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DIPLOMARBEIT
Master Thesis

Impact Problems of Elastic Structures Discretized as Multi-Degree-of-Freedom Systems

ausgeführt zum Zwecke der Erlangung des akademischen Grades
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unter der Leitung von

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Abstract

One of the main influence, which in the modeling and calculation of a structure must be taken into account, is an extraordinary loading case. This work deals with exceptional excitation of the following events

- Impact of road vehicles

- Impact of fork lift trucks

- Impact of rail equipments

- Impact of ships

- Hard landing by helicopters on (roof-) structures

In high-buildings the impact in following cases should be recognized:

- Multi-storey car parks

- Structures with approved transport of vehicles or fork lift trucks and

- Structures, which be placed at the side of rail transport.

This thesis is dedicated to the Impact Problems of Elastic Structures Discretized as Multi-Degree-of-Freedom Systems. Here we derive numerical models and algorithms to analyze the response of the system to impact and also two different types of excitation.

Kurzfassung

Eine der wichtigsten Parameter, die bei der Modellierung und Berechnung einer Struktur berücksichtigt werden muss, ist die sogenannte außergewöhnliche Einwirkung. Diese Arbeit behandelt außergewöhnliche Lastfälle für folgende Ereignisse:

- Anprall von Straßenfahrzeugen
- Anprall von Gabelstaplern
- Anprall von Eisenbahnfahrzeugen
- Anprall von Schiffen
- Harte Landung von Helikoptern auf (Dach-) Konstruktionen

Im Hochbau sind Anpralllasten in folgenden Fällen anzusetzen:

- Parkhäuser
- Bauwerke mit zugelassenem Verkehr von Fahrzeugen oder Gabelstaplern und
- Bauwerke, die an Straßenverkehr oder Schienenverkehr angrenzen.

Die vorliegende Arbeit widmet sich den Anprall-Problemen von elastischen Strukturen, diskretisiert als Mehrfreiheitsgradsysteme. Numerischer Modelle und Algorithmen werden entwickelt, um die Systemantwort zufolge Anprall zu analysieren. Dabei werden zwei verschiedene Arten der Stoßmodellierung betrachtet.

Acknowledgment

This master's thesis marks the completion of my graduate studies in Civil Engineering at the Vienna University of Technology. I want to express my thanks to all those who helped me in various ways to achieve this goal.

I want to address special thanks to Ao. Univ. Prof. Dipl.-Ing. Dr.techn. Rudolf Heuer for the excellent supervision. Further special thanks go to all the staff members of the Center of Mechanics and Structural Dynamics who helped me out wherever they could.

Finally, I would like to thank my parents for always being there for me and supporting me and special thanks to Maryam Petersen.

Dedicated to

† Peter Petersen

Contents

INTRODUCTION

In mechanics, an impact is a high force or shock applied over a short time period when two or more bodies collide. Such a force or acceleration usually has a greater effect than a lower force applied over a proportionally longer period of time. The effect depends critically on the relative velocity between the bodies under consideration.

In the first part of this thesis the velocity of colliding object depending on the position of collision is calculated and also the response of 2DOF damped and undamped systems to the collision are determined.

As well known, in many practical situations, the dynamic excitation is neither harmonic nor periodic. Thus we are interested in studying the dynamic response of 2DOF systems to impact-like excitation. In the second part of this thesis, a general procedure is developed to analyze the response of the 2DOF system to the two types of excitation, which are rectangular and half cycle sine pulse force. Also the maximum response to each of these forces is graphically presented.

2 Modeling

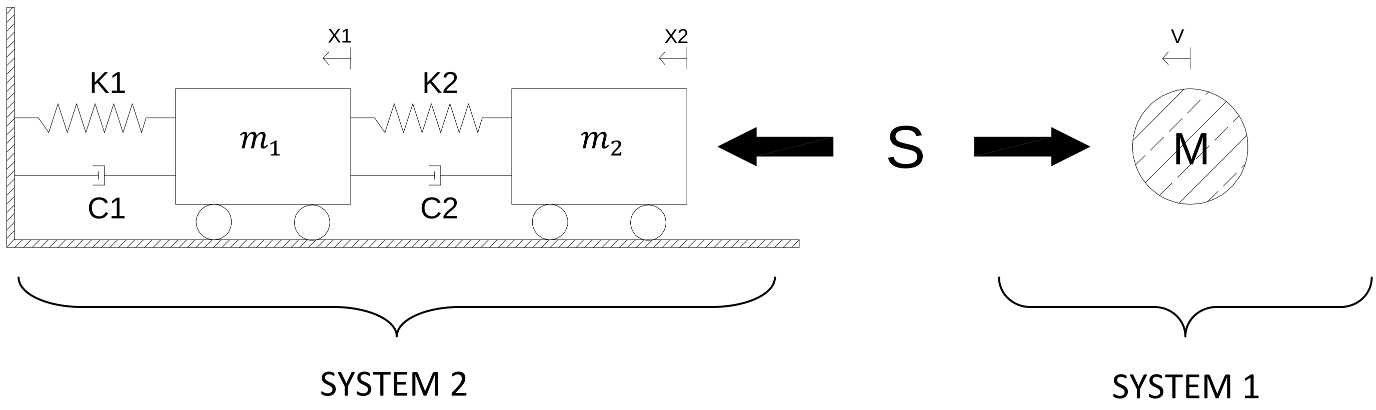

Figure 2.1 Mechanical model to describe the collision

2.1 Collision object

M Mass of system 1

S Impulse

V Velocity of the object before impact

V' Velocity of the object after impact

2.2 System with 2DOF

m_1 Mass of body 1

m_2 Mass of body 2

k_1 Stiffness of body 1

k_2 Stiffness of body 2

c_1 Damping of body 1

c_2 Damping of body 2

V_2 Velocity of body 2 before impact

V_2' Velocity of body 2 after impact

3 Impact

3.1 Introduction

Impact is a process of momentum exchange between two colliding bodies within a short time of contact. With respect to a single impacted body or structure, the loading in such a process acts with high intensity during this short period of time. As a result, the initial velocity distribution is rapidly changed (even pressure wave loadings, eg following an explosion, are events of that category).In this following calculation, the reaction of the structure after impact with consideration of two extreme cases is analyzed.

An elastic collision is an encounter between two bodies in which the total kinetic energy of the two bodies after the encounter is equal to their total kinetic energy before the encounter. Elastic collisions occur only if there is no net conversion of kinetic energy into other forms.

$$T = T^{'} \longrightarrow \Delta E = 0 \tag{3.1}$$

An inelastic collision is a collision in which the kinetic energy is not conserved.

$$V = V_2^{'} \longrightarrow T - T^{'} = max(\Delta E) \tag{3.2}$$

3.1.1 Coefficient of restitution

The coefficient of restitution (COR), or bounciness of an object, is a fractional value representing the ratio of velocities after and before an impact. An object with a COR of 1 collides elastically, while an object with a COR ≡ β< 1 collides inelastically. For the limiting case COR ≡ β = 0, the object effectively "stops" at the surface of collision.

$$V_2' - V' = \beta(V - V_2) \tag{3.3}$$

The energie dissipation is given by the difference of the kinetic energies before and after the impact

$$\Delta T = T - T^{'} \tag{3.4}$$

3.2 The idealized elastic collision of an undamped 2DOF system

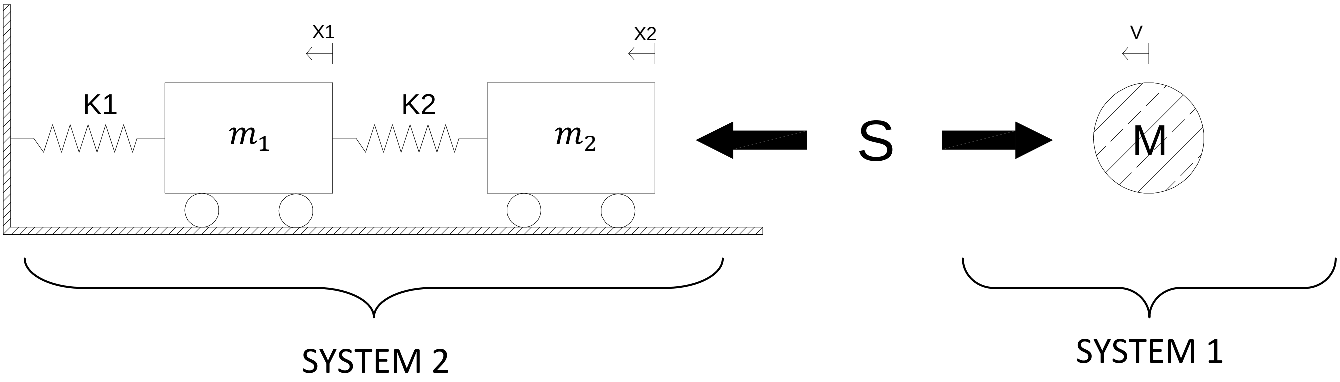

Figure 3.1 Impacting of two masses.

No mechanical energy is dissipated in this case and the conservation of mechanical energy of the two colliding bodies, Eq. (3.5), holds during impact [1],[2],[3].

$$T = T' \tag{3.5}$$

The kinetic energy of the system in general form is written as

$$T(t) = \frac{1}{2}\sum_j m_j \dot{x}_j^2 \tag{3.6}$$

The kinetic energy of system 1 and system 2 before impact

$$T = \frac{1}{2}MV^2 + \frac{1}{2}MV_2^2 \tag{3.7}$$

The kinetic energy of system 1 and system 2 after impact

$$T' = \frac{1}{2}MV'^2 + \frac{1}{2}MV_2'^2 \tag{3.8}$$

Initial condition of the system in case of an elastic collision

$$\beta = 1,\ V_2 = 0 \tag{3.9}$$

Velocity of m_2 immediately after impact is defined as

$$V_2' = \dot{x}_2(t = 0) = \dot{x}_{20} \tag{3.10}$$

As shown in Fig. 3.1, the free body diagram at impact is considered with impulse –S and S acting on the mass *M* and m_2, respectively. The momentum relation is applied to each system to render

$$m_2V_2 + m_2V_2^{'} = S \tag{3.11}$$

$$MV^{'} - MV = -S \tag{3.12}$$

For the case $V_2 = 0$, elimination of S gives

$$MV - MV^{'} = m_2V_2^{'} \tag{3.13}$$

The conservation of energy of the gross system renders

$$T = \frac{1}{2}MV^2 = T^{'} = \frac{1}{2}MV^{'2} + \frac{1}{2}MV_2^{'2} \tag{3.14}$$

Expansion of the difference of squares gives

$$m_2\left(V_2^{'}\right)\left(V_2^{'}\right) + M\left(V^{'} + V\right)\left(V^{'} - V\right) = 0 \tag{3.15}$$

and by substituting of Eqs. (3.11) and (3.12), the impulse becomes a common factor, S≠0, and cancels

$$\left(-V_2^{'}\right)(S) + \left(V^{'} + V\right)(-S) = 0 \tag{3.16}$$

A linear equation results and replaces the nonlinear energy relation

$$V^{'} = V_2^{'} - V \tag{3.17}$$

Substituting Eq. (3.13) into (3.17) gives the velocity of body 2 after impact

$$V_2^{'} = \dot{x}_{20} = \frac{2V}{\left(1 + \frac{m_2}{M}V_2^{'}\right)} \tag{3.18}$$

3.3 The collision with general β of an undamped 2DOF system [4]

As in section 3.1.1 descried, for a general collision, which is between elastic and inelastic collision, the coefficient of restitution is given by

$$\beta = \frac{V_2^{'} - V^{'}}{V - V_2} \tag{3.19}$$

in which

$$0 \le \beta \le 1.0 \tag{3.20}$$

Initial condition of the system

$$V_2 = 0 \tag{3.21}$$

together with Eq. (3.19) becomes

$$(V_2' - V') = \beta V \tag{3.22}$$

As shown in Fig. 3.1, the free body diagram at impact is considered with impulse –S and S acting on the mass *M* and m_2, respectively. Again the momentum relation is applied and leads to

$$m_2 V_2 + m_2 V_2^{'} = S \tag{3.23}$$

$$MV^{'} - MV = -S \tag{3.24}$$

Elimination of S results in

$$MV - MV^{'} = m_2 V_2^{'} \tag{3.25}$$

Substituting Eq. (3.22) into (3.25) gives the velocity of body 2 after impact

$$V_2^{'} = \dot{x}_{20} = \frac{(1+\beta)V}{1 + \frac{m_2}{M}} \tag{3.26}$$

3.4 The Lagrange equation [1]

The kinetic energy T when expressed in a set of generalized coordinates $q_j(j = 1,2,\ldots,n)$ is, in general, a function of the generalized displacement q_j as well as generalized velocities $\dot{q}_j$.

$$T = T(\dot{q}_1, \dot{q}_2, .\,.\,, \dot{q}_n; q_1, q_2, \ldots, q_n) \tag{3.27}$$

It follows that

$$\delta T = \frac{\partial T}{\partial \dot{q}_1}\delta\dot{q}_1 + \frac{\partial T}{\partial \dot{q}_2}\delta\dot{q}_2 + \cdots + \frac{\partial T}{\partial \dot{q}_n}\delta\dot{q}_n$$
$$+ \frac{\partial T}{\partial q_1}\delta q_1 + \frac{\partial T}{\partial q_2}\delta q_2 + \cdots + \frac{\partial T}{\partial q_n}\delta q_n \tag{3.28}$$

Or, in more concise form

$$\delta T = \sum_{j=1}^{n}\left(\frac{\partial T}{\partial \dot{q}_n}\delta\dot{q}_n + \frac{\partial T}{\partial q_n}\delta q_n\right) \tag{3.29}$$

The generalized forces may be obtained from the forces F_j by the principle of virtual work.

$$\sum_{j=1}^{n} Q_j \delta q_j = \sum_{j=1}^{n} F_j \delta x_j \tag{3.30}$$

We recall that $F_j, j = 1,2,\ldots,n$ represent all forces acting on the rigid masses and, hence, may include internal, as well as external, forces. Q_j represent the generalized forces. Each side of Eq. (3.30) represents the virtual work δW done by forces Q_j or F_j on virtual displacements δq_j or δx_j, respectively,

$$\delta W = \sum_{j} Q_j \delta q_j \tag{3.31}$$

Now, we proceed with the development of Lagrange's equations in generalized coordinates, starting with Hamilton's principle:

$$\int_{t_1}^{t_2}(\delta T+\delta W)dt=\int_{t_1}^{t_2}\sum_j(\frac{\partial T}{\partial \dot{q}_j}\delta\dot{q}_j+\frac{\partial T}{\partial \dot{q}_j}\delta q_j+Q_j\delta q_j)dt \tag{3.32}$$

in which δT and δW are substituted from Eqs.(3.29) and (3.31), respectively. Consider the first term in Eq. (3.32)

$$\int_{t_1}^{t_2}\sum_j\frac{\partial T}{\partial \dot{q}_j}\delta\dot{q}_j dt=\sum_j\int_{t_1}^{t_2}\frac{\partial T}{\partial \dot{q}_j}\delta q_j dt$$

$$=-\int_{t_1}^{t_2}\sum_j\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)\delta q_j dt \tag{3.33}$$

Remembering that

$$\frac{\partial T}{\partial \dot{q}_j}\delta q_j|_{t_1}^{t_2}=0 \quad , \ j=1,2,\dots,n \tag{3.34}$$

Using the identity (3.33) in Eq. (3.32) we obtain

$$\int_{t_1}^{t_2}\sum_j\{-\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)+\frac{\partial T}{\delta q_j}+Q_j\}\delta q_j dt=0 \tag{3.35}$$

Since the q_j stand for the generalized coordinates, δq_j are arbitrary except at $t=t_1$ and $t=t_2$, at which instants they are set equal to zero. Consequently, the expression in the brackets of Eq. (3.33) must vanish.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial \dot{q}_j}=Q_j \tag{3.36}$$

There are n of these equations $(j=1,2,\dots,n)$, the Lagrange equations expressed in generalized coordinates.

The generalized force Q_j is generally considered to be composed of three parts

$$Q_j=Q_{Aj}+Q_{Ej}+Q_{Dj} \tag{3.37}$$

where

Q_{Aj} ... external force

Q_{Ej} ... internal elastic force

Q_{Dj} ... damping force; may be internally or externally acting.

Considering the elastic force components Q_{Ej}, with potential U (strain energy), Castigliano theorem gives

$$Q_{Ej} = -\frac{\partial U}{\partial q_j} \tag{3.38}$$

Using relations (3.37) and (3.38), the Lagrange Equation may be written in a modified

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial \dot{q}_j} + \frac{\partial U}{\partial \dot{q}_j} = Q_{Aj} + Q_{Dj} \tag{3.39}$$

3.4.1 Damping forces

The term of Q_{Dj} represents the *j-th* generalized damping force. The expression for Q_{Dj} in terms of damping force F_{Dj} in the constrained x_j coordinate system can be derived from the principle of virtual work. The virtual work δW_D done by damping forces F_{Dj} along virtual displacements δx_j in the x coordinate system is given by

$$\delta W_D = \sum_j F_{Dj} \delta x_j \tag{3.40}$$

We now apply a coordinate transformation

$$x_j = x_j(q_1, q_2, \ldots q_n) \tag{3.41}$$

$$j = 1,2, \ldots, n$$

in order to project the coordinates x_j to generalized coordinates q_j. Then

$$\delta x_j = \sum_{j=1}^{n} \frac{\partial x_j}{\partial q_j} \, \delta q_j \tag{3.41}$$

and Eq.(3.40) becomes

$$\partial W_D = F_{Dj} \sum_j \frac{\partial x_j}{\partial q_j} \partial W_D \delta q_j \tag{3.42}$$

Interchanging the order of summation and rearranging, we write

$$\partial W_D = \sum_j \delta q_j \sum_j F_{Dj} \frac{\partial X_j}{\partial q_i} \tag{3.43}$$

The virtual work δW_D can also be expressed as the sum of the work done by the generalized damping forces Q_{Dj} along their corresponding virtual displacements δq_j

$$\delta W_D = \sum_j Q_{Dj}\, \delta q_j \tag{3.44}$$

Comparing Eqs. (3.43) and (3.44) we write

$$Q_{Dj} = \sum_j F_{Dj} \frac{\partial x_j}{\partial q_j} \tag{3.45}$$

$$j = 1,2, \ldots, n$$

Proceeding in a similar manner, the *j-th* generalized applied force Q_{Aj} on the right – hand side of Eq. (3.39) can be expressed in the form

$$Q_{Aj} = \sum_j F_{Aj} \frac{\partial x_j}{\partial q_j} \tag{3.46}$$

In which $F_{Aj}(j = 1,2, \ldots, n)$ are the applied forces in the constrained $x_j(j = 1,2, \ldots, n)$ coordinate system.

3.5 2DOF damped system after impact at m_2

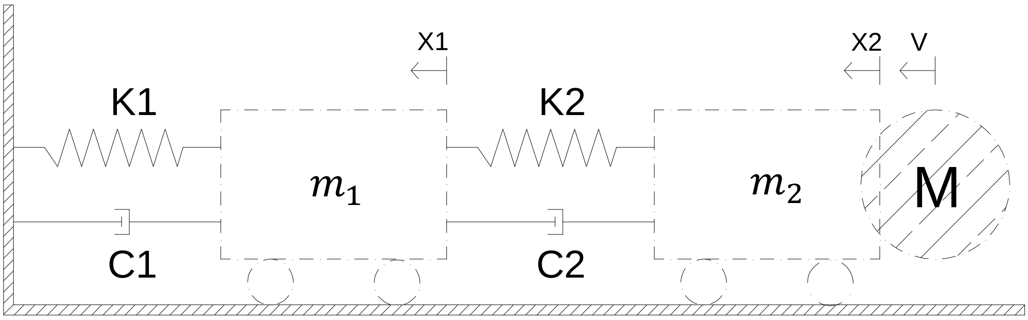

Figure 3.2 A damped system after impact

The Lagrange equation in generalized coordinate is given by

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial \dot{q}_j} + \frac{\partial U}{\partial \dot{q}_j} = Q_{Aj} + Q_{Dj} \tag{3.47}$$

The kinetic energy of system 2 in x coordinates is written as

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 \tag{3.48}$$

And further

$$\frac{\partial T}{\partial \dot{x}_1} = \frac{1}{2} m_1 \ddot{x}_1 \tag{3.49}$$

and

$$\frac{\partial T}{\partial \dot{x}_2} = \frac{1}{2} m_2 \ddot{x}_2 \tag{3.50}$$

The strain energy of system 2 becomes

$$V = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2(x_2 - x_1)^2 = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2(x_2^2 - 2x_1x_2 + x_1^2) \qquad (3.51)$$

and

$$\frac{\partial V}{\partial x_1} = k_1x_1 + \frac{1}{2}k_2(-2x_2 + 2x_1) \qquad (3.52)$$

$$\frac{\partial V}{\partial x_2} = \frac{1}{2}k_2(2x_2 - 2x_1) \qquad (3.53)$$

The damping forces of body 1 and body 2 are

$$F_{D1} = -c_1\dot{x}_1 - c_2(\dot{x}_1 - \dot{x}_2) = -c_1\dot{x}_1 - c_2\dot{x}_1 + c_2\dot{x}_2 \qquad (3.54)$$

$$F_{D2} = -c_2(\dot{x}_2 - \dot{x}_1) = -c_2\dot{x}_2 + c_2\dot{x}_1 \qquad (3.55)$$

Finally, the Lagrange equation is of the form

$$m_1\ddot{x}_1 + k_1x_1 + k_2x_1 - k_2x_2 + c_1\dot{x}_1 + c_2\dot{x}_1 - c_2\dot{x}_2 = 0 \qquad (3.56)$$

and

$$m_2\ddot{x}_2 + k_2x_2 - k_2x_1 + c_2\dot{x}_2 - c_2\dot{x}_1 = 0 \qquad (3.57)$$

Reformulation gives

$$\sum_{i=1}^{n} m_{ri}\,\ddot{x}_i + \sum_{i=1}^{n} c_{ri}\,\dot{x}_i + \sum_{i=1}^{n} k_{ri}x_i = 0 \qquad (3.58)$$

and in matrix form

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{\widetilde{M}} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}}_{\tilde{C}} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}}_{\widetilde{K}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (3.59)$$

3.5.1 Modal Analysis [4],[5],[6]

The eigenvalues and eigenvectors are required for various different analyses. They are used to covert the equations of motion to n independent equations of motion which are exactly the same form as the single degree of freedom (SDOF) equation of motion. The multi degree of freedom equations of motion are transformed to the modal coordinate system. The solution of these uncoupled equations and superposing the modal contribution are referred to as the modal superposition method.

The initial condition of system 2

$$\vec{x}(t=0) \quad \text{and} \quad \dot{\vec{x}}(t=0) \tag{3.60}$$

prescribed in vector form

$$\vec{x}(t=0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \dot{\vec{x}}(t=0) = \begin{bmatrix} 0 \\ \dot{x}_{20} \end{bmatrix} \tag{3.61}$$

The natural frequencies $\omega_1, \omega_2$of the system are determined from the condition

$$Det[\widetilde{K} - \omega^2 \widetilde{M}] = 0 \tag{3.62}$$

The natural mode shapes $\overrightarrow{\emptyset_1}, \overrightarrow{\emptyset_2}$ of the system result from

$$[\widetilde{K} - \omega^2 \widetilde{M}]\overrightarrow{\emptyset_k} = \vec{0} \tag{3.63}$$

Frequently they are normalized as

$$\overrightarrow{\emptyset_1} = \begin{bmatrix} 1 \\ \alpha_1 \end{bmatrix} \quad , \overrightarrow{\emptyset_2} = \begin{bmatrix} 1 \\ \alpha_2 \end{bmatrix} \tag{3.64}$$

In this part the coupled equations of motion are transformed into a set of uncoupled equations, each uncoupled equation is analogous to the equation of motion for a SDOF system, and can be solved in the same way. The response of the system is expanded as:

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, q_k(t) \tag{3.65}$$

$q_k(t)$ is the generalized coordinate representing the variation of the response in mode k with time.

For 2DOF system in matrix form

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} \tag{3.66}$$

the equation of motion for body 1 becomes

$$\ddot{q}_1(t) + 2\xi_1^* \omega_1 \dot{q}_1(t) + \omega_1^2 q_1 = 0 \tag{3.67}$$

and for body 2

$$\ddot{q}_2(t) + 2\xi_2^* \omega_2 \dot{q}_2(t) + \omega_2^2 q_2 = 0 \tag{3.68}$$

Due to the orthogonality of the modes, the generalized masses and stiffnesses can be computed from the following relations:

Generalized masses

$$m_1^* = \overrightarrow{\emptyset_1}^T \widetilde{M} \, \overrightarrow{\emptyset_1} \tag{3.69}$$

$$m_2^* = \overrightarrow{\emptyset_2}^T \widetilde{M} \, \overrightarrow{\emptyset_2} \tag{3.70}$$

Generalized stiffness

$$k_1^* = \overrightarrow{\emptyset_1}^T \widetilde{K} \, \overrightarrow{\emptyset_1} \tag{3.71}$$

$$k_2^* = \overrightarrow{\emptyset_2}^T \widetilde{K} \, \overrightarrow{\emptyset_2} \tag{3.72}$$

A common type of damping used in the nonlinear incremental analysis of structures is to assume that the damping matrix is proportional to the mass and stiffness matrices,

$$\tilde{C} = \eta \, \widetilde{M} + \delta \, \widetilde{K} \tag{3.73}$$

This type of damping is normally referred to as Rayleigh damping. In mode superposition analysis the damping matrix must have the following properties in order for the modal equations to be uncoupled:

$$2\omega_n\xi_n\, m_n^* = \emptyset_n^T\, \tilde{C}\, \emptyset_n \tag{3.74}$$

Due to the orthogonality properties of the mass and stiffness matrices, this equation can be rewritten as

$$2\omega_n\xi_n = \eta + \delta\, \omega_n^2 \tag{3.75}$$

It is apparent that modal damping can be specified exactly at only two frequencies in order to solve for η and δ in the above equation.

Generalized damping

$$c_1^* = \overrightarrow{\emptyset_1}^T\, \tilde{C}\, \overrightarrow{\emptyset_1} \tag{3.76}$$

$$c_2^* = \overrightarrow{\emptyset_2}^T\, \tilde{C}\, \overrightarrow{\emptyset_2} \tag{3.77}$$

Damping ratios

$$\xi_1^* = \frac{c_1^*}{2\sqrt{k_1^* m_1^*}} \tag{3.78}$$

$$\xi_2^* = \frac{c_2^*}{2\sqrt{k_2^* m_2^*}} \tag{3.79}$$

The response of the damped system, becomes

$$q_k = e^{-\xi_k^*\omega_k t}(A_k Cos\omega_{Dk}t + B_k Sin\omega_{Dk}t) \tag{3.80}$$

with

$$\omega_{Dk} = \omega_k\sqrt{1 - \xi_{\mathrm{k}}^{*2}} \tag{3.81}$$

Substituting Eq. (3.80) in (3.65) gives

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, (e^{-\xi_k^*\omega_k t}(A_k Cos\omega_{Dk}t + B_k Sin\omega_{Dk})) \tag{3.82}$$

where A_k and B_k are $2N$ constants of integration. These can be expressed in terms of initial conditions

$$A_k = q_k(0) \tag{3.83}$$

$$B_k = \frac{\dot{q}_k(0) + \xi_k^* \omega_k q_k(0)}{\omega_{Dk}} \tag{3.84}$$

Calculation the velocity of the system in generalized coordinates

$$\overrightarrow{\dot{x}_0} = \widetilde{\emptyset}\, \overrightarrow{\dot{q}_0} \tag{3.85}$$

in matrix form

$$\begin{bmatrix} 0 \\ \dot{x}_{20} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} \dot{q}_{10} \\ \dot{q}_{20} \end{bmatrix} \tag{3.86}$$

and

$$\begin{bmatrix} \dot{q}_{10} \\ \dot{q}_{20} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \dot{x}_{20} \end{bmatrix} \tag{3.87}$$

finally

$$= \frac{1}{\alpha_{2-}\alpha_1} \begin{bmatrix} \alpha_2 & -1 \\ -\alpha_1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \dot{x}_{20} \end{bmatrix} = \frac{\dot{x}_{20}}{\alpha_{2-}\alpha_1} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \tag{3.88}$$

With respect to the initial conditions the displacement of the system is written as

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, q_k(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k} \dot{q}_{k0} \frac{e^{-\xi_k^* \omega_k t}}{\omega_k} Sin\omega_k t \tag{3.89}$$

as in matrix form

$$\vec{x}(t) = \begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} \dot{q}_{10} \dfrac{e^{-\xi_1^* \omega_1 t}}{\omega_1} Sin\omega_1 t \\ \dot{q}_{20} \dfrac{e^{-\xi_2^* \omega_2 t}}{\omega_2} Sin\omega_2 t \end{bmatrix} \tag{3.90}$$

Substituting Eq. (3.88) in Eq. (3.90),

$$= \frac{\dot{x}_{20}}{\alpha_{2-}\alpha_1} \begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} -\dfrac{e^{-\xi_1^* \omega_1 t}}{\omega_1} Sin\omega_1 t \\ \dfrac{e^{-\xi_2^* \omega_2 t}}{\omega_2} Sin\omega_2 t \end{bmatrix} \tag{3.91}$$

finally the displacement of the system becomes in matrix form

$$\vec{x}(t) = \frac{\dot{x}_{20}}{\alpha_{2-}\alpha_1} \begin{bmatrix} -\dfrac{e^{-\xi_1^* \omega_1 t}}{\omega_1} Sin\omega_1 t + \dfrac{e^{-\xi_2^* \omega_2 t}}{\omega_2} Sin\omega_2 t \\ -\alpha_1 \dfrac{e^{-\xi_1^* \omega_1 t}}{\omega_1} Sin\omega_1 t + \alpha_2 \dfrac{e^{-\xi_2^* \omega_2 t}}{\omega_2} Sin\omega_2 t \end{bmatrix} \tag{3.92}$$

3.6 2DOF undamped system after impact at m_2

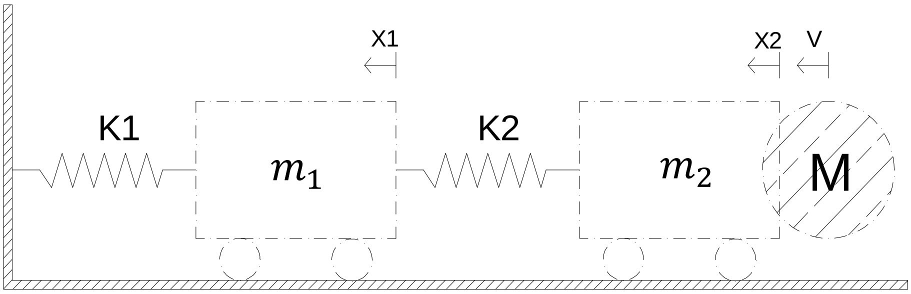

Figure 3.3 An undamped system after impact

The equation of motion, Eq.(3.59), for an undamped system in matrix form reads

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{\widetilde{M}} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}}_{\widetilde{K}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3.93}$$

From Eq.(3.82) the displacement of the system is written as

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k} \left[q_k(0) Cos\omega_k t + \frac{\dot{q}_k(0)}{\omega_k} Sin\omega_k t \right] \tag{3.94}$$

In case of a 2DOF system

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} \tag{3.95}$$

the initial condition of system 2

$$\vec{x}(t = 0) \quad \text{and} \quad \dot{\vec{x}}(t = 0) \tag{3.96}$$

in vector form are

$$\vec{x}(t=0)=\begin{bmatrix}0\\0\end{bmatrix} \qquad \dot{\vec{x}}(t=0)=\begin{bmatrix}0\\\dot{x}_{20}\end{bmatrix} \tag{3.97}$$

With respect to initial conditions the displacement of the system is rewritten as

$$\vec{x}(t)=\sum_{k=1}^{2}\overrightarrow{\emptyset_k}\,q_k(t)=\sum_{k=1}^{2}\overrightarrow{\emptyset_k}\,\frac{\dot{q}_{k0}}{\omega_k}Sin\omega_k t \tag{3.98}$$

as in matrix form

$$\vec{x}(t)=\begin{bmatrix}1 & 1\\\alpha_1 & \alpha_2\end{bmatrix}\begin{bmatrix}\frac{\dot{q}_{10}}{\omega_1}Sin\omega_1 t\\\frac{\dot{q}_{20}}{\omega_2}Sin\omega_2 t\end{bmatrix} \tag{3.99}$$

Substituting Eq. (3.88) in Eq. (3.99),

$$=\frac{\dot{x}_{20}}{\alpha_{2-}\alpha_1}\begin{bmatrix}1 & 1\\\alpha_1 & \alpha_2\end{bmatrix}\begin{bmatrix}-\frac{Sin\omega_1 t}{\omega_1}\\\frac{Sin\omega_2 t}{\omega_2}\end{bmatrix} \tag{3.100}$$

Finally the displacement of the system in matrix form is written as

$$\vec{x}(t)=\frac{\dot{x}_{20}}{\alpha_{2-}\alpha_1}\begin{bmatrix}\frac{Sin\omega_2 t}{\omega_2}-\frac{Sin\omega_1 t}{\omega_1}\\\alpha_2\frac{Sin\omega_2 t}{\omega_2}-\alpha_1\frac{Sin\omega_1 t}{\omega_1}\end{bmatrix} \tag{3.101}$$

3.7 2DOF damped system after impact at m_1

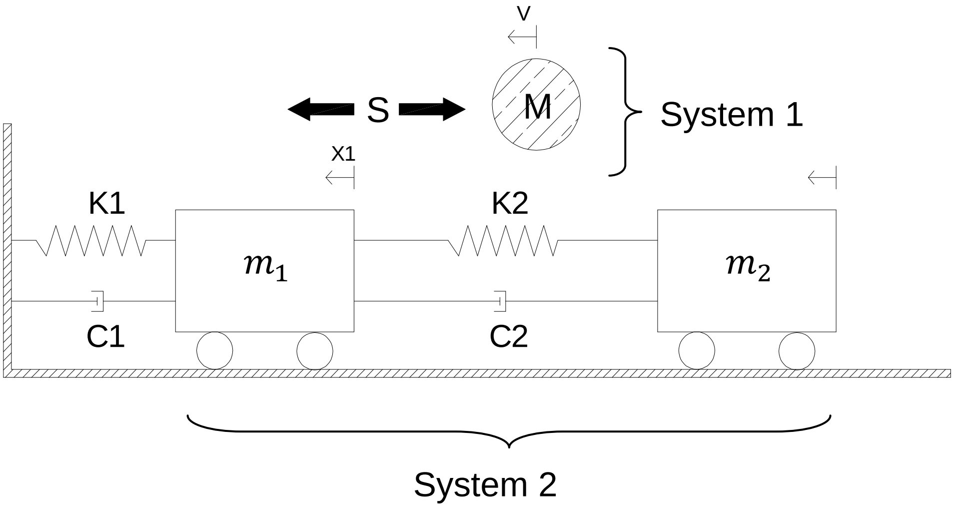

Figure 3.4 A damped system after impact

3.7.1 Collision with general β

$$\beta = \frac{V_1^{'} - V^{'}}{V - V_1} \tag{3.102}$$

where

$$0 \leq \beta \leq 1.0 \tag{3.103}$$

and the initial condition of the system

$$V_1 = 0 \tag{3.104}$$

Thus Eq. (3.102) becomes

$$(V_1' - V') = \beta V \tag{3.105}$$

As shown in Fig. 3.4 the free body diagram at impact is considered with impulse –S and S acting on the mass *M* and m_1, respectively. The momentum relation is applied to render

$$m_1 V_1 + m_1 V_1^{'} = S \tag{3.106}$$

$$MV^{'} - MV = -S \tag{3.107}$$

Elimination of S results in

$$MV - MV' = m_1 V_1' \tag{3.108}$$

Substituting Eq. (3.105) into (3.108) gives the velocity of body 1 after impact

$$V_1' = \dot{x}_{10} = \frac{(1+\beta)V}{1 + \frac{m_1}{M}} \tag{3.109}$$

The Lagrange equation in matrix form is written as

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{\widetilde{M}} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}}_{\tilde{C}} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}}_{\widetilde{K}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3.110}$$

After calculating the natural frequencies and eigenvectors of the system by means of modal analysis then we can write the equation of motion in generalized coordinates, and declared, the general solution of Eq. (3.110) is given by superposition of response in individual modes. Thus

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, q_k(t) \tag{3.111}$$

as in matrix form

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} \tag{3.112}$$

The initial conditions of system 2 are

$$\vec{x}(t=0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \dot{\vec{x}}(t=0) = \begin{bmatrix} \dot{x}_{10} \\ 0 \end{bmatrix} \tag{3.113}$$

Calculation the velocity of system in generalized coordinate

$$\overrightarrow{\dot{x}_0} = \widetilde{\emptyset}\,\overrightarrow{\dot{q}_0} \tag{3.114}$$

as

$$\begin{bmatrix} \dot{x}_{10} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} \dot{q}_{10} \\ \dot{q}_{20} \end{bmatrix} \tag{3.115}$$

and

$$\begin{bmatrix} \dot{q}_{10} \\ \dot{q}_{20} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix}^{-1} \begin{bmatrix} \dot{x}_{10} \\ 0 \end{bmatrix} \tag{3.116}$$

Finally

$$= \frac{1}{\alpha_{2-}\alpha_1} \begin{bmatrix} \alpha_2 & -1 \\ -\alpha_1 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_{10} \\ 0 \end{bmatrix} = \frac{\dot{x}_{10}}{\alpha_{2-}\alpha_1} \begin{bmatrix} \alpha_2 \\ -\alpha_1 \end{bmatrix} \tag{3.117}$$

From Eq.(3.82) displacement of the system is written as

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, q_k(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k} \dot{q}_{k0} \frac{e^{-\xi_k^* \omega_k t}}{\omega_k} Sin\omega_k t \tag{3.118}$$

In matrix form

$$\vec{x}(t) = \begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} \dot{q}_{10} \dfrac{e^{-\xi_1^* \omega_1 t}}{\omega_1} Sin\omega_1 t \\ \dot{q}_{20} \dfrac{e^{-\xi_2^* \omega_2 t}}{\omega_2} Sin\omega_2 t \end{bmatrix} \tag{3.119}$$

Substituting Eq. (3.117) in Eq. (3.119),

$$= \frac{\dot{x}_{10}}{\alpha_{2-}\alpha_1}\begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix}\begin{bmatrix} \alpha_2 \dfrac{e^{-\xi_1^*\omega_1 t}}{\omega_1} Sin\omega_1 t \\ -\alpha_1 \dfrac{e^{-\xi_2^*\omega_2 t}}{\omega_2} Sin\omega_2 t \end{bmatrix} \tag{3.120}$$

Finally the displacement of the system in matrix form is written as

$$\vec{x}(t) = \frac{\dot{x}_{10}}{\alpha_{2-}\alpha_1}\begin{bmatrix} \alpha_2 \dfrac{e^{-\xi_1^*\omega_1 t}}{\omega_1} Sin\omega_1 t - \alpha_1 \dfrac{e^{-\xi_2^*\omega_2 t}}{\omega_2} Sin\omega_2 t \\ \alpha_1\alpha_2 \dfrac{e^{-\xi_1^*\omega_1 t}}{\omega_1} Sin\omega_1 t - \alpha_1\alpha_2\alpha_2 \dfrac{e^{-\xi_2^*\omega_2 t}}{\omega_2} Sin\omega_2 t \end{bmatrix} \tag{3.121}$$

3.8 2DOF undamped system after impact at m_1

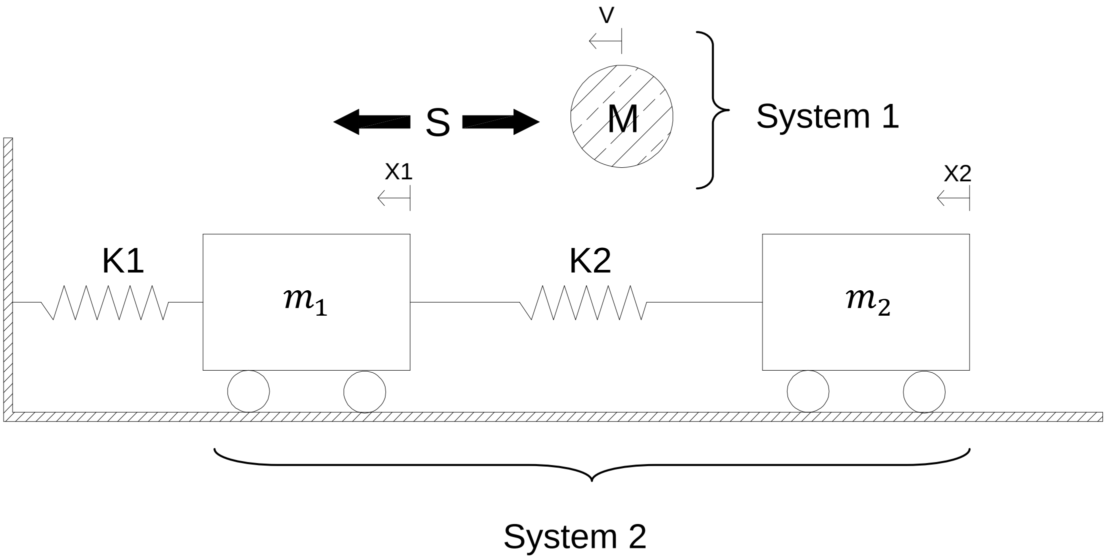

Figure 3.5 An undamped system after impact

As in section 3.5 the Lagrange equation in matrix form is written as

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{\widetilde{M}} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}}_{\widetilde{K}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3.122}$$

After calculating the natural frequencies and modes of the system with help of modal analysis now we can write the equation of motion in generalized coordinate, as we declared the general solution of Eq. (3.122) is given by a superposition of response in individual modes. Thus

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, q_k(t) \tag{3.123}$$

From Eq.(3.82) displacement of the system is written as

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, q_k(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k} \frac{\dot{q}_{k0}}{\omega_k} Sin\omega_k t \tag{3.124}$$

In matrix form

$$\vec{x}(t) = \begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} \frac{\dot{q}_{10}}{\omega_1} Sin\omega_1 t \\ \frac{\dot{q}_{20}}{\omega_2} Sin\omega_2 t \end{bmatrix} \tag{3.125}$$

Substituting Eq. (3.117) in Eq. (3.125),

$$= \frac{\dot{x}_{10}}{\alpha_{2-}\alpha_1} \begin{bmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} \alpha_2 \frac{Sin\omega_1 t}{\omega_1} \\ -\alpha_1 \frac{Sin\omega_2 t}{\omega_2} \end{bmatrix} \tag{3.126}$$

Finally the displacement of the system in matrix form is written as

$$\vec{x}(t) = \frac{\dot{x}_{10}}{\alpha_{2-}\alpha_1} \begin{bmatrix} \alpha_2 \frac{Sin\omega_1 t}{\omega_1} - \alpha_1 \frac{Sin\omega_2 t}{\omega_2} \\ \alpha_1\alpha_2 \frac{Sin\omega_1 t}{\omega_1} - \alpha_1\alpha_2 \frac{Sin\omega_2 t}{\omega_2} \end{bmatrix} \tag{3.127}$$

4. Numerical examples

4.1 2DOF damped system after impact at m_2

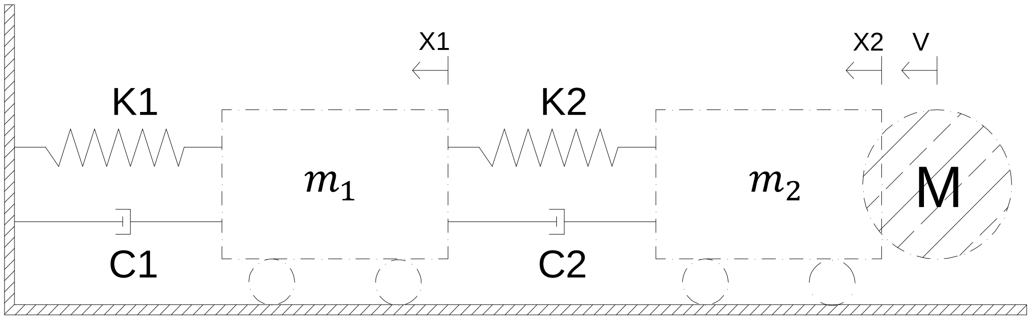

Figure 4.1 A damped system after impact

$m_1 = 20000\ [kg]$ $K_1 = 7000000\ [N/m]$ $C_1 = 200\ [Ns/m]$

$m_2 = 15000\ [kg]$ $K_2 = 6000000\ [N/m]$ $C_2 = 3000\ [Ns/m]$

$M = 5000\ [kg]$ $V = 17\ [m/s]$ $\beta = 0.6$

The Lagrange equation for system in term of coordinate x is

$$\underbrace{\begin{bmatrix} 20000 & 0 \\ 0 & 15000 \end{bmatrix}}_{\widetilde{M}} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 3200 & -3000 \\ -3000 & 3000 \end{bmatrix}}_{\tilde{C}} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1.3 \times 10^7 & -6 \times 10^6 \\ -6 \times 10^6 & 6 \times 10^6 \end{bmatrix}}_{\widetilde{K}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (4.1)$$

4.1.1 Modal analysis

Natural frequencies:

$$\omega_1 = 12.5191 \quad \left[\frac{rad}{s}\right] \qquad \omega_2 = 29.8877 \quad \left[\frac{rad}{s}\right] \quad (4.2)$$

Eigevectors:

$$\overrightarrow{\emptyset_1} = \begin{bmatrix} 1 \\ 1.644 \end{bmatrix} \qquad \overrightarrow{\emptyset_2} = \begin{bmatrix} 1 \\ -0.8109 \end{bmatrix} \tag{4.3}$$

Generalized mass matrix:

$$\widetilde{m}^* = \begin{bmatrix} 60553 & 0 \\ 0 & 29863.6 \end{bmatrix} \quad [kg] \tag{4.4 }$$

Generalized stiffness matrix:

$$\tilde{k}^* = \begin{bmatrix} 9.4903 \times 10^6 & 0 \\ 0 & 2.66764 \times 10^7 \end{bmatrix} \quad \left[\frac{N}{m}\right] \tag{4.5}$$

Generalized damping matrix:

$$\tilde{c}^* = \begin{bmatrix} 1445.15 & 0 \\ 0 & 10038.2 \end{bmatrix} \quad \left[\frac{Ns}{m}\right] \tag{4.6}$$

Damping ratio:

$$\xi_1^* = \frac{c_1^*}{2\sqrt{k_1^* m_1^*}} \quad = \quad 0.000953179$$

$$\xi_2^* = \frac{c_2^*}{2\sqrt{k_2^* m_2^*}} \quad = \quad 0.00562329 \tag{4.7}$$

Velocity of body 2 after impact:

$$V_2^{'} = \dot{x}_{20} = \frac{(1+\beta)V}{1+\frac{m_2}{M}} \quad = \quad 6.80 \quad \left[\frac{m}{s}\right] \tag{4.8}$$

The equation of motion for body 1 and body 2 in generalized coordinates:

$$\ddot{q}_1(t) + 2\xi_1^*\omega_1\dot{q}_1(t) + \omega_1^2 q_1 = 0$$

$$\ddot{q}_2(t) + 2\xi_2^*\omega_2\dot{q}_2(t) + \omega_2^2 q_2 = 0 \tag{4.9}$$

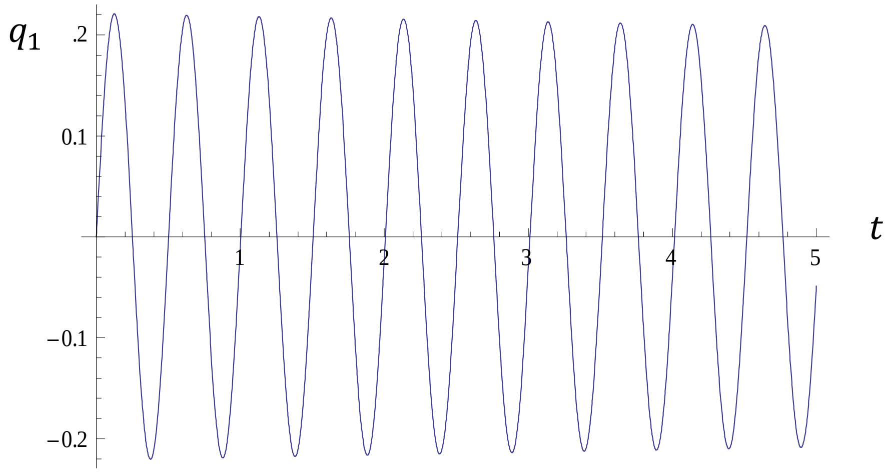

Figure 4.2 Deflection of body 1 after impact $0 \leq t \leq 5.0\ [s]$

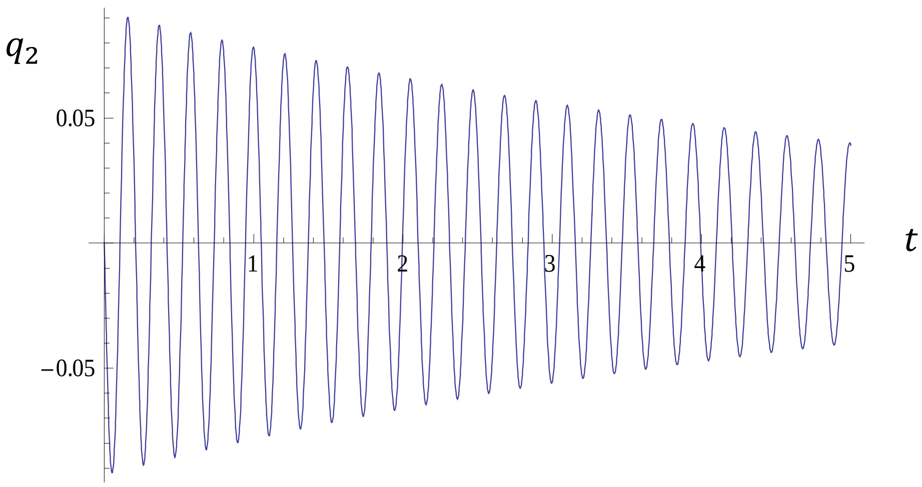

Figure 4.3 Deflection of body 2 after impact $0 \leq t \leq 5.0\ [s]$

Displacement of the system

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, q_k(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, \dot{q}_{k0} \frac{e^{-\xi_k^* \omega_k t}}{\omega_k} Sin\omega_k t \qquad (4.10)$$

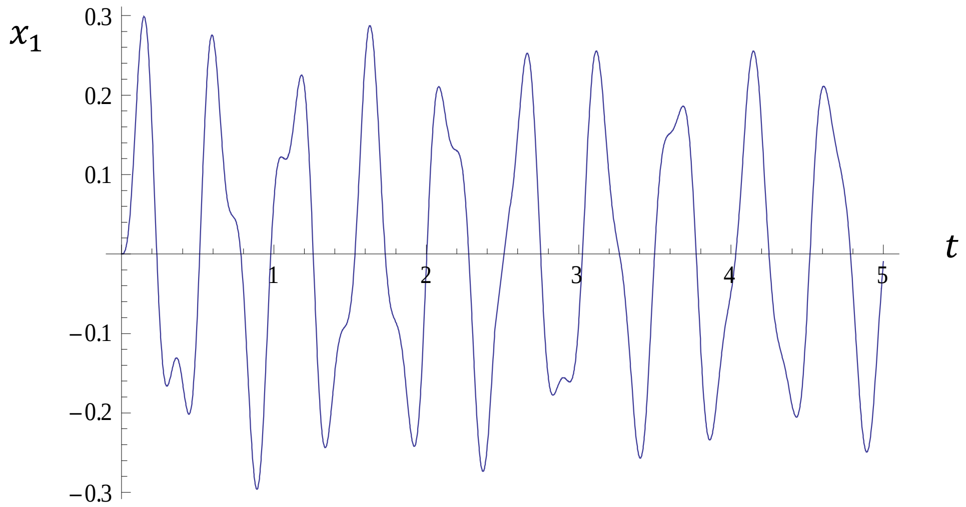

Figure 4.4 Displacement of the system, x_1

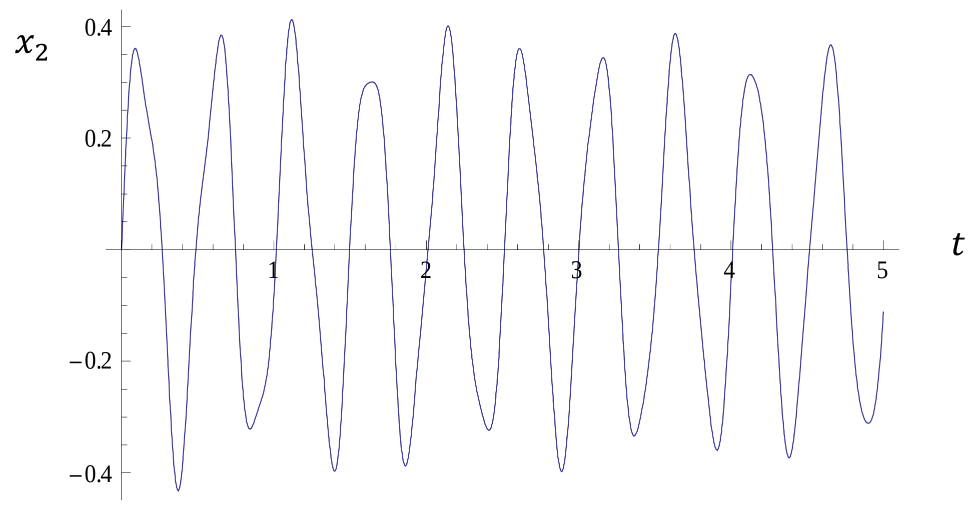

Figure 4.5 Displacement of the system, x_2

4.2 2DOF undamped system after impact at m_2

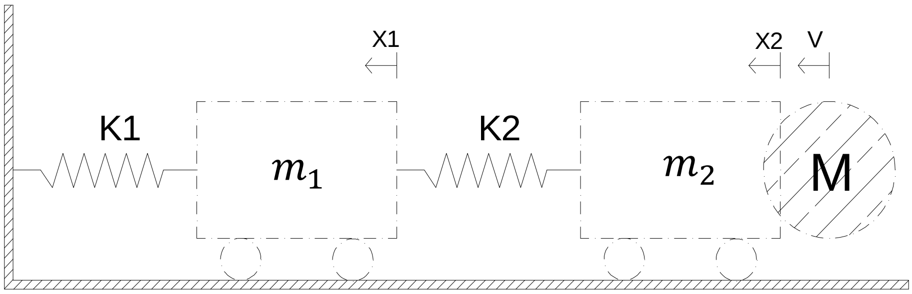

Figure 4.6 An undamped system after impact

$m_1 = 20000\ [kg]$ $\quad K_1 = 7000000\ [N/m]$

$m_2 = 15000\ [kg]$ $\quad K_2 = 6000000\ [N/m]$

$M = 5000\ [kg]$ $\quad V = 17\ \left[\frac{m}{s}\right]$ $\quad \beta = 0.6$

The Lagrange equation for system in term of coordinate x is

$$\underbrace{\begin{bmatrix} 20000 & 0 \\ 0 & 15000 \end{bmatrix}}_{\widetilde{M}} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1.3 \times 10^7 & -6 \times 10^6 \\ -6 \times 10^6 & 6 \times 10^6 \end{bmatrix}}_{\widetilde{K}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{4.11}$$

4.2.1 Modal analysis

Natural frequencies:

$$\omega_1 = 12.5191 \quad \left[\frac{rad}{s}\right] \qquad \omega_2 = 29.8877 \quad \left[\frac{rad}{s}\right] \tag{4.12}$$

Eigevectors:

$$\overrightarrow{\emptyset_1} = \begin{bmatrix} 1 \\ 1.644 \end{bmatrix} \qquad \overrightarrow{\emptyset_2} = \begin{bmatrix} 1 \\ -0.8109 \end{bmatrix} \tag{4.13}$$

Generalized mass matrix:

$$\widetilde{m}^* = \begin{bmatrix} 60553 & 0 \\ 0 & 29863.6 \end{bmatrix} \quad [kg] \tag{4.14}$$

Generalized stiffness matrix:

$$\tilde{k}^* = \begin{bmatrix} 9.4903 \times 10^6 & 0 \\ 0 & 2.66764 \times 10^7 \end{bmatrix} \quad \left[\frac{N}{m}\right] \tag{4.15}$$

Velocity of body2 after impact:

$$V_2' = \dot{x}_{20} = \frac{(1+\beta)V}{1+\frac{m_2}{M}} = 6.80 \quad \left[\frac{m}{s}\right] \tag{4.16}$$

The equation of motion for body 1 and body 2 in generalized coordinates:

$$m_1^*\ddot{q}_1 + k_1^* q_1 = 0$$

$$m_2^*\ddot{q}_2 + k_2^* q_2 = 0 \tag{4.17}$$

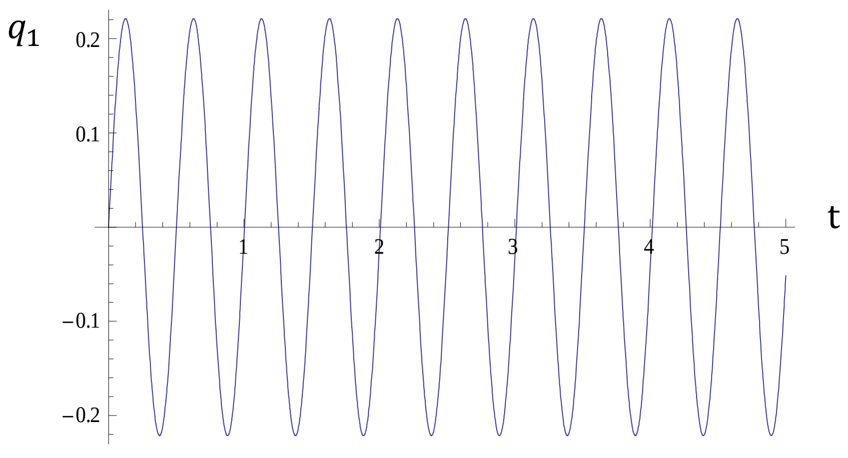

Figure 4.7 Deflection of body 1 after impact $0 \le t \le 5.0\ [s]$

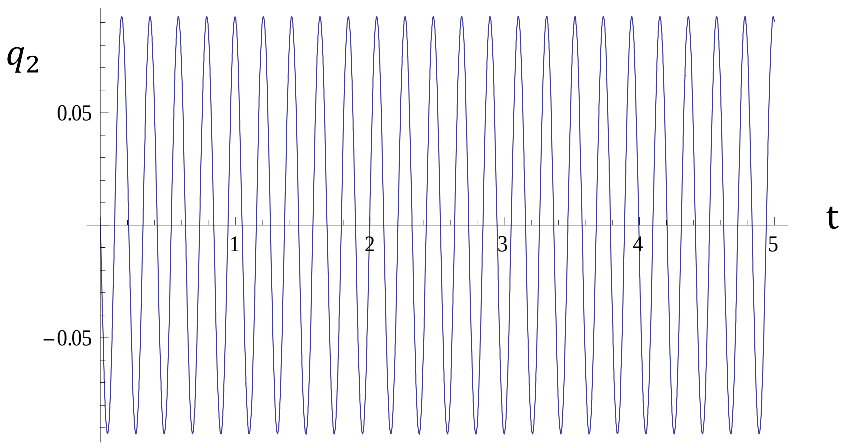

Figure 4.8 Deflection of body 2 after impact $0 \le t \le 5.0\ [s]$

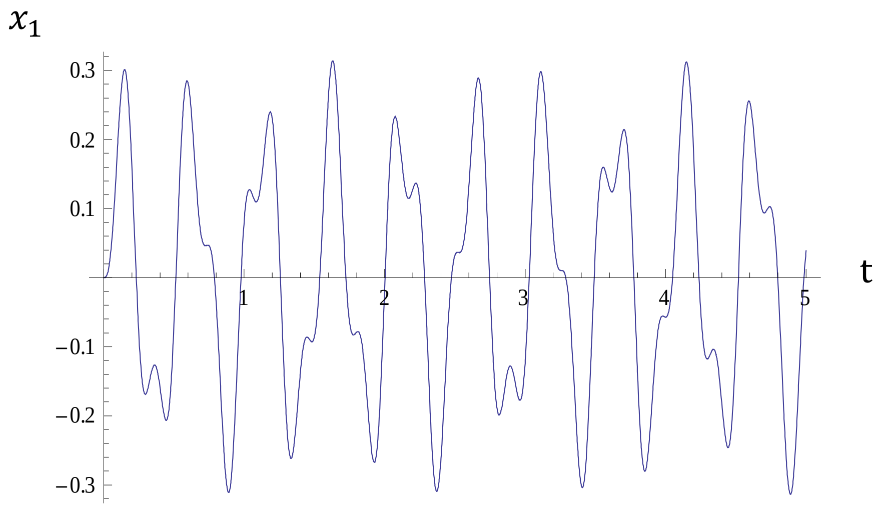

Figure 4.9 Displacement of the system, x_1

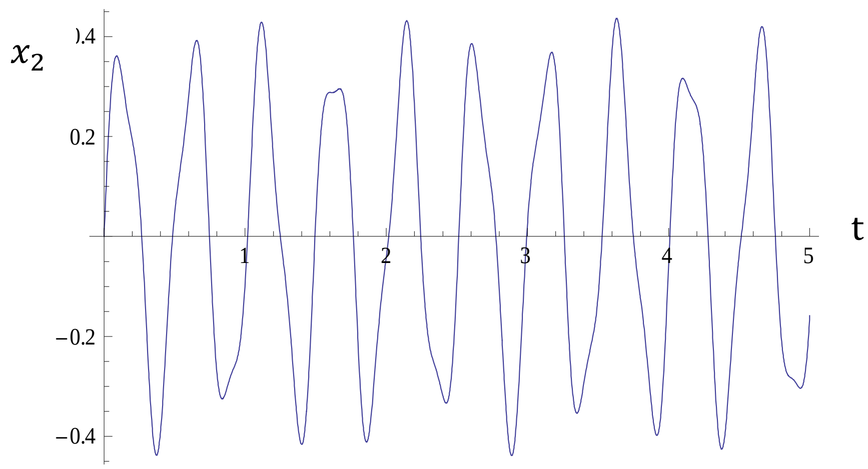

Figure 4.10 Displacement of the system, x_2

4.3 2DOF damped system after impact at m_1

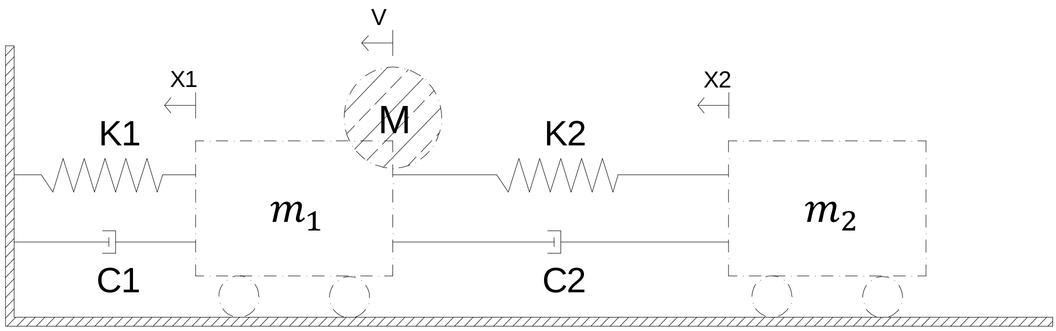

Figure 4.11 A damped system after impact

$m_1 = 20000\ \ [kg]$ $\quad K_1 = 7000000\ \ [N/m]$ $\quad C_1 = 200\ \ \ [Ns/m]$

$m_2 = 15000\ \ [kg]$ $\quad K_2 = 6000000\ \ [N/m]$ $\quad C_2 = 3000\ \ [Ns/m]$

$M = 5000\ \ [kg]$ $\quad V = 17\ \ [m/s]$ $\quad \beta = 0.6$

The Lagrange equation for system in term of coordinate x is

$$\underbrace{\begin{bmatrix} 20000 & 0 \\ 0 & 15000 \end{bmatrix}}_{\widetilde{M}} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 3200 & -3000 \\ -3000 & 3000 \end{bmatrix}}_{\tilde{C}} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1.3\times 10^7 & -6\times 10^6 \\ -6\times 10^6 & 6\times 10^6 \end{bmatrix}}_{\widetilde{K}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{4.18}$$

4.3.1 Modal analysis

Natural frequencies:

$$\omega_1 = \ 12.5191 \quad \left[\frac{rad}{s}\right] \qquad\qquad \omega_2 = \ 29.8877 \quad \left[\frac{rad}{s}\right] \tag{4.19}$$

Eigevectors:

$$\overrightarrow{\emptyset_1} = \begin{bmatrix} 1 \\ 1.644 \end{bmatrix} \qquad \overrightarrow{\emptyset_2} = \begin{bmatrix} 1 \\ -0.8109 \end{bmatrix} \tag{4.20}$$

Generalized mass matrix:

$$\widetilde{m}^* = \begin{bmatrix} 60553 & 0 \\ 0 & 29863.6 \end{bmatrix} \quad [kg] \tag{4.21}$$

Generalized stiffness matrix:

$$\tilde{k}^* = \begin{bmatrix} 9.4903 \times 10^6 & 0 \\ 0 & 2.66764 \times 10^7 \end{bmatrix} \left[\frac{N}{m}\right] \tag{4.22}$$

Generalized damping matrix:

$$\tilde{c}^* = \begin{bmatrix} 1445.15 & 0 \\ 0 & 10038.2 \end{bmatrix} \quad \left[\frac{Ns}{m}\right] \tag{4.23}$$

Damping ratio:

$$\xi_1^* = \frac{c_1^*}{2\sqrt{k_1^* m_1^*}} \quad = \quad 0.000953179$$

$$\xi_2^* = \frac{c_2^*}{2\sqrt{k_2^* m_2^*}} \quad = \quad 0.00562329 \tag{4.24}$$

Velocity of body1 after impact:

$$V_1^{'} = \dot{x}_{10} = \frac{(1+\beta)V}{1+\frac{m_1}{M}} \quad = \quad 5.44 \quad \left[\frac{m}{s}\right] \tag{4.25}$$

The equation of motion for body 1 and body 2 in generalized coordinates:

$$\ddot{q}_1(t) + 2\xi_1^*\omega_1\dot{q}_1(t) + \omega_1^2 q_1 = 0$$

$$\ddot{q}_2(t) + 2\xi_2^*\omega_2\dot{q}_2(t) + \omega_2^2 q_2 = 0 \tag{4.26}$$

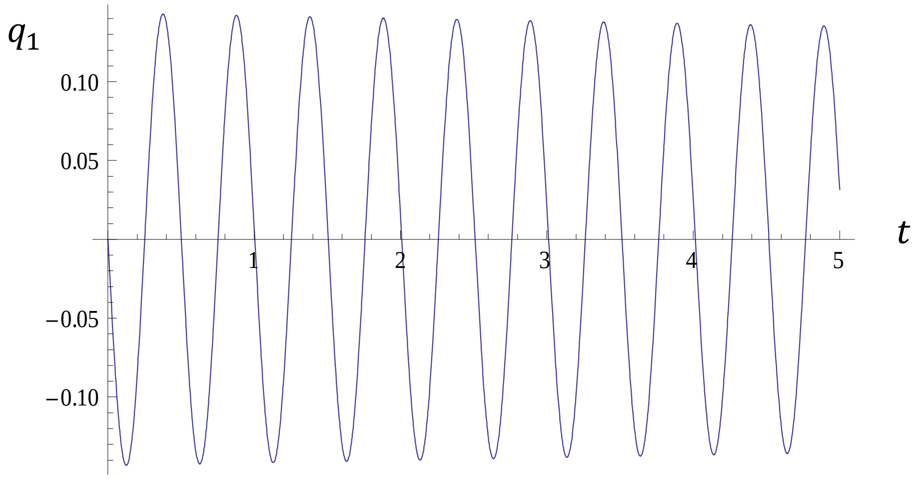

Figure 4.12 Deflection of body 1 after impact $0 \leq t \leq 5.0\ [s]$

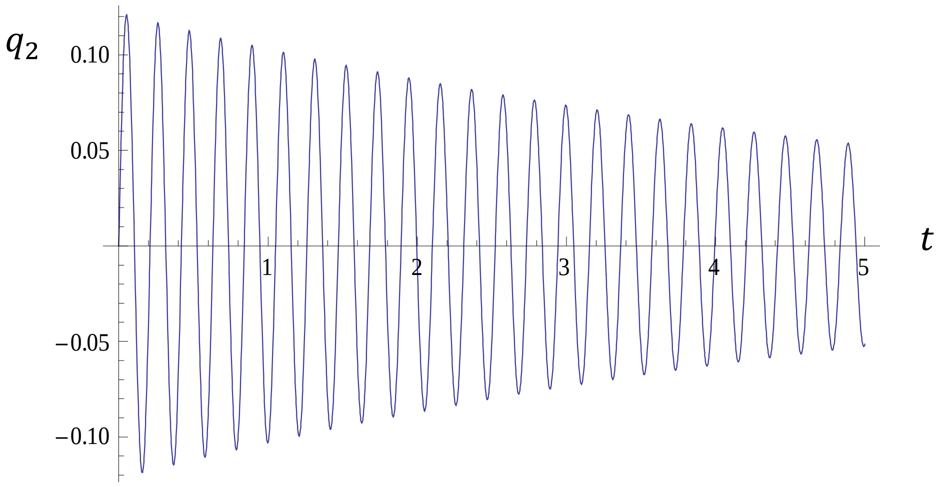

Figure 4.13 Deflection of body 2 after impact $0 \leq t \leq 5.0\ [s]$

Displacement of the system

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, q_k(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, \dot{q}_{k0} \frac{e^{-\xi_k^* \omega_k t}}{\omega_k} Sin\omega_k t \tag{4.27}$$

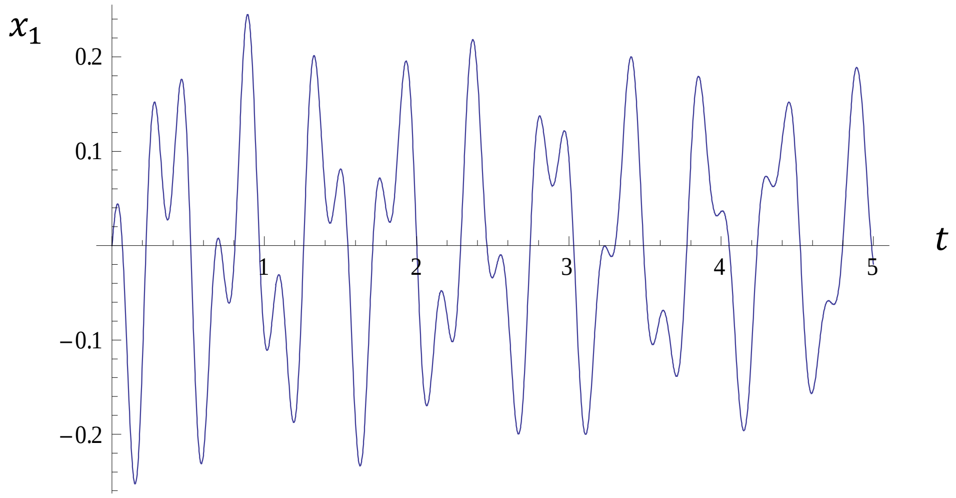

Figure 4.14 Displacement of the system, x_1

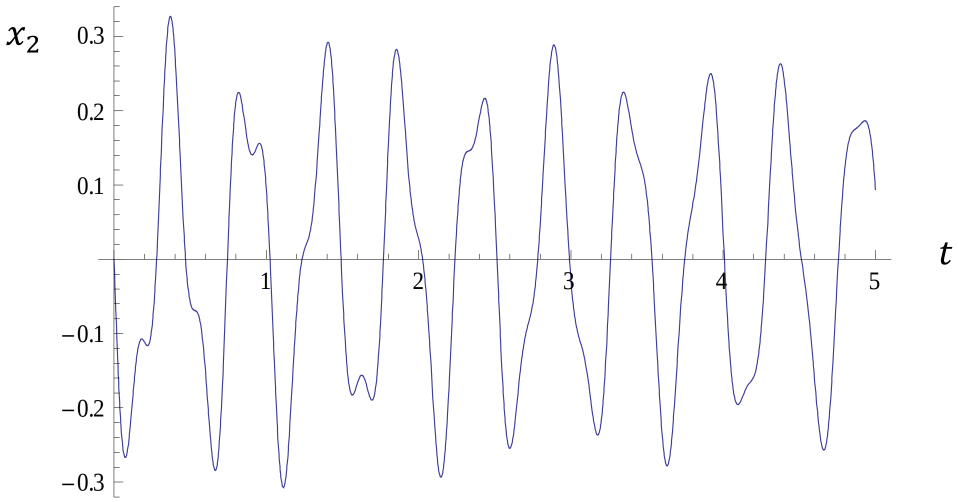

Figure 4.15 Displacement of the system, x_2

4.4 2DOF undamped system after impact at m_1

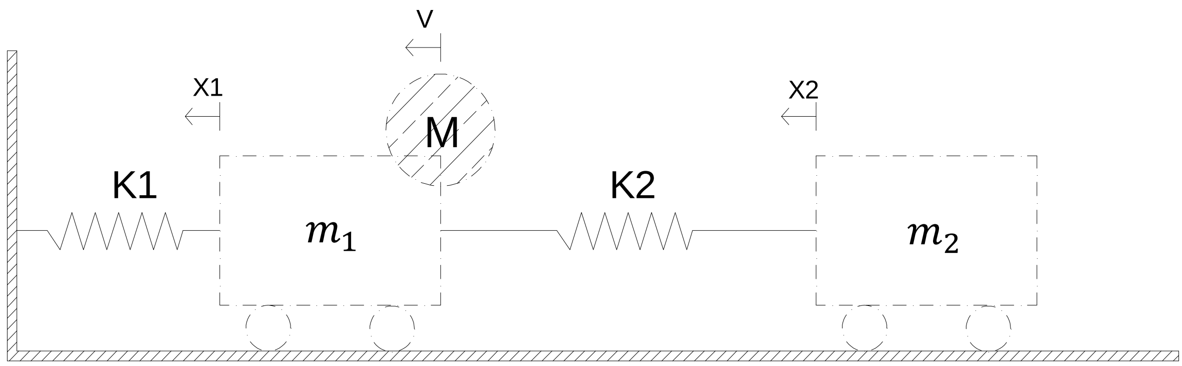

Figure 4.16 An undamped system after impact

$m_1 = 20000\ [kg]$ $K_1 = 7000000\ [N/m]$

$m_2 = 15000\ [kg]$ $K_2 = 6000000\ [N/m]$

$M = 5000\ [kg]$ $V = 17\ \left[\frac{m}{s}\right]$ $\beta = 0.6$

The Lagrange equation for system in term of coordinate x is

$$\underbrace{\begin{bmatrix} 20000 & 0 \\ 0 & 15000 \end{bmatrix}}_{\widetilde{M}} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1.3 \times 10^7 & -6 \times 10^6 \\ -6 \times 10^6 & 6 \times 10^6 \end{bmatrix}}_{\widetilde{K}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{4.28}$$

4.4.1 Modal analysis

Natural frequencies:

$$\omega_1 = 12.5191 \quad \left[\frac{rad}{s}\right] \qquad \omega_2 = 29.8877 \quad \left[\frac{rad}{s}\right] \tag{4.29}$$

Eigevectors:

$$\overrightarrow{\emptyset_1} = \begin{bmatrix} 1 \\ 1.644 \end{bmatrix} \qquad \overrightarrow{\emptyset_2} = \begin{bmatrix} 1 \\ -0.8109 \end{bmatrix} \tag{4.30}$$

Generalized mass matrix:

$$\widetilde{m}^* = \begin{bmatrix} 60553 & 0 \\ 0 & 29863.6 \end{bmatrix} \quad [kg] \tag{4.31}$$

Generalized stiffness matrix:

$$\tilde{k}^* = \begin{bmatrix} 9.4903 \times 10^6 & 0 \\ 0 & 2.66764 \times 10^7 \end{bmatrix} \quad [N/m] \tag{4.32}$$

Velocity of body1 after impact:

$$V_1' = \dot{x}_{10} = \frac{(1+\beta)V}{1+\frac{m_1}{M}} = 5.44 \quad \left[\frac{m}{s}\right] \tag{4.33}$$

The equation of motion for body 1 and body 2 in generalized coordinates:

$$m_1^* \ddot{q}_1 + k_1^* q_1 = 0$$

$$m_2^* \ddot{q}_2 + k_2^* q_2 = 0 \tag{4.34}$$

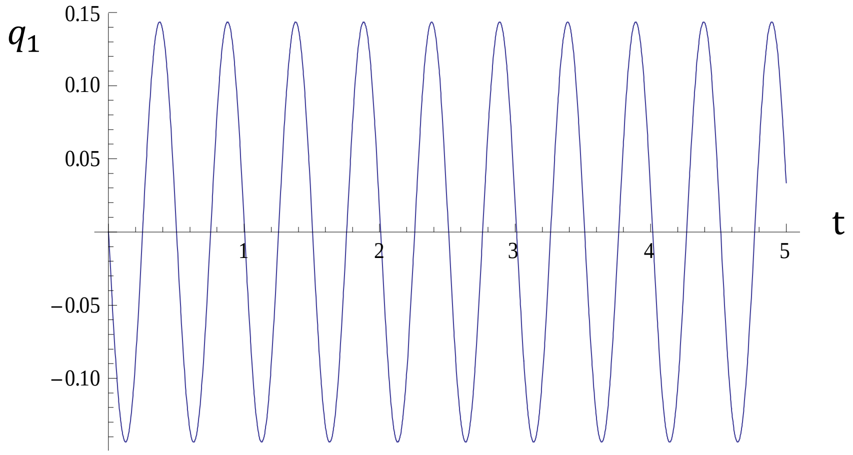

Figure 4.17 Deflection of body 1 after impact $0 \le t \le 5.0\ [s]$

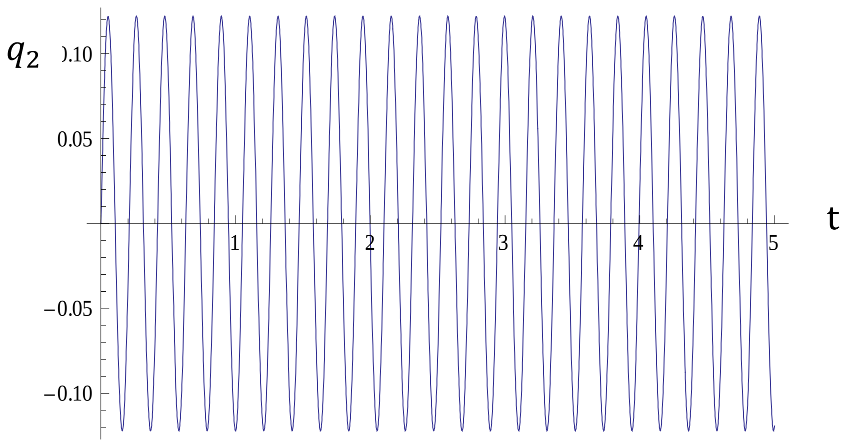

Figure 4.18 Deflection of body 2 after impact $0 \leq t \leq 5.0$ $[s]$

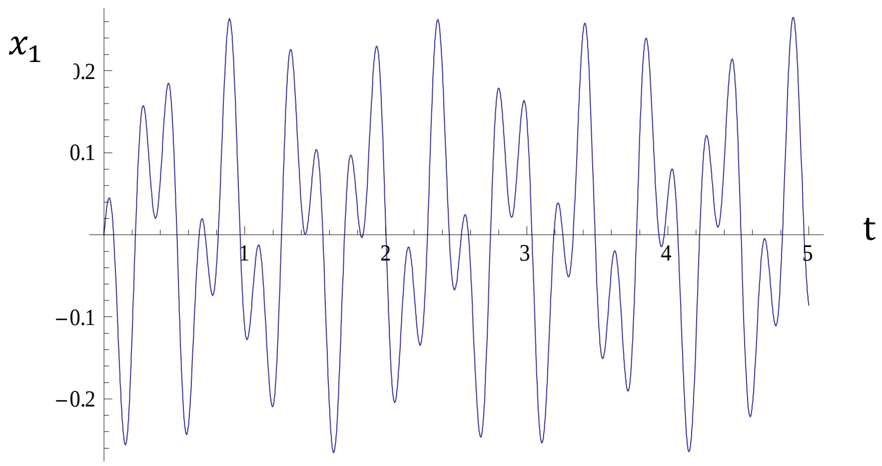

Figure 4.19 Displacement of the system, x_1

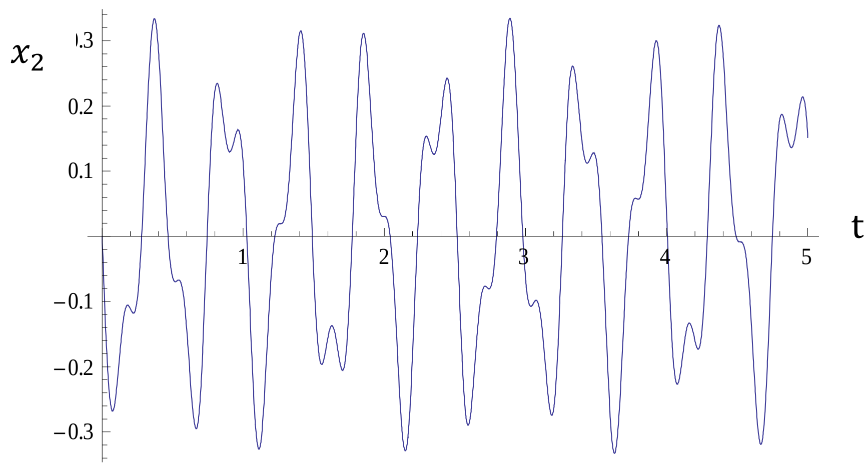

Figure 4.20 Displacement of the system, x_2

5. Response to Pulse Excitation [5]

In many practical situations the dynamic excitation is neither harmonic nor periodic. Thus we are interested in considering the dynamic response of 2DOF systems to excitations that consist of essentially a single pulse. Air pressures generated on a structure due to aboveground blasts or explosions are essentially a single pulse and can usually be idealized by simple shapes such as those shown in Fig. (5.1).

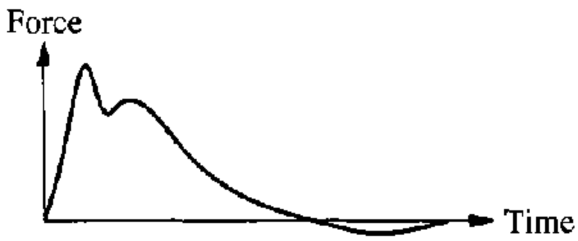

Figure 5.1 Pulse excitation

The response of the system to pulse excitations, in general, does not reach steady-state condition; the effect of initial conditions must be considered. The response of the system to such pulse excitation can be determined by one of several analytical methods: (1) the classical method for solving differential equations, (2) evaluating Duhamel's integral and (3) expressing the pulse as the superposition of two or more simpler functions for which response solutions are already available or easier to determine.

We prefer to use Duhamel's integral in evaluating the response of MDOF systems to pulse forces because it's closely tied to the dynamic of the system. Using Duhamel's integral the response to pulse forces will be determined in two phases. The first is the force vibration phase that covers the duration of the excitation. The second is the free vibration phase, which follows the end of the pulse force.

The Duhamel's integral in terms of coordinate x reads:

$$x(t) = e^{-\xi\omega_0 t}\left(x_0 Cos\omega_D t + \frac{\dot{x}_0 + x_0\xi\omega_0}{\omega_D} Sin\omega_D t\right) + \frac{1}{m\omega_D}\int_0^t f(\tau)e^{-\xi\omega_0(t-\tau)} Sin\,\omega_D(t-\tau)d\tau \tag{5.1}$$

$f(\tau)$...stands for the external forcing function

5.1 Response of undamped 2DOF system to rectangular impulse force

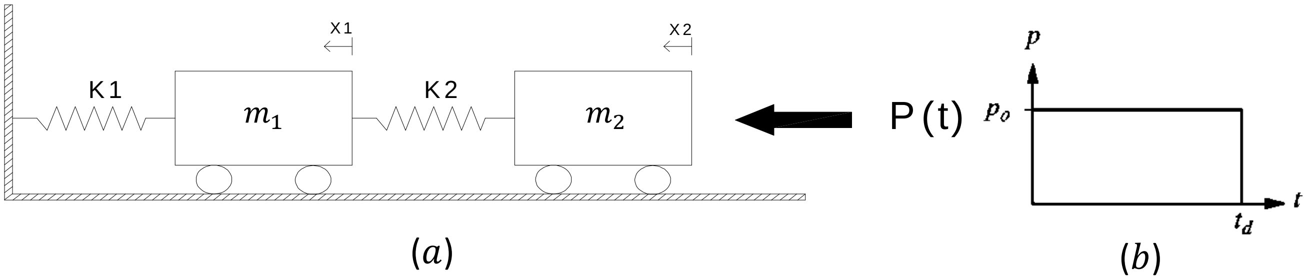

Figure 5.2 (a) 2DOF system; (b) Rectangular pulse force

$m_1 = 20000 \ [kg]$ $\qquad$ $K_1 = 7000000 \ [N/m]$

$m_2 = 15000 \ [kg]$ $\qquad$ $K_2 = 6000000 \ [N/m]$

$$\mathrm{P(t)} = \begin{cases} \mathrm{P_0} & 0 \le \mathrm{t} \le \mathrm{t_d} \\ 0 & t_d \le t \end{cases}$$

$\mathrm{P_0} = 1000 \quad [N]$

$t_d = 5.0 \quad [s]$

The Lagrange equation for a system in terms of coordinate x is

$$\underbrace{\begin{bmatrix} 20000 & 0 \\ 0 & 15000 \end{bmatrix}}_{\widetilde{M}} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1.3 \times 10^7 & -6 \times 10^6 \\ -6 \times 10^6 & 6 \times 10^6 \end{bmatrix}}_{\widetilde{K}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ \mathrm{P(t)} \end{bmatrix}}_{\vec{F}} \tag{5.2}$$

5.1.1 Modal analysis

Natural frequencies:

$$\omega_1 = 12.5191 \quad \left[\frac{rad}{s}\right] \qquad\qquad \omega_2 = 29.8877 \quad \left[\frac{rad}{s}\right] \tag{5.3}$$

Eigenvectors:

$$\overrightarrow{\emptyset_1} = \begin{bmatrix} 1 \\ 1.644 \end{bmatrix} \qquad\qquad \overrightarrow{\emptyset_2} = \begin{bmatrix} 1 \\ -0.8109 \end{bmatrix} \tag{5.4}$$

Generalized mass matrix:

$$\widetilde{m}^* = \begin{bmatrix} 60553 & 0 \\ 0 & 29863.6 \end{bmatrix} \quad [kg] \tag{5.5}$$

Generalized stiffness matrix:

$$\tilde{k}^* = \begin{bmatrix} 9.4903 \times 10^6 & 0 \\ 0 & 2.66764 \times 10^7 \end{bmatrix} \quad [N/m] \tag{5.6}$$

Generalized forces:

$$f_1^* = \overrightarrow{\emptyset_1}^T \vec{F} \quad = \quad 1644.24 \quad [N]$$

$$f_2^* = \overrightarrow{\emptyset_2}^T \vec{F} \quad = \quad -810.91 \quad [N] \tag{5.7}$$

The equation of motion for body 1 and body 2 in generalized coordinates

$$m_1^* \ddot{q}_1 + k_1^* q_1 = 0$$

$$m_2^* \ddot{q}_2 + k_2^* q_2 = 0 \tag{5.8}$$

With at-rest initial conditions

$$x(t = 0) = \dot{x}(t = 0) = 0 \tag{5.9}$$

The analysis is organized in two phases;

1. Forced vibration phase. During this phase, the system is subjected to a step force. The response of system is calculated as follows

$$x(t) = \frac{1}{m\omega_D}\int_0^t f(\tau) Sin\,\omega_D(t-\tau)d\tau \tag{5.10}$$

The Duhamel's integral for body 1in terms of coordinate $q_{1,i}$

$$q_{1,1}(t) = \frac{1}{m_1^*\omega_1}\int_0^{t_d} f_1^* Sin\,\omega_1(t_d-\tau) \tag{5.11}$$

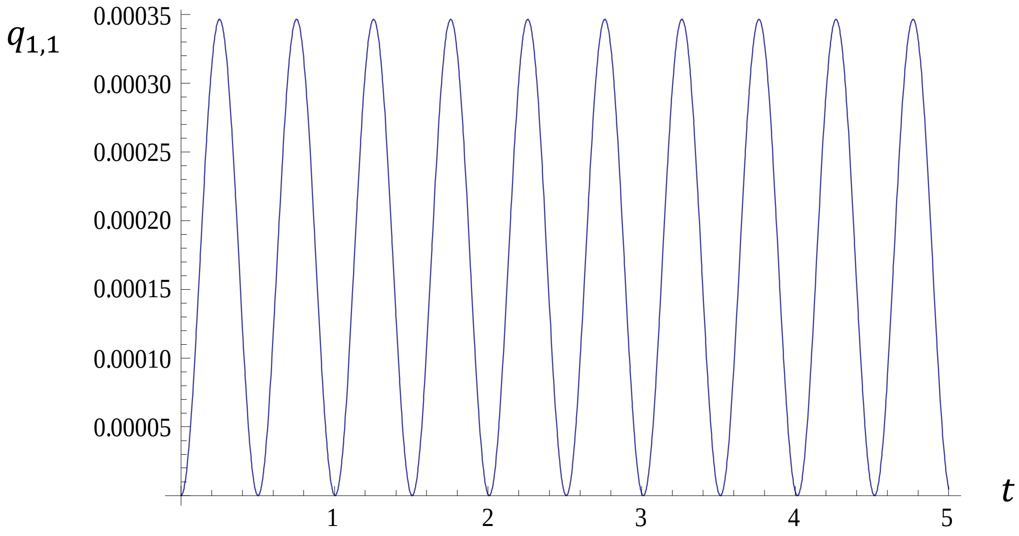

Figure 5.3 Deflection of body 1 after impact $0 \le t \le 5.0\ [s]$

2. Free vibration phase. After the force ends at t_d ,the system undergoes free vibration,

$$q_{1,2}(t) = q_{1,t_d} Cos\omega_1 t + \frac{\dot{q}_{1,t_d}}{\omega_1} Sin\omega_1 t \tag{5.12}$$

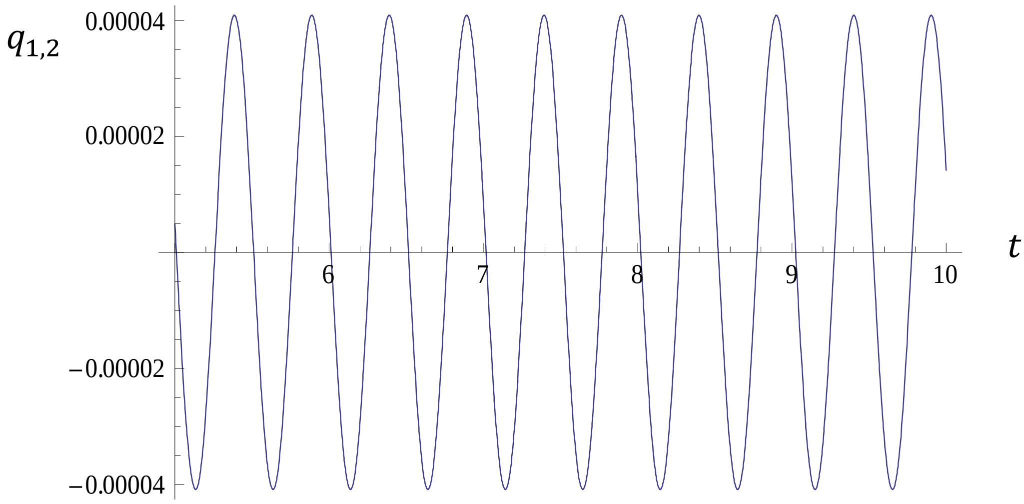

Figure 5.4 Deflection of body 1 after impact $5.0 \le t \le 10.0\ [s]$

The Duhamel's integral for body 2in terms of coordinate $q_{2,i}$

1. Phase

$$q_{2,1}(t) = \frac{1}{m_2^* \omega_2} \int_0^{t_d} f_2^* Sin\, \omega_2 (t_d - \tau) d\tau \tag{5.13}$$

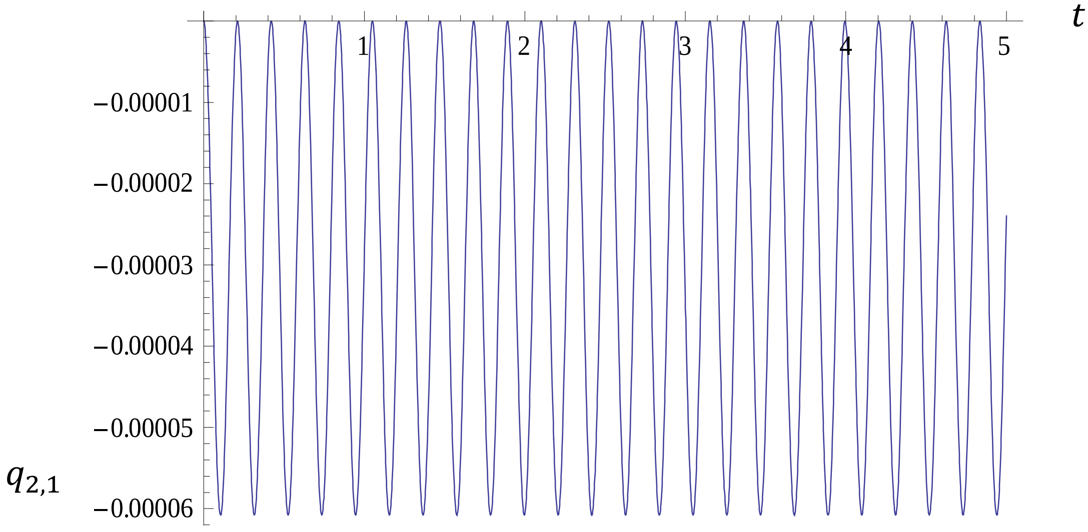

Figure 5.5 Deflection of body 2 after impact $0 \leq t \leq 5.0\ [s]$

2. Phase

$$q_{2,2}(t) = q_{2,t_d} Cos\omega_2 t + \frac{\dot{q}_{2,t_{d}t_d}}{\omega_2} Sin\omega_2 t \tag{5.14}$$

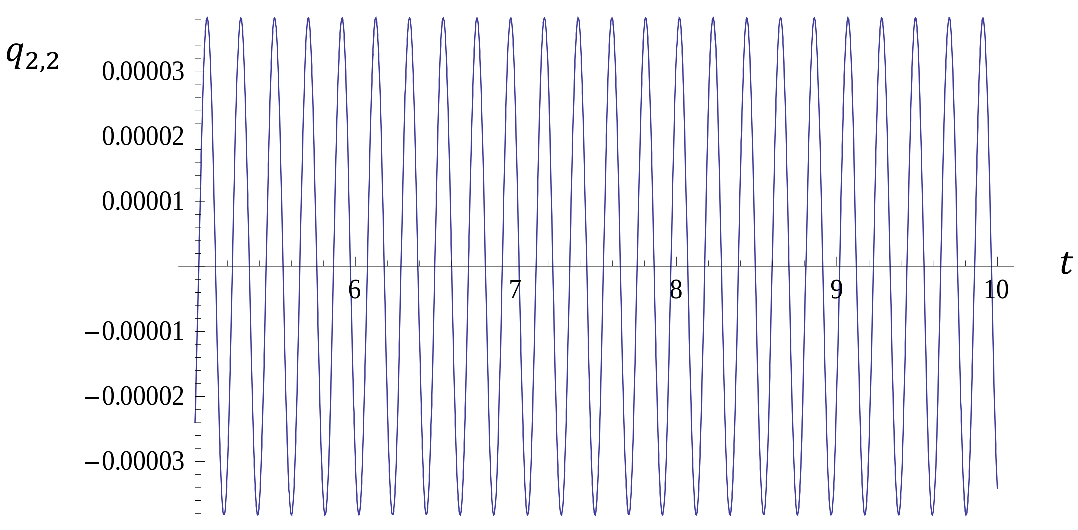

Figure 5.6 Deflection of body 2 after impact $5.0 \leq t \leq 10.0\ [s]$

Displacement of the system

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, q_k(t) \tag{5.15}$$

1. Phase

Figure 5.7 Displacement of the system, $x_{1,1}$ $0 \leq t \leq 5.0\ [s]$

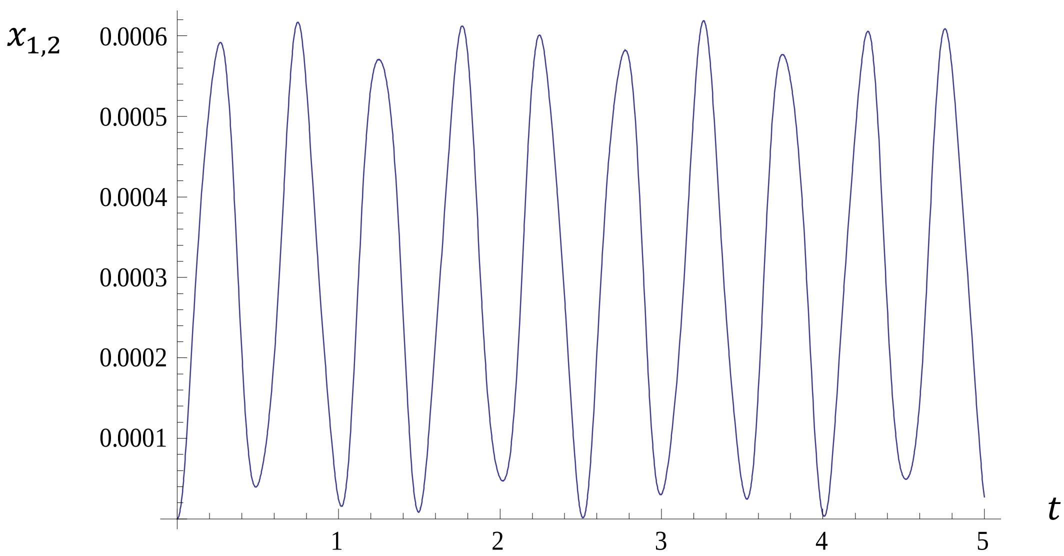

Figure 5.8 Displacement of the system, $x_{1,2}$ $0 \leq t \leq 5.0\ [s]$

2. Phase

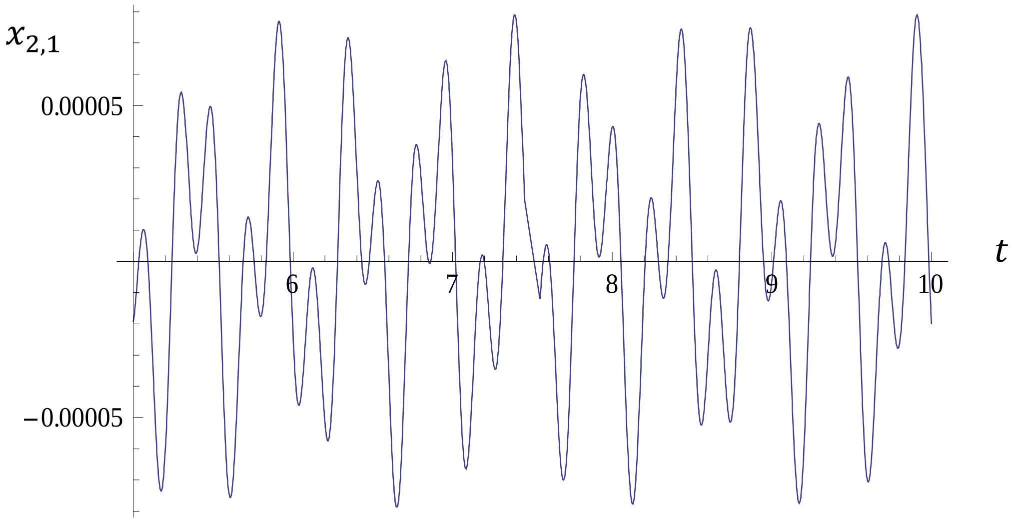

Figure 5.9 Displacement of the system, $x_{2,1}$ $5.0 \leq t \leq 10.0\ [s]$

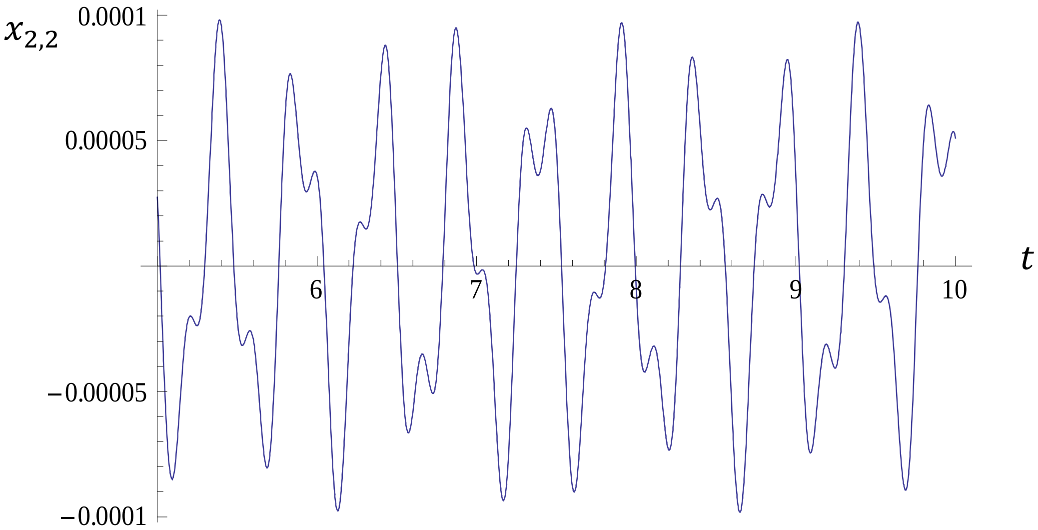

Figure 5.10 Displacement of the system, $x_{2,2}$ $5.0 \leq t \leq 10.0\ [s]$

5.2 Response of undamped 2DOF system to half cycle Sine pulse force

The next pulse under consideration is a half sinusoidal force-distribution. The response analysis procedure for this pulse is the same as developed in section. 5.1 for a rectangular pulse. In this part we study the responses of the system by changing the duration of the pulse.

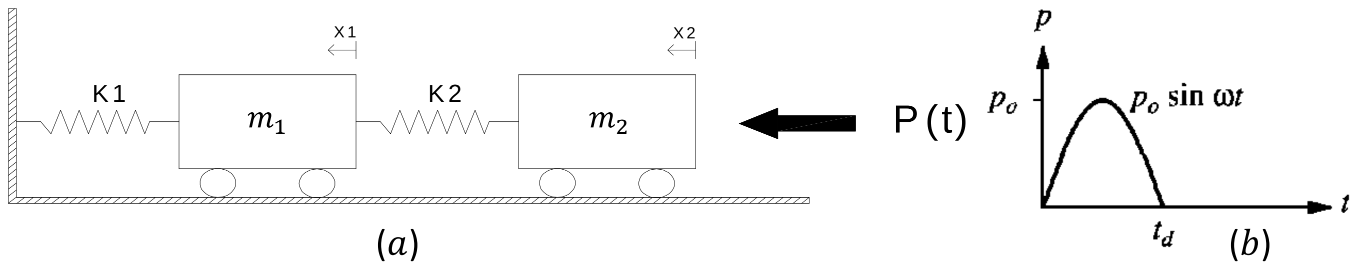

Figure 5.11 (a) 2DOF system; (b) Half cycle sine pulse force

5.2.1 Case 1: $t_d = 5.0\ [s]$

$P_0 = 1000 \quad [N]$

$t_d = 5.0 \quad [s]$

$$P(t) = \begin{cases} P_0.\mathrm{Sin}(\pi t/t_d) & 0 \le t \le t_d \\ 0 & t_d \le t \end{cases}$$

Initial conditions

$$x(t=0) = \dot{x}(t=0) = 0 \tag{5.16}$$

1. Forced vibration phase (body 1)

The Duhamel's integral for body 1in terms of coordinate $q_{1,i}$

$$q_{1,1}(t) = \frac{1}{m_1^* \omega_1} \int_0^{t_d} f_1^* Sin\, \omega_1 (t_d - \tau) d\tau \tag{5.17}$$

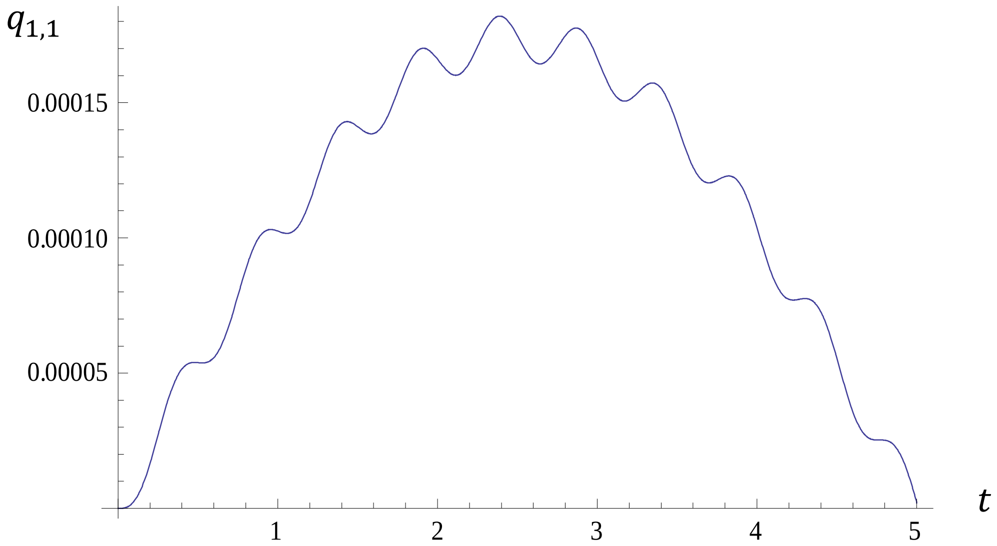

Figure 5.12 Deflection of body 1 after impact $\quad 0 \le t \le 5.0\ [s]$

2. Free vibration phase. After the force ends at t_d, the system undergoes free vibration,

$$q_{1,2}(t) = q_{1,t_d} Cos\omega_1 t + \frac{\dot{q}_{1,t_d}}{\omega_1} Sin\omega_1 t \tag{5.18}$$

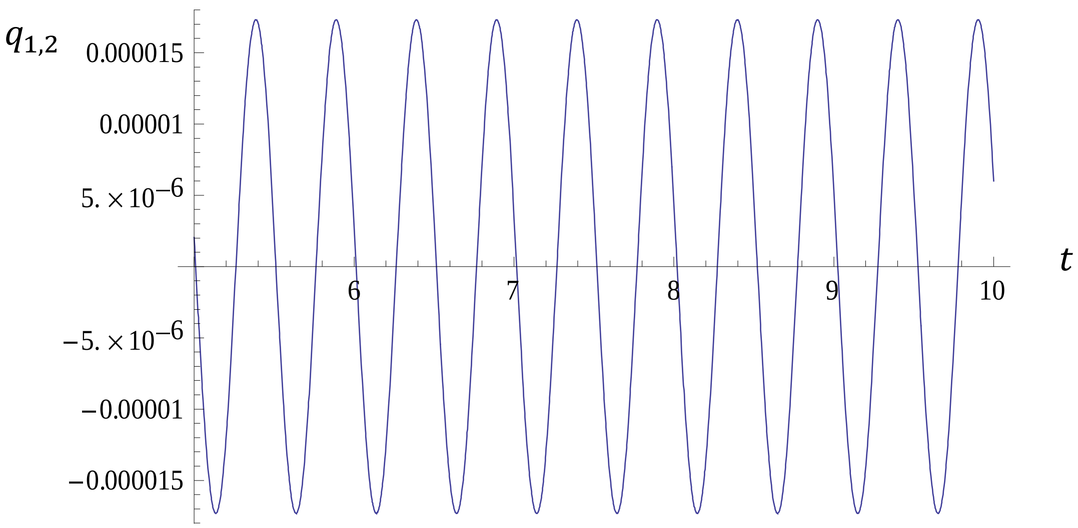

Figure 5.13 Deflection of body 1 after impact $\quad 5.0 \le t \le 10.0\ [s]$

The Duhamel's integral for body 2 in terms of coordinate $q_{2,i}$

1. Phase

$$q_{2,1}(t) = \frac{1}{m_2^* \omega_2} \int_0^{t_d} f_2^* Sin\, \omega_2 (t_d - \tau) d\tau \tag{5.19}$$

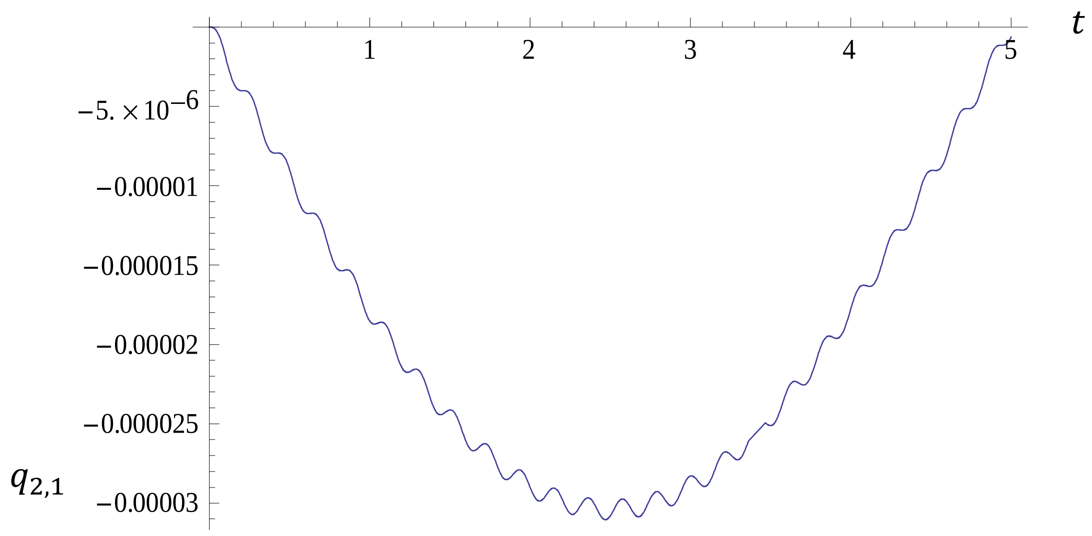

Figure 5.14 Deflection of body 2 after impact $0 \leq t \leq 5.0\ [s]$

2. Phase

$$q_{2,2}(t) = q_{2,t_d} Cos\omega_2 t + \frac{\dot{q}_{2,t_{d}t_d}}{\omega_2} Sin\omega_2 t \tag{5.20}$$

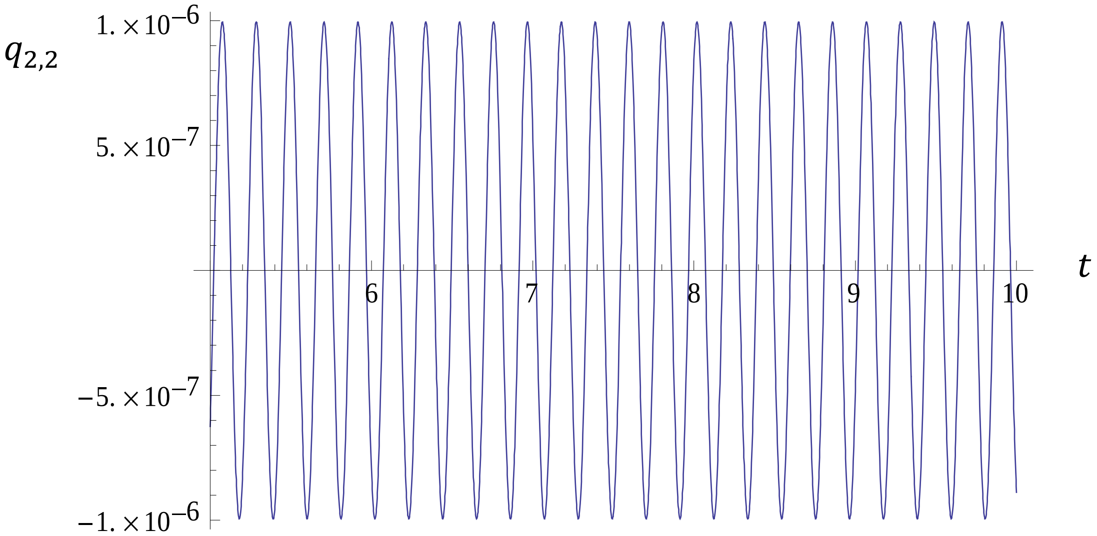

Figure 5.15 Deflection of body 2 after impact $5.0 \leq t \leq 10.0\ [s]$

Displacement of the system

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, q_k(t) \tag{5.21}$$

1. Phase

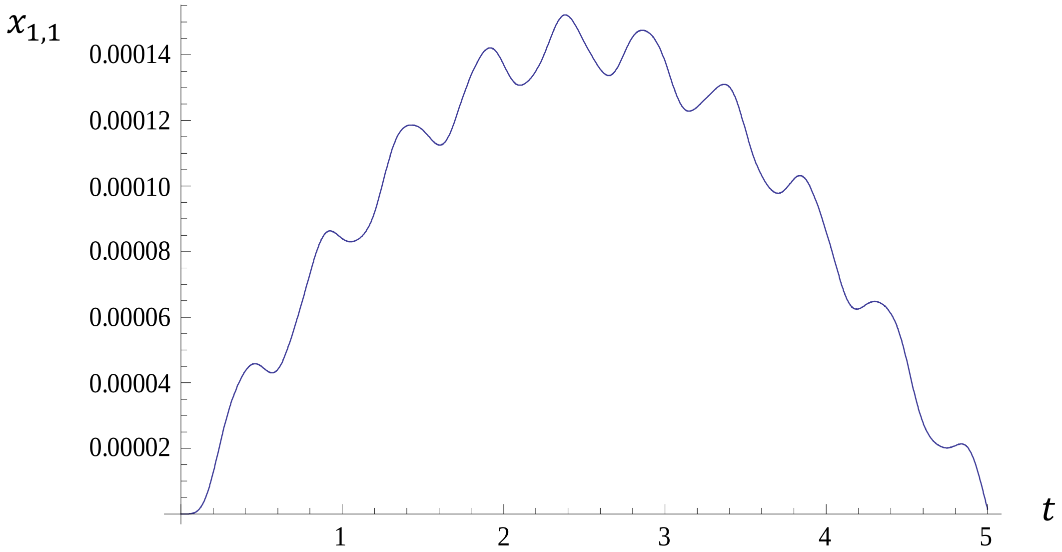

Figure 5.16 Displacement of the system, $x_{1,1}$ $0 \le t \le 5.0\ [s]$

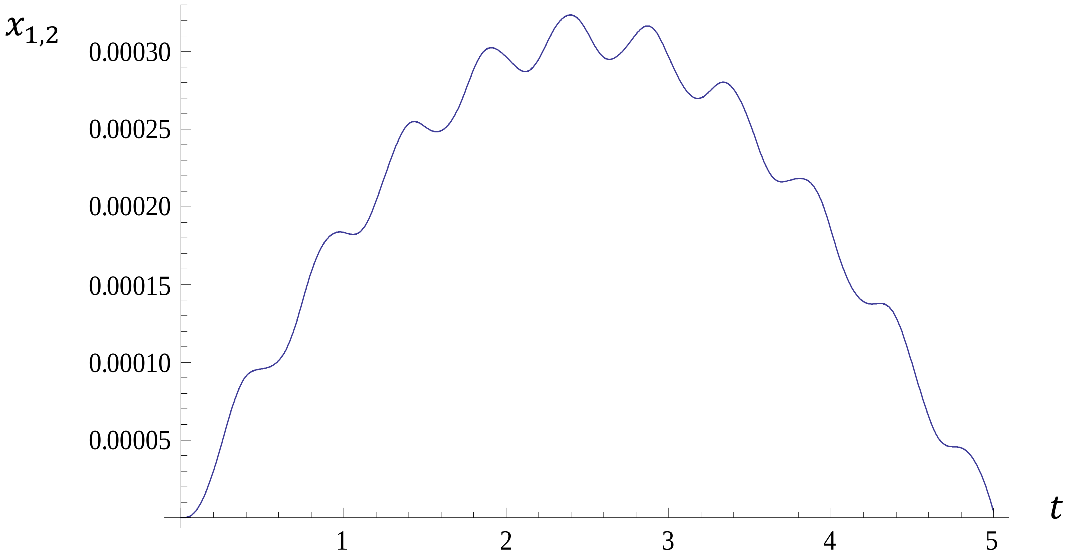

Figure 5.17 Displacement of the system, $x_{1,2}$ $0 \le t \le 5.0\ [s]$

2. Phase

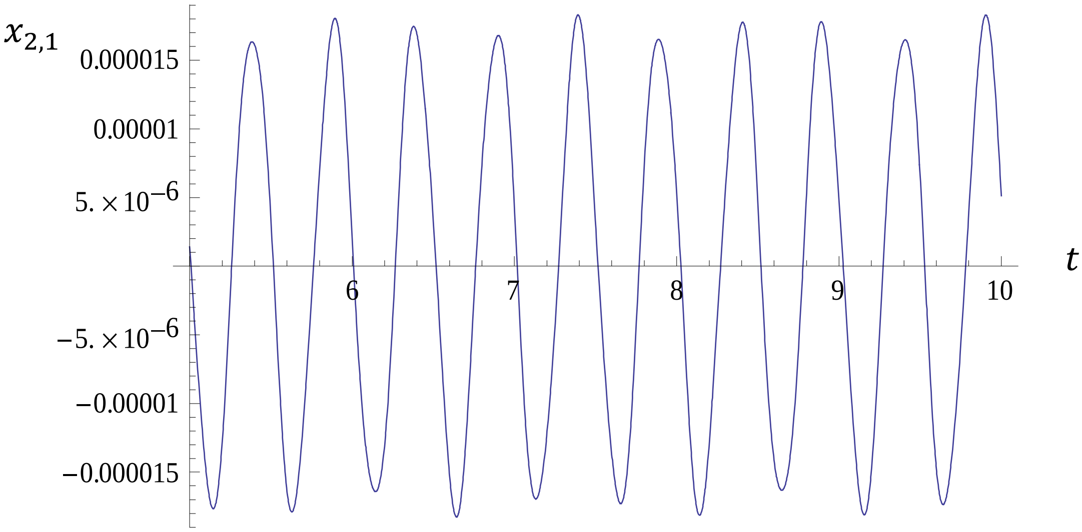

Figure 5.18 Displacement of the system, $x_{2,1}$ $5.0 \leq t \leq 10.0$ $[s]$

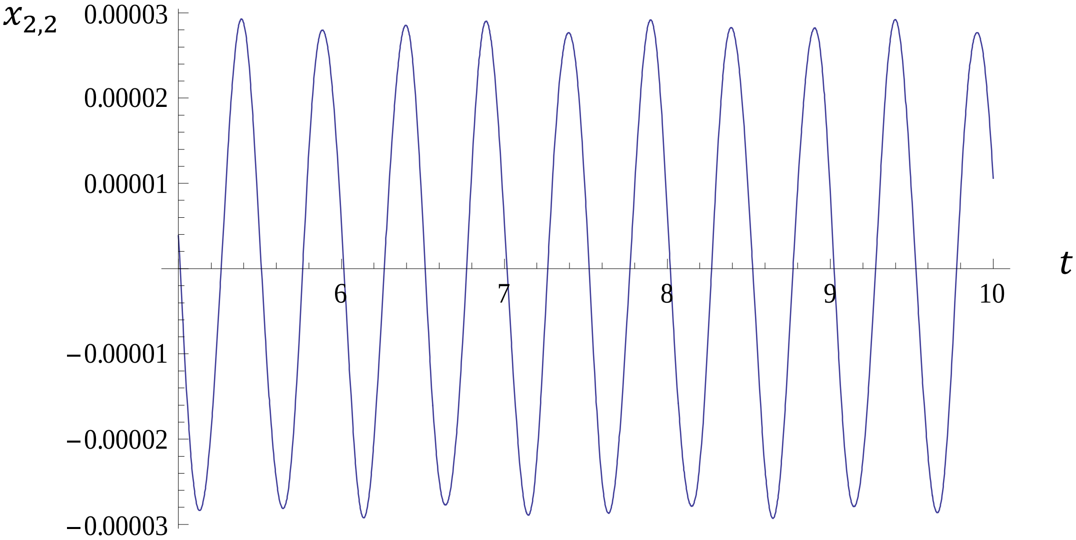

Figure 5.19 Displacement of the system, $x_{2,2}$ $5.0 \leq t \leq 10.0$ $[s]$

5.2.2 Case 2: $t_d = 0.5$ $[s]$

$P_0 = 1000$ $[N]$ $t_d = 0.5$ $[s]$

1. Forced vibration phase (Body 1)

The Duhamel's integral for body 1in terms of coordinate $q_{1,i}$

$$q_{1,1}(t) = \frac{1}{m_1^* \omega_1} \int_0^{t_d} f_1^* Sin\, \omega_1 (t_d - \tau) d\tau \tag{5.22}$$

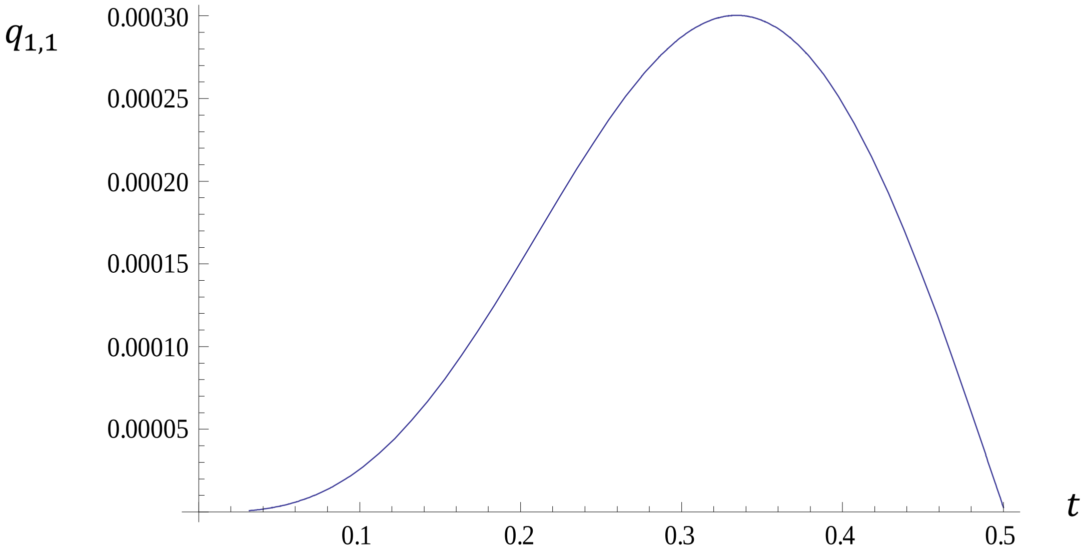

Figure 5.20 Deflection of body 1 after impact $0 \leq t \leq 0.5\ [s]$

2. Free vibration phase. After the force ends at t_d, the system undergoes free vibration,

$$q_{1,2}(t) = q_{1,t_d} Cos\omega_1 t + \frac{\dot{q}_{1,t_d}}{\omega_1} Sin\omega_1 t \tag{5.23}$$

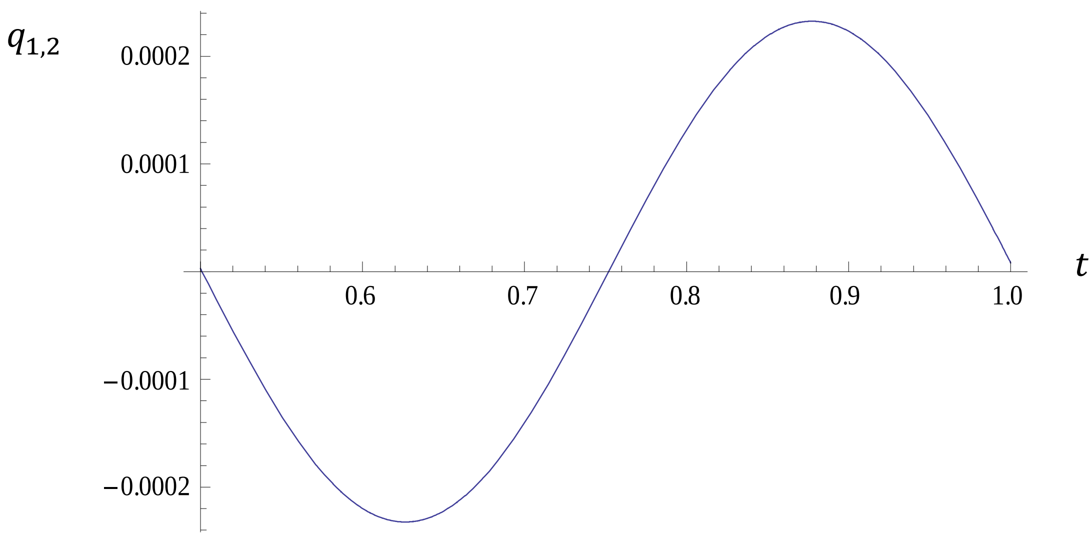

Figure 5.21 Deflection of body 1 after impact $0.5 \leq t \leq 1.0\ [s]$

The Duhamel's integral for body 2 in terms of coordinate $q_{2,i}$

1. Phase

$$q_{2,1}(t) = \frac{1}{m_2^* \omega_2} \int_0^{t_d} f_2^* Sin\, \omega_2 (t_d - \tau) d\tau \tag{5.24}$$

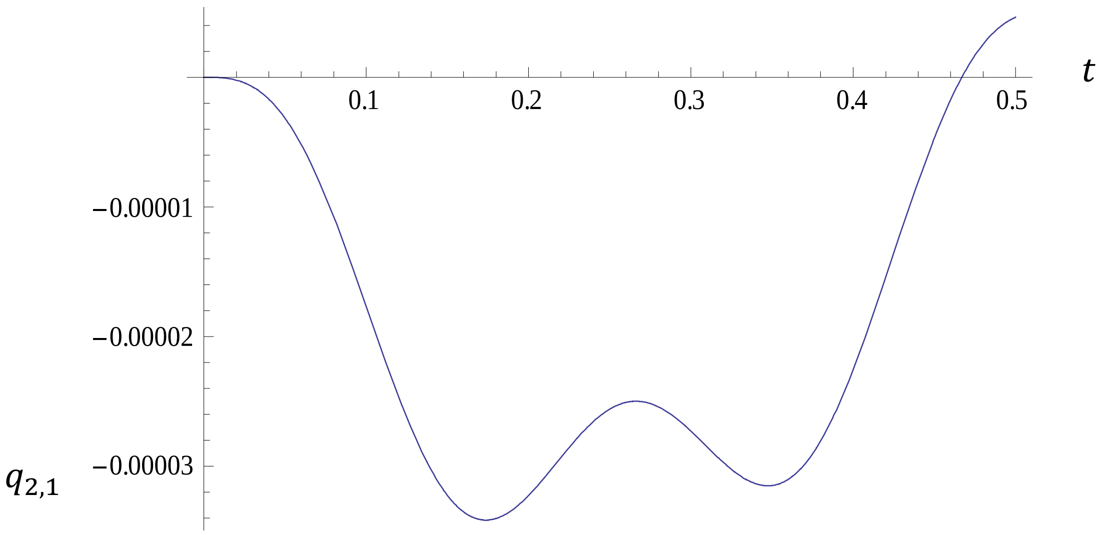

Figure 5.22 Deflection of body 2 after impact $0 \le t \le 0.5$ $[s]$

2. Phase

$$q_{2,2}(t) = q_{2,t_d} Cos \omega_2 t + \frac{\dot{q}_{2,t_{d}t_d}}{\omega_2} Sin \omega_2 t \tag{5.25}$$

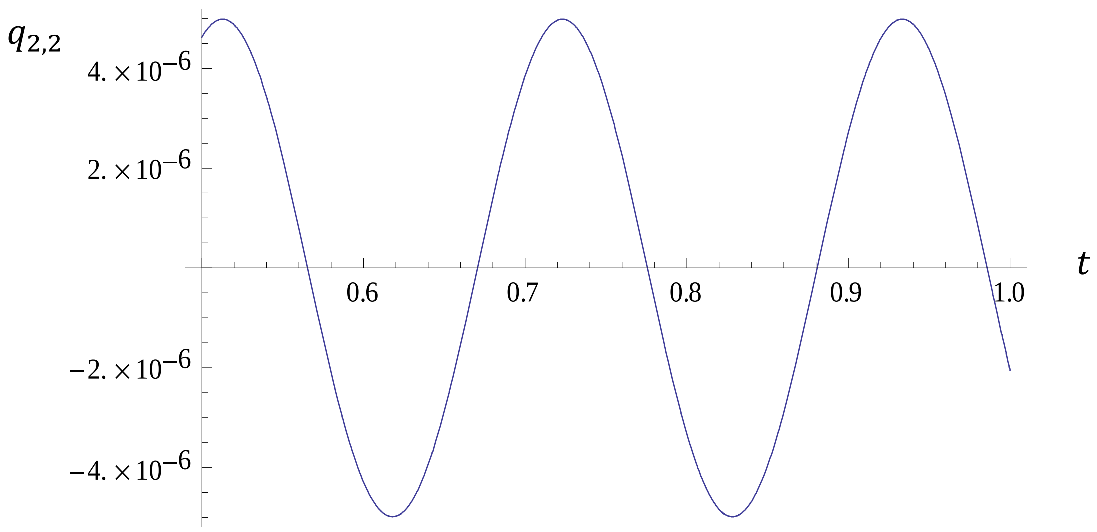

Figure 5.23 Deflection of body 2 after impact $0.5 \le t \le 1.0$ $[s]$

Displacement of the system

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, q_k(t) \tag{5.26}$$

1. Phase

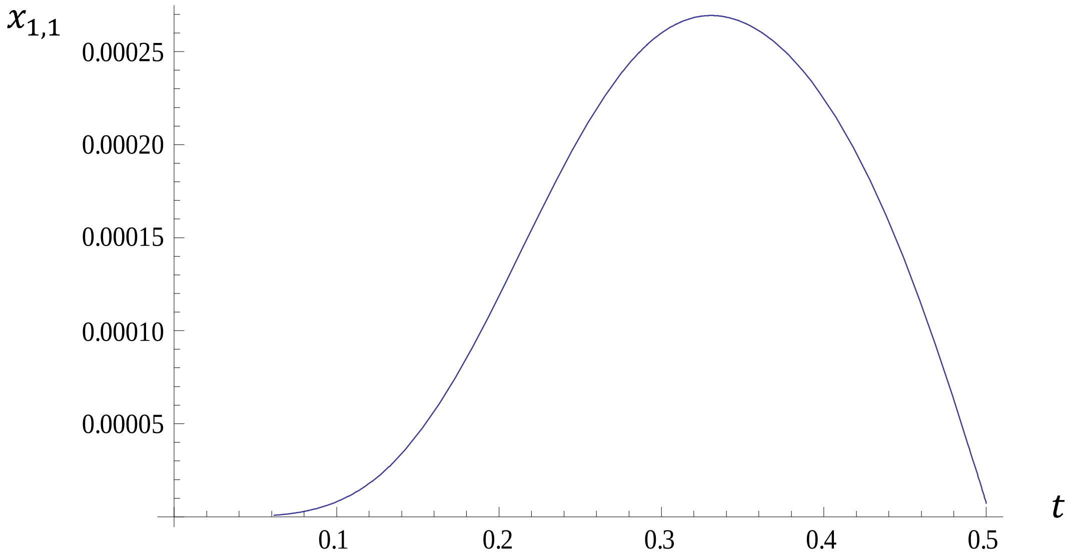

Figure 5.24 Displacement of the system, $x_{1,1}$ $0 \leq t \leq 0.5$ $[s]$

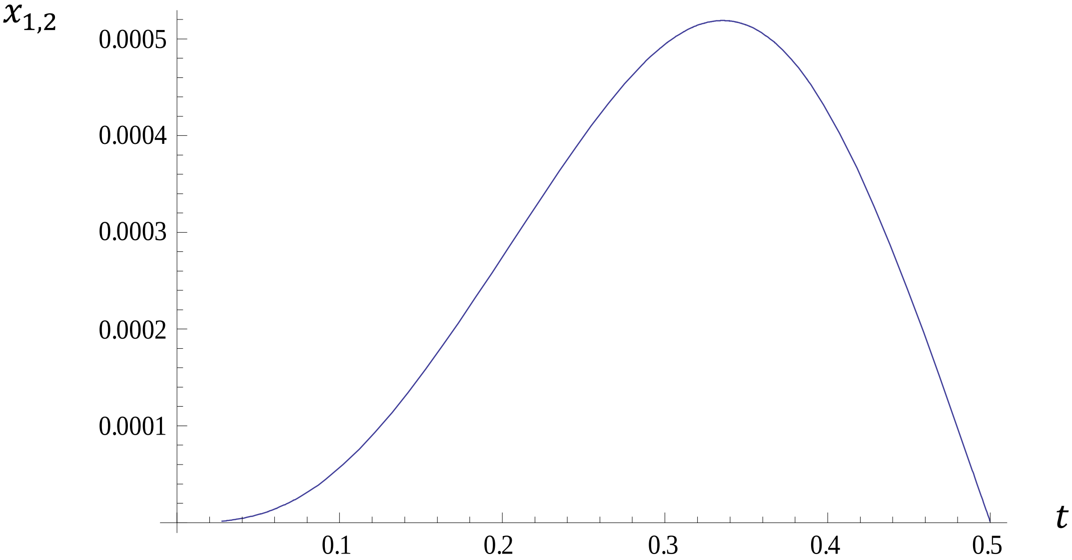

Figure 5.25 Displacement of the system, $x_{1,2}$ $0 \leq t \leq 0.5$ $[s]$

2. Phase

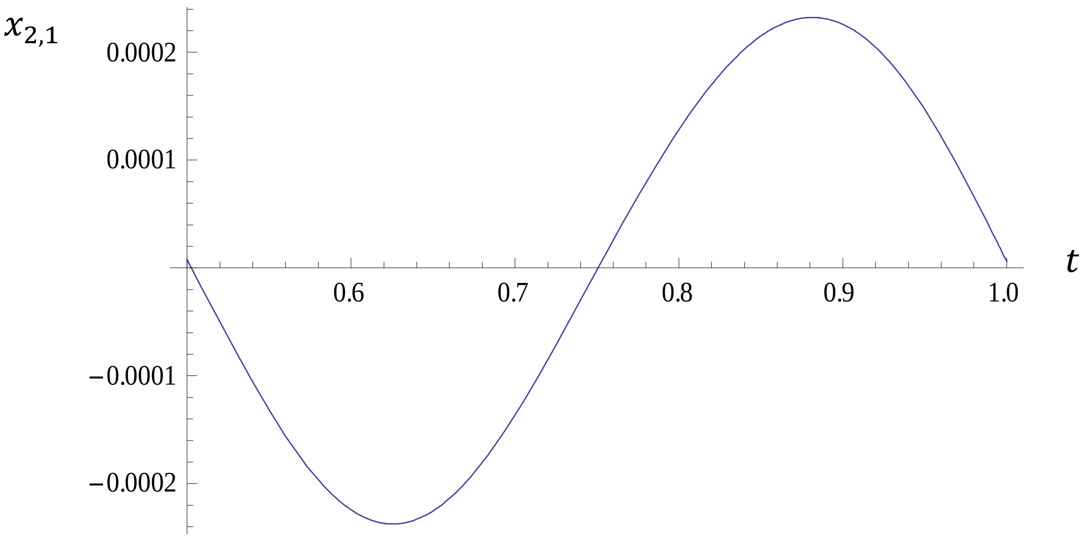

Figure 5.26 Displacement of the system, $x_{2,1}$ $0.5 \leq t \leq 1.0$ $[s]$

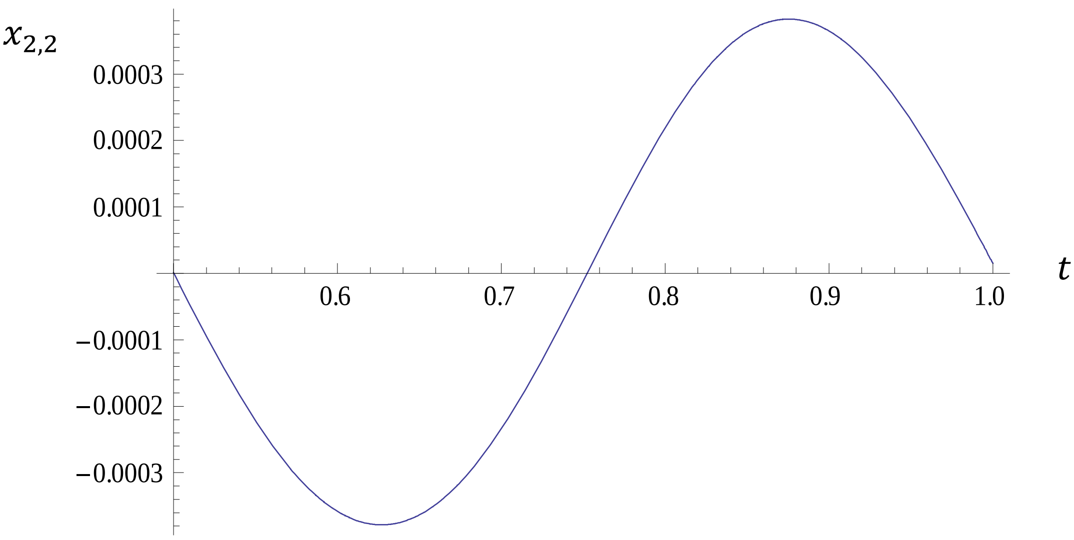

Figure 5.27 Displacement of the system, $x_{2,2}$ $0.5 \leq t \leq 1.0$ $[s]$

5.3 Response of damped 2DOF system to Half cycle Sine pulse force

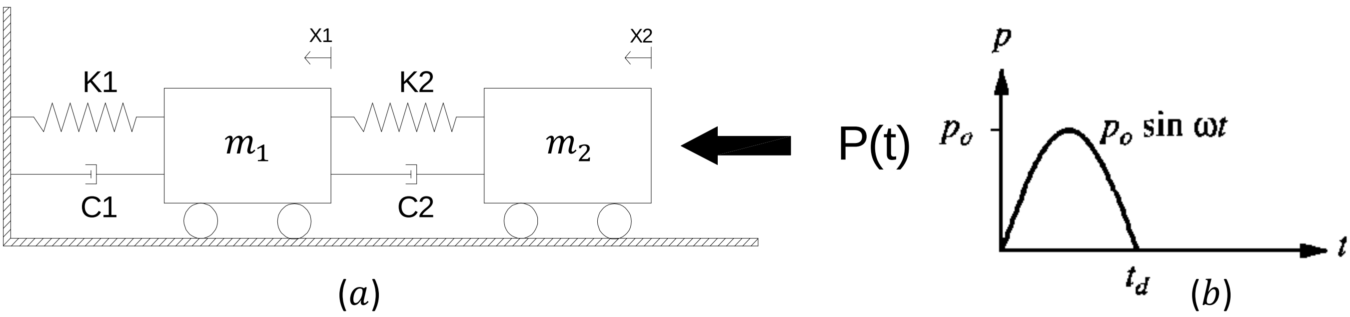

Figure 5.28 (a) 2DOF damped system; (b) Half cycle Sine pulse force

In this section we want to compute the response of damped system with three different values of damping ratio to half cycle Sine pulse force.

$m_1 = 20000\ [kg]$ $\qquad K_1 = 7000000\ [N/m]$

$m_2 = 15000\ [kg]$ $\qquad K_2 = 6000000\ [N/m]$

$$P(t) = \begin{cases} P_0.\mathrm{Sin}(\pi t/t_d) & 0 \le t \le t_d \\ 0 & t_d \le t \end{cases}$$

$P_0 = 1000 \quad [N]$

$t_d = 0.50 \quad [s]$

A) $\xi = 0.2$ $\qquad$ B) $\xi = 0.5$ $\qquad$ C) $\xi = 0.8$

5.3.1 Modal analysis

Natural frequencies:

$$\omega_1 = 12.5191 \quad \left[\frac{rad}{s}\right] \qquad \omega_2 = 29.8877 \quad \left[\frac{rad}{s}\right] \qquad (5.27)$$

$$\omega_D = \omega\sqrt{1-\xi}$$

(A) $\left[\frac{rad}{s}\right]$ (B) $\left[\frac{rad}{s}\right]$ (C) $\left[\frac{rad}{s}\right]$

$\omega_{1D} = 12.2661$ $\omega_{1D} = 12.2661$ $\omega_{1D} = 12.2661$

$\omega_{2D} = 29.2838$ $\omega_{2D} = 29.2838$ $\omega_{2D} = 29.2838$

(5.28)

Eigenvectors:

$$\overrightarrow{\emptyset_1} = \begin{bmatrix} 1 \\ 1.644 \end{bmatrix} \qquad \overrightarrow{\emptyset_2} = \begin{bmatrix} 1 \\ -0.8109 \end{bmatrix} \tag{5.29}$$

Generalized mass matrix:

$$\widetilde{m}^* = \begin{bmatrix} 60553 & 0 \\ 0 & 29863.6 \end{bmatrix} \quad [kg] \tag{5.30}$$

Generalized stiffness matrix:

$$\tilde{k}^* = \begin{bmatrix} 9.4903 \times 10^6 & 0 \\ 0 & 2.66764 \times 10^7 \end{bmatrix} \quad [N/m] \tag{5.31}$$

Generalized forces:

$$f_1^* = \overrightarrow{\emptyset_1}^T \vec{F} \quad = \quad 1644.24 \quad [N]$$

$$f_2^* = \overrightarrow{\emptyset_2}^T \vec{F} \quad = \quad -810.91 \quad [N] \tag{5.32}$$

1. Forced vibration phase (Body 1)

The Duhamel's integral for body 1in terms of coordinate $q_{1,i}$

$$q_{1,1}(t) = \frac{1}{m_1^* \omega_{1D}} \int_0^t f_1^* e^{-\xi \omega_1 (t-\tau)} Sin\, \omega_{1D}(t-\tau) d\tau \qquad (5.33)$$

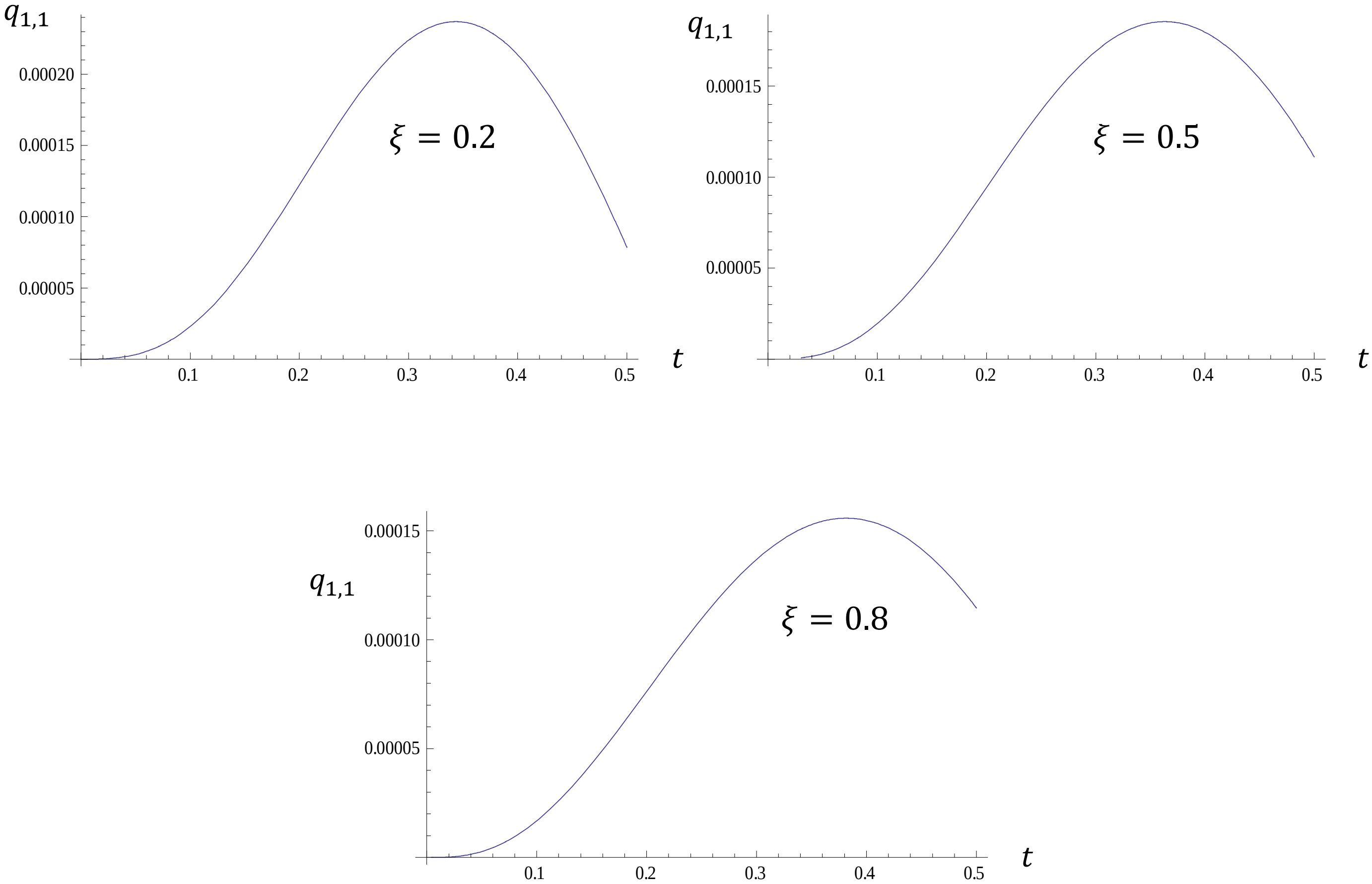

Figure 5.29 Deflection of body 1 after impact $0 \leq t \leq 0.5\ [s]$

2. Free vibration phase. After the force ends at t_d, the system undergoes free vibration,

$$q_{1,2}(t) = \left(q_{1,t_d} Cos\omega_{1D} t + \frac{\dot{q}_{1,t_d} + q_{1,t_d} \xi \omega_1}{\omega_{1D}} Sin\omega_{1D} t \right) \qquad (5.34)$$

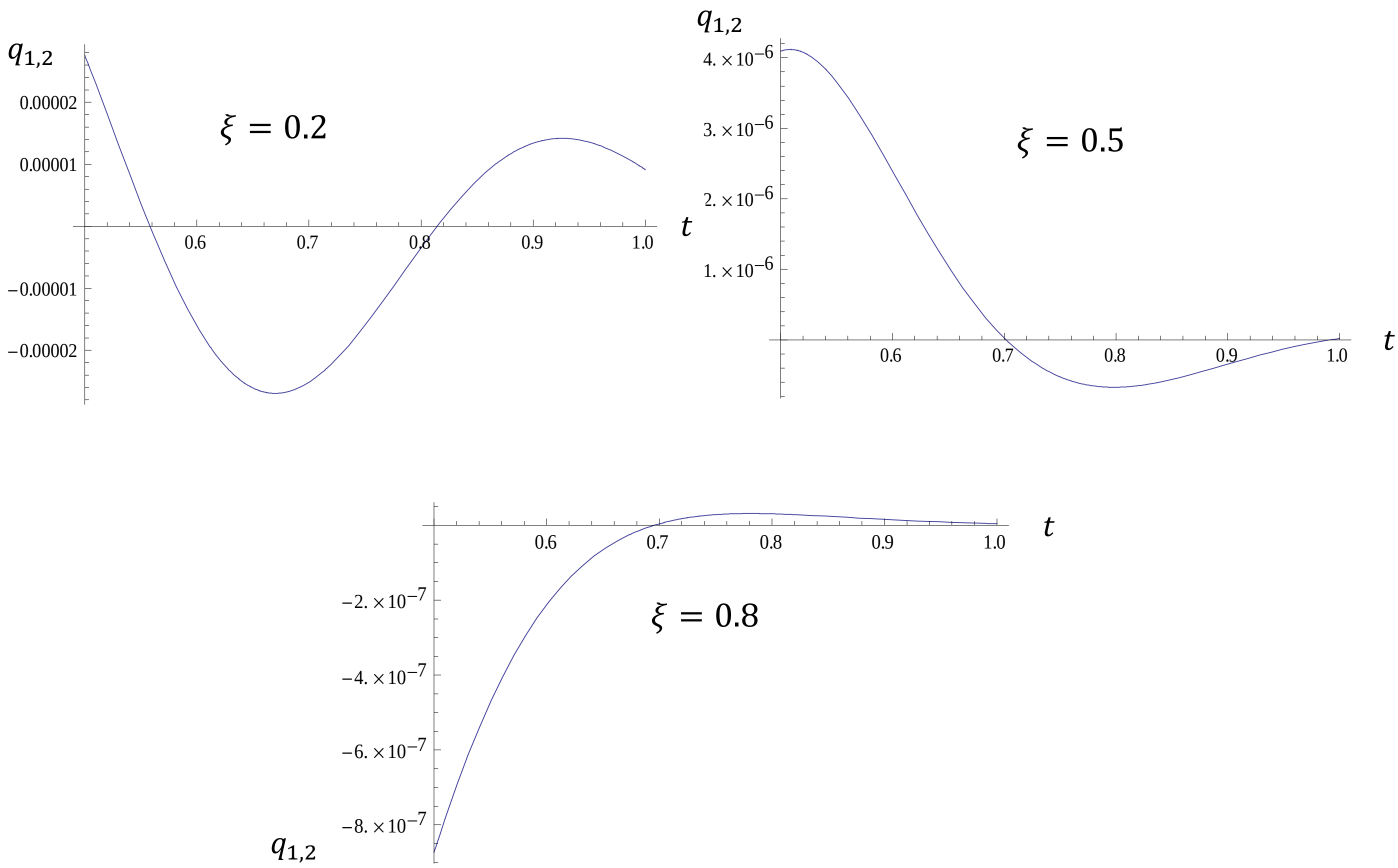

Figure 5.30 Deflection of body 1 after impact $0.5 \leq t \leq 1.0\ [s]$

The Duhamel's integral for body 2 in terms of coordinate $q_{2,i}$

1. Phase

$$q_{2,1}(t) = \frac{1}{m_2^* \omega_{2D}} \int_0^t f_2^* e^{-\xi\omega_2(t-\tau)} Sin\, \omega_{2D}(t-\tau) d\tau \tag{5.35}$$

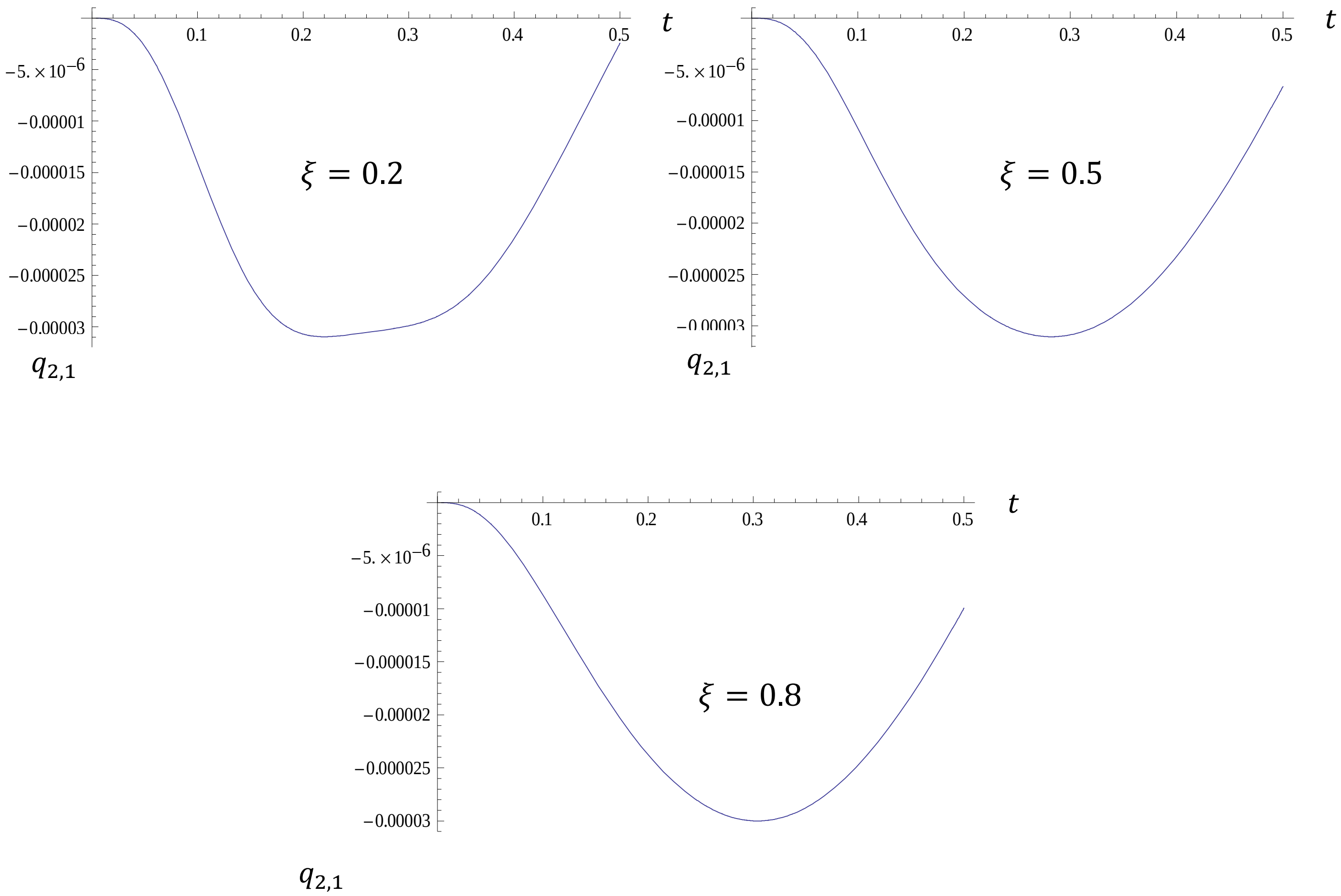

Figure 5.31 Deflection of body 2 after impact $0 \le t \le 0.5\ [s]$

2. Phase

$$q_{2,2}(t) = \left(q_{2,t_d} Cos\omega_{2D} t + \frac{\dot{q}_{2,t_d} + q_{2,t_d}\xi\omega_2}{\omega_{2D}} Sin\omega_{2D} t\right) \qquad (5.36)$$

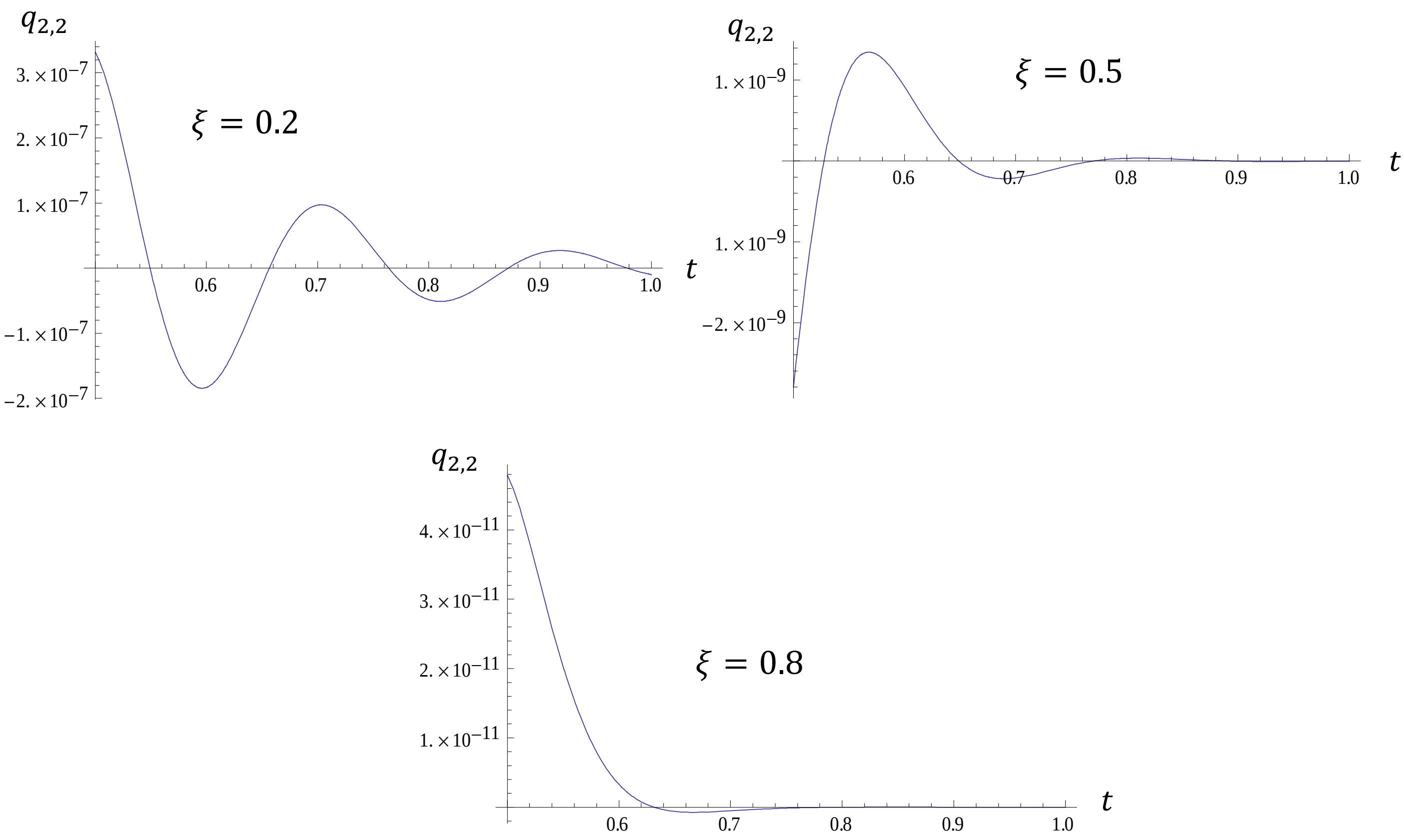

Figure 5.32 Deflection of body 2 after impact $0.5 \le t \le 1.0\ [s]$

Displacement of the system

$$\vec{x}(t) = \sum_{k=1}^{2} \overrightarrow{\emptyset_k}\, q_k(t) \tag{5.37}$$

1. Phase

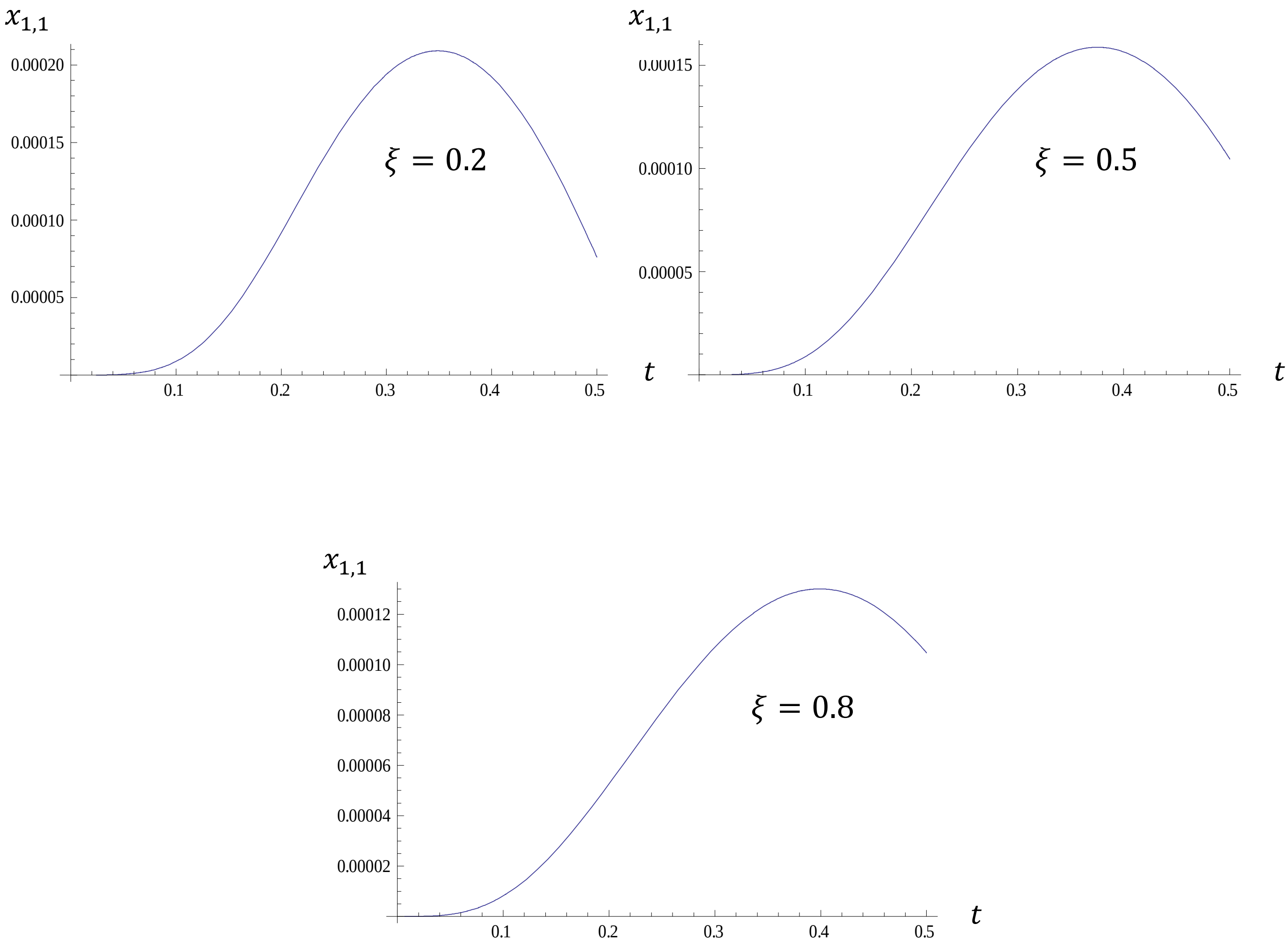

Figure 5.33 Displacement of the system, $x_{1,1}$ $\quad 0 \leq t \leq 0.5\ [s]$

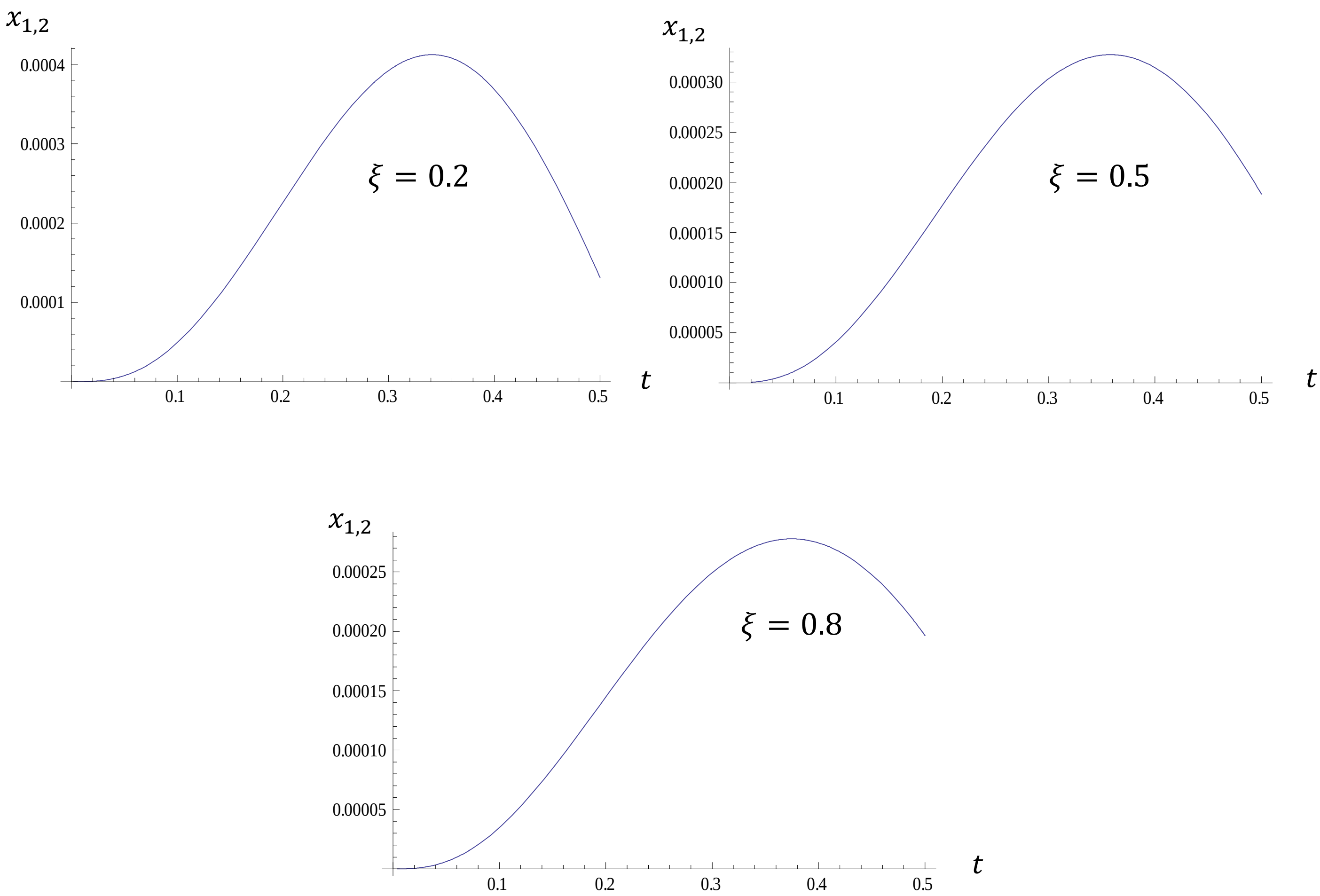

Figure 5.34 Displacement of the system, $x_{1,2} \quad 0 \leq t \leq 0.5\ [s]$

2. Phase

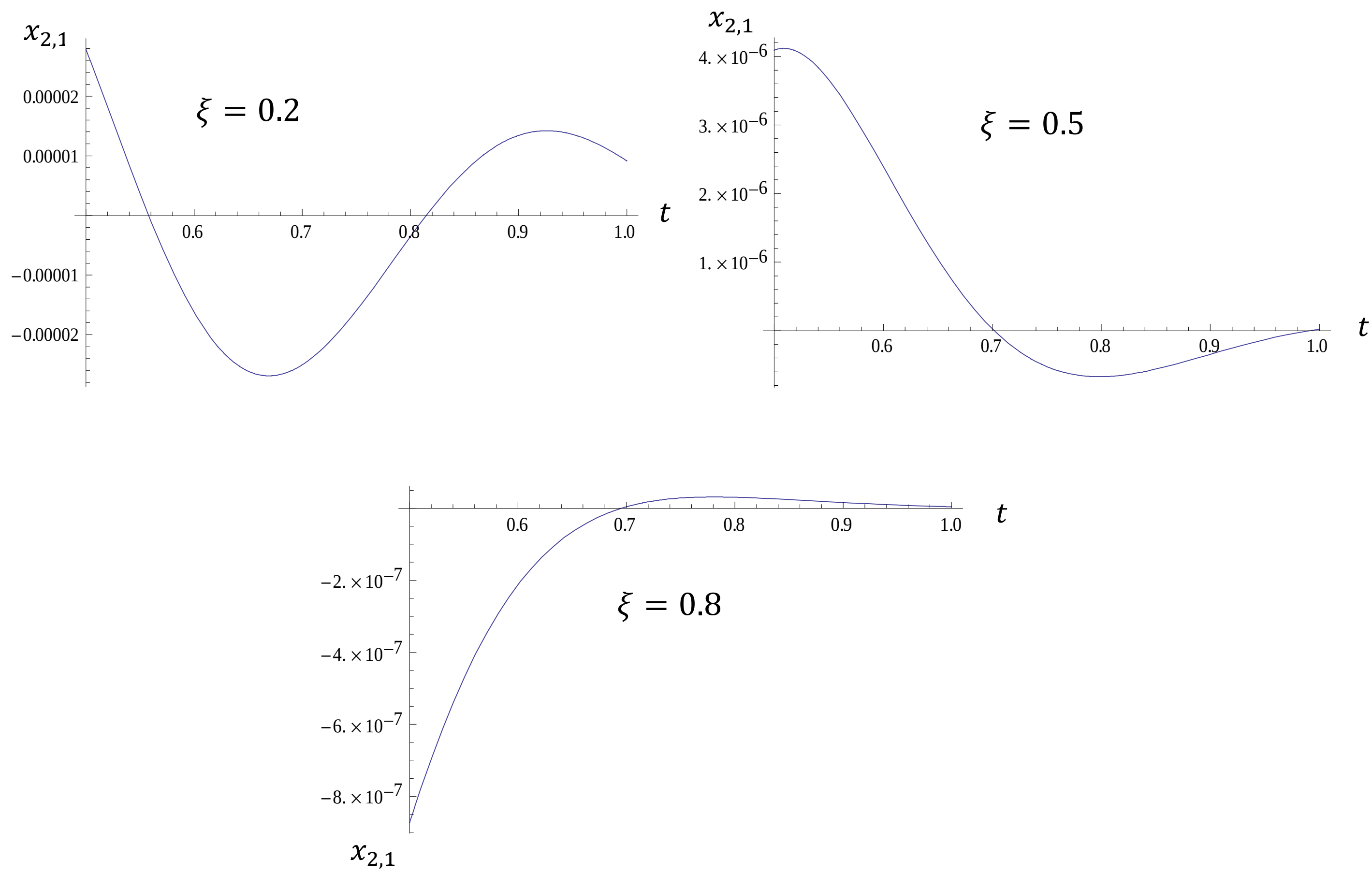

Figure 5.35 Displacement of the system, $x_{2,1}$ $\quad 0.5 \le t \le 1.0\ [s]$

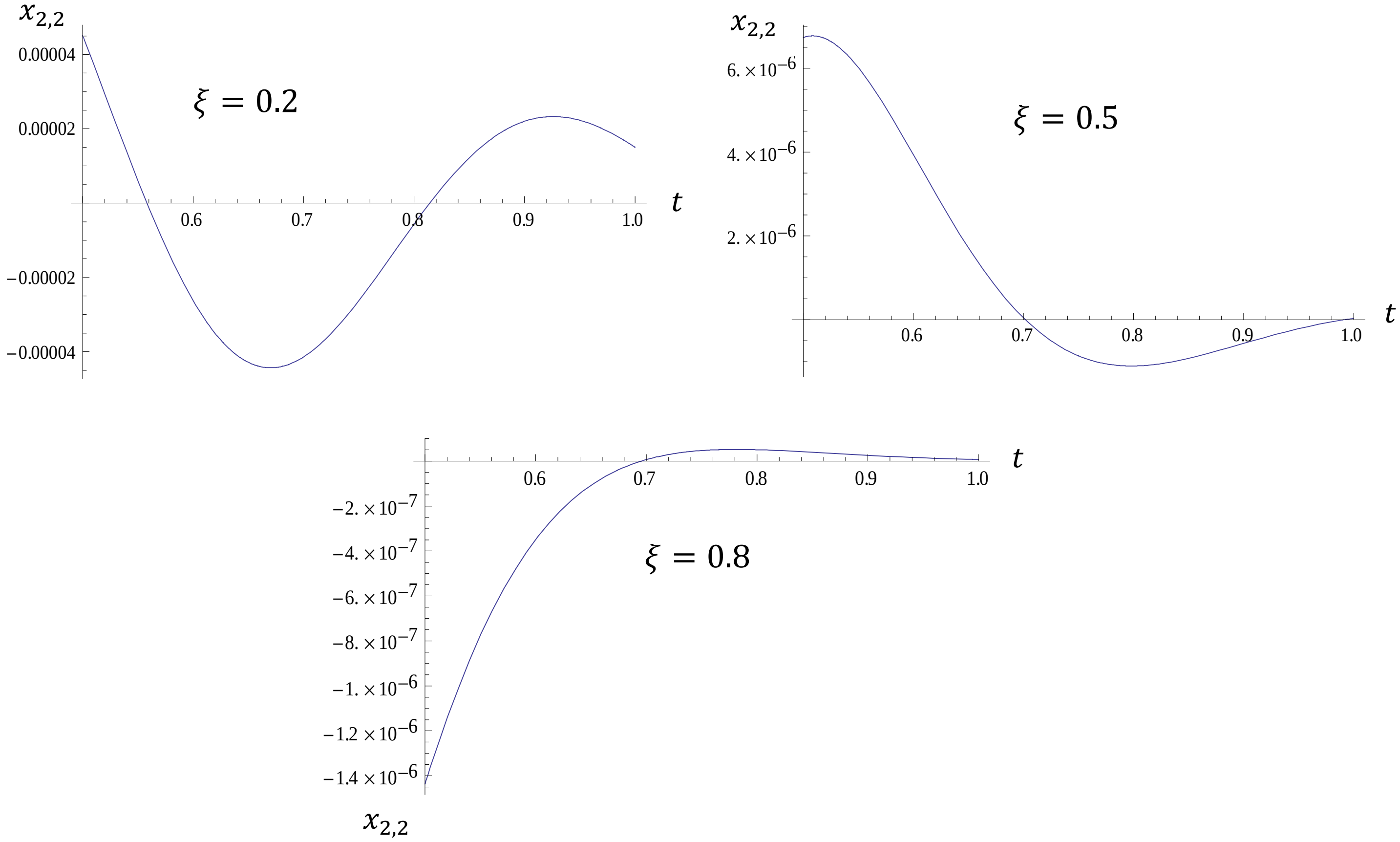

Figure 5.36 Displacement of the system, $x_{2,2}$ $\quad 0.5 \leq t \leq 1.0\ [s]$

Final Remarks

The goal of this study was to simulate and calculate the deflection of an undamped as well as a damped system with 2DOF caused by elastic collision and by general collision. The response of the system according to two different collision positions was calculated.

In the second part of this thesis, a general procedure was developed to analyze the response of the 2DOF system to two types of excitation, rectangular and half cycle sine pulse force. Additionally the time variation of the response to half cycle sine pulse force is studied, and graphically shows that the maximum response is as a function of t_d/T_n, the ratio of pulse duration to natural vibration period. In the last part is the effect of damping on the response to a single pulse excitation demonstrated.

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