DIPLOMARBEIT

Stability of Climate Coalitions

A numerical approach using the CWS model

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Abstract

This diploma thesis summarizes the accumulated background knowledge and the work conducted during the collaboration in the CLIMNEG project at the Center for Operations Research and Econometrics (CORE) at the Université catholique de Louvain in Belgium. Within this project an integrated assessment model of the climate and the economy is used to study the economic behavior and the environmental impacts of international climate coalitions. The results should support policy makers on governmental and intergovernmental level with information to understand the mechanisms of such climate coalitions. During this work the model and its computational implementation have been refined and extended to 18 regions. After a broad view on the necessary background to understand the model and the stability analysis of the climate coalitions, the model itself, and the new results are presented. A closer look is taken at the difficulties occurring with the extension to 18 regions and how it has been dealt with them. Further the use of the model to perform a stability analysis of the coalitions is described. Finally methods are given, which should help to achieve insights from this analysis in the next steps. The project is still work in progress and therefore this thesis has to be seen as a snapshot of one part within a bigger framework. Besides basic knowledge of the project, a reader should gain detailed insight in some of the scientific work which has been done during this thesis.

Zusammenfassung

Diese Diplomarbeit fasst das erlangte Hintergrundwissen und die durchgeführten Tätigkeiten während der Mitarbeit an dem Forschungsprojekt CLIMNEG am Center for Operations Research and Econometrics (CORE) an der Université catholique de Louvain in Belgien zusammen. In diesem Projekt wird ein mathematisches Modell des Weltklimas und der Wirtschaft verwendet, um das Verhalten von Klimakoalitionen und die dadurch entstehenden Umwelteinflüsse zu analysieren. Die Ergebnisse sollen Entscheidungsträgern auf nationaler und internationaler Ebene mit Informationen über Mechanismen solcher Klimakoalitionen unterstützen. Während dieser Arbeit wurde das Modell und seine rechnergestützte Umsetzung verfeinert und auf 18 Regionen erweitert. Nach einem breiten Überblick über das - für das Verständnis des Modells und der Stabilitätsuntersuchung der Klimakoalitionen nötige Hintergrundwissen, werden das Modell und die erhaltenen Ergebnisse präsentiert. Näher eingegangen wird auf die Schwierigkeiten die sich durch die Erweiterung auf die 18 Regionen ergeben und wie mit diesen umgegangen wurde. Weiters wird die Verwendung des Modells für die Stabilitätsuntersuchung der Koalitionen beschrieben. Schlussendlich werden noch Methoden vorgeschlagen, wie in den nächsten Schritten Einsichten aus der Stabilitätsuntersuchung gewonnen werden können. Das Projekt ist nach wie vor im Gange, deshalb soll diese Arbeit lediglich als Momentaufnahme eines Teils eines größeren Rahmens gesehen werden. Neben einem soliden Basiswissen über das Projekt, sollen dem Leser auch detaillierte Einsichten über die durchgeführten wissenschaftlichen Arbeiten während dieser Diplomarbeit gegeben werden.

Contents

1 Preface

Whoever believes the climate issues
can be solved within the next ten years
may leave the room.

Michael Oppenheimer to conference audience, Rio 1992

Global warming has posed a serious threat to our and our children's life. The speed of actual global warming is a distinguishable evidence of a global climate change induced by mankind. At the moment it is not fully possible to estimate the impact of this climate change to human dimensions. However, because of the first indications (hurricanes, floods, droughts, etc.), we have to fear the worst. In order to get a better overall picture of the impact of human doing on the global climate and finally the impact on human socio-economic systems has become an important scientific topic. Therefore, the scientific community is working intensely on this task.

Since climate change and its impact on human socio-economic systems is an extremely complex problem, many scientific disciplines are needed to provide various tools to analyze its issues. On the one hand it is important to understand the underlying coherences and to collect and process data on the global climate system. On the other hand due to the complexity of the global climate system a model-based approach is the most sensitive method to analyze these systems.

In a first approach, scientists use simulation models to describe various scenarios. These scenarios were employed to showcase the hazards coming along with global climate changes in order to create public and political awareness of this serious threat to human dimensions. Descriptive simulation models deliver insight to a predetermined situation; however, mathematical models allow a prescriptive analysis. For instance, Operations Research (OR) is the scientific field that concentrates on model-based approach to provide decision support.

This thesis presents an approach where OR-methods are used to tackle climate issues in connection with economic questions. More precisely, a Game Theory approach using an integrated assessment model is used to analyze climate coalitions .

The content of this thesis is based on the work of the environmental group at the Center for Operations Research and Econometrics (CORE) of the Université catholique de Louvain (UCL), located in Louvain-la-Neuve (Belgium). I had the opportunity to work with this group during one semester on the CLIMNEG project[1]. In this thesis I will present the background, the project and the work that has been done during this semester.

What is the approach dealing with the climate issue used in the CLIMNEG project?

Since 2000 the CLIMNEG World Simulation (CWS) model, which is a nonlinear model of the interaction of the worlds economies and climate change, is one central element in the project. For different global coalition structures on an aggregated country level, the achievable welfare for the countries choosing an optimal path of emissions abatement is calculated. This is done with the modeling language GAMS and a solver for non-linear optimization problems. These achievable welfare levels are further used to determine which coalition structures are stable. The aim is to figure out what the underlying factors of

[1]I am deeply grateful to all the people who made this collaboration possible and that joyful!

Figure 1: placement of the work

stability of climate coalitions are, and how efficient these coalitions would be in preventing climate change.

Within the CLIMNEG project much work on this has already been done, many results have been published but one is continuously working to improve the model, to apply new methods and to gain more insights. The aim is providing policy makers on governmental and intergovernmental level with information, making them understand the mechanisms, and supporting them towards an environmentally compatible future.

In Chapter 2 some background knowledge in environmental economics, game theory and modeling is presented to provide information which is necessary to understand the CWS model described in Chapter 3. Besides a detailed description of the model itself, this chapter contains its implementation in GAMS, its recent updates, output and interpretation. Since the whole project is work in progress, it is just a snapshot of the current state, including a short presentation of the work which has been done during this thesis. Finally Chapter 4 presents approaches on the actual stability analysis of climate coalitions using the output gained from the model.

Since these are all pieces within a bigger framework, this thesis will not give any major results, but the reader should be provided with a substantiated insight into the project and how this interdisciplinary area of research provides assistance to deal with the climate change issue.

2 The background

To get started some background has to be given. First of all a more general discussion on environmental economics and climate change is summarized. Afterwards the necessary mathematical background on operations research and game theory is developed and finally these two topics will be brought together in some first modeling results.

2.1 Environmental economics - climate change

The general statement which will be discussed in this section with different side topics is the following:

"The economy affects the environment, the environment affects the economy."

Environmental economics is a subfield of economics concerned with environmental issues such as air pollution, water quality, toxic substances, solid waste, and global warming. They all have in common the concept of externalities, which are impacts (positive or negative) on any party which did not decide on getting impacted.[2] This work will focus on climate change and global warming which have recently become widely discussed problems. There is interaction between climate change and the economy. The simplest form of this interaction is given in Figure 2. The economy influences the global climate through emissions E. They cause a temperature change ΔT which affects again the economy. Therefore two questions should be answered more in detail first. How does the economy affect climate change? And how does climate change affect the economy?

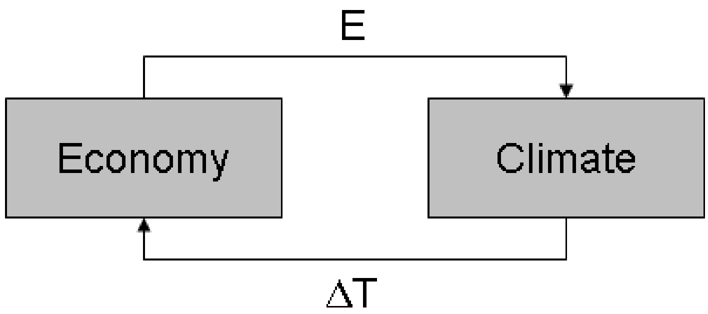


Figure 2: interaction between climate and the economy

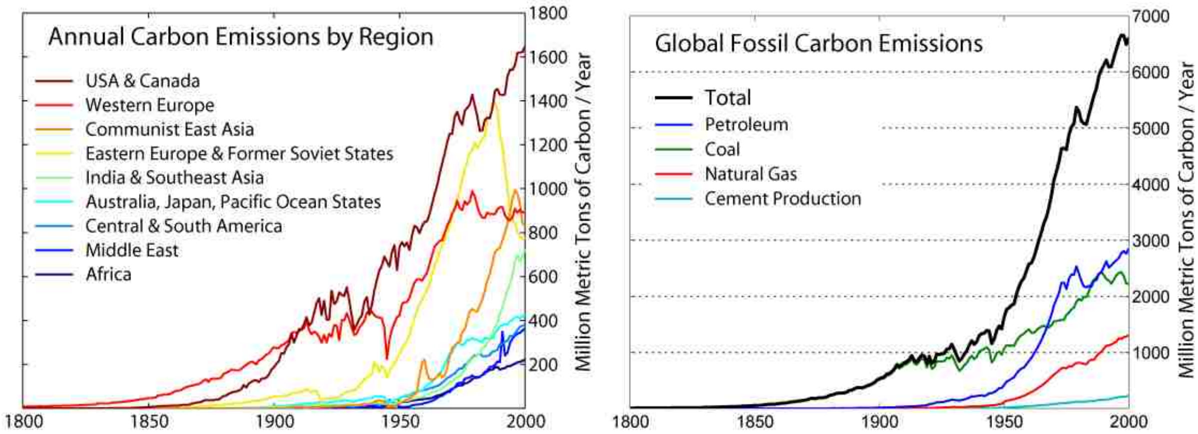
2.1.1 How does the economy affect climate change?[3]

Today it is evident that the main driver of the recent climate change and the phenomenon of global warming is the man made emission of greenhouse gases (GHG), especially carbon dioxide (CO_2). These emissions are an unmeant byproduct of our society and economy. The human-induced CO_2 emissions represent about 3% of total CO_2 emissions. Notwithstanding their small share, they create an important imbalance in the so-called carbon cycle. They cause a higher atmospheric CO_2 concentration which is very persistent and

[2] compare Section 2.1.4
[3] compare [15] and [20]

affects the extremely sensitive climate system through an increase of the greenhouse effect of the atmosphere. Evidence shows clearly that the average temperature on Earth has increased by more than 0.6 °C over the 20th century. Climate simulations have shown that most of the observed warming over the past 50 years is likely due to greenhouse gases from human activities. Scientists are very confident, that these higher CO_2 concentrations are also leading to a future increase in global temperature, but there is still uncertainty about the size of the impact. It has been estimated by IPCC[4] that a doubling of the CO_2 concentrations (which is likely to happen already within the next 100 years) will lead to a temperature increase of approximately 1.5 °C to 4.5 °C. Recent research however, suggests that the upper bound of this estimate could reach 11 °C.

This human induced emissions can also be seen as input for our economy, which is necessary to create a macroeconomic output. To get an idea of the amount of the recent human-induced carbon, Figure 3 gives the development of annual emissions by region and by source.



(a) by regions

(b) by source

Figure 3: carbon emmissions[5]

These emissions lead to a higher carbon concentration in the atmosphere, resulting to more radiative forcing and therefore a global temperature increase.

2.1.2 How does climate change affect the economy?

There are a lot of different and also interacting effects that climate change can have on the economy. An excellent overview is given in chapter 5 of the Stern Review on the Economics of Climate Change[6].

[4] the World Meteorological Organization (WMO) and the United Nations Environment Programme (UNEP) established the Intergovernmental Panel on Climate Change (IPCC) in 1988. Its role is to assess on a comprehensive, objective, open and transparent basis the latest scientific, technical and socio-economic literature produced worldwide relevant to the understanding of the risk of human-induced climate change, its observed and projected impacts and options for adaptation. Compare [20] and www.ipcc.ch

[5] taken from http://www.globalwarmingart.com/wiki with data from the Carbon Dioxide Information Analysis Center

[6] [23], available at http://www.hm-treasury.gov.uk

In general, climate change will have some positive effects for a few developed countries (such as Canada, Russia and Scandinavia) for moderate amounts of warming, but will become very damaging at the high temperatures that threaten the world in the second half of this century.

Warming will have strong impacts on the water availability and on agriculture, which is highly sensitive to climate change. Considering energy there will be a shift from heating demand to cooling demand in summer, while climate change could also disrupt energy production. Further, tourism will be affected. Due to heat waves and water reduction, much tourism will shift northwards. Mountain regions that rely on snow for winter recreation may experience significant declines in income. The impacts will become more damaging from north to south, a reason why the poorest countries will be the most vulnerable to climate change. Also the political framework, financial markets and trade routes will be affected by climate change. Further climate change is likely to increase significantly migratory pressures on developed countries.

Moreover there will be higher costs due to extreme weather events, such as storms, floods, droughts and heatwaves. Two examples for recent extreme weather events:

- Hurricane Katrina (2005)

  1,300 people died, \$125 billion in economic losses (~1.2% of US GDP)

- European Heatwave (2003)

  35,000 people died, \$15 billion damage in farming, livestock and forestry

Only the costs of such extreme events could reach 0.5-1% of world GDP by the middle of the century.

All these examples show how the environment affects the economy through climate change, therefore the output of an economy Y can be seen as a function of capital K, labor L (as commonly known) and additionally the environmental quality E.

$$Y = Y(K, L, E)$$

Quantifying the economical impacts of climate change is quite hard. For estimations a lot of assumptions have been done and the range of possible cases is large. Figure 4 gives an example for estimated changes of real GDP in the USA due to a temperature change, which goes from clearly negative to positive impacts. The truth lies somewhere in between. However for higher temperature increases the effects on the economy are clearly negative.

Therefore modeling the impacts of climate change is a sophisticated challenge, especially since the effects appear with long lags , which requires forecasting over a century or more. It is also a well known fact that only a small portion of the cost of climate change between now and 2050 can be realistically avoided, because of inertia in the climate system.

Nevertheless for modeling it is necessary to make assumptions and quantify these economical impacts or damages. This can be done by specifying damage functions which give the monetary damage for a region using the temperature change ΔT as argument. (In more simple models the emissions or the carbon stock are used as arguments.)

$$D = D(\Delta T)$$

[7] taken from [23]

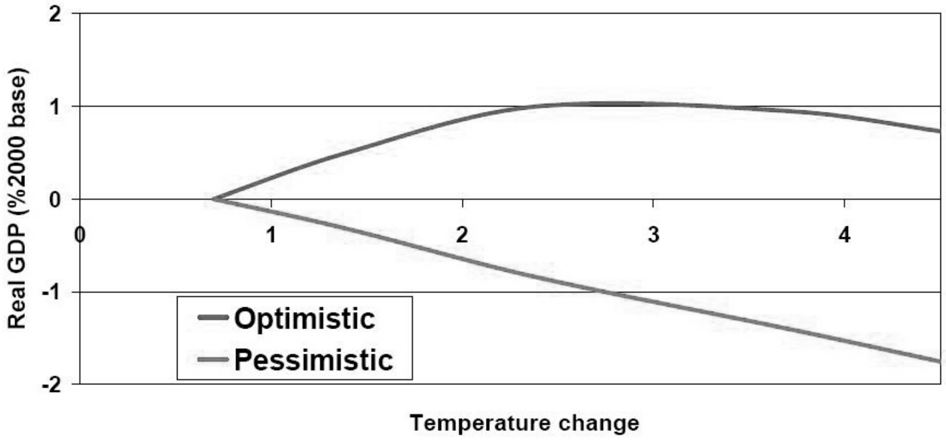


Figure 4: GDP change through temperature change[7]

Simply subtracting this damage is one possible way of considering the environmental quality in the macroeconomic output.

2.1.3 Integrated assesment models (IAMs)[8]

In order to assess policy options, the scientific and economic aspects of climate change are combined since the 1990ies in so-called integrated assessment models (IAMs).

An integrated assessment model can be broadly defined as any model which combines scientific and socio- economic aspects of climate change primarily for the purpose of assessing policy options for climate change control.[9]

They simulate the process of human-induced climate change, from emissions of GHGs to the socio-economic impacts of climate change (see Figure 5). In the real climate-human system, there will be feedbacks between many links in this chain. Significant for such models are large uncertainties. These uncertainties are physical as well as economical, concerning the key parameters of the climate change process and the future economic development. Moreover, these uncertainties are not constant because of gaining new insights about the physical process of climate change as well as better estimates for abatement costs and the cost of protecting ourselves against damages.

Although a multi-dimensional view of economic and social goals would be more appropriate than a narrowly monetary one, models that can measure climate change damage in monetary terms have an important role.

Three important IAMs are:[11]

- The "Mendelson" model (1998)
- The "Tol" model (2002)
- The "Nordhaus" model (2000)

[8]compare [23] chapter 6
[9]compare [18]
[10]taken from [23]
[11]compare [23] p.147

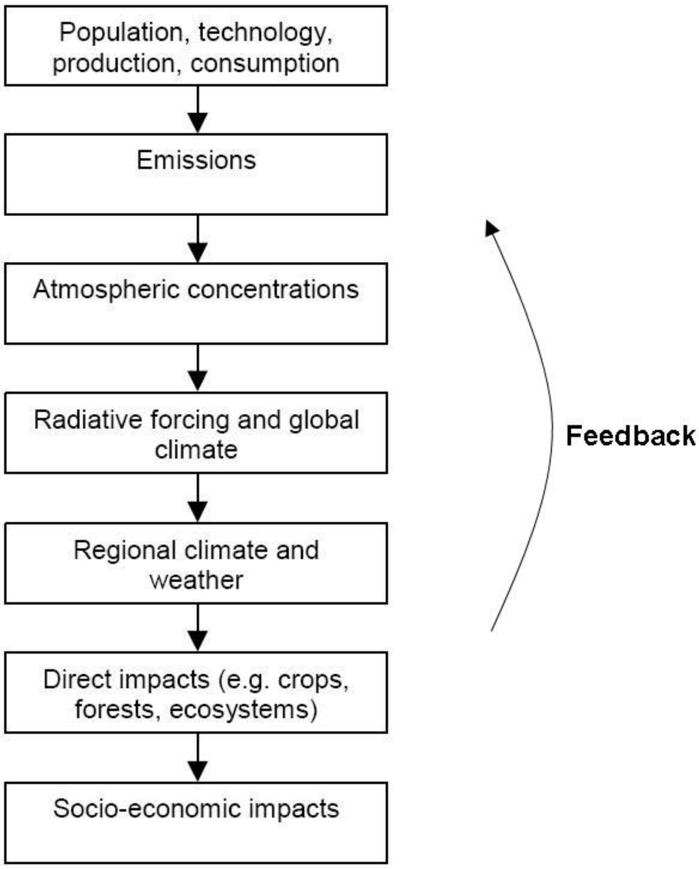


Figure 5: modeling in IAMs[10]

They estimate the cost of climate change in percent of world GDP with up to -11% for $6\,^{\circ}\mathrm{C}$ warming. Nevertheless existing models omit many potentially very important impacts and factors such as social and political instability and cross-sectoral impacts.

The CWS model presented in Chapter 3 is based on the Nordhaus model.

2.1.4 Externalities and public goods[12]

Dealing with environmental economics has one crucial point, externalities. The theory of negative externalities is a foundation of environmental economics. The most important contributions have been done by Pigou (1920), Coase (1960) and Baumol and Oates (1988) and should be summarized to a short extent in this section.

Externalities arise when certain actions of parties have unintended external (indirect) effects on other parties. In the literature many definitions of externalities can be found. One general and widely known definition is the one given by Baumol:

Definition (Externalities[13]). An externality is present whenever the two following conditions hold:

- The utility of some individuals include real (nonmonetary) variables, whose values

[12]compare [2]
[13]compare [2] p.17

are chosen by others without particular attention to the effects on the welfare of these individuals.

- The decision maker, whose activity affects the utility levels of others, does not receive (pay) in compensation for this activity an amount equal in value to the resulting benefits (or costs) to others.

But even Baumol himself notes that the definition can be shortened to its first part.

Externalities can be positive (e.g. technological spillovers) or negative (e.g. emissions). It is clear that externalities are always present in the area of environmental economics. Emissions of greenhouse gases are a prime example for an activity generating a negative externality since the action of one party causes harmful effects on others. It can be said that social cost is higher than private cost, since the burden on the whole society is bigger than just the burden on the party generating the negative externality. This divergence in private and social cost results in inefficiency in resource allocation. Producers of externalities do not have any incentive to take into account the effects of their actions on others. In a competitive market economy, private optimum output is determined at the point where marginal private cost equals marginal private benefit. When a negative externality occurs, the marginal social cost will be higher than the marginal private cost and hence the private optimal level of output will be higher than the social optimal output. This situation is illustrated in Figure 6 for the case of emissions as an externality. Considering just the private costs C_p the optimal emission level e_p^* will be higher than the optimal emission level e_s^* with considering the social costs C_s. (This relation will be illustrated more in detail in Section 2.3.1.)

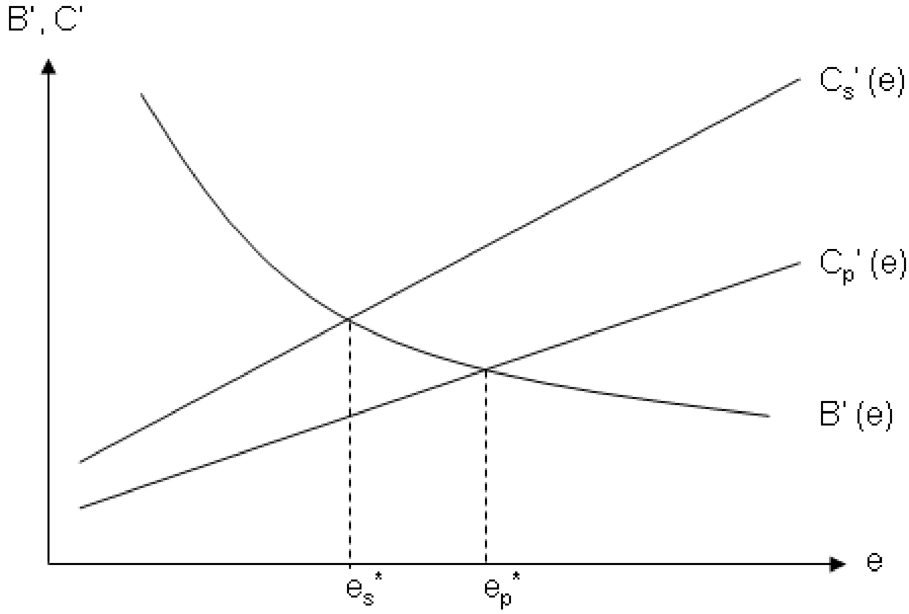


Figure 6: private and social optimum

There is a need to internalize externalities in the decisions of individuals so that social optimal levels of outputs and private optimal levels of outputs are the same. This can be done in two ways:

- setting a tax equal to the marginal damage

  This would be a traditional approach on the problem. An organization above the parties (if such an organization exists) introduces costs for the activity causing the externality, a so-called Pigouvian tax. (This is done indirectly for example with the EU-ETS described in Section 2.1.5)

- organizing private negotiations towards an efficient allocation of resources

  Coase[14] provides a different approach: instead of the producer of the negative externality harming the victim, a tax would harm the producer. The real question that has to be answered is: should the producer be allowed to harm the victim (through the negative externality) or should the victim be allowed to harm the producer (with a tax)? The problem is to avoid the most serious harm. The Coase Theorem basically suggests that an efficient solution can be achieved independently from the ownership rights. The reasoning is that if the emitter of the negative externality has the property rights, any victim will be willing to pay the emitter an amount up to the value of the damage being caused, to make the emitter reduce the externality. Similarly, if the victim has the rights, he will not allow the emitter to produce more than the social optimal amount, as the damage caused to the victim is greater than any payment the emitter would be willing to make. The establishment of property rights thus creates a framework for bargaining and for the achievement of the socially optimal outcome.

  More applicable to the issue discussed in this work, these payments to compensate harm can also be seen as transfers, as an outcome of private negotiations among parties to achieve an efficient social optimum. These transfers require cooperation. Such a group of cooperating players can be called a coalition.

Many externalities have the character of public goods, which are goods that are non-rival and non-excludable. This means that consumption of the good by one individual does not reduce the amount of the good available for consumption by others and that no one can be excluded from enjoying the benefits they generate. Greenhouse gas emission are therefore a global public bad in the sense that regardless of where the emissions are generated, they affect all persons on the earth and the ecosystem as a whole. Whereas emissions are a public bad, reducing emissions of greenhouse gases is an example of providing a pure public good.

2.1.5 International environmental agreements (IEAs) and free-riding

Global environmental problems such as the climate change involve many countries (or parties). As mentioned in the previous section, private negotiations and therefore cooperation is one way to be efficient in the presence of externalities. At the global level it is even the only possible way since there is no organization above the countries which could implement a tax. Such cooperation between countries leads to international environmental agreements.

Such agreements do already exist. Two well known examples which deal with emissions are the Kyoto Protocol and the European Emission Trading Scheme which will be described

[14] compare [9]

here.

- The Kyoto Protocol [15]

  The Kyoto Protocol is an international and legally binding agreement to reduce greenhouse gases emissions worldwide, made under the United Nations Framework Convention on Climate Change (UNFCCC). Countries that ratify this protocol commit to reduce their emissions of carbon dioxide and five other greenhouse gases, or engage in emissions trading if they maintain or increase emissions of these gases. It was agreed on 11 December 1997 at the 3rd Conference of the Parties to the treaty when they met in Kyoto, and entered into force on 16 February 2005. Governments are separated into two general categories: developed countries, referred to as Annex I countries (who have accepted greenhouse gas emission reduction obligations and must submit an annual greenhouse gas inventory); and developing countries, referred to as Non-Annex I countries (who have no greenhouse gas emission reduction obligations but may participate in the Clean Development Mechanism).

  The three Kyoto mechanisms are:

  – clean development mechanism (CDM)

    It aims at implementing projects that reduce emissions in non-Annex I Parties, or absorb carbon through afforestation or reforestation activities, in return for certified emission reductions. It also assist the host Parties in achieving sustainable development and contributing to the ultimate objective of the Convention.

  – Joint Implementation (JI)

    Under JI, an Annex I Party may implement an emission-reducing project or a project that enhances removals by sinks in the territory of another Annex I Party and count the resulting emission reduction units towards meeting its own Kyoto target.

  – Emissions trading

    This allows Annex I Parties to acquire units from other Annex I Parties.

  All three mechanisms under the Kyoto Protocol are based on the Protocol´s system for the accounting of targets. Under this system, the amount to which an Annex I Party must reduce its emissions over the five year commitment period is divided into units each equal to one tonne of carbon dioxide equivalent. These assigned amount units, besides other units defined by the Protocol provide the basis for the Kyoto mechanisms by enabling a Party to gain credit from action taken in other Parties that may be counted towards its own emissions target.

  In the Kyoto Protocol the industrialized countries agreed on reducing their greenhouse gas emissions till 2012 by 5.2% on average compared to the level of 1990.[16] According to IAMs this is a step in the right direction but it is far not enough.[17]

  Since the 2008-2012 Kyoto commitment period is coming to its end soon one objective of recent research in environmental economics is to provide knowledge for the

[15] compare http://unfccc.int
[16] compare [24] p.13
[17] compare [6]

ongoing Post-Kyoto negotiations on greenhouse gas emissions.

- The European Emission Trading Scheme (EU ETS)

  Launched in January 2005, the EU ETS is the world's largest companylevel "cap-and-trade" system for the trade of CO_2 emissions. At its heart is the common trading "currency" of emission allowances. One allowance gives the right to emit one tonne of CO_2. To each installation in the system a certain amount of allowances is allocated. A limit or "cap" on the number of allowances allocated creates a scarcity which is needed for a trading market to emerge. The price of allowances is determined by supply and demand as in any other market. Companies that keep their emissions below the level of their allowances can sell their excess allowances at this price. Those facing difficulty in remaining within their emissions limit have a choice between taking measures to reduce their emissions (such as investing in more efficient technology or using a less carbon-intensive energy source), buying the extra allowances they need at the market rate, or a combination of the two, whichever is cheapest. This ensures that emissions are reduced in the most cost-effective way. Because CO_2 now has a price, companies have incentives to identify cost-effective ways to reduce their emissions. The Emission Trading Scheme is therefore linking the Joint Implementation (JI) and the Clean Development Mechanism (CDM) from the Kyoto protocol.

  In the first trading period, from 2005 to 2007, the system covers only CO_2 emissions from large emitters in the industry (combustion plants, oil refineries, coke ovens, iron and steel plants and factories making cement, glass, lime, bricks, ceramics, pulp and paper). Some 10,500 installations in the 27 Member States are covered. They account for around 50% of the EU's total CO_2 emissions and about 40% of its overall greenhouse gas emissions. When several countries revealed registries indicating that their industries had been allocated more allowances than they could use, trading prices crashed from about 30€/ton to an all time low of 0.03€ at the beginning of December 2007. In the second trading period, from 2008 to 2012, emissions of nitrous oxide are also being included.

Other examples for IEAs are the ASEAN Agreement on Transboundary Haze Pollution, the Basel Convention and the Montreal Protocol on Substances That Deplete the Ozone Layer.

Countries will sign an international environmental agreement only if it is in their best interest to do so and so it is just a logical consequence if a country free-rides on such an agreement if it is economically better of in doing so. Free riding means taking advantage of the through the agreement generated benefits without participating, which is possible since these benefits are in general public goods and not excludable. In fact every country faces a strong free riding incentive when it has to decide on ratifying a environmental agreement and hope the others will do the effort.

This issues of free-riding and the former discussed externalities deal with cooperation of parties, which leads to Game Theory as a tool for analyzing the behavior of individuals and coalitions.

2.2 Game theory - coalitional games and stability concepts

In this sections some game theory which is important for the following work is summarized. It goes from basic definitions up to recently developed concepts in coalitional game theory and stability concepts. To make the reading easier throughout the whole section and also for the following chapters a constant notation is used.

2.2.1 Cooperative games

Cooperative games are a branch of game theory where players can negotiate about what to do in the game before the game is played. In these kind of games the idea of coalitions is a central element.

First of all it has to be clarified what is meant by a coalition in abstract terms.

Definition (coalition[18])**.** For a n-person game, we shall let $N = \{1, 2, \ldots, n\}$ be the set of all players. Any nonempty subset S of N is called a coalition.

The payoff or welfare which the ith player obtains in the game is denoted with $W_i(e)$ and is a function of the strategy vector $e = (e_1, \ldots, e_n)$ where e_i denotes the strategy of the ith player.

So far this says nothing about any cooperation among players, but for the further work it will be assumed that players within one coalition cooperate with each other.

The extreme cases that can occur are either that the players do not cooperate at all (play as singletons) or that there is total cooperation among all the players (grand coalition). These two extreme cases are reflecting two important situations. Whereas absence of any cooperation will lead to a Nash equilibrium, in the case of total cooperation a Pareto optimal situation will be achieved.

A vector of strategies is a Nash equilibrium if no player can do better by unilaterally changing his or her strategy.

Definition(Nash equilibrium[19])**.** A Nash equilibrium is the strategy vector $\bar{e} = (\bar{e}_1, \ldots \bar{e}_n)$ which is characterized by

$$W_i(\bar{e}_1, \ldots \bar{e}_i, \ldots \bar{e}_n) \geq W_i(\bar{e}_1, \ldots, \bar{e}_{i-1}, e_i, \bar{e}_{i+1}, \ldots \bar{e}_n)$$

for all $i = 1 \ldots n$. This level of welfare shall be denoted as $\bar{W}_i$.

The elements of the Nash strategy vector can also be described as

$$\bar{e}_i = \arg\max_{e_i} W_i(\bar{e}_1, \ldots, \bar{e}_{i-1}, e_i, \bar{e}_{i+1} \ldots, \bar{e}_n)$$

which leads to the method of obtaining the Nash equilibrium by an iterative process.

Every player is trying to maximize his own payoff, taking the strategies of the other players as given and not considering the consequences of the own strategy on them. In contrast with the grand coalition, the players act as one player, adapting their strategies

[18] compare [21]
[19] compare[16] p.535

to gain a maximized common welfare. The so achieved welfare for each player should be denoted as W_i^*.

$$e_i^* = \arg\max_{e_i} \sum_{j \in N} W_j(e_1^*, \ldots, e_{i-1}^*, e_i, e_{i+1}^* \ldots, e_n^*)$$

The so achieved situation is among the Pareto optimal strategies, which are characterized by the non-existence of any possibility to increase the outcome of one player without decreasing the outcome of any other player.

Formally Pareto optimality is defined as following:

Definition(Pareto optimal[20])**.** A strategy vector $e^* = (e_1^*, \ldots e_n^*)$ is called Pareto optimal, when for all other possible strategy vectors $e = (e_1, \ldots e_n)$ either

$$W_i(e_1, \ldots e_n) = W_i(e_1^*, \ldots e_n^*)$$

for all $i = 1 \ldots, n$ holds, or there exists at least one $j \in 1, \ldots, n$ so that

$$W_j(e_1, \ldots e_n) < W_j(e_1^*, \ldots e_n^*)$$

The concept of Pareto efficiency is often used by economists as a normative objective for society. It entails the idea that resources should not be wasted.

Although for the aggregated welfare levels

$$\sum_{i \in N} W_i^* \geq \sum_{i \in N} \bar{W}_i$$

holds, it is not ensured that $W_i^* \geq \bar{W}_i$ is true for all $i \in N$.

But besides these two extreme cases, there are also intermediate cases. Some players can form a coalition S. e_S will denote now the strategy vector of the members of the coalition S, whereas $e_{N \setminus S}$ will denote the strategy vector of the non-members. In case of one forming coalition, there must be an assumption on the behavior of the other players. A common assumption (the so-called γ-assumption) is that the other players maximize their own welfare as singletons which defines the so-called Partial-agreement Nash equilibrium (PANE).

Definition (Partial-agreement Nash equilibrium (PANE)[21])**.** Given a coalition $S \subseteq N$, a partial-agreement Nash equilibrium with respect to S is the strategy vector $\tilde{e} = (\tilde{e}_S, \tilde{e}_{N \setminus S})$ which is characterized by:

- $\tilde{e}_S = \arg\max_{e_S} \sum_{i \in S} W_i(e_S, \tilde{e}_{N \setminus S})$
- $\tilde{e}_i = \arg\max_{e_i} W_i(\tilde{e}_S, e_{N \setminus S})$ for all $i \in N \setminus S$

Therefore the PANE simply says that the players play Nash strategies (maximizing just their own pay-off), with the players in the coalition S acting jointly as one player. It is easy to see that the concept of the PANE can also be applied for several coalitions existing at the same time, by just treating each coalition as a Nash player.

[20]compare[16] p.541
[21]compare [17] p.247

These concepts (Nash equilibrium, Pareto optimality and PANE) will be illustrated in a minimal model for environmental economics in Section 2.3.2.

Further we will be interested in the utility that can be obtained by one of these coalitions. This utility can be divided among the members of S in any possible way. To express the amount of this utility the characteristic function is used, which gives the capacities of different coalitions.

Definition(characteristic function[22]). The characteristic function $v : P(N) \to \mathbb{R}$ provides for each possible coalition $S \subseteq N$ the information about the highest obtainable aggregate payoff, which is called the worth of the coalition.

$$v(S) = \max_{e_S} \sum_{i \in S} W_i(e_S, e_{N \setminus S})$$

The different assumptions (such as the γ-assumption) that can be done on the behavior of these outsiders are discussed in Section 2.2.2.

(Another approach instead of making general assumptions on the behavior of the other players, is to make the worth of a coalition depended on the entire coalitional structure of the game. This approach, called the partition function, is introduced in [3].)

It is straight forward to see that for fixed strategies $e_{N \setminus S}$ for the characteristic function holds the property of superadditivity:

$$v(S_1 \cup S_2) \geq v(S_1) + v(S_2) \text{ for } S_1 \cap S_2 = \emptyset$$

This property is important for the domination of the grand coalition. But one has to be aware of the fact that in games with externalities the strategies $e_{N \setminus S}$ might not be fixed. Outsiders are likely to adapt their strategies, which could lead to the violation of superadditivity.

At this point it has to be pointed out that whereas $\sum_{i \in S} W_i$ denotes the actual assigned welfare to the coalition S, $v(S)$ denotes the maximal achievable welfare of the coalition S. It is clear that not any allocation with a payoff vector $W = (W_1, \dots, W_n)$ will really occur in the game. There are two conditions which W must satisfy to get any chance of actually emerging. These are individual rationality and group rationality. A payoff vector which satisfies both conditions is called an imputation.

Definition (imputation[23]). An imputation (for an n-person game) is a vector $W = (W_1, \dots, W_n)$ satisfying

- individual rationality IR

$$W_i \geq v(\{i\}) \text{ for all } i \in N$$

- group rationality GR

$$\sum_{i \in N} W_i \geq v(N)$$

[22] compare [17] p. 245

[23] compare [21] p. 214

The motivations therefore are quite simple. If any player expects to be better off as an individual, than in a given allocation, this allocation will not remain. If it is assumed that the group rationality is not satisfied, then the players could form the grand coalition, and distribute the achieved surplus equally, which would make every player better off.

Further it seems intuitively clear that for an allocation to be stable it is also necessary that no possible coalition S has to profit from leaving the allocation and forming separately. This idea motivates the definition of the core.

Definition (core[24])**.** An imputation is said to be in the core of a game when it satisfies also the property

- coalitional rationality CR

$$\sum_{i \in S} W_i = v(S) \text{ for all } S \subset N$$

It is to be noted that CR now implies IR, since a single individual is by definition a coalition too. Therefore GR and CR are sufficient for defining the core.

The core, as a concept of the cooperative game theory, represents an important branch of the literature of coalitions. Stability of allocations (not only coalitions) can be naturally defined with this concept.

2.2.2 Stability concepts[25]

It is a well known fact that when a particular allocation is Pareto efficient, it does not imply that all players are better off compared to a Nash equilibrium. Any player who is worse off will accept an agreement that propose to implement such a coalition. Therefore it is important to be able to determine which allocations are stable. This question is not that simple to answer since there are different interpretations of stability.

In the following, the most important stability concepts for IEA should be discussed. But first we have to clarify what is meant by stability - stability of coalitions or allocations? In the papers which inspire this work (mainly [4] and [14]) it is exclusively talked about one coalition against the "rest of the world". The question may be raised why limiting oneself to this structure of one coalition S and the other players acting as singletons and not considering the family of all possible partitions that contain S? This is mainly due to the small number and complexity of ways to treat such games in the literature. Besides the fact that the number of possibilities rises enormous (see Section 3.6), it is difficult to treat the data and find suitable stability concepts which could be applied.

It has also to be said that the analysis of formed groups taken as given, can be carried forward to the analysis how they got formed. This is a highly advanced and a still quite unexplored topic, which will not be discussed here.

In the current literature about coalitions in environmental economics, two stability concepts are mainly used: the core stability and the internal-external stability. Both define the stability of a certain payoff vector $W = (W_1, \ldots, W_n)$ assigned to the players.

[24] compare [21] p. 219

[25] compare [5], [8]

Core stability

As it can be seen from the definition of the core in Section 2.2.1, the core property is that if any individual or group of players considers deviating from it, the best it can do leads to a less welfare than what it gets in the allocation. Therefore the allocations in the core will last and can be called stable in the core sense.
Definition (core stability)**.** A payoff vector assigned to the players $W = (W_1, \ldots, W_n)$ is called to be stable in the core sense if it lies in the core of the game. That is satisfying the properties

- group rationality GR
$$\sum_{i \in N} W_i \geq v(N)$$
- coalitional rationality CR
$$\sum_{i \in S} W_i = v(S) \text{ for all } S \subset N$$

It is important to note that when strict superadditivity

$$v(S_1 \cup S_2) > v(S_1) + v(S_2) \text{ for } S_1 \cap S_2 = \emptyset$$

holds, group rationality can be only achieved with total cooperation. In these cases, core stability can be only achieved with strategies chosen jointly by the members of the grand coalition.

It has also been mentioned before that the crucial element in the concept of core stability is the chosen characteristic function, which represents the worth of a coalition. But in order to calculate this worth with the presence of externalities, also the predicted actions of the nonmembers must been taken into account. Different assumptions in this respect lead to different concepts of characteristic functions such as the α-, β- and γ-characteristic functions.

- α- and β-characteristic functions [26]

  A frequently made assumption is that the non-coalition members adopt those strategies that are least favorable to the coalition, this puts the coalition members down to their minimax or maximin payoffs. This leads to the α- and β-characteristic functions.
$$v^{\alpha}(S) = \min_{e_{N \setminus S}} \max_{e_S} \sum_{i \in S} W_i(e_S, e_{N \setminus S})$$
$$v^{\beta}(S) = \max_{e_S} \min_{e_{N \setminus S}} \sum_{i \in S} W_i(e_S, e_{N \setminus S})$$
  This includes that the non-members will punish the coalition members as hard as possible, which seems not very realistic in the international environmental context.

  In global emission game context both characteristic functions are identical.

[26] compare[17] p. 246

- γ-characteristic function

  Contrary to the α- and β-characteristic functions, the concept of the γ-characteristic function (introduced in [7]) assumes that when a coalition S forms, the nonmembers would not take any particular coalitional action against it, but would adopt only their individually best strategies. This strategy is called partial-agreement Nash equilibrium with respect to S (PANE w.r.t. S) which is defined in Section 2.2.1.

  $$v^{\gamma}(S) = \max_{e_S} \sum_{i \in S} W_i(e_S, \tilde{e}_{N \setminus S})$$

  This approach assumes that if one or several countries attempt to free-ride on an assigned allocation, the other countries do not cooperate among themselves anymore, to make the free rider see that he is better off within the assigned allocation. Therefore free riders are threatened by the total absence of cooperation and therefore by their Nash payoff (or PANE payoff in case of a deviating coalition). This threat induces stability.

Therefore the α-, β- and γ core are defined as the core where the characteristic function is given by v^{α}, v^{β} and v^{γ}.

In practice, it is difficult to characterize the entire set of imputations which lie in the core. Therefore the practical analysis will be restricted to check whether a certain payoff vector lies in the core and represents therefore a stable situation in the core sense or not.

Since for environmental issues it is more realistic, for the rest of this work only the concept of the γ-core will be used.

Internal and external stability

Since this concept allows only one coalition it can be talked about the stability of a coalition. A coalition S is called internally stable when there is no incentive for a signatory (insider) to leave the coalition. That is when the payoff he gets in the coalition W_i^S is higher then the payoff he would get as a singleton $W_i^{S \setminus \{i\}}$, assuming that the coalition $S \setminus \{i\}$ remains. And the same coalition is called externally stable when there is no incentive for a non-signatory (outsider) to join the coalition. That is when the payoff he gets as singleton W_i^S is higher then the payoff he would get as a member of the coalition W_i^S, assuming that the coalition $S \cup \{i\}$ forms.

Definition(Internal and external stability[27])**.** A coalition S is called internally and externally stable if for the assigned payoff vector $W = (W_1, \ldots, W_n)$ the following properties hold:

- internal stability of coalition S (IS)

  $$W_i^S \geq W_i^{S \setminus \{i\}} \text{ for all } i \in S$$

- external stability of coalition S (ES)

  $$W_i^S \geq W_i^{S \cup \{i\}} \text{ for all } i \notin S$$

[27] compare [8]

This includes the assumption that the deviation of one player does not lead to any reaction of the other players. A coalition in this stability concept would tolerate free riding.

It is further assumed that besides the payoff vector with the coalition W^S, also the payoff vectors $W^{S\setminus\{i\}}$ and $W_i^{S\cup\{i\}}$ are known, which leads again to an assumption on the behavior of the players. To be consistent, for the work the γ-assumption will be used again.

It is reasonable to modify the concept of external stability, since in reality a violation of this condition would not lead necessarily to a collapse of the coalition. This can be seen considering the case that one player has an incentive to enter the coalition, but the current members would be worse off. They will not let him join the coalition which would lead to a stable situation.

The so-called "exclusive membership" external stability of a certain coalition requires the members of S to disagree with the accession of an outsider wanting to join the coalition. Otherwise, if they agree, the outsider would be accepted and therefore the coalition would change. "Agree" can be modeled in different ways, like majority voting (half of the original members of S plus 1 have to agree with accession, i.e. must be better off in $S \cup \{i\}$ than in S) and unanimity voting (all members of S have to be better off).

Definition (Exclusive membership external stability)**.** A coalition S is called externally stable with exclusive membership if for a player i for whom ES is violated the following property is violated too:

- exclusive membership external stability with unanimity voting (EMES-UV)

$$W_j^{S\cup\{i\}} > W_j^S \text{ for all } j \in S$$

- exclusive membership external stability with majority voting (EMES-MV)

$$W_j^{S\cup\{i\}} > W_j^S \text{ for all } j \in T \text{ where } T \subset S \text{ with } |T| > \frac{|S|}{2}$$

A last concept is the so-called potential internal stability. A coalition is said to be potentially internally stable if it can guarantee to all its members at least their free rider payoff.

Definition (potential internal stability PIS[28])**.**

$$\sum_{i\in S} W_i^S \geq \sum_{i\in S} W_i^i$$

Therefore a certain coalition which is not internally stable (IS), is potentially internally stable when the common payoff of the coalition can be redistributed in such a way that this coalition becomes internally stable. With other words, potentially internally stable agreements are coalitions that generate sufficient cooperation surplus to cover the free riding claims of all its members.

[28]compare [4]

2.2.3 Transfers[29]

If a given allocation is not stable it might be possible to shift pay offs from players to others, to achieve a new allocation which could be stable then. Such a shift of pay offs is called a transfer. Hence in transferable utility (TU) games transfers can be used to stabilize allocations and lower free riding incentives. Three transfer schemes have main importance for the later work. The Chander-Tulkens transfers, the Eyckmans-Finus transfers, and emission trading.

- Chander-Tulkens transfers[30]

  The motivation for this transfer scheme is that in reality, considering the global emissions game with total cooperation, the achieved payoff vector without transfers is not stable in the core sense (shown in [14]). In particular there exists at least one player i for whom $\bar{W}_i > W_i^*$ holds (where $\bar{W}_i$ denotes the payoff in the absence of cooperation). The Chander-Tulkens transfers redistribute the surplus from cooperation towards core stability, which can even get ensured under certain conditions.

  A reallocation of the surplus is introduced, so that after transfers each player achieves the following welfare level:

  $$W_i^{CT} = \bar{W}_i + \delta_i \Delta_W$$

  in which

  $$\Delta_W = \sum_{j \in N} (W_j^* - \bar{W}_j)$$

  stands for the joint global surplus of cooperation in the grand coalition compared to the equilibrium as singletons. Each player gets a positive share δ_i of the global surplus from cooperation, where $\sum_{i \in N} \delta_i = 1$. Hence all will be better off than with acting as singletons. Individual rationality will be therefore satisfied. It is easy to see that starting from an imputation W^*, group rationality is also satisfied since $\sum_{i \in N} \delta_i = 1$ and thus the transfer scheme leads to an imputation again.

  Chander and Tulkens have further shown that if the damage through the negative externality is linear, and if the chosen δ_i for each i reflects the share of the countries marginal climate change damages, applying the transfer scheme on an imputation (which is achieved in the case of total cooperation), the resulting allocation is not only individually rational, but also coalitionally rational. Therefore even the grand coalition (which is without transfers not necessarily stable in the core sense) can be stabilized in the core sense with the CT transfers, as long the linearity of the damage holds.

- Eyckmans-Finus transfers[31]

  Eyckmans and Finus presented a transfer scheme which is especially designed to counter free riding incentives of members of a coalition S: the Almost Ideal Charing

[29] compare [11]
[30] introduced in [7]
[31] introduced in [12]

Scheme (AISS) makes potentially internally stable coalitions internally stable. The idea is simply to give every member of S at least his free rider payoff, and to distribute the remaining surplus to the members. With the notation used before, after applying the AISS each player in the coalition S achieves the following welfare level:

$$W_i^{AISS} = W_i^{S\setminus\{i\}} + \delta_i \Delta_W^{AISS}$$

in which

$$\Delta_W^{AISS} = \sum_{j \in S} (W_j^S - W_j^{S\setminus\{j\}})$$

represents the surplus compared to the aggregated freerider payoffs.

A property of AISS is that, given any potentially internally stable coalition, with suitable chosen δ_i it stabilizes the coalition internally. Further, applying AISS on a PIS coalition S, the property individual rationality (IR) is satisfied for all players $i \in N$.

- Emission trading

  A third, conceptual different approach to transfer welfare is given for the specific case of emissions. It is assumed that within the coalition there is a market where the players can trade their emissions at a certain price p^E. More precise, not the emissions itself are traded, but the additional abatement efforts. Therefore it is necessary to assign each player a certain amount of permitted emissions $\hat{e}_i$ (e.g. grandfathering if the permits are in proportion to past emissions). If now a player i emits less than $\hat{e}_i$ he is assumed to sell his additional abatement efforts. The received payment can be interpreted as a transfer, since over all the players they sum up to zero.

$$W_i^{ET} = W_i + p^E(\hat{e}_i - e_i)$$

2.3 Modeling and general insights

With the background gained so far some first optimization models in the field of environmental economics dealing with emissions should be given. The objective is to see how modeling in this area works and how some game theoretical concepts are applied. But first it has to be clarified what is meant by an optimization model.

Definition (optimization model[32]). Optimization models represent problem choices as decision variables and seek values that maximize or minimize objective functions of the decision variables subject to constraints on variable values expressing the limit.

Talking about models in environmental economics, the decision variables will be the emissions (either directly or in form of an abatement rate) to maximize the welfare which is the objective. The only constraint for now will be that the emissions have to be positive.

In the following, the simplest form of such a model will be applied in different ways.

2.3.1 A first 2-countries model

First a two player model should be used to illustrate the basic mechanisms and the difference between Nash equilibrium and Pareto optimum.

Assuming the world has only two countries. The welfare W_1 of country 1 is the difference of its output Y_1 generated with its own emissions e_1 and the damage D_1 it suffers from, which is a function of its own emissions e_1 and also the emission of the other country e_2. (Since the game is symmetric, all statements can be also seen for country 2.)

$$W_1(e_1, e_2) = Y_i(e_1) - D_i(e_1, e_2)$$

It is assumed that the marginal productivity of emissions is decreasing and therefore the output function Y_1 is concave in emissions e_1. Contrary the damage function D_1 is assumed to be convex in emissions e_1. It is straightforward to see that for maximizing the welfare W_1 the marginal output has to be equal to the marginal damage which is illustrated in Figure 7.

An increase in the emissions of country 2 from e_2 to $e_2 \uparrow$ leads to an upwards shift and a steeper slope of D_1. Therefore the reaction of country 1 would be to lower its emissions e_1. In that sense the optimal choice of the emissions e_1^{opt} of country 1 can be seen as a function of the emissions e_2 of country 2 and vice versa. These functions are called the reaction functions of the players.

More insight provides a look on the situation in the e_1-e_2-plane given in Figure 8. The curves represent the isoquants of constant welfare for the two countries for each strategy pair (e_1, e_2). Further their reaction functions

$$e_i^{opt}(e_j) = \max_{e_i} W_i(e_i, e_j)$$

can be seen.

If each country just reacts in its optimal way on the emissions of the other country, the Nash equilibrium $\bar{e}$ will prevail, which is at the intersection of the two reaction functions.

[32] [22] p.4

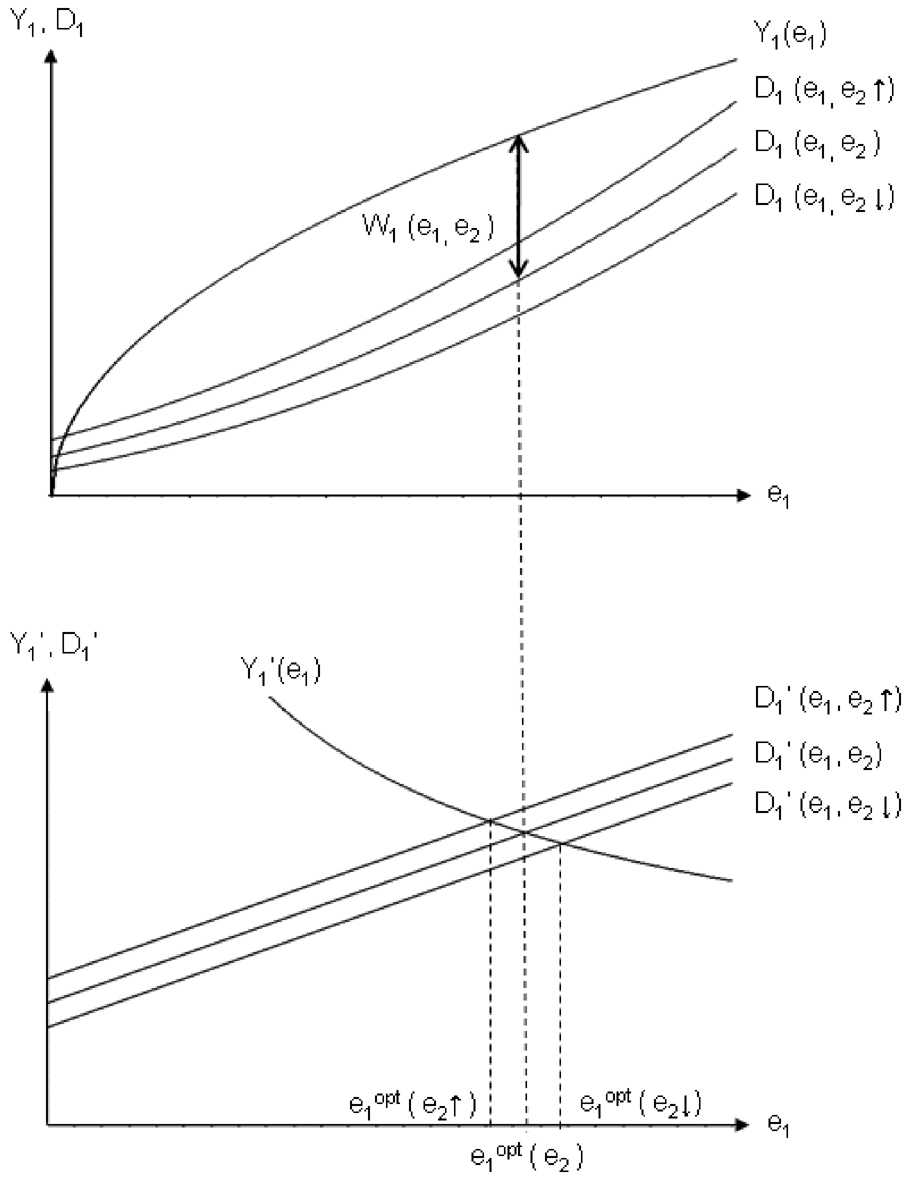


Figure 7: maximizing the welfare

But in the figure it is easy to see that there exist situations where both countries are better off (where they have higher welfare). Some of these strategies are among the Pareto optimal situations, which are described by the curve e^*. They have in common the non existence of any possibility to increase the outcome of one player without decreasing the outcome of the other player (which requires that the isoquants touch each other at exactly one point). Among these Pareto optimal situations there is one strategy which maximizes $\sum_{i\in\{1,2\}}$, the social optimum. (Where exactly depends on the slopes of the welfare levels.) There exists further a stable subset of the Pareto optimal strategies where $W_i^* > \bar{W}_i$ (and also $e_i^* < \bar{e}_i$) holds for $i = 1, 2$.

From the figure it can be also easily seen that in this simple model a unique Nash equilibrium exists when for the slopes of the reaction functions $-1 < \partial e_i^{opt}/\partial e_j < 1$ holds. For more complicated models the uniqueness of the Nash equilibrium is far not that easy to proof.

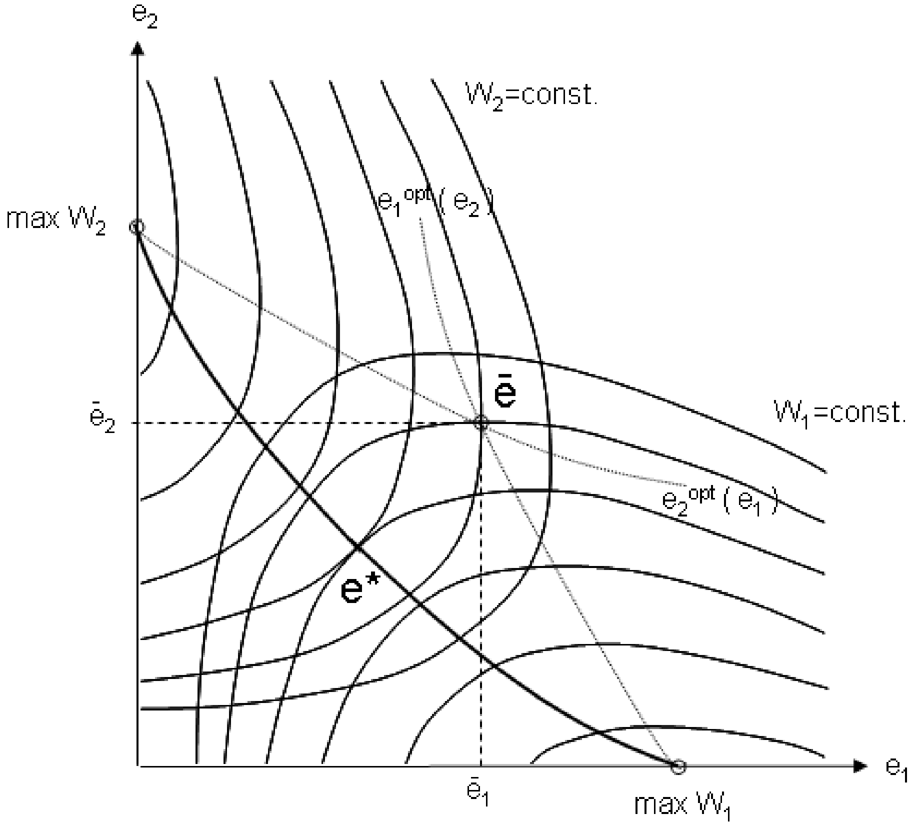


Figure 8: Nash strategy and Pareto optimal strategies

To get closer to the real world, this little model will be extended to n countries in the following section.

2.3.2 Extension to n countries[33]

As before, the players in this model are the n countries (or regions) of the world (indexed with $i = 1, \ldots, n$), with a welfare level W_i which each country wants to maximize. The benefit or output $Y_i(e_i)$ of each country can be described as a strictly concave function of emissions e_i. With the total sum of emissions $\sum_{i=1}^{n} e_i$, damages in the countries are generated which are given by increasing damage cost functions $D_i(e_w)$, where $e_w = \sum_{i=1}^{n} e_i$. For simplicity, in this model the damage cost functions will be assumed to be linear in e_w. Therefore $D_i(e_w) = d_i e_w$. With this, the welfare of the ith country can be written as

$$W_i(e_i, e_w) = Y_i(e_i) - d_i e_w$$

where $d_i > 0$.

The two extreme situations which will first be looked at are the absence of cooperation (national optimum) and total cooperation (world optimum).

- national optimum

  If every country acts as a single player, the optimum is achieved by maximizing its

[33] compare [6] and [13]

own welfare

$$\max_{e_i} W_i(e_i, e_w) = Y_i(e_i) - d_i(e_w)$$

with respect to its strategy e_i. If all countries adopt such a behavior, a Nash equilibrium $\bar{e}$ between the countries prevails. The first order condition for each country looks now as following:

$$Y_i'(\bar{e}_i) = d_i$$

This condition states that every country will let its emissions reach a level $\bar{e}_i$ such that the national benefits from the last ton emitted exactly equals the national damage it entails. This level of welfare should be denoted as $\bar{W}_i = W_i(\bar{e}_i, \bar{e}_w)$.

- world optimum

  Now it should be assumed that the whole world acts jointly and forms the grand coalition. World optimality can thus be reached by maximizing the sum of the welfare of all the countries

  $$\max_{e_1,\ldots,e_n} \sum_{i\in N} W_i(e_i, e_w) = \sum_{i\in N} Y_i(e_i) - \sum_{i\in N} d_i e_w$$

  with respect to the n variables $e_1, \ldots, e_n$, which leads to a Pareto optimal solution e^*

  Therefore the first order conditions for a maximum are given by the following system of equations:

  $$Y_i'(e_i^*) = \sum_{j\in N} d_j \text{ for all } i \in N$$

  Now all the climate damages are taken into account for each individual decision, therefore in the world optimum damages of emissions are perfectly internalized. This level of welfare should be denoted as $W_i^* = Wi(e_i^*, e_w^*)$.

  Further it can be seen that

  $$Y_i'(e_i^*) = Y_j'(e_j^*)$$

  holds for any pair of countries i and j. This says that the cost of the last ton of greenhouse gases cut should be equalized over all countries, which is known as cost efficiency. It is impossible to decrease the total costs of global emission reduction target by altering the burden sharing. In particular, low cost countries should perform relatively more effort than high cost countries.

By this it can be easily seen that the equilibrium emissions in a non cooperative system are larger than the emissions in the world optimum. Since $\sum_{j=1}^{n} d_j > d_i$ for all i, by comparing the first order conditions it follows that $Y_i'(e_i^*) > Y_i'(\bar{e}_i)$. The assumption that $Y_i(e_i)$ is concave leads to $\bar{e}_i > e_i^*$ for each i.

But now it should be also looked at the cases between the national and the world optimum: the situation where only some countries cooperate and form a coalition S.

- optimum with coalitions

  Let N denote the set of all countries of the world and $S \subset N$ be any subset of countries called a coalition. Then under the γ-assumption, the members of S could obtain their best outcome with maximizing their joint welfare, whereas the other countries $i \notin S$ maximize just their own payoff. This leads to the PANE defined in Section 2.2.1.

  $$\max_{e_i|i\in S} \sum_{i\in S} W_i(e_i, e_w) = \sum_{i\in S} Y_i(e_i) - \sum_{i\in S} d_i e_w \text{ for } S$$

  $$\max_{e_i} W_i(e_i, e_w) = Y_i(e_i) - d_i(e_w) \text{ for all } i \notin S$$

  The first order conditions for a maximum are given by the following system of equations:

  $$Y_i'(\tilde{e}_i) = \sum_{j\in S} d_j \text{ for all } i \in S$$

  $$Y_i'(\tilde{e}_i) = d_i \text{ for all } i \notin S$$

  Analog as before it can be seen that for the global emissions $e_w^* > \tilde{e}_w > \bar{e}_w$ holds.

  At this point it has to be noted that in such games with negative externalities outsiders always gain from a growing coalition. This is due lower global emissions e^w when a coalition gets bigger, which has only a positive effect on the outsiders.

While it is straightforward to define and characterize a world optimum and the optimum with coalitions in theory, implementing it is undoubtedly difficult in practice for several reasons:

- determining optimal emissions requires knowledge and agreement on the aggregate marginal damage costs $\sum_{i\in S} d_i$ and the countries marginal benefit from emissions $Y_i'(e_i)$,
- in reality the world optimum without transfers is in general not stable (shown in [14]), therefore transfers between the countries must be organized,
- if reductions in emission $(\bar{e}_i - e_i^*)$ are very large, they are not feasible (the Kyoto protocol requires only relatively small reductions).

The CWS model which will be introduced in Chapter 3 is basically an extension of this simple model. Whereas this model was static, the CWS model is a discrete time model. The welfare level will be determined by discounted consumption

$$Z_i = Y_i(e_i) - D_i \sum_{j=1}^{n} e_j$$

which will be expanded with investment I_i to archive a Ramsey type model and abatement costs C_i to take also into account the cost for reducing the emissions. A further step towards a Ramsey model will be that the output will depend on capital and labor $Y_i(K_i, L_i)$. And whereas the emissions e_i have been seen so far as the control variables, in the CWS model the control variables will be the investments I_i and the abatement rates μ, which are leading implicitly to the emissions E_i.

2.3.3 Optimal control[34]

In this section the simple model from before will be analyzed as a continuous optimal control problem.

Two unrealistic restrictions should be released. First, to choose the level of emissions is a dynamic problem, since the emissions can vary over time. Second, the damage is not really a function of emissions, but rather depend on the atmospheric emission stock $m(t)$ at time t which determines the global temperature. It will be dealt with only one player, the grand coalition, which has to decide on an optimal emissions path $e(t)$. Looking at this model as a continuous optimal control problem, will provide some insights of equilibria with different benefit and cost functions.

The formulation of the former used model as optimal control problem looks as following:

$$\max_{e(t)} W = \int_0^T (Y(e) - D(m)) \exp(-rt)$$

$$\dot{m} = e - \delta m$$

Again the welfare W to be maximized consists of the production $Y(e)$ dependent on the emissions level and the damage function $D(m)$, which is now a function of the atmospheric emission stock $m(t)$. The change in this stock $\dot{m}$ is determined by the emissions minus the natural carbon reduction which is given by the rate δ.

The Hamilton Function of this problem follows to

$$H = Y(e) - D(m) + \lambda(e - \delta m)$$

with the complementary slack variable λ.

The conditions for maximizing W are:

$$H_e = Y' + \lambda = 0$$

$$\dot{\lambda} = r\lambda - H_m = \lambda(r + \delta) + D'$$

Expressing λ and $\dot{\lambda}$ from the first condition and insertion in the second condition forms together with $\dot{m}$ the differential equation system:

$$\dot{e} = (r + \delta)\frac{Y'}{Y''} - \frac{D'}{Y''}$$

$$\dot{m} = e - \delta m$$

The isoclines of this system are given by:

$$\dot{m} = 0 \Leftrightarrow e = \delta m$$

$$\dot{e} = 0 \Leftrightarrow (r + \delta)Y' = D'$$

In the simplest case the production $Y(e)$ can be assumed to be concave in e and the damage $D(m)$ convex in m as before. The resulting phase diagram is given in Figure 9.

[34] the theory of optimal control and Pontryagin's maximum principle is taken from [16]

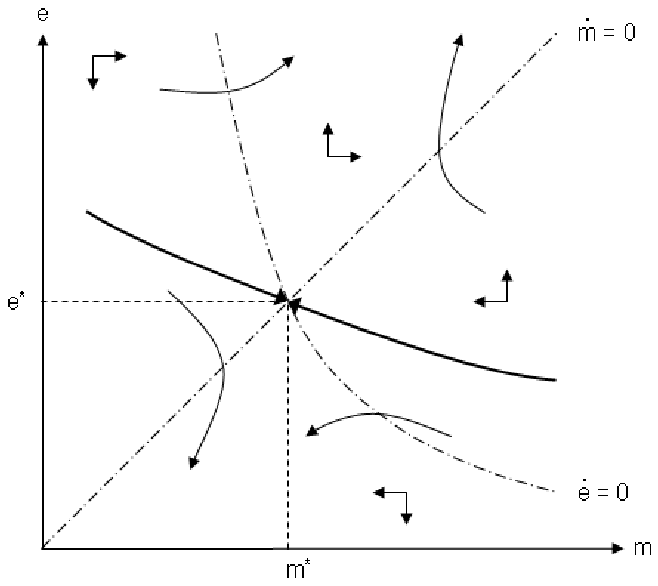


Figure 9: phase diagram

The system has one saddle point where the optimal trajectory leads to. The result is not really surprising: If there is a low atmospheric carbon stock, the highest welfare will be obtained with higher emissions than the equilibrium emissions e^*. If the carbon stock is higher than the equilibrium carbon stock m^* the emissions should be lower till the saddle point as an equilibrium is reached.

This analysis should be done in the same way with another damage function, which is given in Figure 10.

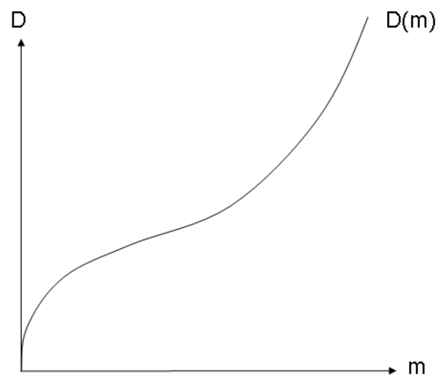


Figure 10: other damage function

It assumes that the first rise in the carbon concentration and therefore temperature increase harms the economy hard, but it will adapt so the damage curve gets flattened. But if the carbon concentration gets too high the economy collapses and the damage increases rapidly.

The related phase diagram is given in Figure 11.

Here three points of equilibrium occur (the stability properties of the different equilibria

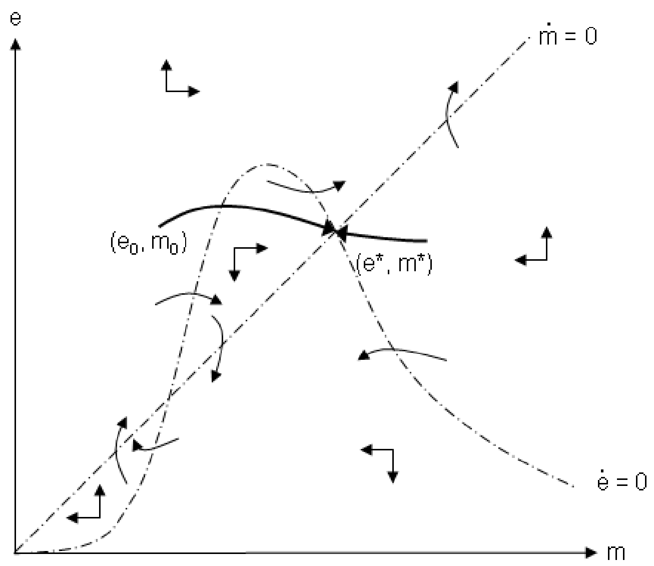


Figure 11: phase diagram for other damage function

are determined by choosing suitable functional forms and looking at the determinant of the corresponding Jacobian matrix.):

- one saddle point with the same properties as before
- one saddle point at $(0, 0)$, which says that if there is no carbon stock at all it is optimal to remain there (which is not the case in the real world)
- one unstable node in between

To transfer this to the real world, it is a reasonable assumption that the global carbon stock m_0 is lower than the values m^* in the equilibrium. Therefore a further increase in emissions and carbon stock can be expected. At a certain point the emissions should go down again to e^* in order to reach the equilibrium carbon stock m^*.

The conclusion of this analysis is that the number and types optimal equilibria depend highly on the underlying assumptions on the shape of the damage and the production function.

3 The CWS model

In this chapter, the CWS model, a central element of this work, will be presented in detail. Besides the model itself, also the implementation in GAMS and output will be given. The model and its applications are always work in progress, therefore the following pages will be rather a snapshot of the recent version.

3.1 Background - Goals and Development[35]

CLIMNEG (for CLIMate NEGociations) is an interdisciplinary research program on the economics and science of climate change that originated in 1996. It was funded by the Belgian Federal Office for Scientific, Technical and Cultural Affairs. The subject of this project is the exploration of the potential for post-Kyoto climate regimes with respect to two key issues:

- How could stable coalitions of countries emerge tomitigate climate change significantly?
- What could be the contribution of technological progress for a sustainable climate?

The partners of the project are the Center for Operations Research and Econometrics (UcL-CORE) and the Institut d'Astronomie et de Geophysique Georges Lemaître (UcL-ASTR) at the Université catholique de Louvain, and the Europese Hogeschool St Aloysius (EHSAL, Brussels).

A central element in the project is the CLIMNEG World Simulations (CWS) model, which is an integrated assessment model derived from the seminal RICE model[36]. The CWS model was introduced by Eyckmans and Tulkens in 2003 ([14]). Since that it has been used for a number of publications mainly related to climate coalitions. Over the course of years a lot of improvements to the model have been done. Although the basic mechanisms in the model remain the same, the calibration of the model and the computation in GAMS have been changed considerably. A recent big step is to refine the regional aggregation of the model from 6 to 18 regions.

Besides the general results of an IAM, the prediction of climate change through economic activities, the main goal of the recent research with the CWS model is to gain insights on what determines stable coalitions or allocations. Therefore the CWS model provides data which are further used to analyze the stability properties of coalitions.

[35] compare http://www.climneg.be

[36] the RICE model is introduced in [19]

3.2 The model itself[37]

The CWS model is a non-linear, time discrete and deterministic multi-region integrated assessment model. The used variables and their units are summarized in Table 1. (PPP is used for the exchange rates.)

E	carbon emissions (billion tons of carbon per year)
μ	carbon emission abatement (%)
M^{AT}	atmospheric carbon concentration (billion tons of carbon)
M^{UO}	upper ocean and vegetation carbon concentration (billion tons of carbon)
M^{LO}	lower ocean carbon concentration (billion tons of carbon)
F	radiative forcing (Watt per m^2)
T^L	temperature change lower ocean (°C compared to 1800)
T^E	temperature change atmosphere (°C compared to 1800)
Y	production (billion US$ 2000)
Z	consumption (billion US$ 2000)
I	investment (billion US$ 2000)
C	abatement costs (billion US$ 2000)
D	damage costs (billion US$ 2000)
K	capital stock (billion US$ 2000)

Table 1: names and units of variables

The economic part is a long term dynamic multi-region Ramsey type of optimal growth model with endogenous capital accumulation driven by assumptions on regional technological progress, population growth and time preference. Emissions of carbon are a function of economic output, exogenous technological progress and endogenous emission abatement policies. This model of the world economy and emission processes is coupled to a carbon cycle model and a climate model. Although it is a dynamic model, because of its time discrete form, it will be treated as static in that way that the equations and control variables are given for every time step.

The principle of the CWS model is quite simple and follows the models introduced in Section 2.3. The world is divided into n regions. Each region i runs its economy independent from the others (trade is excluded), causing at every time t emissions $E_{i,t}$. The aggregated emissions of all the regions affect the climate. First, in the carbon cycle, the emissions influence the carbon concentration in the atmosphere M_t^{AT} which determines further a global temperature change T_t^E. This temperature change has again impact on the economies, which experience damage through higher temperatures. This basic cycle is illustrated in Figure 12.

What the regions can do to control this cycle is to abate part of their carbon emissions which is represented by their abatement rates $\mu_{i,t}$. This causes abatement costs in the short run but leads to less damage costs through reduced temperature change in the long run. The crucial point hereby is that the abatement of emissions in one region affects the global temperature and therefore the damage costs for all regions.

[37]compare [10], [14] and [4]
further the recent changes in the model which are not published yet are given.

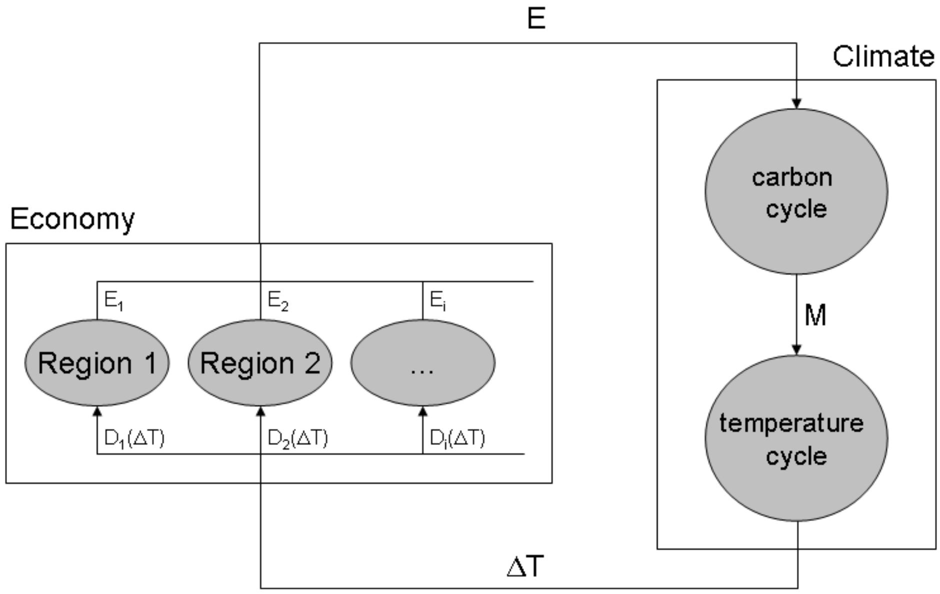


Figure 12: basic scheme of the CWS model

What is happening now in detail?

Each region i maximizes its total welfare W_i which is defined as its discounted aggregated utility per capita $U_{i,t}$ with discount rate $\rho_{i,t}$, times its population $L_{i,t}$ over the whole time horizon $t = 0, \dots, T$. Therefore

$$\max_{\mu_{i,t}, I_{it}} W_i = \sum_{t=0}^{T} \frac{L_{i,t} U_{i,t}}{(1+\rho_i)^t} \tag{1}$$

using the control variables abatement rate $\mu_{i,t}$ and investment $I_{i,t}$. The abatement rate $\mu_{i,t} \in [0, 1]$ is the ratio at which the emissions will be reduced. It is assumed that the regions can decide on this rate with policies and measures (e.g. taxes, subsidies and other financial incentives, research and development on renewable energy,...).

Therefore it can be said that the players payoffs are the welfare levels W_i of the countries at time T and the players strategies are the chosen decision variables $\mu_{i,t}$ and $I_{i,t}$.

For the utility function a form with constant elasticity of marginal utility per capita has been chosen. However, in the current implementation of the model η is set to 0, therefore the welfare is just the discounted consumption. But with this form a wide range of other utility functions (as mentioned in [23]) can be easily applied.

$$U_{i,t} = \frac{1}{1-\eta} \left(\frac{Z_{i,t}}{L_{i,t}} \right)^{1-\eta} \quad \text{for } \eta \neq 1 \tag{2a}$$

$$U_{i,t} = \log \left(\frac{Z_{i,t}}{L_{i,t}} \right) \quad \text{for } \eta = 1 \tag{2b}$$

This maximization is done subject to the following system consisting of the economic part and the climate part.

3.2.1 The economy

The following equations describe the economy of a region i as a Ramsey type growth model.

$$Y_{i,t} = A_{i,t} K_{i,t}^{\gamma} L_{i,t}^{1-\gamma} \tag{3}$$

$$K_{i,t+1} = (1 - \delta_K)^{10} K_{i,t} + 10 I_{i,t} \tag{4a}$$

$$\rho_i K_{i,T} \leq I_{i,T} \tag{4b}$$

$$Z_{i,t} = Y_{i,t} - I_{i,t} - C_i(\mu_{i,t}) - D_i(T_t^E) \tag{5}$$

The generated macroeconomic output $Y_{i,t}$ in equation (3) is given by a Cobb Douglass function of capital $K_{i,t}$ and the population $L_{i,t}$ with an overall productivity parameter $A_{i,t}$ which represents technological progress over time. The growth of population and the productivity parameter are hereby exogenous. Since the exponents of K and L sum up to 1, the production function has constant returns to scale.

Equation (4) describes the development of the capital $K_{i,t}$ under the capital depreciation rate δ_K and the accumulating investments $I_{i,t}$ (in time steps of 10 years). Equation (4b) is just necessary to prevent that in the last but one period the whole capital stock will be consumed. The condition ensures that the capital stock remains the same as in the previous period.

Finally equation (5), the budget equation, describes the use of the output $Y_{i,t}$, where consumption results to be what is left after investment, abatement costs $C_i(\mu_{i,t})$ and damage costs $D_i(T_t^E)$.

Essential for the model are the abatement cost and the damage cost functions.

$$C_i(\mu_{i,t}) = -Y_{i,t} c_i [(1 - \mu_{i,t}) \log(1 - \mu_{i,t}) + \mu_{i,t}] \tag{6}$$

$$D_i(T_t^E) = Y_{i,t} \Theta_{i,1} \left(\frac{T_t^E}{2.5} \right)^{\Theta_{i,2}} \tag{7}$$

The abatement costs $C_i(\mu_{i,t})$ in equation (6) describe the monetary efforts which have to be done to archive an abatement rate of $\mu_{i,t}$. It is derived from the desired marginal cost function

$$C_i'(\mu_{i,t}) = Y_{i,t} c_i \log(1 - \mu_{i,t})$$

with $c_i < 0$. It is constructed so, that it is strictly increasing and strictly convex in abetment $\mu_{i,t}$, with $\lim_{\mu \to 1} C_i' = \infty$, which makes 100% abatement unaffordable. Also the abatement costs $C_i(\mu_{i,t})$ itself are strictly increasing and strictly convex in abetment $\mu_{i,t}$.

The damage costs $D_i(T_t^E)$ in equation (7) are the amounts of damage in monetary terms which the region i faces by an average global temperature change of T_t^E. Herby $\Theta_{i,1}$ can be interpreted as the damage as ratio of GDP, through a temperature increase of 2.5 °C. The function is strictly increasing and strictly convex in temperature change T_t^E (which follows from $\Theta_{i,2} > 1$).

The carbon emissions $E_{i,t}$ which are the link to the climate part of the model are proportional to production. The exogenous emissions to output ratio $\sigma_{i,t}$ declines exogenously over time due to an assumed energy efficiency increase. Emissions can be reduced at a rate $\mu_{i,t} \in [0,1]$ in every period.

$$E_{i,t} = \sigma_{i,t}(1-\mu_{i,t})Y_{i,t} \tag{8}$$

3.2.2 The climate

Further the model contains an environmental part, which is transferring the aggregated carbon emissions $\sum_{i=1}^{n} E_{i,t}$ into a temperature change in the atmosphere T_t^E.

Climate change is a complex physical process which requires the cooperation of different disciplines for modeling the interaction of the different physical, chemical, and biological systems. Compared to other models, the climate in the CWS model is rather simple but still requires some explanation to non climatologists. A complication in modeling climate change is that the effects are not uniform. The temperature change differs from region to region. Nevertheless the climate model of the CWS model calculates just an average temperature change in a simplified form. Thus the regional differences have to be incorporated in the regional damage cost functions. The climate model used for the CWS model has basically two parts: the carbon cycle and the temperature cycle.

In the carbon cycle the atmospheric carbon concentration M_t^{AT} is changing according to the aggregated carbon emissions $\sum_{i=1}^{n} E_{i,t}$.

$$M_{t+1}^{AT} = M_t^{AT} + 10(b_{11}M_t^{AT} + b_{21}M_t^{UO} + \sum_{i=1}^{n} E_{i,t}) \tag{9a}$$

$$M_{t+1}^{UO} = M_t^{UO} + 10(b_{12}M_t^{AT} + b_{22}M_t^{UO} + b_{32}M_t^{LO}) \tag{9b}$$

$$M_{t+1}^{LO} = M_t^{LO} + 10(b_{23}M_t^{UO} + b_{33}M_t^{LO}) \tag{9c}$$

It is represented by the interaction of the atmospheric carbon accumulation process M_t^{AT}, the upper ocean and vegetation carbon accumulation process M_t^{UO}, and the lower ocean carbon accumulation process M_t^{LO}.

The temperature cycle determines finally the average temperature change in the atmosphere T_t^E, which is the absolute change in °C compared to the pre-industrial temperature in 1800.

$$T_{t+1}^{E} = \frac{1}{1+c_1\lambda+c_1c_3}T_t^E + c_1(F_{t+1} + c_3T_{t+1}^L) \tag{10a}$$

$$T^L_{t+1} = T^L_t + c_4(T^E_t - T^L_t) \tag{10b}$$

$$F_t = \frac{\log(M^{AT}_t/M_0)}{\log(2)} F^{2\times} + F^{other}_t \tag{11}$$

The temperature process in the atmosphere T^E_t consists of the temperature increase in the previous period, a summand taking into account the radiative forcing F_t and a summand considering the temperature process in the ocean T^L_t. λ is a climate feedback factor. It is defined as the ratio of the computed surface temperature change taking into account feedbacks to the temperature change calculated assuming no feedbacks. (Such feedbacks occur for example through clouds, ice fields, atmospheric water vapour content,...) The temperature process in the ocean is explained as the temperature increase in the previous period and the weighted difference of the temperature increase in the atmosphere and in the ocean. Finally, the link to the carbon cycle is given in equation (11) where the radiative forcing (in Watt per square meter) is described as a function of the ratio of the current carbon concentration to the initial carbon concentration M_0 in 2000. Herby $F^{2\times}$ is the forcing with a carbon concentration doubling and F^{other}_t is the forcing of other gases and aerosols which is given as an exogenous time series.

A further problem of this implementation in the CWS model is that the time step in the total model is 10 years, which is a very rough time step for a climate model. Therefore there is work done currently on this part of the model to improve its quality.

The set of equations (1) to (12) is finally determining the CWS model, which is summarized in Figure 13. The parameters of the model, the initial values and the endogenous variables are given in appendix A.

3.2.3 Remarks and additional features

Besides this basic framework some remarks should be done on the CWS model.

- Free riding correction[38]

  The rest of the world is treated as one region (ROW). So it is assumed, that all the countries within this formal region have as common objective their aggregated utility which they adapt to their common damage function. But this assumption is not realistic, since these countries will not act as one decision maker (namely the region ROW) and consider their common damage. Therefore, to be more realistic the damage of this region which is used for the model solving has to be reduced in the case when the rest of the world is not in any other coalition. This is simply done by dividing the multiplicative parameter $\Theta_{ROW,1}$ in the damage function $D_{ROW}(T^E_t)$ by the so-called "free riding correction" - factor n_{row}. It represents approximately the number of countries in the region ROW.

  $$D_{ROW}(T^E_t) = \frac{\Theta_{ROW,1}}{n_{ROW}} {T^E_t}^{\Theta_{ROW,2}} \text{ if } ROW \notin S$$

[38] compare also [19]

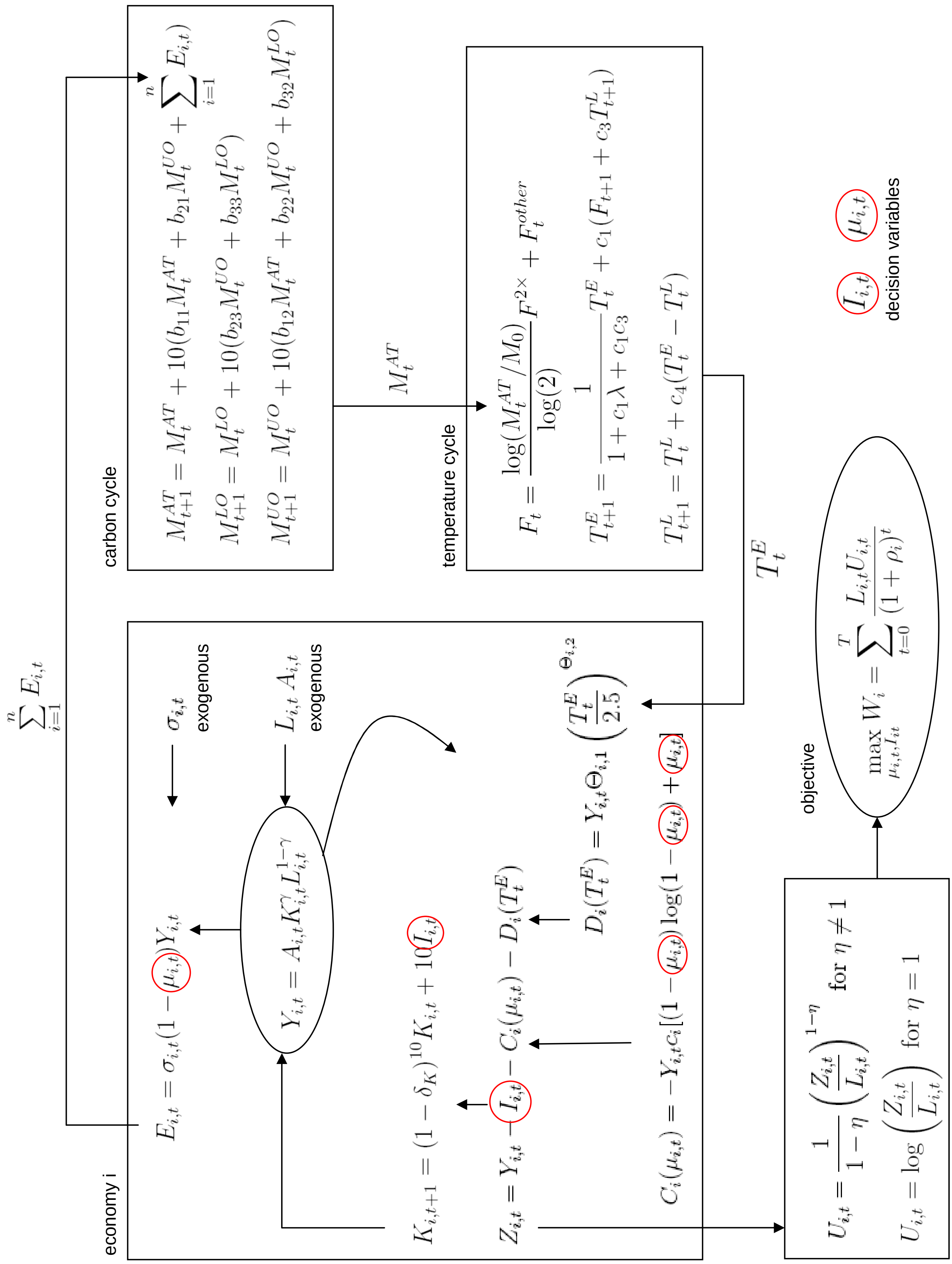


Figure 13: the CWS model

- Deforestation

  Besides the carbon emissions caused by production, also the emissions through deforestation are taken into account. Deforestation is the conversion of forested areas to non-forest land for use, mainly in the tropical countries. The loss of wood releases a lot of carbon stored in the trees back to the atmosphere, which accounts for up to one third of human induced CO_2 emissions. These emissions are entering the model as exogenous time series added to the emissions through production. (In the current implementation this feature, although it is implemented is set to inactive.) Therefore in the remaining part of this thesis it will not be considered.

- Bounds

  For certain endogenous variables, reasonable bounds are defined to reduce the search area when optimizing the model. They cannot be called constraints of the model since they are chosen in such a way that they cannot be reached. These bounds are defined trough intuition and testing and are changing with the model. Therefore the actual values are not given here.

- Uniqueness of the Nash equilibrium

  Unfortunately there cannot be given a proof of uniqueness of the Nash equilibrium reached with the model. The literature provides only statements on uniqueness for very specific cases of nonlinear models.[39] The difficulty arises through the multidimensional decision vector of the players. In the CWS model the decision vector of each player i consists of his abatement rate $\mu_{i,t}$ and his investment $I_{i,t}$ at each time step t. So far there has been only one equilibrium detected. Also numerical testing with different start vectors did not lead to an other Nash equilibrium. Another major argument for neglecting this issue is that the economic interpretation of the found equilibrium has always been reasonable. Therefore it is simply assumed that a Nash equilibrium exists and that it is unique.

- Dynamic coalitions

  One further remark should be done on the used assumption that coalitions in the CWS model are by definition coalitions over the whole time period. This assumption is not very realistic since incentives joining the one or the other coalition could change over time. But so far only first concepts to deal with these dynamic coalitions, which would enlarge the set of possibilities enormous, have been introduced in the literature.

[39] compare [1]

3.3 Implementation in GAMS

The General Algebraic Modeling System (GAMS) to be found at *www.gams.com* is a modeling system for mathematical programming and optimization. It consists of a language compiler and a number of integrated solvers. GAMS is tailored for complex, large scale modeling applications, and allows the user to build large maintainable models that can be adapted quickly to new situations. The CWS model got implemented in GAMS 2000 by Johan Eyckmans. Since that, many adaptations have been done.

During this work, the code has been restructured and all the provisional arrangements which have been done in the last years have been neatly implemented. A further improvement was an upgrade from a 6-regions structure to a flexible structure with 18 regions, which allows flexibility in the treatment of allocations or coalitions and an optimal interface for handling different scenarios.

In this section the basic framework of the current code, sketched in Figure 14, should be explained. Parts of the recent version of the code can be found in Appendix C.1.

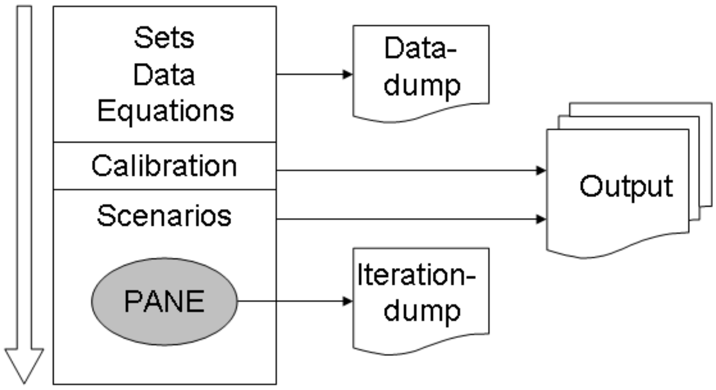


Figure 14: code structure of the CWS model

The input consists of the set definitions, the data and the equations. Sets are the basic building blocks of a GAMS model, corresponding exactly to the indices in the algebraic representations of models. Especially players and time steps are defined here. The players are the former 6, now 18 regions given in Section 3.4 and the time range is from 2000 till 2300 with steps of ten years, therefore 34 periods.

In the data input, initial values, growth rates and other necessary parameters are defined and assigned. Also the trend data for population, output and the emission/output ratio which are exogenous for the model are calculated here through a functional form (given in Section 3.4 and Appendix A). For new estimates or for testing different parameters (for example discount rates) changes have to be done here. These input data (especially the generated time series) are written to a dump file for giving the possibility to inspect these data later on.

The very core of the model are the equations. These are the equations (1)-(12) listed in Section 3.2. Further the aggregation of equations to models is done here. (The solver solves models, which are sets of equations. Specifically for the calibration there are different models for the economy and the climate necessary.)

In the next step a initialization and calibration of the model is done in order to provide a good starting position for later scenarios (compare Appendix B). First a Ramsey model (just the economy part of the equations) is solved for every region separately, disallowing abatement and damage through climate change. This is done several times to achieve equilibrium values for all the variables. After the Ramsey model, the climate model is solved separately. It takes the emissions out of the Ramsey model as given and assigns values to all the climate variables. With the so achieved temperature change, the consumption calculated in the Ramsey model is corrected by the damage costs. The results of this so called business as usual (BAU) scenario are written to an output file. Also the bounds for the variables in the model (as described in Section 3.2.3) are defined here in order to narrow the search area for the solver.

Now the main calculations are done. Scenarios can be chosen, for which a solution will be calculated. With scenarios are meant different forms of allocations. The main scenarios are the NASH scenario (absence of cooperation - every region acts as a singleton) and the COOP scenario (total cooperation - grand coalition). Further the model can be fed with other scenarios like all allocations with one coalition against singletons or all the partitions. Since the main scenarios are of major interest, they will be practically calculated every time when the program runs. For these also special output files are generated.

For other scenarios a key-matrix has to be fed in. This key matrix (taken from another file which has to be modified or changed for different scenarios) consists simply of one allocation per line. For all these allocations in the key matrix the equilibrium will be calculated and the relevant variables displayed in an output file.

For the representation of a coalition in this key matrix two ways are possible. These will be described now. For simplicity the case of only 6 players $\{A, B, C, D, E, F\}$ is discussed.

- binary key for only one coalition S

  In the case of only one coalition for each allocation a binary key of n digits can be given. The ith digit is 1 if the ith player joins the coalition and 0 otherwise.

  e.g. players B,C and F form a coalition whereas players A, D and E stay as singletons, the key would be (011001).

- alphanumeric key for multiple coalitions

  In the case with multiple coalitions a binary key is not sufficient anymore. A suitable key therefore is to put on the ith digit the number of the coalition the ith player joins. Also singletons are treated here as a coalition. To exclude multiple representations of an allocation in this key, there has to be introduced the rule, that the digits have to appear in a lexicographical order.

  e.g. players A and F form a coalition $S1$, players B and C form another coalition $S2$ whereas players D and E stay as singletons, the key would be (122341).

  When exceeding 9 players, the digits have either to contain letters or more digits have to be reserved for each player.

During this work the code has been modified so that the program can handle both representations. How the program works with these keys is explained more detailed in Section 3.4.

For the calculation of the solution for each scenario the partial Nash equilibrium with respect to a coalition (PANE), described in Section 2.2.1, is used. What is done practically in the PANE algorithm for every allocation is the following:

- take the first coalition $S1$
- fix the control variables $I_{i,t}$ and $\mu_{i,t}$ for all regions i which are not in the coalition
- set the objective function to the welfare level of this coalition (the sum of the welfare levels of the members)

  This is done basically by having the objective function $\sum_{i \in S_1} w_i W_i$, where w_i, the current welfare weight, is set to 1 if the region is in the coalition $S1$ and 0 otherwise. (With this framework there are also other welfare weights possible, where the welfare levels of different countries count differently.)
- maximize the welfare level of this coalition

  Therefore the model (with fixed variables and a utility function according to the current step in the current scenario) is solved with GAMS/MINOS[40], which is a general purpose nonlinear programming (NLP) solver, designed to find solutions that are locally optimal.
- do the same for the next coalition (including singletons)
- repeat this loop over all the coalitions until the stop criterion or a prespecified upper limit of iterations is reached (the stop criterion is more precisely discussed in Section 3.7)

This is done for each scenario. After finishing the PANE for one scenario, the current values of the variables of further interest are written to the output and the optimization of the next scenario takes these values as starting values.

A lot of code deals with the preparation and generation of the output and dump files. These are basically commands to put specific strings and values at a specific place in a specific text file. The exact specification of the output changes with the requirements and therefore it will not be explained in detail here.

But what is also done in the output preparation is the final calculation of values of interest. While solving the model, just the variables used in the model are directly assigned with values. These are:

- economic variables: W_i, $U_{i,t}$, $Z_{i,t}$, $Y_{i,t}$, $I_{i,t}$, $K_{i,t}$, $D_{i,t}$, $C_{i,t}$, $E_{i,t}$, $\mu_{i,t}$
- climate variables: M_t^{AT}, M_t^{UO}, M_t^{LO}, F_t, T_t^E, T_t^L

All other values of interest must be calculated afterwards with these values and the input data (initial values and exogenous variables in the form of calculated trends). Examples for such values of interest are ratios, marginal values and also welfare levels after transfers. The generated output is summarized in Section 3.5.

[40]compare http://www.gams.com/solvers/solvers.htm

3.4 The update to 18 regions

An important update which has been done during this work, was to extend the model from 6 to 18 regions. This was necessary on the one hand to make the region "rest of the world" smaller in order to be more precise, and on the other hand to have a larger number of possible coalitions to gain more general results about their stability.

So far the code was explicitly written for 6 regions. For every scenario 6 coalitions got formed (which is the maximum number of coalitions in one allocation with 6 regions) which occurred explicitly in the code. To overcome this problem and to make the code also suitable for any size of regions, the "partition matrix" got introduced as a useful instrument to deal with an arbitrary number of regions in a flexible way.

The partition matrix is a 2-dimensional binary key representing the given allocation in a unique way - a $n \times n$-matrix where the columns indicate the players and the rows the coalitions. When the ith player joins the jth coalition the value at (i, j) equals 1. Otherwise it equals 0.

For the example used above, (122341) the partition matrix would be

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

With this partition matrix as a tool, it is easy to formulate all the actions done in the code as loops, which reduces massively the size and readability of the code. The loop over all coalitions is done as a loop over all the rows which have elements different from 0.

Whereas the partition matrix is used for computing, the 1-dimensional keys discussed in Section 3.3 are still more practical to illustrate an allocation in the input and output. Therefore, as soon as the different scenarios are entering the programm through the key-matrix (in a 1-dimensional key), they get immediately translated into the partition matrix. Herby the programm can deal with both representations (binary key and alphanumerical key). The following code segment shows how this is been done:

```
PM(N,N1) = 0;
PM(N,N1) = 1$(key(part,N1) EQ ord(N));
*for 1-coalition notation (0,1)
count=2 ;
if(sum(N1,key(part,N1)) eq 0,
    count=1 ;
) ;
loop(N1,
    if(key(part,N1) eq 0,
   PM(N2,N1)$(ord(N2) EQ count)=1 ;
   count=count+1 ;
    ) ;
) ;
```

The second important part of the update to 18 regions concerns the data of the regions, which enter the programm through a new data file. The data mining has been done mainly by other people in the project. The aggregation of country data to the regions is performed in an separate Excel sheet. These so gained pure input data will not be discussed here.

For the population $L_{i,t}$, the productivity $A_{i,t}$, and the carbon intensity $\sigma_{i,t}$, time series are generated. These are based on the initial value of a variable X_0, the initial growth rate X_0^G and asymptotic value X_T.

Before the time series have been calculated with the growth rate X^G and the depreciation of the growth rate X^D as following:

$$X_t = L_0 e^{X_t^G}$$

$$X_t^G = X^{G_0}(1 - e^{-X^D \cdot (t-1)}) \text{ with } X^{G_0} = \log(X^{X_T/X_0})$$

$$X^D = -\log(1 - \frac{\log(1 + \frac{X_0'}{100})}{X_0^G})$$

The disadvantage of this method is, that the path of the variable X_t is required to be monotone. When this was not the case, the functional form or even the values have been changed manually. To avoid this problem in the new version, as functional form simply a polynomial of degree 3 is used.

The requirements on the polynomial are given by fixed values of X_0, X_0', and X_T (with T the last period) and $X_T' = 0$ (to model something like asymptotic behavior). Whereby the initial slope of the variable X_0' is simply given as $X_0' = X_0 X_0^G$, with the initial growth rate X_0^G.

With this conditions the coefficients of the polynomial follow to:

$$c_0 = X_0$$

$$c_1 = X_0'$$

$$c_2 = \frac{-X_0' - \frac{3}{T}(X_T - X_0 - X_0'T)}{-T}$$

$$c_3 = \frac{-X_0' - \frac{2}{T}(X_T - X_0 - X_0'T)}{T^2}$$

The path of the exogenous variable X_t can be finally calculated to:

$$X_t = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

This is done for the variables $A_{i,t}$ and $\sigma_{i,t}$. These two variables represent high uncertainties in the model since both affect through

$$E_{i,t} = \sigma_{i,t}(1 - \mu_{i,t}) A_{i,t} K_{i,t}^{\gamma} L_{i,t}^{1-\gamma}$$

directly the emissions $E_{i,t}$, and the asymptotic values for both are more less rough guesses. Whereas the starting values can get calculated directly, it is assumed that the these variables converge by the end of the considered period for all the regions to one value.

$A_{i,t}$ and $\sigma_{i,t}$ are chosen in that way, that the resulting emissions are conform to the results from other predictions (such as from IPCC).

One further remark has to be done on the fact, that it could happen that thought the functional form of the polynomial, the values fall below σ_T or exceed A_T. To prevent this, in that case the values get simply cut, which is economically reasonable.

$$\sigma_{i,t} = \max(\sigma_{i,t}, \sigma_T)$$

$$A_{i,t} = \min(A_{i,t}, A_T)$$

For the population $L_{i,t}$ there is more data available. The UN gives projections of population by country from 2000 till 2300 in steps of 50 years. In order to fit this values a polynomial interpolation is used to generate the time series $L_{i,t}$.

The following code segment shows how the Neville's algorithm is implemented in GAMS to calculate the values of $L_{i,t}$ with a polynomial with degree 6.

```
PARAMETERS
POP(N,PD)           regional population at time PD
POPO(N,PD)          original values
L(N,T)              population
POPT(PD)            times of given population;
* fitting polynomial with neville's algorithm
POPO(N,PD) = POP(N,PD);
loop(T,
    loop(PD1$(ord(PD1) LT card(PD)),
        loop(PD2$(ord(PD2) LE (card(PD)-ord(PD1))),
            POP(N,PD2) = (ord(T)-POPT(PD2))*POP(N,PD2+1) -
            (ord(T)- sum(PD$(ord(PD) eq ord(PD1)+ord(PD2)), POPT(PD)))*POP(N,PD2);
            POP(N,PD2) = POP(N,PD2)/
            (sum(PD$(ord(PD) eq ord(PD1)+ord(PD2)), POPT(PD))-POPT(PD2));
         );
    );
L(N,T) = POP(N,"L_2000");
POP(N,PD) = POPO(N,PD);
);
```

Figure 15 shows the so gained world population.

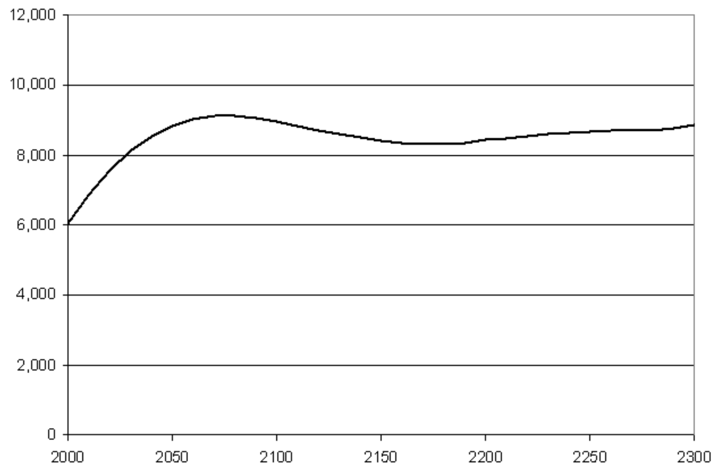


Figure 15: world population (million people)

All the used data can be found in Appendix A.

The free riding correction, discussed in Section 3.2.3 has been set from 100 (for six regions) to 25, in order to consider that the number of countries in ROW has been reduced.

The full list of the regions and their composition is given in Table 2.

label	name	composition
CAN	Canada	
USA	USA	
JPN	Japan	Japan, South Korea
EU	European Union	EU15
OEU	Other Europe	Iceland, Norway, Switzerland
CEA	Central Eastern Associates	Bulgaria, Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Romania, Slovakia, Slovenia
FSU	Former Soviet Union	Armenia, Azerbaijan, Belarus, Georgia, Kazakhstan, Kyrgyzstan, Moldova, Russian Federation, Tajikistan, Turkmenistan, Ukraine, Uzbekistan
AUZ	Australasia	Australia, New Zealand
MED	Mediterranean	Algeria, Egypt, Israel, Lebanon, Morocco, Syria, Tunisia, Turkey
MEA	Middle East	Bahrain, Iran, Jordan, Kuwait, Oman, Saudi Arabia, United Arab Emirates, Yemen
AFR	Africa	Angola, Benin, Botswana, Burkina-Faso, Burundi, Cameroon, Cape Verde, Central African Republic, Chad, Comoros, Congo, Republic of Congo, Djibouti, Equatorial Guinea, Eritrea, Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea Bissau, Ivory coast, Kenya, Lesotho, Madagascar, Malawi, Mali, Mauritania, Mauritius, Mozambique, Namibia, Niger, Nigeria, reunion, Rwanda, Senegal, Sierra-Leone, South Africa, Sudan, Swaziland, Tanzania,Togo, Uganda, Zambia, Zimbabwe
CHN	China	China, Hong Kong
IND	India	
RAS	Rest of Asia	Bangladesh, Cambodia, Laos, Mongolia, Nepal, Pakistan, Papua New Guinea, Sri-Lanka
EAS	Eastern Asia	Indonesia, Malaysia, Philippines, Singapore, Thailand, Vietnam
LAM	Latin America	Mexico, Brazil, Venezuela, Peru, Argentina, Chile, Uruguay, Paraguay
LAO	Latin America other	Bolivia, Colombia, Costa-Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Haiti, Honduras, Jamaica, Nicaragua, Panama, Trinidad & Tobago, Bahamas, Belize, Guyana, Suriname
ROW	Rest of the world	Afghanistan, Cuba, Libya, Iraq, ... $\sim$ 25 countries (not assignable or with incomplete data)

Table 2: the 18 regions

3.5 Use, output and interpretation

This section will provide some insights about how to use the model, which output is generated and how to interpret this output. It is not dealt with stability yet, since the CWS model just produces the output which is used later on as the input for the stability analysis.

Figure 16 illustrates what the CWS model is doing looking at it as a black box. With the given data, which represents actually the model specification it produces for user defined coalition structures output.

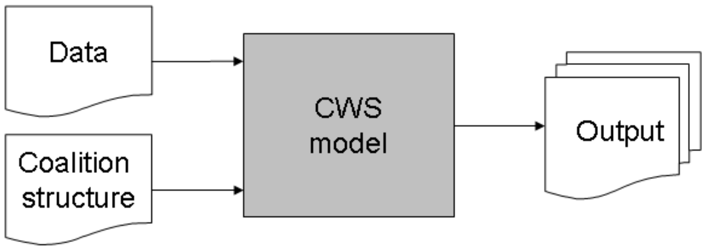


Figure 16: the CWS model as a black box

There are two types of output. The detailed output which is basically only used for the main scenarios (BAU, NASH, COOP) to get insight about the situation at a given allocation and the output summary to compare some key data for a whole set of allocations.

3.5.1 The main scenarios

For each of the main scenarios (NASH, COOP and also BAU) a separate output file for the detailed output is generated. This detailed output contains the following variables at the solution for each time period:

- Global Variables
  - carbon cycle and Climate: μ, E, M, F, T^L, T^E
  - world economy expenditures: Y, Z, I, C, D (absolute and % of Y)
  - world economy growth rates: dY, dX, dZ, dI, dC, dD
  - world emissions: E/Y, E, dE, E/L
  - world shadow prices: MC

    The marginal abatement cost MC is the price of the last abated ton CO_2. Since C_i is a function of the abatement rate μ_i and the Output Y_i some calculation is necessary to transform C to a function of abated CO_2. The first derivative leads then to the result.

    $$MC_{i,t} = \frac{1}{\sigma_{i,t}} c_i \log(1 - \mu_{i,t})$$

  - discounted utility for each region

  - discounted per capita utility for each region including the sum over all periods and the welfare weights

- regional variables (for each region i)
  - regional expenditure: Y, Z, I, C, D (absolute and % of Y)
  - regional growth rates: dY, dX, dZ, dI, dC, dD
  - regional production: A, dA, K, L, w

    Hereby the wage w is calculated as following (market equilibrium):

$$w = \frac{\partial Y}{\partial L} = \frac{(1-\gamma)Y}{L}$$

  - regional shadow prices: ψ, λ, MU, MC

    ψ is the marginal value of equation (4),
    λ is the marginal value of equation (5) and
    MU is the discounted marginal utility $\frac{1}{(\rho+1)^t}\frac{\partial U}{\partial(Z/L)}$

  - regional emissions: σ, E/Y, E, dE, E/L, μ
  - regional utility statistics: R, $R \cdot Z$, Z/L, $R \cdot Z/L$, U, $R \cdot U$

    including the sums over $R \cdot Z$, $R \cdot Z/L$ and $R \cdot U$

    where R is the discount factor

$$R_{i,t} = \frac{1}{(1+\rho_i)^t}$$

Now some basic results for the main scenarios should be given and interpreted. In the following it will be looked at a comparison of global variables of the three main scenarios BAU (no abatement), NASH (absence of cooperation - every region acts as singleton) and COOP (total cooperation - grand coalition). The importance of these main scenarios is given by the fact that in case of superadditivity (which is assumed to hold here) they represent the boundaries of what can be achieved with coalitions. No coalition can do better than the grand coalition in COOP and scenarios with coalitions will not be worse than NASH or even BAU.

It has to be said that the CWS model has not the aim to be a prediction model. The calibration (through $A_{i,t}$ and $\sigma_{i,t}$) has been done in that way, that the global emissions are consistent with other predictions (such as from IPCC). Still the following results give insights about behavior of regions and global effects.

- carbon and climate

  A first comparison is done in Figure 17 for the aggregated optimal CO_2 emissions. It can be seen that for all the scenarios a sharp increase followed by a decrease to a level double as high then nowadays is expected. This can be explained by the endogenous growth of output, which will be later compensated with a higher carbon efficiency. Optimal behavior of the regions just optimizing their own welfare in the NASH would lead to only a little improvement compared to the BAU. This could be also interpreted that the actual current emissions of the regions are close to their national optimum. A markable reduction of the emissions can be achieved with total cooperation COOP.

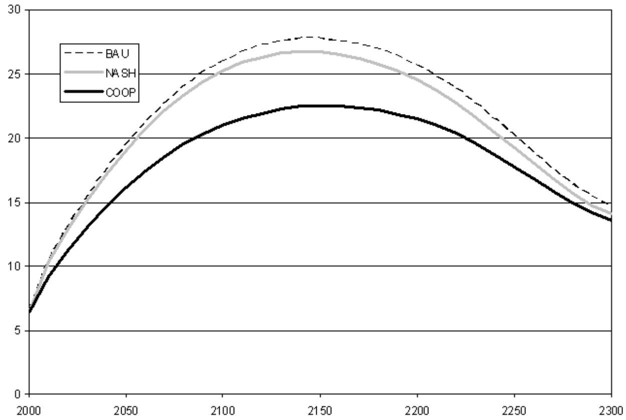


Figure 17: emissions E (billion tons carbon per year)

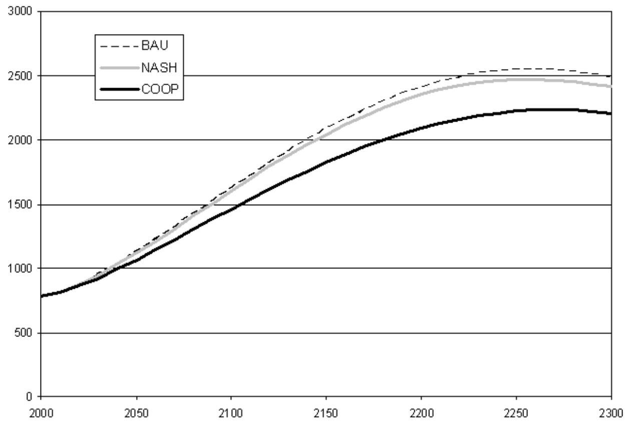


Figure 18: carbon stock M^{AT} (billion tons carbon)

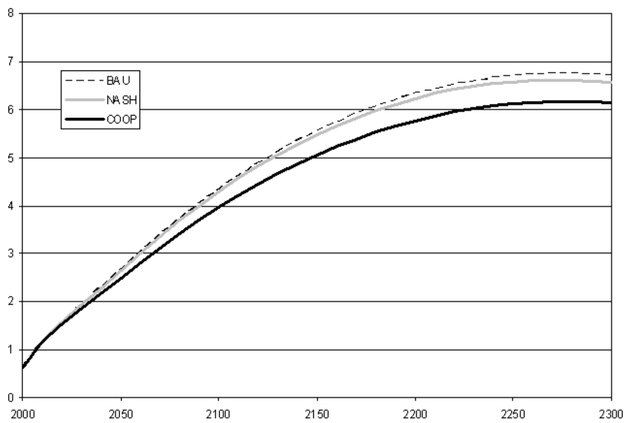


Figure 19: temperature change T^E (degrees Celsius compared to 1800)

The resulting carbon stock in the atmosphere is given in Figure 18. Here the time lag from the emissions to the carbon concentration can be seen. Nevertheless, in all three scenarios the carbon concentration will reach its maximum within the viewed period. With cooperation the carbon stock will grow to around 10% less than in the other scenarios.

The therefore predicted global temperature increase is displayed in Figure 19. From this it can be said that pure economical behavior will cause a certain climate change in form of a temperature increase of about 6 °C in any case. Still, optimal behavior under cooperation will internalize the externalities and reduce the impacts on the global climate. This reduce in global warming only through cooperation would be around 0.5 °C. One might say that this is a marginal improvement, but one has also to consider that this improvement can be achieved without cutting back on consumption.

- control variables

  Now a closer look should be taken on the optimal values of the control variables which are leading to this result in the climate. From Figure 20 can be seen that the total investment is hardly depending on cooperation. It ensures mainly an optimal growth path for the economies.

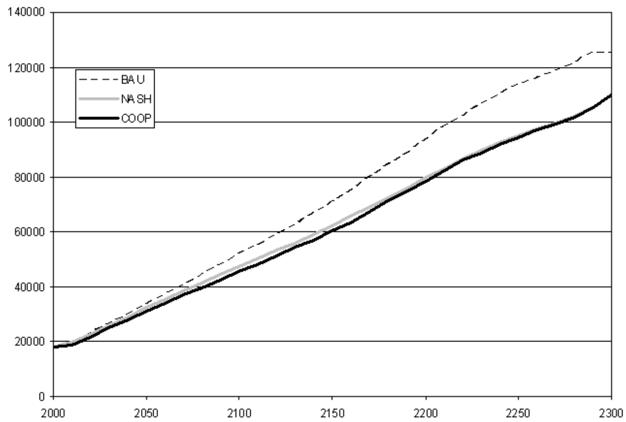


Figure 20: total investment I (billion 2000 US$)

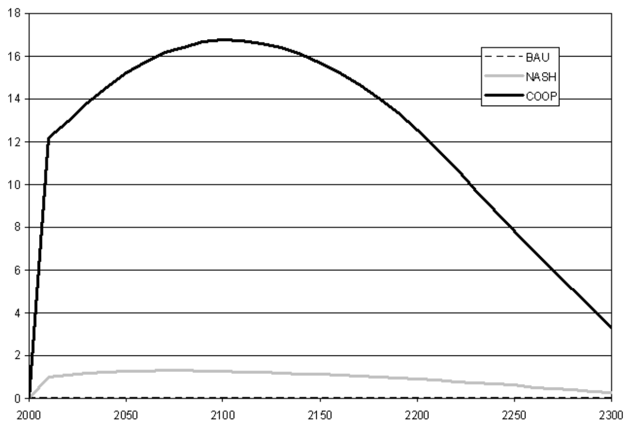


Figure 21: total abatement rate μ (%)

The global abatement rates are given in Figure 21. They reflect the shares of through the output generated emissions which have to be abated in the optimum. For the BAU no abatement takes place per definition. An abatement rate from a little more than 1% is optimal in the NASH scenario, which indicates again that the current situation is close to the national optimum. Contrary to that in the COOP scenario significant abatement should take place. There, the optimal abatement path reaches around 17% and decreases afterwards again which is mainly due to the exogenous carbon efficiency which makes additional abatement less important.

Here is interesting which countries have to do the efforts in abatement. The optimal paths by region are given in Figure 22 for NASH and in Figure 23 for COOP.

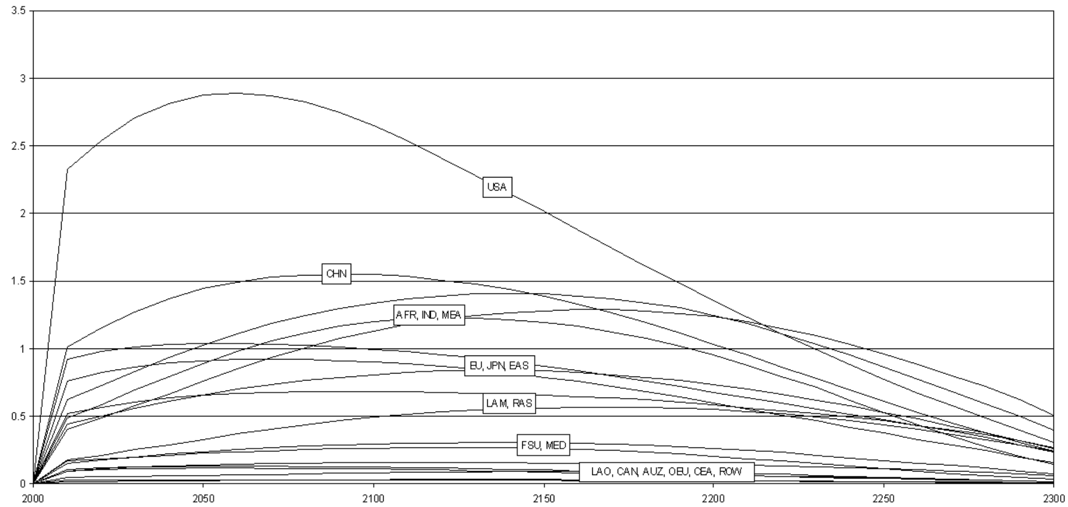


Figure 22: abatement rate μ per region in NASH scenario (%)

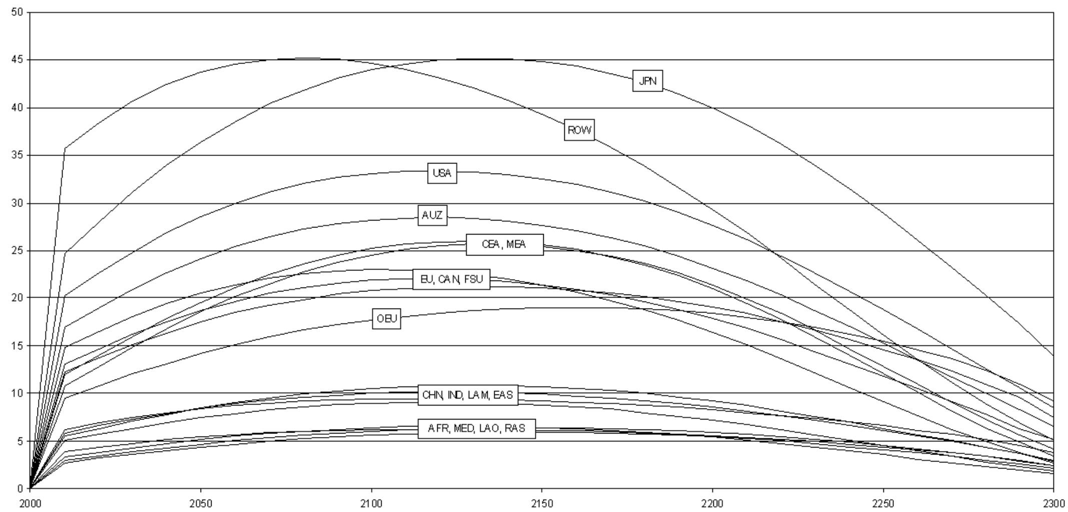


Figure 23: abatement rate μ per region in COOP scenario (%)

Acting just in self interest without cooperation the USA are supposed to do a higher abetment effort than the other regions. This can be also interpreted in that way, that the USA are currently not even acting in a national optimal way. For the other regions the abetment rate stays below 1.5%.

The shapes of the single curves in the COOP scenario look very similar, whereas higher developed countries are supposed to do higher efforts of up to 45%. One should not try to interpret the abatement rate of the region ROW, since this region lacks of data and has to be seen more as a dump.

- Output Y

  Further will be looked at the composition of the output $Y = Z + I + D + C$ for BAU at Figure 24, the NASH at Figure 25 and the COOP at Figure 26.

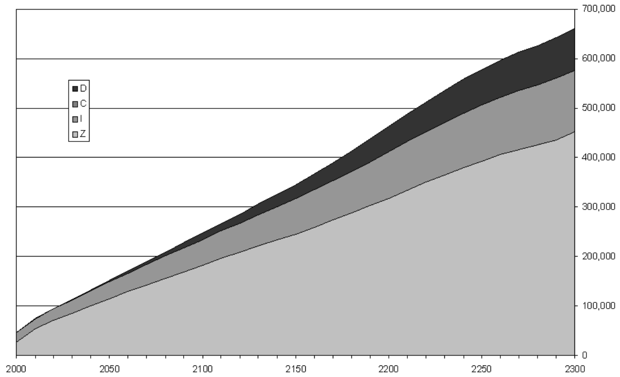


Figure 24: output composition BAU (billion 2000 US$)

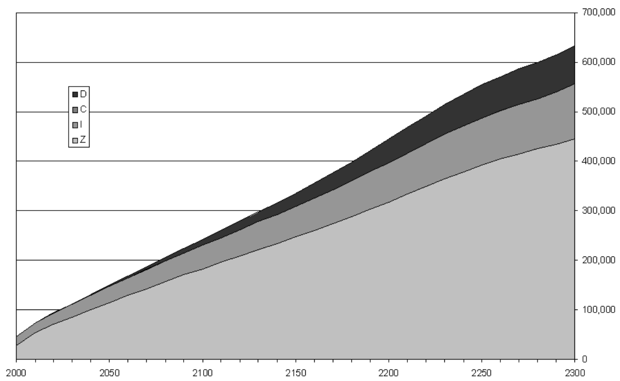


Figure 25: output composition NASH (billion 2000 US$)

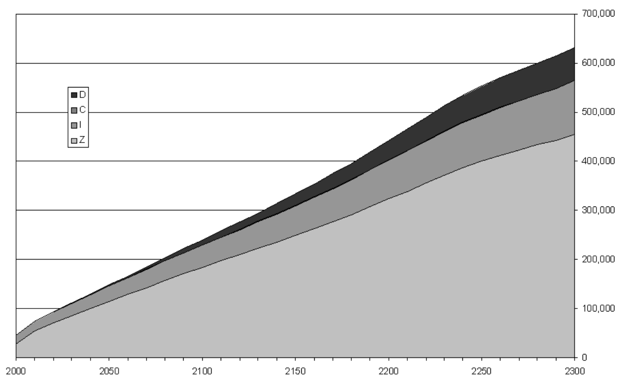


Figure 26: output composition COOP (billion 2000 US$)

In these pictures there is not really a shift in the output composition visible. The fraction of the abetment cost C is in all scenarios marginal. More information is provided by Figure 27 and Figure 28. Here the output compositions in the different scenarios are compared in 2100 and 2300.

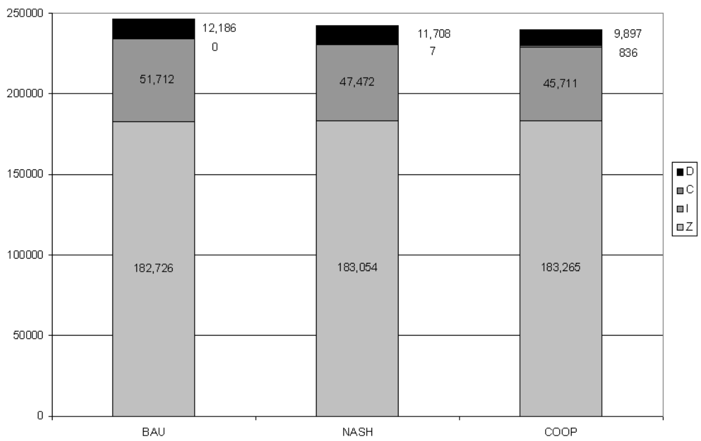


Figure 27: output composition in 2100 (billion 2000 US$)

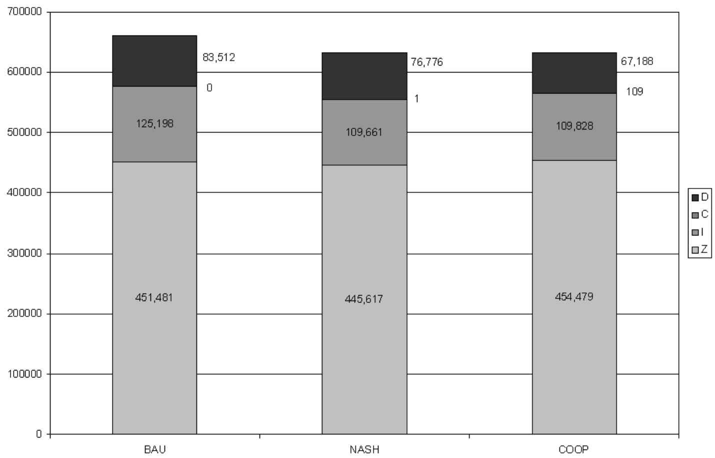


Figure 28: output composition in 2300 (billion 2000 US$)

It can be clearly seen that optimal behavior and cooperation reduce as expected the damage and increase the consumption. The gain in global consumption through cooperation is less then 1%, nevertheless it is a gain additional to a better environmental quality with just marginal abatement costs.

- Welfare W

  Finally the total welfare levels of the regions are given in Figure 29. The absolute values are useful to see which the bigger regions in terms of welfare are. But more interesting is the relative improvement compared to the BAU scenario, given in Figure 30.

  Although the aggregated welfare in COOP is clearly higher than in NASH, some regions are worse off in COOP compared to NASH. In general developing regions profit more from cooperation, whereas high developed countries even loose. Therefore the grand coalition is first hand instable and will not form. As explained in Section

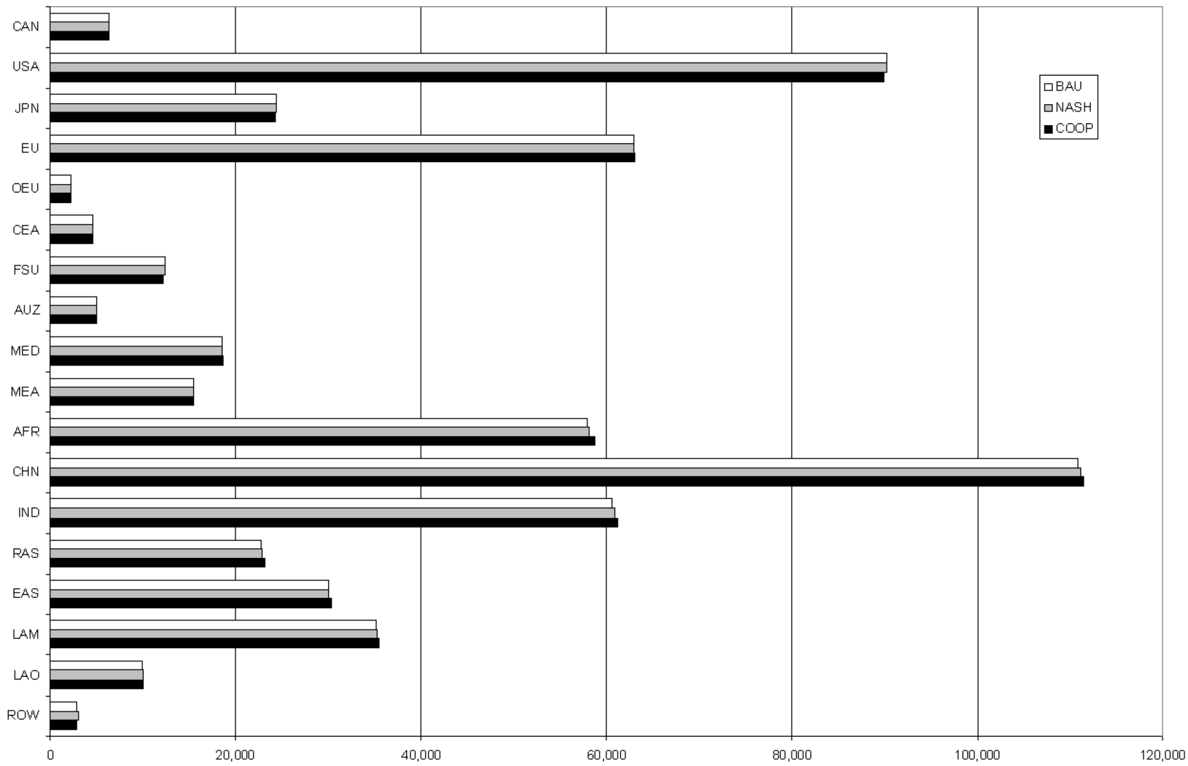


Figure 29: welfare levels

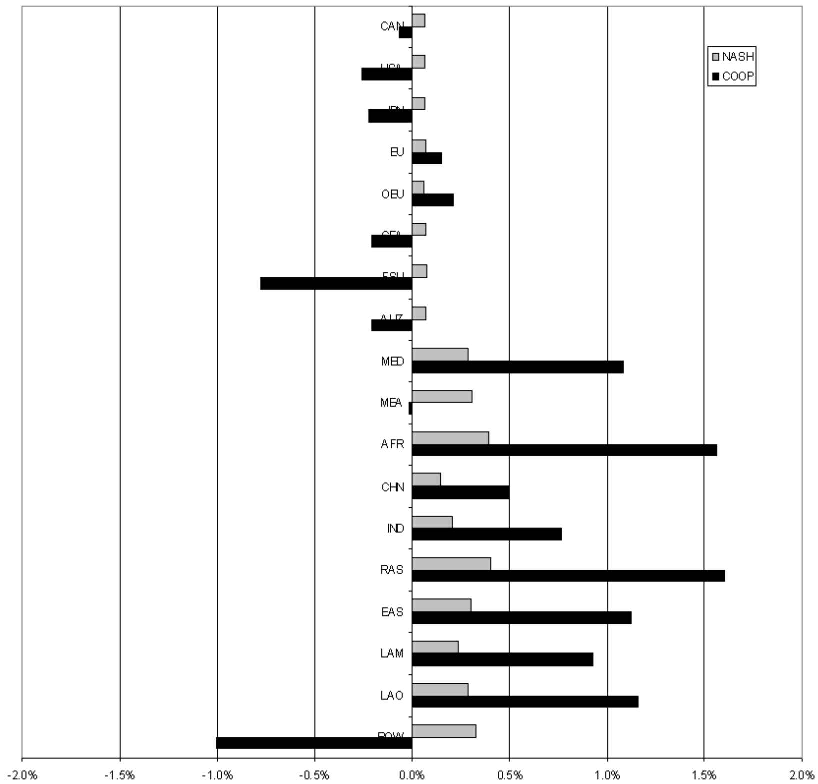


Figure 30: relative improvement in welfare

2.2.3, transfers (in specific Chander-Tulkens transfers) can be used to improve the stability of the grand coalition.

That the grand coalition can be stabilized at least in the sense of individual rationality (IR) can be seen from the fact that the aggregated welfare level in the COOP scenario is around 0.3% higher than in the NASH scenario. This represents the economical gain from worldwide cooperation in the climate issue.

At this point it has also to be said that these absolute statements are highly sensitive to the uncertain parameters in the model (such as the carbon efficiency, the productivity, the damage function, the abatement cost function,...). Nevertheless, a good idea of the difference between total cooperation and non-cooperation is given.

So far has been only talked about the main scenarios. Also a first remark on stability has been done. In the following, especially in Chapter 4 the cases in between the main scenarios with certain coalitions are the object of study. The therefor necessary output is given in the allocations summary.

3.5.2 Allocations summary

When it is about comparing a set of allocations, which is needed for the stability analysis in Chapter 4, there is far less output important. This necessary output is given in the form of a table and contains for every allocation the following values:

- the welfare W_i for every region i
- the aggregated welfare W
- number of iterations and solver status to see if the solver reached a solution
- the sum of the total emissions E over the whole time
- the carbon concentration M^{AT} in the last period T
- the temperature difference T^E in the last period T

In the following an example of such an allocation summary is given. The whole table is defined as a new matrix "dataS" for the possibility of using the data directly in the stability analysis, described in Chapter 4. (The reason why the key is split into 3 parts is simply that 18 digits are too much for a matrix entry.)

```
table dataS(part,*)
   key1    key2    key3    CAN               ROW       WORLD        it  st  sumE      M2300     T2300
1  000000  000000  000000  9936.973   .....  2441.133  345937.336   5   2   1985.618  5824.92   9.95
2  011111  111111  111111  10221.356  .....  2454.838  351425.924   3   2   568.835   1318.285  3.998
3  101111  111111  111111  10136.614  .....  2456.747  350785.042   5   2   697.159   1580.919  4.881
4  110111  111111  111111  10197.372  .....  2455.537  351413.023   2   2   581.074   1340.749  4.085
5  111011  111111  111111  10173.471  .....  2456.462  351241.396   3   2   622.498   1425.407  4.385
6  111101  111111  111111  10209.513  .....  2454.681  351448.208   2   2   562.402   1305.908  3.95
7  111110  111111  111111  10205.921  .....  2454.093  351365.941   4   2   570.472   1328.801  4.023
8  111111  011111  111111  10202.808  .....  2453.561  351317.077   3   2   573.168   1332.181  4.04
9  111111  101111  111111  10207.656  .....  2454.462  351412.289   3   2   568.18    1324.023  4.005
.. ......  ......  ......  .........  .....  ........  ..........   .   .   .......   ........  .....
```

This allocations summary is done for different transfer schemes, which generates different output files. The application of these transfer schemes is done after solving the model and right before generating the output. The transfer schemes apply only within the members of a coalition S. (Therefore it works also for multiple coalitions.) Besides the welfare levels without transfers, the welfare levels with two different transfer schemes (compare Section 2.2.3) are implemented here.

- Chander Tulkens transfers

  First the shares δ_i are calculated so that they reflect the share of the regions' marginal climate change damages within the coalition S.

$$\delta_i = \frac{\sum\limits_{t=0}^{T} \frac{\partial D_i}{\partial T^E}|_t}{\sum\limits_{j \in S} \sum\limits_{t=0}^{T} \frac{\partial D_j}{\partial T^E}|_t}$$

  With the consumption levels $\bar{Z}$ from the NASH scenario, the welfare surplus Δ_W in the coalition S through the allocation can be calculated.

$$\Delta_W = \sum_{i \in S} \sum_{t=0}^{T} \frac{1}{(1+\rho)^t}(Z_{i,t} - \bar{Z}_{i,t})$$

  With this the welfare after transfers W^{CT} can be calculated.

$$W_i^{CT} = W_i - \sum_{t=0}^{T} \frac{1}{(1+\rho)^t}(Z_{i,t} - \bar{Z}_{i,t}) + \delta_i \Delta_W$$

- emission trading (grandfathering with respect to BAU)

  It is assumed that regions which abate more than the weighted average can sell this surplus of abated CO_2 to regions which abate less at a certain price P_t^E. Therefore the received transfers $T_{i,t}$ follow to

$$T_{i,t} = P_t^E(\hat{E}_{i,t} - E_{i,t})$$

  where $E_{i,t} = Y_{i,t}\sigma_{i,t}(1 - \mu_{i,t})$ and $\hat{E}_{i,t}$ denotes the assigned emissions as a share of the total emissions within the coalition.

$$\hat{E}_{i,t} = \delta_{i,t} \sum_{i \in S} E_i$$

  In detail these shares δ_i are calculated so that they reflect the regions' BAU-emissions in S. The more a region used to emit, the more emission allowances are assigned to it (grandfathering).

$$\delta_{i,t} = \frac{\sigma_{i,t} Y_{i,t}}{\sum_{j \in S} \sigma_{j,t} Y_{j,t}}$$

  Further the price P_t^E of a ton CO_2 is defined as the weighted marginal abatement costs $MC_{i,t}$

$$P_t^E = \delta_{i,t} MC_{i,t}$$

  And finally payoffs after transfers $Z_{i,t}^{ET}$ can be calculated and summed up to get the welfare level after transfers.

$$Z_{i,t}^{ET} = Z_{i,t} + P_t^E \sigma_{i,t} Y_{i,t}(\mu_{i,t} - \sum_{j \in S} \mu_{j,t} \delta_{j,t})$$

The almost ideal sharing scheme (AISS) (compare Section 2.2.3) is not implemented here since for this the free rider payoffs of the regions would be needed. Further remarks on this are given in Section 4.1.

With this allocation summary the necessary output data is prepared for further use. It can be produced for any certain number of allocations. But how many different allocations are there?

3.6 A remark on combinatorics related to coalitions

As said in the previous sections, the program can be feeded with a set of scenarios (allocations) for which the results are summarized in an output file which is used later on for the stability analysis. A problem which occurs here is that the number of possible scenarios rises enormously with the number of regions. In this section the numbers of these possible allocations for a certain amount of players should be derived.

First there can be considered allocations with one coalition S and all other players acting as singletons. The number of possible different coalitions for n players, and therefore different allocations is

$$\sum_{k=1}^{n} \binom{n}{k}$$

which is simply derived with the formula for combinations without repetitions as the sum of all coalitions with 1 member, all coalitions with 2 members, up to all coalitions with n members. The same result can be also reached with $2^n - 1$, since the possible n-player allocations (with only one coalition S) can be represented as the binary numbers with n digits. (Excluding the allocation without any coalition leads to the result.)

But to be complete, there must be considered that several coalitions can exist at the same time next to each other. To cover all the possibilities of these allocations, all partitions of the set of players have to be taken into account. A partition of a set N is a set of nonempty subsets of N such that every element i in N is in exactly one of these subsets. The number of ways a set of n elements can be partitioned into nonempty subsets is called the Bell number and is denoted B_n[41]. The calculation of the Bell numbers is not that easy.

Dobinski's formula gives the nth Bell number

$$B_n = \frac{1}{e} \sum_{k=0}^{\infty} \frac{k^n}{k!}.$$

The problem here is the infinite sum. Therefore the recursive relation

$$B_{n+1} = \sum_{k=0}^{n} B_k \binom{n}{k}$$

might be more useful. An easy numerical algorithm to calculate the Bell Numbers is the so called "Bell Triangle".

[41] compare http://mathworld.wolfram.com/BellNumber.html

players	coalitions	allocations
1	1	1
2	3	2
3	7	5
4	15	15
5	31	52
6	**63**	**203**
7	127	877
8	255	4.140
9	511	21.147
10	1.023	115.975
11	2.047	678.570
12	4.095	4.213.597
13	8.191	27.644.437
14	16.383	190.899.322
15	32.767	1.382.958.545
16	65.535	10.480.142.147
17	131.071	82.864.869.804
18	**262.143**	**682.076.806.159**

Table 3: possible coalition and allocations

In Table 3 the number of all possible coalition (including singletons) and the number of all possible allocations, which are the set partitions, are given for a fixed number of up to 18 players.

The original version of the CWS model deals with 6 regions (therefore 6 players), where it is easy to calculate, analyze and also visualize the results of all the coalitions and even allocations. The extended version of the CWS model has now 18 regions, which cause mainly two problems:

- The computing time which is needed to solve all the possible scenarios is getting much too high for the current used methods. Section 3.7 is dealing with the evaluation of the computing time methods to downsize this time.
- With 6 regions there was a simple list of the allocations and their stability possible, which could be analyzed by simply looking at it. With the now gained enormous amount of possible allocation this is not possible anymore. There have to be found ways to focus on a certain set of scenarios and also ways to perform a reasonable analysis. With this issue deals Chapter 4.

3.7 Computing time

It is a reasonable and tested assumption that the computing time is mainly determined by the number of solver calls during the run of the programm. But what determines the number of the solver calls? For each allocation the solver is so long called sequential for every player till the stop criterion is reached. One loop over all the players is called an iteration. Besides to the numbers of players (which is given) the computing time is proportional to the number of iterations per allocation, which should be the focus for improvement in the computing time.

To extract the time to a sperate output file at the begin of the programm the new parameter "timestart" is initialized with the current system time in seconds.

```
file timetest /output\timetest.dmp/ ;
put timetest ;
parameter timestart ;
timestart=jnow*86400
put (jnow*86400-timestart) /;
```

After each allocation the key of the allocation, the number of needed iterations, and the time since starting the programm gets written to the output file.

```
put timetest ;
loop(N1, put key(part,N1):1:0) ;
put icount:4:0 ;
put (jnow*86400-timestart):>10 /;
```

The result of a first test with randomly chosen allocations is that on the average around 6 iterations which an average time of around 17 seconds are needed for one allocation. Therefore the calculation time for one allocation is around 100 seconds. Considering the enormous number of possible allocations derived in Section 3.6, a day of 86.400 seconds, which allows the calculation of less than 1000 allocations seems to be rather short. Even considering "only" the 262.143 allocations with just one coalition around one year is needed to calculate them all.

Although it will hardly be possible to calculate all the possibilities reducing the computing time is still an important objective.

The following approaches should be analyzed:

- creating a better stop criterion
- putting the allocations in an optimal order
- calling the solver for each player in an optimal order
- reducing the needed memory with not writing output.

In the following the results of some experiments are provided. It has been looked at 30 randomly chosen allocations with one coalition. For each test run are the average number of iterations for each allocation, and the average time needed for one iteration given. As a benchmark the current values are:

	iter	sec/iter	sec/allocation
current implementation	6.2	16.2	100.4

It has to be said that these values vary at each run of the programm to the extend of around up to 10%. Nevertheless this testing gives a rough idea in which scale improvements in the computing time can be done.

The stop criterion

The stop criterion is used for stopping the iterations when the solution is close enough to the optimum.

The current stop criterion is the following:

$$\sqrt{\sum_i \sum_t (\mu_{i,t}^k - \mu_{i,t}^{k-1})^2} \leq tol$$

where $\mu_{i,t}^k$ denotes the abatement rate of region i in period t after the kth iteration and the divergence tolerance tol is set to 10^{-3}.

The main point of criticism herby is that it is looked at the change of the decision variables $\mu_{i,t}$, whereas the objective values which are used for the analysis later on are the welfare levels W_i. Therefore it would be reasonable to implement a stop criterion of the following form:

$$\sqrt{\sum_i (W_i^k - W_i^{k-1})^2} \leq tol$$

In the following table the results of the experiments with different stop criterions are provided. In the first row are the benchmark values for the original stop criterion focusing on μ with $tol = 10^{-3}$ followed by the stop criterions focusing on W_i with different tolerances from 10^{-2} to 10^1.

	iter	sec/iter	sec/allocation
current implementation	6.2	16.2	100.4
$tol = 10^{-2}$	6.3	15.4	97.3
$tol = 10^{-1}$	4.6	18.2	83.0
$tol = 10^{0}$	3.2	21.7	68.7
$tol = 10^{1}$	2.6	24.9	63.7

Interesting is that while the number of iterations is decreasing, the average needed time for an iteration is increasing. This might be due to the fact that the later iterations which have already better starting values for the model go faster. Nevertheless, important is that the needed time per allocation is going down.

Although the improvement in the number of iterations is only achieved by releasing strongly the tolerance tol, the advantage of a stop criterion considering W_i is that the tolerance affects the outcome directly. Therefore it can be set by making reasonable requirements to the welfare levels, which was not possible before, since the relationship of welfare level and abetment rate is not easy to describe.

The welfare levels are at a dimension of 10^4 to 10^5, a tolerance of 10^0 is therefore far enough and hence a reasonable choice. It is expected to lower the calculation time by around 1/3 of the original value. To make this criterion also resistant to future scaling in the welfare levels, the tolerance is formulated in relative terms as 0.1% of the smallest welfare from the BAU scenario.

$$\sqrt{\sum_i (W_i^k - W_i^{k-1})^2} \leq 10^{-4} \min_{i \in N}(W_i^{BAU})$$

One point which has to be taken care of is, that even if the changes in the welfare level are within the tolerance, there could still be significant deviations in other variables. This does not matter for the stability analysis, where just the welfare level is used, but it is important for the economical analysis of the main scenarios (BAU, NASH, COOP) where the actual path of the variables plays a role.

For making sure that this is not the case, a simple test is performed. For the original and for the new stop criterion the main scenarios are calculated. For this two stop criterions the paths of one variable, which is to be expected to have a high deviation, are compared. The used variable is the abatement rate μ (this is also the reason why this variable has been used for the original stop criterion). The relative deviations have been compared for the total abatement and exemplary for one region in the NASH and the COOP scenario. The biggest deviation has been observed in the total abatement for the COOP scenario. This highest deviation is around 0.2% compared to the original value, which can be neglected. Therefore the new proposed stop criterion is used from now on.

Putting allocations in an optimal order

For testing if the order of the allocation can have an influence on the computing time another set of allocations has been taken. Starting from the grand coalition with 18 regions, one by one the players every player leaves the coalition till all regions play as singletons. After this all these allocations are calculated again the other way round. The average values of these tests are given in the following table:

	iter	sec/iter	sec/allocation
random allocations	6.2	16.2	100.4
shrinking coalitions	4.1	10.0	40.9
growing coalitions	3.1	11.0	33.6

The result is that compared to a randomized order of allocations, a order where similar allocations are following each other can decrease the computing time enormously. What weakens this result is that in general the allocations are not chosen randomly anyways.

Optimal order of solver calls

For this we have to recall the partition matrix described in Section 3.4.

The algorithm goes row by row through the matrix and optimizes the welfare for the current player which is represented by the row. The order of the the players is therefore exogenous. An improvement could be done by sorting the players after their influence on the system. This can be done by simply switching the rows in the partition matrix, so that the importance decreases by the row number. For a first test the importance of a coalition will be approximated with its initial production $\sum_{j \in S_i} Y_{0,j}$.

This is done with the following code element for each partition matrix.

```
sortvector(N) =sum(N1,PM(N,N1)*Y0(N1));
count=1;
while(sum(N,sortvector(N)) ne 0,
    loop(N,
        if( sortvector(N) eq smax(N1,sortvector(N1)), maxpos=ord(N) );
     ) ;
    loop(N,loop(N1,
        if(ord(N)=count and ord(N1)=maxpos,
            PM2(N,N2)= PM(N1,N2);
            sortvector(N1)=0;
    );
     ) ; ) ;
    count=count+1;
) ;
```

	iter	sec/iter	sec/allocation
current implementation	6.2	16.2	100.4
optimal order	6.3	21.8	137.2

The time which is needed is significantly higher. An explanation for this unexpected result could be that the chosen approximation for the importance of a coalition was not ideal. Since there is no indiction for choosing another reasonable sorting criterion, this approach for improving the computing time is rejected.

Not writing output

One way to improve the computation time could be to reduce the written output in order to gain more free memory for the computation. For testing this all the output commands have been set inactive.

	iter	sec/iter	sec/allocation
current implementation	6.2	16.2	100.4
without output	4.3	18.5	80.3

There is an expected decrease in needed time per allocation. (Still, the fact that this decrease is mainly achieved through less iterations per allocation is surprising.) Nevertheless, the improve is too less to skip the output

Summarizing, with an optimal stop criterion and a reasonable order of the allocations the needed computing time for one allocation is expected to be 30-60 seconds.

4 Stability analysis

So far the necessary background in game theory has been given and the CWS model has been introduced also as a tool for generating output for further analysis. In this chapter should be explained how the output from the CWS model is used to perform stability analysis and some thoughts for further investigations should be provided.

First it will be explained how the stability gets checked and afterwards some ideas will be collected how to gain insights about the underlying factors of stability of coalitions. The need for new approaches is given by the dramatically increased number of possible allocations which cannot be anymore analyzed by just looking at them. These methods should extract insights in the following steps of the CLIMNEG project.

4.1 Checking stability

The output of the GAMS implementation of the CWS model should be used now to check the stability of certain coalitions. What is needed therefore are just the payoff vectors $W = (W_1, \dots W_n)$ of the allocations. These payoffs are the regions' welfare levels at the end of the period. The payoff vectors are in the allocations summary (explained in Section 3.5.2) already calculated for specific transfer schemes. Therefore, one should talk of stability under a specific transfer scheme.

For calculating whether coalitions are stable or not another GAMS routine is used. It takes the payoff vectors in the output files as its input and produces as output a vector over all the analyzed coalitions with a 1 if the coalition is stable and a 0 if the coalition is unstable. This is done for different concepts of stability, explained in Section 2.2.2.

One remark has to be done on the fact that because of calculating the welfare of the coalitions with the PANE algorithm, the joint payoffs are already the best feasible outcome under the $\gamma-$assumption. Therefore

$$\sum_{i \in S} W_i = v^{\gamma}(S)$$

is ensured.

Although the CWS model is ready to produce the output for allocations with multiple coalitions, in the stability analysis will be dealt so far only with allocations with only one coalition.

What exactly is done to check the different stability concepts should be explained and illustrated with an example now. Basically it is just comparing the payoffs of the regions in a specific allocation, e.g. (011001), with the gained payoff in certain other allocations.

- core stability

  Since in the case of the CWS model the payoffs can be assumed to be strictly super additive (has been checked once manually and is reasonable for a realistic emission game), core stability can be only reached with the payoff vector of the grand coalition W^N and not with the payoff vector of any other partition W^P, for which the condition of group rationality (GR) in the definition of the core would be violated.

$$v^{\gamma}(N) = \sum_{i \in N} W_i^N > \sum_{i \in N} W_i^P \text{ for any partition } P \neq N$$

To check whether the grand coalition is stable in the core sense, the welfare levels of all the coalitions have to be compared to the aggregated welfare levels of the players of these coalitions in the grand coalition. Hence the actual condition of γ-core stability is the following:

$\sum_{i \in S} W_i^N \geq \sum_{i \in S} W_i^S$ for all $S \subseteq N$

- individual rationality IR

  An allocation is individual rational when each region achieves a higher welfare level than in the case without cooperation.

  e.g. $W_i^{(011001)} \geq W_i^{(000000)}$ for all $i \in N$

  Since strict additivity is given in the optimal welfare levels in the CWS model, with certain transfers (Chander Tulkens transfers or almost ideal sharing scheme, explained in 2.2.3) IR can always get achieved.

- internal stability IS

  A coalition is internally stable when every coalition member has a higher welfare level inside the coalition than outside.

  e.g. $W_2^{(011001)} \geq W_2^{(001001)}$
  $W_3^{(011001)} \geq W_3^{(010001)}$
  $W_6^{(011001)} \geq W_6^{(011000)}$

- external stability ES

  A coalition is externally stable when every non-member has a higher welfare level outside the coalition than inside.

  e.g. $W_1^{(011001)} \geq W_1^{(111001)}$
  $W_4^{(011001)} \geq W_4^{(011101)}$
  $W_5^{(011001)} \geq W_5^{(011011)}$

- potential internal stability PIS

  A coalition is potentially internally stable when the joint payoffs of the coalition members exceed their joint free rider payoffs.

  e.g. $\sum_{i \in \{2,3,6\}} W_i^{(011001)} \geq W_2^{(001001)} + W_3^{(010001)} + W_6^{(011000)}$

- exclusive membership external stability with unanimity voting (EMES-UV)

  A coalition violates exclusive membership external stability with unanimity voting, when all insiders of a coalition gain higher payoffs when letting the player who violates the external stability in. (In the example player one is violating the external stability and wants to join the coalition.)

e.g. $W_i^{(111001)} \geq W_i^{(011001)}$ for all $i \in \{2, 3, 6\}$

- exclusive membership external stability with majority voting (EMES-MV)

  A coalition violates exclusive membership external stability with majority voting, when the majority of the insiders of a coalition gain higher payoffs when letting the player who violates the external stability in. (In the example player one is violating the external stability and wants to join the coalition.)

  e.g. $W_i^{(111001)} \geq W_i^{(011001)}$ for at least two $i \in \{2, 3, 6\}$

It is importat to note, that if the coalitions with s members should be checked whether if they are stable or not (in the sense of internal and external stability), also the payoff vectors of the coalitions with size $s-1$ and $s+1$ have to be given. For the specific case of 18 regions the numbers of different coalitions with s regions (which are simply the binomial coefficients 18 over s) and the number of allocations needed to calculate their stability are given in Table 4.

size	# of coalitions	# needed allocations
0	1	0
2	153	970
3	816	4.029
4	3.060	12.444
5	8.568	30.192
6	18.564	58.956
7	31.824	94.146
8	43.758	124.202
9	48.620	136.136
10	43.758	124.202
11	31.824	94.146
12	18.564	58.956
13	8.568	30.192
14	3.060	12.444
15	816	4.029
16	153	987
17	18	172
18	1	19

Table 4: needed allocations for determining stability

The stability of either very small or very large coalitions can be checked therefore with much less computational efforts than other coalitions, which is quite an important argument recalling the computation time of 30-60 seconds for one allocation.

The GAMS routine used for checking the stability, written originally by Johan Eyckmans in 2000 and adapted during this work, is given in Appendix C.2. Besides the implementation for the check of the core stability, one important change which has been done during this work, was to implement the possibility that not all the needed data is available to check the stability for a certain allocation. Now the program detects these cases and calculates the stability only where this is possible with the given input data.

As an output, the routine provides a detailed analysis of the stability for each allocation with the exact values of the therefore done comparisons. *D* denotes hereby the allocations through deviating of a certain player. An example for such an detailed output is given in the following:

```
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
partition number =  25
binary key       = 010000001
structure = {USA,ROW}
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++  ...  ++++++++++++++++++++++++++++++++++++++++++++
                           CAN         USA         JPN     ...       FSU         AUZ          ROW
S= 25    010000001   10213.310  134141.104   40310.821     ...   3308.870    2427.971   58837.305
D= 53    110000001   10209.518-                            ...
D= 10    000000001              132529.311-                ...
D= 80    011000001              134270.660+  40336.411+    ...                          58705.851-      1/2
D= 85    010100001              134444.837+                ...                          58486.371-      1/2
D= 89    010010001                                         ...
D= 92    010001001                                         ...
D= 94    010000101                                         ...   3273.679-
D= 95    010000011                                         ...               2418.050-
D=  3    010000000                                         ...                          59067.045+
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++  ...  +++++++++++++++++++++++++++++++++++++++++++
IR =0                                    1                 ...                              0
IS =0                                    1                 ...                              0
ES =0                        1                       0     ...      1           1
ESUNA =1                     1                       1     ...      1           1
ESMAJ =1                     1                       1     ...      1           1
IES=0
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
PIS=1     uFR=  191596.356     uS=  192978.409      diff=    1382.053
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
```

A further output is a summary table of all the analyzed allocations. These summary tables table will be used later on to do some further analysis. In the following an example for such a summary is given:

nr	key	IR	PIS	IS	ES	I&ES	ESUNA	I&ESUNA	ESMAJ	I&ESMAJ
1	000000000	1	1	1	0	0	0	0	1	1
2	100000000	1	1	1	0	0	0	0	0	0
3	010000000	0	0	0	0	0	0	0	0	0
4	001000000	1	1	1	0	0	0	0	0	0
5	000100000	1	1	1	0	0	0	0	0	0
6	000010000	1	1	1	0	0	0	0	0	0
7	000001000	1	1	1	0	0	0	0	0	0
8	000000100	1	1	1	0	0	0	0	0	0
9	000000010	1	1	1	0	0	0	0	0	0
10	000000001	1	1	1	0	0	0	0	0	0
11	110000000	0	1	0	0	0	0	0	0	0
12	101000000	0	1	0	0	0	0	0	0	0
13	100100000	0	1	0	0	0	1	0	1	0
14	100010000	0	1	0	0	0	0	0	0	0
15	100001000	0	1	0	0	0	1	0	1	0
16	100000100	0	1	0	0	0	0	0	0	0
17	100000010	0	1	0	0	0	0	0	0	0
18	100000001	1	1	1	0	0	1	1	1	1
19	011000000	1	1	1	0	0	0	0	0	0
20	010100000	1	1	1	1	1	1	1	1	1
..	....	.	.	.	.	.	.	.	.	.

Besides these main outputs, some statistics about the number of checked and stable coalitions, and the result of the check on core stability is given (in case all the necessary allocations to prove this have been available).

4.2 Getting Insights

So far a table can be achieved which indicates if a certain allocation is stable in the sense of an arbitrary stability concept. But considering the number of possible allocations a statement about the stability of a certain allocation is not really practical.

One objective of further research is, to figure out what makes a coalition stable or unstable. Therefore some ideas to extract general statements from this potentially huge data set should be given. It is a quest for underlying economical factors with the goal of general statements concerning this issue. A short outlook for approaches for further investigation is presented in this section.

It has to be mentioned that the stability of a certain coalition does not say much about the contribution of this coalition to the climate issue. It can be just said that its efficiency will be somewhere in between the NASH and the COOP scenario. The CLIMNEG working paper "Efficiency vs. stability of climate coalitions" ([4]) deals with this topic whereas here it will be neglected. The focus of this chapter will be exclusively on stability.

A further question which has to be answered first is, which stability concept should be used for an upcoming analysis? As it is mentioned before, core stability can be only archived in the grand coalition and is therefore not a proper concept to study stability of general coalitions. Looking at IS and ES, for the case of climate coalitions, ES has quite low importance. This is simply due to the fact that a climate coalition still will merge even if there are outsiders which would like to join. They will not prevent the coalition from forming since they would be better of themselves. The same is valid for exclusive membership external stability. So far it seems to be reasonable to look just at IS. The in Section 2.2.3 presented almost ideal sharing scheme (AISS) has the property to make PIS coalitions internally stable and to ensure also individual rationality to all the players. Therefore it is most reasonable to use PIS as the stability concept for further analysis.

4.2.1 Omitting one player

One experimental approach, where the results are still easily possible to depict, is to start from the situation with total cooperation and to observe what happens by omitting just one region. Therefore it will be looked at the allocation with total cooperation and at all coalitions with 17 members. Table 5 shows the output table of the performed stability check for these allocations. The therefore used transfer scheme are the Chander-Tulkens transfers. By looking at the table it can be seen, that non of the analyzed allocations is potentially internally stable. But still there are some interesting statements possible. Basically there are two types of coalitions:

- externally stable coalitions

  The regions which are here the outsiders do not want to join the coalition.

- not externally stable coalition, which are neither exclusive membership externally stable

  Here, the outsiders would like to join the coalition and would be even accepted by the coalition members.

key	IR	PIS	IS	ES	I&ES	UV	I&UV	MV	I&MV
111111111111111111	1	0	0	1	0	1	0	1	0
011111111111111111	1	0	0	1	0	1	0	1	0
101111111111111111	1	0	0	1	0	1	0	1	0
110111111111111111	1	0	0	1	0	1	0	1	0
111011111111111111	1	0	0	1	0	1	0	1	0
111101111111111111	1	0	0	0	0	0	0	0	0
111110111111111111	1	0	0	0	0	0	0	0	0
111111011111111111	1	0	0	1	0	1	0	1	0
111111101111111111	1	0	0	0	0	0	0	0	0
111111110111111111	1	0	0	1	0	1	0	1	0
111111111011111111	1	0	0	0	0	0	0	0	0
111111111101111111	1	0	0	1	0	1	0	1	0
111111111110111111	1	0	0	0	0	0	0	0	0
111111111111011111	1	0	0	1	0	1	0	1	0
111111111111101111	1	0	0	1	0	1	0	1	0
111111111111110111	1	0	0	1	0	1	0	1	0
111111111111111011	1	0	0	1	0	1	0	1	0
111111111111111101	1	0	0	1	0	1	0	1	0
111111111111111110	1	0	0	1	0	1	0	1	0

Table 5: stability of coalitions wit 17 members

Interesting are the outsiders of the coalitions in the first category, because they are not in favor of the grand coalition.

Thinking about ES and EMES, of further interest are coalitions which violate ES but satisfy EMES. In this case there exists an outsider who would like to join the coalition, but who is not accepted by the insiders of the coalition. With other words this outsider pulls the existing coalition down. Looking for such cases some regions could be identified as "unwanted regions" on which further some economical analysis could be done to determine the reasons for this.

4.2.2 Statistical analysis[42]

One promising approach for figuring out which parameters influence the stability of coalitions is regression analysis. Since the response variable is binary (1 for stable, 0 for unstable) logistic regression can be used.

In general, logistic regression is a model used for prediction of the probability of occurrence of an event. This probability is modeled as a function of a set of explanatory variables, a constant, and an error term. The error term is treated as a random variable and is assumed to be logistic distributed. It represents unexplained variation in the dependent variable. The parameters are estimated so as to give a "best fit" of the data, which is evaluated by the Maximum Likelihood.

Therefore the probability that a coalition S_i with certain parameters is stable, is modeled

[42]the theory of logistic regression is taken from the course notes for "CE 5724 Analytical Techniques in Transportation", City College New York, fall term 2006

as the following function of parameters $x_{1,i} \dots x_{k,i}$ of the coalition.

$$Pr(s(S_i) = 1) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_{1,i} + \dots + \beta_k x_{k,i})}}$$

Herby the function $s : P(N) \rightarrow (0,1)$ (with $P(N)$ the powerset of N) assigns each coalition $S_i \subseteq N$ the value 1 if it is stable (in a certain sense, here PIS) and 0 if not.

The regression coefficients $\beta_1 \dots \beta_k$ gained from the logistic regression, describe the size of the contribution of the belonging parameters $x_{1,i} \dots x_{k,i}$. A positive regression coefficient means that that factor increases the probability of an allocation to be stable, while a negative regression coefficient means that that factor decreases this probability.

For a more precise interpretation, the odds that a coalition is stable, is defined as the ratio of the probability that it is stable to the probability that it is unstable.

$$Odds(s(S_i) = 1) = \frac{Pr(s(S_i) = 1)}{Pr(s(S_i) = 0)}$$

With this follows

$$Odds(s(S_i) = 1) = e^{\beta_0 + \beta_1 x_{1,i} + \dots + \beta_k x_{k,i}}$$

which makes easy to see that e^{β_j} is the factor by that the odds of stability increases if the parameter $x_{j,i}$ increases by 1.

Besides the detecting of underlying factors of stability these results can be also used to determine if a certain coalition is expected to be stable or not.

To perform such an analysis it has to be thought of parameters of the coalition which might explain its tendency to be stable. These parameters should be that general that they can be applied for all coalitions. First hand the following groups of parameters should be considered:

- size

  The size of the coalition can be measured in different economic variables (such as the number of members, aggregated population L, aggregated output Y, ...). To be more general, the size of a coalition with respect to the variable X, should be given relatively to the whole world.

  $$size = \frac{\sum_{i \in S} X_i}{\sum_{i \in N} X_i}$$

- homogeneity

  The homogeneity in a certain economic variable X (such as L, Y, $growth$,...) could be simply measured by its variance.

  $$\sigma^2 = \frac{1}{|S|} \sum_{i \in S} (X_i - \overline{X_S})^2$$

  where $\overline{X_S}$ denotes $\frac{1}{|S|} \sum_{i \in S} X_i$

- level

  Of interest could also be the relative level of a certain economic variable (such as L, Y, $growth$,...). A measure of the level of a variable X could be the average of this variable in the coalition S compared to the overall average.

  $$level = \frac{\overline{X_S}}{\overline{X_N}}$$

In one of the next steps of the project some economical experience and intuition will be needed to pick the most promising parameters from this set.

A further investigation could be done on ways to measure the stability on a continuous scale, which would also make general regression possible instead of just binary regression

4.2.3 Trajectories

It can be assumed (although so far not analytically proofed) that when a certain coalition is stable that also its subcoalitions are likely to be stable. And also the other way round, that if a coalition is constructed of stable subcoalitions, it is likely to be stable too.

In Table 6 all the stable coalitions, sorted by their size are given. This example has been generated in a test run using only 9 regions to have a still easy manageable outcome.

$\|S\| = 2$	$\|S\| = 3$	$\|S\| = 4$	$\|S\| = 5$	$\|S\| = 6$
100000001	101000001	101010001	101010101	101010111
011000000	100010001	101000101	101010011	
010100000	100000101	101000011	101000111	
001100000	100000011	100010101	100010111	
001000001	001010001	100010011	001010111	
000100001	001000101	100000111	000110111	
000011000	001000011	001010101		
000010100	000110001	001010011		
000010010	000100101	001000111		
000010001	000100011	000110101		
000001010	000010110	000110011		
000000110	000010101	000100111		
000000101	000010011	000010111		
000000011	000000111			

Table 6: example - stable coalitions

It can be easily seen that in this example the stable coalitions are not totally independent from each other. Looking at the biggest stable coalition, 5 of the 6 stable coalitions with 5 members are subcoalitions of it, and 10 of the 13 stable coalitions with 4 members are subcoalitions of it.

Although such statements are quite interesting, they are not really satisfying yet since it will get much more complex with 18 regions, and even than the question could be asked, so what?

It can be asked now for the share of direct subcoalitions (subcoalitions with one player less than the original coalition) of a certain stable coalition S_i which are stable itself. The number of direct subcoalitions is herby simply given by $|S_i|$ (which follows from just omitting each member).

$$\Phi^{S_i} = \frac{\sum\limits_{S_j|(S_j \subset S_i \wedge |S_j|=|S_i|-1)} s(S_j)}{|S_i|}$$

An average of this number over all stable coalitions gives an indicator of the existence of such trajectories.

One could be also interested in the minimal absolute number of direct subcoalitions which are stable. This could be seen as a empirically determined condition for obtaining a stable coalition. Such a statement can be used to focus with the computation of coalitions on a smaller subset in order to decrease computing time.

$$\min_{S_i|s(S_i)=1} \sum_{S_j|(S_j \subset S_i \wedge |S_j|=|S_i|-1)} s(S_j)$$

These figures should provide first information if the approach with trajectories is promising and worth to explore further.

4.2.4 Key regions

It can be also searched for regions which tend to stabilize or destabilize a coalition. In the former example in Table 6, the last region seems to be important for the stability. Only by omitting the last region from the biggest stable coalition the remaining coalition is not stable anymore.

To formalize such an observation it can be looked on this in two ways:

- the share of coalitions including the region i which are stable

  For this some more combinatorics is needed, in specific the number of all possible coalitions of the size k in a set of n players, which include a certain region. This is simply given by

  $$\binom{n-1}{k-1}$$

  Therefore the total number of possible coalitions in a set of n players, including a certain region is

  $$\sum_{k=2}^{n} \binom{n-1}{k-1}$$

  With this, the share of coalitions including the region i which are stable, follows to

  $$\Phi_i^1 = \frac{\sum\limits_{S_j|i \in S_j} s(S_j)}{\sum\limits_{k=2}^{n} \binom{n-1}{k-1}}$$

- the share of stable coalitions which include region i

  The other way round is somewhat easier since the number of all stable coalitions is simply given by $\sum_{S_j \subseteq N} s(S_j)$.

$$\Phi_i^2 = \frac{\sum_{S_j | i \in S_j} s(S_j)}{\sum_{S_j \subseteq N} s(S_j)}$$

By comparing these shares for all the regions i it can be easily seen which regions tend to contribute the most to the stability of coalitions and which the least.

Further this method could get adapted to search for key coalitions in the same way.

5 Conclusion

The CLIMNEG project of the environmental group at CORE is still in progress. The work which has been done during this thesis is just a part within the whole framework. So far one can draw following conclusions:

The most relevant part of this work for the CLIMNEG project, was to update the CWS model and its computational implementation. This update is described in Chapter 3. A lot of work has been done in cleaning the GAMS code to a necessary minimum which required very detailed knowledge of what is going on in the model, and also a lot of skills in GAMS programming which have been achieved during this work. This was followed by a number of improvements to make the code more efficient. Finally the update to 18 regions leaded to changes at the very heart of the code. Also new data mining has been performed as well as some changes in the functional form of the model. Therefore, it can be said that during this work, the implementation of the model has been brought to a new standard which provides a solid basis for the work which will be done with the model in near future. Nevertheless, currently there is still some work done to perform the fine tuning of the model, in order to achieve a version which satisfies economists as well as climatologists.

One main issue through this update to 18 regions is the massively increased number of possible cooperations between the regions. A remark on combinatorics provides these numbers of possible allocations and coalitions of a game with a certain number of players as an additional result. Since these numbers are huge for 18 players, improvements in computing time had to be done. Different approaches have been analyzed on their effectiveness.

The notes on stability analysis in Chapter 4 can be seen as an outlook for the next steps in the project. It is an objective to calculate the solutions for all the allocations with one coalition. On one commonly used PC this would last about two month. Nevertheless, the use of faster computers and splitting the work to several PCs will make this task possible in a reasonable time. Meanwhile it has to be decided on the methods to gather information from the so gained data. Some promising ideas are given in this work, but it is still unknown if they really bring the desired results.

Further this work provides a short documentation of the model and the project (Chapter 3), and the essential background which is needed to understand the work done with the model (Chapter 2). Its aim is to provide future collaborators of the project with a first insight.

Personally, it has been a very interesting experience to see how applied science works and how knowledge and tools from different areas are brought together. Although the scientific work, done on climate coalitions,is still of a manageable amount compared to other fields, it is impressive how many different ideas and approaches are published. It is a continuous quest for insights where each answer brings up even more questions.

A Data

In this section the input data for the CWS model is given.

A.1 general scalars

δ_K	depreciation rate of capital per year	0.10
n_{ROW}	free riding parameter ROW	25
η	inequality aversion	0.0
γ	capital elasticity in output	0.25
ρ	discount rate	0.25

Table 7: general scalars

A.2 Initial data for each country

Y_0	reference PPP GDP 2000 (billion 2000 US$)
Y_0^G	growth rate of PPP GDP 2000
E_0^F	carbon emissions from fuel uses 2000 (btC) (national total excluding land use changes)
E_0^G	growth rate of carbon emissions 2000
K_0	capital stock 2000 (billion 2000 US$)

	Y_0	Y_0^G	E_0^F	E_0^G	K_0/Y_0
CAN	846.397	0.0290	0.142	0.0120	1.80
USA	9764.800	0.0330	1.581	0.0180	1.44
JPN	4069.047	0.0211	0.472	0.0247	2.43
EU	9523.671	0.0227	0.891	0.0026	1.80
OEU	386.140	0.0211	0.023	0.0087	2.00
CEA	1049.825	-0.0207	0.195	0.0287	1.40
FSU	1443.094	0.0151	0.617	-0.0143	1.40
AUZ	567.245	0.0349	0.100	0.0218	1.60
MED	1207.926	0.0376	0.167	0.0344	1.20
MEA	829.763	0.0322	0.227	0.0560	1.20
AFR	1986.814	0.0179	0.151	0.0220	0.60
CHN	5147.692	0.1005	0.928	0.0220	1.15
IND	2402.087	0.0550	0.282	0.0520	0.76
RAS	603.570	0.0436	0.044	0.0441	0.70
EAS	1742.654	0.0488	0.209	0.0316	1.20
LAM	3052.046	0.0339	0.299	-0.0057	1.30
LAO	568.414	0.0328	0.054	-0.0014	0.70
ROW	73.275	0.0269	0.046	-0.0276	1.00

Table 8: parameters for abatement and damage cost functions

A.3 Trend data

Data for generating the exogenous time series $L_{i,t}$, $A_{i,t}$ and $\sigma_{i,t}$.

L	population (million people)
A_0	initial productivity 2000
A_T	regional asymptotic productivity
A_0^G	initial regional productivity growth rate per decade (2000-2010)
σ_0	initial emission-GDP ratio 2000 (kg of carbon per US$)
σ_T	regional asymptotic emission-GDP ratio (kg of carbon per US$)
σ_0^G	initial regional emission-GDP growth rate per decade (2000-2010)
L_0^G	initial regional population growth rate per decade (2000-2010)

	L_{2000}	L_{2050}	L_{2100}	L_{2150}	L_{2200}	L_{2250}	L_{2300}
CAN	30.769	39.085	36.234	37.143	38.539	39.781	40.876
USA	285.003	408.695	437.155	452.753	470.045	483.033	493.038
JPN	150.035	149.273	129.508	127.312	130.811	135.102	138.907
EU	377.335	369.771	329.931	337.381	351.943	364.959	376.588
OEU	11.928	11.035	9.625	9.810	10.209	10.575	10.909
CEA	68.676	53.942	43.754	44.620	46.369	47.840	49.083
FSU	282.353	237.159	195.727	194.445	201.065	207.183	212.088
AUZ	22.937	30.072	28.828	29.207	30.362	31.430	32.427
MED	231.016	382.892	374.994	345.456	347.702	358.343	367.063
MEA	119.994	271.882	332.016	309.935	300.545	307.006	314.059
AFR	640.874	1506.429	1928.166	1781.772	1710.777	1754.598	1799.205
CHN	1282.022	1404.613	1189.580	1157.178	1209.072	1255.308	1294.001
IND	1016.938	1531.438	1458.360	1308.190	1304.534	1342.329	1371.709
RAS	348.978	731.179	808.509	720.066	698.309	715.730	730.934
EAS	477.183	666.491	622.944	590.337	604.074	622.611	637.573
LAM	382.068	547.054	510.927	476.548	485.323	501.611	515.348
LAO	120.851	204.049	207.721	184.710	181.228	187.130	192.235
ROW	131.688	149.633	134.911	128.250	130.973	134.971	138.202

Table 9: UN projection of population in million people

A_T	regional asymptotic productivity	20
σ_T	regional asymptotic emission-GDP ratio (kg of carbon per US$)	0.020

Table 10: asymptotic values

For the polynomial interpolation needed values:

$$A_0 = \frac{Y_0}{K_0^\gamma L_{2000}^{1-\gamma}}$$

$$A_0^G = \frac{1}{2}\left(\frac{(1+Y_0^G)^{10(1-\gamma)}}{(1+L_0^G)^{1-\gamma}} - 1\right) \text{ with } L_0^G = \frac{L_{2010} - L_{2000}}{L_{2000}}$$

$$\sigma_0 = 1000\frac{E_0^F}{Y_0}$$

$$\sigma_0^G = \frac{1}{10}((1+E_0^G)^{10} - (1+Y_0^G)^{10})$$

The divisions by 10 in σ_0^G and 2 in A_0^G are done to lower the effect of the current growth rate, which is quite radical.

A.4 Parameters for abatement and damage cost functions

θ_1	intercept damage function
θ_2	exponent damage function
c	parameter of abatement function

	θ_1	θ_2	c
CAN	0.01102	2.0	−0.20
USA	0.01102	2.0	−0.12
JPN	0.01174	2.0	−0.07
EU	0.01174	2.0	−0.12
OEU	0.01174	2.0	−0.10
CEA	0.01174	2.0	−0.29
FSU	0.00857	2.0	−0.44
AUZ	0.01102	2.0	−0.16
MED	0.02093	2.0	−0.69
MEA	0.02093	2.0	−0.38
AFR	0.02093	2.0	−0.47
CHN	0.01523	2.0	−0.59
IND	0.01523	2.0	−0.35
RAS	0.02093	2.0	−0.41
EAS	0.02093	2.0	−0.33
LAM	0.02093	2.0	−0.35
LAO	0.02093	2.0	−0.39
ROW	0.02093	2.0	−0.23

Table 11: parameters for abatement and damage cost functions

A.5 Parameter values of the climate system

Here the input parameters for the climare system are given

M_0	initial atmospheric CO_2 concentration in 1800 (btC)	590
M_0^{AT}	initial atmospheric CO_2 concentration in 2000 (btC)	783
M_0^{UO}	initial upper ocean and vegetation CO_2 concentration in 2000 (btC)	807
M_0^{LO}	initial lower ocean CO_2 concentration in 2000 (btC)	19238
b_{11}	carbon cycle transition matrix coefficient	-0.033384
b_{22}	carbon cycle transition matrix coefficient	-0.039103
b_{33}	carbon cycle transition matrix coefficient	-0.000422
b_{12}	carbon cycle transition matrix coefficient	0.033384
b_{21}	carbon cycle transition matrix coefficient	0.027607
b_{23}	carbon cycle transition matrix coefficient	0.011496
b_{32}	carbon cycle transition matrix coefficient	0.000422

Table 12: parameter values carbon cycle

T_0^L	initial temperature change lower ocean (°C compared to 1800)	0.108
T_0^E	temperature change atmosphere (°C compared to 1800)	0.622
$F^{2\times}$	forcing with a carbon concentration doubling	4.030
$T^{2\times}$	temperature stabilization with a carbon concentration doubling	3.300
λ	climate feedback factor ($F^{2\times}$ divided by $T^{2\times}$)	0.914
c_1	coefficient for upper level	1.7
c_3	transfer coefficient upper to lower level	0.794
c_4	transfer coefficient for lower level	0.03609

Table 13: parameter values temperature cycle

forcing of other gases and aerosols

$$F_t^{other} = 0.21833 + 0.013767t - 0.000063774t^2 + 0.000000058979t^3$$

B Model initialization

In the following the initialization and calibration of the model is formalized.

B.1 Initializing the economy

- disabling climate system, costs and damages: $C = 0$, $D = 0$,$T^E = 0$, $\mu = 0$
- capital stock from first order conditions (compare [14])

$$K_{i,t} = \left(\frac{\frac{1+\rho_{i,t}}{1+\rho_{i,t-1}} - (1-\delta_K)^{10}}{10\gamma A_{i,t} L_{i,t}^{1-\gamma}} \right)^{\frac{1}{\gamma-1}}$$

- production from production function

$$Y_{i,t} = A_{i,t} K_{i,t}^{\gamma} L_{i,t}^{1-\gamma}$$

- investment from capital accumulation equation

$$I_{i,t} = \frac{1}{10}(K_{i,t+1} - (1-\delta_K)^{10} K_{i,t})$$

- consumption from GDP definition

$$Z_{i,t} = Y_{i,t} - I_{i,t}$$

- solving the economy model several times for each region separately
- relaxing C, D ,T^E

B.2 Initializing the climate system

- calculating emissions

$$E_{i,t} = \sigma_{i,t}(1 - A_{i,t})Y_{i,t}$$

- initializing the carbon stocks

$$M_{t+1}^{AT} = M_t^{AT} + 10(b_{11} M_t^{AT} + \sum_i E_{i,t}) + b_{21} M_t^{UO})$$

$$M_{t+1}^{UO} = M_t^{UO} + 10(b_{12} M_t^{AT} + b_{22} M_t^{UO} + b_{32} M_t^{LO})$$
$$M_{t+1}^{LO} = M_t^{LO} + 10(b_{33} M_t^{LO} + b_{23} M_t^{UO})$$

- initializing the temperature

$$F_t = \frac{\log(M_t^{AT}/M_0)}{\log(2)} F^{2\times} + F_t^{other}$$

$$T_{t+1}^E = \frac{1}{1 + c_1\lambda + c_1 c_3}(T_t^E + c_1(F_{t+1} + c_3 T_{t+1}^L))$$
$$T_{t+1}^L = T_t^L + c_4 T_t^E - T_t^L$$

- solving the climate model several times

After that the consumption has to be corrected by the damage.

$$D_{i,t} = Y_{i,t}\Theta_{i,1}\left(\frac{T_t^E}{2.5}\right)^{\Theta_{i,2}}$$

$$Z_{i,t} = Z_{i,t} - D_{i,t}$$

C Code

This section provides the GAMS code of the CWS model and the code used for performing the stability analysis.

C.1 CWS model

In this section parts of the source code of the CWS model for GAMS are given.

It contains the following files:

cws.gms	run file, choice of scenarios and key matrix
data_18.inp	sets, general parameters, initial values, trends
equations.inp	definition of variables, equations and models
datadump.inc	writes input data to dump
calib.inc	calibration of the model and setting of bounds
extreme.scn	scenarios NASH and COOP
scenarios.scn	through the key matrix arbitrary chosen scenarios
key_xxx.inp	key matrix
pane.sol	PANE algorithm
cwsm.out	generating of detailed output of BAU, NASH, and COOP scenarios
totab.inc	storing values from BAU, NASH, and COOP scenarios for comparison
tabgen.out	generating a output for comparing BAU, NASH, and COOP scenarios
scenarios.out	preparing output files for allocations summary
scenarios2.out	filling output files for allocations summary
head1.inc	head for output files

Table 14: files of the CWS code

The bold written files are given now.

C.1.1 cws.gms

```
$ontext
============================================================================

                       CLIMNEG WORLD SIMULATION MODEL

                  (c) 2000 "original" by Johan Eyckmans
                      2005 "back to the basics" by Mirabelle Muuls
                      2006 "basis 2000" by François Gerard
                      2008 "upgrade" by Paul Holzweber

============================================================================
Johan Eyckmans (johan.eyckmans@econ.kuleuven.be)
Mirabelle Muuls (m.p.muuls@lse.ac.uk)
François Gerard (fgerard@gmail.com)
Paul Holzweber (paul.holzweber@gmx.at)
============================================================================
GAMS program file: CWS.GMS
last update: 03/05/2008
============================================================================
=> The program writes ASCII output files:
   *.dmp      dump file for data and iteration process
   *.txt      output different scenarios
============================================================================
$offtext

$title Climneg World Simulation Model
$inlinecom {* *}
$offupper
$offsymxref offsymlist offuellist offuelxref

***** sets and data input **************************************************

$batinclude include\data_18.inp ;
$batinclude include\datadump.inc ;

***** definition variables and equations **********************************

$batinclude include\equations.inp ;

***** options **************************************************************

OPTION ITERLIM  = 99999;
OPTION LIMROW   = 0;
OPTION LIMCOL   = 0;
OPTION SOLPRINT = OFF;
OPTION RESLIM   = 99999;
OPTION NLP      = minos5 ;

ramsey.optfile=1;
co2clim.optfile=1;
crice.optfile=1;

*** calibration ***********************************************************

$batinclude include\calib.inc ;
divWtol = 10**(-4)*smin(N,sum(T, RR(N,T)*L(N,T)*U.L(N,T))) ;
```

```
*** preparing file for iteration dump ************************************

file ITERATION /output\ITERATION.dmp/ ;
ITERATION.PS=70 ;
ITERATION.NR=0 ;
ITERATION.NW=cow ;
ITERATION.ND=dec ;
parameter col column counter ;
col = 1 ;
parameter line line counter ;
line = 1 ;
put ITERATION ;
$batinclude include\head1.inc "ITERATION REPORT" ;
put "iteration tolerance:",       @30, divWtol       / ;
put "iteration limit:",           @30, itlim  :cow:0 / ;
loop(iter$(ord(iter) LE paw), put "=") ;
put // ;

*** scenarios ************************************************************

$batinclude include\EXTREME.SCN ;

$batinclude include\SCENARIOS.SCN "KEY_testCOALITIONS_18";

**************************************************************************

$label end
```

C.1.2 equations.inp

```
***** VARIABLES ***********************************************************

variables
*** economy
W             welfare
U(N,T)        utility
C(N,T)        cost of abatement
D(N,T)        climate change damage
Z(N,T)        consumption
Y(N,T)        output
K(N,T)        capital stock
I(N,T)        investment
X(N,T)        transfers
A(N,T)        emission control rates
TE(T)       regional temperature
E(N,T)        annual CO2 emissions (GtC)
*** climate
M_AT(T)       carbon concentration in atmosphere (b.t.c.)
M_UO(T)       carbon concentration in shallow oceans (b.t.c.)
M_LO(T)       carbon concentration in lower oceans (b.t.c.)
FORC(T)       radiative forcing (Watt per square meter)
TL(T)         ocean temperature
*** fake objective function
fobj          fake objective value
;

positive variables
C, D, Z, Y, K, I, A, E, M_AT, M_UO, M_LO, FORC, TE, TL
;

***** EQUATIONS ***********************************************************

equations
*** economy
defW          definition welfare
defUTIL1(N,T) definition CRA utility
defUTIL2(N,T) definition logaritmic utility
defY(N,T)     definition production function
defC(N,T)     definition abatement cost function
defD(N,T)     definition climate change damage function
budget(N,T)   budget constraint
accK(N,T)     accumulation capital stock
defKter(N,T)  definition terminal condition of K
defE(N,T)     carbon emissions process
*** climate
MM_AT(T)      atmospheric carbon accumulation process
MM_UO(T)      upper ocean and vegetation carbon accumulation process
MM_LO(T)      lower ocean carbon accumulation process
defFORC(T)    radiative forcing process
defTE(T)    atmospheric temperature process
defTL(T)      ocean temperature process
*** fake objective function
fakeobj       fake objective function
;
```

```
*** economy
defW..             W         =E= sum((N,T), WW(N)*RR(N,T)*L(N,T)*U(N,T)) ;
defUTIL1(N,T)$(ia NE 1)..
                   U(N,T)    =E= (1/(1-ia))*(Z(N,T)/L(N,T))**(1-ia) ;
defUTIL2(N,T)$(ia EQ 1)..
                   U(N,T)    =E= LOG(Z(N,T)/L(N,T)) ;
defY(N,T)..        Y(N,T)    =E= AL(N,T)*K(N,T)**GAMA*L(N,T)**(1-GAMA) ;
defC(N,T)..        C(N,T)    =E= Y(N,T)*B1(N)*((1-A(N,T))*log(1-A(N,T))+A(N,T)) ;
defD(N,T)..        D(N,T)    =E= Y(N,T)*A1(N)*(TE(T)/2.5)**A2(N) ;
budget(N,T)..      Y(N,T)    =E= Z(N,T) + I(N,T) + C(N,T) + D(N,T) ;
accK(N,T+1)..      K(N,T+1)  =E= tint*I(N,T)+((1-DK)**tint)*K(N,T) ;
defKter(N,TT)..    R(N,TT)*K(N,TT) =L= I(N,TT) ;
defE(N,T)..       E(N,T)    =E= SIGMA(N,T)*(1-A(N,T))*Y(N,T) + ETREE(N,T) ;
*** climate
MM_AT(T+1)..       M_AT(T+1) =E= M_AT(T) + tint*(M_AT(T)*b11 + sum(N, E(N,T))
                                + M_UO(T)*b21) ;
MM_UO(T+1)..       M_UO(T+1) =E= M_UO(T) + tint*(M_AT(T)*b12 + M_UO(T)*b22
                                + M_LO(T)*b32) ;
MM_LO(T+1)..       M_LO(T+1) =E= M_LO(T) + tint*(M_LO(T)*b33 + M_UO(T)*b23) ;
defFORC(T)..       FORC(T)   =E= F2X*(LOG(M_AT(T)/M00)/LOG(2)) + RFOgas(T) ;
defTE(T+1)..      TE(T+1) =E= (1/(1 + C1*LAM + C1*C3))*(TE(T)
                                + C1*(FORC(T+1) + C3*TL(T+1))) ;
defTL(T+1)..       TL(T+1)   =E= TL(T) + C4*(TE(T) - TL(T)) ;
*** fake objective function
fakeobj..          fobj      =E= 1000 ;

***** BOUNDS **************************************************************

*** economy
Y.LO(N,T)     =   0.8*Y0(N) ;
Y.UP(N,T)     =  500.0*Y0(N) ;
Z.LO(N,T)     = EPS ;
K.LO(N,T)     = EPS ;
E.LO(N,T)     = EPS ;
I.LO(N,T)          = EPS ;
I.UP(N,T)          = Y.UP(N,T) ;
*** climate
M_AT.LO(T)       = 0.99*M0 ;
TE.LO(T)    = 0.99*T0 ;
TL.LO(T)      = 0.99*TL0 ;

***** INITIAL VALUES ******************************************************

*** economy
Y.FX(N,TI)    = Y0(N) ;
K.FX(N,TI)    = K0(N) ;
A.FX(N,TI)    = 0 ;
E.FX(N,TI)    = E0(N) ;
*** climate
M_AT.FX(TI)   = M_AT2000 ;
M_UO.FX(TI)   = M_UO2000 ;
M_LO.FX(TI)   = M_LO2000 ;
TE.FX(TI)     = T0 ;
TL.FX(TI)     = TL0 ;
```

```
***** MODELS **************************************************************

model RAMSEY  / defW, defUTIL1, defUTIL2, defY, defC, defD, budget, accK,
                defKter / ;

model CO2CLIM / defE, MM_AT, MM_UO, MM_LO, defFORC, defTL, defTE, fakeobj / ;

model CRICE   / defW, defUTIL1, defUTIL2, defY, defC, defD, budget, accK,defKter,
                defE, MM_AT, MM_UO, MM_LO, defFORC, defTL, defTE / ;
```

C.1.3 calib.inc

```
* disabling climate system, costs and damages
A.FX(N,T)  = 0 ;
TE.FX(T) = 0 ;
C.FX(N,T)  = 0 ;
D.FX(N,T)  = 0 ;

*** initializing economy
* no transfers
X.FX(N,T)  = 0 ;
* capital stock from first order conditions
K.L(N,T)$(ord(T) GT 1) =
    ( ((RR(N,T-1)/RR(N,T)) - (1-DK)**TINT) /
    (TINT*GAMA*AL(N,T)*(L(N,T)**(1-GAMA))) )**(1/(GAMA-1)) ;
* production from production function
Y.L(N,T)   = AL(N,T)*(K.L(N,T)**GAMA)*(L(N,T)**(1-GAMA)) ;
* investment from capital accumulation equation
I.L(N,T)   = (1/TINT) * (K.L(N,T+1) - ((1-DK)**TINT)*K.L(N,T)) ;
* consumption from GDP definition
Z.L(N,T)   = Y.L(N,T) - I.L(N,T) ;

Z.LO(N,T) = Y.L(N,T)/2;
I.LO(N,T) = Y.L(N,T)/10;
I.LO(N,TT)= Y.L(N,TT)/50;

* solving model several times for each region separately

loop(iter$(ord(iter) LT 3),
    loop(N1,
        {* constructing weight vector *}
        WW(N) = 0 ;
        WW(N)$(ord(N) EQ ord(N1)) = 1 ;
        solve RAMSEY using NLP maximizing W ;
        {* recording multiplier capital accumulation constraint *}
        PSI(N,T)$(ord(N) EQ ord(N1))
            = tint*abs(accK.M(N,T+1)) ;
        PSI(N,T)$(ord(N) EQ ord(N1) AND ord(T) EQ card(T))
            = abs(defKter.M(N,T)) ;
        {* recording multiplier resource constraint *}
        LAMBDA(N,T)$(ord(N) EQ ord(N1))
            = abs(budget.M(N,T)) ;
    ) ;
) ;

* relaxing C,D,TE
Z.LO(N,T)   = 0 ;
I.LO(N,T)   = 0 ;
C.LO(N,T)   = 0 ;
C.UP(N,T)   = +INF ;
D.LO(N,T)   = 0 ;
D.UP(N,T)   = +INF ;
TE.LO(T)  = T0 ;
TE.UP(T)  = +INF ;
TE.FX(TI) = T0 ;

* initializing carbon cycle
E.FX(N,T)     = SIGMA(N,T)*(1-A.L(N,T))*Y.L(N,T) ;
```

```
M_AT.L(T+1)   = M_AT.L(T)+tint*(M_AT.L(T)*b11+sum(N, E.L(N,T))+M_UO.L(T)*b21);
M_UO.L(T+1)   = M_UO.L(T)+tint*(M_AT.L(T)*b12+M_UO.L(T)*b22+M_LO.L(T)*b32);
M_LO.L(T+1)   = M_LO.L(T)+tint*(M_LO.L(T)*b33+M_UO.L(T)*b23);
M.L(T+1)      = M_AT.L(T+1) ;

* initializing climate module
FORC.L(T)   = F2X*(LOG(M.l(T)/M00)/LOG(2))+RFOgas(T) ;
TE.L(T+1) = (1/(1+C1*LAM+C1*C3))*(TE.L(T) + C1*(FORC.L(T+1)+C3*TL.L(T+1))) ;
TL.L(T+1)   = TL.L(T) + C4*(sum(N, TE.L(T))/card(N) - TL.L(T)) ;

* solving carbon cycle and climate submodule several times

loop(iter$(ord(iter) LT 3),
    solve CO2CLIM using NLP maximizing fobj ;
) ;

* calculating climate change damage and correcting consumption
D.L(N,T) = Y.L(N,T)*A1(N)*(TE.L(T)/2.5)**A2(N) ;
Z.L(N,T) = Z.L(N,T) - D.L(N,T) ;
WW(N) = 1 ;

$batinclude include\cwsm.out DETAIL_BAU "Bussiness As Usual" ;
$batinclude include\totab.inc "BAU" ;

parameters
YUP(N,T)  upper bound GDP,
YLOW(N,T) lower bound GDP,
ILOW(N,T) lower bound investment,
IUP(N,T)  upper bound investment,
ALOW(N,T) lower bound abatement,
AUP(N,T)  upper bound abatement,
ELOW(N,T) lower bound emissions,
EUP(N,T)  upper bound emissions;

YLOW(N,T) = 0.90*Y.L(N,T) ;
YUP(N,T)  = 1.00*Y.L(N,T) ;
ILOW(N,T) = EPS ;
IUP(N,T)  = 1.00*Y.L(N,T) ;
ALOW(N,T)$(ord(T) NE 1) = 0.000001 ;
AUP(N,T)$(ord(T) NE 1)  = 1 ;
ELOW(N,T) = EPS ;
EUP(N,T)  = 1.00*E.L(N,T) ;

Y.LO(N,T) = YLOW(N,T) ;
Y.UP(N,T) = YUP(N,T) ;
I.LO(N,T) = ILOW(N,T) ;
I.UP(N,T) = IUP(N,T) ;
E.LO(N,T) = ELOW(N,T) ;
E.UP(N,T) = EUP(N,T) ;
A.LO(N,T) = ALOW(N,T) ;
A.UP(N,T) = AUP(N,T) ;
A.FX(N,T)$(ord(T) EQ 1) = 0 ;

A.L(N,T)$(ord(T) NE 1)  = EPS ;
```

C.1.4 extreme.scn

```
*** NASH scenario **********************************************************

* constructing coalition
PM(N1,N2) = 0;
PM(N1,N2) = 1$(ord(N1) EQ ord(N2));

* free riding correction for ROW
A1("ROW") = A1("ROW") / nROW ;

put ITERATION ;
put "NASH-Scenario" / ;

* solving for coalition equilibrium
$batinclude include\pane.sol ;

* restoring free riding correction for ROW
Z.L("ROW",T) = Z.L("ROW",T) + D.L("ROW",T) ;
A1("ROW")    = A1("ROW") * nROW ;
D.L("ROW",T) = Y.L("ROW",T)*A1("ROW")*(TE.L(T)/2.5)**A2("ROW") ;
Z.L("ROW",T) = Z.L("ROW",T) - D.L("ROW",T) ;

* writing output
WW(N) = 1 ;
$batinclude include\cwsm.out DETAILS_NASH "voluntary provision equilibrium";
$batinclude include\totab.inc "NASH" ;

Y.LO(N,T) = 0.75*Y.L(N,T) ;
Y.UP(N,T) = 1.00*Y.L(N,T) ;

E.LO(N,T)     = 0.20*E.L(N,T) ;
E.UP(N,T)     = 1.00*E.L(N,T) ;

*** COOP scenario **********************************************************

* constructing coalition
WW(N) = 1 ;
PM(N1,N2) = 0;
PM(N1,N2) = 1$(ord(N1) EQ 1);

* first period similar as in the NASH scenario
I.FX(N,TI)=I.L(N,TI);

put ITERATION ;
put "COOP-Scenario" / ;

* solving for coalition equilibrium
$batinclude include\pane.sol ;

* writing output
WW(N) = 1 ;
$batinclude include\cwsm.out DETAILS_COOP "Pareto efficient solution" ;
$batinclude include\totab.inc "COOP" ;

*** writing ASCII output for import in EXCEL ****************************
$batinclude include\tabgen.out TABFIG ;
```

C.1.5 scenarios.scn

```
*************************
*** SCENARIOS scenario ***
*************************

*** preparing file for per member partition function
$batinclude include\SCENARIOS.OUT ;

*** including all partition keys
$batinclude include\%1.inp ;

*************** loop over keys ***************

parameter cardSROW cardinality coalition containing ROW ;

parameter count general counter;
count=0;

loop(part,
    PM(N,N1) = 0;
    PM(N,N1) = 1$(key(part,N1) EQ ord(N));

*for 1-coalition notation (0,1)
    count=2 ;
    if(sum(N1,key(part,N1)) eq 0,
count=1 ;
    ) ;
    loop(N1,
        if(key(part,N1) eq 0,
    PM(N2,N1)$(ord(N2) EQ count)=1 ;
    count=count+1 ;
        ) ;
    ) ;

* free riding correction
    cardsROW = 0 ;
    loop(N,
        if (PM(N,"ROW") = 1,
                 cardsROW = sum(N1, PM(N,N1));
        );
    ) ;
    if (cardsROW EQ 1,
         A1("ROW") = A1("ROW")/nROW;
    );

* writing partition structure in iteration dump file*****
    put ITERATION ;
    put "coalition = { " ;
    loop(N1, put key(part,N1):>3, ",") ;
    put @(ITERATION.cc-1) " }" / ;

$batinclude include\PANE.SOL ;

$batinclude include\SCENARIOS2.OUT ;

) ;
```

C.1.6 pane.sol

```
divW = 1000 ;
icount = 0 ;

{* iteration loop *}
loop(iter$((divW GT divWtol) AND (icount LT itlim)),
    icount = icount + 1 ;

    {* recording last abatement vector *}
    WLAST(N) = sum(T, RR(N,T)*L(N,T)*U.L(N,T)) ;

    {* writing iteration number to dump *}
    col=1 ;
    line=1 ;
    putpage ;
    put #line ;
    PUT @col, "iteration ", icount:3:0 / ;
    put @col, "-------------" / ;
    line=line+2 ;
    {* writing years to dump *}
    PUT #(line+1) ;
    loop(T, PUT @col, (1980+10*ord(T)):4:0 /) ;
    put @col, "Util" / ;
    col = col + 6 ;

    {* loop over coalitions *}
    loop(N1,
        if(sum(N2,PM(N1,N2)) GE 1,
            {* construct coalition *}
            nash(N)  = NO ;
            nash(N)  = YES$(PM(N1,N) EQ 1) ;
            nashc(N) = NOT nash(N) ;
            {* assign weights *}
            WW(N)       = 0 ;
            WW(nash(N)) = 1 ;
            WW(nash(N)) = WW(nash)/sum(N2, WW(N2)) ;
            {* fixing strategies complement of Nash player *}
            A.FX(nashc,t) = A.L(nashc,t) ;
            I.FX(nashc,t) = I.L(nashc,t) ;
            {* solving welfare maximization problem insiders *}
            solve CRICE using NLP maximizing W ;
            {* free strategies complement *}
            A.UP(nashc,t)   = AUP(nashc,T) ;
            A.LO(nashc,t)   = ALOW(nashc,T) ;
            I.UP(nashc,t)   = IUP(nashc,t) ;
            I.LO(nashc,t)   = ILOW(nashc,t) ;
        ) ;

        {* recording multiplier capital accumulation constraint player N1 *}
        PSI(N1,T)$(ord(T) LT card(T)) = tint*abs(accK.M(N1,T+1)) ;
        PSI(N1,T)$(ord(T) EQ card(T)) = abs(defKter.M(N1,T)) ;

        {* recording multiplier resource constraint player N1 *}
        LAMBDA(N1,T) = abs(budget.M(N1,T)) ;

        {* writing emission abatement vector to dump *}
        PUT #line ;
```

```
        PUT @col N1.TL:>4 ;
        PUT #(line+1)
        loop(T,
             put @col A.L(N1,T):7:6 ;
             put$(abs(A.M(N1,T)) GT EPS) "*":>1 ;
             put / ;
        ) ;
        {* writing utility level to dump *}
        put @col sum(T, RR(N1,T)*L(N1,T)*U.L(N1,T)):8:6 / ;
        {* writing model status to dump *}
        put @col CRICE.modelstat:8:0 / ;
        col = col + 10 ;
    ) ;

    {* calculate divergence between two last iterations *}
     divW = sqrt( sum(N, (sum(T,
     RR(N,T)*L(N,T)*U.L(N,T))-WLAST(N))*(sum(T, RR(N,T)*L(N,T)*U.L(N,T))-WLAST(N))) ) ;

    {* recording divergence *}
    divWhist(iter1)$(ord(iter1) EQ ord(iter)) = divW ;

    {* recording model status *}
    modelstat(iter1)$(ord(iter1) EQ ord(iter)) = CRICE.modelstat ;

    {* writing divergence to dump *}
    line = line + card(T) + 3 ;
    put #line ;
    put "divergence: ", divW:12:10 / ;
    put #(line+1) ;
    loop(iter1$(ord(iter1) LE paw), put "=") ;
    line = line + 2 ;
) ;

put / ;
put "ITERATION REPORT" / ;
put "----------------" / ;
put "iter":>5, "  ", "stat":>5, "  ", "divW":>cow / ;
loop(iter$(ord(iter) LE icount),
    put ord(iter):5:0,
    put "  " ,
    put modelstat(iter):5:0,
    put "  " ,
    put divWhist(iter) / ;
) ;
put / ;
put "divtol =", @15, divWtol // ;
if(icount LE itlim AND divW LT divWtol,
    loop(iter2$(ord(iter2) LE paw), put "=") ;
    put / ;
    put "===> Convergence reached after", icount:4:0, " iterations." / ;
    loop(iter2$(ord(iter2) LE paw), put "=") ;
    put / ;
else
    put "== WARNING == WARNING == WARNING == WARNING == WARNING == WARNING ==" / ;
    put "===> NO convergence reached after", icount:4:0, " iterations!" / ;
    put "== WARNING == WARNING == WARNING == WARNING == WARNING == WARNING ==" / ;
) ;
put // ;
```

C.2 Checking stability

```
$ontext
============================================================================

                       CLIMNEG WORLD SIMULATION MODEL

                  (c) 2000 "original" by Johan Eyckmans
                      2008 "upgrade" by Paul Holzweber

============================================================================
GAMS program file: stability.GMS
last update: 03/05/2008
============================================================================
this program tests
     => Internal (IS) and External (ES) Stability of single coalitions
     => Exclusive membership
     => Core stability
only binary inputs are allowed!!!
============================================================================
$offtext

$TITLE INTERNAL AND EXTERNAL STABILITY AND EXCLUSIVE MEMBERSHIP
$inlinecom {* *}
$offupper
$offsymxref offsymlist offuellist offuelxref

***** data input ***********************************************************

$batinclude include\data_18.inp ;

*** input keys
$batinclude include\KEY_topCOALITIONS_18.INP ;

*** input payoffs
$batinclude output\PMPF18_2.TXT ;

***** creating output files ************************************************

file stability /output\stability18_2.txt/ ;
put stability ;
*$include \include\head1.inc ;

***** additional parameters ************************************************

parameters
found        found searched key
S(N)
domNash(N)   individual dominance wrt Nash equilibrium
indrat(N)    individual rationality
instab(N)    individual internal stability
exstab(N)    individual external stability
exstabUNA(N) exclusive membership unanimity
exstabMAJ(N) exclusive membership majority
DNC(part)    coalition dominance wrt Nash equilibrium
IR(part)     individual rationality
GR(part)     group ratonality of grand coalition
PIS(part)    potentially internal stability
```

```
IS(part)     internal stability
ES(part)     external stability
IES(part)    internal and external stability
ESUNA(part) exclusive membership unanimity vote external stability
ESMAJ(part) exclusive membership majority vote external stability
keyS(N)      key coalition S
keyD(N)      key deviating coalition D
uS(N)        utility vector in coalition S
uT(N)        utility vector after transfers
uFR(N)       free riding payoff
uNC(N)       utility vector in Nash equilibrium
uGC(N)       utility vector in grand coalition
uPAR(N)      utiltiy vector Pareto efficient allocation
uDEV(N)         utility when deviating
uSDEV        utility when NOT deviating
key2(N,N)
keyS2(N,N)
keySb(N)     binary key
keyDb(N)     binary key
pos
cntVIOLIS(N) counter violations internal stability
cntVIOLES(N) counter violations external stability
winner(N)    vector of winners for exclusive membership
;

***** initialization ******************************************************

*** parameters
cntVIOLIS(N) = 0 ;
cntVIOLES(N) = 0 ;
IR(part)   = 0 ;
IS(part)   = 0 ;
ES(part)   = 0 ;
IES(part)  = 0 ;

*nash payoffs
loop(part,
    if (sum(N, key(part,N)) eq 0, uNC(N) = dataS(part,N));
);
*grand coalition payoffs
loop(part,
    if (sum(N, key(part,N)) eq card(N), uGC(N) = dataS(part,N));
);

***** main program ********************************************************

*** CHECK INTERAL AND EXTERNAL STABILITY SINGLE COALITIONS ****************

loop(part,
    display "================ NEW PARTITION ================= " ;
    {* recording key and utility levels of partition to be tested *}
    keySb(N) = key(part,N) ;
    uS(N)     = dataS(part,N) ;
    {* initializing variables *}
    uFR(N)       = 0 ;
    indrat(N)     = 0 ;
    instab(N)     = 1 ;
    exstab(N)     = 1 ;
```

```
exstabUNA(N) = 1 ;
exstabMAJ(N) = 1 ;
{* construct single coalition S *}
S(N) = NO ;
S(N) = YES$(keySb(N) EQ 1);
{* writing header for partition into dump file *}
put // ;
loop(iter$(ord(iter) LE (28+card(N)*12)), put "+") ;
put / ;
put "partition number = ", ord(part):3:0 / ;
put "binary key       = " ;
loop(N, put keySb(N):1:0) ;
put / ;
{* writing partition structure to dump file *}
put "structure = {" ;
    loop(N,
        if(keySb(N) EQ 1, put N.TL:>3, "," ) ;
    ) ;
put "}" / ;
loop(iter$(ord(iter) LE(28+card(N)*12)), put "+") ;
put / ;
{* writing header for payoff comparisons *}
put @28 ;
loop(N, put N.TL:>12 ) ;
put / ;
put "S=", ord(part):3:0 ;
put @10 ;
loop(N, put keySb(N):1:0) ;
loop(N,
    put uS(N):12:3;
) ;
put / ;
{* BEGIN LOOP OVER PLAYERS *}
loop(N1,
    {* INDIVIDUAL RATIONALITY *}
    if(uS(N1) GE uNC(N1),
        indrat(N1) = 1 ;
    ) ;
    {* INTERAL AND EXTERNAL STABILITY *}
    keyDb(N) = keySb(N) ;
    if(S(N1),
        {* INTERNAL STABILITY *}
        {* deviator opts out *}
        keyDb(N1) = 0 ;
    else
        {* EXTERNAL STABILITY *}
        {* deviator joins *}
        keyDb(N1) = 1 ;
    ) ;
    put "D=" ;
    {* locate pay off *}
    pos = 0 ;
    uDEV(N) =0 ;
    loop(part1$(NOT pos),
    found = YES ;
        loop(N,
            if(key(part1,N) NE keyDb(N),
                found= NO ;
```

```
        ) ;
    ) ;
    if(found ,
       uDEV(N) = dataS(part1,N) ;
       pos = ord(part1) ;
    ) ;
) ;
put pos:3:0 ;
put @10 ;
loop(N, put keyDb(N):1:0) ;
put @(ord(N1)*12+16) ;
put uDEV(N1):12:3 ;
uFR(N1)$S(N1) = uDEV(N1) ;
    if(uDEV(N1) EQ 0,
    if(S(N1),
        instab(N1) = 2 ;
    else
        exstab(N1) = 2 ;
    ) ;
     else
if(uS(N1) GE uDEV(N1),
    put "-" ;
else
    put "+" ;
    if(S(N1),
        instab(N1) = 0 ;
    else
        exstab(N1) = 0 ;
        {* NEW: EXCLUSIVE MEMBERSHIP *}
        winner(N) = 0 ;
        loop(N$S(N),
            if(uDEV(N) GE uS(N),
               winner(N) = 1 ;
            ) ;
       ) ;
       {* unanimity vote *}
        if(prod(N$S(N), winner(N)) EQ 1,
           exstabUNA(N1) = 0 ;
        ) ;
        {* majority vote *}
        if(sum(N$S(N), winner(N)) GE floor(card(S)/2)+1,
            exstabMAJ(N1) = 0 ;
        ) ;
        loop(N$S(N),
            put @(ord(N)*12+17) ;
            put uDEV(N):11:3 ;
            put$winner(N) "+" ;
            put$(NOT winner(N)) "-" ;
       ) ;
        put @(30+card(N)*12) ;
        put sum(N$S(N), winner(N)):1:0, "/", card(S):1:0 ;
    ) ;
) ;
) ;
put / ;
) ;
{* CLOSE LOOP OVER PLAYERS *}
loop(iter$(ord(iter) LE (28+card(N)*12)), put "+") ;
```

```
        put / ;
        loop(N,
            if(S(N) AND NOT instab(N),
                cntVIOLIS(N) = cntVIOLIS(N) + 1 ;
            ) ;
            if(NOT S(N) AND NOT exstab(N),
                cntVIOLES(N) = cntVIOLES(N) + 1 ;
            ) ;
        ) ;
        IR(part)     = prod(N$S(N), indrat(N)) ;
        IS(part)     = prod(N$S(N), instab(N)) ;
        ES(part)     = prod(N$(NOT S(N)), exstab(N)) ;
        IES(part)    = IS(part)*ES(part) ;
        ESUNA(part) = prod(N$(NOT S(N)), exstabUNA(N)) ;
        ESMAJ(part) = prod(N$(NOT S(N)), exstabMAJ(N)) ;
        put "IR =", IR(part):1:0 ;
        loop(N$S(N), put @(ord(N)*12+15) indrat(N):1:0) ;
        put / ;
        put "IS =", IS(part):1:0 ;
        loop(N$S(N), put @(ord(N)*12+15) instab(N):1:0) ;
        put / ;
        put "ES =", ES(part):1:0 ;
        loop(N$(NOT S(N)), put @(ord(N)*12+15) exstab(N):1:0) ;
        put / ;
        put "ESUNA =", ESUNA(part):1:0 ;
        loop(N$(NOT S(N)), put @(ord(N)*12+15) exstabUNA(N):1:0) ;
        put / ;
        put "ESMAJ =", ESMAJ(part):1:0 ;
        loop(N$(NOT S(N)), put @(ord(N)*12+15) exstabMAJ(N):1:0) ;
        put / ;
        put "IES=", IES(part):1:0 ;
        put / ;
        loop(iter$(ord(iter) LE (28+card(N)*12)), put "+") ;
        put / ;
        if(smin(N$S(N), uFR(N)) eq 0,
            PIS(part) = 2 ;
            put "PIS=X" ;
            else
            if(sum(N$S(N), uFR(N)) LE sum(N$S(N), uS(N)),
               PIS(part) = 1 ;
               put "PIS=1" ;
            else
               PIS(part) = 0 ;
               put "PIS=0" ;
            ) ;
        );
        put "     uFR=", sum(N$S(N), uFR(N)):12:3 ;
        put "     uS=", sum(N$S(N), uS(N)):12:3 ;
        put "     diff=", (sum(N$S(N), uS(N))-sum(N$S(N), uFR(N))):12:3 ;
        put / ;
        loop(iter$(ord(iter) LE (28+card(N)*12)), put "+") ;
        put // ;
);
{* CLOSE LOOP PARTITIONS *}

put // ;

***      CHECKING CORE STABILITY OF GRAND COALITION  ***************************
```

```
parameters
coal(N) coalition which is searched for
;

coal(N)=0;

scalar here one if the coalitions is here zero if not \0\;
scalar allhere one if all coalitions are here zero if not \1\;

allhere=1;

{* Checking if all coalitions are here *}
scalar dec0 number of possible coalitions in dec system;
scalar dec;

dec0=2**card(N)-1;

scalar i counter;
for (i = 0 to dec0,
    dec=i;
    loop (N,
        coal(N)=dec-2*floor(dec/2);
        dec= floor(dec/2);
    );
    here=0;
    loop(part,
             if (sum(N, abs(coal(N)-key(part,N))) eq 0, here=1);
    );
    allhere=allhere*here;
);

GR(part)  = 0 ;
loop(part,
    uS(N)     = dataS(part,N) ;
    keySb(N) = key(part,N) ;
    if(sum(N,uS(N)*keySb(N)) LE sum(N,uGC(N)*keySb(N)),
         GR(part) = 1 ;
    );
);

*** WRITING A SUMMARY ******************************************************

put "========== S U M M A R Y ==========" // ;
put// ;
put "Internal & external stability check for single coalitions" // ;

put "nr"      :>5,
    "key"     :>22,
    "IR"      :>8,
    "PIS"     :>8,
    "IS"      :>8,
    "ES"      :>8,
    "I&ES"    :>8,
    "ESUNA"   :>8,
    "I&ESUNA" :>8,
    "ESMAJ"   :>8,
    "I&ESMAJ" :>8,
```

```
{*    "PI&ESUN" :>8,
    "PI&ESMA" :>8 *} ;
put / ;
loop(iter$(ord(iter) LE 103), put "-") ;
put / ;
loop(part,
    put ord(part):5:0 ;
    put @(27-card(N)+1) ;
    loop(N,
        put key(part,N):1:0 ;
    );
    put IR(part)    :8:0 ;
    if (PIS(part) GT 1, put "X":>8 ; else put PIS(part):8:0 ; ) ;
    if (IS(part) GT 1, put "X":>8 ; else put IS(part):8:0 ; ) ;
    if (ES(part) GT 1, put "X":>8 ;else put ES(part):8:0 ; ) ;
    if (IES(part) GT 1, put "X":>8 ; else put IES(part):8:0 ; ) ;
    if (ESUNA(part) GT 1, put "X":>8 ; else put ESUNA(part):8:0 ; ) ;
    if ((ESUNA(part)*IS(part)) GT 1, put "X":>8 ;
    else put (ESUNA(part)*IS(part)):8:0 ; ) ;
    if (ESMAJ(part) GT 1, put "X":>8 ; else put ESMAJ(part):8:0 ; ) ;
    if ((ESMAJ(part)*IS(part)) GT 1, put "X":>8 ;
    else put (ESMAJ(part)*IS(part)):8:0 ; ) ;
    put / ;
) ;
loop(iter$(ord(iter) LE 103), put "-") ;
put / ;
put @28,
put sum(part$(IR(part) EQ 1), IR(part))   :8:0 ;
put sum(part$(PIS(part) EQ 1), PIS(part))  :8:0 ;
put sum(part$(IS(part) EQ 1), IS(part))   :8:0 ;
put sum(part$(ES(part) EQ 1), ES(part))   :8:0 ;
put sum(part$(IES(part) EQ 1), IES(part))  :8:0 ;
put sum(part$(ESUNA(part) EQ 1), ESUNA(part)):8:0 ;
put sum(part$(ESUNA(part)*IS(part) EQ 1), ESUNA(part)*IS(part)):8:0 ;
put sum(part$(ESMAJ(part) EQ 1), ESMAJ(part)):8:0 ;
put sum(part$(ESMAJ(part)*IS(part) EQ 1), ESMAJ(part)*IS(part)):8:0 ;
put / ;
loop(iter$(ord(iter) LE 103), put "-") ;
put /// ;

put "Core stability check" // ;
put "====================" // ;

if(allhere eq 1,
    put "all coalitions here" ;
    else
    put "coalitions are missing!!!!!!!!" ;
) ;
put / ;
if((card(part)- sum(part, GR(part))) eq 0,
    put "GR of grand coalitions satisfied" ;
    else
    put "GR of grand coalitions violated in " ;
    put (card(part)- sum(part, GR(part))) ;
    put "cases!!!" ;
    put /// ;
) ;
```

References

[1] T. Basar and G. J. Olsder. *Dynamic Noncooperative Game Theory.* Academic Press, 1982.

[2] W. J. Baumol and W. E. Oates. *The theory of environmental policy.* Camebridge University Press, 2nd edition, 1988.

[3] F. Bloch. *Non-Cooperative Models of Coalition Formation in Games with Spillovers.*

[4] T. Brechet, F. Gerard, and H. Tulkens. *Efficiency vs. stability of climate coalitions: a conceptual and computational appraisal.* CORE discussion paper No. 2007/3, 2007.

[5] P. Chander. *The γ-core and Coalition Formation.* Fondazione Eni Enrico Mattei, Working Paper No. 2003.94, October, 2003.

[6] P. Chander, J.-P. van Ypersele H.Tulkens, and S. Willems. *The Kyoto Protocol: An Economic and Game Theoretic Interpretation.* CESifo Working Paper No. 229, December, 1999.

[7] P. Chander and H. Tulkens. *The core of an economy with multilateral environmental externalities.* International Journal of Game Theory,Vol. 26, No. 3, pp. 379-401, October, 1997.

[8] P. Chander and H. Tulkens. *Cooperation, Stability and self-enforcement in international environmental agreements: a conceptual discussion.* CORE discussion paper No. 2006/03, 2006.

[9] R. H. Coase. *The problem of social cost.* Journal of Law and Economics, Volume 3, October, 1960.

[10] J. Eyckmans and C. Bertrand. *Integrated assessment of carbon and sulphur emissions, simulations with the CLIMNEG model.* Katholieke Universiteit Leuven, Working Paper Series No. 2000-8, 2000.

[11] J. Eyckmans, H. Tulkens editors B. Willams, J. Eyckmans, and S. Prost. *Optimal International Climate Agreements, collection Economic Aspects of Climate Change Policy).* acco, 2005.

[12] J. Eyckmans and M. Finus. *An almost ideal sharing scheme for coalition games with externalities.* ClimNeg Working paper 62, 2004.

[13] J. Eyckmans and H.Tulkens. *Optimal and Stable International Climate Agreements.*

[14] J. Eyckmans and H. Tulkens. *Simulating coalitionally stable burden sharing agreements for the climate change problem.* Resource and Energy Economics, Elsevier, vol. 25(4), pp. 299-327, October, 2003.

[15] J. Eyckmans, B. Willams, and J.P. van Ypersele. Climate change: Challenges for the world.

[16] G. Feichtinger and R.F. Hartl. *Optimale Kontrolle oekonomischer Prozesse - Anwendung des Maximumprinzips in den Wirtschaftswissenschaften.* Walter de Gruyter, 1986.

[17] M. Finus. *Game Theory and International Environmental Cooperation.* Edward Elgar Publishing, 2001.

[18] D.L. Kelly and C.D. Kolstad. *Integrated Assessment Models For Climate Change Control.* University of California Santa Barbara - Department of Economics 31-98, 1998.

[19] W. Nordhaus and Z. Yang. *A Regional Dynamic General-Equilibrium Model of Alternative Climate-Change Strategies.* The American Economic Review, Vol. 86, No. 4, pp. 741-765, September, 1996.

[20] Intergovernmental Panel on Climate Change. *4th Assessment Report - Climate Change 2007).* IPCC, 2007.

[21] G. Owen. *Game Theory.* Academic Press, 3rd edition, 2001.

[22] R. Rardin. *Optimization in Operations Research.* Prentice Hall, 1997.

[23] N. Stern et al. *Stern Review: The Economics of Climate Change.* HM Treasury, 2006.

[24] H. Wiesmeth. *Umweltökonomie.* Springer, 2003.