growth

## 5.4 GMM and Abel's model

The state-price deflator of Abel's "Catching up with the Joneses" is given by

$$m_{t+1}(\theta) = e^{-\delta} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{C_t}{C_{t-1}} \right)^{\kappa(\gamma-1)}$$

and uses three parameters  $\delta, \gamma$  and  $\kappa$ .

Similar to the previous section the orthogonality conditions are then given as

$$\mathbb{E} \left\{ \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{C_t}{C_{t-1}} \right)^{\kappa(\gamma-1)} R_{t+1} - 1 \right] \otimes z_t \right\} = 0. \quad (5.19)$$

where  $R_{t+1}$  denotes the vector of returns on the ATX and the Vibor/Euribor and  $z_t$  is the respective instrument vector.

The sample orthogonality conditions based on  $T$  observations are given by

$$g_T(\theta) = \frac{1}{T} \sum_{t=1}^T \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{C_t}{C_{t-1}} \right)^{\kappa(\gamma-1)} R_{t+1} - 1 \right] \otimes z_t. \quad (5.20)$$

The parameter  $\kappa$  is not estimated during the procedure, but the estimation is done for different values of  $\kappa$ . Even though Abel limits  $\kappa$  to  $0 < \kappa \leq 1$  in [2] we use some values greater than 1 as a comparison. The considered values for  $\kappa$  are 0.5, 0.75, 0.95, 1, 2 and 5. The parameter vector to be estimated is therefore still given by  $\theta = [\beta, \gamma]'$ , the instrument sets used are the same seven sets as given in the previous section.



## Chapter 6

# Results

### 6.1 Summary statistics

#### Returns and consumption

Using the time series for returns on the ATX, the Vibor/Euribor as a risk-free rate and consumption growth as depicted in Figures 4.4, 4.11 and 6.1, Fable 6.1 states the summary statistics for version 1 of the simple model.

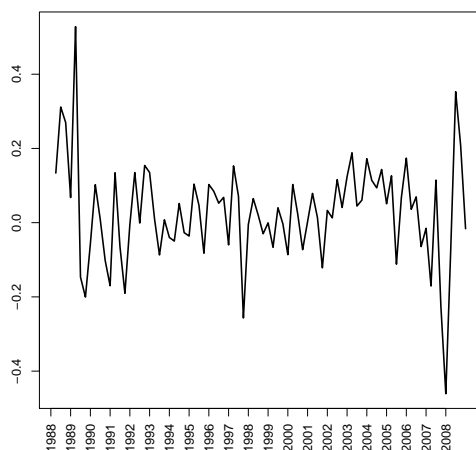


Figure 6.1: Returns on the ATX - 1Q88-4Q09

1Q1989 - 4Q2009	Mean	Std.Dev	Autocorr.
Return on Stocks	11.080	27.220	0.201
Return on Bonds	3.721	1.226	0.968
Consumption Growth	1.375	1.664	-0.335

Table 6.1: Summary statistics for the simple model version 1

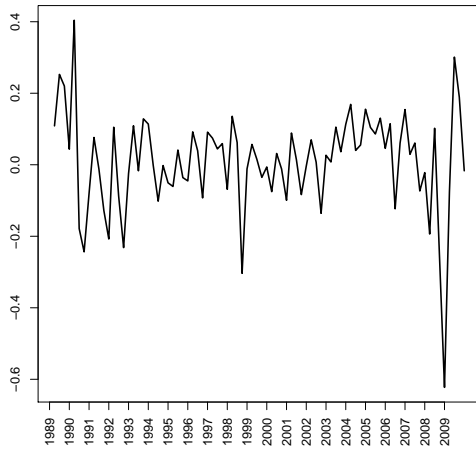


Figure 6.2: Log-returns on the ATX - 1Q89-4Q09

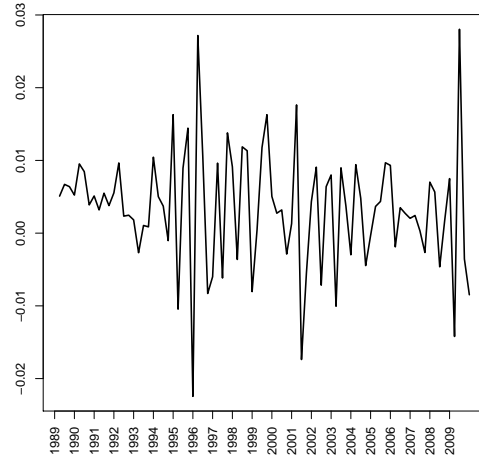


Figure 6.3: Log-consumption growth rates - 1Q89-4Q09

In Austria the average annual return<sup>1</sup> on the ATX accounts for 11.808% and is therefore high when considering the period from 1Q1989 to 4Q2009. The standard deviation of risky assets is high as well, returns on the ATX have a volatility of 27.220%. First-order autocorrelation has a value of 0.201. Returns on bonds are low compared to risky returns and account for 3.721% with a low standard deviation of 1.226%. The first-order autocorrelation is high accounting for 0.968. Consumption growth has a mean of 1.375% and a low standard deviation of 1.664%. First-order autocorrelation of consumption growth is negative and has a value of  $-0.335$ .

### Logarithmic returns and consumption

For the simple model version 2 the returns and consumption growth are considered in logarithmic terms and are depicted in Figures 6.2 and 6.3. Table 6.2 reports the summary statistics for version 2 of the simple model.

1Q1989 - 4Q2009	Mean	Std.Dev	Autocorr
Return on Stocks	7.239	27.722	0.225
Return on Bonds	3.696	1.211	0.968
Consumption Growth	1.359	1.659	-0.336

Table 6.2: Summary statistics for the simple model version 2

As expected, the figures are in the same range as in version 1 of the simple model. The average return on risky assets accounts for 7.239% with a standard

<sup>1</sup>To annualize the quarterly data, means were multiplied by 400 to obtain annual percentage points, standard deviations by 200 as suggested by Campbell in [5].

deviation of 27.722% and a first-order autocorrelation of 0.225. The mean of the risk-free asset has a value of 3.696% and is again much smaller than returns on the risky assets. The standard deviation is low and accounts for 1.211%. First-order autocorrelation is the same as in version 1. Consumption growth has a value of 1.359% with a standard deviation of 1.659%. Autocorrelation is again negative and accounts for  $-0.336$ .

## 6.2 The simple model

### Version 1

Using the summary statistics of version 1 of the simple model and substituting them into (3.7)

$$\text{EER}_{i,t+1} = \text{E}_t[R_{i,t+1}] - R_t^f \approx \gamma \sigma_C \rho_t \left[ R_{i,t+1}, \frac{C_{t+1}}{C_t} \right] \sigma_t[R_{i,t+1}],$$

we get the following result for version 1 of the simple model.

Country	Period	$E\tilde{E}R$	$\tilde{R}^f$	$\tilde{\sigma}_R$	$\tilde{\sigma}_C$	$\tilde{\rho}$	$\gamma$	$\hat{\gamma}$
AUT	89Q1-09Q4	7.359	3.720	27.220	1.664	0.152	106.6	16.25

Table 6.3: The equity premium puzzle in Austria - version 1

### Version 2

Considering again the more often used version 2 of the simple model and equation (3.10)

$$\text{eer}_{i,t+1} + \frac{1}{2} \text{Var}_t[r_{i,t+1}] = \gamma \sigma_C \rho_t \left[ \ln \frac{C_{t+1}}{C_t}, r_{i,t+1} \right] \sigma_t[r_{i,t+1}]$$

it is now possible to append the results for Austria to Table 3.1.

Country	Period	$\widetilde{\text{eer}}$	$\tilde{r}^f$	$\tilde{\sigma}_R$	$\tilde{\sigma}_C$	$\tilde{\rho}$	$\gamma$	$\hat{\gamma}$
USA	74Q2-98Q3	8.071	0.896	15.271	1.071	0.205	240.6	49.3
FRA	73Q2-98Q3	8.308	2.715	23.175	2.922	-0.093	neg	12.3
GER	78Q4-97Q3	8.669	3.219	20.196	2.447	0.029	599.5	17.5
ITA	71Q2-98Q1	4.687	2.371	27.068	1.665	-0.006	neg	10.4
CH	82Q2-98Q4	14.898	1.393	21.878	2.123	-0.112	neg	32.1
NL	97Q2-98Q3	11.421	3.377	16.901	2.510	0.032	850.0	26.9
SWE	70Q1-99Q2	11.539	1.995	23.518	1.851	0.015	1713.2	26.5
UK	70Q1-99Q1	9.169	1.255	21.198	2.511	0.093	186.0	17.2
AUT	89Q1-09Q4	7.405	3.696	27.722	1.659	0.117	209.0	24.4

Table 6.4: The equity premium puzzle in selected countries including Austria

The first column of Table 6.4 reports the left hand side of (3.10), this is the average excess log-return derived by subtracting the risk-free rate from returns on the ATX and adjusting it for half of the variance of the returns on the ATX. As mentioned before, this adjustment arises from the fact that expectations of log-returns are considered. The expected excess return for the Austrian

data set is within the range of the other countries given in Table 3.1. The second column shows the risk-free rate, which is slightly higher than in the other countries and is comparable with risk-free returns in Germany or the Netherlands. The third column gives the annualized standard deviation of the log-excess return on the ATX, which is comparable with the highest of all values in Italy. The next column reports the annualized standard deviation of consumption growth, which is again in the range of Italy and therefore mid-range when compared to the other countries. Column five gives the correlation of consumption growth and the returns on risky assets, which is relatively high, but still not as high as in the US. The relative risk aversion  $\gamma$  obtained using these figures accounts for 209.0. Setting the correlation to 1 gives a  $\gamma$  of 24.4 which is in the range of the other countries but still too high to be a realistic value which would be between 1 and 10.

### The equity premium puzzle

When using the figures  $\tilde{\sigma}_R, \tilde{\sigma}_C$  and  $\tilde{\rho}$  derived from Austrian data as stated in Table 6.4 it is possible to calculate the expected excess return for reasonable values of  $\gamma$  ranging from 1 to 10. The first column in Table 6.5 headed by  $\widetilde{\text{eer}}(\gamma)$  reports the expected excess return when using equation (3.10), the second column headed by  $\widetilde{\text{eer}}(\hat{\gamma})$  uses a correlation  $\tilde{\rho}$  equal to 1, which is most favorable to the model and yields in the highest possible values for the expected excess return.

$\gamma$	$\widetilde{\text{eer}}(\gamma)$	$\widetilde{\text{eer}}(\hat{\gamma})$	$\gamma$	$\widetilde{\text{eer}}(\gamma)$	$\widetilde{\text{eer}}(\hat{\gamma})$
1	0.054%	0.460%	6	0.323%	2.760%
2	0.011%	0.920%	7	0.376%	3.219%
3	0.161%	1.380%	8	0.431%	3.679%
4	0.215%	1.840%	9	0.484%	4.139%
5	0.269%	2.300%	10	0.538%	4.599%

Table 6.5: Expected excess returns for reasonable  $\gamma$ 's

It can be noted, that the equity premium puzzle is present in the Austrian data set, as the excess returns, when considering  $\gamma$  ranging from 1 to 10, is significantly lower than the historical average of 7.405%. Hence, like in other countries, the simple model fails to explain the high returns on stocks.

Using a correlation of 1 the values for  $\widetilde{\text{eer}}$  are higher, but still too low to resolve the equity premium puzzle. When allowing for high values of the relative risk aversion coefficient the equity premium puzzle can be solved, however, this can lead to the risk-free rate puzzle, which is described in the next section.

### The risk-free rate puzzle

Calculating the risk-free rate as given in (3.9)

$$r_t^f = \delta + \gamma \bar{g} - \frac{1}{2} \gamma^2 \sigma_C^2,$$

for reasonable values of  $\gamma$  between 1 and 10 and a subjective time preference rate  $\delta$  equal to 0.9 the following result can be obtained:

$\gamma$	$\tilde{r}^f(\gamma)$	$\gamma$	$\tilde{r}^f(\gamma)$
1	2.259%	6	9.049%
2	3.617%	7	10.406%
3	4.976%	8	11.763%
4	6.334%	9	13.120%
5	7.692%	10	14.476%

Table 6.6: Risk-free rates for reasonable  $\gamma$ 's

To get the historical risk-free rate of 3.696% a relative risk aversion between 3 and 4 is needed. Higher values of  $\gamma$  result in an unrealistically high risk-free rate, hence, when solving the equity premium puzzle by allowing a higher risk aversion, we run into the risk-free rate puzzle as it was noted in other countries as well.

As a result, like in other countries the simple power utility model is not able to solve the considered asset pricing puzzles simultaneously.

### 6.3 Abel's model

Considering Abel's "Catching up with the Joneses" and the summery statistics stated in section 6.1 the equity premium puzzle remains the same as in section 6.2 but we can have a look at the risk-free rate given in (3.16) by

$$r_t^f = \delta + \gamma \bar{g} - \frac{1}{2} \gamma^2 \sigma_c^2 - \kappa(\gamma - 1) \bar{g}.$$

As there is an additional term depending on  $\gamma$  and  $\kappa$  which decreases the risk-free rate we can hope for a lower value compared to the simple model. Using again reasonable values for  $\gamma$  ranging from 1 to 10,  $\delta = 0.9$  and  $0 < \kappa \leq 1$  as suggested by Abel in [2] the following risk-free rates were obtained.

$\gamma$	$\kappa = 0.5$	$\kappa = 0.75$	$\kappa = 0.9$	$\kappa = 1$
1	2.259%	2.259%	2.259%	2.259%
2	2.938%	2.598%	2.394%	2.258%
3	3.617%	2.937%	2.530%	2.258%
4	4.295%	3.276%	2.664%	2.257%
5	4.974%	3.615%	2.799%	2.256%
6	5.652%	3.953%	2.934%	2.254%
7	6.329%	4.291%	3.068%	2.252%
8	7.007%	4.628%	3.201%	2.250%
9	7.684%	4.966%	3.335%	2.248%
10	8.361%	5.303%	3.468%	2.245%

Table 6.7: Risk-free rates for reasonable  $\gamma$ 's - Abel model

Compared to Table 6.6 the risk-free rates are lower, and therefore allow for a higher relative risk aversion  $\gamma$ . The case  $\kappa = 1$  is not realistic for Austrian data as the risk-free rate decreases with increasing  $\gamma$  and the highest value is given by 2.259%. For  $\kappa = 0.9$  the historical value of 3.696% was reached with  $\gamma$  between 11 and 12 and for  $\kappa = 0.95$  a value around  $\gamma = 24$  was needed, which was obtained using equation (3.10) with a correlation of  $\rho = 1$ . For  $\kappa > 1$  negative values for the risk-free rate were obtained, which confirms the restriction of  $\kappa \leq 1$ .

## 6.4 Results of the GMM estimation

For the simple model and Abel's habit formation model the parameters  $\beta$  and  $\gamma$  were formally estimated with the R package "gmm". In order to obtain starting values for the procedure, a grid search was done using the moment conditions in (5.4) and the corresponding instrument sets. The identity matrix was used as weighting matrix and the values with the lowest  $TJ_T$  statistic were selected as the initial parameters. Two different ranges corresponding to the values for  $\gamma$  and  $\hat{\gamma}$  stated in Table 6.4 were considered, which resulted in similar starting values. The sourcecode conducting the estimation is partly given in Appendix B.

### 6.4.1 GMM estimation of the simple model

Given the state-price deflator of the simple model  $\zeta_{t+1}/\zeta_t = e^{-\delta}(C_{t+1}/C_t)^{-\gamma}$  with  $\beta = e^{-\delta}$  the following sections describe the estimates obtained for the parameters  $\gamma$  and  $\beta$  using the two-step as well as the iterative version of GMM.

#### Two-step estimation

For the original two-step version proposed by [9] and described in section 5.2 the estimated optimal weighting matrix  $\hat{S}$  was used in the second step to estimate the parameters. The parameter space was restricted to  $\beta \in (0, \infty)$  as the subjective time preference rate  $\delta$  is then derived as the negative of the logarithm of  $\beta$ . Moreover,  $\gamma \in (0, \infty)$  is required as only a positive parameter  $\gamma$  results in a concave utility function  $u$ .

I-Set	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
1	1.04133	0.02730	18.72439	12.19299	3.60708	0.16471
2	1.00644	0.02796	12.64402	8.81056	1.12010	0.57118
3	1.05315	0.02242	16.44676	7.59717	0.65374	0.72118
4	1.02210	0.02097	10.64091	5.58863	4.98324	0.28902
5	1.04056	0.01752	17.08042	5.18848	5.82537	0.21257
6	1.02795	0.01721	11.19497	4.03124	1.66066	0.79785
7	1.03329	0.01728	14.18698	3.81370	6.59640	0.35979

Table 6.8: Two-step GMM results - simple model

The first column of Table 6.8 headed by  $\hat{\beta}$  states the estimates obtained for the parameter  $\beta$ , the second column shows the corresponding standard deviations. It follows, that the values for  $\beta$  are in the range around 1. As a result the corresponding subjective time preference rates  $\delta$  are low, which means that investors are unwilling to substitute consumption over time. This reduces their desire to borrow and, recalling the risk-free rate puzzle,  $\beta > 1$  could help



to obtain a lower risk-free rate. However, it can be noted that the risk-free rate puzzle is still present as  $\delta$  is only slightly negative for  $\beta$  slightly greater than 1, and hence there is more or less no effect on the risk-free rate. The third column headed by  $\hat{\gamma}$  gives the estimated relative risk aversion coefficient  $\hat{\gamma}$ , which is greater than 10 for all instrument sets. The next column states the corresponding standard deviations. As these standard deviations are high, no evidence can be given for a  $\gamma$  in the desired range from 1 to 10. On the other hand, the values are smaller than the value of 209.0 derived by using the simple equation in the previous section. The fifth column shows the test statistic  $TJ_T$  for Hansen's test of overidentifying restrictions and the sixth column the corresponding p-value. On conventional significance levels the  $J_T$  test does not reject the simple model in any case. Instrument set 4 shows the smallest value of  $\hat{\gamma}$  which is just slightly above 10. The corresponding standard deviation, however, is 5.58863. The corresponding  $\hat{\beta}$  is given by 1.02210 with a standard deviation of 0.02097.

### Iterative estimation

For the iterative GMM estimation of the simple model the weighting matrix was again the estimated optimal weighting matrix  $\hat{S}$ . The procedure is said to converge if the distance  $\|\hat{\theta}_1 - \hat{\theta}_2\|$  gets smaller than 0.1 during 100 iterations. In a first step, the parameter space was restricted to  $\gamma \in (0, \infty)$  and  $\beta \in (0, \infty)$ , because only positive values make sense for the model. However, for instrument set 3 the procedure reached this minimal possible value and stopped before converging. Table 6.9 shows the results when restricting the parameter space as mentioned above, Table 6.10 shows several steps of the procedure for instrument set 3 without this restriction.

Results with a restricted parameter space						
I-Set	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
1	0.98373	0.00281	0.18836	0.67191	20.29000	0.00004
2	1.01213	0.01441	6.87589	4.29595	0.73905	0.69106
3	0.99083	0.00157	0.00000	0.06443	1.62033	0.44478
4	1.00328	0.00516	4.29924	1.59557	5.56470	0.23410
5	0.99699	0.00165	2.13148	0.47489	5.83402	0.21189
6	0.99485	0.00000	0.01952	0.00005	42099	0
7	0.99838	0.00205	2.77760	0.58196	10.43194	0.10760

Table 6.9: Iterative GMM results simple model - restricted parameter space

The values and standard deviations obtained by the iterated GMM are lower than those obtained by using the two-step GMM, and some of the values can be assumed to lie in the range from 1 to 10. Except for instrument set 3, where the minimal possible value of 0 was reached, the estimators of the relative risk

aversion parameter are in the range of  $\hat{\gamma} \in [0.01952, 6.87589]$ . with relatively low standard deviations compared to the two-step GMM results.  $\hat{\beta}$  is close to 1 for all instrument sets and the corresponding standard deviations are low. However, when using instrument set 1 and 6 the model is rejected by Hansen's  $J_T$  test of overidentifying restrictions. Interestingly, these are the instrument sets which resulted in positive values for  $\hat{\gamma}$  that are smaller than 1. For instrument set 3 the procedure stopped when  $\hat{\gamma}$  reached 0, at this point the model was not rejected on the conventional significance levels.

Results with an unrestricted parameter space						
Instrument Set 3						
It.	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
1	0.99850	0.00449	2.90980	1.37420	0.84864	0.65421
2	0.99136	0.00135	0.31097	0.17787	1.19705	0.54962
3	0.99042	0.00309	-0.10676	0.15025	1.25899	0.53286
4	0.99069	0.0262	-0.16219	0.17836	2.96771	0.22676
5	0.99030	0.00239	-0.18495	0.17737	0.87619	0.64526
6	0.99028	0.00240	-0.18353	0.17693	0.96464	0.61735
7	0.99028	0.00240	-0.18353	0.17693	0.96464	0.61735

Table 6.10: Iterative GMM results simple model - instrument set 3

Considering Table 6.10, by using an unrestricted parameter space allowing  $\beta \in (-\infty, \infty)$  and  $\gamma \in (-\infty, \infty)$ , the iterative GMM procedure converged after 7 iterations using instrument set 3. After the third iteration the value for  $\hat{\gamma}$  was negative, after the sixth iteration the value did not change any more. The corresponding standard deviation is about 0.17. Even though the value obtained for the relative risk aversion parameter is not a meaningful result when expecting a value between 1 and 10, it can be noted, how fast the values decrease when allowing for several iterations. This was also noticed when using the other instrument sets.

#### 6.4.2 GMM estimation of Abel's model

The formal parameter estimation for Abel's model was done in the same way as the parameters of the simple model were estimated. The state-price deflator  $\zeta_{t+1}/\zeta_t = e^{-\delta} (C_{t+1}/C_t)^{-\gamma} (C_t/C_{t-1})^{\kappa(\gamma-1)}$  with  $\beta = e^{-\delta}$  was used to set up the moment conditions. The parameter space for  $\beta$  and  $\gamma$  was restricted to positive values. The parameter  $\kappa$  was not estimated by GMM but the procedure was done for several different values of  $\kappa$ . As section 6.3 had shown that reasonable risk-free rates for higher values of  $\gamma$  are only possible with  $\kappa < 1$ , but close to 1 special attention was paid to these values.

**Two-step estimation**

Instrument Set 1						
$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	0.09089	0.10215	203.21277	58.38272	12.44662	0.00198
0.75	0.00507	0.02157	274.27317	147.93145	9.27257	0.00969
0.95	0.00093	0.00758	298.38253	246.79546	8.59672	0.01359
1	0.00720	0.06856	232.46774	285.18086	6.16176	0.04591
2	0.52115	0.18362	47.07854	15.96607	12.30173	0.00213
5	0.56968	0.19928	18.32984	7.71102	13.60378	0.00111

Instrument Set 2						
$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	0.99204	0.00801	16.86304	7.78775	0.62890	0.73019
0.75	0.96793	0.02142	-2.38150	19.33878	0.53777	0.76423
0.95	0.97582	0.01335	16.56703	6.06453	0.60665	0.73836
1	0.97290	0.01438	16.28777	6.05215	0.60713	0.73818
2	0.92372	0.04170	12.95372	5.94754	0.62944	0.72999
5	0.86569	0.08407	7.63369	3.92398	0.67754	0.71264

Instrument Set 3						
$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	1.01522	0.00844	9.87555	2.01563	1.70678	0.42597
0.75	1.00551	0.00786	8.34032	1.75695	1.72224	0.42269
0.95	0.99982	0.00796	7.38310	1.58279	1.73445	0.42012
1	0.99863	0.00802	7.17816	1.54206	1.73701	0.41958
2	0.98465	0.00893	4.74531	1.05397	1.75267	0.41631
5	0.97207	0.01189	2.77244	0.59690	1.71334	0.42457

Instrument Set 4						
$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	0.14302	0.08130	182.32420	36.37687	16.39326	0.00253
0.75	0.00468	0.00891	276.14657	74.04099	9.63823	0.04698
0.95	0.29618	0.10266	111.59014	18.42171	19.03100	0.00078
1	0.52680	0.10521	80.60771	13.08808	21.34600	0.00027
2	0.81097	0.05944	25.85157	6.65207	23.36800	0.00011
5	0.83048	0.08268	9.69075	3.72128	23.52600	0.00010

Instrument Set 5						
$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	0.85583	0.00876	-9.75652	2.19394	70.25700	0.00000
0.75	0.91861	0.00620	-5.39801	1.40039	59.89900	0.00000
0.95	1.00289	0.00644	11.95365	2.40582	27.15300	0.00002
1	1.00038	0.00643	11.28891	2.28978	25.86200	0.00003
2	0.98106	0.00926	5.12027	1.14676	19.95200	0.00051
5	0.97209	0.01013	2.56687	0.51540	18.59660	0.00094

Instrument Set 6						
$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	1.02162	0.00682	12.11324	2.40943	2.59304	0.62806
0.75	1.00859	0.00539	10.19533	2.14069	2.66207	0.61587
0.95	1.00022	0.00555	8.80111	1.89245	2.74427	0.60149
1	0.99845	0.00571	8.48536	1.83134	2.76430	0.59800
2	0.98099	0.00813	4.79621	1.05523	2.92559	0.57035
5	0.97029	0.01041	2.61188	0.49576	2.77698	0.59581

Instrument Set 7						
$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	1.00120	0.05198	95.96013	11.99766	87.84200	0.00000
0.75	0.93570	0.04498	75.90787	10.53907	72.95100	0.00000
0.95	0.97185	0.01848	33.68463	6.22041	44.28400	0.00000
1	0.96840	0.01750	30.55280	5.77540	42.53600	0.00000
2	0.95456	0.01281	8.69935	1.94310	30.09500	0.00004
5	0.95461	0.01805	3.76497	0.84077	24.63100	0.00040

Table 6.11: Two-step GMM results - Abel model

The two-step estimation gives varying results for different instrument sets. For instrument set 1 and  $\kappa$  lying between 0 and 1 the parameter  $\hat{\gamma}$ , considered together with the corresponding standard deviation, is unrealistically high. For  $\kappa > 1$ ,  $\hat{\gamma}$  and the corresponding standard deviations get lower the higher the value of  $\kappa$ .  $\hat{\gamma}$  reaches a value around 18 with a standard deviation of about 7 for the highest value of  $\kappa$ . The model was rejected by Hansen's  $J_T$  test for all values of  $\kappa$  on conventional significance levels except for  $\kappa = 0.95$  and  $\kappa = 1$  on the 10% significance level. Similar results appear for instrument sets 4 and 7. For  $0 < \kappa \leq 1$ ,  $\hat{\gamma}$  and the corresponding standard deviations are high, but with  $\kappa > 1$ , they decrease. For the higher values of  $\kappa$  we get some realistic values for  $\hat{\gamma}$ . However, the model using these instrument sets was

clearly rejected by Hansen's  $J_T$  test. Instrument set 5 shows negative values of  $\hat{\gamma}$  and relatively low standard deviations for  $0 < \kappa < 1$ . For  $\kappa \geq 1$  the estimates for  $\gamma$  are in a reasonable range, however, the model was rejected for all values of  $\kappa$  when using this instrument set. For instrument sets 2 the model is not rejected for any value of  $\kappa$ . The values for  $\hat{\gamma}$  are mostly around 16 with standard deviations of about 6-7. The most reasonable result is obtained when using  $\kappa = 5$ . Instrument sets 3 and 6 resulted in relatively low values for  $\hat{\gamma}$ , which are below 12, and low standard deviations, even with  $0 < \kappa \leq 1$ . Again  $\hat{\gamma}$  and the standard deviations decreases when increasing  $\kappa$  and the model was not rejected by Hansen's  $J_T$  test. What most instrument sets have in common is that the estimated relative risk aversion parameter  $\hat{\gamma}$  and the corresponding standard deviations decrease for an increasing  $\kappa$ . This contradicts with the findings in section 6.3 where  $\kappa \in (0, 1]$  was assumed in order to derive positive risk-free rates. The most reasonable values when restricting  $\kappa$  and assuming that  $\gamma$  decreases with increasing  $\kappa$  are given for instrument set 3 or instrument set 6 with  $\kappa \in [0.75, 0.95]$ .

### Iterative estimation

Instrument Set 1						
$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	0.00001	0.00004	430.39135	103.21911	5305.2	0.00000
0.75	0.00019	0.00086	376.73913	156.54715	9.76621	0.00757
0.95	0.00024	0.00193	336.14423	243.27555	4.997481	0.08219
1	0.00009	0.00090	354.91695	277.46733	4.79326	0.09102
2	0.00685	0.02843	154.79951	82.18833	4.19541	0.12274
5	0.03074	0.02507	55.24878	9.96218	2.33140	0.31170

Instrument Set 2						
$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	1.00462	0.00954	10.71609	6.56832	0.59825	0.74147
0.75	0.99362	0.00537	11.81118	5.64662	0.59167	0.74391
0.95	0.98334	0.00886	12.27402	5.63702	0.59465	0.74280
1	0.98055	0.01017	12.43581	5.66384	0.59795	0.74158
2	0.92954	0.04026	12.10838	5.89547	0.62263	0.73248
5	0.86569	0.08407	7.63373	3.92398	0.67754	0.71264

Instrument Set 3

$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	0.99277	0.00116	0.16843	0.04199	1.46167	0.48151
0.75	0.99353	0.00116	0.21904	0.04429	1.44448	0.48566
0.95	0.99407	0.00117	0.25327	0.04668	1.44560	0.48540
1	0.99420	0.00117	0.26257	0.04721	1.44623	0.48524
2	0.99628	0.00127	0.39316	0.05618	1.45210	0.48382
5	0.99935	0.00161	0.62456	0.05196	1.47264	0.47887

Instrument Set 4

$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5*	0.67469	0.011326	95.37656	15.49157	3484.6	0.00000
0.75*	0.34234	0.11239	120.37275	19.82806	2853.8	0.00000
0.95*	-0.0009	0.00028	409.95312	95.64646	30.09600	0.00001
1*	0.00111	0.00240	285.93176	70.21232	12.86732	0.01194
2*	-0.03831	0.00148	-21.88347	7.10883	19.30400	0.00068
5	0.81098	0.08484	10.58423	3.77070	22.86600	0.00013

Instrument Set 5

$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	0.99267	0.00116	0.11511	0.04222	15.86985	0.00320
0.75	0.99349	0.00116	0.17048	0.04691	15.11668	0.00447
0.95	0.99407	0.00117	0.21000	0.05042	14.55359	0.00572
1	0.99421	0.00118	0.21952	0.05119	14.42904	0.00604
2	0.99637	0.00128	0.37346	0.06002	13.15455	0.01545
5	0.99948	0.00166	0.61128	0.05579	12.88527	0.01185

Instrument Set 6

$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	0.99543	0.00061	0.12078	0.04448	9.36441	0.05261
0.75	0.99599	0.00070	0.17789	0.04840	8.93571	0.06273
0.95	0.99653	0.00078	0.22325	0.05089	9.85533	0.04294
1	0.99663	0.00079	0.23304	0.05147	9.88319	0.04244
2	0.99799	0.00112	0.38850	0.05762	11.31743	0.02322
5	1.00004	0.00174	0.57426	0.06345	6.99330	0.13624

Instrument Set 7						
$\kappa$	$\hat{\beta}$	Std.Dev	$\hat{\gamma}$	Std.Dev	$TJ_T$	p-Value
0.5	0.99165	0.06071	103.29577	12.70494	47.54700	0.00000
0.75	0.92022	0.04743	78.82301	10.70742	43.58900	0.00000
0.95	0.99153	0.00084	1.03180	0.03403	73.76400	0.00000
1	0.99184	0.00083	1.05746	0.03783	74.19400	0.00000
2	0.99705	0.00117	0.37261	0.06017	25.78300	0.00024
5	0.99912	0.00174	0.58079	0.06317	24.07463	0.00051

Table 6.12: Iterative GMM results - Abel model

Using the iterative GMM estimation the results vary even more than the results obtained by the two-step method. Instrument set 1 shows again too high values for  $\hat{\gamma}$  and the corresponding standard deviations as to be able to derive a meaningful result, especially in the range  $0 < \kappa \leq 1$ . For these values the model was rejected by Hansen's  $J_T$  test at least at the 10% significance level. With  $\kappa > 1$  the model was not rejected, but the relative risk aversion parameter considered together with the standard deviation is still too high. Instrument set 2 resulted in values for  $\hat{\gamma}$  of about 10 and standard deviations of around 6, and the model was not rejected in any case. Neither was instrument set 3, with  $\hat{\gamma}$  being positive but smaller than 1 and small standard deviations of about 0.05. For instrument set 4 the  $\kappa$ 's marked with \* resulted either in negative values for one of the parameters  $\hat{\beta}$  and  $\hat{\gamma}$  or even both, or in positive values but rejection of the model by Hansen's  $J_T$  test depending on the number of iterations used. The values stated are the results after 50 iterations. Instrument set 5 shows small values of  $\hat{\gamma}$  and the corresponding standard deviations, however, the model was rejected by the  $J_T$  test for  $0 < \kappa \leq 1$  and close to rejection for  $\kappa > 1$ . A similar situation is given for instrument set 6, where the relative risk aversion is positive and smaller than 1, and the model was mostly rejected at least for significance levels below 10%. Instrument set 7 shows high values for  $\hat{\gamma}$ , standard deviations of at least 10 and a rejection of the model in the range  $0 < \kappa \leq 0.75$ . For  $\kappa > 0.75$  the values are much smaller with standard deviations below 1, but the model was still strictly rejected by Hansen's  $J_T$  test. Using the iterative GMM a decreasing  $\hat{\gamma}$  for increasing  $\kappa$  is no longer given.

### 6.4.3 Simple model versus Abel model

When comparing the simple model to the Abel model using the seven different instrument sets it can be noted, that the Abel model was rejected more often. For the two-step GMM the simple model was not rejected at all, the Abel model was rejected for instrument set 1, 4, 5 and 7 which is actually more often than it is not rejected. Using the iterative GMM the simple model was rejected for instrument set 1 and 6, instrument set 3 resulted in a negative value for  $\hat{\gamma}$  with a standard deviation below 1. Abel's model was not rejected for instrument set 2 and 3 but no reasonable values could be obtained.

The value of  $\kappa$  does not significantly influence the rejection of a model. Moreover, when looking at the two-step GMM results of the simple model mostly a high  $\kappa$  in Abel's model is needed in order to obtain values in the range of the simple model estimates. As  $\hat{\gamma}$  decreases with increasing  $\kappa$  this again contradicts with the assumption  $\kappa \in (0, 1)$ .

As a result, it was not possible to provide evidence for the Abel model outperforming the simple model when using GMM for estimation of the parameters  $\gamma$  and  $\beta$ .



## Chapter 7

# Concluding remarks

The aim of this thesis was to empirically test whether consumption-based capital asset pricing models are able to explain the movements of stock and bond prices on the Austrian capital market. Using the Austrian ATX as the risky asset, the Vibor/Euribor as the risk-free rate and private aggregate consumption for describing consumption growth, the simple power utility model [13] and Abel's more general "Catching up with the Joneses" [1] were examined. The main empirical findings are as follows.

First, when testing the assumptions made for the models and GMM it could be noted, that not all of these assumptions were satisfied when using Austrian data. Returns on the ATX and consumption growth failed to be lognormally distributed as it is needed to derive closed-form expressions for the state-price deflator and hence for the stock and bond prices. In contrast, returns on the ATX satisfied the stationarity assumption needed for applying GMM, which was unfortunately not true for consumption growth and the risk-free rate.

Second, using the time series mentioned above resulted in the well-known equity premium puzzle, as the historical excess return of 7.405% could only be obtained with an unrealistically high relative risk aversion parameter  $\gamma$ . Allowing for such a high parameter the risk-free rate puzzle was encountered since it was not possible to obtain the low value of 3.696% with the high  $\gamma$ . Using the more general Abel model allowed the use of higher relative risk aversion coefficients without encountering the risk-free rate puzzle for selected values of  $\kappa$ , measuring the importance of the additionally introduced consumption-benchmark.

Third, estimating the parameters  $\beta$  and  $\gamma$  and testing the overall fit of the models using the two version of GMM as suggested by Hansen [9] resulted in different findings for the two different models and different instrument sets. Considering the simple model and the two-step GMM the relative risk aversion coefficient was lower when compared to the initial findings using the simple equation for the excess return. The model was not rejected in any case by Hansen's  $J_T$  goodness-of fit test. However, the parameter  $\gamma$  was found to have

a too high standard deviation as to be a meaningful result. The iterative GMM resulted in more appropriate figures for  $\gamma$  for some but not all instrument sets. Turning to Abel's "Catching up with the Joneses", the model was rejected for several instrument sets. For the others, reasonable values for  $\gamma$  were mostly obtained for  $\kappa > 1$ , which contradicts with the assumption made by Abel in [2]. The results for the iterative GMM were even less encouraging since the model was rejected for five of seven instrument sets and the results of the remaining were not satisfying either. As a result, using GMM did not provide evidence for the Abel model performing better than the simple power utility model.

Further work following this analysis could contain a refinement of the data set, especially of the time series for the risk-free rate and consumption as these time series were not stationary. Additionally, having consumption only available for the period from 1Q88 to 4Q09 restricted the length of the time series used considerably. Furthermore it would be more appropriate to deduct the value of consumption of durable goods from private aggregate consumption. Additionally, the testing and estimation of more complex models could be a challenge and would hopefully result in improved findings as it was noticed in several empirical studies.

## Appendix A

# Results on the lognormal distribution

A random variable  $Y$  is said to be lognormally distributed if the random variable  $X = \ln Y$  is normally distributed. If we have

$$X = \ln Y \sim N(m, s^2)$$

the probability density function for  $X$  is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi s^2}} \exp \left\{ -\frac{(x - m)^2}{2s^2} \right\}, x \in \mathbb{R}.$$

**Theorem A. 1** *The probability density function for  $Y$  is given by*

$$f_Y(y) = \frac{1}{\sqrt{2\pi s^2} y} \exp \left\{ -\frac{(\ln y - m)^2}{2s^2} \right\}, y > 0$$

and  $f_Y(y) = 0$  for  $y \leq 0$ .

**Proof:** The transformed density function of a random variable  $Y = H(X)$ , with  $H(x)$  a monotonically increasing function of  $x$  and the unique inverse given by  $x = H^{-1}(y)$ , is derived with the following result:

$$f_Y(y) = f_X(H^{-1}(y)) |H^{-1'}(y)|$$

with  $x = H^{-1}(y) = \ln y$  and  $H^{-1'}(y) = 1/y$  we have that

$$f_Y(y) = \frac{1}{\sqrt{2\pi s^2}} \exp \left\{ -\frac{(\ln y - m)^2}{2s^2} \right\} \frac{1}{y}.$$

□

**Theorem A. 2** For  $X \sim N(m, s^2)$  and  $\gamma \in \mathbb{R}$  the following holds true

$$\mathbb{E}[e^{-\gamma X}] = \exp \left\{ -\gamma m + \frac{1}{2} \gamma^2 s^2 \right\}.$$

**Proof:** Per definition we have

$$\mathbb{E}[e^{-\gamma X}] = \int_{-\infty}^{+\infty} e^{-\gamma x} \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{(x-m)^2}{2s^2}} dx.$$

Rewriting the exponent leads to

$$\begin{aligned} \mathbb{E}[e^{-\gamma X}] &= e^{-\gamma m + \frac{1}{2} \gamma^2 s^2} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{1}{2s^2} [(x-m)^2 + 2\gamma(x-m)s^2 + \gamma^2 s^4]} dx \\ &= e^{-\gamma m + \frac{1}{2} \gamma^2 s^2} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{(x-[m-\gamma s^2])^2}{2s^2}} dx \\ &= e^{-\gamma m + \frac{1}{2} \gamma^2 s^2}, \end{aligned}$$

where the last equality is due to the fact that the expression of the integral is again a density function of a  $N(m - \gamma s^2, s^2)$  distributed random variable and is therefore equal to 1.  $\square$

Using  $\gamma = -1$  the mean of a lognormally distributed random variable  $Y = e^X$  can easily be computed:

$$\mathbb{E}[Y] = \mathbb{E}[e^X] = \exp \left\{ m + \frac{1}{2} s^2 \right\}.$$

With  $\gamma = -2$  we get

$$\mathbb{E}[Y^2] = \mathbb{E}[e^{2X}] = e^{2(m+s^2)},$$

so that the variance of  $Y$  is given by

$$\begin{aligned} \text{Var}[Y] &= \mathbb{E}[Y^2] - (\mathbb{E}[Y])^2 \\ &= e^{2(m+s^2)} - e^{2m+s^2} \\ &= e^{2m+s^2} (e^{s^2} - 1). \end{aligned}$$

# Appendix B

## Sourcecodes

### B.1 Consumption

```
#####  
##### CONSUMPTION #####  
#####  
  
#select correct file  
setwd("C:/Users/Martin/Documents/R")  
  
#Load data concerning consumption  
ConData<-read.table("ConData.txt",header=TRUE,sep="\t",dec = ",")  
  
#save copy of original data  
y<-length(ConData[,1])  
CData<-array(1,dim=c(y,5))  
  
varianz<-function(x){n=length(x);var(x)*(n-1)/n}  
stdabw<-function(x){n=length(x);sqrt(var(x)*(n-1)/n)}  
  
#Consumption deflator based on 2005  
for(i in 1:y){  
  if(ConData[i,1]==20051)basis<-ConData[i,3]}  
  for(i in 1:y){  
    CData[i,5]<-round(100/basis*ConData[i,3],digits=1)}  
  
#Consumption deflator based on 1990  
for(i in 1:y){  
  if(ConData[i,1]==19901)basis<-ConData[i,3]}  
  for(i in 1:y){  
    CData[i,3]<-round(100/basis*ConData[i,3],digits=1)}  
  
#calculate per capita consumption in EUR instead of Mio EUR  
for(j in 1:y){  
  CData[j,2]<-ConData[j,2]/ConData[j,4]*100000}
```

```
#Consumption Growth based on quarter of the previous year
for(j in 5:y){
  CData[j,1]<-CData[j,2]/CData[j-4,2]}

#Consumption Growth previous quarter
for(j in 5:y) {
  CData[j,4] <- CData[j,2]/CData[j-1,2]}

ConGrowth<-CData[c(-1,-2,-3),]

write.table(ConGrowth,file="ConsumptionGrowth",quote=FALSE,sep="\t",dec=",")

#####
#####

#Load seasonally adjusted consumption data
Con_ad<-read.table("Consumption_adj.txt",header=TRUE,sep="\t",dec=".")
ConData[,2]<-Con_ad
y<-length(ConData[,1])

#calculate per capita consumption in EUR instead of Mio EUR
for(j in 1:y){
  CData[j,2]<-ConData[j,2]/ConData[j,4]*100000}

#Consumption Growth based on quarter of the previous year
for(j in 5:y){
  CData[j,1]<-CData[j,2]/CData[j-4,2]}

#Consumption Growth based on quater of previous quarter
for(j in 5:y){
  CData[j,4]<-CData[j,2]/CData[j-1,2]}

ConGrowth_ad<-CData[c(-1,-2,-3),]

write.table(ConGrowth_ad,file="ConsumptionGrowth",quote=FALSE,sep="\t",dec=",")
```

## B.2 Population

```
#####
##### POPULATION #####
#####

#select correct file
setwd("C:/Users/Martin/Documents/R")

#Load data concerning population
PopulationData<-read.table("PopulationData.txt",header=FALSE,sep="\t",dec=",")
```

```
varianz<-function(x){n=length(x);var(x)*(n-1)/n}
stdabw<-function(x){n=length(x);sqrt(var(x)*(n-1)/n)}

beginyear<-1988
endyear<-2009
years<-endyear-beginyear+1
quarters<-years*4

#calculate difference between years
Diff<-array(dim=c(years,1))
for(j in 1:years){
  Diff[j]<-round((PopulationData[j+1,2]-PopulationData[j,2])/4,digits=0)}

#first column containing quarters/years
QData<-array(beginyear:endyear,dim=c(quarters,2))
QData[,1]<-sort(QData[,1])
for(j in 1:quarters){
  QData[j,1]<-QData[j,1]*10+1*j%%4+1}
QData[,1]<-sort(QData[,1])
QData[,2]<-0

#compute quarterly figures for population
j<-0
repeat{QData[j*4+1,2]<-PopulationData[j+1,2];
  j<-j+1;if(j==years) break}

i<-0
repeat{for(j in 2:4){
  QData[i*4+j,2]<-QData[i*4+j-1,2]+Diff[i+1]};
  i<-i+1;if(i==years) break}

PopData<-QData[c(-1,-2,-3),2]

write.table(PopData,file="PopulationData",quote=FALSE,sep="\t",dec=",")
```

### B.3 ATX and Market Value

```
#####
##### ATX #####
#####

# select correct file
setwd("C:/Users/Martin/Documents/R")

# Load data concerning Atx Stock Prices
AtxData<-read.table("AtxData.txt",header=FALSE,sep="\t",dec=",")

beginyear<-1988
```

```
endyear<-2009
countdays<-length(AtxData[,1])
countquarters<-(endyear - beginyear +1)*4

varianz<-function(x){n=length(x);var(x)*(n-1)/n}
stdabw<-function(x){n=length(x);sqrt(var(x)*(n-1)/n)}

#first column containing quarters/years
QData<-array(beginyear:endyear,dim=c(countquarters,2))
QData[,1]<-sort(QData[,1])
for(j in 1:countquarters){
  QData[j,1]<-QData[j,1]*10+1*j%4+1}
QData[,1]<-sort(QData[,1])

#calculate average quarterly values
QData[,2]<-0
j<-beginyear
i<-1
repeat k<-3
repeat{quater<-NULL
year<-subset(AtxData,as.numeric(substr(AtxData[,1],7,10))==j);
quater<-subset(year,as.numeric(substr(year[,1],4,5))==k);
QData[i,2]<-mean(quater[,2]);
i<-i+1;k<-k+3;if(k>12) break}
j<-j+1;if(j==endyear+1) break}

AtxData<-QData[c(-1,-2,-3),]

write.table(AtxData,file ="AtxData",quote=FALSE,sep="\t",dec=",")
```

The market value was derived in exactly the same way as the ATX.

## B.4 Vibor/Euribor

```
#####
##### VIBOR/EURIBOR #####
#####

# select correct file
setwd("C:/Users/Martin/Documents/R")

# Load data concerning Vibor Overnight Rate
ViborOnData<-read.table("ViborOnData.txt",header=TRUE,sep="\t",dec=",")

beginyear<-1989
endyear<-2009
countdays<-length(ViborOnData[,1])
countquarters<-(endyear-beginyear +1)*4
Time<-array(dim = c(countdays,5))
```



```
#allocation of months in quarters
for(j in 1:countdays){
  Time[j,1]<-as.numeric(substr(ViborOnData[j,1],7,10));
  Time[j,2]<-as.numeric(substr(ViborOnData[j,1],4,5))}
for(j in 1:countdays){
  Time[j,1]<-Time[j,1]*10+if (Time[j,2]<4)
  1
  else if(Time[j,2]<7)
  2
  else if(Time[j,2]<10)
  3
  else 4}

#append data
Time[,3]<-ViborOnData[,2]

#first column containing quarters/years
QData<-array(beginyear:endyear,dim=c(countquarters,3))
QData[,1]<-sort(QData[,1])
for(j in 1:countquarters){
  QData[j,1]<-QData[j,1]*10+1*j%%4+1}
QData[,1]<-sort(QData[,1])

#calculate averages
QData[,2]<-0
QData[,3]<-0
#sum up values for a quarter
j<-1
repeat{for(i in 1:countdays){
  if(Time[i,1]==QData[j,1])
  QData[j,2]<-QData[j,2]+Time[i,3]};
  j<-j+1;if(j==countquarters+1) break}
#number of values of the quater
j<-1
repeat{for(i in 1:countdays){
  if(Time[i,1]==QData[j,1])QData[j,3]<-QData[j,3]+1};
  j<-j+1;if(j==countquarters+1) break}

#average = sum values for a quater/number of values
for(i in 1:countquarters){
  QData[i,2]<-((QData[i,2]/QData[i,3]/100+1)^(1/4)-1)*100}

ViborData<-append(0,QData[,2])

write.table(ViborData,file="ViborOnData",quote=FALSE,sep="\t",dec=",")
```

## B.5 Dividends

```
#####
##### DIVIDENDS #####
#####

# select correct file
setwd("C:/Users/Martin/Documents/R")

#Load Data
DividendData<-read.table("DividendData.txt",header=FALSE,sep="\t",dec=",")
n<-nrow(DividendData)

#dividends absolute
if(max(dim(DividendData))==max(dim(MarketValueData))){
  for(i in 1:n){
    DividendData[i,2]<-DividendData[i,2]*MarketValueData[i,2]/100/12}}

beginyear <- 1988
endyear <- 2009

#first column containing quaters/years
QData<-array(beginyear:endyear,dim=c(countquarters,2))
QData[,1]<-sort(QData[,1])
for(j in 1:countquarters){
  QData[j,1]<-QData[j,1]*10+1*j%4+1}
QData[,1]<-sort(QData[,1])

#calculate quaterly values by summing up the monthly
QData[,2]<-0

j<-beginyear
i<-1
repeat{k<-1
  m<-array(0,dim=c(12,1))
  q<-array(0,dim=c(12,1))
  year1<-NULL
  year2<-NULL
  month1<-NULL
  month2<-NULL
  repeat{#select data year j, month k
    year1<-subset(DividendData,as.numeric(substr(DividendData[,1],7,10))==j);
    year2<-subset(MarketValueData,as.numeric(substr(MarketValueData[,1],7,10))==j)
    month1<-subset(year1,as.numeric(substr(year1[,1],4,5))==k);
    month2<-subset(year2,as.numeric(substr(year2[,1],4,5))==k);
    #sum up end-of-month data in vector m
    if(k>1)
      m[k,1]<-m[k-1,1]+month1[length(month1[,1]),2]
    else
```

```

m[k,1]<-month1[length(month1[,1]),2]

if(k>1)
q[k]<-q[k-1]+mean(month2[,2])
else
q[k]<-mean(month2[,2])
k<-k+1;if(k == 13) break}

#calculate summed up quaterly values
QData[i,2]<-m[3,1] #/q[3,1]*100
QData[i+1,2]<-(m[6,1]-m[3,1])#/(q[6,1]-q[3,1])*100
QData[i+2,2]<-(m[9,1]-m[6,1])#/(q[9,1]-q[6,1])*100
QData[i+3,2]<-(m[12,1]-m[9,1])#/(q[12,1]-q[9,1])*100
i<-i+4;
j<-j+1;if(j==endyear+1) break}

DivData<-QData[c(-1,-2,-3),2]
write.table(DivData,file="DividendData",quote=FALSE,sep="\t",dec=",")

```

## B.6 Summary statistics and $\gamma$

```

#####
##### SUMMARY STATISTICS #####
#####

#select correct file
setwd("C:/Users/Martin/Documents/R")

# Load data
PopulationData<-read.table("PopulationData",header=TRUE,sep="\t",dec=",")
AtxData<-read.table("AtxData",header=TRUE,sep="\t",dec=",")
MarketValueData<-read.table("MarketValueData",header=TRUE,sep="\t",dec=",")
DividendData<-read.table("DividendData",header=TRUE,sep="\t",dec=",")
ViborOnData<-read.table("ViborOnData",header=TRUE,sep="\t",dec=",")
ConsumptionData<-read.table("ConsumptionGrowth",header=TRUE,sep="\t",dec=",")

beginyear<-1988
endyear<-2009
countdays<-length(AtxData[,1])
countquarters<-(endyear-beginyear +1)*4
y<-nrow(AtxData)

#generate data matrix
CombinedData<-array(dim=c(max(dim(AtxData)),9))
colnames(CombinedData)<-c("TIME","POP","ATX","DIV","RFR","CSP","CPI05","CPI90","MV")
CombinedData[,1]<-AtxData[,1]
CombinedData[,2]<-PopulationData[,1]
CombinedData[,3]<-AtxData[,2]

```

```

CombinedData[,4]<-DividendData[,1]
CombinedData[,5]<-ViborOnData[,1]
CombinedData[,6]<-ConsumptionData[,2]
CombinedData[,7]<-ConsumptionData[,5]
CombinedData[,8]<-ConsumptionData[,3]
CombinedData[,9]<-MarketValueData[,1]

write.table(CombinedData,file="CombinedData",quote=FALSE,sep="\t",dec=",")

#inflation adjustment of ATX and RFR - 2009 to 1990
for(i in 1:y){
  CombinedData[i,3]<-CombinedData[i,3]/CombinedData[i,8]*100;
  CombinedData[i,5]<-CombinedData[i,5]/CombinedData[i,8]*100}
#inflation adjustment of Consumption - 2005 to 1990
for(i in 1:y){
  CombinedData[i,6]<-CombinedData[i,6]*CombinedData[i,8]/CombinedData[i,7]}

#####
#####

varianz<-function(x){n=length(x);var(x)*(n-1)/n}
stdabw<-function(x){n=length(x);sqrt(var(x)*(n-1)/n)}

#Calculation (log)consumption growth

x<-NULL
i<-2
repeat{ x[i-1]<-(CombinedData[i,6]/CombinedData[i-1,6]);
  i<-i+1; if(i== max(dim(CombinedData))+1) break}
consumptiongrowth<-x

x<-NULL
i<-2
repeat{ x[i-1]<-log(CombinedData[i,6]/CombinedData[i-1,6]);
  i<-i+1; if(i== max(dim(CombinedData))+1) break}
logconsumptiongrowth<-x

meanC<-mean(consumptiongrowth-1)*400
standarddeviationC<-stdabw(consumptiongrowth-1)*200
varC <- standarddeviationC^2
autocorrC <- acf(consumptiongrowth)

logmeanC<-mean(logconsumptiongrowth)*400
logstandarddeviationC<-stdabw(logconsumptiongrowth)*200
logvarC<-logstandarddeviationC^2
logautocorrC<-acf(logconsumptiongrowth)

#####
#####

```

```
#Calculation returns on risky and risk-free asset

x<-array(0,dim=c(y-1,1))
i<-1
repeat{ x[i]<-(CombinedData[i+1,3]/CombinedData[i,3]+CombinedData[i+1,4]/
CombinedData[i,9]-1);
i<-i+1;if(i==y) break}
return<-x

RiskFreeData<-combinedData[c(-1),5]/100

riskyreturn<-mean(return)*400
riskfreereturn<-mean(RiskFreeData)*400
standarddeviationriskfree<-stdabw(RiskFreeData)*200
expectedexcessreturn<-(riskyreturn-riskfreereturn)
standarddeviationR<-stdabw(return)*200
correlation<-cor(return, consumptiongrowth)
autocorrR<-acf(return)
autocorrRF<-acf(RiskFreeData)

#Calculation LOG-returns

logreturnr<-log(1+return)
logreturnrf<-log(1+RiskFreeData)

logriskyreturn<-mean(logreturnr)*400
logstandarddeviationrisky<-stdabw(logreturnr)*200
logriskfreereturn<-mean(logreturnrf)*400
logstandarddeviationriskfree<-stdabw(logreturnrf)*200
logeer<-logreturnr-logreturnrf
logstandarddeviationexcessreturn<-stdabw(logeer)*200
logvarhalf<-((stdabw(logeer)*2)^2)/2*100
logexpectedexcessreturn<-mean(logeer)*400+logvarhalf
logcorrelation<-cor(logreturnr,logconsumptiongrowth)
logautocorrR<-acf(logreturnr)
logautocorrRF<-acf(logreturnrf)

#generate matrix for gmm

DataGmm <- array(dim = c(y-1, 3))
DataGmm[,1]<-logreturnr+1
DataGmm[,2]<-logreturnrf+1
DataGmm[,3]<-consumptiongrowth

write.table(DataGmm,file="DataGmm",quote=FALSE,sep="\t",dec=",")
```

```
#####
#####
```

```
#Calculation of gamma

gamma<-(expectedexcessreturn/100)/((standarddeviationR/100)*
(standarddeviationC/100)*correlation)
gammahat<-(expectedexcessreturn/100)/((standarddeviationR/100)*
(standarddeviationC/100))
loggamma<-(logexpectedexcessreturn+logvarhalf)/100/
((logstandarddeviationrisky/100)*(standarddeviationC/100)*logcorrelation)
loggammahat<-loggamma*logcorrelation
```

## B.7 GMM estimation

The GMM procedure described below was run for each instrument set. The code differs only in the definition of function `g`, where the appropriate instrument set needs to be appended to the variable `s`.

```
#####
##### GMM #####
#####

#select correct file
setwd("C:/Users/Martin/Documents/R")

library("gmm")

#load data
GmmData<-read.table("DataGmm",header=TRUE,sep="\t",dec=",")

T<-nrow(GmmData)
r<-GmmData[-T,1] #risky return lagged
rf<-GmmData[-T,2] #risk-free return lagged
c<-GmmData[-T,3] #consumption growth lagged

#data matrix
x<-as.matrix(GmmData[-1,])

I<-c("ident") #weights matrix = identity matrix
O<-c("optimal") # weiths matrix = optimal matrix

#####
##### function g(tet,x) as in (3.7) SIMPLE MODEL #####
#####

g1<-function(tet,x){
s<-(tet[1]*x[,3]^(-tet[2]))*x[,1:2]-1)
s1<-cbind(s,s*r)
return(s1)}
```

```
#####
#### GRID SEARCH - SIMPLE MODEL #####
#####

#search for minimum objective function value

K1<-matrix(nrow = 500)

for(i in 1:50){
  for(j in 1:10){
    K1[(i-1)*10+j] <- gmm(g=g1, x = x ,wmatrix = I,
    t0 = c(beta = 0.8+j*2/100, gamma = i))[2]}}
K1<-cbind(1:500,unlist(K1))
K1<-K1[order(K1[,2]),]

#####
#### CALCULATION - STARTING VALUES #####
#####

gamma1<-floor(K1[1,1]/10)+1
beta1<-0.8+2*(K1[1,1]-(gamma1-1)*10)/100
if(beta1==0.8) beta1<-1

#####
#### GMM ESTIMATION - SIMPEL MODEL #####
#####

#####
#"Instrument set 1"#
#####
beta1
gamma1
print(res1<-gmm(g=g1,x=x,t0=c(beta=beta1,gamma=gamma1),wmatrix=0,
type="twoStep",optfct=c("nlminb"),lower=c(0,0),upper=c(Inf,Inf)))
summary(res1)
```

The source code for the iterative GMM has the same body except for the final calculation which is given for instrument set 1 as an example as follows.

```
#####
#"Instrument set 1"#
#####
beta1
gamma1
print(res1<-gmm(g=g1,x=x,t0=c(beta=beta1,gamma=gamma1),wmatrix=0,
type="iterative",crit=0.1,optfct=c("nlminb"),lower=c(0,0),upper=c(Inf,Inf),
itermax=imax))
summary(res1)
```

The Abel model was estimated in the same manner, but with function  $g$  as follows. The procedure was done for different parameters  $\kappa$ .

```
#####  
#### Function g(tet,x) Abel Model as in (5.16) #####  
#####  
  
g1<-function(tet,x){  
  s<-tet[1]*x[,3]^(-tet[2])*x[,4]^(k*(tet[2]-1))*x[,1:2]-1  
  s1<-cbind(s,s*r)  
  return(s1)}
```



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