DISSERTATION

Robust Indirect Adaptive Fuzzy Control for a Class of Uncertain Nonlinear Systems

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Abstract

This thesis enhances the indirect adaptive fuzzy control approach in following directions: The proposed approach is less conservative concerning the stability of the plant zero dynamics and allows an optimal controller design, the attenuation of measurement noise and the inclusion of linear observers. An automatic control for a class of nonlinear uncertain MIMO systems is developed, which requires only less information about the controlled process. However, if there is any linguistic process model description from experts available this knowledge in form of fuzzy if-then rules can be included during the controller design. The robustness and performance of the proposed tracking control will be shown based on fuzzy model error compensation and the attenuation of external disturbances as well as measurement noise in their impact on the control error. A Lyapunov stability proof is stated which guarantees all included signals to be bounded and the fuzzy parameters convergence to their optimal values. Appropriate projection algorithms were chosen to restrict the parameters of the adaptive fuzzy systems within constraint sets. A SPR Lyapunov design approach allows the inclusion of linear state observers in the control concept and a dynamic fuzzy rule activation method remedies the phenomenon which is called the curse of dimensionality. Simulations of an inverse pendulum system and a magnetic levitation system were carried out to show the practicability and high performance of the proposed approach. A short discussion of the qualities and limitations of the proposed method concludes the thesis.

Kurzfassung

Diese Doktorarbeit erweitert die Möglichkeiten der Robusten Indirekt Adaptiven Fuzzy Regelungen in vielerlei Hinsicht: Die Konservativität des Verfahrens wird reduziert indem die betrachteten nichtlinearen MIMO Strecken auf jene Klasse der Eingangs-Ausgangs linearisierbaren Systeme erweitert wird bei der zum Nachweis der Stabilität der Nulldynamik nur das Anfahren von abgeschlossenen Mengen anstatt der restriktiven exponentiellen Stabilität erforderlich ist. Es bietet im weiteren die Möglichkeit der Einbringung von Optimalitätskriterien in den Reglerentwurf, der Unterdrückung des Messrauschens und der Einbringung von linearen Beobachtern in das Regelkonzept. Durch die Flexibilität und Lernfähigkeit des vorgeschlagenen adaptiven Verfahrens ist es im Gegensatz zu klassischen robusten Reglungen auch möglich dynamische Systeme mit einer vorgegebenen Genauigkeit automatisiert zu regeln bei denen sowohl die funktionelle Ausformung der statischen Nichtlinearitäten als auch deren Schranken unbekannt sind. Falls linguistische Beschreibungen des Prozessmodells in Form von Expertenwissen vorliegen, kann dieses Wissen durch Fuzzy Wenn-Dann Regeln beim Reglerentwurf eingebunden werden um dadurch die Performance der Regelung zusätzlich zu erhöhen. Die Robustheit und Performance der Folgereglung hinsichtlich der Kompensation von Fuzzy Approximierungsfehlern und der Unterdrückung der Auswirkungen von externen Störgrößen und des Messrauschens auf die Regelabweichung wird gezeigt. Die Stabilität des Verfahrens und die Parameterkonvergenz wird mit Hilfe des Ansatzes von speziellen Lyapunov-Funktionen nachgewiesen und es wird die Beschränktheit aller im Regelkonzept

vorkommenden Signale gezeigt. Dazu werden geeignete Projektionsalgorithmen implementiert, um die veränderlichen Parameter der linear parametrierten Adaptiven Fuzzy Systeme zu beschränken. Ein weiterer Ansatz, der strikt positiv reale MIMO Systeme berücksichtigt, garantiert bei der Einbindung von linearen Beobachtern in das Regelkonzept die Existenz von Lyapunov stabilen Lösungen. Die Implementierung einer Methode welche die Fuzzy-Regeln dynamisch aktiviert ermöglicht es, den Realisierungsaufwand hinsichtlich notwendiger Rechenleistung und erforderlichem Speicherbedarf deutlich zu senken und damit den sogenannten „Fluch der Dimensionalität", der beim Approximieren von MIMO Strecken durch Fuzzy Systeme auftritt, zu entschärfen. Die Anwendbarkeit des Verfahrens auf nichtlineare, elektromechanische Ein- und Mehrgrößensysteme wird anhand von Simulationen der Regelung eines Invertierten Pendels und einer Magnetschweberegelung aufgezeigt. Eine kurze Diskussion und Bewertung des Verfahrens schließen die Arbeit ab.

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Chapter 1

1 Introduction

1.1 Motivation

Control of Multi-Input Multi-Output (MIMO) nonlinear dynamic systems is one of the most challenging tasks for many control engineers, especially when the system is required to manoeuvre very quickly in an environment of uncertainties and imprecision. In the last few decades, adaptive control strategies have undergone rapid development leading to global stability and tracking results for a reasonably large class of nonlinear systems (*Sastry, S., Bodson, M., 1989,* Prentice Hall, *Slotine, J.J.E. and Li, W., 1991, Ioannou, P.A., and Sun, J., 1996*).

Conventional adaptive controllers based on nonlinear control laws can achieve fine control and compensate partially unknown system dynamics. However, their complexity always grows exponentially with the number of unknown parameters, which leads to heavy computational burden and hinders real-time applications (*Sastry, S., Bodson, M., 1989,* Prentice Hall). Although variable-structure control strategy using sliding mode is an effective way to deal with uncertainties in nonlinear systems, without modification the chattering phenomenon due to switching operations greatly affects the accuracy of the tracking performance. Moreover, in the design of a sliding-mode controller, mathematical models of the system and the bound of uncertainties need to be known in advance (*Ge, S.S., et al., 1998*). Hence, there is a need for model-free control strategies with learning and adaptive ability.

In the last few decades, much research effort has been directed towards design of intelligent controllers using fuzzy logic and neural networks. Fuzzy Inference Systems (FISs) and Artificial Neural Networks (ANNs) are basically model-free estimators and dynamical systems. They share the common capability of improving the intelligence of a system, working in an uncertain and imprecise environment. To a certain extent, both techniques have been proven to be very powerful in the discipline of system modelling and control, especially when the controlled system is hard to be modelled mathematically, or when the controlled system has large uncertainties and strong nonlinearities. Therefore, fuzzy logic and neural networks have been greatly adopted in model-free adaptive control of nonlinear systems (*Zadeh, L.A., 1965, Takagi, T., Sugeno, M., 1985, Wang, L.X., 1994*).

Although FISs and ANNs are formally similar, there are significant differences between them. FISs are structured numerical estimators (*Lin, C.T., 1994*). They are derived from highly formalized insights about the structure of categories found in the real world and articulated from fuzzy logic rules as a kind of expert knowledge. They base their decisions on inputs in the form of linguistic variables derived from membership functions which are formulas used to determine the fuzzy set to which a value belongs and the degree of membership in that set. The variables are then matched with the preconditions of linguistic fuzzy logic rules, and the response of each rule is obtained through fuzzy implication. To perform compositional rule of inference, the response of each rule is weighted according to the confidence or degree of membership of its inputs, and the centroid of the responses is calculated to generate the appropriate output. Currently, a major research problem concerning fuzzy logic systems is how to determine the fuzzy logic rules and membership functions systematically.

The ANNs, on the other hand, are trainable dynamical systems (*Lin, C.T., 1994*) whose learning, noise tolerance, and generalization abilities grow out of their connectionist structures, their dynamics, and their distributed data representation. ANNs have a large number of highly interconnected processing nodes (neurons) which have the ability to learn and generalize from training patterns or data. Like human beings, they can perform pattern-matching tasks. Currently, due to the fact that the internal layers of ANNs play an extremely important role, the research on ANNs focuses on the determination of the structure and the size of a network.

Fuzzy logic systems, e.g. the fuzzy logic systems proposed in this thesis with center average defuzzifier, product-inference rule, singleton fuzzifier and Gaussian membership functions, can be represented as a three-layer feedforward network. Therefore, adaptive fuzzy systems are a promising approach of getting both, the benefits of FISs and ANNs and solving their respective problems. An adaptive fuzzy system is a fuzzy logic system equipped with a training algorithm, where the fuzzy logic system is constructed from a collection of fuzzy if-then rules, and the training algorithm adjusts the parameters of the fuzzy logic system based on numerical input-output pairs. Conceptually, adaptive fuzzy systems combine linguistic and numerical information in the following way. Because fuzzy logic systems are constructed from fuzzy if-then rules, linguistic information (in form of fuzzy if-then rules) can be directly incorporated; on the other hand, numerical information (in form of input-output pairs) is incorporated by training the fuzzy logic system to match the input output pairs. The most fundamental difference between adaptive fuzzy systems instead of ANNs is that the former takes linguistic information explicitly into consideration and makes use of it in a systematic way, whereas the latter does not.

The theme of this research revolves around such needs of developing an adaptive intelligent modelling- and control system, which can cope with very complex uncertain nonlinear systems. For modelling and control of a class of input-ouput linearizable uncertain nonlinear systems I propose the use of adaptive fuzzy systems. The applied adaptive fuzzy systems are incorporating two different rule sets - one rule set in form of linguistic fuzzy if-then rules for FIS initialization and a second rule set including (within certain limits) arbitrary parameterized fuzzy sets - whose parameters (of both rule sets) are then adapted by an appropriate training algorithm. Even if no linguistic descriptions are available for FIS initialization the proposed control scheme is still able to perform a good tracking control. Hence, an automatic control for a class of nonlinear uncertain MIMO systems is developed which requires only less information about the controlled process. To maximize the degree of flexibility in the context of modelling unknown plant nonlinearities, an indirect (model based) adaptive fuzzy control scheme was chosen as basis for this development. The applied adaptive fuzzy systems consist of linear parameterised Fuzzy Basis Functions as proposed in (*Wang, L.X., 1994*) to guarantee the compatibility to linguistic information of human experts in form of fuzzy if-then rules, the ability to approximate any given nonlinear function to arbitrary accuracy and to develop simple adaptive laws which support a fast parameter convergence and online-learning. However, beside the plant estimation also the control aspect is accommodated in this thesis. Concepts of classical control schemes like Variable Structure Systems (VSS) control, H_∞ Control, Optimal Control and Observer-based Control are incorporated in the development of the robust indirect adaptive fuzzy control. A Lyapunov stability proof is stated which guarantees in combination with appropriate projection algorithms that all included signals are bounded as well as the fuzzy parameters convergence to their optimal values, respectively. Finally, to consider practical limitations of processor power and storage capacity in real-time implementations a dynamic rule activation method is proposed to reduce the number of active fuzzy rules. Summarized, the robust indirect adaptive control scheme which is introduced in this thesis integrates several concepts, each with beneficial properties, to enable a superior tracking control for a class of uncertain nonlinear SISO and MIMO plants. Simulations of an inverse pendulum system and a magnetic levitation system were carried out to show the practicability and high performance of the proposed approach.

1.2 Major Contributions of the Thesis

In this thesis a Robust Indirect Adaptive Fuzzy Controller (RIAFC) is proposed for tracking control of a class of technical input-output linearizable, nonlinear SISO and MIMO plants which contain large uncertainties. Moreover, in a second step the RIAFC concept is extended by a linear state observer to build a novel Observer-based Robust Indirect Adaptive Fuzzy Control (ORIAFC). The thesis integrates several concepts, each with beneficial properties, to enable a superior tracking control and enhances the indirect adaptive fuzzy control approach in following directions:

- The proposed approach is less conservative concerning bounded tracking in minimum-phase systems. Instead of calling for exponential stability, a strong form of stability, the RIAFC concept weakens that restriction and expands the class of controllable plants to systems with exponentially attractive zero dynamics.

- Most of the adaptive fuzzy control schemes proposed in the literature do not take in account the influence of measurement noise on the performance of the tracking control. During the development of the RIAFC and the ORIAFC the impact of measurement noise on the performance of the control is explicitly derived. Moreover, with the help of the ORIAFC this impact can be significantly attenuated.

- The RIAFC and the ORIAFC schemes guarantee that all signals involved are bounded, and provide the fuzzy modelling error cancellation by a VSS control term and the bounded external disturbances as well as measurement noise attenuation with H_∞ performance, obtained by a Riccati-like equation. A rigorous Lyapunov stability proof is stated to show the stability of both schemes.

- Linear parameterized adaptive fuzzy systems based on Fuzzy Basis Functions are applied to guarantee the ability of incorporating linguistic information of human experts for FIS initialization and to develop simple adaptive laws which support a fast parameter convergence and online-learning. Special projection algorithms are chosen to restrict the parameters of the adaptive fuzzy systems within constraint sets.

- The existence of stable solutions for the ORIAFC is shown by a Strict Positive Real Lyapunov (SPR-Lyapunov) design approach, based on the famous Kalman-Yakubovich-Popov lemma. A proof for the MIMO case seems to be unique in H_∞ based adaptive fuzzy control literature.

- In comparison to many other adaptive fuzzy control schemes the ORIAFC does neither require that all system states are available for measurement, nor the measured output signals or the reference trajectories to be smooth, necessarily.

- In accompany with increasing complexity of the fuzzy systems also the burden for computation and storage effort grows dramatically. Especially concerning the hardware realisation of fuzzy controls, where processor power and storage capability is limited, the so called "curse of dimensionality" is a critical point in the design of fuzzy controls. By applying a Dynamic Fuzzy Rule Activation Method this phenomenon can be significantly weakened.

Simulation results of an SISO inverted pendulum system and a MIMO magnetic levitation system demonstrate the effectiveness and robustness of the proposed ORIAFC. In particular, the MIMO magnetic levitation simulation is considering several hardware specifications and constraints of an approved real-time experiment and is therefore very close to machine level.

1.3 Outline of the Thesis

This thesis is organized in seven chapters, each of the chapters is devoted to a particular sub issue. Much of the material of this thesis is derived from my publications in IJAA (*Jambrich, G., 2004, Jambrich, G., 2005*). A summary of the content of each chapter is given here:

- Chapter 1 presents motivations as well as contributions of the thesis and gives a brief outline of each chapter in this thesis.

- Chapter 2 introduces briefly the definitions of fuzzy logic, fuzzy sets, fuzzy if-then rules and fuzzy inference systems. In this chapter the relevance of FIS in and beyond control engineering is shown, and several examples of practical applications are listed. The popular FIS-types aree described in detail and compared with each other. In particular, the advantages of Fuzzy Basis Function based FIS are highlighted.

- Chapter 3 gives an overview of the field of adaptive fuzzy control. In this chapter a classification scheme is stated within the majority of adaptive fuzzy control architectures, inclusive the Robust Indirect Adaptive Fuzzy Controller developed in chapter 5, can be arranged. The qualities and limitations of these architectures are presented and on the basis of these observations guidelines for the design of an adaptive fuzzy control are derived. Also the transitions to Chapter 5 are pointed out.

- Chapter 4 describes how a class of input-affine, general nonlinear systems can be linearized exactly. By first applying a coordinates transformation to it, and second linearizing the system in canonical normal form via special control laws. If the sum of relative degrees of the subsystems is equal to the system order the above described transformations lead to a linear system, which can be stabilized by classical linear controllers. If the sum of relative degrees of the subsystems is less than the system order also the internal dynamics of the plant have to be considered. In general, systems with stable zero dynamics are assumed to guarantee the practicability of the input-output linearization method.

- Chapter 5 represents the kernel of this thesis. This Chapter describes the design of the robust indirect adaptive fuzzy control scheme for MIMO nonlinear dynamic systems based on the observations of Chapters 1-4, derives the Robust Indirect Adaptive Fuzzy Controller (RIAFC) by applying input-output linearization combined with H_∞- and VSS control techniques, and presents the convergence- and stability analysis of the RIAFC. Moreover, the RIAFC is extended by a linear state observer to build an Observer-based Robust Indirect Adaptive Fuzzy Control (ORIAFC). To guarantee the existence of solutions for the observer-based control, a SPR Lyapunov Design Approach is introduced. Modified Adaptation Laws are defined to maintain the adaptive parameters of the adaptive FISs inside predefined constraint sets. Finally a Dynamic Rule Activation Method is proposed to remedy the phenomenon which is known as “the curse of dimensionality”. Comparisons with some existing adaptive fuzzy controllers are also carried out to further demonstrate its superior performance.

- Chapter 6 presents simulation examples to show the practicability and high performance of the algorithms developed in Chapter 5. First the ORIAFC is applied to control an inverted pendulum (SISO model) to track a given reference trajectory. Second, the positions of magnet disks of a MIMO magnetic levitation system are controlled by the RIAFC, and for comparison the ORIAFC is applied to control the MIMO magnetic levitation system.

- Chapter 7 concludes the thesis with a short discussion of the qualities and limitations of the proposed method and suggests directions for further research.

Chapter 2

2 Review of Fuzzy Logic and Fuzzy Inference Systems

During the last few decades, Fuzzy Inference Systems (FISs) have emerged as one of the most active and fruitful areas for research in the application of fuzzy set theory. Fuzzy logic has found a variety of applications in various fields ranging from industrial process control to medical diagnostics and securities trading. Most notably, FISs have been applied to control nonlinear, time-varying, ill defined systems, and systems whose dynamics are not exactly known such as servomotor position control and robot arm control systems. Hybrid architectures like Fuzzy-PI / Fuzzy-PD / Fuzzy-PID controllers were proposed which can provide better results than the conventional non-fuzzy controllers alone (*Yao, L., Lin, et al., 2002*). Furthermore, FIS have been applied to complex decision making or diagnostic systems (*Zadeh, L.A., 1965, Takagi, T., Sugeno, M., 1985, Wang, L.X., 1994*). FIS have strong foundation in mathematical theory. Combining multi-valued logic, probability theory and artificial intelligence, it is a control/decision methodology that simulates human thinking by incorporating imprecision which is inherent in all physical systems. In general, the FIS is best applied to nonlinear, time-varying, ill-defined systems which are too complex for conventional control theory to be applied. The FIS is a model-free estimator and it deals with the relationship of the output to the input, lumping many parameters together.

2.1 Fuzzy Logic

Fuzzy logic first proposed by Lotfi Zadeh in 1965 (*Zadeh, L.A., 1965*) is primarily concerned with the representation of the sort of imprecise knowledge which is common in natural systems. It facilitates representations in digital computers of some kind of knowledge through the use of fuzzy sets. On this basis, fuzzy logic uses logical operators to collate and integrate the knowledge in order to approximate the kind of reasoning common in natural intelligence.

A Fuzzy Inference System (FIS) is a computation framework based on the concepts of fuzzy sets, fuzzy if-then rules and fuzzy reasoning. FISs are known by other names such as fuzzy rule-based systems, fuzzy models or simply fuzzy systems. The essential part of the FIS is a set of linguistic rules related by the dual concepts of fuzzy implication and the compositional rule of inference. Intrinsically, the FIS provides an algorithm, which can convert the linguistic rules based on expert knowledge into an automatic control action. Many experiments have shown that FISs yield results far more superior to those obtained by conventional approaches. In particular, the methodology of FISs appears very useful when the processes are too complex for analysis by conventional quantitatively techniques or when the available sources of information are interpreted qualitatively, inexactly or uncertainly. Thus, FISs may be viewed as a step towards a rapprochement between conventional precise mathematical paradigms and human-like decision making (*Wang, L.X., 1994, Passino, K.M. and Yurkovich, S.,1998)*.

2.2 Fuzzy Set

Conventional set theory is based on the premise that an element either belongs to or does not belong to a given set. Fuzzy set theory takes a less rigid view and allows elements to have degrees of membership of a particular set such that elements are not restricted to either being in or out of a set but are allowed to be "somewhat" in. In many cases, this is a more natural approach. For example, consider the case of describing the atmospheric temperature as being "hot". If one was to express this concept in conventional set theory, one would be forced to designate a distinct range of temperatures, such as 30°C and over, as belonging to the set *hot*, i.e. *hot*=[30,∞)°C. This seems contrived because any temperature which falls just slightly outside this range would not be a member of the set, even though a human being may not be able to distinguish between it and one which is just inside the set.

In fuzzy set theory, a precise representation of imprecise knowledge is not enforced since strict limits of a set are not required to be defined, instead a *membership-function* is defined. A membership function describes the relationship between a variable and the degree of membership of the fuzzy set that corresponds to particular values of that variable. This degree of membership is defined in terms of a number between 0 and 1, inclusive, where 0 implies total absence of membership, 1 implies complete membership, and any value between implies partial membership of the fuzzy set. This may be written as $mf(x)\in[0\ \ 1]$ for $x\in U$, where U is the *universe of discourse* which defines the total range of interest over which the variable x should be defined.

For example, to define membership of the fuzzy set, *hot*, a function which rises from 0 to 1 over the range 25°C to 35°C may be used, i.e.

$$mf(x)=\begin{cases} 0 & x<25°C \\ 1+\dfrac{x-35}{10} & 25\le x\le 35°C \\ 1 & x>35°C \end{cases}. \tag{2-1}$$

This implies that 20°C is not hot, 27°C is a bit hot, 30°C is quite hot, and 40°C is truly hot. Specific measurable values, such as 27, 30 and 40, are often referred to as *crisp* values of *fuzzy singletons*, to distinguish them from *fuzzy* values, such as *hot*, which are defined by a fuzzy set. Fuzzy values are sometimes also called *linguistic* values. This definition is more reflective of human or linguistic interpretations of temperatures and hence better approximates such concepts.

Two most commonly used membership functions are:

- Triangular Membership Function

$$mf(x) = \begin{cases} 1 - \dfrac{|x-c|}{\sigma} & if\ |x-c| \le \sigma \\ 0 & otherwise \end{cases}. \quad (2\text{-}2)$$

- Gaussian Membership Function

$$mf(x) = exp\left[-\frac{(x-c)^2}{\sigma^2} \right], \quad (2\text{-}3)$$

where c and σ are called the center and width of the fuzzy set respectively. The membership function shapes are shown in Fig. 2-1.

While seeming imprecise to a human being, fuzzy sets are mathematically precise in that they can be fully represented by exact numbers. They can therefore be seen as a method of tying together human and machine knowledge representation. Given that such a natural method of representing information in a computer exists, information processing methods can be applied to it by the use of FISs.

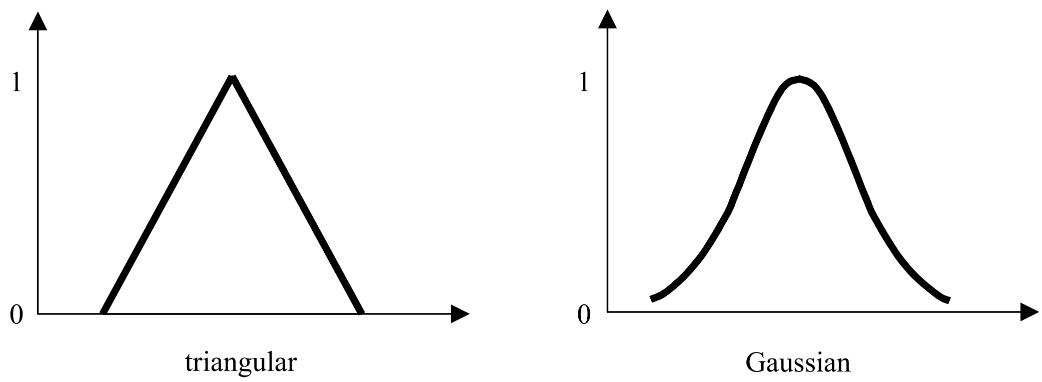

Fig. 2-1: Two common fuzzy membership function shapes

2.3 Fuzzy If-Then Rules

FISs are essentially rule-based expert systems, which comprise a collection of rules. Each rule defines a desired action when a particular combination of fuzzy values occurs. The rules are defined as "if-then" logical expressions:

rule^j : if (x_i is F_i^j) then (y_k is G_k^j) (2-4)

where x_i : $i=1...N_i$ and y_k : $k=1...N_o$ are input an output linguistic variables respectively, F_i^j : $i=1...N_i$, $j=1...N_r$ and G_k^j : $k=1...$ N_o, $j=1...N_r$ are linguistic variables or labels of fuzzy sets characterized by appropriate membership functions $mf_i^j(x_i)$ and $mf_k^j(y_k)$ respectively. An example of the fuzzy if-then rule that describes a simple fact is

if (*pressure* is *high*) then (*volume* is *small*) (2-5)

where *pressure*, *volume*, *high* and *small* are all linguistic variables. Besides that, *high* and *small* are also labels of fuzzy sets that are characterized by membership functions.

Since the output linguistic variables of a Multi-Input Multi-Output (MIMO) system are independent, a MIMO FIS can be represented as a collection of Multi-Input Single-Output (MISO) FISs by decomposing the above rule into N_o sub-rules with G_k: $k=1...N_o$ as the single consequence of the *k*th sub-rule (*Wang, L.X., 1994*). For notational simplicity, the MISO FISs is considered in the rest of the paper.

Another form of fuzzy if-then rules has fuzzy sets direclty involved only in the premise part. This form of fuzzy if-then rules can be categorized into two models, namely Simplified Model and Takagi-Sugeno-Kang model.

Simplified Model (S-model): In the S-model, a fuzzy singleton is used for the output (*Wang, L.X., 1994*), i.e.

$$\text{rule}^j: \ \text{if } (x_i \text{ is } F_i^j) \text{ then } (y \text{ is } C^j) \qquad (2\text{-}6)$$

where C^j is a fuzzy singleton (center of the fuzzy set G^j) .

A general form of the S-model-type fuzzy if-then rules, including more than one input linguistic variable, is the following:

$$\text{rule}^j: \ \text{if } (x_1 \text{ is } F_1^j) \text{ and } \cdots \text{ and } (x_{N_i} \text{ is } F_{N_i}^j) \text{ then } (y \text{ is } C^j)\,. \qquad (2\text{-}7)$$

Takagi-Sugeno-Kang model (TSK-model): The TSK model was proposed by Takagi, Sugeno and Kang (*Takagi, T., Sugeno, M., 1985*) in an effort to develop a systematic approach for generating fuzzy rules from a given input-output data set. The typical model output is a linear combination of the input variables, i.e.

$$\text{rule}^j: \ \text{if } (x_i \text{ is } F_i^j) \text{ then } (y \text{ is } k_0^j + k_1^j x_1 + \ldots + k_{N_i}^j x_{N_i}) \qquad (2\text{-}8)$$

where k_i^j: $i=0,1...N_i$, $j=1...N_r$ are real-valued parameters. If no input variables are considered in the consequence part, the TSK-model is exactly the same as the S-model.

Therefore, the S-model can be considered as a special case of the TSK-model. Experiments show that the TSK-model has advantages like (*MathWorks Inc., Manual of Fuzzy Logic Toolbox*): computational efficiency, compatibility with linear, adaptive and optimization techniques, and continuity of output surface. However, a big weak point of this fuzzy logic system is that the *then* part of the rule is not fuzzy; therefore, it does not provide a natural framework to incorporate fuzzy rules from human experts.

Both types of fuzzy if-then rules have been extensively used in both modelling and control. Through the use of linguistic labels and membership functions, a fuzzy if-then rule can easily capture the spirit of a "rule of thumb" used by human beings (*Jang, J.S.R., et al., 1997*). From another point of view, due to the qualifiers on the premise parts, each fuzzy if-then rule is actually a local description of the system under consideration as illustrated in Fig. 2-2, (*Zadeh, L.A., 1965*). On the contrary, conventional approaches of system modelling operate on the entire scope to find global functional or analytical structure of a nonlinear system.

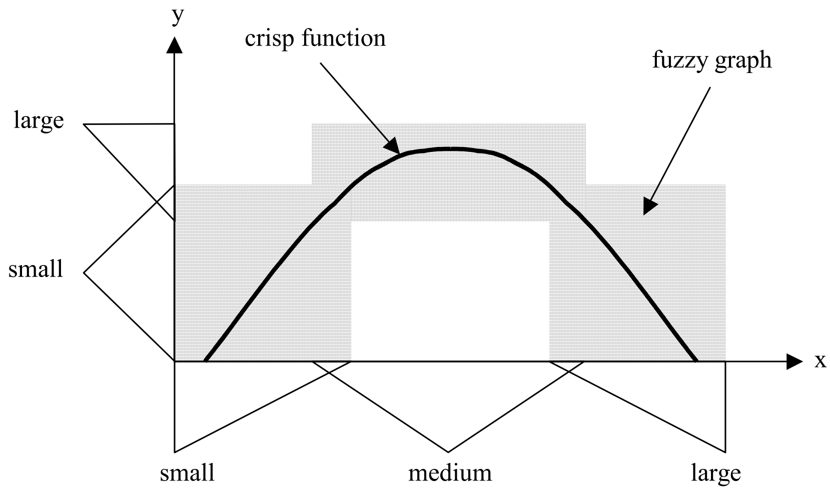

Fig. 2-2: Fuzzy approximation of a nonlinear system

2.4 Fuzzy Inference Systems (FISs)

A FIS (see Fig. 2-3) can be defined as a system which transforms or maps one collection of fuzzy or crisp values to another collection of fuzzy or crisp values. This mapping process is performed by five parts:

- Fuzzifier – Converts a set of crisp variables into a set of fuzzy variables to enable the application of logical rules.

- Rule Set – Stores a set of logical if-then rules defined on the fuzzy variables.

- Label Set – Stores a set of membership functions of fuzzy rules used in the rule set.

- Inference Mechanism – An algorithm which is used for calculating to which extent each rule is activated for a given input pattern. The combination of the rule set, the label set and the inference mechanism may be described as the *reasoning* of the FIS.

- Defuzzifier – Converts a set of fuzzy variables into crisp values in order to enable the output of the FIS to be applied to another non-fuzzy system. If a crisp output is not required then defuzzification is not necessary.

The steps of fuzzy reasoning, i.e. inference operations upon fuzzy if-then rules, performed by FISs are:

- Compare the input variables with the membership functions on the premise part to obtain the membership values or compatibility measures of each linguistic label. This step is often called fuzzification.

- Combine (through a specific T-norm operator, usually multiplication or minimum) the membership values of the premise part to get firing strength of each rule.

- Generate the qualified consequence (either fuzzy or crisp) of each rule depending on the firing strength.

- Aggregate the qualified consequence to produce a crisp output. This step is called defuzzification.

Several types of fuzzy reasoning have been proposed in the literature (*Takagi, T., Sugeno, M., 1985, Lee, C., 1990*). Depending on the type of fuzzy reasoning, most FISs can be classified into the following four types.

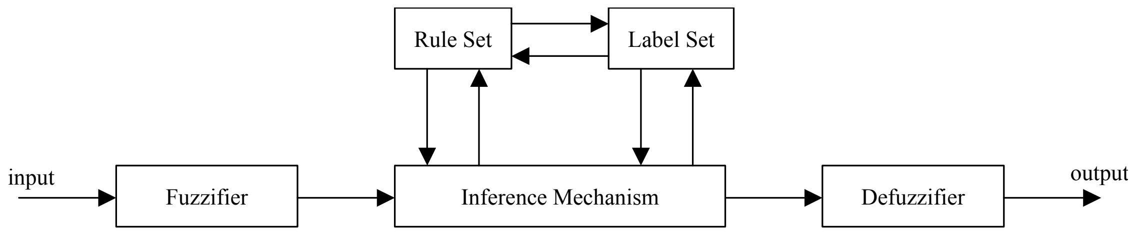

Fig. 2-3: Fuzzy Inference System

2.4.1 Tsukamoto-type FIS

In the Tsukamoto-type FIS (*Tsukamoto, Y., 1979)*, the overall output y is the weighted average of the *j*th rule's crisp output, w^j induced by the *j*th rule's firing strength, f^j (the *product* or *minimum* of the degree of match with the premise part) and output monotonic membership functions $mf^j(y)$, i.e.

$$y = \frac{\sum_{j=1}^{N_r} f^j w^j}{\sum_{j=1}^{N_r} f^j} \qquad (2\text{-}9)$$

where f^j is calculated by the T-norm operation, e.g.

- Intersection:

$$f^j = min[mf_1^j(x_1)\, mf_2^j(x_2) \ldots mf_{N_i}^j(x_{N_i})] \tag{2-10}$$

- Algebraic Product:

$$f^j = \prod_{i=1}^{N_i} mf_i^j(x_i)\,. \tag{2-11}$$

Fig. 2-4 illustrates the reasoning procedure for a two-input two-rule system. Since each rule infers a crisp output, the Tsukamoto-type FIS aggregates each rule's output by the method of weighted average and thus avoids the time-consuming process of defuzzification. However, the Tsukamoto-type FIS is not used often since it is not as transparent as the FIS-types which are introduced next. Since the reasoning mechanism of the Tsukamoto-type FIS does not follow strictly the compositional rule of inference, the output is always crisp even when the inputs are fuzzy.

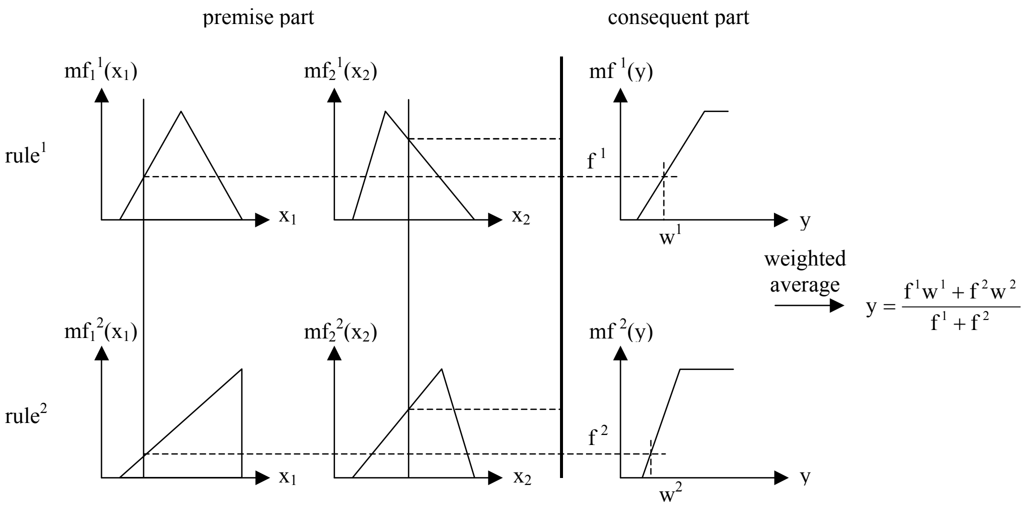

Fig. 2-4: Tsukamoto-type fuzzy reasoning

2.4.2 Mamdani-type FIS

In the Mamdani-type FIS (*Mamdani, E.H., Assilian, S.,1975*), the overall fuzzy output $\mu(y)$ is derived by applying a *maximum* operation to the qualified fuzzy output (each of which is equal to the minimum of firing strength f^j and the output membership function of each rule $mf^j(y)$). Fig. 2-5 shows the fuzzy reasoning procedure for a two-input two-rule Mamdani-type FIS. Various defuzzification schemes have been proposed to choose the final crisp output y based on the overall fuzzy output $\mu(y)$; some of them are *centroid of area* (COA), *mean of maximum* (MOM), *bisector of area* (BOA), *maximum criterion*, etc. (*Lee, C., 1990, Jang, J.S.R., et al., 1997, Mamdani, E.H., Assilian, S.,1975*). The COA method is the most widely adopted defuzzification strategy, which is reminiscent of the calculation of expected values of probability distributions, i.e. the crisp output y is

$$y = \frac{\int \mu(y)\, y\, dy}{\int \mu(y)} . \tag{2-12}$$

The calculation needed to carry out any of the above-mentioned defuzzification operations is time-consuming unless special hardware support is available (*Jang, J.S.R., et al., 1997*). Furthermore, these defuzzification operations are not easily subject to rigorous mathematical analysis, so most of the studies are based on experimental results. This leads to the propositions of other types of FISs that do not need defuzzification at all, such as Tsukamoto, TSK and Fuzzy Basis Function (FBF)-type FISs.

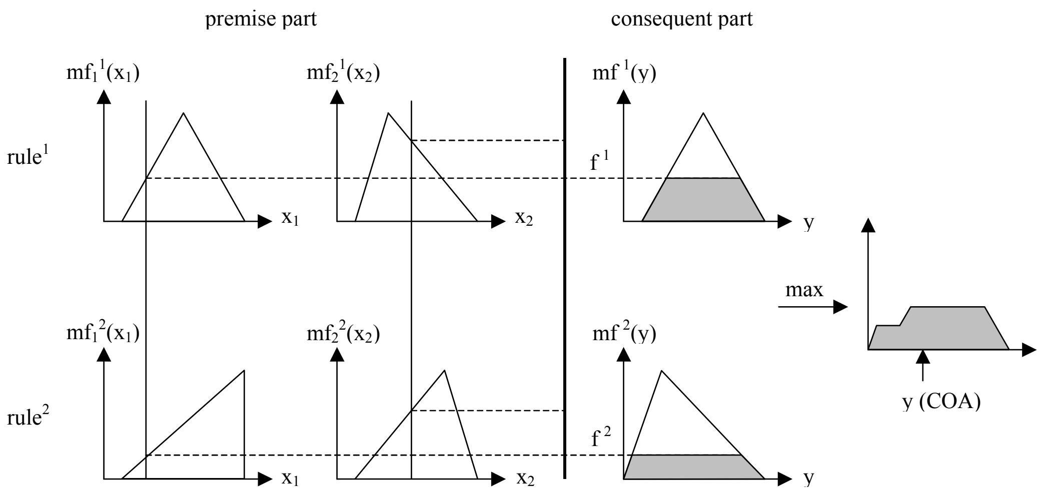

Fig. 2-5: Mamdani-type fuzzy reasoning

2.4.3 TSK-type FIS

TSK-model fuzzy if-then rules (*Takagi, T., Sugeno, M., 1985*), which are described in Section 2.3, can be used to implement FISs. The output of each rule is a linear combination of input variables plus a constant term, and the final crisp output, y is the weighted average of each rule's output, w^j, i.e.

$$y = \frac{\sum_{j=1}^{N_r} f^j w^j}{\sum_{j=1}^{N_r} f^j} \tag{2-13}$$

where the firing strength f^j is calculated according to (2-11) and w^j is computed as

$$w^j = k_0^j + k_1^j x_1 + \ldots + k_{N_i}^j x_{N_i} . \tag{2-14}$$

Fig. 2-6 shows the fuzzy reasoning procedure for a two-input two-rule TSK-type FIS. Since each rule has a crisp output, the overall output is obtained via *weighted average*, thus avoiding the time-consuming process of defuzzification required in a Mamdani-type FIS. In practice, the *weighted average* operator is sometimes replaced with the *weighted sum* operator (i.e. $y=\sum_{j=1}^{N_r} f^j w^j$ in (2-13)) to reduce computation further, especially in the training of a FIS. As mentioned in Section 2.3, a crucial drawback of this fuzzy logic system is that the *then* part of the rule is not fuzzy; therefore, it does not provide a natural framework to incorporate fuzzy rules from human experts.

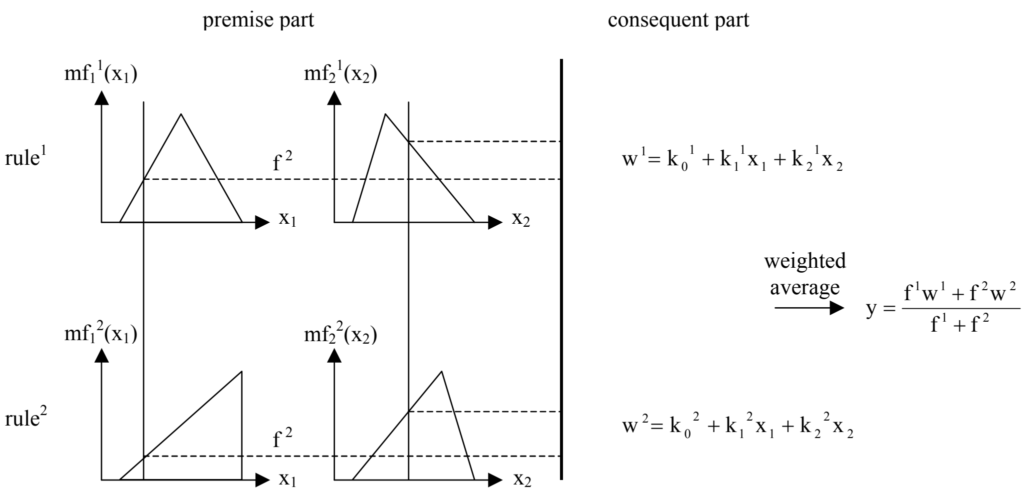

Fig. 2-6: TSK-type fuzzy reasoning

2.4.4 Fuzzy Basis Function Expansions

Fuzzy Basis Function Expansions were introduced by Wang (*Wang, L.X*, *1992, Wang, L.X., Mendel, J.M., 1992*, *Wang, L.X.*, *1993, Wang, L.X., 1994, Wang, L.X.,1998*). His work in the last decade of the past century was formative for the following research in the field of adaptive fuzzy systems and control and has numerously been reflected in the fuzzy control literature.

Definition 2-1: *The fuzzy logic systems with center average defuzzifier, product-inference rule and singleton fuzzifier are of the following form* (*Wang, L.X., 1994*)*:*

$$y=\frac{\sum_{j=1}^{N_r} f^j w^j}{\sum_{j=1}^{N_r} f^j}=\frac{\sum_{j=1}^{N_r}\left(\prod_{i=1}^{N_i} mf_i^j(x_i)\right)w^j}{\sum_{j=1}^{N_r}\left(\prod_{i=1}^{N_i} mf_i^j(x_i)\right)} \tag{2-15}$$

where the firing strength f^j is calculated according to (2-11) and w^j is the point at which $mf^j(y)$ achieves its maximum value and we assume that $mf^j(w^j)=1$.

Furthermore, the $mf_i^j(x_i) = a_i^j\, exp(-\frac{1}{2}(\frac{x - \bar{x}_i^j}{\sigma_i^j})^2)$ *are assumed to be Gaussian membership functions and* a_i^j, $\bar{x}_i^j$, σ_i^j *and* w^j *are adjustable design parameters with the constraints* $w^j \in V$, $a_i^j \in (0,1)$, $\bar{x}_i^j \in U_i$, *and* $\sigma_i^j > 0$. *U_i and V are closed sets, called universes of discourse in input and output space. We assume that* $a_i^j = 1$ *because intuitively we can assume that* mf_i^j *achieves 1 at some point.*

■

Theorem 2-1: *Universal Approximation Theorem (UAT) [Wang]. For any given real continuous function g on a compact set* $U \subset R^n$ *and arbitrary ε>0, there exists a fuzzy logic system f in form of (2-15) such that*

$$\sup_{\boldsymbol{x} \in U} \left| f(\boldsymbol{x}) - g(\boldsymbol{x}) \right| < \varepsilon . \qquad (2\text{-}16)$$

For proof and the extension to discrete functions refer also to (*Wang, L.X., 1994*).

■

Remarks 2-1: to UAT (*Wang, L.X., 1994*):

- This theorem provides a justification for applying the fuzzy logic systems to almost any nonlinear modelling problems. It also provides an explanation for the practical successes of the fuzzy logic systems in engineering applications.
- This theorem is just an existence theorem; that is, it shows that there exists a fuzzy system (2-15) that can uniformly approximate any given function to arbitrary accuracy. How to find such a fuzzy logic system is another question. Although we use this theorem as one justification for using the fuzzy logic systems, the importance of this theorem should not be overemphasized because many other types of functions are also universal approximators, including the simple polynomials. What should be emphasized is the capability of the fuzzy logic systems to incorporate linguistic information in a natural and systematic way – a unique advantage of the fuzzy logic systems which is not shared by other types of universal approximators, including polynomials, neural networks and so on.

■

Definition 2-2: *Define Fuzzy Basis Functions (FBF) as*

$$\varphi_j = \frac{\prod_{i=1}^{N_i} mf_i^j(x_i)}{\sum_{j=1}^{N_r} \prod_{i=1}^{N_i} mf_i^j(x_i)} . \qquad (2\text{-}17)$$

Then the fuzzy logic system (2-15) is equivalent to an FBF expansion

$$y = \sum_{j=1}^{N_r} \varphi_j \theta_j \qquad (2\text{-}18)$$

where $\theta_j = w^j$ *are constants.*

■

Fig. 2-7 shows the fuzzy reasoning procedure for a two-input two-rule FBF expansion-type FIS.

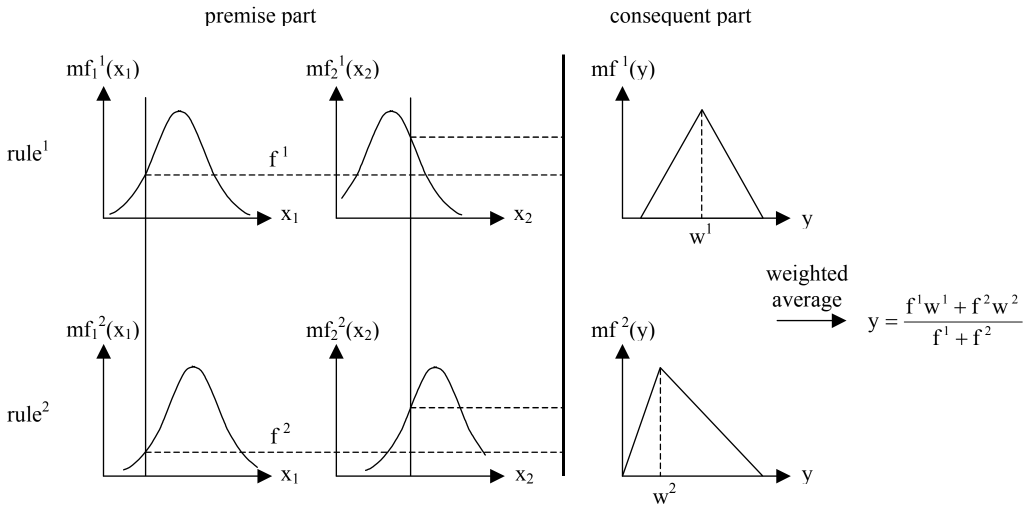

Fig. 2-7: FBF expansion-type fuzzy reasoning

From (2-18) and (2-7) we see that an FBF expansion corresponds to a fuzzy if-then rule. Specifically, an FBF for a rule can be determined as follows: first, calculate the product of all membership functions for the linguistic terms in the if part of the rule, and call it a *pseudo* FBF for the rule; then after calculating the pseudo-FBFs for all N_r rules, the FBF for the *j*th rule is determined by dividing the pseudo-FBF for the *j*th rule by the sum of all the N_r pseudo-FBFs. An FBF can either be determined based on a given linguistic rule as previously, or generated based on a numerical input-output pair (*Wang, L.X., 1994*).

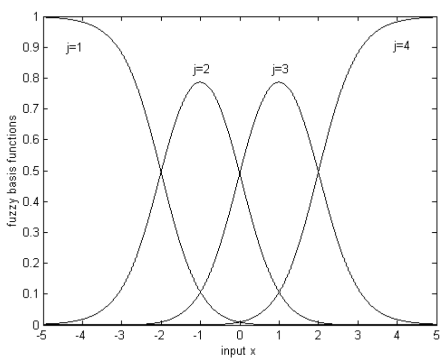

Fig. 2-8: An example of Fuzzy Basis Functions

We consider now a simple one-dimensional example of Fuzzy Basis Functions (that is, $N_i=1$). Suppose that we have four fuzzy rules in form of (2-7) with $mf^j(x) = exp(-\frac{1}{2}(x-\bar{x}^j)^2)$, $\bar{x}^j = -3, -1, 1, 3$ for $j=1, 2, 3, 4$, respectively (note that FBFs are determined only based on the "if parts" of the rules). Therefore, $\varphi_j(x) = exp(-\frac{1}{2}(x-\bar{x}^j)^2) / \sum_{i=1}^{4} exp(-\frac{1}{2}(x-\bar{x}^j)^2)$, which are plotted in Fig. 2-8 from left to right for $j=1, 2, 3, 4$, respectively.
From Fig. 2-8 we see an interesting property of the FBFs: the φ_j whose centers are inside the interval [-3, 3] (which contains all the center) look like Gaussian functions whereas the φ_j whose centers are on the boundaries of the interval [-3, 3] look like sigmoid functions (*Cybenko, G.,1989*). It is known in neural network literature that Gaussian radial basis functions are good at characterizing local properties, whereas neural networks with sigmoid nonlinearities are good at characterizing global properties (*Lippmann, R., 1991*). Our FBFs seem to combine the advantages of both the Gaussian radial basis functions and the sigmoid neural networks. Specifically, for regions in the input space *U* which have sampling points, the FBFs cover them with Gaussian-like functions so that higher resolution can be obtained for the FBF expansion over these regions. On the other hand, for regions in *U* which have no sampling points, the FBFs cover them with sigmoid-like functions which have been shown to have good global properties (*Cybenko, G., 1989*, *Lippmann, R., 1991*).
Equation (2-17) defines only one kind of FBF, that is, it defines the FBF for fuzzy systems with *center average* defuzzifier, *product*-inference, *singleton* fuzzifier and Gaussian membership function. Other fuzzy systems can have other forms of FBFs; for example, the fuzzy systems with *minimum* inference have an FBF in form of (2-17) with product operation replaced by minimum operation; however, the basic idea remains the same, that is, to view a fuzzy system as a linear combination of functions which are defined as FBFs. Different FBFs have different properties.

The similarity of adaptive fuzzy systems of FBF and TSK-type FISs to neural networks were shown in (*Jambrich, G., 2004*). The most important advantage of using fuzzy basis function functions rather than polynomials , radial basis functions, neural networks, and so on is that a linguistic fuzzy if-then rule is naturally related to a fuzzy basis function. Linguistic fuzzy if-then rules can often be obtained from human experts who are familiar with the system under consideration. These linguistic rules are very important and often contain information which is not included in the input-output pairs obtained by measuring the outputs of a system for test inputs because the test inputs my not be rich enough to excite all the modes of a system.

Linguistic rules can be incorporated in FBF expansions as following:

$$\text{rule}^j: \quad \text{if } (x_1 \text{ is } F_1^j) \text{ and} \cdots \text{and } (x_{N_i} \text{ is } F_{N_i}^j), \text{ then } (y \text{ is } G^j) \qquad (2\text{-}19)$$

where F_i^j and G^j are fuzzy sets in R, $j=1...N_r$. The FBFs φ_j are constructed according to Definition 2-2 and the points at which the output membership functions $mf^j(y)$ achieve their maximum values are set to θ_j.

E.g. consider the system $\dot{x}_1 = x_2, \quad \dot{x}_2 = f(x_1,x_2) + g(x_1,x_2)u$. Suppose that $(x_1,x_2)=(0,0)$ is an equilibrium point of this system and there is no control; that is, $u=0$. Then the first linguistic rule describing $f(x_1,x_2)$ can be stated as

$$\text{rule}^1: \quad \text{if } (x_1 \text{ is } \textit{near zero}) \text{ and } (x_2 \text{ is } \textit{near zero}), \text{ then } (f(x_1,x_2) \text{ is } \textit{near zero}). \qquad (2\text{-}20)$$

"*near zero*" is a fuzzy set with center zero, since only the centers of the then part fuzzy sets are used in the fuzzy systems, the $mf^j(y)$ do not need to be specified in detail; that is, knowing their centers is sufficient. More information about incorporating fuzzy descriptions of the plant or fuzzy control rules into adaptive fuzzy controllers is given in Chapter 3.

Using fuzzy basis function expansions, we can easily combine two sets of rules:

- one rule set generated from measured (sampled) input-output data pairs of the unknown plant or an existing stabilizing controller (e.g. over an appropriate Least Squares algorithm (*Wang, L.X., 1994*)), or a rule set created from within constraints randomly parameterized fuzzy sets which shall (after training) describe the unknown plant or controller (*Wang, L.X., 1994*),

- and another rule set obtained from linguistic fuzzy if-then rules describing the unknown plant or controller (FIS initialization).

into a single fuzzy basis function expansion, which is therefore constructed using both numerical and linguistic information in a uniform fashion. Especially, the combination of a rule set in form of linguistic fuzzy if-then rules and another rule set including arbitrary parameterized fuzzy sets, whose parameters (of both rule sets) are then adapted by an appropriate training algorithm is successfully applied in this thesis.

Chapter 3

3 Review of Adaptive Fuzzy Control

Section 2.4 describes fuzzy inference systems in a general case. This Chapter will explain the method by which a FIS may be utilized for the purpose of automatic control. This will be done for the purpose of providing a basis from which adaptation methods for fuzzy control may be described. Secondly, in this Chapter, an overview of existing methods of adaptive fuzzy control will be presented.

3.1 Fuzzy Controller Design

How can a fuzzy controller design be performed? As shown in Fig. 3-1, the procedure of fuzzy controller design reflects its pragmatic nature by the reliance upon intuitive knowledge and an experimental-oriented approach. Firstly, the objectives of the design process are determined. Secondly, an intuitive understanding of the nature of the process to be controlled must be obtained. The accuracy of the intuitive understanding is vital since the fuzzy controller will be built based upon this. It may be obtained by any means available such as personal experience, advice from experts, mathematical analysis and experimentation. Once the designer is confident that he or she has sufficient understanding of the plant and the method by which it may be controlled, a suitable type of FIS, such as Tsukamoto-type, Mamdani-type, TSK-type or Fuzzy Basis Functions Expansions (as used in this work), must be selected. Once this has been done, an iterative procedure aimed at achieving the original design objectives begins. This iterative procedure begins with the selection of sensor signals which the designer feels contain sufficient information about the plant to allow the fuzzy controller to infer the correct control signal. The ranges of these inputs are then determined and fuzzy sets defining concepts such as high, medium, and low, are created. A set of "if-then" rules and a set of appropriate labels are then defined on the basis of the fuzzy sets and a defuzzification method, such as the centroid method described in Section 2.4, is selected. This completes the definition of a fuzzy controller; this controller must then be tested and improved further until the design objectives are met. Once the design objectives for the controller have been met, the system may be commissioned.
Unfortunately, there still remain some drawbacks of a conventional (non-adaptive) fuzzy controller design. They can be summarized as following:

- While intuitive understanding can provide a good method of initializing membership functions and consequences of a fuzzy controller, it usually provides clues about how to fine-tune them because there are many independent parameters and it may take considerable time to discover ways of altering them to improve a working solution. In addition, a given plant may change its characteristics over time and require re-tuning. Re-tuning and fine-tuning may be done automatically using methods of adaptive fuzzy control which will be described in the following Section.

- Fuzzy controllers are supposed to work in situations where there is a large uncertainty or unknown variation in plant parameters and structures. Using static fuzzy controllers it is difficult to state stability conditions and to maintain a consistent performance over the whole operating range of the closed loop control system (*Wang, L.X., 1994*).

- FIS design is a subjective procedure which is adopted to express domain expert's knowledge. However, transferring expert knowledge into a useable knowledge base is time-consuming and nontrivial (*Lee, C., 1990*).

- Moreover, the dependency on human introspection and experience results in some problems because, even for human experts, their knowledge is often incomplete and episodic rather than systematic. Thus, the completeness and logical consistency of a set of fuzzy if-then rules which is only provided from human knowledge (even from human experts) can hardly be guaranteed if complex systems in the presence of the above mentioned uncertainties are under consideration (*Leichtfried, J., Heiss, M., 1995*).

- MIMO fuzzy systems with high dimensionality often suffer from the problem of "curse of dimensionality" due to the rapid increase of fuzzy rules (*Wang, L.X., 1998*). High complexity of fuzzy systems leads to problems in practical realisations because of limited processor power and storage space.

Hence, bringing the learning abilities of Adaptive FISs (AFIS) to fuzzy controls may provide a more promising approach.

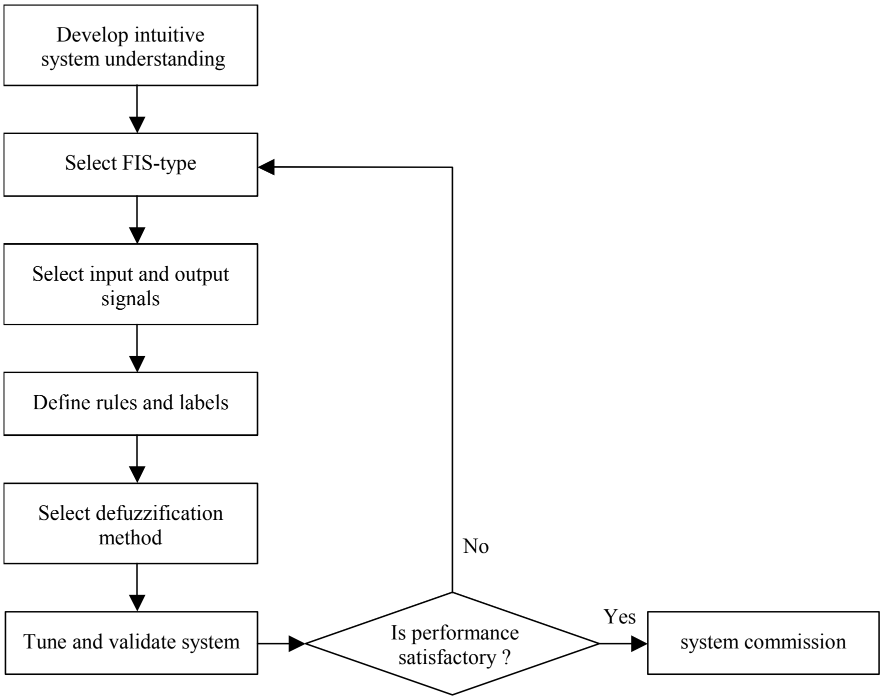

Fig. 3-1: Design procedure of a fuzzy controller

3.2 Adaptive Fuzzy Control

What is Adaptive Fuzzy Control? Roughly speaking, if a controller is constructed from adaptive fuzzy systems (recall that an adaptive fuzzy system is a fuzzy logic system equipped with a training (adaptation) algorithm), it is called an adaptive controller. An adaptive controller can be a single adaptive fuzzy system, or it can be constructed from several adaptive systems.
How does an adaptive fuzzy controller compare with a conventional adaptive controller? The most important advantage of adaptive fuzzy control is that adaptive fuzzy controllers are capable of incorporating linguistic fuzzy information from human operators, whereas conventional adaptive controllers cannot. This is especially important for the systems with a high degree of uncertainty, such as chemical processes, aircraft, and so on, because although these systems are difficult to control from control theoretical point of view, they are often successfully controlled by human operators. How can human operators successfully control such a complex system without a mathematical model in their minds? If we ask a human operator what their control strategies are, they may just tell us a few control rules in fuzzy terms and some linguistic descriptions about the behaviour of the system under various conditions, which are, of course, also in fuzzy terms. Although these fuzzy control rules and descriptions are not precise and may not be sufficient for constructing a successful controller, they provide very important information about how to control the system and how the system behaves. Adaptive fuzzy control provides a tool for making use of the fuzzy information in a systematic and efficient manner.
How can adaptive fuzzy controllers be classified? In (*Narendra, K.S., Parthasarathy, K., 1990*) the adaptive fuzzy controllers are classified according to two criteria: (1) whether the adaptive controller can incorporate fuzzy control rules or fuzzy descriptions about the

system, and (2) whether the fuzzy logic systems in the adaptive fuzzy controller are linear or nonlinear in their adjustable parameters. These classifications are detailed in the next two Subsections.

3.2.1 Direct and Indirect Adaptive Fuzzy Control

In conventional adaptive control literature, adaptive controllers are classified into two categories (*Wang, L.X., 1994*): direct and indirect adaptive controllers. In direct adaptive control, the parameters are directly adjusted to reduce some norm of the output error, see Fig. 3-2. In indirect adaptive control, the parameters of the plant are estimated and the controller is chosen assuming that the estimated parameters represent the true values of the plant parameters, see Fig. 3-3.
In fuzzy control, linguistic information from human experts can be classified into two categories:

- Fuzzy control rules which state in what situations which control actions should be taken (for example, we often use the following fuzzy if-then rule to drive a car: “If the speed is slow, then apply more force to the accelerator,” where slow and more are labels of fuzzy sets).

- Fuzzy if-then rules which describe the behaviour of the unknown plant (for example, we can describe the behaviour of a car using the fuzzy if-then rule: “If you apply more force to the accelerator, then the speed of the car will increase,” where “more” and “increase” are labels of fuzzy sets).

Interestingly enough, adaptive fuzzy controllers which make use of these two classes of linguistic information correspond to the direct and indirect adaptive control schemes, respectively. More specifically, direct adaptive fuzzy controllers use fuzzy logic systems as controllers. On the other hand, indirect adaptive controllers use fuzzy logic systems to model the plant and construct the controllers assuming that the fuzzy logic systems represent the true plant; therefore, fuzzy if then rules describing the plant can be directly incorporated into the indirect adaptive fuzzy controller. Formally, we have the following definition:

- If an adaptive fuzzy controller uses fuzzy logic systems as controllers, it is called a *direct adaptive fuzzy controller*. A direct adaptive fuzzy controller can incorporate fuzzy control rules directly into itself.

- If an adaptive fuzzy controller uses fuzzy logic systems as model of the plant, it is called an indirect adaptive fuzzy controller. An indirect adaptive fuzzy controller can incorporate fuzzy descriptions of the plant (in terms of fuzzy if-then rules) directly into itself.

The adaptive fuzzy controller development in Chapter 5 of this thesis is build upon the classical approach of (*Wang, L.X., 1994*) and extensions of other researchers on the same topic. However, there also exist a lot of other adaptive fuzzy architectures which are summarized in Section 3.3. Studying the literature it is noticeable, that the majority of classical direct adaptive fuzzy control schemes based on (*Wang, L.X., 1994*) share, conditional upon their mathematical structure, a common weak point: If e.g. a simple nonlinear SISO plant in from of $\dot{x}_1 = x_2, \quad \dot{x}_2 = f(x_1, x_2) + g(x_1, x_2)u$ is assumed, the nonlinear function g must be known exactly for the design of a direct adaptive control (*Chen, B.S., et al., 1996*). This drawback also occurs if MIMO plants are under consideration. On the other hand, based on this assumption there exist direct schemes, which can use special rule-base optimization methods by identifying the inverse of the plant and are therefore well suited for MIMO real time applications (*Gao, Y., et al., 2003)*. This means there is a trade-off between the flexibility to dominate a larger class of uncertain systems and the computational (hardware) effort for numerical calculations. The adaptive fuzzy controller development in Chapter 5 poses an intermediate between these two approaches. Therefore it is based on the flexible indirect adaptive approach and uses a Dynamic Rule Activation Method to weaken the problem of the “curse of dimensionality”.

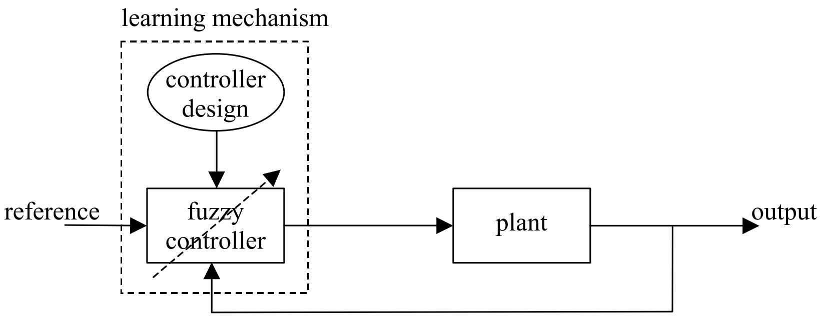

Fig. 3-2: Direct adaptive fuzzy control architecture

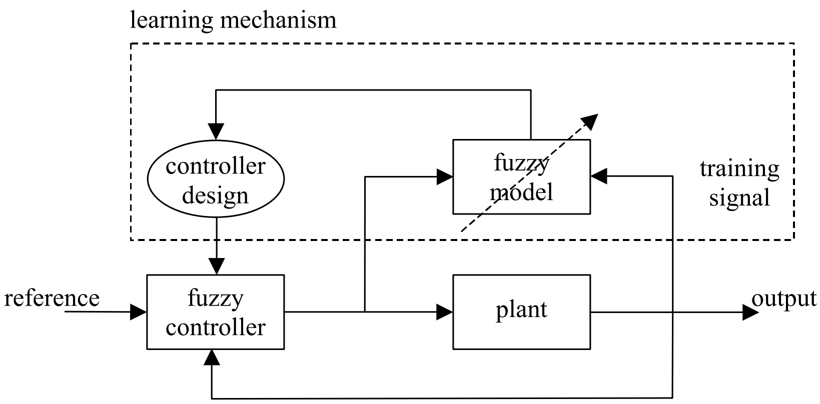

Fig. 3-3: Indirect adaptive fuzzy control architecture

3.2.2 First and Second Types of Adaptive Fuzzy Control

Training algorithms for the fuzzy logic system can be quite different depending upon whether the fuzzy logic system is linear or nonlinear in its adjustable parameters. If the fuzzy system is linear in its adjustable parameters, then it is easier to find an optimal fuzzy logic system. However, because the searching space is limited for the fuzzy logic systems which are linear in their adjustable parameters, the optimal fuzzy logic system in searching space may not be good enough. On the other hand, if the fuzzy logic system is nonlinear in its adjustable parameters, then it is more difficult to find an optimal fuzzy logic system. However, if such an optimal fuzzy logic system can be found, its performance should be good because the searching space is large. Thus the performance, complexity, and adaptive law of an adaptive fuzzy controller can be quite different depending on whether the fuzzy logic system in the adaptive controller is linear or nonlinear in their adjustable parameters. Therefore adaptive fuzzy controllers can be classified into two types:

- If the fuzzy logic systems used in an adaptive fuzzy controller are linear in their adjustable parameters, this adaptive fuzzy controller is called a *first-type adaptive fuzzy controller*.

- If the fuzzy logic systems used in an adaptive fuzzy controller are nonlinear in their adjustable parameters, this adaptive fuzzy controller is called a *second-type adaptive fuzzy controller*.

Notice that both first type and second type of adaptive fuzzy controllers are nonlinear adaptive fuzzy controllers. Now the formula of the fuzzy logic systems used in first and second types of adaptive fuzzy controllers are specified.

In the first-type adaptive fuzzy controller, the following fuzzy logic systems are used:

$$y = \sum_{j=1}^{N_r} \varphi_j \theta_j = \boldsymbol{\varphi}^T \boldsymbol{\theta} \tag{3-1}$$

where $\boldsymbol{\theta}=(\theta_1, \ldots, \theta_{Nr})^T$, $\boldsymbol{\varphi}=(\varphi_1, \ldots, \varphi_{Nr})^T$, φ_l is the Fuzzy Basis Function (Chapter 2) defined by

$$\varphi_j = \frac{\prod_{i=1}^{N_i} mf_i^j(x_i)}{\sum_{j=1}^{N_r} \prod_{i=1}^{N_i} mf_i^j(x_i)} . \tag{3-2}$$

θ_j are adjustable parameters, and mf_i^j are given membership functions. Clearly, (3-1) is equivalent to (2-18) assuming that mf_i^j are given; that is, mf_i^j will not change during the adaptation (training) procedure. The mf_i^j can be Gaussian, triangular, or any other type of membership functions. For the adaptive fuzzy control design in Chapter 5 Gaussian membership functions were chosen.

In the second-type adaptive fuzzy controller, the following fuzzy logic system are used:

$$y = \frac{\sum_{j=1}^{N_r} \left(\prod_{i=1}^{N_i} exp(-\frac{1}{2}(\frac{x-\bar{x}_i^j}{\sigma_i^j})^2 \right) w^j}{\sum_{j=1}^{N_r} \left(\prod_{i=1}^{N_i} exp(-\frac{1}{2}(\frac{x-\bar{x}_i^j}{\sigma_i^j})^2 \right)} \qquad (3\text{-}3)$$

where w^j, $\bar{x}_i^j$ and σ_i^j are adjustable parameters. Clearly, (3-3) is (2-15) with $a_i^j = 1$.

Equation (3-1) and (2-15) can be analysed from two points of view. First, if all the parameters a_i^j, $\bar{x}_i^j$, σ_i^j in φ_j are viewed as free design parameters, then FBF expansion (3-1) is nonlinear in the parameters. In order to specify such a FBF expansion, nonlinear optimisation techniques must be used, for example, use a back-propagation algorithm (*Wang, L.X., 1994*). On the other hand, all the parameters in φ_j can be fixed at the very beginning of the FBF expansion design procedure, so that the only free design parameters are θ_j; in this case, the function y of (3-1) is linear in the parameters. The advantages of this point of view are that

- Very efficient linear parameter estimation methods can be used, for example, the Gram-Schmidt Orthogonal Least Squares (OLS) algorithm (*Wang, L.X., 1994*) or the Least Mean Squares (LMS) algorithm (*Heiss, M., et al., 1994*) to design the FBF expansions,

- Relatively simpler adaptive laws to adjust the parameters of the adaptive fuzzy controller can be constructed,

- The convergence of the adaptation procedure is expected to be faster because we are not concerned with complicated nonlinear search problems,

- Therefore the performance of the controller is less sensitive to the initial values of the parameters and

- The performance of controller is more independent of an controller initialisation with linguistic rules.

Because of the numerous advantages mentioned above, the adaptive fuzzy controller design in Chapter 5 of this thesis is based on the first-type adaptive fuzzy controllers using FBF expansions which are linear in the parameters.

3.3 Adaptive Fuzzy Control Architectures

With only a few exceptions, which will be discussed in this Section, the purpose of an Adaptive Fuzzy Inference System (AFIS) in an adaptive controller is to perform function approximation. For Fuzzy Systems trained by means of associating error signals with particular input signals, the aim is to try to achieve a small error and the sources of input signals are selected with a view of achieving this aim. The function which an AFIS will perform is therefore wholly dependent on the way which error signal is calculated, or in other words, the learning algorithm that the AFIS is adopted with. The adaptive fuzzy controller classification scheme used in this Section is designed on the basis of observation. Various adaptive fuzzy controller architectures, according to their counterparts of classical (non-fuzzy) control theory, will therefore be grouped considering the methods of the AFIS learning algorithm. Fig. 3-4 shows the hierarchical classification scheme used in this literature survey. These control architectures are generally FIS independent. Most FIS architectures (discussed in Section 2.4) and learning algorithms can be used, although some may be more suitable than others. The degree with which a particular FIS satisfies these properties determines its suitability for online modelling and control. It does not solely depend on the FISs modelling capacity or on the learning algorithm, but a combination of these two factors.

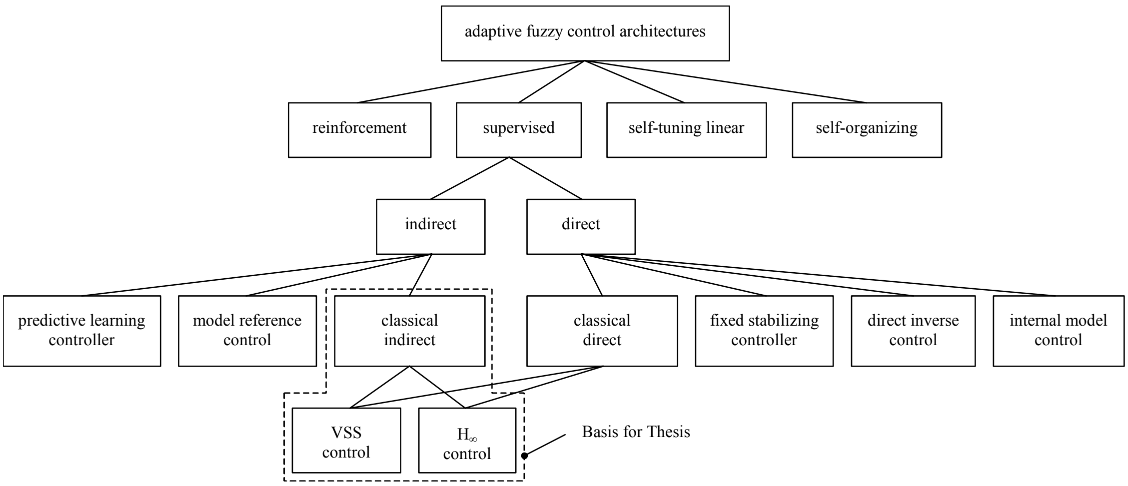

Fig. 3-4: Classification of adaptive fuzzy control architectures based on AFIS learning methods

3.3.1 Supervised Control Architecture

Supervised learning algorithms need to be posed within specific modelling and control architectures, and some of the most popular ones are described in this Section. One of the problems in formulating an online learning controller is that the desired control signal is rarely available and generally only the desired plant output can be used to train the controller.

As mentioned in Section 3.2.1 there are two classical approaches which have been formulated in the adaptive control field: direct and indirect schemes (*Wang, L.X., 1994*). A direct adaptive control scheme builds an explicit model of the desired controller, whereas an indirect scheme produces a model of the plant and synthesizes the control law, using a predefined optimisation/inversion calculation. For instance, the majority of adaptive fuzzy controllers have been direct, as a fuzzy rule base is used as a controller and there exists a performance index which relates errors of the plant's output to errors of the control signal in order to update the rule base as discussed in Section 3.1. In contrast, most of the fuzzy-neural controllers have been indirect, as an explicit plant model is generally constructed.

However, there also exist a lot of hybrid control schemes, including explicit fuzzy plant models and fuzzy controllers, where it is not easy to associate these schemes to a direct or an indirect approach. Hybrid schemes pose an intermediate between the two classical directions, but to associate them to the hierarchical classification scheme of Fig. 3-4 there is made following additional definition: If the adaptive fuzzy controller of a hybrid scheme directly uses the inverse model of the plant (in form of fuzzy if-then control rules) then these scheme is counted among the direct adaptive fuzzy approaches.
Especially mentionable are following subgroups of the classical supervised learning architectures (direct and indirect) based on two control methods, namely: (1) VSS (Variable Structure Systems) Control and there in particular Sliding Mode Control and (2) H_∞ Control.

3.3.1.1 VSS Control

A suitable way of tackling with uncertainties without the use of complicated models is to introduce Variable Structure Systems (VSS) theory based components into the system structure. Because of its robustness, VSS Control has successfully been applied to a wide variety of systems having uncertainties in the representative system models. The philosophy of the control strategy is simple, being based on two goals. First, the system is forced toward desired dynamics, second, the systems is maintained on that differential geometry. In the literature the former dynamics are named reaching mode, while the latter is called sliding mode. The control strategy borrows its name from the latter dynamic behaviour, and is called Sliding Mode Control (SMC). The idea behind Sliding Mode Control is that any state vector can be driven toward the so called sliding surface and then maintained on it while forcing the error dynamics toward the origin. Including Fuzzy Control techniques in SMC a new research area accrued, namely Fuzzy Sliding Mode Control (FSMC). In FSMC the Fuzzy Inference Systems are introduced for smoothing the controller action (chattering phenomena) and to facilitate the implementation of linguistic knowledge in VSS.

3.3.1.2 H_∞ Control

The H_∞-design approach plays a crucial role in adaptive fuzzy control because it considers robust and optimal control issues in one control concept. Using H_∞-techniques the influence of external disturbances and fuzzy modelling errors on the control error can be attenuated to infinite small levels and therefore the performance and robustness of adaptive fuzzy controls can be increased significantly. Moreover, the H_∞-design approach based on Lyapunov-stability theory leads to Riccati-like matrix equations and considers therefore also optimal control goals like minimization of the control effort and the control error via certain cost functions. It should also be mentioned that this approach can result in controller actions which temporarily tend to high controller output values. However, in practical applications the actuator energy is always limited and therefore there is a natural trade-off between control performance and control effort.

Note, that the indirect adaptive fuzzy controller design in Chapter 5 of this thesis includes both, VSS- as well as H_∞-design techniques and thus represents a simple method to combine the advantages of both approaches. Beside the above described classical adaptive fuzzy controller architectures (direct and indirect) there also exist other architectures (compare to Fig. 3-4) which are representing the state of the art. These architectures are described in the following.

3.3.1.3 Predictive Learning Control

Model-based predictive learning control schemes attempt to formulate a control strategy by assessing the effect of their actions for many time-steps into the future and selecting the current "optimal" control action, which is then applied to the plant. The architecture requires the development of a forward plant model using an adaptive fuzzy emulator, a performance function to evaluate the effect of the control action and an optimization technique which can determine the best control action. This is illustrated in Fig. 3-5, where a learning adaptive fuzzy controller has also been included, so that after sufficient training, the full optimization calculation does not need to be calculated and the computing resources can be allocated to other tasks. If the plant is time-varying, the model is generally adaptive, although the optimisation calculations may give very poor closed-loop control if the process model is inaccurate.

When the plant model is accurate and the performance function and the search strategy are appropriately chosen, this control scheme can provide excellent closed-loop control. However, the multi-step ahead optimisation calculation is generally very expensive and is only applied to systems which are not time critical. Many simplifications of the above architecture can be performed which makes this technique more suitable for real-time control.

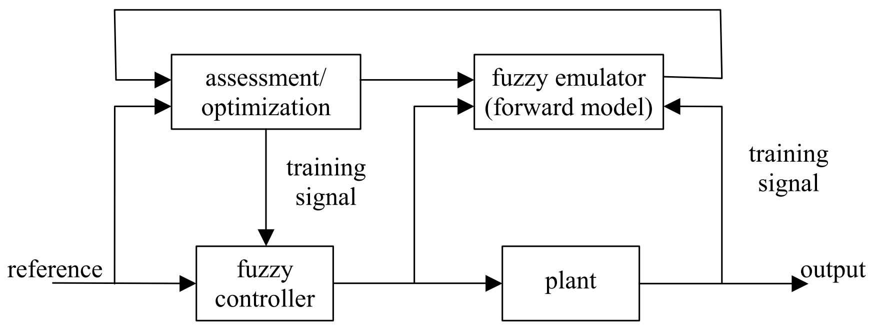

Fig. 3-5: A predictive learning control architecture

3.3.1.4 Model Reference Control

Model reference fuzzy controllers are the fuzzy control equivalence of the linear Model Reference Adaptive System (MRAS) controllers, and are shown in Fig. 3-6. The control objective is to adjust the control signal in a stable manner so that the plant's output asymptotically tracks the reference model's output. The performance of this algorithm depends on the choice of a suitable reference model and the derivation of an appropriate learning mechanism.

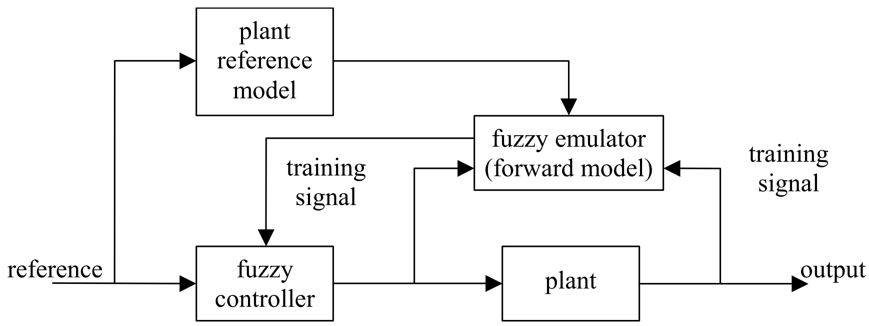

Fig. 3-6: A model reference control architecture

3.3.1.5 Fixed Stabilizing Controller

One of the simplest direct learning control scheme is shown in Fig. 3-7, where a fixed stabilizing linear feedback controller is used to train a adaptive fuzzy controller to learn the inverse model of the plant. The linear controller is designed so that the closed loop system is stable in every operating region and the control signal provides a training signal for the learning module. The performance of the closed loop system depends on the current operating point, although iterative training of the adaptive fuzzy controller gradually improves its performance online. As the operating point changes, the learning controller builds up a nonlinear model of the desired control surface, such that when the plant returns to the original operating point, the learned response has not been forgotten and it can be improved upon. This requires a learning module which is temporally stable and it should be highlighted that learning about one area in the input space affects the knowledge stored in a different region minimally. Despite the algorithms simplicity, this approach has one main drawback due to the design of the fixed linear controller. It has been claimed that a algorithm is robust with respect to the design of this controller, although the rate of convergence of the learning module depends on the quality of the training signal. A learning module is slow to adapt when the linear controller is performing poorly.

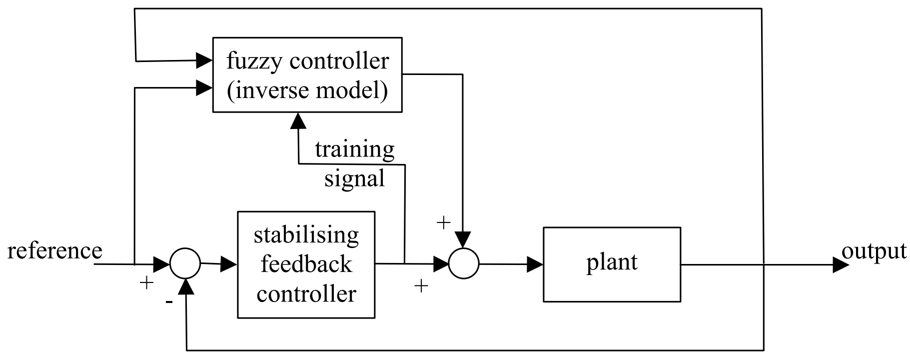

Fig. 3-7: A direct control architecture with a fixed stabilising controller

3.3.1.6 Direct Inverse Control

Inverse Controllers make use of fuzzy systems to identify a function which is exact inverse of the plant function. It is an intermediate between a direct and indirect scheme, but because it directly supplies fuzzy control rules, it is closer to the direct approach. It involves using two adaptive fuzzy systems: a controller and an emulator of the inverse plant model as shown in Fig. 3-8. At each time instant, the emulator is used to reconstruct a control signal. Since the actual control signal is already known, the adaptive fuzzy controller and emulator can be adapted using the error signal between the real control signal and the reconstructed one. If the function required of the inverse controller is equivalent to that required of the inverse model of the plant a single adaptive fuzzy system may be used both controller and emulator structures. Some recent studies on this scheme can be found in (*Omatu, S., et al.., 1996*).

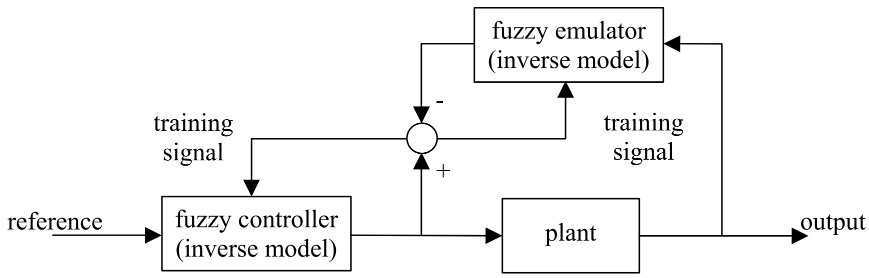

Fig. 3-8: A direct inverse control architecture

3.3.1.7 Internal Model Control

Internal model control uses a structure similar to the predictive learning control scheme, as shown in Fig. 3-9. an adaptive fuzzy emulator is used to model the plant directly, receiving the applied control signal, rather than the reference signal which is used in the model reference control scheme. The error between the forward model and the measured plant output is used as a feedback signal and this is passed to the adaptive fuzzy controller, which is generally designed to be an inverse plant model. Therefore, analogue to direct inverse control, this architecture is also counted among the direct schemes. The adaptive fuzzy systems used to model the plant can be trained using standard fuzzy modelling schemes (*Omatu, S., et al.., 1996*) and may be adapted online. Many theoretical stability results about internal model control loops are available, although they generally make the assumption that the open loop system is stable. Despite this assumption, it is claimed that this approach extends readily to nonlinear systems and yields robustness and stability analysis.

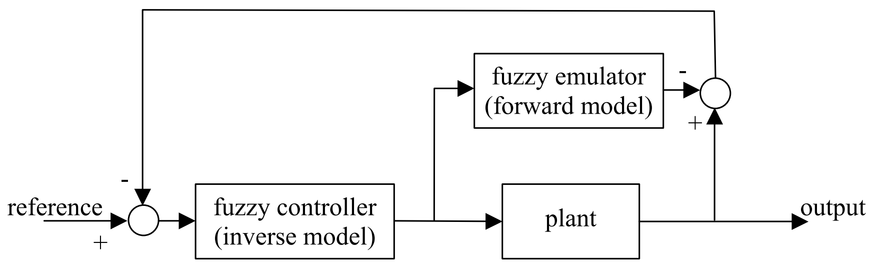

Fig. 3-9: An internal model control architecture

3.3.2 Reinforcement Control Architecture

The main distinguish feature of this type of controller is that it uses reinforcement learning instead of the typical supervised learning. Reinforcement learning may roughly be described as “learning with a critic as opposed to learning with a teacher”; a teacher tells a student how to do something, whereas a critic only tell someone if they have done well or badly.
A reinforcement learning controller is this adapted on the basis of a reinforcement signal from a critic which does not provide information about the extent of the error of the controller, but merely rates how well it has performed. The concept is shown in Fig. 3-10. Over the past ten years, there has been a greater theoretical understanding of the overall system, as well as a growing number of simulations and applications that employ modified versions of this technique (*Berenji, H.R., Khedkar, P., 1992*).

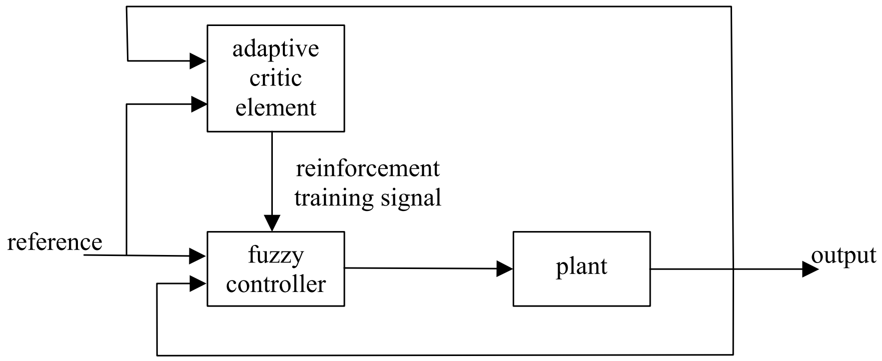

Fig. 3-10: A reinforcement control architecture

3.3.3 Self-Tuning Linear Control Architecture

This novel parameter self-tuning adaptive fuzzy control architecture attempts to exploit the ability of adaptive fuzzy systems to learn an arbitrary functional relationship. There are many reasons for utilization an intelligent gain-scheduling type approach: widespread industrial acceptance of linear feedback controllers, many theoretical and practical results about robustness and closed-loop stability and their low implementation are available to produce algorithms which can be used to calculate the parameters for both offline and online control. Successful systems would result in reduced commissioning costs and posses the ability to adapt to time-varying process dynamics.
All of these approaches assume previous knowledge about the plant’s structure, as this simplifies the problem. One example is the idea of extracting the feature labels which describe the closed-loop system’s response. The desired closed-loop response is expressed as a set of feature labels, which are compared with the labels describing the system’s closed loop step response. This error label vector is passed to the AFIS which predicts a change in the PID parameters so that the system’s response will be closer to the desired one, as depicted in Fig. Fig. 3-11. The ability to synthesis a set of PID parameters online, using only input/output data, has been investigated for many years. The potential payback from such a system which can increase the robustness of PID controllers is large.

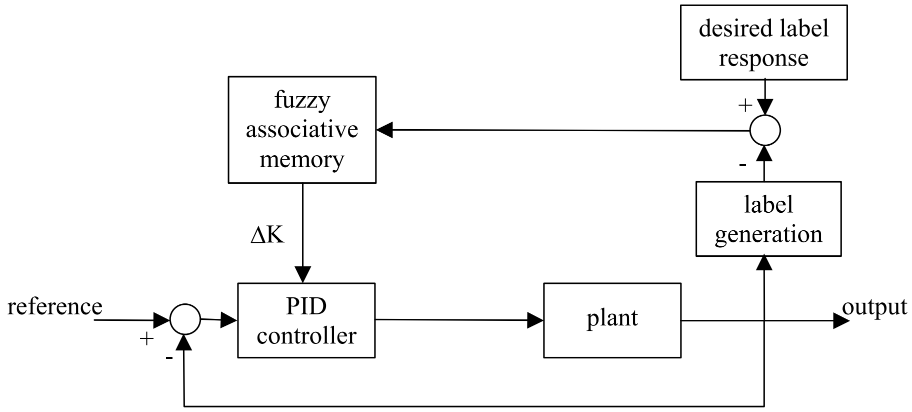

Fig. 3-11: A self-tuning linear control architecture

3.3.4 Self-organizing Fuzzy Control

In contrast with the model-based approaches, the learning-based methodology tries to emulate the human's learning ability by means of operating the process repeatedly, and thus the process behaviour is not explicitly taken into account. Here, past experience is of particular importance and is utilized intensively for creating workable control schemes. A notable example belonging to this class is a so called linguistic self-organizing controller (SOC), (*Procyk, T., Mamdani, E., 1979*). The architecture of the SOC is illustrated in Fig. 3-12. A performance feedback loop is added to a basic fuzzy controller. On-line measured performance of the control system indicated by a Performance Index Table (PIT) is used to create or modify the rule base. Starting from an empty rule-base the SOC is able to construct a suitable rule base via which the resulting control performance is acceptable in the sense of following the PIT measure. Thus with a little a priori knowledge about the process, the rule-base is self organized with the learning process. Following the work of (*Procyk, T., Mamdani, E., 1979*), the performance of SOC has been investigated intensively and some improved algorithms and design procedures as well as successful applications have been reported. The SOC has the important advantage of self-acquiring the required knowledge without relying on human experts. Like the model-based case, however, the SOC works in high-dimensional space even when the process itself is a single variable system and therefore, as would be expected, it has difficulty in handling multivariable systems.

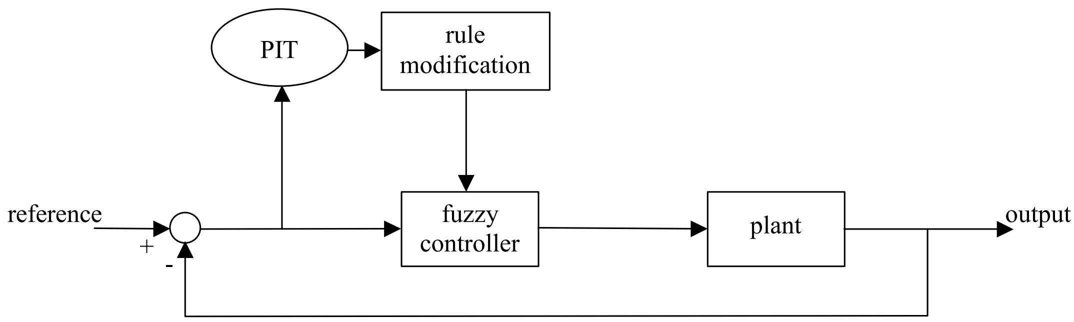

Fig. 3-12: A self-organizing fuzzy control architecture

Chapter 4

4 Input-Output Linearization

This Chapter describes how a class of input-affine, general nonlinear systems can be linearized exactly by first applying a coordinates transformation to it, and second linearizing the system in canonical normal form via special control laws. If the sum of relative degrees of the subsystems is less than the system order also the internal dynamics of the plant have to be considered. Input-output linearization leads to a linear system, which can be stabilized by classical linear controllers. As mentioned in the introduction, the input-output linearization method is used as basis for the in Chapter 5 developed adaptive fuzzy control schemes.

4.1 System description of the nonlinear MIMO plant

Assume that a nonlinear MIMO plant is represented by the following set of equations

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}'(\boldsymbol{x}) + \sum_{i=1}^{m} \boldsymbol{g}_i'(\boldsymbol{x}) u_i \\ y_1 &= h_1(\boldsymbol{x}) \\ &\vdots \\ y_m &= h_m(\boldsymbol{x}) \end{aligned} \qquad (4\text{-}1)$$

which is equivalent to the compressed form

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}'(\boldsymbol{x}) + \boldsymbol{G}'(\boldsymbol{x})\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{h}(\boldsymbol{x}) \\ \boldsymbol{G}'(\boldsymbol{x}) &= \left[\boldsymbol{g}_1'(\boldsymbol{x}) \quad \cdots \quad \boldsymbol{g}_m'(\boldsymbol{x})\right]. \end{aligned} \qquad (4\text{-}2)$$

$\boldsymbol{x} \in \boldsymbol{U} \subset \boldsymbol{R}^n$ is the state vector of the nonlinear system, $\boldsymbol{u}, \boldsymbol{y}$ are the plant inputs and outputs respectively, $\boldsymbol{u}, \boldsymbol{y} \in \boldsymbol{R}^m$ (number of outputs is equal to the number of inputs), the vector fields $\boldsymbol{f}', \boldsymbol{g}_1', \ldots, \boldsymbol{g}_m' : \boldsymbol{U} \subset \boldsymbol{R}^n \to \boldsymbol{R}^n$ and the functions $h_i : \boldsymbol{U} \subset \boldsymbol{R}^n \to \boldsymbol{R}$ are assumed to be *smooth*.

4.2 Coordinates Transformations

Definition 4-1: *The system (4-1) has a (vector) relative degree* $\boldsymbol{r}_u = [r_1, \ldots, r_m]^T$ *in a region*

$U_0 \subset U$ *if* $\forall \boldsymbol{x} \in U_0$

(i)

$$L_{\boldsymbol{g'}_j} L_{\boldsymbol{f'}}^k h_i(\boldsymbol{x}) = 0 \qquad (i, j=1,2,\ldots,m;\ 0 \leq k < r_{i-1}) \tag{4-3}$$

or equivalently

$$L_{\boldsymbol{g'}_j} h_i(\boldsymbol{x}) = L_{\boldsymbol{g'}_j} L_{\boldsymbol{f'}} h_i(\boldsymbol{x}) = \cdots = L_{\boldsymbol{g'}_j} L_{\boldsymbol{f'}}^{r_i-2} h_i(\boldsymbol{x}) = 0 \tag{4-4}$$

where e.g. $L_{\boldsymbol{f'}}^k h_i(\boldsymbol{x})$ *means the* k^{th} *Lie-derivative of the scalar valued function* h_i *with respect to the vector valued function* $\boldsymbol{f'}$.

(ii) the $m \times m$ *matrix*

$$\boldsymbol{G}(\boldsymbol{x}) = \begin{bmatrix} L_{\boldsymbol{g}'_1} L_{\boldsymbol{f'}}^{r_1-1} h_1(\boldsymbol{x}) & \cdots & L_{\boldsymbol{g}'_m} L_{\boldsymbol{f'}}^{r_1-1} h_1(\boldsymbol{x}) \\ L_{\boldsymbol{g}'_1} L_{\boldsymbol{f'}}^{r_2-1} h_2(\boldsymbol{x}) & \cdots & L_{\boldsymbol{g}'_m} L_{\boldsymbol{f'}}^{r_2-1} h_2(\boldsymbol{x}) \\ \vdots & \vdots & \vdots \\ L_{\boldsymbol{g}'_1} L_{\boldsymbol{f'}}^{r_m-1} h_m(\boldsymbol{x}) & \cdots & L_{\boldsymbol{g}'_m} L_{\boldsymbol{f'}}^{r_m-1} h_m(\boldsymbol{x}) \end{bmatrix} \tag{4-5}$$

is not singular.

$$L_{\boldsymbol{g'}_j} L_{\boldsymbol{f'}}^{r_i-1} h_i(\boldsymbol{x}) \neq 0 \qquad \text{for at least one } (j=1,2,\ldots,m) \tag{4-6}$$

is only implied by, but not equivalent to (ii).

■

Proposition 4-1: *Suppose a system has a (vector) relative degree* $\boldsymbol{r}_u=[r_1,\ldots,r_m]^T$ $\forall \boldsymbol{x} \in \boldsymbol{U}_0 \subset \boldsymbol{U}$. *Then* $\forall \boldsymbol{x}_0 \in \boldsymbol{U}_0$

$$r = \sum_{i=1}^{m} r_i = r_1 + \cdots + r_m \leq n\,. \tag{4-7}$$

Set, for i=1,2,…,m

$$\begin{aligned} \psi_1^i(\boldsymbol{x}) &= h_i(\boldsymbol{x}) \\ \psi_2^i(\boldsymbol{x}) &= L_{\boldsymbol{f'}} h_i(\boldsymbol{x}) \\ &\vdots \\ \psi_{r_i}^i(\boldsymbol{x}) &= L_{\boldsymbol{f'}}^{r_i-1} h_i(\boldsymbol{x})\,. \end{aligned} \tag{4-8}$$

If $r = r_1 + \cdots + r_m < n$, it is always possible to find n-r more functions $\psi_{r+1}(\boldsymbol{x}),...,\psi_n(\boldsymbol{x})$ such that the mapping

$$\boldsymbol{\Psi}(\boldsymbol{x}) = [\psi_1^1(\boldsymbol{x}),...,\psi_{r_1}^1(\boldsymbol{x}),...,\psi_1^m(\boldsymbol{x}),...,\psi_{r_m}^m(\boldsymbol{x}),\psi_{r+1}(\boldsymbol{x}),...,\psi_n(\boldsymbol{x})]^T \tag{4-9}$$

has a Jacobian matrix which is nonsingular $\forall \boldsymbol{x}_0 \in \boldsymbol{U}_0$ and therefore qualifies as a local coordinates transformation in a neighborhood of $\boldsymbol{x}_0$. The value at $\boldsymbol{x}_0$ of these additional functions can be chosen arbitrarily. Moreover, if the distribution

$$\boldsymbol{Y} = span\{\boldsymbol{g}'_1,...,\boldsymbol{g}'_m\} \tag{4-10}$$

is involutive near $\boldsymbol{x}_0$, it is always possible to choose $\psi_{r+1}(\mathbf{x}),...,\psi_n(\mathbf{x})$ in such way that

$$L_{\boldsymbol{g}'_j}\psi_i(\boldsymbol{x}) = 0 \qquad (r+1 \le i \le n;\ j = 1,2,...,m). \tag{4-11}$$

Proof: refer to (*Isidori, A., 1989*).

■

As a matter of fact, differentiating with respect to time, one obtains, e.g. for the first set of new coordinates

$$\begin{aligned} \frac{d\psi_1^1}{dt} &= \psi_2^1(t) \\ &\vdots \\ \frac{d\psi_{r_i-1}^1}{dt} &= \psi_{r_i}^1(t) \\ \frac{d\psi_{r_i}^1}{dt} &= L_{f'}^{r_i}h_i(\boldsymbol{x}(t)) + \sum_{j=1}^{m} L_{\boldsymbol{g}'_j}L_{f'}^{r_i-1}h_i(\boldsymbol{x}(t))u_j(\boldsymbol{x}). \end{aligned} \tag{4-12}$$

Note that the coefficient that multiply $u_j(t)$ in the latter equation is exactly equal to the *(1,j)* entry of the matrix $\boldsymbol{G}(\boldsymbol{x})$.

Set now

$$\begin{aligned} &\boldsymbol{\xi}^i = [\xi_1^i,\xi_2^i,...,\xi_{r_i}^i] = [\psi_1^i(\boldsymbol{x}),\psi_2^i(\boldsymbol{x}),...,\psi_{r_i}^i(\boldsymbol{x})] \qquad (i=1,2,...,m) \\ &\boldsymbol{\xi} = [\boldsymbol{\xi}^1,\boldsymbol{\xi}^2,...,\boldsymbol{\xi}^m]^T \\ &\boldsymbol{\eta} = [\eta_1,\eta_2,...,\eta_{n-r}]^T = [\psi_{r+1}(\boldsymbol{x}),\psi_{r+2}(\boldsymbol{x}),...,\psi_n(\boldsymbol{x})]^T \end{aligned} \tag{4-13}$$

and

$$\begin{aligned} &g_{ij}(\boldsymbol{\xi},\boldsymbol{\eta}) = L_{\boldsymbol{g}'_j}L_{f'}^{r_i-1}h_i(\boldsymbol{\Psi}^{-1}(\boldsymbol{\xi},\boldsymbol{\eta})) \qquad (i,j=1,2,...,m) \\ &f_i(\boldsymbol{\xi},\boldsymbol{\eta}) = L_{f'}^{r_i}h_i(\boldsymbol{\Psi}^{-1}(\boldsymbol{\xi},\boldsymbol{\eta})) \qquad (i=1,2,...,m). \end{aligned} \tag{4-14}$$

Then the equations in question can be rewritten as

$$
\begin{aligned}
\dot{\xi}_1^i &= \xi_2^i \\
&\vdots \\
\dot{\xi}_{r_i-1}^i &= \xi_{r_i}^i \\
\dot{\xi}_{r_i}^i &= f_i(\xi,\boldsymbol{\eta}) + \sum_{j=1}^{m} g_{ij}(\xi,\boldsymbol{\eta})u_j \\
y_i &= \xi_1^i \qquad (i=1,2,\ldots,m).
\end{aligned}
\tag{4-15}
$$

If the distribution spanned by the vector fields $\boldsymbol{g}_1'(\boldsymbol{x}),\ldots,\ \boldsymbol{g}_m'(\boldsymbol{x})$ is not *involutive* (most general case), we can only write generically, with a vector notation

$$
\dot{\boldsymbol{\eta}} = \boldsymbol{q}(\xi,\boldsymbol{\eta}) + \sum_{i=1}^{m} p_i(\xi,\boldsymbol{\eta})u_i = \boldsymbol{q}(\xi,\boldsymbol{\eta}) + \boldsymbol{p}(\xi,\boldsymbol{\eta})\boldsymbol{u} = \chi(\xi,\boldsymbol{\eta})\,. \tag{4-16}
$$

Otherwise, if the distribution in question is involutive, it is always possible to choose the remaining set of coordinates $\psi_{r+1}(\boldsymbol{x}),\ldots,\psi_n(\boldsymbol{x})$ in such way as to obtain an equation of the type

$$
\dot{\boldsymbol{\eta}} = \boldsymbol{q}(\xi,\boldsymbol{\eta})\,. \tag{4-17}
$$

The equations (4-15) and (4-16) characterize the *normal form* of the equations describing (locally around a point $\boldsymbol{x}_0$) a nonlinear system with m inputs and m outputs having a (vector) relative degree $\boldsymbol{r}_u=[r_1,\ldots,r_m]^T$ at $\boldsymbol{x}_0$. The vector $\boldsymbol{\xi}$ represents the so called *external dynamics* which can be linearized according to Section 4.3. The vector $\boldsymbol{\eta}$ represents the *internal dynamics* and $\boldsymbol{\eta}$ is unobservable from the output. Note that – in equation (4-14) - the coefficients $g_{ij}(\xi,\eta)$ are exactly the entries of the matrix (4-5), with $\boldsymbol{x}$ replaced by $\boldsymbol{\Psi}^1(\xi,\eta)$, and the coefficients $f_i(\xi,\eta)$ are the entries of a vector

$$
\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} L_{f'}^{r_1}h_1(\boldsymbol{x}) \\ L_{f'}^{r_2}h_2(\boldsymbol{x}) \\ \vdots \\ L_{f'}^{r_m}h_m(\boldsymbol{x}) \end{bmatrix} \tag{4-18}
$$

again with $\boldsymbol{x}$ replaced by $\boldsymbol{\Psi}^1(\xi,\eta)$.

$\boldsymbol{\Psi}(\boldsymbol{x})$ is called a *diffeomorphism* on $\boldsymbol{U}_0$, or a *local diffeomorphism*. Given a nonlinear state space realization of an input-affine system (4-1) a diffeomorphism $\boldsymbol{\Psi}(\boldsymbol{x})$ can be used to perform a coordinate transformation which converts the nonlinear system into the normal form equations (4-15) and (4-16). Because of the existence of $\boldsymbol{\Psi}^1$ (nonsingularity) we can always recover the original state space representation if $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ are known.

If the system (4-1) has a strong uniform relative degree vector $\boldsymbol{r}_u=[r_1,\dots,r_m]^T$ $\forall \boldsymbol{x} \in \boldsymbol{R}^n$ ($\boldsymbol{U} = \boldsymbol{R}^n$) a diffeomorphism $\boldsymbol{\Psi(x)}$ for all $\boldsymbol{x} \in \boldsymbol{R}^n$ can be found and therefore $\boldsymbol{\Psi(x)}$ is called a *global diffeomorphism* on $\boldsymbol{R}^n$.

Notice that not all systems have a well-defined uniform relative degree, e.g. if $L_{g'_j} L_{f'}^{r_i-1} h_i(\boldsymbol{x}) = 0$ at a point $\boldsymbol{x}_0 \in \boldsymbol{R}^n$ and $L_{g'_j} L_{f'}^{r_i-1} h_i(\boldsymbol{x}) \neq 0$ elsewhere (*Sastry, S., Isidori, A., 1989*).

4.3 Input-Output Linearization via static feedback linearizing control law

The equations in (4-15) may also be written as

$$\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_{f'}^{r_1} h_1(\boldsymbol{x}) \\ \vdots \\ L_{f'}^{r_m} h_m(\boldsymbol{x}) \end{bmatrix} + \boldsymbol{G}(\boldsymbol{x}) \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}, \quad \begin{matrix} y_1 = h_1(\boldsymbol{x}) \\ \vdots \\ y_m = h_m(\boldsymbol{x}) \end{matrix} . \tag{4-19}$$

If $\boldsymbol{G(x)} \in \boldsymbol{R}^{m \times m}$ is bounded away from singularity, the state feedback control law

$$\boldsymbol{u}(\boldsymbol{x}) = -\boldsymbol{G}^{-1}(\boldsymbol{x}) \begin{bmatrix} L_{f'}^{r_1} h_1(\boldsymbol{x}) \\ \vdots \\ L_{f'}^{r_m} h_m(\boldsymbol{x}) \end{bmatrix} + \boldsymbol{G}^{-1}(\boldsymbol{x}) \boldsymbol{v} \tag{4-20}$$

including a linear stabilizing control term $\boldsymbol{v}$ which has appropriately to be designed, yields to the closed-loop decoupled, linear system

$$\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} \tag{4-21}$$

and in addition if the system is not fully linearizable ($r = r_1 + \cdots + r_m < n$) to the internal dynamics

$$\dot{\boldsymbol{\eta}} = \boldsymbol{q}(\boldsymbol{\xi}, \boldsymbol{\eta}) + \boldsymbol{p}(\boldsymbol{\xi}, \boldsymbol{\eta}) \boldsymbol{u} . \tag{4-22}$$

Once linearization has been achived and the internal dynamics are stable, any further control objective such as model matching, pole placement, tracking may be easily met. The feedback law (4-20) is referred as a *static-state feedback linearizing control law*.

Remarks 4-1: If $\boldsymbol{G(x)}$ defined in (4-5) is singular, linearization may still be achived using dynamic state feedback. The development may be followed by using integrators before some inputs; exact conditions under which linearization may be achived by dynamic state feedback are given, for instance, in (*Sastry, S., Isidori, A., 1989*).

■

Definition 4-2: *Given a dynamical system of the form (4-15) and (4-16) the autonomous equation*

$$\dot{\boldsymbol{\eta}} = \boldsymbol{q}(\boldsymbol{0},\boldsymbol{\eta}) + \boldsymbol{p}(\boldsymbol{0},\boldsymbol{\eta})\boldsymbol{u} \qquad (4\text{-}23)$$

is called the zero dynamics.

Because of the analogy with the linear system case, systems whose zero dynamics are stable are said to be minimum phase (Marquez, J.M., 2003).

■

Let $\boldsymbol{u}^*(\boldsymbol{\xi},\boldsymbol{\eta})$ be the linearizing control law, i.e.,

$$\boldsymbol{u}^*(\boldsymbol{\xi},\boldsymbol{\eta}) = -[\boldsymbol{G}(\boldsymbol{\xi}(t),\boldsymbol{\eta}(t))]^{-1}\boldsymbol{f}(\boldsymbol{\xi}(t),\boldsymbol{\eta}(t))\,. \qquad (4\text{-}24)$$

Using this control in the equations for $\boldsymbol{\eta}$ and assuming that $\boldsymbol{0}\in\boldsymbol{R}^n$ is the equilibrium point of the undriven system (i.e., $\boldsymbol{f}(\boldsymbol{0})=\boldsymbol{0}$) and $h_1(\boldsymbol{0})=\ldots=h_m(\boldsymbol{0})=0$, we see that the subspace $\{(\boldsymbol{0},\ \boldsymbol{\eta})\}\subset\boldsymbol{R}^n$ is an invariant subspace and the zero dynamics are the dynamics of

$$\dot{\boldsymbol{\eta}} = \boldsymbol{q}(\boldsymbol{0},\boldsymbol{\eta}) + \boldsymbol{p}(\boldsymbol{0},\boldsymbol{\eta})\boldsymbol{u}^* = \chi(\boldsymbol{0},\boldsymbol{\eta})\,. \qquad (4\text{-}25)$$

The idea of Definition 4-2 is that of solving first the Problem of *Zeroing the Output* $\boldsymbol{y}$, i.e. to find initial conditions and inputs consistent with the constraint that the output function $\boldsymbol{y}(t)$ is identically zero. If the output has to be zero for all t, then necessarily the initial state of the system must be set to a value that $\boldsymbol{\xi}(0)=\boldsymbol{0}$ where $\boldsymbol{\eta}(0)=\boldsymbol{\eta}^0$ can be chosen arbitrarily. According to the value of $\boldsymbol{\eta}^0$, the input must be set as

$$\boldsymbol{u}(t) = -[\boldsymbol{G}(\boldsymbol{0},\boldsymbol{\eta}(t))]^{-1}\boldsymbol{f}(\boldsymbol{0},\boldsymbol{\eta}(t)) \qquad (4\text{-}26)$$

with

$$\dot{\boldsymbol{\eta}}(t) = \boldsymbol{q}(\boldsymbol{0},\boldsymbol{\eta}) - \boldsymbol{p}(\boldsymbol{0},\boldsymbol{\eta})[\boldsymbol{G}(\boldsymbol{0},\boldsymbol{\eta}(t)]^{-1}\boldsymbol{f}(\boldsymbol{0},\boldsymbol{\eta}(t))\,. \qquad (4\text{-}27)$$

Moving from these calculations to a *coordinate-free setting*, the reader will have no difficulties in realizing that, in order to yield $\boldsymbol{y}(t)=\boldsymbol{0}$ for all times , the system must evolve on the subset

$$\boldsymbol{Z}^* = \left\{\boldsymbol{x}\in\boldsymbol{R}^n : L_{f'}^k h_i(\boldsymbol{x}) = 0,\ 0\le k\le r_i - 1,\ i = 1,2,\ldots,m\right\} \qquad (4\text{-}28)$$

under the effect of an input $\boldsymbol{u}(t)$ which is a solution of the equation

$$\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{G}(\boldsymbol{x}(t))\boldsymbol{u}(t) = \boldsymbol{0} \tag{4-29}$$

so that the state feedback becomes to

$$\boldsymbol{u}^*(\boldsymbol{x}) = -\boldsymbol{G}^{-1}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x}). \tag{4-30}$$

We notice that setting $\boldsymbol{y} \equiv \boldsymbol{0}$ in equations (4-15) and (4-16) we have that

$$\begin{aligned} \boldsymbol{y} \equiv \boldsymbol{0} \quad &\Leftrightarrow \quad \boldsymbol{\xi} \equiv \boldsymbol{0} \\ &\Leftrightarrow \quad \boldsymbol{u}(t) = -[\boldsymbol{G}(\boldsymbol{0},\boldsymbol{\eta}(t))]^{-1}\boldsymbol{f}(\boldsymbol{0},\boldsymbol{\eta}(t)) \\ &\Rightarrow \quad \dot{\boldsymbol{\eta}} = \boldsymbol{\chi}(\boldsymbol{0},\boldsymbol{\eta}). \end{aligned} \tag{4-31}$$

Thus the zero dynamics can be defined as the internal dynamics of the system when the output is kept identically zero by a suitable input function. This means that the zero dynamics can be determined without transforming the system into normal form (*Marquez, J.M., 2003*).

Summary 4-1: The stability properties of the zero dynamics play a very important role whenever input-output linearization is applied. Input-output linearization is achieved via partial cancellation of nonlinear terms. Two cases should be distinguished:

$r = r_1 + \cdots + r_m = n$: If the sum of relative degrees of the subsystems is equal to the order of the whole system, then the nonlinear system can be fully linearized and, input-output linearization can be successfully applied. This analysis, of course, ignores at the moment robustness issues that always arise as a result of imperfect modeling.

$r = r_1 + \cdots + r_m < n$: If the sum of relative degrees of the subsystems is lower than the order of the whole system, then only the external dynamics of order r are linearized. The remaining $n–r$ states are unobservable from the output. The stability properties of the internal dynamics is determined by the zero dynamics. Thus, whether input-output linearization can be applied successfully depends on the stability of the zero dynamics. If the zero dynamics are not asymptotically stable, then input-output linearization does not produce a control law of any practical use.

■

Chapter 5

5 Design of the Robust Indirect Adaptive Fuzzy Control Scheme for MIMO Nonlinear Dynamic Systems

This Chapter describes the development of the robust adaptive fuzzy control scheme for controlling a class of MIMO nonlinear dynamic systems, derives the Robust Indirect Adaptive Fuzzy Controller (RIAFC) based on H_∞- and VSS control techniques and presents the convergence and stability analysis of the RIAFC. Moreover, the RIAFC is extended by a linear state observer to build an Observer-based Robust Indirect Adaptive Fuzzy Control (ORIAFC). To guarantee the existence of solutions for the observer-based control a SPR Lyapunov Design Approach is introduced. Modified Adaptation Laws are defined to maintain the adaptive parameters of the AFISs inside pre-defined constraint sets. Finally, a Dynamic Rule Activation Method is proposed to remedy the phenomenon which is known as the "curse of dimensionality". Comparisons with some existing adaptive fuzzy controllers are also carried out to further demonstrate its superior performance.

5.1 Motivation and Development

Fuzzy control methodologies have emerged in recent years as promising ways to approach nonlinear control problems. Fuzzy control, in particular, had an impact on the control community because of its simple approach to use heuristic control knowledge for nonlinear control problems. In very complicated situations, where the plant parameters are perturbed or the plant dynamics of systems are too complex for a mathematical model to describe, adaptive schemes have to be used online to gather data and adjust the control parameters automatically (*Procyk, T., et al., 1979*, *Lee, C., 1990*, *Driankov, D., et al., 1993*, *Layne, J.R., et. al., 1993*, *Kwong, W.A., et al., 1996, Gegov, A.E., et al., 1995*). However, no stability conditions were provided so far for these adaptive approaches. Based on the universal approximation theorem (*Wang, L.X., et al., 1992*) stable direct and indirect adaptive fuzzy control schemes were first developed to control unknown nonlinear systems with closed loop stability given by Lyapunov function method (*Wang, L.X.*, *1993, Wang, L.X.*, *1994*). Second, several stable adaptive fuzzy control schemes were introduced for controlling Single-Input-Single-Output (SISO) nonlinear systems (*Chen, B.S., et al., 1996, Spooner, J.T., et al., 1996, Sue, C.Y., et al., 1994, Tong, S.C., et al., 1999, Chai, T.Y., et al., 1999*). In these adaptive control schemes, the controllers are generally composed of two main components. One is the fuzzy logic system for rough tuning. The other one a kind of robust compensator, such as supervisory control (*Wang, L.X.*, *1993*), H_∞ control

(*Chen, B.S., et al., 1996*), sliding-mode control (*Spooner, J.T., et al., 1996*, *Sue, C.Y.,et al., 1994*, *Tong, S.C., et al., 1999*, *Chai, T.Y., et al. 1999*), or the combination of the latter two, for fine-tuning. Recently, several stable adaptive fuzzy control schemes were developed for Multi-Input-Multiple-Output (MIMO) nonlinear systems (*Li, Q.G., et al., 1997*, *Tong, S.C., et al., 2000*, *Chang, Y.C.*, *2000*, *Ordonez, R., et al., 1999*, *Zhang, et al., 2000*, *Chang, Y.C., et al., 1997*). However, these adaptive control techniques are limited to the MIMO nonlinear systems whose states are available for measurement. In practical situations, the state variables are often unavailable in nonlinear systems. Thus, an output feedback or an observer-based adaptive fuzzy control is required for such complicated applications.

This motivated me to investigate an adaptive Robust Indirect Adaptive Fuzzy Control (RIAFC) based on H_∞- and VSS control techniques and to extend this scheme to an Observer-based Robust Indirect Adaptive Fuzzy Control (ORIAFC) for a class of nonlinear MIMO plants, based on the previous work (*Li, Q.G., et al., 1997*, *Tong, S.C., et al., 2000*, *Chang, Y.C.*, *2000*). But unlike to the state-of-the-art, the Observer-based Robust Indirect Adaptive Fuzzy Control does neither require all system states to be available for measurement nor the measured output signals or the reference trajectories to be smooth, necessarily. Adaptive fuzzy logic systems are used to approximate the unknown vector functions and then a state observer is constructed, upon which the indirect adaptive fuzzy control system can be developed to control the MIMO system and maintain the system stability. Being the auxiliary compensation, an H_∞ control and a Variable Structure Systems (VSS) control are designed to improve the system performance by suppressing the influence of external disturbance as well as measuring noise and removing the fuzzy approximation error. Thus, the proposed Observer-based Robust Indirect Adaptive Fuzzy Control can guarantee the closed-loop stability, and also attenuate the influence of the matching error, external disturbance and measurement noise to a small level. In addition, by the use of a dynamic fuzzy rule activation method the phenomenon which is called “the curse of dimensionality” can be significantly weakened.

The salient features of the proposed RIAFC are summarized as follows:

- Nonlinear Dynamic Plants: Including the class of dynamic SISO and MIMO plants with exponentially attractive zero dynamics the developed indirect adaptive fuzzy control method of this thesis is less conservative compared with other methods.

- Online adaptive learning: No prescribed training models are needed for online-learning. The RIAFC can learn adaptively from the measured signals sequentially.

- Fast adaptation and learning: The proposed AFISs which are linear in their adjustable parameters allow the use of efficient learning methods. The tuning of the adjustable AFIS parameters is done automatically by Modified Adaptation Laws which guarantee that all parameters are bounded. Moreover, the parameters are modified without using any iteration methods.

- Ease of incorporating expert knowledge: Expert knowledge can easily be incorporated into the RIAFC in form of fuzzy if-then rules describing the plant.

- Fast convergence of tracking error: Fast adaptation- and learning speed of the RIAFC enables any controlled plant of the in Section 5.2.1 defined system class to track the desired trajectory very quickly.

- Adaptive control: The parameters of the RIAFC are self-adaptive in the presence of high complexity, uncertainties and imprecision so as to maintain high control performance.

- Robust Control: Asymptotic stability of the control system is established using the Lyapunov theorem. H_∞- and VSS control techniques are applied to improve the system performance by suppressing the influence of external disturbance as well as measuring noise and removing the fuzzy approximation error.

- Reduction of activated fuzzy rules: By the use of a Dynamic Fuzzy Rule Activation Method the phenomenon which is called "the curse of dimensionality" can be significantly weakened. Therefore only that fuzzy rules were activated whose fuzzy basis function values are greater than a given threshold.

- Measurement noise: The impact of measurement noise on the performance of the closed-loop control system is evaluated in a mathematical rigorous fashion.

The ORIAFC offers besides the advantages of the RIAFC following superior features:

- Availability of system states: The ORIAFC does not require all system states to be available for measurement. Only the plant outputs are assumed to be available for measurement.

- Smoothness of measured output signals or reference trajectories: The ORIAFC does neither require the measured (noisy) output signals nor the reference trajectories to be smooth, necessarily.

- Measurement noise: The impact of measurement noise on the performance of the closed-loop control system is evaluated in a mathematical rigorous fashion. Moreover, applying an observer-based approach in combination with H_∞-control techniques the influence of measurement noise on the performance of the control can be attenuated to a small level.

All algorithms which are presented in this thesis can also successfully be applied if SISO plants have to be controlled. To guarantee a clear presentation, the control schemes are only for the general MIMO case explicitly developed. However, a guidance how to simplify the notations for SISO case can be found in (*Chen, B.S., et al., 1996, Spooner, J.T., et al., 1996, Sue, C.Y., et al., 1994, Tong, S.C., et al., 1999, Chai, T.Y., et al., 1999*). In Chapter 6 a SISO simulation is carried out, where the pendulum angle of an inverted pendulum system is successfully controlled by the ORIAFC.

Following **control objectives** are faced for the controller design of the RIAFC and ORIAFC:

Determine a robust feedback control $\boldsymbol{u}=\boldsymbol{u}(\boldsymbol{\xi}|\boldsymbol{\Theta})$ *(based on fuzzy logic systems (3-1)) and an adaptive law for adjusting the parameter vector such that the following conditions are met:*

(i) All the signals and the estimated fuzzy parameters are uniformly bounded,

(ii) For a given disturbance attenuation level $\rho>0$ *the following* H_∞ *tracking performance is achieved:*

$$\int_0^T \boldsymbol{e}^T \boldsymbol{Q}\boldsymbol{e}dt \leq \boldsymbol{e}^T(0)\boldsymbol{P}\boldsymbol{e}(0) + \frac{1}{\gamma}\tilde{\boldsymbol{\Theta}}^T(0)\tilde{\boldsymbol{\Theta}}(0) + \rho^2 \int_0^T \boldsymbol{d}''^T \boldsymbol{d}''dt \,. \qquad (5\text{-}1)$$

$\boldsymbol{Q}=\boldsymbol{Q}^T>\boldsymbol{0}$, $\boldsymbol{P}=\boldsymbol{P}^T>\boldsymbol{0}$ *are positive definite weighting matrices,* $\boldsymbol{e}$ *is the control error vector,* $\gamma>0$ *is an adaptation gain,* $\tilde{\boldsymbol{\Theta}}$ *is the parameter approximation error vector,* $T \in [0,\infty)$ *and* $\boldsymbol{d}'' \in \boldsymbol{L}_2[0,T]$ *is the combined disturbance vector which is assumed to be square integrable.*

■

If the system starts with the initial conditions $\boldsymbol{e}(0)=\boldsymbol{0}$, $\tilde{\boldsymbol{\Theta}}(0)=\boldsymbol{0}$ then the performance in (5-1) can be rewritten as

$$\sup_{\boldsymbol{d}''\in \boldsymbol{L}_2[0,T]} \frac{\|\boldsymbol{e}\|_{\boldsymbol{Q}}^2}{\|\boldsymbol{d}''\|_2^2} \leq \rho^2 \,. \qquad (5\text{-}2)$$

Equation (5-2) with the notations $\|\boldsymbol{e}\|_{\boldsymbol{Q}}^2 = \int_0^T \boldsymbol{e}^T\boldsymbol{Q}\boldsymbol{e}dt$ and $\|\boldsymbol{d}''\|_2^2 = \int_0^T \boldsymbol{d}''^T\boldsymbol{d}''dt$ means that the L_2*- gain* from $\boldsymbol{d}''$ to the tracking error $\boldsymbol{e}$ must be equal to or less ρ.

5.2 Robust Indirect Adaptive Fuzzy Control Scheme

5.2.1 MIMO Nonlinear Plant Dynamics

Based on the observations and definitions of Chapter 4, the class of MIMO plants which can be controlled by the RIAFC and their mathematical descriptions are stated in this Section. Compact notations of these descriptions are presented to allow a better understanding of the developed algorithms.

The class of MIMO plants which can be controlled by the RIAFC can be defined by following assumption:

Assumption 5-1: *The system (4-1) has a uniform strong relative degree vector* $\boldsymbol{r}_u=[r_1,\dots,r_m]^T$ *over a compact set* $\boldsymbol{x}\in\boldsymbol{U}\subset\boldsymbol{R}^n$ *and its zero dynamics are exponentially attractive. Further* $\boldsymbol{\chi}(\boldsymbol{\xi},\boldsymbol{\eta})$ *in (4-16) is assumed to be Lipschitz in* $\boldsymbol{\xi},\boldsymbol{\eta}$.

■

Simply spoken, Assumption 5-1 guarantees bounded tracking for a class of feedback linearizable MIMO plants whose zero dynamics are stable. The definition for exponentially attractive zero dynamics will be given in the context of Bounded Tracking in Minimum-Phase Systems (Proposition 5-1) in Section 5.2.3.

For the design of the RIAFC only the external dynamics are relevant because the internal dynamics are unobservable from the output. Therefore equations (4-15) can be rewritten as

$$\begin{array}{l} \dot{\xi}_1^i=\xi_2^i \\ \quad\vdots \\ \dot{\xi}_{r_i-1}^i=\xi_{r_i}^i \end{array}, \qquad \begin{array}{l} y_1=\xi_1^1 \\ \quad\vdots \\ y_m=\xi_1^m \end{array} \qquad (i=1,2,..,m)$$

$$\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix}=\begin{bmatrix} f_1(\boldsymbol{\Psi}^{-1}(\xi,\boldsymbol{\eta})) \\ \vdots \\ f_m(\boldsymbol{\Psi}^{-1}(\xi,\boldsymbol{\eta})) \end{bmatrix}+\boldsymbol{G}(\boldsymbol{\Psi}^{-1}(\xi,\boldsymbol{\eta}))\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}+\begin{bmatrix} d_1(t) \\ \vdots \\ d_m(t) \end{bmatrix} \tag{5-3}$$

$$=\begin{bmatrix} f_1(\boldsymbol{x}) \\ \vdots \\ f_m(\boldsymbol{x}) \end{bmatrix}+\boldsymbol{G}(\boldsymbol{x})\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}+\begin{bmatrix} d_1(t) \\ \vdots \\ d_m(t) \end{bmatrix}$$

including a disturbance vector ***d(t)*** which is representing an external bounded disturbance. The state vector $\boldsymbol{x}$ and the output vector $\boldsymbol{y}$ are assumed to measurable. The new state vector $\boldsymbol{\xi}$ cannot be measured since the $L_{f'}^{r_i}h_i(\boldsymbol{x})$ in (4-20) are not explicit available. But note, because of measuring $\boldsymbol{y}$ also the new state variables ξ_1^i and as a result the whole vector $\boldsymbol{\xi}$ is known by calculating the derivatives of the variables ξ_1^i (derivate r_i-1 times). Therefore e.g. DT_1-elements with the Laplace transfer function

$$G(s)=\frac{sT_D}{1+sT_1}, \qquad T_D,\ T_1>0 \tag{5-4}$$

could be used, for practical aspects and constraints please refer to Section 5.3 (state observers). However, in the following the vector $\boldsymbol{f}$ and the matrix $\boldsymbol{G}$ are assumed to be partially or completely unknown and in particular the vectors $\boldsymbol{d}$, and the unobservable state vector $\boldsymbol{\eta}$ are assumed to be unknown.

For compact notations define

$$\begin{aligned} \boldsymbol{A} &= diag[\boldsymbol{A}_1,\dots,\boldsymbol{A}_m], \\ \boldsymbol{B} &= diag[\boldsymbol{b}_1,\dots,\boldsymbol{b}_m], \\ \boldsymbol{C}^T &= diag[\boldsymbol{c}_1,\dots,\boldsymbol{c}_m], \end{aligned} \tag{5-5}$$

and

$$\boldsymbol{A}_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B}_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \tag{5-6}$$

$$\boldsymbol{C}_i = [1 \quad 0 \quad \cdots \quad 0 \quad 0],$$

$$\boldsymbol{A}_i \in \boldsymbol{R}^{r_i \times r_i},\ \boldsymbol{b}_i \in \boldsymbol{R}^{r_i \times 1},\ \boldsymbol{c}_i \in \boldsymbol{R}^{1 \times r_i}.$$

Then equations (5-3) can be formulated as

$$\begin{aligned} \dot{\boldsymbol{\xi}} &= \boldsymbol{A}\boldsymbol{\xi} + \boldsymbol{B}[\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{d}] \\ \boldsymbol{y} &= \boldsymbol{C}^T\boldsymbol{\xi} \end{aligned}. \tag{5-7}$$

5.2.2 Lyapunov Stability

A very important aspect of the analysis of adaptive controllers is the investigation of their stability. In this thesis, the Second Method of Lyapunov (*Lyapunov, A.M., Fuller, A.T., 1992*) will be utilized for this purpose. This method is expressed as follows:

Theorem 5-1 *(Lyapunov Stability Theorem): Let* $\boldsymbol{x}=\boldsymbol{0}$ *be an equilibrium point of* $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$, $\boldsymbol{f} : D \to R^n$, *and let* $V : D \to R$ *be a continuously differentiable function such that*
(i) $V(\boldsymbol{0})=0$,
(ii) $V(\boldsymbol{x})>0$ *in* $D-\{\boldsymbol{0}\}$,
(iii) $\dot{V}(\boldsymbol{x}) \le 0$ *in* $D-\{\boldsymbol{0}\}$,
thus $\boldsymbol{x}=\boldsymbol{0}$ *is stable.*
■

A stronger stability condition, which is applied in Section 5.2.4 and Section 5.3.4, represents *asymptotic stability*:

Theorem 5-2 *(Assymptotic Stability Theorem): Under the conditions of Theorem 5-1,* $V(\cdot)$ *is such that*

(i) $V(\mathbf{0})=0$,
(ii) $V(\mathbf{x})>0$ *in* $D-\{\mathbf{0}\}$,
(iii) $\dot{V}(\mathbf{x})<0$ *in* $D-\{\mathbf{0}\}$,
thus $\mathbf{x}=\mathbf{0}$ *is asymtotically stable.*

■

The biggest problem with the Lyapunov function method is that it is often difficult to find a suitable Lyapunov function. There is no single systematic procedure which will always finde one (if it exists), however, there are a number of methods form guessing them. Two of such methods are the variable gradient method of Schultz and Gibson (*Schultz, G.D. and Gibson, J.E., 1962*) and the Zubovs method (*Zubov, V.I., 1957*).

5.2.3 Robust Indirect Adaptive Fuzzy Controller (RIAFC)

An interesting application of the notion of normal form and minimum-phase property is following one. Assume the control $\boldsymbol{v}$ in (4-20) is chosen so that system outputs $y_i(t)$ track the reference trajectories defined by $y_{M,i}(t)$, $i=1,2,\ldots,m$, with the help of linear external controllers v_i, i.e.

$$\begin{aligned} &\boldsymbol{v} = [v_1,\ldots,v_m]^T \\ &v_i = y_{M,i}^{(r_i)} - k_{ir_i}(y_i^{(r_i-1)} - y_{M,i}^{(r_i-1)}) - \ldots - k_{i1}(y_i - y_{M,i}) \\ &(i = 1,2,\ldots,m;\ j = 1,2,\ldots,r_i) \end{aligned} \qquad (5\text{-}8)$$

with the coefficients k_{ij} chosen such that the polynomials

$$s_i^{(r_i)} + k_{ir_i} s_i^{(r_i-1)} + \ldots + k_{i1} = 0 \qquad (5\text{-}9)$$

are Hurwitz polynomials and the tracking errors are defined as

$$e_i = y_i - y_{M,i}\,. \qquad (5\text{-}10)$$

The resulting error dynamics (=error system) follow then to

$$\begin{aligned} e_1^{(r_1)} + k_{1r_1} e_1^{(r_1-1)} + \ldots + k_{11} e_1 = 0 \\ \vdots \qquad\qquad\qquad . \\ e_m^{(r_m)} + k_{mr_m} e_m^{(r_m-1)} + \ldots + k_{m1} e_m = 0 \end{aligned} \qquad (5\text{-}11)$$

If the coefficients $k_{i,j}$ are chosen such that all the polynomials in (5-11) are Hurwitz, then we can conclude that $lim_{t\to\infty} e_i(t) = 0$ - a main objective in control.

Remarks 5-1: The reference trajectories $y_{M,i}(t)$ are assumed to have the property of standard smoothness to guarantee bounded derivatives up to order r_i.■

Proposition 5-1: *Bounded Tracking in Minimum-Phase Systems* (*Sastry, S., Isidori, A.., 1989): Assume that the zero dynamics of the nonlinear system (4-1) or equivalently (4-15) and (4-16) are exponentially stable. Further assume that* $\boldsymbol{\chi}(\boldsymbol{\xi},\boldsymbol{\eta})$ *in (4-16) is Lipschitz in* $\boldsymbol{\xi},\boldsymbol{\eta}$*. Then the control law* (4-20) *results in bounded tracking [i.e.,* $\boldsymbol{x}\in\boldsymbol{R}^n$ *bounded and* $y_i(t)\rightarrow y_{M,i}(t)$*], provided that* $\boldsymbol{y}_{M,i}, \dot{\boldsymbol{y}}_{M,i}, \ldots, \boldsymbol{y}_{M,i}^{(r_i)}$ *are bounded.*

Proof: Converse Lyapunov theorem, refer to (*Sastry, S., Isidori, A.., 1989)*.

■

Remarks 5-2:

- Proposition 5-1 establishes that a bounded input to the exponentially stable, unobservable dynamics yields a bounded state trajectory $\boldsymbol{\eta}$.

- The hypothesis of Proposition 5-1 calls for a strong form of stability – exponential stability; in fact, counterexamples to the Proposition exist if the zero-dynamics are not exponential stable, for example, if some of the eigenvalues of $\partial/\partial\boldsymbol{\eta}\,\boldsymbol{\chi}(\boldsymbol{0},\boldsymbol{\eta})$ lie on the $j\omega$-axis.

- The hypothesis of Proposition 5-1 can, however be weakened substantially by requiring only that all trajectories of (4-25) are eventually *attracted to a compact set*, by requiring as one necessary condition the existence of a Lyapunov function $V(\boldsymbol{\eta})$ which fulfills

$$\frac{dV}{d\boldsymbol{\eta}}\cdot\boldsymbol{\chi}(\boldsymbol{0},\boldsymbol{\eta})\le-\alpha\|\boldsymbol{\eta}\|_F^2 \qquad \text{only for } \|\boldsymbol{\eta}\|_F\ge R \qquad (5\text{-}12)$$

  with the weakening that requiring that the left equation has only to hold outside a ball of radius R. We refer this condition as *exponential boundedness of the zero dynamics.*
  Proof: Converse Lyapunov theorem, (*Sastry, S., Isidori, A.., 1989)*.

■

The observations above lead directly to the definition of the class of MIMO pants which can be controlled by the RIAFC in form of Assumption 5-1 in Section 5.2.1. Moreover, they guarantee bounded tracking for this class of MIMO plants if control law (4-20) is applied to the feedback linearizable external dynamics, assuming that the zero dynamics are exponentially attractive.

In practical implementations of exactly linearizing control laws, the chief drawback is that they are based on exact cancellations of nonlinear terms. If there is any uncertainty, the resulting input-output equation is not linear. I suggest the use of adaptive fuzzy control to get asymptotically exact cancellation, the advantages of this concept were already summarized in Section 3.2.

Since the functions $f_i(\boldsymbol{x})$ and $g_{i,j}(\boldsymbol{x})$ are unknown and $d_i \neq 0$ in our problem, the ideal control (4-20) cannot be applied. In this situation, our purpose is to approximate $f_i(\boldsymbol{x})$ and $g_{i,j}(\boldsymbol{x})$ by using scalar fuzzy logic systems $\hat{f}_i(\boldsymbol{x}|\boldsymbol{\theta}_{f,i})$ and $\hat{g}_{ij}(\boldsymbol{x}|\boldsymbol{\theta}_{g,ij})$ defined as according to (3-1), with linear paramterization, as

$$\begin{aligned}\hat{f}_i(\boldsymbol{x}|\boldsymbol{\theta}_{f,i}) &= \boldsymbol{\varphi}_{f,i}(\boldsymbol{x})^T\boldsymbol{\theta}_{f,i},\\ \hat{g}_{ij}(\boldsymbol{x}|\boldsymbol{\theta}_{g,ij}) &= \boldsymbol{\varphi}_{g,ij}(\boldsymbol{x})^T\boldsymbol{\theta}_{g,ij}\end{aligned} \tag{5-13}$$

where $\boldsymbol{\varphi}_{f,i}(\boldsymbol{x}),\boldsymbol{\theta}_{f,i} \in \boldsymbol{R}^{m_f}; \boldsymbol{\varphi}_{g,ij}(\boldsymbol{x}),\boldsymbol{\theta}_{g,ij} \in \boldsymbol{R}^{m_G}$ and m_f, m_G are the total number of fuzzy rules for modelling the functions f_i and g_{ij}, respectively.

The controller is now chosen as a certainty equivalence control

$$\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = \begin{bmatrix} \hat{g}_{11}(\boldsymbol{x}|\boldsymbol{\theta}_{g,11}) & \cdots & \hat{g}_{1m}(\boldsymbol{x}|\boldsymbol{\theta}_{g,1m}) \\ \vdots & \ddots & \vdots \\ \hat{g}_{m1}(\boldsymbol{x}|\boldsymbol{\theta}_{g,m1}) & \dots & \hat{g}_{mm}(\boldsymbol{x}|\boldsymbol{\theta}_{g,mm}) \end{bmatrix}^{-1} \left(-\begin{bmatrix} \hat{f}_1(\boldsymbol{x}|\boldsymbol{\theta}_{f,1}) \\ \vdots \\ \hat{f}_m(\boldsymbol{x}|\boldsymbol{\theta}_{f,m}) \end{bmatrix} + \begin{bmatrix} \nu_1 \\ \vdots \\ \nu_m \end{bmatrix} \right). \tag{5-14}$$

***Assumption 5-2**: It is pre-assumed that the matrix $\hat{\boldsymbol{G}}(\boldsymbol{x}|\boldsymbol{\theta}_{g,ij}) = matrix(\hat{g}_{ij})$ is invertible (non singular). Later this assumption will be relaxed by the projection algorithm of 5.5 – Modified adaptation laws.*

■

Due to existing fuzzy approximation errors, external disturbances and measurement noise, only a certainty equivalence control cannot ensure the stability of the closed-loop system. Therefore it is necessary to add a robust compensator to attenuate the disturbance effect on the system outputs and a VSS (Variable Structure Systems) control term to compensate the fuzzy approximation errors. The resulting fuzzy control law is

$$\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = \hat{\boldsymbol{G}}^{-1}(\boldsymbol{x}|\boldsymbol{\theta}_{g,ij}) \left(-\begin{bmatrix} \hat{f}_1(\boldsymbol{x}|\boldsymbol{\theta}_{f,1}) \\ \vdots \\ \hat{f}_m(\boldsymbol{x}|\boldsymbol{\theta}_{f,m}) \end{bmatrix} + \begin{bmatrix} \nu_1 \\ \vdots \\ \nu_m \end{bmatrix} + \begin{bmatrix} u_{h,1} \\ \vdots \\ u_{h,m} \end{bmatrix} + \begin{bmatrix} u_{s,1} \\ \vdots \\ u_{s,m} \end{bmatrix} \right). \tag{5-15}$$

The exact formulation of the control terms $u_{h,i}$ and $u_{s,i}$ will be stated in Section 5.2.4 as a result of the stability analysis.

Denote

$$\begin{aligned}\boldsymbol{E}_1 &= [e_1,\dots,e_m]^T\\ \boldsymbol{y}_M &= [y_{M,1},\dots,y_{M,m}]^T\\ \boldsymbol{y}_M^{(r^*)} &= \left[y_{M,1}^{(r_1)},\dots,y_{M,m}^{(r_m)}\right]^T\end{aligned} \tag{5-16}$$

and

$$\begin{aligned}\boldsymbol{\xi} &= \left[\xi_1^1,\dots,\xi_1^{1(r_1-1)},\dots,\xi_1^m,\dots,\xi_1^{m(r_m-1)}\right]^T = \left[\xi_1,\xi_2,\dots,\xi_r\right]^T,\ \boldsymbol{\xi}\in\boldsymbol{R}^{r\times 1}\\ \boldsymbol{Y}_M &= \left[y_{M,1},\dots,y_{M,1}^{(r_1-1)},\dots,y_{M,m},\dots,y_{M,m}^{(r_m-1)}\right]^T.\end{aligned} \tag{5-17}$$

The error vector e is assumed as

$$\boldsymbol{e} = \boldsymbol{\xi} - \boldsymbol{Y}_M = [e_1,\dots,e_1^{(r_1-1)},\dots,e_m,\dots,e_m^{(r_m-1)}]. \tag{5-18}$$

Also the fuzzy models (5-13) can be rewritten in compact form

$$\begin{aligned}\hat{\boldsymbol{f}} &= \boldsymbol{\Phi}_f(\boldsymbol{x})\boldsymbol{\Theta}_f\\ \hat{\boldsymbol{G}} &= \boldsymbol{\Phi}_G(\boldsymbol{x})\boldsymbol{\Theta}_G\end{aligned} \tag{5-19}$$

where

$$\begin{aligned}\boldsymbol{\Phi}_f(\boldsymbol{x}) &= diag\left[\boldsymbol{\varphi}_{f,1}{}^T,\dots,\boldsymbol{\varphi}_{f,m}{}^T\right]\\ \boldsymbol{\Phi}_G(\boldsymbol{x}) &= \left[\boldsymbol{\Phi}_{G,1},\dots,\boldsymbol{\Phi}_{G,m}\right]\\ \boldsymbol{\Phi}_{G,i}(\boldsymbol{x}) &= diag\left[\boldsymbol{\varphi}_{g,i1}{}^T,\dots,\boldsymbol{\varphi}_{g,im}{}^T\right]\\ \boldsymbol{\Theta}_f &= \left[\boldsymbol{\theta}_{f,1}{}^T,\dots,\boldsymbol{\theta}_{f,m}{}^T\right]^T\\ \boldsymbol{\Theta}_G &= diag\left[\boldsymbol{\Theta}_{G,1},\dots,\boldsymbol{\Theta}_{G,m}\right]\\ \boldsymbol{\Theta}_{G,i} &= \left[\boldsymbol{\theta}_{g,i1}{}^T,\dots,\boldsymbol{\theta}_{g,im}{}^T\right]^T.\end{aligned} \tag{5-20}$$

Including the (unknown) bounded noise vectors $\boldsymbol{n}_y(t)$ and $\boldsymbol{n}_x(t)$ which are representing the noise due to measurement of the outputs and the original states, where $\boldsymbol{n}_y(t)$ is assumed to have the property of standard smoothness

$$\begin{matrix}\bar{y}_1 = y_1 + n_{y,1}(t) = \xi_1^1 + n_{y,1}(t) = \bar{\xi}_1^1 & & \bar{x}_1 = x_1 + n_{x,1}(t)\\ \vdots & , & \vdots\\ \bar{y}_m = y_m + n_{y,m}(t) = \xi_1^m + n_{y,m}(t) = \bar{\xi}_1^m & & \bar{x}_n = x_n + n_{x,n}(t)\end{matrix} \tag{5-21}$$

the error vector $\boldsymbol{e}$ can be reformulated as

$$\begin{aligned}\bar{e}_i &= \bar{y}_i - y_{M,i} = \bar{\xi}_1^i - y_{M,i} = e_i + n_{y,i}\\ \bar{\boldsymbol{e}} &= \bar{\boldsymbol{\xi}} - \boldsymbol{Y}_M = \left[\bar{e}_1,\dots,\bar{e}_1^{(r_1-1)},\dots,\bar{e}_i,\dots,\bar{e}_i^{(r_i-1)},\dots,\bar{e}_m,\dots,\bar{e}_m^{(r_m-1)}\right] = \boldsymbol{e} + \boldsymbol{N}_y\\ \boldsymbol{N}_y &= \left[n_{y,1},\dots,n_{y,1}^{(r_1-1)},\dots,n_{y,i},\dots,n_{y,i}^{(r_i-1)},\dots,n_{y,m},\dots,n_{y,m}^{(r_m-1)}\right].\end{aligned} \tag{5-22}$$

Remarks 5-3: If required, the $\bar{y}_i$ could be filtered by a low-pass filter or another slew-rate limiter to bound the derivatives of the noise signals.

■

The control law (4-15) can be rewritten as follows

$$\boldsymbol{u} = \hat{\boldsymbol{G}}^{-1}(\bar{\boldsymbol{x}} \mid \boldsymbol{\Theta}_G)[-\hat{\boldsymbol{f}}(\bar{\boldsymbol{x}} \mid \boldsymbol{\Theta}_f) + \boldsymbol{y}_M^{(r^*)} - \boldsymbol{K}^T\bar{\boldsymbol{e}} + \boldsymbol{u}_h + \boldsymbol{u}_s] \tag{5-23}$$

with

$\boldsymbol{K}^T = [\boldsymbol{k}_r, \boldsymbol{k}_{r-1}, ..., \boldsymbol{k}_1]$ as feedback gain matrix to make the characteristic polynomials of $\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}^T$ to be Hurwitz , because *(**A**,**B**)* is *controllable* concerning the state vector $\boldsymbol{\xi}$, and the fuzzy models

$$\begin{aligned} \hat{\boldsymbol{f}}(\bar{\boldsymbol{x}} \mid \boldsymbol{\Theta}_f) &= \boldsymbol{\Phi}_f(\bar{\boldsymbol{x}})\boldsymbol{\Theta}_f \\ \hat{\boldsymbol{G}}(\bar{\boldsymbol{x}} \mid \boldsymbol{\Theta}_G) &= \boldsymbol{\Phi}_G(\bar{\boldsymbol{x}})\boldsymbol{\Theta}_G . \end{aligned} \tag{5-24}$$

Definition 5-1: *Controllability: (**A**,**B**) is controllable if* $rank(\boldsymbol{A} \vdots \boldsymbol{AB} \vdots \boldsymbol{A}^2\boldsymbol{B} \vdots \cdots \vdots \boldsymbol{A}^{r-1}\boldsymbol{B}) = r$.

■

In general the grid of rules and therefore the complexity of the fuzzy models grows dramatically with the order of the plant (order of the state vector) and among experts this phenomenon is called “the curse of dimensionality”. Concerning practical realizations of complex fuzzy systems Section 5.6 – Dynamic fuzzy rule activation method – shows how the real number of activated fuzzy rules can effectively be reduced.

During this Section the structures of an adaptive fuzzy controller and the corresponding adaptive fuzzy systems were defined. The missing adaptation laws for the parameter update of the fuzzy systems and the precise form of the control terms $u_{h,i}$ and $u_{s,i}$ are stated in the next Section.
Fig. 5-1 gives an overview of the Robust Indirect Adaptive Fuzzy Controller (RIAFC) - based tracking control scheme.

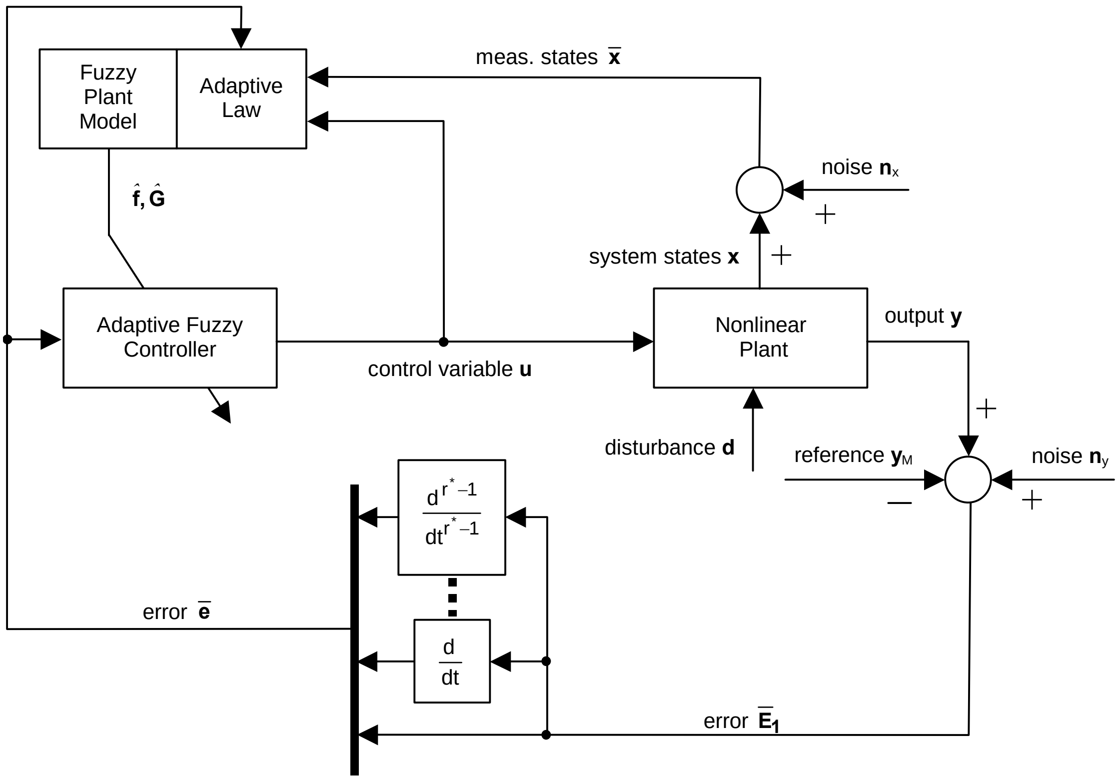

Fig. 5-1: RIAFC-based tracking control scheme

5.2.4 Stability- and Convergence Analysis of the RIAFC

Substituting (5-23) in (5-7) and set $A_0 = A - BK^T$ yields

$$\dot{e} = A_0 e + B[\, f(x) - \hat{f}(\bar{x} \mid \Theta_f) + (G(x) - \hat{G}(\bar{x} \mid \Theta_G))u + u_h + u_s + d - K^T N_y \,] \,. \quad (5\text{-}25)$$

The disturbance vector d and the vector $-K^T N_y$ can be collected in one combined disturbance vector

$$d' = d - K^T N_y \,. \quad (5\text{-}26)$$

Define the *optimal parameter vector* Θ_f^* and the *optimal parameter matrix* Θ_G^*

$$\begin{aligned} \Theta_f^* &= arg \min_{\Theta_f \in \Omega_f} \left\{ \sup_{x \in U_1, \bar{x} \in U_2} \left\| f(x) - \hat{f}(\bar{x} \mid \Theta_f) \right\|_F \right\} \\ \Theta_G^* &= arg \min_{\Theta_G \in \Omega_G} \left\{ \sup_{x \in U_1, \bar{x} \in U_2} \left\| G(x) - \hat{G}(\bar{x} \mid \Theta_G) \right\|_F \right\} \end{aligned}, \quad (5\text{-}27)$$

the minimum (fuzzy) approximation error ω

$$\begin{aligned}\boldsymbol{\omega} &= \boldsymbol{f}(\boldsymbol{x}) - \hat{\boldsymbol{f}}(\boldsymbol{x}\,|\,\boldsymbol{\Theta}_f^{\,*}) + \hat{\boldsymbol{f}}(\overline{\boldsymbol{x}}\,|\,\boldsymbol{\Theta}_f^{\,*}) - \hat{\boldsymbol{f}}(\boldsymbol{x}\,|\,\boldsymbol{\Theta}_f^{\,*}) + (\boldsymbol{G}(\boldsymbol{x}) - \hat{\boldsymbol{G}}(\boldsymbol{x}\,|\,\boldsymbol{\Theta}_G^{\,*}) \\ &+ \hat{\boldsymbol{G}}(\overline{\boldsymbol{x}}\,|\,\boldsymbol{\Theta}_G^{\,*}) - \hat{\boldsymbol{G}}(\boldsymbol{x}\,|\,\boldsymbol{\Theta}_G^{\,*}))\boldsymbol{u} = \boldsymbol{\Delta f} + \boldsymbol{\Delta G u}\end{aligned} \tag{5-28}$$

and the estimation errors

$$\begin{aligned}\tilde{\boldsymbol{\Theta}}_f &= \boldsymbol{\Theta}_f - \boldsymbol{\Theta}_f^{\,*} \\ \tilde{\boldsymbol{\Theta}}_G &= \boldsymbol{\Theta}_G - \boldsymbol{\Theta}_G^{\,*}.\end{aligned} \tag{5-29}$$

It is pre-assumed that $\boldsymbol{x}$, $\overline{\boldsymbol{x}}$, $\boldsymbol{\Theta}_f$, and $\boldsymbol{\Theta}_G$ belong to compact sets $\boldsymbol{U}_1$, $\boldsymbol{U}_2$, $\boldsymbol{\Omega}_f$, $\boldsymbol{\Omega}_G$, respectively, which are defined as

$$\begin{aligned}\boldsymbol{U}_1 &= \left\{\boldsymbol{x} \in \boldsymbol{R}^n : \|\boldsymbol{x}\|_F \le \boldsymbol{M}_x\right\} \\ \boldsymbol{U}_2 &= \left\{\overline{\boldsymbol{x}} \in \boldsymbol{R}^n : \|\overline{\boldsymbol{x}}\|_F \le \boldsymbol{M}_{\overline{x}}\right\} \\ \boldsymbol{\Omega}_f &= \left\{\boldsymbol{\Theta}_f \in \boldsymbol{R}^{m_f m\times 1} : \|\boldsymbol{\Theta}_f\|_F \le \boldsymbol{M}_{\boldsymbol{\Theta}_f}\right\} \\ \boldsymbol{\Omega}_G &= \left\{\boldsymbol{\Theta}_G \in \boldsymbol{R}^{m_G m^2\times m} : \|\boldsymbol{\Theta}_G\|_F \le \boldsymbol{M}_{\boldsymbol{\Theta}_G}\right\}.\end{aligned} \tag{5-30}$$

Equation (5-25) turns out as

$$\dot{\boldsymbol{e}} = \boldsymbol{A}_0\boldsymbol{e} + \boldsymbol{B}[-\boldsymbol{\Phi}_f\tilde{\boldsymbol{\Theta}}_f - \boldsymbol{\Phi}_G\tilde{\boldsymbol{\Theta}}_G\boldsymbol{u} + \boldsymbol{\Delta f} + \boldsymbol{\Delta G u} + \boldsymbol{u}_h + \boldsymbol{u}_s + \boldsymbol{d}'] \tag{5-31}$$

Consider the Lyapunov function candidate

$$\begin{aligned}V &= \frac{1}{2}\boldsymbol{e}^T\boldsymbol{P}\boldsymbol{e} + \frac{1}{2\gamma_f}\tilde{\boldsymbol{\Theta}}_f^{\,T}\tilde{\boldsymbol{\Theta}}_f + \frac{1}{2\gamma_G}tr(\tilde{\boldsymbol{\Theta}}_G^{\,T}\tilde{\boldsymbol{\Theta}}_G) \\ \boldsymbol{P} &= \boldsymbol{P}^T > 0 \quad \gamma_f, \gamma_G > 0.\end{aligned} \tag{5-32}$$

Following notations are made for simplification: $\hat{\boldsymbol{f}} \overset{\Delta}{=} \hat{\boldsymbol{f}}(\overline{\boldsymbol{x}}\,|\,\boldsymbol{\Theta}_f)$ and $\hat{\boldsymbol{G}} \overset{\Delta}{=} \hat{\boldsymbol{G}}(\overline{\boldsymbol{x}}\,|\,\boldsymbol{\Theta}_G)$.

The time derivative of V is

$$\begin{aligned}\dot{V} &= \frac{1}{2}\boldsymbol{e}^T(\boldsymbol{A}_0^{\,T}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_0)\boldsymbol{e} + \boldsymbol{u}_h^{\,T}(\boldsymbol{I} + \boldsymbol{\Delta G}\hat{\boldsymbol{G}}^{-1})^T\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e} + \boldsymbol{d}'^T\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e} \\ &+ \boldsymbol{u}_s^{\,T}(\boldsymbol{I} + \boldsymbol{\Delta G}\hat{\boldsymbol{G}}^{-1})^T\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e} + [\boldsymbol{\Delta F} + \boldsymbol{\Delta G}\hat{\boldsymbol{G}}^{-1}(-\hat{\boldsymbol{f}} + \boldsymbol{y}_M^{(r^*)} - \boldsymbol{K}^T\boldsymbol{e})]^T\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e} \\ &- \tilde{\boldsymbol{\Theta}}_f^{\,T}\boldsymbol{\Phi}_f^{\,T}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e} - \boldsymbol{u}^T\tilde{\boldsymbol{\Theta}}_G^{\,T}\boldsymbol{\Phi}_G^{\,T}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e} + \frac{1}{2\gamma_f}\dot{\tilde{\boldsymbol{\Theta}}}_f^{\,T}\tilde{\boldsymbol{\Theta}}_f + \frac{1}{2\gamma_G}tr(\dot{\tilde{\boldsymbol{\Theta}}}_G^{\,T}\tilde{\boldsymbol{\Theta}}_G).\end{aligned} \tag{5-33}$$

Assumption 5-3: *There exist positive constants κ, M_g and M_e such that*

$$\left|\lambda_i [\Delta \boldsymbol{G}(\boldsymbol{x},\overline{\boldsymbol{x}})\hat{\boldsymbol{G}}^{-1}(\overline{\boldsymbol{x}}\,|\,\boldsymbol{\Theta}_G)]\right| \le \kappa, \qquad 0 \le \kappa < 1$$
$$\left\|\Delta \boldsymbol{G}(\boldsymbol{x},\overline{\boldsymbol{x}})\hat{\boldsymbol{G}}^{-1}(\overline{\boldsymbol{x}}\,|\,\boldsymbol{\Theta}_G)\right\|_\infty = \max_i \left\{ [\sum_{j=1}^{p} \left| [\Delta \boldsymbol{G}(\boldsymbol{x},\overline{\boldsymbol{x}})\hat{\boldsymbol{G}}^{-1}(\overline{\boldsymbol{x}}\,|\,\boldsymbol{\Theta}_G)]_{ij} \right| \right\} \le M_g,$$
$$0 \le M_g < 1 \tag{5-34}$$
$$\left| [\Delta \boldsymbol{F}(\boldsymbol{x},\overline{\boldsymbol{x}}) + \Delta \boldsymbol{G}(\boldsymbol{x},\overline{\boldsymbol{x}})\hat{\boldsymbol{G}}^{-1}(\overline{\boldsymbol{x}}\,|\,\boldsymbol{\Theta}_G)(-\hat{\boldsymbol{f}}(\overline{\boldsymbol{x}}\,|\,\boldsymbol{\Theta}_f) + \boldsymbol{y}_M^{(r^*)} - \boldsymbol{K}^T\boldsymbol{e})]_i \right| \le M_e.$$

■

Assume the H_∞ and VSS control terms as

$$\boldsymbol{u}_h = -\frac{1}{2}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e}, \quad \boldsymbol{R} = diag[r_1,..,r_m] \in \boldsymbol{R}^{m\times m}, \ \ r_1,...,r_m > 0$$
$$\boldsymbol{u}_s = -\frac{M_e}{1-M_g}\boldsymbol{sign}(\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e}) = -k\boldsymbol{sign}(\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e}). \tag{5-35}$$

Considering that only the biased vector $\overline{\boldsymbol{e}} = \boldsymbol{e} + \boldsymbol{N}_y$ can be measured and implemented in (5-35) instead of $\boldsymbol{e}$, equation (5-33) results with $\dot{\tilde{\boldsymbol{\Theta}}}_f = \dot{\boldsymbol{\Theta}}_f$ and $\dot{\tilde{\boldsymbol{\Theta}}}_G = \dot{\boldsymbol{\Theta}}_G$ ($\boldsymbol{\Theta}_f^{\,*}$ and $\boldsymbol{\Theta}_G^{\,*}$ are assumed to be constants)

and using

$$\boldsymbol{u}_s^{\,T}(\boldsymbol{I} + \Delta\boldsymbol{G}\hat{\boldsymbol{G}}^{-1})^T\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e} + [\Delta\boldsymbol{F} + \Delta\boldsymbol{G}\hat{\boldsymbol{G}}^{-1}(-\hat{\boldsymbol{f}} + \boldsymbol{y}_M^{(r^*)} - \boldsymbol{K}^T\boldsymbol{e})]^T\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e} \le 0 \tag{5-36}$$

as follows

$$\dot{V} \le \frac{1}{2}\boldsymbol{e}^T[\boldsymbol{A}_0^{\,T}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_0 - \boldsymbol{P}\boldsymbol{B}(1-\kappa)\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}]\boldsymbol{e} + \boldsymbol{d}''^T\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e} - \tilde{\boldsymbol{\Theta}}_f^{\,T}\boldsymbol{\Phi}_f^{\,T}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e}.$$
$$-\boldsymbol{u}^T\tilde{\boldsymbol{\Theta}}_G^{\,T}\boldsymbol{\Phi}_G^{\,T}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e} + \frac{1}{2\gamma_f}\dot{\tilde{\boldsymbol{\Theta}}}_f^{\,T}\tilde{\boldsymbol{\Theta}}_f + \frac{1}{2\gamma_G}tr(\dot{\tilde{\boldsymbol{\Theta}}}_G^{\,T}\tilde{\boldsymbol{\Theta}}_G) \tag{5-37}$$
$$\boldsymbol{d}'' = \boldsymbol{d}' - \frac{1}{2}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}\,\boldsymbol{N}_y.$$

If the parameter ideal update laws are chosen to

$$\dot{\boldsymbol{\Theta}}_f = \gamma_f\boldsymbol{\Phi}_f^{\,T}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e}$$
$$\dot{\boldsymbol{\Theta}}_{G,i} = \gamma_G\boldsymbol{\Phi}_{G,i}^{\,T}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{e}u_i, \quad (i = 1,2,...,m) \tag{5-38}$$

the Lyapunov derivative can be simplified to

$$\dot{V} \le \frac{1}{2}\boldsymbol{e}^T [\boldsymbol{A_0}^T \boldsymbol{P} + \boldsymbol{PA_0} - \boldsymbol{PB}(1-\kappa)\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}]\boldsymbol{e} + \boldsymbol{d}''^T\boldsymbol{B}^T\boldsymbol{Pe} . \qquad (5\text{-}39)$$

Remarks 5-4:

- The adaptation rates $\gamma_{f,i}$, $\gamma_{G,ij}$ can be properly chosen for each element $\hat{f}_i$ and $\hat{g}_{ij}$ individually. Only for simplified notation they were stated as γ_f, γ_G.
- Exactly observed, also the update laws are biased by noise but due to low-pass filtering of the resulting functions $\hat{\boldsymbol{f}}$ and $\hat{\boldsymbol{G}}$ this effect can again be eliminated.

■

Proposition 5-2: *For a given positive-definite matrix* $\boldsymbol{Q}$ *there exists a unique positive-definite solution* $\boldsymbol{P}$ *for the Riccati-like matrix equation*

$$\begin{aligned} &\boldsymbol{A_0}^T \boldsymbol{P} + \boldsymbol{PA_0} - \boldsymbol{PB}[(1-\kappa)\boldsymbol{R}^{-1} - \frac{1}{\rho^2}\boldsymbol{I}]\boldsymbol{B}^T\boldsymbol{P} + \boldsymbol{Q} = \boldsymbol{0}, \\ &\boldsymbol{P},\boldsymbol{Q} \in \boldsymbol{R}^{r\times r}, \quad \boldsymbol{P} = \boldsymbol{P}^T > \boldsymbol{0}, \boldsymbol{Q} = \boldsymbol{Q}^T > \boldsymbol{0} \end{aligned} \qquad (5\text{-}40)$$

if and only if $(1-\kappa)\boldsymbol{R}^{-1} - \frac{1}{\rho^2}\boldsymbol{I} > 0$.

■

Combining (5-39)and (5-40) results in

$$\begin{aligned} \dot{V} &\le -\frac{1}{2}\boldsymbol{e}^T\boldsymbol{Qe} - \frac{1}{2\rho^2}\boldsymbol{e}^T\boldsymbol{PBB}^T\boldsymbol{Pe} + \boldsymbol{d}''^T\boldsymbol{B}^T\boldsymbol{Pe} \\ &= -\frac{1}{2}\boldsymbol{e}^T\boldsymbol{Qe} - \frac{1}{2}[\frac{1}{\rho}\boldsymbol{B}^T\boldsymbol{Pe} - \rho\boldsymbol{d}'']^T[\frac{1}{\rho}\boldsymbol{B}^T\boldsymbol{Pe} - \rho\boldsymbol{d}''] + \frac{1}{2}\rho^2\boldsymbol{d}''^T\boldsymbol{d}'' \\ &= -\frac{1}{2}\boldsymbol{e}^T\boldsymbol{Qe} + \frac{1}{2}\rho^2\boldsymbol{d}''^T\boldsymbol{d}'' . \end{aligned} \qquad (5\text{-}41)$$

As long as $\boldsymbol{d}'' \in L_2$ and the fuzzy parameters $\boldsymbol{\Theta}_f$ and $\boldsymbol{\Theta}_G$ are bounded (refer to Section 5.5 – Modified adaptation laws) it is included that $\boldsymbol{e}, \bar{\boldsymbol{x}}, \boldsymbol{x}, \boldsymbol{u} \in L_\infty$ and $lim_{t\to\infty} \boldsymbol{E}_1 = \boldsymbol{0}$, (*Wang, L.X.*, *1993*).
Thus the control objective (i) is realized.

Integrating inequality (5-41) from $t=0$ to $t=T$ yields

$$V(T) - V(0) \le -\frac{1}{2}\int_0^T \boldsymbol{e}^T\boldsymbol{Qe}dt + \frac{1}{2}\rho^2\int_0^T \boldsymbol{d}''^T\boldsymbol{d}''dt \qquad 0 \le T < \infty . \qquad (5\text{-}42)$$

Since $V(T) \geq 0$, inequality (5-42) implies

$$\frac{1}{2}\int_0^T \boldsymbol{e}^T \boldsymbol{Q}\boldsymbol{e}dt \leq \frac{1}{2}\boldsymbol{e}^T(0)\boldsymbol{P}\boldsymbol{e}(0) + \frac{1}{2\gamma_f}\tilde{\boldsymbol{\Theta}}_f^{\,T}(0)\tilde{\boldsymbol{\Theta}}_f(0) + \frac{1}{2\gamma_G}tr[\tilde{\boldsymbol{\Theta}}_G^{\,T}(0)\tilde{\boldsymbol{\Theta}}_G(0)] + \frac{1}{2}\rho^2\int_0^T \boldsymbol{d}''^T\boldsymbol{d}''dt \qquad (5\text{-}43)$$

which represents the H_∞-criterion given in equation (5-1) for a pre-described attenuation level ρ. Finally, also the control objective (ii) is realized.

Remarks 5-5:

To avoid discontinuity of $\boldsymbol{sign(B^T Pe)}$ which could yield undesirable chattering phenomenon and excite high-frequency unmodeled modes (*Ioannou, P.A., and Sun, J., 1996*), the ***sign(.)*** function is replaced by saturation functions $sat[(.)_i]$, defined as

$$sat\left[(\boldsymbol{B}^T\boldsymbol{Pe})_i\right] = \begin{cases} sign\left[(\boldsymbol{B}^T\boldsymbol{Pe})_i\right] & if\ \left|(\boldsymbol{B}^T\boldsymbol{Pe})_i\right| > \alpha \\ \frac{1}{\alpha}(\boldsymbol{B}^T\boldsymbol{Pe})_i & if\ \left|(\boldsymbol{B}^T\boldsymbol{Pe})_i\right| \leq \alpha \end{cases} \qquad (5\text{-}44)$$

$\alpha > 0$, $i = 1,2,...,m$

or smooth functions $tanh\left[\frac{(\boldsymbol{B}^T\boldsymbol{P}_2\boldsymbol{e})_i}{\alpha}\right]$, $\alpha > 0$, $i = 1,2,...,m$ (5-45)

where the function "*tanh*" means the "hyperbolic tangent" of its argument and α is a small design constant in order to remedy the control chattering.
■

In summarizing the above discussions, the design algorithm for the RIAFC is described as follows (Tab. 5-1).

[Step.1]	Select the feedback gain matrix $\boldsymbol{K}$ such that the matrix $\boldsymbol{A}$-$\boldsymbol{BK}^T$ is a Hurwitz matrix. Choose a positive-definite matrix $\boldsymbol{Q}$ and select the desired attenuation level ρ, the weighting matrix $\boldsymbol{R}$ and the constant κ to solve Lyapunov equation (5-40) in order to get a positive-definite matrix $\boldsymbol{P}$.
[Step.2]	Choose appropriate values for the controller parameters k, α and in (5-35), (5-45), and the parameters for fuzzy modelling $\gamma_{f,i}, \gamma_{G,ij}$, β_1, δ_1, b_{ijk}, c_{ijk}, δ_2 in (5-107)-(5-113).
[Step.3]	Construct the fuzzy sets for $\boldsymbol{x}$. Then solve the fuzzy basis matrices $\boldsymbol{\Phi}_f(\boldsymbol{x}), \boldsymbol{\Phi}_G(\boldsymbol{x})$.
[Step.4]	Obtain the control law (5-23) and solve the modified adaptive laws (5-38) by maintaining the adjustable fuzzy parameter matrices $\boldsymbol{\Theta}_f$ and $\boldsymbol{\Theta}_G$ in a certain constraint region in analogy to (5-107)-(5-113).

Tab. 5-1: Design algorithm for the RIAFC based adaptive fuzzy control system

5.3 Extension by a Linear State Observer

5.3.1 Motivation

Including a linear state observer (e.g. a Luenberger observer) the performance and practicability of controls can be improved significantly, (*Boukezzoula, R.., et al., 2004, Mohanlal, P.P., et al. ,2004*). Especially if there is some quantification noise content included in the output signals of a digital control system (without observer) and the main problems are caused by the necessary derivations for building the state vector, (*Koller, G., 1999*). If there is a lot of noise contained in the measured output signals a Kalman-filter could be introduced to reduce the effects of the measurement noise on the control system (not considered here). If the whole state vector or some elements of it are unavailable for measurement then a linear state observer is absolute required to build up the state-space control system.

The problem statement can be divided in three case studies:

- Case study 1:

  The whole state vector $\boldsymbol{x}$ or some elements of $\boldsymbol{x}$ are unavailable, but the output vector $\boldsymbol{y}$ is assumed to be available for measurement. Then the control law (5-23) can no longer be used to control the nonlinear system (5-7). There also may be some quantification noise to consider due to the measurement of $\boldsymbol{y}$. Applying a linear state observer the new system state $\boldsymbol{\xi}$ and the error vector $\boldsymbol{e}$ are replaced by their estimates $\hat{\boldsymbol{\xi}}$ and $\hat{\boldsymbol{e}}$.
  This case study is objectively the most general and most challenging case which is studied in this thesis, and will therefore in detail be examined in the mathematical investigation below.

- Case study 2:

  State vector $\boldsymbol{x}$ and the output vector $\boldsymbol{y}$ are still available for measurement, but due to quantification of the measured output signals (practical aspect) and the following DT_1-elements (5-4) for calculation of the state vector $\boldsymbol{\xi}$ the higher order elements of $\boldsymbol{\xi}$ and as a result also the control signal $\boldsymbol{u}$ may contain a lot of noise which could lead to high stress for the hardware components, a loss of performance and in extreme situations to instability of the closed loop control system. Using a linear state observer these derivatives are not needed anymore and the system state $\boldsymbol{\xi}$ and the error vector $\boldsymbol{e}$ are again replaced by their estimates $\hat{\boldsymbol{\xi}}$ and $\hat{\boldsymbol{e}}$.
  The mathematical investigation can be build up like the one in Case study 1 but with one exception: Because of the availability of the measured state vector $\boldsymbol{x}$, this vector can directly be used as an input for modelling the fuzzy systems instead of $\hat{\boldsymbol{\xi}}$. This should result in smaller fuzzy approximation errors concerning the approximation of the nonlinear functions $\boldsymbol{f}$ and $\boldsymbol{G}$ by the estimated fuzzy functions $\hat{\boldsymbol{f}}$ and $\hat{\boldsymbol{G}}$.

- Case study 3:

  The nonlinear system is already given in normal form (5-46) and a coordinates transformation was not necessary. Then the state vector $\boldsymbol{\xi}$ does not appear in the state-space equations anymore and the nonlinear MIMO system in normal form depends only on the state vector $\boldsymbol{x}$.

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}[\,\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{d}\,] \\ \boldsymbol{y} &= \boldsymbol{C}^T\boldsymbol{x}\,.\end{aligned} \qquad (\text{ 5-46 })$$

  In case of unavailability of the state vector $\boldsymbol{x}$ for measurement, the control law (5-23) cannot, and if there is some noise content in the measured outputs of the plant, control law (5-23) should not directly be applied (refer to Case study 2). A linear state observer can be used instead and the system state $\boldsymbol{x}$ and the error vector $\boldsymbol{e}$ are here replaced by their estimates $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{e}}$. The next steps are chosen according to Case study 1.

5.3.2 MIMO Nonlinear Plant Dynamics

Based on the observations and assumptions of Chapter 4 the mathematical descriptions of MIMO plants to be controlled by the ORIAFC are derived. Compact notations these descriptions were stated to allow a better understanding of the developed algorithms and to support a time-efficient implementation in computer simulations and hardware implementations, respectively. Note, the stated MIMO plant descriptions can easily be simplified if SISO plants are under consideration. Therefore refer to (*Chen, B.S., et al., 1996, Spooner, J.T., et al., 1996, Sue, C.Y., et al., 1994, Tong, S.C., et al., 1999, Chai, T.Y., et al., 1999*).

Assumption 5-4: *It is assumed that the system (4-1) has a uniform strong relative degree vector* $\boldsymbol{r}_u=[r_1,\dots,r_m]^T$ *over a compact set* $\boldsymbol{x}\in\boldsymbol{U}\subset\boldsymbol{R}^n$ *and its zero dynamics are exponentially attractive. Moreover, the nonlinear functions* $\boldsymbol{f}$ *and* $\boldsymbol{G}$ *are assumed to be independent of the unobservable state vector* $\boldsymbol{\eta}$.

■

Then equations (5-3) can be simplified to

$$\begin{array}{lll} \dot{\xi}_1^i=\xi_2^i & y_1=\xi_1^1 & \\ \vdots \quad , & \vdots & (i=1,2,..,m) \\ \dot{\xi}_{r_i-1}^i=\xi_{r_i}^i & y_m=\xi_1^m & \end{array}$$

$$\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix}=\begin{bmatrix} f_1(\boldsymbol{\Psi}^{-1}(\xi)) \\ \vdots \\ f_m(\boldsymbol{\Psi}^{-1}(\xi)) \end{bmatrix}+\boldsymbol{G}(\boldsymbol{\Psi}^{-1}(\xi))\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}+\begin{bmatrix} d_1(t) \\ \vdots \\ d_m(t) \end{bmatrix} \tag{5-47}$$

■

Following notations are made for simplification: $\boldsymbol{f}\overset{\Delta}{=}\boldsymbol{f}(\boldsymbol{\Psi}^{-1}(\xi))$ and $\boldsymbol{G}\overset{\Delta}{=}\boldsymbol{G}(\boldsymbol{\Psi}^{-1}(\xi))$.

5.3.3 Observer-based Robust Indirect Adaptive Fuzzy Control (ORIAFC)

The system state $\boldsymbol{\xi}$ and the error vector $\boldsymbol{e}$ are replaced by their estimates $\hat{\boldsymbol{\xi}}$ and $\hat{\boldsymbol{e}}$

$$\hat{\boldsymbol{e}}=\hat{\boldsymbol{\xi}}-\boldsymbol{Y}_M=[\hat{e}_1,\dots,\hat{e}_1^{(r_1-1)},\dots,\hat{e}_m,\dots,\hat{e}_m^{(r_m-1)}] \tag{5-48}$$

and the controller is designed as

$$\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}=\hat{\boldsymbol{G}}^{-1}(\hat{\boldsymbol{\xi}}\,|\,\boldsymbol{\theta}_{g,ij})\left(-\begin{bmatrix} \hat{f}_1(\hat{\boldsymbol{\xi}}\,|\,\boldsymbol{\theta}_{f,1}) \\ \vdots \\ \hat{f}_m(\hat{\boldsymbol{\xi}}\,|\,\boldsymbol{\theta}_{f,m}) \end{bmatrix}+\begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix}+\begin{bmatrix} u_{h,1} \\ \vdots \\ u_{h,m} \end{bmatrix}+\begin{bmatrix} u_{o,1} \\ \vdots \\ u_{o,m} \end{bmatrix}+\begin{bmatrix} u_{s,1} \\ \vdots \\ u_{s,m} \end{bmatrix}\right) \tag{5-49}$$

including $\boldsymbol{v}=\boldsymbol{y}_M{}^{(r^*)}-\boldsymbol{K}^T\hat{\boldsymbol{e}}$ with

$\boldsymbol{K}^T=[\boldsymbol{k}_r,\boldsymbol{k}_{r-1},\dots,\boldsymbol{k}_1]$ as feedback gain matrix to make the characteristic polynomials of $\boldsymbol{A}_0=\boldsymbol{A}-\boldsymbol{B}\boldsymbol{K}^T$ to be Hurwitz, because *(A,B)* is controllable (the matrix definitions are the same as in Section 5.2.1) .

The control term $\boldsymbol{u}_h$ is again responsible for attenuate the disturbance effect on the system outputs, $\boldsymbol{u}_o$ is the feedback control for $\hat{\boldsymbol{e}}$ and need to be designed and $\boldsymbol{u}_s$ is a sliding-mode control to compensate fuzzy approximation errors. $\hat{\boldsymbol{f}}(\hat{\xi} \mid \boldsymbol{\Theta}_f)$ and $\hat{\boldsymbol{G}}(\hat{\xi} \mid \boldsymbol{\Theta}_G)$ be of the following form

$$\begin{aligned} \hat{\boldsymbol{f}} &= \boldsymbol{\Phi}_f(\hat{\xi})\boldsymbol{\Theta}_f \\ \hat{\boldsymbol{G}} &= \boldsymbol{\Phi}_G(\hat{\xi})\boldsymbol{\Theta}_G \,. \end{aligned} \tag{5-50}$$

The dimensions are the same as in (5-19).

Assumption 5-5: *It is again pre-assumed that the matrix* $\hat{\boldsymbol{G}}(\boldsymbol{x} \mid \boldsymbol{\theta}_{g,ij}) = matrix(\hat{g}_{ij})$ *is invertible (non singular). Later this assumption will be relaxed by the projection algorithm of Section 5.5 - Modified adaptation laws.*

■

As described in RIAFC design stage, the grid of rules and therefore the complexity of the fuzzy models grows dramatically with the order of the plant (order of the state vector $\boldsymbol{\xi}$). To remedy this problem Section 5.6 "Dynamic fuzzy rule activation method" shows how the real number of activated fuzzy rules can effectively be reduced.

Substituting (5-49) in (5-47) yields

$$\begin{aligned} \dot{\boldsymbol{e}} &= \boldsymbol{A}\boldsymbol{e} - \boldsymbol{B}\boldsymbol{K}^T\hat{\boldsymbol{e}} + \boldsymbol{B}[\boldsymbol{f} - \hat{\boldsymbol{f}} + (\boldsymbol{G} - \hat{\boldsymbol{G}})\boldsymbol{u} + \boldsymbol{u}_h + \boldsymbol{u}_o + \boldsymbol{u}_s + \boldsymbol{d}] \\ \boldsymbol{E}_1 &= \boldsymbol{C}^T\boldsymbol{e} \,. \end{aligned} \tag{5-51}$$

Thus, we have converted the tracking problem into the regulation problem of designing a state observer for estimating the error vector $\boldsymbol{e}$ in (5-51) in order to regulate $\boldsymbol{E}_1$ to zero.

Design the (linear) observer as follows

$$\begin{aligned} \dot{\hat{\boldsymbol{e}}} &= \boldsymbol{A}\hat{\boldsymbol{e}} - \boldsymbol{B}\boldsymbol{K}^T\hat{\boldsymbol{e}} + \boldsymbol{K}_o(\boldsymbol{E}_1 - \hat{\boldsymbol{E}}_1) \\ \hat{\boldsymbol{E}}_1 &= \boldsymbol{C}^T\hat{\boldsymbol{e}} \,, \end{aligned} \tag{5-52}$$

where $\boldsymbol{K}_o{}^T = [\boldsymbol{k}_1^o, \boldsymbol{k}_2^o, \ldots, \boldsymbol{k}_r^o]$ is the observer gain matrix to make sure that the characteristic polynomials of $\boldsymbol{A}_1 = \boldsymbol{A} - \boldsymbol{K}_o\boldsymbol{C}^T$ are strict Hurwitz, because $(\boldsymbol{C}^T, \boldsymbol{A})$ is *observable*.

Definition 5-2:

Observability: ($\boldsymbol{C}^T$,$\boldsymbol{A}$) is controllable if $rank\begin{pmatrix} \boldsymbol{C}^T \\ \boldsymbol{C}^T\boldsymbol{A} \\ \vdots \\ \boldsymbol{C}^T\boldsymbol{A}^{n-1} \end{pmatrix} = r$.

■

The interaction between control and state-observer can be described by the coupled differential equations

$$\begin{pmatrix} \dot{\boldsymbol{e}} \\ \dot{\hat{\boldsymbol{e}}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{A} & -\boldsymbol{B}\boldsymbol{K}^T \\ \boldsymbol{K}_0\boldsymbol{C}^T & \boldsymbol{A}-\boldsymbol{B}\boldsymbol{K}^T-\boldsymbol{K}_o\boldsymbol{C}^T \end{pmatrix}\begin{pmatrix} \boldsymbol{e} \\ \hat{\boldsymbol{e}} \end{pmatrix} + \begin{pmatrix} \boldsymbol{B} \\ \boldsymbol{0} \end{pmatrix}\breve{\boldsymbol{u}} \tag{5-53}$$

with the virtual input

$$\breve{\boldsymbol{u}} = \boldsymbol{f} - \hat{\boldsymbol{f}} + (\boldsymbol{G} - \hat{\boldsymbol{G}})\boldsymbol{u} + \boldsymbol{u}_h + \boldsymbol{u}_o + \boldsymbol{u}_s + \boldsymbol{d} \tag{5-54}$$

and the resulting system matrix, a 2n x 2n Hurwitz-stable coefficients matrix

$$\begin{pmatrix} \boldsymbol{A} & -\boldsymbol{B}\boldsymbol{K}^T \\ \boldsymbol{K}_0\boldsymbol{C}^T & \boldsymbol{A}-\boldsymbol{B}\boldsymbol{K}^T-\boldsymbol{K}_o\boldsymbol{C}^T \end{pmatrix} \tag{5-55}$$

which shows the same eigenvalues as the characteristic polynomials of $\boldsymbol{A}_0$ and $\boldsymbol{A}_1$.

Applying the Laplacetransform (Laplace-variable "*s*") on (5-52) and setting the initial condition $\hat{\boldsymbol{e}}(0) = 0$ results in

$$\begin{aligned} \hat{\boldsymbol{e}}(s) &= (s\boldsymbol{I} - \boldsymbol{F})^{-1}\boldsymbol{K}_o\boldsymbol{C}^T\boldsymbol{e}(s) \\ \boldsymbol{F} &= (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}^T - \boldsymbol{K}_o\boldsymbol{C}^T) \end{aligned} \tag{5-56}$$

Including the (unknown) bounded noise vector $\boldsymbol{n}_y(t)$ which is representing the noise due to measurement of the outputs, where $\boldsymbol{n}_y(t)$ does not need to have the property of standard smoothness, necessarily:

$$\begin{aligned} \bar{y}_1 &= y_1 + n_{y,1}(t) = \xi_1^1 + n_{y,1}(t) = \bar{\xi}_1^1 \\ &\vdots \\ \bar{y}_m &= y_m + n_{y,m}(t) = \xi_1^m + n_{y,m}(t) = \bar{\xi}_1^m \end{aligned} . \tag{5-57}$$

The tracking error vector $\bar{\boldsymbol{E}}_1$ can be formulated as

$$\bar{e}_i = \bar{y}_i - y_{M,i} = \bar{\xi}_1^i - y_{M,i} = e_i + n_{y,i} \tag{5-58}$$

$$\overline{\boldsymbol{E}}_1 = [\bar{e}_1,\dots,\bar{e}_m]^T = \boldsymbol{E}_1 + \boldsymbol{n}_y .$$

Note, it is also not required that the reference trajectories $y_{M,i}(t)$ are smooth, necessarily.

Fig. 5.1 gives an overview about the resulting Observer-based Robust Indirect Adaptive Fuzzy Control (ORIAFC).

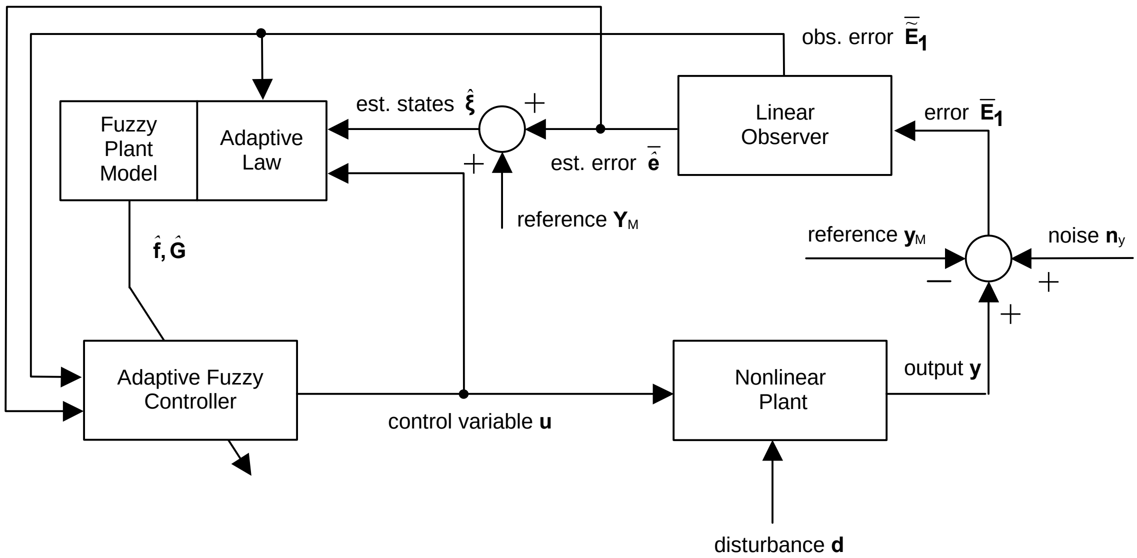

Fig. 5-2: Observer-based Robust Indirect Adaptive Fuzzy Control (ORIAFC)

Feed in $\overline{\boldsymbol{E}}_1$ into the observer the biased output follows to

$$\begin{aligned}&\hat{\bar{\boldsymbol{e}}}(s) = (s\boldsymbol{I} - \boldsymbol{F})^{-1}\boldsymbol{K}_o\boldsymbol{C}^T\boldsymbol{e}(s) + (s\boldsymbol{I} - \boldsymbol{F})^{-1}\boldsymbol{K}_o\boldsymbol{n}_y = \hat{\boldsymbol{e}}(s) + \boldsymbol{J}(s)\boldsymbol{n}_y(s)\\ &\boldsymbol{J}(s) = (s\boldsymbol{I} - \boldsymbol{F})^{-1}\boldsymbol{K}_o\end{aligned} \qquad (5\text{-}59)$$

assuming that a bounded Laplace-transformed variable $\boldsymbol{n}_y(s)$ exists. $\boldsymbol{J}(s)$ represents a known, stable transfer matrix.

According to the properties between the Laplacetransform L and the inverse Laplace-transform L^{-1} of the product, $\boldsymbol{J}(s)\boldsymbol{n}_y(s)$ is characterised by the convolution integral of the matrix- respectively vector-valued functions $\boldsymbol{J}(t)$ and $\boldsymbol{n}_y(t)$ in time domain:

$$\begin{aligned}&L^{-1}\{\boldsymbol{J}(s)\boldsymbol{n}_y(s)\} = \varepsilon_H(t)\boldsymbol{J}(t) * \boldsymbol{n}_y(t) = \int_{0-}^{t}\boldsymbol{J}(t-\tau)\boldsymbol{n}_y(\tau)d\tau\\ &\varepsilon_H(t-\tau_0) = \begin{cases}0 & t \le \tau_0\\ 1 & t > \tau_0\end{cases} \qquad \varepsilon_H(t)\text{...Heaviside step} .\end{aligned} \qquad (5\text{-}60)$$

Summarizing, the noise vector $\boldsymbol{n}_y$ can, from the mathematical point of view, be equivalently transformed from the input to the output of the observer by the linear transfer matrix $\boldsymbol{J}(s)$.

Substituting (5-49), including $\boldsymbol{v} = \boldsymbol{y}_M{}^{(r^*)} - \boldsymbol{K}^T \bar{\boldsymbol{e}}(t)$, in (5-47) yields

$$\dot{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{e} - \boldsymbol{B}\boldsymbol{K}^T \hat{\boldsymbol{e}} + \boldsymbol{B}[\,\boldsymbol{f} - \hat{\boldsymbol{f}} + (\boldsymbol{G} - \hat{\boldsymbol{G}})\boldsymbol{u} + \boldsymbol{u}_h + \boldsymbol{u}_o + \boldsymbol{u}_s + \boldsymbol{d} - \varepsilon_H(t)\boldsymbol{K}^T \boldsymbol{J}(t) * \boldsymbol{n}_y(t)]$$

$$\boldsymbol{E}_I = \boldsymbol{C}^T \boldsymbol{e}\,. \tag{5-61}$$

The disturbance vector $\boldsymbol{d}$ and the vector $-\varepsilon(t)\boldsymbol{K}^T \boldsymbol{J}(t) * \boldsymbol{n}_y(t)$ can be collected in one combined disturbance vector

$$\boldsymbol{d}' = \boldsymbol{d} - \varepsilon_H(t)\boldsymbol{K}^T \boldsymbol{J}(t) * \boldsymbol{n}_y(t)\,. \tag{5-62}$$

Defining the observation error as

$$\tilde{\boldsymbol{e}} = \boldsymbol{e} - \hat{\boldsymbol{e}} \tag{5-63}$$

and subtracting (5-52) from (5-61) results in

$$\dot{\tilde{\boldsymbol{e}}} = (\boldsymbol{A} - \boldsymbol{K}_0 \boldsymbol{C}^T)\tilde{\boldsymbol{e}} + \boldsymbol{B}[\,\boldsymbol{f} - \hat{\boldsymbol{f}} + (\boldsymbol{G} - \hat{\boldsymbol{G}})\boldsymbol{u} + \boldsymbol{u}_h + \boldsymbol{u}_o + \boldsymbol{u}_s + \boldsymbol{d}'\,]\,. \tag{5-64}$$

If we take a closer look at the interaction between the $\boldsymbol{e}$ and $\tilde{\boldsymbol{e}}$

$$\begin{pmatrix} \dot{\boldsymbol{e}} \\ \dot{\tilde{\boldsymbol{e}}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}^T & \boldsymbol{B}\boldsymbol{K}^T \\ \boldsymbol{0} & \boldsymbol{A} - \boldsymbol{K}_o \boldsymbol{C}^T \end{pmatrix} \begin{pmatrix} \boldsymbol{e} \\ \tilde{\boldsymbol{e}} \end{pmatrix} + \begin{pmatrix} \boldsymbol{B} \\ \boldsymbol{B} \end{pmatrix} \breve{\boldsymbol{u}}$$

$$\breve{\boldsymbol{u}} = \boldsymbol{f} - \hat{\boldsymbol{f}} + (\boldsymbol{G} - \hat{\boldsymbol{G}})\boldsymbol{u} + \boldsymbol{u}_h + \boldsymbol{u}_o + \boldsymbol{u}_s + \boldsymbol{d}' \tag{5-65}$$

the representative system matrix, a 2n x 2n Hurwitz-stable coefficients matrix, can be identified as

$$\begin{pmatrix} \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}^T & \boldsymbol{B}\boldsymbol{K}^T \\ \boldsymbol{0} & \boldsymbol{A} - \boldsymbol{K}_o \boldsymbol{C}^T \end{pmatrix}. \tag{5-66}$$

Ignoring the nonlinearity due to the input $\breve{\boldsymbol{u}}$ for the moment, the characteristic polynomials of control and observer are described by the product

$$det(s\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K}^T) \cdot det(s\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{K}_o \boldsymbol{C}^T)\,. \tag{5-67}$$

This would mean that the assumptions for $\boldsymbol{K}$ and $\boldsymbol{K}_o$ and therefore the controller- and observer design could be separated (separation principle for linear systems). In the following Lyapunov stability proof it will be shown, that the Lyapunov candidate (5-73) is indeed a Lyapunov function which stabilizes the overall nonlinear system (5-47) and allows the separability of controller- and observer design similar to the separation principle for linear systems.

Remarks 5-6: In the linear control literature, e.g. in (*Weinmann, A., 1995*), it is a well-known fact that the *eigenvalues* of the observer should be located more left in the complex "*s* - half plane" than the eigenvalues of the control system without observer. However, observers with too far left located eigenvalues tend to a derivative action, which is not desired. If there is a lot of measurement noise contained in the output signals a Kalman-filter could be introduced to reduce the effects of the measurement noise on the control system (not considered here). More details about controls with stochastic optimal prediction, which is not part of this thesis, can be found in (*Weinmann, A., 1995*, *Perez, C., et al., 2004, Mohanlal, P.P., et al., 2004*).

5.3.4 Stability- and Convergence Analysis of the ORIAFC

Define the *optimal parameter vector* $\boldsymbol{\Theta}_f^*$ and the *optimal parameter matrix* $\boldsymbol{\Theta}_G^*$

$$\begin{aligned}\boldsymbol{\Theta}_f^* &= arg \min_{\boldsymbol{\Theta}_f \in \boldsymbol{\Omega}_f} \left\{ \sup_{\xi \in \boldsymbol{U}_1, \hat{\xi} \in \boldsymbol{U}_2} \left\| \boldsymbol{f}(\xi) - \hat{\boldsymbol{f}}(\hat{\xi} \mid \boldsymbol{\Theta}_f) \right\|_F \right\} \\ \boldsymbol{\Theta}_G^* &= arg \min_{\boldsymbol{\Theta}_G \in \boldsymbol{\Omega}_G} \left\{ \sup_{\xi \in \boldsymbol{U}_1, \hat{\xi} \in \boldsymbol{U}_2} \left\| \boldsymbol{G}(\xi) - \hat{\boldsymbol{G}}(\hat{\xi} \mid \boldsymbol{\Theta}_G) \right\|_F \right\},\end{aligned} \tag{5-68}$$

the minimum (fuzzy) approximation error $\boldsymbol{\omega}$

$$\begin{aligned}\boldsymbol{\omega} &= \hat{\boldsymbol{f}}(\hat{\xi} \mid \boldsymbol{\Theta}_f^*) - \hat{\boldsymbol{f}}(\xi \mid \boldsymbol{\Theta}_f^*) + \boldsymbol{f}(\xi) - \hat{\boldsymbol{f}}(\xi \mid \boldsymbol{\Theta}_f^*) + [\hat{\boldsymbol{G}}(\hat{\xi} \mid \boldsymbol{\Theta}_G^*) - \hat{\boldsymbol{G}}(\xi \mid \boldsymbol{\Theta}_G^*) \\ &+ \boldsymbol{G}(\xi) - \hat{\boldsymbol{G}}(\xi \mid \boldsymbol{\Theta}_G^*)]\boldsymbol{u} = \Delta \boldsymbol{f} + \Delta \boldsymbol{G}\boldsymbol{u},\end{aligned} \tag{5-69}$$

and the estimation errors

$$\begin{aligned}\tilde{\boldsymbol{\Theta}}_f &= \boldsymbol{\Theta}_f - \boldsymbol{\Theta}_f^* \\ \tilde{\boldsymbol{\Theta}}_G &= \boldsymbol{\Theta}_G - \boldsymbol{\Theta}_G^*.\end{aligned} \tag{5-70}$$

It is pre-assumed that $\boldsymbol{\xi}$, $\hat{\boldsymbol{\xi}}$, $\boldsymbol{\Theta}_f$, and $\boldsymbol{\Theta}_G$ belong to compact sets $\boldsymbol{U}_1$, $\boldsymbol{U}_2$, $\boldsymbol{\Omega}_f$, $\boldsymbol{\Omega}_G$, respectively, which are defined as

$$\begin{aligned}
&\boldsymbol{U}_1 = \left\{\boldsymbol{\xi} \in \boldsymbol{R}^r : \left\|\boldsymbol{\xi}\right\|_F \le \boldsymbol{M}_{\xi}\right\} \\
&\boldsymbol{U}_2 = \left\{\hat{\boldsymbol{\xi}} \in \boldsymbol{R}^r : \left\|\hat{\boldsymbol{\xi}}\right\|_F \le \boldsymbol{M}_{\hat{\xi}}\right\} \\
&\boldsymbol{\Omega}_f = \left\{\boldsymbol{\Theta}_f \in \boldsymbol{R}^{m_f m\times 1} : \left\|\boldsymbol{\Theta}_f\right\|_F \le \boldsymbol{M}_{\boldsymbol{\Theta}_f}\right\} \\
&\boldsymbol{\Omega}_G = \left\{\boldsymbol{\Theta}_G \in \boldsymbol{R}^{m_G m^2\times m} : \left\|\boldsymbol{\Theta}_G\right\|_F \le \boldsymbol{M}_{\boldsymbol{\Theta}_G}\right\}.
\end{aligned} \tag{5-71}$$

Equation (5-64) turns out as

$$\dot{\tilde{\boldsymbol{e}}} = (\boldsymbol{A} - \boldsymbol{K}_o\boldsymbol{C}^T)\tilde{\boldsymbol{e}} + \boldsymbol{B}[-\boldsymbol{\Phi}_f\tilde{\boldsymbol{\Theta}}_f - \boldsymbol{\Phi}_G\tilde{\boldsymbol{\Theta}}_G\boldsymbol{u} + \boldsymbol{\omega} + \boldsymbol{u}_h + \boldsymbol{u}_o + \boldsymbol{u}_s + \boldsymbol{d}'] . \tag{5-72}$$

Consider the Lyapunov function candidate

$$\begin{aligned}
&V = \frac{1}{2}\hat{\boldsymbol{e}}^T\boldsymbol{P}_1\hat{\boldsymbol{e}} + \frac{1}{2}\tilde{\boldsymbol{e}}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} + \frac{1}{2\gamma_f}\tilde{\boldsymbol{\Theta}}_f{}^T\tilde{\boldsymbol{\Theta}}_f + \frac{1}{2\gamma_G}tr(\tilde{\boldsymbol{\Theta}}_G{}^T\tilde{\boldsymbol{\Theta}}_G) \\
&\boldsymbol{P}_1 = \boldsymbol{P}_1^T ,\ \boldsymbol{P}_2 = \boldsymbol{P}_2^T > 0 \quad \gamma_f, \gamma_G > 0 .
\end{aligned} \tag{5-73}$$

The time derivative of V is

$$\begin{aligned}
\dot{V} = {} & \frac{1}{2}\hat{\boldsymbol{e}}^T(\boldsymbol{A}_0{}^T\boldsymbol{P}_1 + \boldsymbol{P}_1\boldsymbol{A}_0)\hat{\boldsymbol{e}} + \hat{\boldsymbol{e}}^T\boldsymbol{P}_1\boldsymbol{K}_o\boldsymbol{C}^T\tilde{\boldsymbol{e}} + \tilde{\boldsymbol{e}}^T\boldsymbol{P}_2\boldsymbol{B}\boldsymbol{u}_o \\
& + \frac{1}{2}\tilde{\boldsymbol{e}}^T(\boldsymbol{A}_1{}^T\boldsymbol{P}_2 + \boldsymbol{P}_2\boldsymbol{A}_1)\tilde{\boldsymbol{e}} + \boldsymbol{u}_h{}^T(\boldsymbol{I} + \Delta\boldsymbol{G}\hat{\boldsymbol{G}}^{-1})^T\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} + \boldsymbol{d}'^T\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} \\
& + \boldsymbol{u}_s{}^T(\boldsymbol{I} + \Delta\boldsymbol{G}\hat{\boldsymbol{G}}^{-1})^T\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} + [\Delta\boldsymbol{F} + \Delta\boldsymbol{G}\hat{\boldsymbol{G}}^{-1}(-\hat{\boldsymbol{f}} + \boldsymbol{y}_M{}^{(r^*)} - \boldsymbol{K}^T\hat{\boldsymbol{e}} + \boldsymbol{u}_0)]^T\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} \\
& - \tilde{\boldsymbol{\Theta}}_f{}^T\boldsymbol{\Phi}_f{}^T\boldsymbol{B}^T\boldsymbol{P}_2\boldsymbol{e} - \boldsymbol{u}^T\tilde{\boldsymbol{\Theta}}_G{}^T\boldsymbol{\Phi}_G{}^T\boldsymbol{B}^T\boldsymbol{P}_2\boldsymbol{e} + \frac{1}{2\gamma_f}\dot{\tilde{\boldsymbol{\Theta}}}_f{}^T\tilde{\boldsymbol{\Theta}}_f + \frac{1}{2\gamma_G}tr(\dot{\tilde{\boldsymbol{\Theta}}}_G{}^T\tilde{\boldsymbol{\Theta}}_G) .
\end{aligned} \tag{5-74}$$

Assumption 5-6: *There exist positive constants κ, M_g and M_e such that*

$$\begin{aligned}
&\left|\lambda_i[\Delta\boldsymbol{G}(\xi,\hat{\xi})\hat{\boldsymbol{G}}^{-1}(\hat{\xi}\,|\,\boldsymbol{\Theta}_G)]\right| \le \kappa , \qquad 0 \le \kappa < 1 \\
&\left\|\Delta\boldsymbol{G}(\xi,\hat{\xi})\hat{\boldsymbol{G}}^{-1}(\hat{\xi}\,|\,\boldsymbol{\Theta}_G)\right\|_\infty = \\
&\max_i\left\{[\sum_{j=1}^{p}\left|[(\Delta\boldsymbol{G}(\xi,\hat{\xi})\hat{\boldsymbol{G}}^{-1}(\hat{\xi}\,|\,\boldsymbol{\Theta}_G)]_{ij}\right|\right\} \le M_g , \quad 0 \le M_g < 1 \\
&\left|[\Delta\boldsymbol{f}(\xi,\hat{\xi}) + \Delta\boldsymbol{G}(\xi,\hat{\xi})\hat{\boldsymbol{G}}^{-1}(\hat{\xi}\,|\,\boldsymbol{\Theta}_G)(-\hat{\boldsymbol{f}}(\hat{\xi}\,|\,\boldsymbol{\Theta}_f) + \boldsymbol{y}_M{}^{(r^*)} - \boldsymbol{K}^T\hat{\boldsymbol{e}} + \boldsymbol{u}_0)]_i\right| \le M_e
\end{aligned} \tag{5-75}$$

and the input output constraint

$$\boldsymbol{P}_2\boldsymbol{B} = \boldsymbol{C} \tag{5-76}$$

is fulfilled.

■

Choosing the H∞ and VSS control terms as

$$\boldsymbol{u}_h = -\frac{1}{2}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}}\ ,\ \ \boldsymbol{R} = diag[\,r_1,..,r_m\,] \in \boldsymbol{R}^{m\times m},\ \ r_1,...,r_m > 0 \tag{5-77}$$

$$\boldsymbol{u}_s = -\frac{M_e}{1-M_g}\boldsymbol{sign}(\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}}\,) = -k\boldsymbol{sign}(\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}}\,) \tag{5-78}$$

and the tuning control term for the observer as

$$\boldsymbol{u}_o = -\boldsymbol{K}_o^{\ T}\boldsymbol{P}_1\hat{\boldsymbol{e}}\,, \tag{5-79}$$

respectively. Considering that only the biased vector $\overline{\tilde{\boldsymbol{E}}}_1 = \boldsymbol{B}^T\boldsymbol{P}_2\overline{\tilde{\boldsymbol{e}}} = \boldsymbol{C}^T\overline{\tilde{\boldsymbol{e}}}$ can be measured and implemented in (5-77)-(5-78) instead of $\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}}$ and $\bar{\hat{\boldsymbol{e}}}$ instead of $\hat{\boldsymbol{e}}$ in (5-79), and using

$$\begin{aligned}&\boldsymbol{u}_s^{\ T}(\,I + \Delta\boldsymbol{G}\hat{\boldsymbol{G}}^{-1}\,)^T\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} + \{\,\Delta\boldsymbol{F} + \Delta\boldsymbol{G}\hat{\boldsymbol{G}}^{-1}[\,-\hat{\boldsymbol{f}} + \boldsymbol{y}_M^{\ (r^*)}\\ &-(\,\boldsymbol{K}^T + \boldsymbol{K}_o^{\ T}\boldsymbol{P}_1\,)\hat{\boldsymbol{e}}\,]\}^T\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} \le 0\end{aligned} \tag{5-80}$$

equation (5-74) results with $\dot{\tilde{\boldsymbol{\Theta}}}_f = \dot{\boldsymbol{\Theta}}_f$ and $\dot{\tilde{\boldsymbol{\Theta}}}_G = \dot{\boldsymbol{\Theta}}_G$ ($\boldsymbol{\Theta}_f^{\ *}$ and $\boldsymbol{\Theta}_G^{\ *}$ are assumed to be constants) as follows

$$\begin{aligned}\dot{V} \le\ &\frac{1}{2}\hat{\boldsymbol{e}}^T(\,\boldsymbol{A}_0^{\ T}\boldsymbol{P}_1 + \boldsymbol{P}_1\boldsymbol{A}_0\,)\hat{\boldsymbol{e}} + \frac{1}{2}\tilde{\boldsymbol{e}}^T[\,\boldsymbol{A}_1^{\ T}\boldsymbol{P}_2 + \boldsymbol{P}_2\boldsymbol{A}_1 - \boldsymbol{P}_2\boldsymbol{B}(1-\kappa\,)\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}_2\,]\tilde{\boldsymbol{e}}\\ &+\boldsymbol{d}''^T\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} - \tilde{\boldsymbol{\Theta}}_f^{\ T}\boldsymbol{\Phi}_f^{\ T}\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} - \boldsymbol{u}^T\tilde{\boldsymbol{\Theta}}_G^{\ T}\boldsymbol{\Phi}_G^{\ T}\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} + \frac{1}{2\gamma_f}\dot{\tilde{\boldsymbol{\Theta}}}_f^{\ T}\tilde{\boldsymbol{\Theta}}_f\\ &+\frac{1}{2\gamma_G}tr(\dot{\tilde{\boldsymbol{\Theta}}}_G^{\ T}\tilde{\boldsymbol{\Theta}}_G\,)\\ \boldsymbol{d}'' =\ &\boldsymbol{d}' - \frac{1}{2}\boldsymbol{R}^{-1}\boldsymbol{n}_y(\,t\,) + \frac{1}{2}\varepsilon_H(\,t\,)\boldsymbol{R}^{-1}\boldsymbol{C}^T\boldsymbol{J}(\,t\,)*\boldsymbol{n}_y(\,t\,) - \varepsilon_H(\,t\,)\boldsymbol{K}_o^{\ T}\boldsymbol{P}_1\boldsymbol{J}(\,t\,)*\boldsymbol{n}_y(\,t\,)\,.\end{aligned} \tag{5-81}$$

Remarks 5-7:

The ***sign****(.)* function is again replaced by saturation functions *sat[(.)$_i$]* of the form

$$sat\left[(\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}})_i\right] = \begin{cases} sign\left[(\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}})_i\right] & if \left|(\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}})_i\right| > \alpha \\ \dfrac{1}{\alpha}(\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}})_i & if \left|(\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}})_i\right| \le \alpha \end{cases} \tag{5-82}$$

$\alpha > 0\,,\ i = 1,2,\ldots,m$

$$or\ smooth\ functions\ \ tanh\left[\frac{(\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}})_i}{\alpha}\right],\ \alpha > 0\,,\ \ i = 1,2,\ldots,m \tag{5-83}$$

where the function "*tanh*" means the "hyperbolic tangent" of its argument and α is a small design constant in order to remedy the control chattering.

■

If the ideal parameter update laws are chosen to

$$\begin{aligned} \dot{\boldsymbol{\Theta}}_f &= \gamma_f \boldsymbol{\Phi}_f{}^T(\hat{\boldsymbol{\xi}})\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} \\ \dot{\boldsymbol{\Theta}}_{G,i} &= \gamma_G \boldsymbol{\Phi}_{G,i}{}^T(\hat{\boldsymbol{\xi}})\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}}u_i\,, \quad (i = 1,2,\ldots,m) \end{aligned} \tag{5-84}$$

the Lyapunov derivative can be simplified to

$$\begin{aligned} \dot{V} \le\ & \frac{1}{2}\hat{\boldsymbol{e}}^T(\boldsymbol{A}_0{}^T\boldsymbol{P}_1 + \boldsymbol{P}_1\boldsymbol{A}_0)\hat{\boldsymbol{e}} + \frac{1}{2}\tilde{\boldsymbol{e}}^T[\boldsymbol{A}_1{}^T\boldsymbol{P}_2 + \boldsymbol{P}_2\boldsymbol{A}_1 - \boldsymbol{P}_2\boldsymbol{B}(1-\kappa)\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}_2]\tilde{\boldsymbol{e}} \\ & + \boldsymbol{d}''^T\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}}\ . \end{aligned} \tag{5-85}$$

Remarks 5-8:

- The adaptation rates $\gamma_{f,i}$, $\gamma_{G,ij}$ can be properly chosen for each element $\hat{f}_i$ and $\hat{g}_{ij}$ individually. Only for simplified notation they were stated as γ_f, γ_G.
- Exactly observed, also the update laws are biased by noise but due to low-pass filtering of the resulting functions $\hat{\boldsymbol{f}}$ and $\hat{\boldsymbol{G}}$ this effect can again be eliminated.

■

Proposition 5-3: *For the given positive-definite matrices $\boldsymbol{Q}_1$, $\boldsymbol{Q}_2$ there exist positive-definite solutions $\boldsymbol{P}_1$, $\boldsymbol{P}_2$ for the following Lyapunov- and Riccati-like matrix equations considering the input-output constraint (5-76)*:

$$\boldsymbol{A}_0{}^T\boldsymbol{P}_1 + \boldsymbol{P}_1\boldsymbol{A}_0 + \boldsymbol{Q}_1 = \boldsymbol{0} \tag{5-86}$$

$$\begin{aligned} & \boldsymbol{A}_1{}^T\boldsymbol{P}_2 + \boldsymbol{P}_2\boldsymbol{A}_1 - \boldsymbol{P}_2\boldsymbol{B}[(1-\kappa)\boldsymbol{R}^{-1} - \frac{1}{\rho^2}\boldsymbol{I}]\boldsymbol{B}^T\boldsymbol{P}_2 + \boldsymbol{Q}_2 = \boldsymbol{0} \\ & \boldsymbol{P}_2\boldsymbol{B} = \boldsymbol{C}\ . \end{aligned} \tag{5-87}$$

Preconditions:

$$\boldsymbol{P}_1, \boldsymbol{Q}_1, \boldsymbol{P}_2, \boldsymbol{Q}_2 \in \boldsymbol{R}^{r\times r},\ \boldsymbol{P}_i = \boldsymbol{P}_i^T > \boldsymbol{0}, \boldsymbol{Q}_i = \boldsymbol{Q}_i^T > \boldsymbol{0} \qquad (i=1,2)$$
$$(1-\kappa)\boldsymbol{R}^{-1} - \frac{1}{\rho^2}\boldsymbol{I} > \boldsymbol{0}. \qquad (5\text{-}88)$$

■

A proof for the existence of stable solutions to Riccati-like matrix equation in (5-87) subject to the input-output constraint (5-76) is given in Section 5.4 – "SPR-Lyapunov design approach". The results of this approach can be summarized as follows:

- If the plant satisfies the Strict Positive Real Lemma (SPR lemma) of Kalman-Yakubovich-Popov, the existence of stable solutions to equations (5-87) can be verified.
- In case the plant does not show SPR-property at once, the control circuit can be transformed into an overall SPR-system in a simple manner.

Combining (5-85) and (5-86)-(5-87) results in

$$\dot{V} = -\frac{1}{2}\hat{\boldsymbol{e}}^T\boldsymbol{Q}_1\hat{\boldsymbol{e}} - \frac{1}{2}\tilde{\boldsymbol{e}}^T\boldsymbol{Q}_2\tilde{\boldsymbol{e}} - \frac{1}{2\rho^2}\tilde{\boldsymbol{e}}^T\boldsymbol{P}_2\boldsymbol{B}\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} + \boldsymbol{d}''^T\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}}$$
$$= -\frac{1}{2}\hat{\boldsymbol{e}}^T\boldsymbol{Q}_1\hat{\boldsymbol{e}} - \frac{1}{2}\tilde{\boldsymbol{e}}^T\boldsymbol{Q}_2\tilde{\boldsymbol{e}} - \frac{1}{2}[\frac{1}{\rho}\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} - \rho\boldsymbol{d}'']^T[\frac{1}{\rho}\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} - \rho\boldsymbol{d}''] + \frac{1}{2}\rho^2\boldsymbol{d}''^T\boldsymbol{d}'' \qquad (5\text{-}89)$$
$$\leq -\frac{1}{2}\hat{\boldsymbol{e}}^T\boldsymbol{Q}_1\hat{\boldsymbol{e}} - \frac{1}{2}\tilde{\boldsymbol{e}}^T\boldsymbol{Q}_2\tilde{\boldsymbol{e}} + \frac{1}{2}\rho^2\boldsymbol{d}''^T\boldsymbol{d}''.$$

As long as $\boldsymbol{d}'' \in L_2$ and the fuzzy parameters $\boldsymbol{\Theta}_f$ and $\boldsymbol{\Theta}_G$ are bounded (refer to Section 5.5 – "Modified adaptation laws, *projection algorithm*"), it is included that $\hat{\boldsymbol{e}}, \tilde{\boldsymbol{e}}, \boldsymbol{\xi}, \hat{\boldsymbol{\xi}}, \boldsymbol{x}, \boldsymbol{u} \in L_\infty$ (*Sastry, S., et al., 1989, Wang, L.X., 1993*). Thus, there are $lim_{t\to\infty}\tilde{\boldsymbol{e}} = \boldsymbol{0}$ and $lim_{t\to\infty}\hat{\boldsymbol{e}} = \boldsymbol{0}$. Since $\boldsymbol{e} = \hat{\boldsymbol{e}} + \tilde{\boldsymbol{e}}$ and $\boldsymbol{E}_1 = \boldsymbol{C}^T\boldsymbol{e}$, there is $lim_{t\to\infty}\boldsymbol{E}_1 = \boldsymbol{0}$.
Thus the control objective (i) is realized.

Integrating inequality (5-89) from $t=0$ to $t=T$ yields

$$V(T) - V(0) \leq -\frac{1}{2}\int_0^T \hat{\boldsymbol{e}}^T\boldsymbol{Q}_1\hat{\boldsymbol{e}}\,dt - \frac{1}{2}\int_0^T \tilde{\boldsymbol{e}}^T\boldsymbol{Q}_2\tilde{\boldsymbol{e}}\,dt + \frac{1}{2}\rho^2\int_0^T \boldsymbol{d}''^T\boldsymbol{d}''dt \qquad (5\text{-}90)$$
$$0 \leq T < \infty.$$

Since $V(T) \geq 0$, inequality (5-90) implies

$$\frac{1}{2}\int_0^T \hat{\boldsymbol{e}}^T \boldsymbol{Q}_1 \hat{\boldsymbol{e}}\,dt + \frac{1}{2}\int_0^T \tilde{\boldsymbol{e}}^T \boldsymbol{Q}_2 \tilde{\boldsymbol{e}}\,dt \le \frac{1}{2}\hat{\boldsymbol{e}}^T(0)\boldsymbol{P}_1\hat{\boldsymbol{e}}(0) + \frac{1}{2}\tilde{\boldsymbol{e}}^T(0)\boldsymbol{P}_2\tilde{\boldsymbol{e}}(0)$$
$$+\frac{1}{2\gamma_f}\tilde{\boldsymbol{\Theta}}_f^T(0)\tilde{\boldsymbol{\Theta}}_f(0) + \frac{1}{2\gamma_G}tr[\tilde{\boldsymbol{\Theta}}_G^T(0)\tilde{\boldsymbol{\Theta}}_G(0)] + \frac{1}{2}\rho^2\int_0^T \boldsymbol{d}''^T\boldsymbol{d}''dt \tag{5-91}$$

which represents an H_∞-criterion like the one as given in equation (5-1) for a pre-described attenuation level ρ. Finally, also the control objective (ii) is realized.

In summarizing the above discussions, the design algorithm for the Robust Observer-based Adaptive Fuzzy Control (ORIAFC) is described as follows (Tab. 5-2):

[Step.1]	Select the observer and feedback gain matrices $\boldsymbol{K}_o$, $\boldsymbol{K}$ such that the matrices $\boldsymbol{A}$-$\boldsymbol{BK}^T$ and $\boldsymbol{A}$-$\boldsymbol{K}_o\boldsymbol{C}^T$ are Hurwitz matrices, respectively. Choose a positive-definite matrix $\boldsymbol{Q}_1$ and solve Lyapunov equation (5-86) in order to get a positive-definite matrix $\boldsymbol{P}_1$.
[Step.2]	Choose appropriate values for the controller parameters k_T, α and $\boldsymbol{R}$ in (5-105), (5-83) and the parameters for fuzzy modelling $\gamma_{f,i}, \gamma_{G,ij}$, β_1, δ_1, b_{ijk}, c_{ijk}, δ_2 in (5-107)-(5-113).
[Step.3]	Solve the state observer in (5-52), then $\hat{\boldsymbol{\xi}} = \boldsymbol{Y}_M + \hat{\boldsymbol{e}}$.
[Step.4]	Construct the fuzzy sets for $\hat{\boldsymbol{\xi}}$. Then, solve the fuzzy basis matrices $\boldsymbol{\Phi}_f(\hat{\boldsymbol{\xi}}), \boldsymbol{\Phi}_G(\hat{\boldsymbol{\xi}})$.
[Step.5]	Select $\boldsymbol{L}(s)$ in (5-98) so, that $\boldsymbol{L}^{-1}(s)$ is a proper stable transfer function matrix and $\boldsymbol{H}(s)\boldsymbol{L}(s)$ to be a proper SPR transfer function matrix.
[Step.6]	Obtain the control law (5-49) and solve the modified adaptive laws (5-106) by maintaining the adjustable fuzzy parameter matrices $\boldsymbol{\Theta}_f$ and $\boldsymbol{\Theta}_G$ in a certain constraint region in analogy to (5-107)-(5-113).

Tab. 5-2: Design algorithm for the Robust Observer-based Adaptive Fuzzy Control (ORIAFC)

5.4 SPR-Lyapunov design approach

Since $\boldsymbol{A}_0 = \boldsymbol{A} - \boldsymbol{BK}^T$ is a Hurwitz matrix with a properly chosen $\boldsymbol{K}$, there exists a positive-definite matrix $\boldsymbol{P}_1$ for Lyapunov-equation in (5-87). Actually Proposition 5-3 depends on whether there exists a positive-definite matrix $\boldsymbol{P}_2$ for the Riccati-like equation in (5-87).

- If a positive-definite $\boldsymbol{P}_2$ exists for (5-87), we have $\boldsymbol{B}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}} = \tilde{\boldsymbol{E}}_1$. The estimates $\hat{\boldsymbol{e}}$ and $\hat{\boldsymbol{E}}_1$ are available to make the proposed adaptive fuzzy control scheme realizable.

- If a positive-definite $\boldsymbol{P}_2$ does not exist for (5-87), then the given system $(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C}^T)$ of (5-64) can be converted into a Strict Positive Real (SPR) system by using a similar way as proposed in (*Kim, Y.H., et al., 1997*).

Definition 5-3, (*Khalil, H.K., IEEE, 1996*): *The transfer matrix* $\boldsymbol{H}(s) \in \boldsymbol{R}^{m\,x\,m}$ *is positive-real if*

- *(1) all elements of* $\boldsymbol{H}(s)$ *are analytic for* $\boldsymbol{Re}(s) > 0$ *and*
- *(2) any pure imaginary pole of any element of* $\boldsymbol{H}(s)$ *is a simple pole and the associated residue matrix of* $\boldsymbol{H}(s)$ *is a positive semidefinite Hermitian matrix* $\boldsymbol{He}[\boldsymbol{H}(s)] = \frac{1}{2}(\boldsymbol{H}(\mathrm{s}) + \boldsymbol{H}^*(\mathrm{s})) \geq \boldsymbol{0}$
- *(3) for all real* ω *for* $j\omega$ *is not a pole of any element of* $\boldsymbol{H}(s)$, *the matrix* $\boldsymbol{H}(j\omega)+\boldsymbol{H}^T(-j\omega)$ *is positive semidefinite.*

The transfer matrix $\boldsymbol{H}(s)$ *is strict positive-real if* $\boldsymbol{H}(s-\varepsilon)$ *is positive real for a small positive constant* $\varepsilon>0$.

■

To satisfy the Riccati-like equation in (5-87) subject to the input-output constraint $\boldsymbol{P_2B}=\boldsymbol{C}$ the linear system $(\boldsymbol{A}_1,\boldsymbol{B},\boldsymbol{C}^T)$ has in addition to satisfy the following Strict Positive Real Lemma (SPR lemma), which is an extension of the famous *Positive Real Lemma (PR lemma) of Kalman-Yakubovich-Popov*.

Lemma 5-1 (*Vidyasagar, M., 1993*): *Consider a system of the form*

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{Ax} + \boldsymbol{Bu} \qquad && \boldsymbol{x} \in \boldsymbol{R}^n, \boldsymbol{u} \in \boldsymbol{R}^m \\ \boldsymbol{y} &= \boldsymbol{Cx} + \boldsymbol{Du} && \boldsymbol{y} \in \boldsymbol{R}^m, m < n \end{aligned} \tag{5-92}$$

and assume that (i) the eigenvalues of $\boldsymbol{A}$ *lie in the left half of the complex plane, (ii)* $(\boldsymbol{A},\boldsymbol{B})$ *is controllable and (iii)* $(\boldsymbol{C},\boldsymbol{A})$ *is observable. Then* $\boldsymbol{H}(\mathrm{s}) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}$ *is SPR if and only if there exist a symmetric positive-definite matrix* $\boldsymbol{P} \in \boldsymbol{R}^{n\,x\,n}$, *and matrices* $\boldsymbol{Q} \in \boldsymbol{R}^{m\,x\,n}$, $\boldsymbol{W} \in \boldsymbol{R}^{m\,x\,m}$, *and* $\varepsilon > 0$ *sufficiently small such that*

$$\begin{aligned} &\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{PA} = -\boldsymbol{Q}^T\boldsymbol{Q} - \varepsilon\boldsymbol{P} \\ &\boldsymbol{B}^T\boldsymbol{P} + \boldsymbol{W}^T\boldsymbol{Q} = \boldsymbol{C} \\ &\boldsymbol{W}^T\boldsymbol{W} = \boldsymbol{D} + \boldsymbol{D}^T. \end{aligned} \tag{5-93}$$

■

In our application $\boldsymbol{D}=\boldsymbol{0}$ and therefore also $\boldsymbol{W}$ has also to vanish, the positive-semidefinite $\boldsymbol{Q}^T\boldsymbol{Q}$ can be replaced by the term $-\boldsymbol{P}_2\boldsymbol{B}[(1-\kappa)\boldsymbol{R}^{-1}-\frac{1}{\rho^2}\boldsymbol{I}]\boldsymbol{B}^T\boldsymbol{P}_2+\boldsymbol{Q}_2\geq\boldsymbol{0}$ which has to be designed at least positive-semidefinite and (5-93) can be reformulated as

$$\begin{aligned}&\boldsymbol{A}_\varepsilon^{\ T}\boldsymbol{P}_2+\boldsymbol{P}_2\boldsymbol{A}_\varepsilon=\boldsymbol{P}_2\boldsymbol{B}[(1-\kappa)\boldsymbol{R}^{-1}-\frac{1}{\rho^2}\boldsymbol{I}]\boldsymbol{B}^T\boldsymbol{P}_2-\boldsymbol{Q}_2\leq\boldsymbol{0}\\&\boldsymbol{P}_2\boldsymbol{B}=\boldsymbol{C}\ .\end{aligned} \tag{5-94}$$

A sufficient small positive constant $\varepsilon>0$ can be found that $\boldsymbol{A}_\varepsilon=(\boldsymbol{A}_1+\frac{\varepsilon}{2}\boldsymbol{I})$ is also Hurwitz, $\boldsymbol{H}(s\text{-}\frac{\varepsilon}{2})=\boldsymbol{C}^T(s\boldsymbol{I}-\boldsymbol{A}_\varepsilon)^{-1}\boldsymbol{B}$ with the state space representation $(\boldsymbol{A}_\varepsilon,\boldsymbol{B},\boldsymbol{C}^T)$ is positive-real and a positive symmetric matrix $\boldsymbol{P}_2$ satisfying (5-94) exists .

Summarized, if and only if $\boldsymbol{H}(s)$ is SPR, for the Riccati-like equation (5-94) which is equal to (5-87) subject to $\boldsymbol{P}_2\boldsymbol{B}=\boldsymbol{C}$ exists a positive-definite solution $\boldsymbol{P}_2$.

Proposition 5-4: *Positive-realness of $\boldsymbol{H}(s)$ is equivalent to the passivity of $(\boldsymbol{A}_1,\boldsymbol{B},\boldsymbol{C}^T)$. The transfer matrix $\boldsymbol{H}(s)$ being SPR restricts the relative degrees r_i of the subsystems $(\boldsymbol{A}_{1,i},\boldsymbol{B}_i,\boldsymbol{C}_i^T)$ to be zero or one and its zeros to be stable (minimum phase).*

Proof: (*Khalil, H.K., 1996*, Prentice-Hall).

■

However, if a positive-definite $\boldsymbol{P}_2$ does not exist for (5-87) because $(\boldsymbol{A}_1,\boldsymbol{B},\boldsymbol{C}^T)$ is not SPR, then $(\boldsymbol{A}_1,\boldsymbol{B},\boldsymbol{C}^T)$ can be converted to a SPR system $(\boldsymbol{A}_1,\boldsymbol{B}_c,\boldsymbol{C}^T)$ by following a similar way as described in (*Kim, Y.H., et al., 1997*). The details are as follows.

First, the output error dynamics of (5-72) can be formulated as

$$\tilde{\boldsymbol{E}}_1=\boldsymbol{H}(\mathrm{s})\breve{\boldsymbol{u}}\ , \tag{5-95}$$

where

$$\begin{aligned}&\boldsymbol{H}(\mathrm{s})=\boldsymbol{C}^T[s\boldsymbol{I}-(\boldsymbol{A}-\boldsymbol{K}_0\boldsymbol{C}^T)]^{-1}\boldsymbol{B}\\&\breve{\boldsymbol{u}}=[-\boldsymbol{\Phi}_f(\hat{\xi})\tilde{\boldsymbol{\Theta}}_f-\boldsymbol{\Phi}_G(\hat{\xi})\tilde{\boldsymbol{\Theta}}_G\boldsymbol{u}+\boldsymbol{\omega}+\boldsymbol{u}_h+\boldsymbol{u}_o+\boldsymbol{u}_s+\boldsymbol{d}']\ .\end{aligned} \tag{5-96}$$

The transfer function $\boldsymbol{H}(s)$ is a known stable transfer function matrix. In order to use the SPR-Lyapunov design approach, (5-96) can be written as

$$\begin{aligned}\tilde{\boldsymbol{E}}_1=&\ \boldsymbol{H}(\mathrm{s})\boldsymbol{L}(s)[-\boldsymbol{L}^{-1}(s)\boldsymbol{\Phi}_f(\hat{\xi})\tilde{\boldsymbol{\Theta}}_f-\boldsymbol{L}^{-1}(s)\boldsymbol{\Phi}_G(\hat{\xi})\tilde{\boldsymbol{\Theta}}_G\boldsymbol{u}+\boldsymbol{L}^{-1}(s)\boldsymbol{\omega}\\&+\boldsymbol{L}^{-1}(s)(\boldsymbol{u}_h+\boldsymbol{u}_o+\boldsymbol{u}_s)+\boldsymbol{L}^{-1}(s)\boldsymbol{d}']\end{aligned} \tag{5-97}$$

with

$$\boldsymbol{L}(s) = diag[L_1(s),...,L_m(s)] \tag{5-98}$$

and

$$L_i(s) = s^{p_i} + b_i s^{p_i - 1} + ... + b_{p_i} \quad (p_i < r_i), i = 1,2,...,m . \tag{5-99}$$

$\boldsymbol{L}(s)$ is chosen so, that $\boldsymbol{L}^{-1}(s)$ is a proper stable transfer function matrix and $\boldsymbol{H}(s)\boldsymbol{L}(s)$ to be a proper SPR transfer function matrix.

Then the state-space realization of (5-97) can be formulated as

$$\begin{aligned} \dot{\tilde{\boldsymbol{e}}}_c &= (\boldsymbol{A} - \boldsymbol{K}_0\boldsymbol{C}^T)\tilde{\boldsymbol{e}}_c + \boldsymbol{B}_c[-\hat{\boldsymbol{\Phi}}_f(\hat{\xi})\tilde{\boldsymbol{\Theta}}_f - \hat{\boldsymbol{\Phi}}_G(\hat{\xi})\tilde{\boldsymbol{\Theta}}_G\boldsymbol{u} + \hat{\boldsymbol{\omega}} + \hat{\boldsymbol{u}}_h + \hat{\boldsymbol{u}}_o \\ &+ \hat{\boldsymbol{u}}_s + \hat{\boldsymbol{d}}] \end{aligned} \tag{5-100}$$

$$\tilde{\boldsymbol{E}}_1 = \boldsymbol{C}^T\tilde{\boldsymbol{e}}_c$$

where

$$\begin{aligned} \boldsymbol{B}_c &= diag[\boldsymbol{B}_{c1},...,\boldsymbol{B}_{cm}] \\ \boldsymbol{B}_{ci} &= [0,0,...,b_{1i},...,b_{p_i}]^T \quad i = 1,2,...,m \end{aligned} \tag{5-101}$$

$$\begin{aligned} \hat{\boldsymbol{\Phi}}_f(\hat{\xi}) &= \boldsymbol{L}^{-1}(s)\boldsymbol{\Phi}_f(\hat{\xi}) \\ \hat{\boldsymbol{\Phi}}_G(\hat{\xi}) &= \boldsymbol{L}^{-1}(s)\boldsymbol{\Phi}_G(\hat{\xi}) \\ \varsigma &= -\boldsymbol{\Phi}_f(\hat{\xi})\tilde{\boldsymbol{\Theta}}_f - \boldsymbol{\Phi}_G(\hat{\xi})\tilde{\boldsymbol{\Theta}}_G\boldsymbol{u} + \boldsymbol{L}(s)\hat{\boldsymbol{\Phi}}_f(\hat{\xi})\tilde{\boldsymbol{\Theta}}_f + \boldsymbol{L}(s)\hat{\boldsymbol{\Phi}}_G(\hat{\xi})\tilde{\boldsymbol{\Theta}}_G\boldsymbol{u} \\ \boldsymbol{\omega}_T &= \boldsymbol{\omega} + \varsigma \\ \hat{\boldsymbol{\omega}} &= \boldsymbol{L}^{-1}(s)\boldsymbol{\omega}_T \end{aligned} \tag{5-102}$$

$$\begin{aligned} \hat{\boldsymbol{u}}_h &= \boldsymbol{L}^{-1}(s)\boldsymbol{u}_h \\ \hat{\boldsymbol{u}}_o &= \boldsymbol{L}^{-1}(s)\boldsymbol{u}_o \\ \hat{\boldsymbol{u}}_s &= \boldsymbol{L}^{-1}(s)\boldsymbol{u}_s \\ \hat{\boldsymbol{d}} &= \boldsymbol{L}^{-1}(s)\boldsymbol{d}' . \end{aligned} \tag{5-103}$$

Assumption 5-7: ς *is assumed to satisfy*

$$\|\varsigma\|_F \le o \qquad \text{with a positive constant } o > 0 . \tag{5-104}$$

■

Assumption 5-7 is reasonable because of the fact $\|\varsigma\|_F \leq o_1\|\tilde{\boldsymbol{\Theta}}_f\|_F + o_2\|\tilde{\boldsymbol{\Theta}}_G\|_F$ and $\|\tilde{\boldsymbol{\Theta}}_f\|_F$, $\|\tilde{\boldsymbol{\Theta}}_G\|_F$ are bounded by the projection algorithm.

Note, that each term in (5-96) need to be filtered by $\boldsymbol{L}^{-1}(s)$. However, as shown later, we can implement this scheme easily by filtering the fuzzy basis functions. Therefore this error dynamics can be only used for analysis purposes.

After this transformation $(\boldsymbol{A} - \boldsymbol{K}_0^T\boldsymbol{C}, \boldsymbol{B}_c, \boldsymbol{C}^T)$ is a SPR system. Since $(\boldsymbol{A} - \boldsymbol{K}_0^T\boldsymbol{C}, \boldsymbol{B}_c, \boldsymbol{C}^T)$ is SPR for the given positive matrix $\boldsymbol{Q}_2$ a positive-definite solution $\boldsymbol{P}_2$ exists for (5-87).

Since $\boldsymbol{B}_c{}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}}_c = \boldsymbol{C}^T\tilde{\boldsymbol{e}}_c = \tilde{\boldsymbol{E}}_1$ is available for feedback control we define

$$\begin{aligned}
\hat{\boldsymbol{u}}_h &= -\frac{1}{2}\boldsymbol{R}^{-1}\boldsymbol{B}_c{}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}}_c\,,\ \boldsymbol{R} = diag[r_1,..,r_m] \in \boldsymbol{R}^{m\times m},\ \ r_1,...,r_m > 0 \\
\hat{\boldsymbol{u}}_o &= -\boldsymbol{K}_o{}^T\boldsymbol{P}_1\hat{\boldsymbol{e}} \\
\hat{\boldsymbol{u}}_s &= -k_T\,\boldsymbol{sign}(\boldsymbol{B}_c{}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}}_c) \qquad\qquad k_T \geq k + o\,.
\end{aligned} \tag{5-105}$$

Following the Lyapunov stability proof once again the ideal parameter update laws results to

$$\begin{aligned}
\dot{\boldsymbol{\Theta}}_f &= \gamma_f\hat{\boldsymbol{\Phi}}_f{}^T(\hat{\xi})\boldsymbol{B}_c{}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}}_c \\
\dot{\boldsymbol{\Theta}}_{G,i} &= \gamma_G\hat{\boldsymbol{\Phi}}_{G,i}{}^T(\hat{\xi})\boldsymbol{B}_c{}^T\boldsymbol{P}_2\tilde{\boldsymbol{e}}_c u_i\,,\quad (i = 1,2,...,m)\,.
\end{aligned} \tag{5-106}$$

5.5 Modified adaptation laws (projection algorithm)

The adaptation laws (5-38) and (5-84) can be modified using a smooth gradient projection algorithm (*Khalil, H.K., 1996, IEEE*) to maintain the adjustable parameters in a certain constraint region defined by

$$\boldsymbol{\Omega}_{01} = \left\{\boldsymbol{\Theta}_f \mid \boldsymbol{\Theta}_f{}^T\boldsymbol{\Theta}_f \leq \beta_1\right\},\quad \boldsymbol{\Omega}_{\theta_f} = \left\{\boldsymbol{\Theta}_f \mid \boldsymbol{\Theta}_f{}^T\boldsymbol{\Theta}_f \leq \beta_1 + \delta_1\right\} \tag{5-107}$$

for some $\beta_1 > 0$ and $\delta_1 > 0$. Define the projection $Proj(\boldsymbol{\Theta}_f, \boldsymbol{\Gamma}_f)$ by

$$Proj(\boldsymbol{\Theta}_f, \boldsymbol{\Gamma}_f) = \begin{cases} \boldsymbol{\Gamma}_f - \dfrac{(\|\boldsymbol{\Theta}_f\|_F^2 - \beta_1)\boldsymbol{\Gamma}_f{}^T\boldsymbol{\Theta}_f}{\delta_1\|\boldsymbol{\Theta}_f\|_F^2}\boldsymbol{\Theta}_f & if\ \|\boldsymbol{\Theta}_f\|_F^2 = \beta_1\ and\ \boldsymbol{\Gamma}_f{}^T\boldsymbol{\Theta}_f > 0 \\ \boldsymbol{\Gamma}_f & otherwise \end{cases} \tag{5-108}$$

for some smooth function $\boldsymbol{\Gamma}_f$.

The resulting modified update law follows to

$$\dot{\boldsymbol{\Theta}}_{f,i} = \gamma_{f,i}[Proj(\boldsymbol{\Theta}_f, \boldsymbol{\Gamma}_f)]_i \qquad 1 \le i \le m \tag{5-109}$$

where $\boldsymbol{\Gamma}_f = \boldsymbol{\Phi}_f{}^T \boldsymbol{B}^T \boldsymbol{Pe}$ for (5-38) and $\boldsymbol{\Gamma}_f = \boldsymbol{\Phi}_f{}^T \boldsymbol{B}^T \boldsymbol{P}\tilde{\boldsymbol{e}}$, for (5-84), $\dot{\boldsymbol{\Theta}}_f = [\dot{\boldsymbol{\Theta}}_{f,1},\dots,\dot{\boldsymbol{\Theta}}_{f,m}]^T$.

In order to implement the robust indirect adaptive control algorithm the matrices $\hat{\boldsymbol{G}}(\boldsymbol{x} \mid \boldsymbol{\Theta}_G)$ and $\hat{\boldsymbol{G}}(\boldsymbol{\xi} \mid \boldsymbol{\Theta}_G)$, respectively must be invertible, refer to Assumption 5-2 and Assumption 5-5 . Now we shall relax this assumption. E.g. the matrix $\hat{\boldsymbol{G}}(\boldsymbol{x} \mid \boldsymbol{\Theta}_G)$ can be shown to be invertible for a class of nonlinear systems by a suitable choice of the regions $\boldsymbol{\Omega}_{02}$ and $\boldsymbol{\Omega}_{\theta_g}$.

Consider the class of MIMO nonlinear systems in which the output y_i is mainly dominated by the input u_i and the effect of u_j for $j \neq i$ on y_i is smaller than of u_i. For such class of systems we can set b_{ijk} and c_{ijk} such that $|\hat{g}_{ii}| \ge \sum_{j=1,j\neq i}^{m} |\hat{g}_{ij}|, \forall\, 1 \le i \le m$. Then, $\hat{\boldsymbol{G}}(\boldsymbol{x} \mid \boldsymbol{\Theta}_G)$ is so-called strictly diagonal dominant and is invertible (*Horn, R.A., Johnson, C.R., 1991*).

Consider the fuzzy system $\hat{g}_{ij}(\boldsymbol{x} \mid \boldsymbol{\theta}_{g,ij}) = \boldsymbol{\varphi}_{g,ij}^T \boldsymbol{\theta}_{g,ij}$, where

$$\boldsymbol{\theta}_{g,ij} = [\theta_{g,ij1},\dots,\theta_{g,ijm_{G,ij}}]^T \in \boldsymbol{R}^{m_{G,ij}} \tag{5-110}$$

and

$$\boldsymbol{\varphi}_{g,ij} = [\varphi_{g,ij1},\dots,\varphi_{g,ijm_{G,ij}}]^T \in \boldsymbol{R}^{m_{G,ij}} \tag{5-111}$$

again. Chose $\boldsymbol{\Omega}_{\theta_g}$ as convex hypercube, i.e. $\boldsymbol{\Omega}_{02} = \{\boldsymbol{\Theta}_G \mid b_{ijk} \le \theta_{g,ijk} \le c_{ijk}\}$ and $\boldsymbol{\Omega}_{\theta_g} = \{\boldsymbol{\Theta}_G \mid b_{ijk} - \delta_2 \le \theta_{g,ijk} \le c_{ijk} + \delta_2\}$ for $1 \le k \le m_{G,ij}$ and $1 \le i, j \le m$ where b_{ijk}, c_{ijk} and $\delta_2 > 0$ can be specified by the designer. The smooth projection algorithm with respect to $\boldsymbol{\Theta}_G$ is obtained as (*Khalil, H.K., 1996, IEEE*)

$$\dot{\theta}_{g,ijk} = \begin{cases} \gamma_{G,ij}\hat{\Gamma}_{g,ijk} & if\ (\theta_{g,ijk} > c_{ijk}\ and\ \Gamma_{g,ijk} > 0) \\ \gamma_{G,ij}\breve{\Gamma}_{g,ijk} & if\ (\theta_{g,ijk} < b_{ijk}\ and\ \Gamma_{g,ijk} < 0) \\ \gamma_{G,ij}\Gamma_{g,ijk} & otherwise\ , \end{cases} \tag{5-112}$$

where

$$\Gamma_{g,ijk} = \varphi_{g,ijk}\boldsymbol{b}_j'^T \boldsymbol{P}\tilde{\boldsymbol{e}} u_i, \text{ with } \boldsymbol{B} = [\boldsymbol{b}_1',\dots,\boldsymbol{b}_m'], \tag{5-113}$$

$\widehat{\Gamma}_{g,ijk} = (1 + \frac{c_{ijk} - \theta_{g,ijk}}{\delta_2})\Gamma_{g,ijk}$ and $\breve{\Gamma}_{g,ijk} = (1 + \frac{\theta_{g,ijk} - b_{ijk}}{\delta_2})\Gamma_{g,ijk}$.

5.6 Dynamic fuzzy rule activation method

At the beginning of the fuzzy model design process the designer must determine a fine grid of fuzzy rules and fuzzy membership functions, considering as model-input all elements of the state vector $\boldsymbol{x}$ or $\boldsymbol{\xi}$, respectively which are relevant for modelling the nonlinear plant matrices $\boldsymbol{f}$ and $\boldsymbol{G}$ in (5-3) and (5-47), respectively. This grid and thus the number of rules grows dramatically with the order of the plant (order of the state vector) and among experts this phenomenon is called the "curse of dimensionality". In accompany with the increasing complexity of the fuzzy systems also the burden for the computation and storage effort grows dramatically. Especially concerning the hardware realisation of fuzzy controls where the processor power and the storage capability is limited, the "curse of dimensionality" is a critical point in the design of fuzzy controls.
However, once the membership functions are determined, the fuzzy systems (5-13) are constructed from the fuzzy IF-THEN rules. The larger the number of fuzzy rules is, the better approximating ability the fuzzy system obtains. That is, there is a trade-off between the number of fuzzy rules on one side and the performance of the fuzzy system respectively the computation load on the other side. In fact, in (5-13) most Fuzzy Basis Functions (FBFs) and thus the corresponding rules show very small values for a given $\boldsymbol{x}$ or $\boldsymbol{\xi}$. Consequently, they do not nearly contribute to the output of the fuzzy systems (5-13). The idea of the dynamic fuzzy rule activation method (*Park, J.H., et al., 2003*) is now, that at initial time the output of the fuzzy systems is estimated by only a small number of fuzzy rules whose FBF-values are greater than a given threshold and unused rules are gradually activated on-line during the control procedure. By using this dynamic rule activation method, we can reduce the computation time, storage space and dynamic order of the adaptive fuzzy systems because only the adjustable parameters corresponding to the activated FBFs are updated. Based on this idea, the kth FBF of $\boldsymbol{\varphi}_{f,i}$, resp. $\boldsymbol{\varphi}_{g,ij}$ in (5-13) is activated after t_0 if the following activation conditions are satisfied:

1) $\varphi_{f,ik}(\boldsymbol{x})$ resp. $\varphi_{g,ijk}(\boldsymbol{x})$ was inactivated for $0 \le t \le t_0$,
2) $\varphi_{f,ik}(\boldsymbol{x}) \ge \varepsilon_{FBF}$ resp. $\varphi_{g,ijk}(\boldsymbol{x}) \ge \varepsilon_{FBF}$

(5-114)

$1 \le i, j \le m$, $1 \le k \le m_{f,i}$ resp. $1 \le k \le m_{g,ij}$
$m_{f,i}$, $m_{g,ij}$ ...total number of fuzzy rules.

where ε_{FBF} represents an activation threshold satisfying $0 \le \varepsilon_{FBF} < 1$.

If there exists a priori knowledge on the newly activated rule, for example, through an offline identification or expert knowledge, we can use that information. Otherwise, the initial values of the adjustable parameters associated with newly activated FBFs can be set simply to zeros. Using this scheme, the output of the fuzzy systems at time t consists of the contribution of the active FBFs. Thus, one could start with a fuzzy system having a few rules and gradually activate fuzzy rules in response to the activation condition being satisfied. By such means, the number of activated fuzzy rules is increased sequentially, which ensures that the function being approximated is learned up to the required levels of accuracy, using only a minimal number of fuzzy rules. The fuzzy system including this dynamic rule activation method needs smaller computation time and storage space but shows almost the same level of approximation accuracy compared with the conventional fuzzy system, whereby the number of fuzzy rules is determined a priori by the grid rule structure and does not change.

Let us divide the FBFs and the corresponding adjustable parameters into two parts:

$$\begin{aligned} \boldsymbol{\varphi}_{f,i}(\boldsymbol{x}) &= \begin{bmatrix} \boldsymbol{\varphi}_{f,iA} \\ \boldsymbol{\varphi}_{f,iP} \end{bmatrix}, \ \boldsymbol{\varphi}_{g,ij}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{\varphi}_{g,ijA} \\ \boldsymbol{\varphi}_{g,ijP} \end{bmatrix} \\ \boldsymbol{\theta}_{f,i} &= \begin{bmatrix} \boldsymbol{\theta}_{f,iA} \\ \boldsymbol{\theta}_{f,iP} \end{bmatrix}, \ \boldsymbol{\theta}_{g,ij} = \begin{bmatrix} \boldsymbol{\theta}_{g,ijA} \\ \boldsymbol{\theta}_{g,ijP} \end{bmatrix}, \end{aligned} \tag{5-115}$$

where $\boldsymbol{\varphi}_{f,iA}$, $\boldsymbol{\varphi}_{g,ijA}$ are activated FBFs and $\boldsymbol{\theta}_{f,iA}$, $\boldsymbol{\theta}_{g,ijA}$ are their corresponding adjustable parameters. $\boldsymbol{\varphi}_{f,iP}$, $\boldsymbol{\varphi}_{g,ijP}$ and $\boldsymbol{\theta}_{f,iP}$, $\boldsymbol{\theta}_{g,ijP}$ are inactivated (passive) FBFs and their corresponding adjustable parameters. By definition, the passive parameter vector and their time derivative are zero vectors, i.e. $\boldsymbol{\theta}_{f,iP} = \dot{\boldsymbol{\theta}}_{f,iP} = \boldsymbol{0}$, $\boldsymbol{\theta}_{g,ijP} = \dot{\boldsymbol{\theta}}_{g,ijP} = \boldsymbol{0}$. Then, we can describe the output of the Fuzzy Logic Systems as

$$\begin{aligned} \hat{f}_i(\boldsymbol{x} \mid \boldsymbol{\theta}_{f,iA}) &= \boldsymbol{\varphi}_{f,iA}(\boldsymbol{x})^T \boldsymbol{\theta}_{f,iA} + \boldsymbol{\varphi}_{f,iP}(\boldsymbol{x})^T \boldsymbol{\theta}_{f,iP} = \boldsymbol{\varphi}_{f,iA}(\boldsymbol{x})^T \boldsymbol{\theta}_{f,iA}, \\ \hat{g}_{ij}(\boldsymbol{x} \mid \boldsymbol{\theta}_{g,ijA}) &= \boldsymbol{\varphi}_{g,ijA}(\boldsymbol{x})^T \boldsymbol{\theta}_{g,ijA} + \boldsymbol{\varphi}_{g,ijP}(\boldsymbol{x})^T \boldsymbol{\theta}_{g,ijP} = \boldsymbol{\varphi}_{g,ijA}(\boldsymbol{x})^T \boldsymbol{\theta}_{g,ijA}. \end{aligned} \tag{5-116}$$

Assuming, without any restriction, the state vector $\boldsymbol{x}$ to be measurable and the measurement noise vectors $\boldsymbol{n}_y(t)$ and $\boldsymbol{n}_x(t)$ to be zero vectors, the *optimal parameter vector* $\boldsymbol{\Theta}^*_{f,A}$ and the *optimal parameter matrix* $\boldsymbol{\Theta}^*_{G,A}$ corresponding to the activated FBFs follow to

$$\begin{aligned} \boldsymbol{\Theta}^*_{f,A} &= arg \min_{\boldsymbol{\Theta}_{f,A} \in \boldsymbol{\Omega}_f} \left\{ \sup_{\boldsymbol{x} \in \boldsymbol{U}_I} \left\| \boldsymbol{f}(\boldsymbol{x}) - \hat{\boldsymbol{f}}(\boldsymbol{x} \mid \boldsymbol{\Theta}_{f,A}) \right\|_F \right\} \\ \boldsymbol{\Theta}^*_{G,A} &= arg \min_{\boldsymbol{\Theta}_{G,A} \in \boldsymbol{\Omega}_G} \left\{ \sup_{\boldsymbol{x} \in \boldsymbol{U}_I} \left\| \boldsymbol{G}(\boldsymbol{x}) - \hat{\boldsymbol{G}}(\boldsymbol{x} \mid \boldsymbol{\Theta}_{G,A}) \right\|_F \right\} \end{aligned} \tag{5-117}$$

and the minimum (fuzzy) approximation error $\boldsymbol{\omega}_A$ yields

$$\boldsymbol{\omega}_A = \boldsymbol{f}(\boldsymbol{x}) - \hat{\boldsymbol{f}}(\boldsymbol{x} \mid \boldsymbol{\Theta}^*_{f,A}) + (\boldsymbol{G}(\boldsymbol{x}) - \hat{\boldsymbol{G}}(\boldsymbol{x} \mid \boldsymbol{\Theta}^*_{G,A}))\boldsymbol{u} = \Delta \boldsymbol{f}_A + \Delta \boldsymbol{G}_A \boldsymbol{u}. \tag{5-118}$$

Recall the *minimum (fuzzy) approximation error* $\boldsymbol{\omega}$ which results if all rules of the rule base are activated ($\varepsilon_{FBF} = 0$)

$$\boldsymbol{\omega} = \boldsymbol{f}(\boldsymbol{x}) - \hat{\boldsymbol{f}}(\boldsymbol{x} \mid \boldsymbol{\Theta}^*_f) + (\boldsymbol{G}(\boldsymbol{x}) - \hat{\boldsymbol{G}}(\boldsymbol{x} \mid \boldsymbol{\Theta}^*_G))\boldsymbol{u} = \Delta \boldsymbol{f} + \Delta \boldsymbol{G} \boldsymbol{u} \tag{5-119}$$

two conclusions are apparent

$$\|\boldsymbol{\omega}_A\|_F \geq \|\boldsymbol{\omega}\|_F \text{ and } \|\boldsymbol{\omega}_A\|_F \to \|\boldsymbol{\omega}\|_F \text{ if } \varepsilon_{FBF} \to 0, \qquad (5\text{-}120)$$

or in other words the approximation accuracy which can be achieved by the rule activation method compared with conventional fuzzy systems shows almost the same level if a sufficient small ε_{FBF} is chosen. On the other hand, if somebody is using the rule activation method but also wants to eliminate the effect of $\boldsymbol{\omega}_A$ on the tracking error of the closed loop control system (keep the same performance level) the constant k in the $\boldsymbol{u}_S$ term of (5-78) has to be raised to compensate the additional approximation error $\Delta\omega = \|\boldsymbol{\omega}\|_F - \|\boldsymbol{\omega}_A\|_F \geq 0$. This means there is a trade-off between the number of fuzzy IF-THEN rules and the control gain in the VSS control algorithm. Inequality $(1-\kappa)\boldsymbol{R}^{-1} - 1/\rho^2\boldsymbol{I} > 0$ provides a sufficient condition to guarantee the solvability of (5-40), (*Tong, S.C., et al., 2000*). The additional approximation error $\Delta\omega$ implies the bounded value κ is bigger and R^{-1} is also bigger. That is, there is also a trade-off between the number of fuzzy IF-THEN rules in $\hat{\boldsymbol{G}}$ and the control gain in the robust H_∞ controller.

Chapter 6

6 SIMULATION EXAMPLES

6.1 SISO Inverted Pendulum System

In this Section we apply the ORIAFC to control an inverted pendulum system (*Wang, L.X., 1994*), also known as cart-pole system, as shown in Fig. 6-1. This experiment is often picked up in the control literature to show the performance of developed control algorithms. The control objective is now to control the pole angle θ to track a given reference trajectory by applying a force F to the cart. The cart position x is not controlled in this example because the simulated plant is assumed to be a SISO system. However, if the cart position is not constraint within small limits there is no contradiction to the real-time experiment. The nonlinear dynamic equations of this plant can be described as

$$\begin{aligned} &(m_c + m_p)\ddot{x} + k\dot{x} + m_p l\,\ddot{\theta}\cos(\theta) - m_p l\,\dot{\theta}^2 \sin(\theta) = -F \\ &(I + m_p l^2)\ddot{\theta} + c\dot{\theta} - m_p g\,l\sin(\theta) = -m_p l\,\ddot{x}\cos(\theta)\,, \end{aligned} \tag{6-1}$$

where F is the applied force to the cart, k is the viscous friction coefficient of the cart, g is the acceleration due to gravity ($g=9.81m/s^2$), I is moment of inertia of the pole about the center of gravity (COG), c is the viscous friction coefficient of the pole, m_c is the mass of the cart, m_p is the mass of the pole which is assumed to be concentrated in the COG and l is the half-length of the pole. We choose $m_c=1kg$, m_p=0.1kg and $l=0.5m$ in the following simulations. The pole is assumed to have a very small diameter in comparison to its length. Then the moment of inertia of the pole can be calculated to $I=m_p l^2/3$. The friction coefficients are assumed to be unknown.

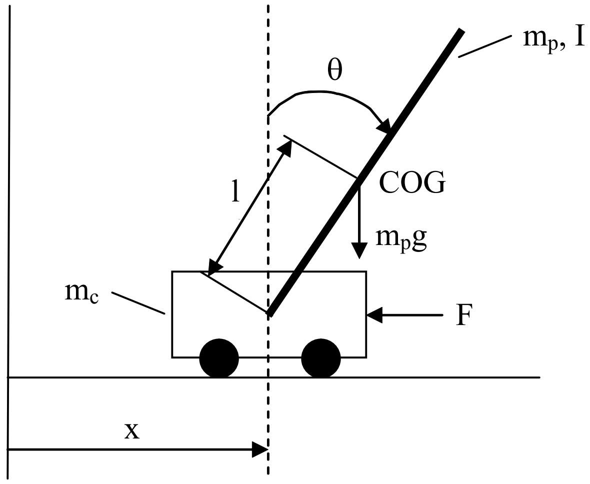

Fig. 6-1: Inverted pendulum system

From the viewpoint of control engineering the SISO plant equations (6-1) inclusive measurement are described in normal form (5-46) as

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}[\,f(\boldsymbol{x}) + g(\boldsymbol{x})u + d\,] \\ \bar{y} &= \boldsymbol{c}^T\boldsymbol{x} + n_y\,. \end{aligned} \qquad (6\text{-}2)$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ \boldsymbol{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \boldsymbol{c} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} \theta & \dot{\theta} \end{bmatrix}^T,\ u = F\ ,\ (y=\theta)$$

$$f(\boldsymbol{x}) = f(x_1, x_2) = \frac{g\,sin(x_1) - \dfrac{m_p l\, x_2^{\,2}\, cos(x_1)\,sin(x_1)}{m_c + m_p}}{l(\dfrac{4}{3} - \dfrac{m_p\, cos^2(x_1)}{m_c + m_p})} \qquad (6\text{-}3)$$

$$g(\boldsymbol{x}) = g(x_1) = \frac{\dfrac{cos(x_1)}{m_c + m_p}}{l(\dfrac{4}{3} - \dfrac{m_p\, cos^2(x_1)}{m_c + m_p})}.$$

Tracking Control with ORIAFC

The external disturbance (including the plant uncertainties due to the unknown friction components) is assumed as $d(t)=3sin(t)\ (rad/s^2)$, the measurement noise $n_y(t)$ (sensor noise, digitization of the sensor signals) is assumed to be white with uniform distribution within the interval *[-5e-3, 5e-3] (rad)* and band-limited by using zero-order-hold with a sampling time of *10ms*. Our control objective is to control the pole angle θ of the cart-pole system to track the reference trajectory $y_M=0.5sin(t)$ *(rad)*. The feedback and observer gain vectors are selected as $\boldsymbol{K}=\boldsymbol{k}=[1296\ \ 72]^T$ and $\boldsymbol{K}o=\boldsymbol{k}o=[90\ \ 2025]^T$, respectively. A matrix $\boldsymbol{Q}=\boldsymbol{diag}([1\ \ 0.1])$ is assumed. The controller parameters are chosen as $k_T=10$, $\alpha=1e\text{-}2$, $R=r=5e\text{-}4$, (minimum achievable attenuation level $\rho_{min}=2.23e\text{-}2$) and the fuzzy modelling parameters as $\gamma_f=3.0e4$, $\gamma_g=10$, $\beta_l=1e4$, $\delta_l=1e3$, $b_k=0$, $c_k=20$, $\delta_2=0.02$. The membership functions for the fuzzy system inputs $\hat{x}_i$, $i=1,2$ are given as Gaussian curve membership functions $\mu_{F_i^l}(\hat{x}_i)=exp[-(\hat{x}_i-\bar{x}^l)^2/(2\sigma^2)]$ where $\sigma=0.35$ and $\bar{x}^l=-1,-0.5,0,0.5,1$ for $l=1,2,3,4,5$, respectively. The inputs $\hat{x}_1,\hat{x}_2$ are therefore normalized and by $\hat{x}_1/(0.6rad)$ and $\hat{x}_2/(0.6rad/sec)$ respectively. The approximated function $\hat{f}$ depends per system definition (6-3) on the inputs $\hat{x}_1$ and $\hat{x}_2$, the function $\hat{g}$ only on the input $\hat{x}_1$. These circumstances reduce the total number of fuzzy rules to *30 rules*. The initial values of the adjustable fuzzy parameters are set to $\theta_{f,k}=0$ for $k=1,2,..,25$ and $\theta_{g,k}=5$ for $k=1,2,..,5$. The initial plant and observer states are given as $\boldsymbol{x}(0)=[0.25rad\ \ 0.05rad/s]^T$ and $\hat{\boldsymbol{x}}(0)=[0rad\ 0rad/s]^T$, respectively. Finally, the filter L^{-1} is chosen to $L^{-1}=1/(s+2)$.

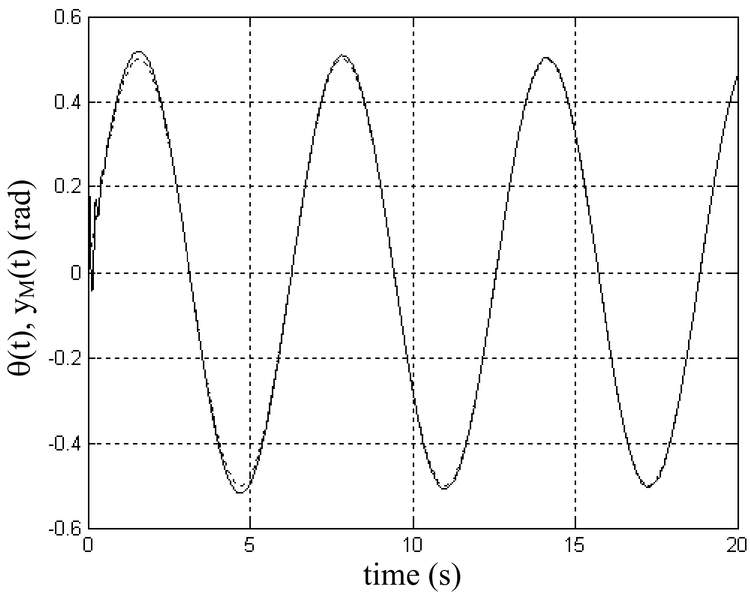

Fig. 6-2: Tracking curves pole angle $\theta(t)$ (solid line), $y_M(t)$ (dashed line)

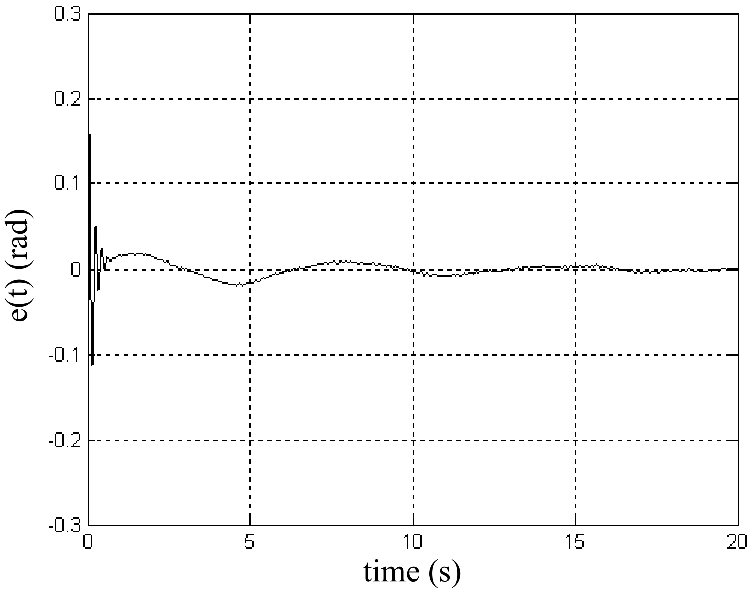

Fig. 6-3: Tracking error *e(t)*

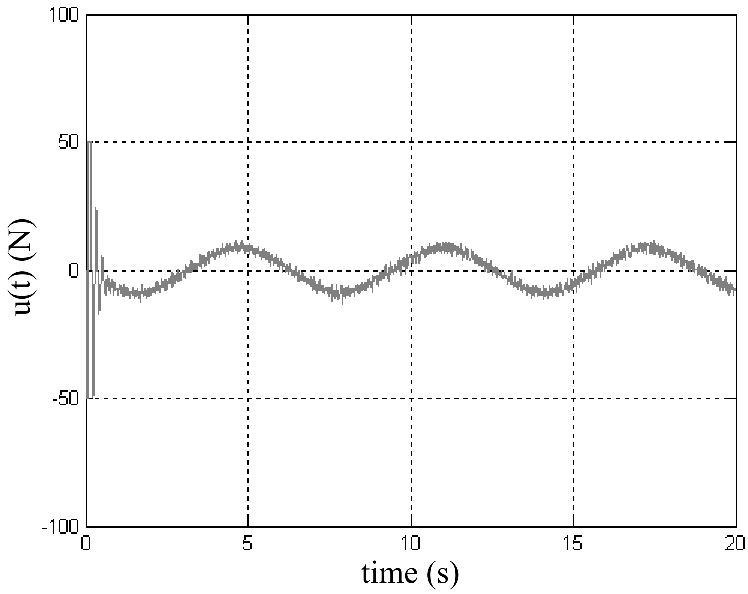

Fig. 6-4: Control variable *u(t)*

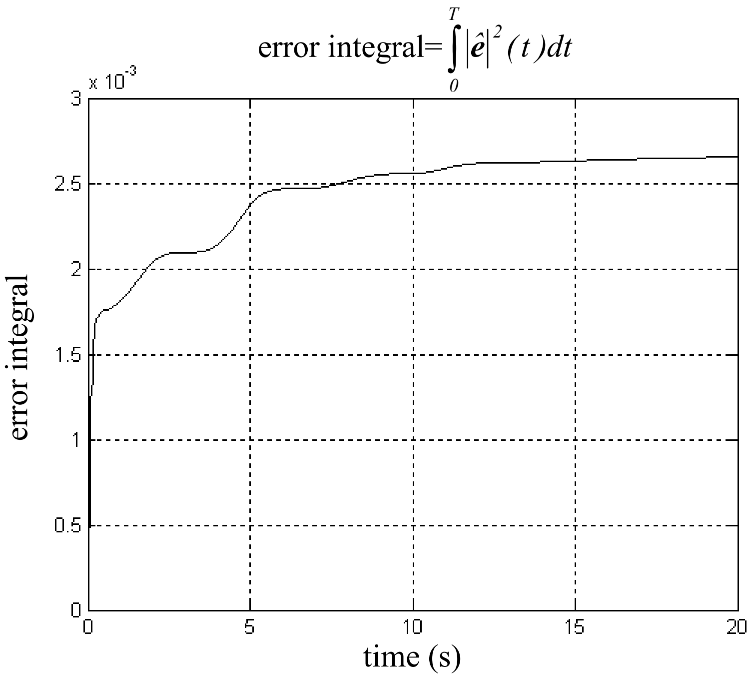

Fig. 6-5: Error integral versus time

The simulation results Fig. 6-2 - Fig. 6-5 (all variables are displayed in SI-units) demonstrate, that the ORIAFC can perform a successful tracking control. The influence of external disturbances and measurement noise on the tracking errors can be attenuated to a small level by decreasing the attenuation level ρ which goes along with choosing a small controller gain r. Also the adaptation speed is superior, although no information about the system nonlinearities was available ("blackstart") and only half of the system states were measurable for the control, respectively. However, if there is additional linguistic information from human experts about the plant nonlinearities f and g in terms of fuzzy IF-THEN rules available, then this information can also easily be implemented due to the open structure of the fuzzy systems (*Wang, L.X*, *1992, Wang, L.X.*, *1993*).

6.2 MIMO Magnetic Levitation System

In this section we apply the H$_\infty$-observer based adaptive fuzzy control system to control a MIMO magnetic levitation system (*EPC, 1991*) as shown in Fig. 6-6. The control objective is now to control the magnet positions y_1 and y_2 of the system to track given reference trajectories. The nonlinear dynamic equations of this plant can be described as

$$\begin{aligned} m\ddot{y}_1 + c_1\dot{y}_1 + F_{m12} &= F_{u11} - F_{u21} - mg \\ m\ddot{y}_2 + c_2\dot{y}_2 - F_{m12} &= F_{u22} - F_{u12} - mg \;. \end{aligned} \tag{6-4}$$

F_{m12} are the magnet forces, F_{uij} for $i,j=1,2$ the actuator forces $c_i\dot{y}_i$ for $i=1,2$ the friction forces, m is the mass of one magnet disk ($m=0.125kg$) and mg the gravity force which affects one magnet disk ($g=9.81m/s^2$). According to the manufacturer, the friction forces are typically small and are therefore not modelled in simulations. In case of real time experiments, the friction forces will be approximated by the adaptive fuzzy systems.

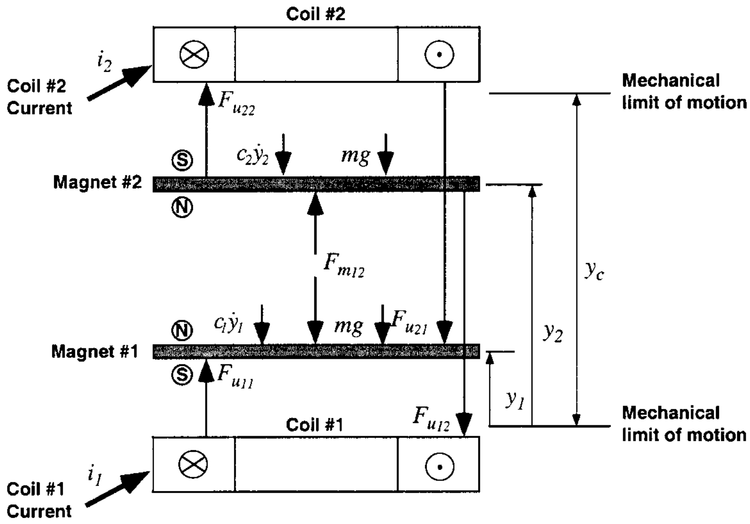

Fig. 6-6: MIMO magnetic levitation system

The magnetic/actuator forces are modeled as having the following forms

$$F_{u11} = \frac{i_1}{a(y_1 + b)^N}, \qquad F_{u12} = \frac{i_1}{a(y_c + y_2 + b)^N},$$
$$F_{u21} = \frac{i_2}{a(y_c - y_1 + b)^N}, \qquad F_{u22} = \frac{i_2}{a(y_c - y_2 + b)^N}, \tag{6-5}$$
$$F_{m12} = \frac{c}{(y_2 - y_1 + d)^N}.$$

i_1 and i_2 are the coil currents of coil1 and coil2 respectively and *a, b, c, d* and *N* are constants which may be determined by numerical modelling of the magnetic configuration or by empirical methods. Typically $3<N<4.5$. In our simulations N was selected as $N=4$, and the parameters *a, b, c, d* were empirically determined as $a=5167A/Nm^4$, $b=0.07984m$, $c=0.00025Nm^4$, $d=0.0407m$. The actuator forces F_{u12} and F_{u21} are generally small compared to F_{u11} and F_{u22} for typical values of coil current and for the magnets in their normal operating range and are therefore, without any restrictions, also omitted in the simulations. The step response of the coils show, that a voltage step of *1 Volt* generates a steady state current of *0.4 Ampere*, this means there is a conversion constant of $k_u=2.5$ Ohm to consider if the actuator is assumed to be a linear voltage-driven current source. The input-output characteristics of the position sensors (Laser Sensors) in Model 730 are nonlinear (static nonlinearities). This effect can be compensated sufficiently by arranging software building blocks, which model the inverse of these functions in the form of nonlinear algebraic equations (*EPC, 1991*), after the analog input blocks. The parameters of this nonlinearity can be determined empirically by following some calibration instructions, also given in (*EPC, 1991*). However, in our simulations exact linearized sensor characteristics are assumed. Special attention needs the rapid heat-up process of the coils due to current excitation. The manufacturer strictly recommends to limit the rms-value of the coil currents i_1 and i_2 to values about $1.2\ A_{eff}$. Short time stresses e.g. $4\ A_{peak}$ for $0.5s$ within a interval of $60s$ are also possible. Both current constraints were successfully maintained in our simulations. Therefore, as minimum requirement the control variables u_i are limited by the interval *[-10,10] (Volt)*. The magnet positions are limited by the interval *[0,0.14] (m)* due to the mechanical constraints.

Considering the simplifications above, from the viewpoint of control engineering the plant equations (6-4) inclusive measurement are described in normal form (5-46) as

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}[\,\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{d}\,] \\ \bar{\boldsymbol{y}} &= \boldsymbol{C}^T\boldsymbol{x} + \boldsymbol{n}_y \end{aligned} \tag{6-6}$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},\ \boldsymbol{C} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\boldsymbol{x} = [y_1 \quad \dot{y}_1 \quad y_2 \quad \dot{y}_2]^T,\ \boldsymbol{u} = [u_1 \quad u_2]^T$$

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{f}(x_1, x_3) = \begin{bmatrix} -\dfrac{c}{m(y_2 - y_1 + d)^N} - g \\ \dfrac{c}{m(y_2 - y_1 + d)^N} - g \end{bmatrix} \tag{6-7}$$

$$\boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{G}(x_1, x_3) = \begin{bmatrix} \dfrac{1}{k_u m\, a(y_1 + b)^N} & 0 \\ 0 & \dfrac{1}{k_u m\, a(y_c - y_2 + b)^N} \end{bmatrix}.$$

6.2.1 Tracking Control with RIAFC

The external disturbance, e.g. due to external axial vibrations which interfere on the apparatus, is assumed as $\boldsymbol{d}(t)=[0.5sin(t);\ 0.1sin(t)]$ *(m/s²)*, the measurement noise $\boldsymbol{n}_y(t)=[n_{y,1};\ n_{y,2}]$ (sensor noise, digitization of the sensor signals) is assumed to be white with uniform distribution within the interval *[-1e-4, 1e-4] (m)* and band-limited by using zero-order-hold with a sampling time of *10ms*. The noise signals of the different position sensors do not correlate with each other. Our control objective is to control the magnet positions y_1 and y_2 of the system to track the reference trajectories

$y_{M,1}$=0.01sin(t)+0.02(1-$e^{-t/0.1}$) (m) and $y_{M,2}$= 0.01sin(t)+0.045(1-$e^{-t/0.1}$)+0.075 (m), respectively. The feedback gain matrix is selected as ***K***=[0.36 0; 1.2 0; 0 0.36; 0 1.2]. A matrix ***Q***=***diag***([5 5 5 5]), an attenuation level ρ=3e-3 and κ=0 are assumed. The controller parameters are chosen as k_T=1, α=1e-4, ***R***=4.5e-4***eye***(2) and the fuzzy modelling parameters as $\gamma_{f,1}$ =3e6, $\gamma_{f,2}$ =6e5, $\gamma_{G,11}$=1.5e5, $\gamma_{G,22}$=1e5, β_1=1e4, δ_1=1e3, b_{11k}= b_{22k}=0 , c_{11k}= c_{22k}=20, δ_2=0.02. The membership functions for the fuzzy system inputs $\hat{x}_i$, *i=1,3* are given as Gaussian curve membership functions $\mu_{F_i^l}(\hat{x}_i) = exp[-(\hat{x}_i - \bar{x}^l)^2/(2\sigma^2)]$ where σ=0.35 and $\bar{x}^l$=-1,-0.5,0,0.5,1 for *l=1,2,3,4,5*, respectively. The inputs $\hat{x}_1, \hat{x}_3$ are therefore normalized and offset-reassessed by ($\hat{x}_1$-0.02m)/0.01m and ($\hat{x}_3$-0.12m)/0.01m respectively. The approximated functions $\hat{f}_1$ and $\hat{f}_2$ depend per system definition (6-7) only on the inputs $\hat{x}_1$ and $\hat{x}_3$, the function $\hat{g}_{11}$ only on the input $\hat{x}_1$ and $\hat{g}_{22}$ only on the input $\hat{x}_3$. These circumstances reduce the total number of fuzzy rules to *35 rules* ($\hat{f}_1$ and $\hat{f}_2$ share the same rule base). The initial values of the adjustable fuzzy parameters are set to $\theta_{f,1k} = -15$ and $\theta_{f,2k} = -5$ for *k=1,2,..,25* and $\theta_{g,11k} = \theta_{g,22k} = 10$ for *k=1,2,..,5*. The initial plant states are given as ***x***(0)=[0m 0m/s 0.075m 0m/s]^T.

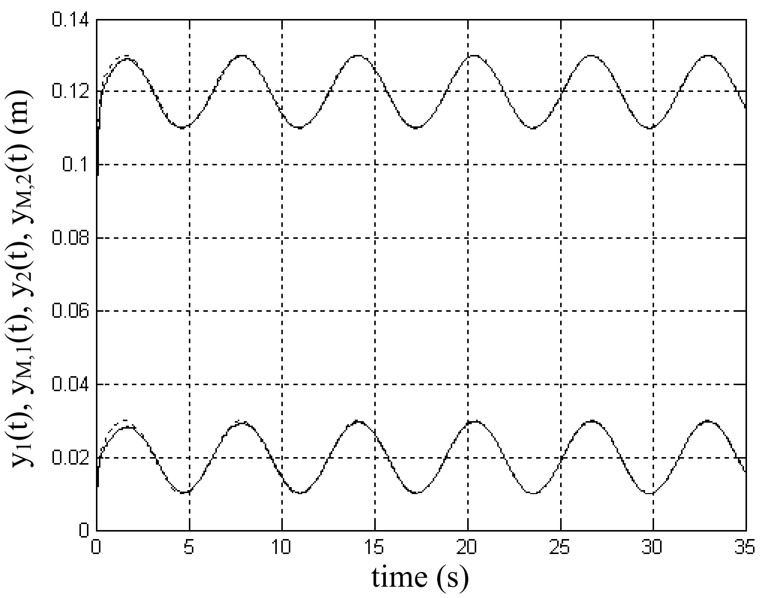

Fig. 6-7: Tracking curves magnet positions $y_1(t)$ (below, solid line), $y_{M,1}(t)$ (below, dashed line) and $y_2(t)$ (top, dashed line), $y_{M,2}(t)$ (top, dashed line)

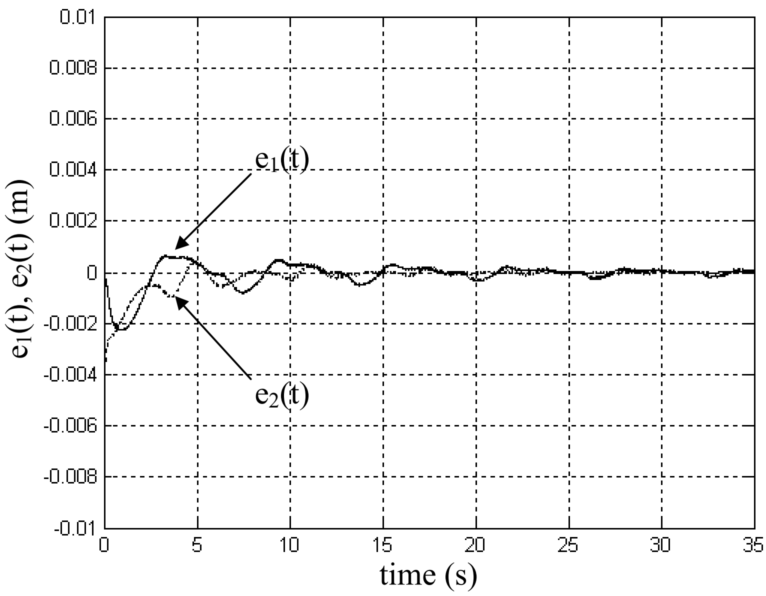

Fig. 6-8: Tracking errors $e_1(t)$ and $e_2(t)$

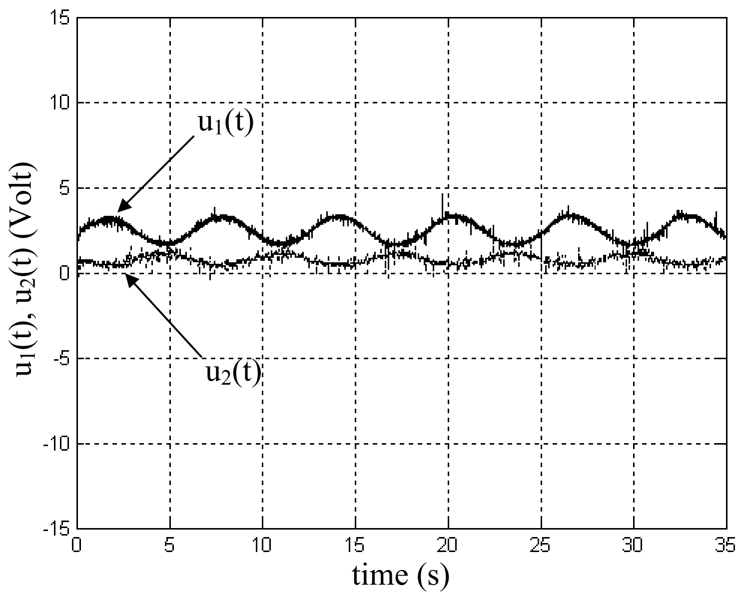

Fig. 6-9: Control variables $u_1(t)$, $u_2(t)$

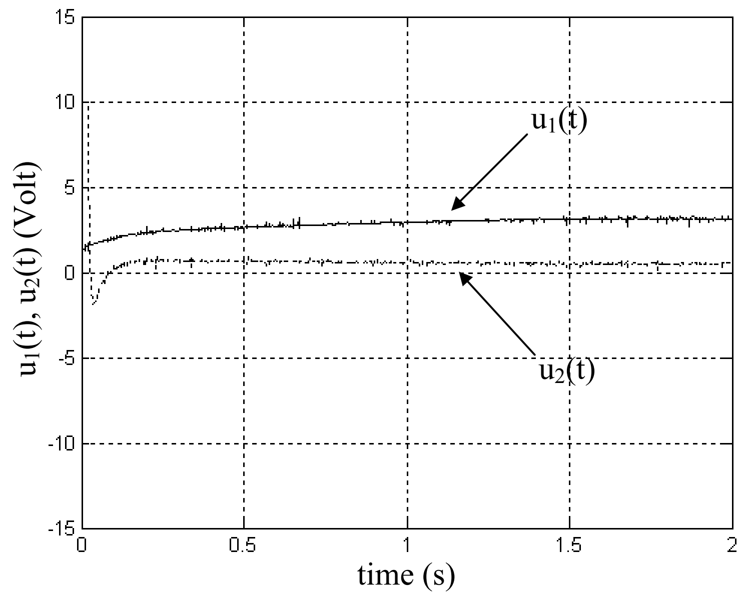

Fig. 6-10: Zoom: Control variables $u_1(t)$, $u_2(t)$

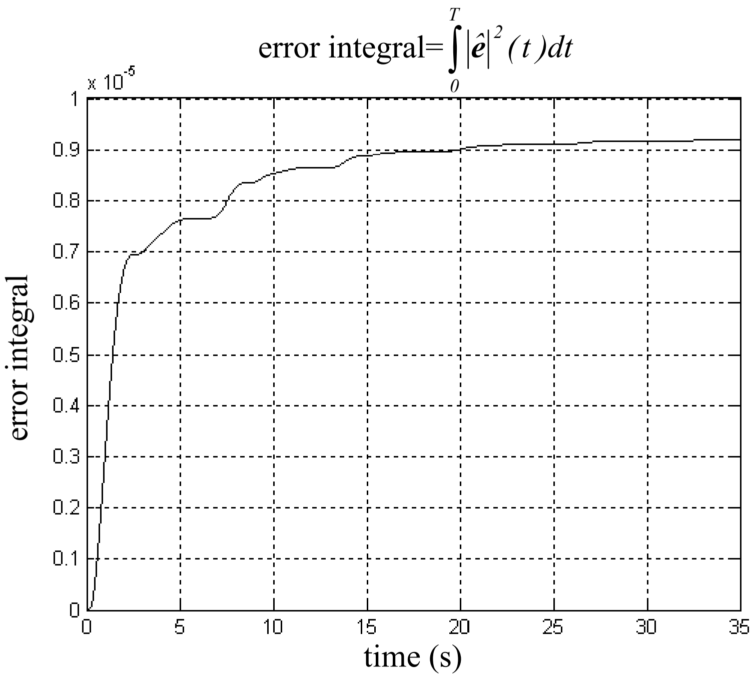

Fig. 6-11: Error integral versus time

The simulation results Fig. 6-7 - Fig. 6-11 (all variables are displayed in SI-units) demonstrate, that the RIAFC can perform a successful tracking control. The influence of external disturbances and measurement noise on the tracking errors can be attenuated to a small level by decreasing the attenuation level ρ which goes along with choosing a controller matrix $\boldsymbol{R}$ with a small Frobenius norm. To reduce the noise level in the control signal Also the adaptation speed is superior, although no exact information about the system nonlinearities was available However, if there is additional linguistic information from human experts about the plant nonlinearities $\boldsymbol{f}$ and $\boldsymbol{G}$ in terms of fuzzy IF-THEN rules available, then this information can also easily be implemented due to the open structure of the fuzzy systems (*Wang, L.X*, *1992, Wang, L.X.*, *1993*).

6.2.2 Tracking Control with ORIAFC

The external disturbance, e.g. due to external axial vibrations which interfere on the apparatus, is assumed as $\boldsymbol{d}(t)=[0.5\sin(t);\ 0.1\sin(t)]\ (m/s^2)$, the measurement noise $\boldsymbol{n}_y(t)=[n_{y,1};\ n_{y,2}]$ (sensor noise, digitization of the sensor signals) is assumed to be white with uniform distribution within the interval $[-1e\text{-}4,\ 1e\text{-}4]\ (m)$ and band-limited by using zero-order-hold with a sampling time *of 10ms*. The noise signals of the different position sensors do not correlate with each other. Our control objective is to control the magnet positions y_1 and y_2 of the system to track the reference trajectories $y_{M,1}=0.01\sin(t)+0.02(1-e^{-t/0.1})\ (m)$ and $y_{M,2}= 0.01\sin(t)+0.12(1-e^{-t/0.1})\ (m)$, respectively. The feedback and observer gain matrices are selected as $\boldsymbol{K}=[2304\ 0;\ 96\ 0;\ 0\ 2304;\ 0\ 96]$ and $\boldsymbol{K}_o=[300\ 0;\ 22500\ 0;\ 0\ 300;\ 0\ 22500]$, respectively. A matrix $\boldsymbol{Q}_1=\boldsymbol{diag}([10\ 10\ 1\ 1])$ is assumed. The controller parameters are chosen as $k_T=1$, $\alpha=1e\text{-}4$, $\boldsymbol{R}=1.25e\text{-}5\boldsymbol{eye}(2)$ (minimum achievable attenuation level $\rho_{min}=3.53e\text{-}3$) and the fuzzy modelling parameters as $\gamma_{f,1}=\gamma_{f,2}=1.6e6$, $\gamma_{G,11}=\gamma_{G,22}=1e5$, $\beta_1=1e4$, $\delta_1=1e3$, $b_{11k}= b_{22k}=0$, $c_{11k}= c_{22k}=20$, $\delta_2=0.02$. The membership functions for the fuzzy system inputs $\hat{x}_i$, $i=1,3$ are given as Gaussian curve membership functions $\mu_{F_i^l}(\hat{x}_i)=exp[-(\hat{x}_i-\bar{x}^l)^2/(2\sigma^2)]$ where $\sigma=0.35$ and $\bar{x}^l=-1,-0.5,0,0.5,1$ for $l=1,2,3,4,5$, respectively. The inputs $\hat{x}_1,\hat{x}_3$ are therefore normalized and offset-reassessed by $(\hat{x}_1\text{-}0.02m)/0.01m$ and $(\hat{x}_3\text{-}0.12m)/0.01m$ respectively. The approximated functions $\hat{f}_1$ and $\hat{f}_2$ depend per system definition (6-7) only on the inputs $\hat{x}_1$ and $\hat{x}_3$, the function $\hat{g}_{11}$ only on the input $\hat{x}_1$ and $\hat{g}_{22}$ only on the input $\hat{x}_3$. These circumstances reduce the total number of fuzzy rules to *35 rules* ($\hat{f}_1$ and $\hat{f}_2$ share the same rule base). The initial values of the adjustable fuzzy parameters are set to $\theta_{f,1k}=\theta_{f,2k}=0$ for $k=1,2,..,25$ and $\theta_{g,11k}=\theta_{g,22k}=10$ for $k=1,2,..,5$. The initial plant and observer states are given as $\boldsymbol{x}(0)=[0m\ 0m/s\ 0.075m\ 0m/s]^T$ and $\hat{\boldsymbol{x}}(0)=[0m\ 0m/s\ 0m\ 0m/s]^T$, respectively. Finally, the filters L_i^{-1} are chosen to $L_i^{-1}=1/(s+5)$ for $i=1,2$ and the activation threshold for the fuzzy rules is $\varepsilon_{FBF}=0.25$.

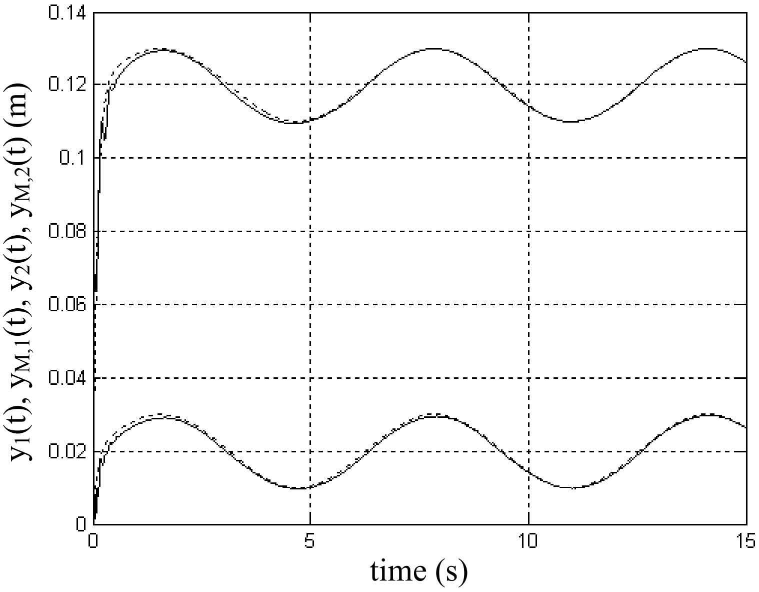

Fig. 6-12: Tracking curves magnet positions $y_1(t)$ (below, solid line), $y_{M,1}(t)$ (below, dashed line) and $y_2(t)$ (top, dashed line), $y_{M,2}(t)$ (top, dashed line)

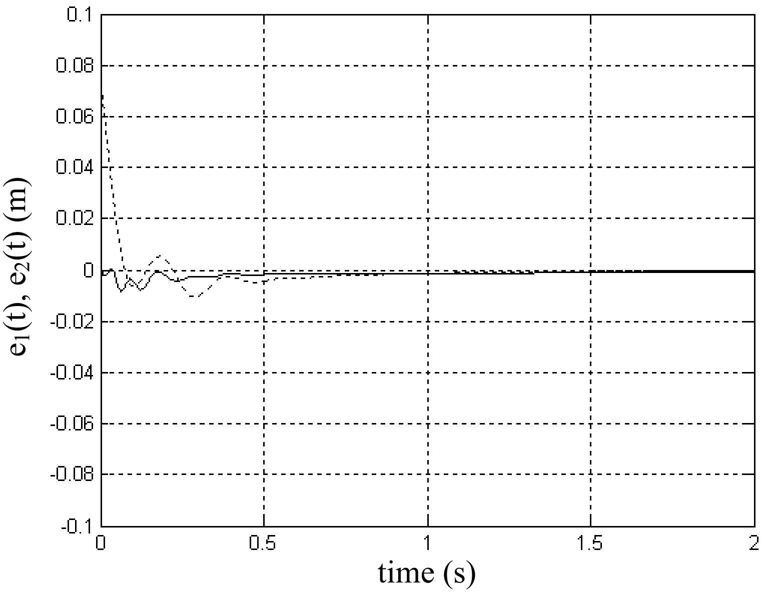

Fig. 6-13: Tracking errors $e_1(t)$ (solid line) and $e_2(t)$ (dashed line)

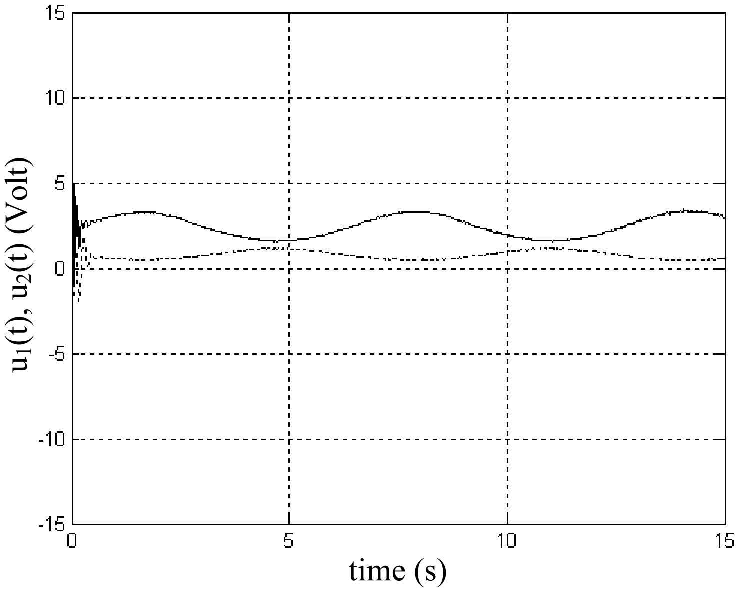

Fig. 6-14: Control variables $u_1(t)$ (solid line), $u_2(t)$ (dashed line)

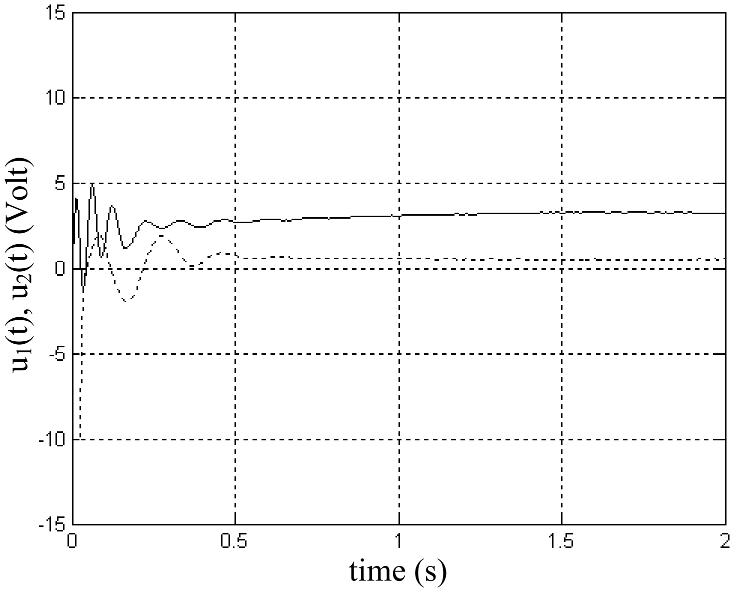

Fig. 6-15: Zoom: Control variables $u_1(t)$ (solid line), $u_2(t)$ (dashed line)

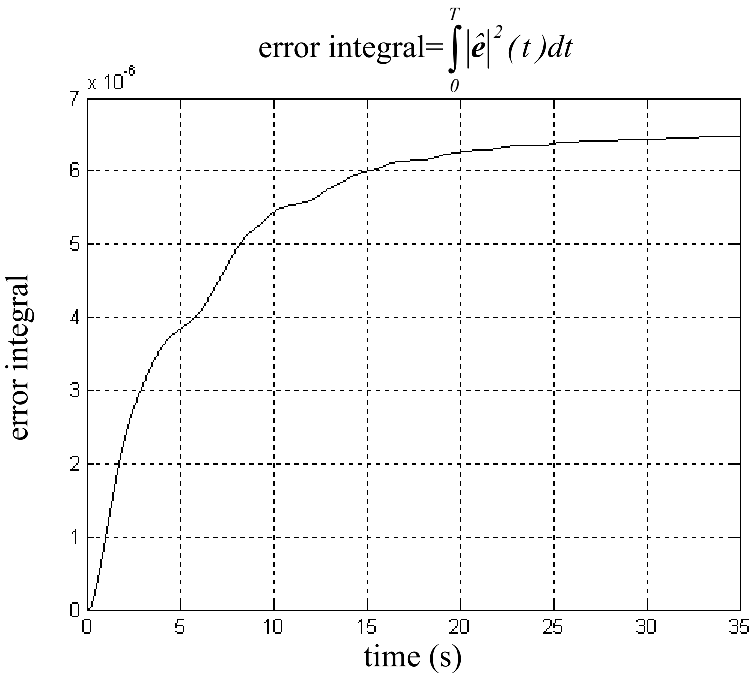

Fig. 6-16: Error integral versus time

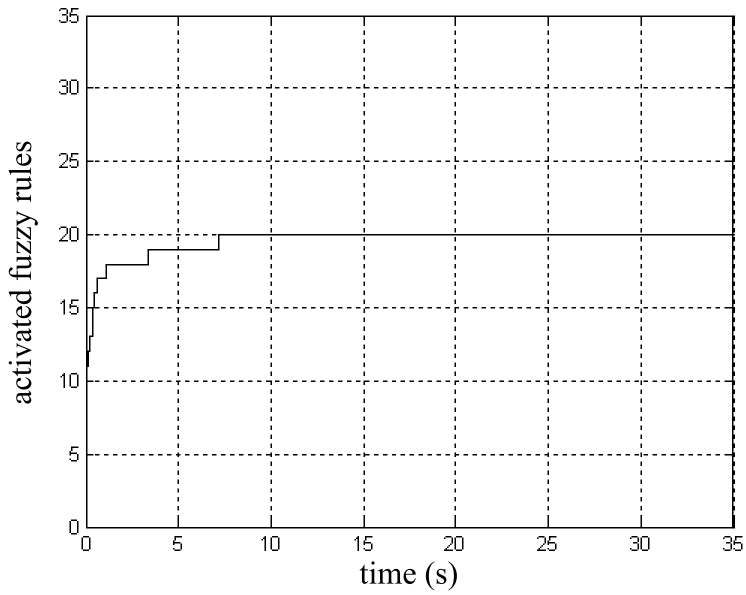

Fig. 6-17: Number of activated fuzzy rules versus time

The simulation results Fig. 6-12 - Fig. 6-17 (all variables are displayed in SI-units) demonstrate, that the ORIAFC can perform a superior tracking control. The influence of external disturbances and measurement noise on the tracking errors can be attenuated to a small level by decreasing the attenuation level ρ which goes along with choosing a controller matrix $\boldsymbol{R}$ with a small Frobenius norm. Moreover, the impact of the measurement noise on the controller output signals $u_i(t)$ was significantly attenuated in comparison to the RIAFC. Also the adaptation speed is superior, although no information about the system nonlinearities was available ("blackstart") and only half of the system states were measurable for the control, respectively. Fig. 6-17 shows, that the total number of necessary fuzzy rules can be reduced significantly (from 35 to 20) by using the proposed dynamic rule activation method. However, if there is additional linguistic information from human experts about the plant nonlinearities $\boldsymbol{f}$ and $\boldsymbol{G}$ in terms of fuzzy IF-THEN rules available, then this information could also easily be implemented due to the open structure of the fuzzy systems (*Wang, L.X*, *1992, Wang, L.X.*, *1993*).

Chapter 7

7 Conclusions and Outlook

7.1 Qualities of the proposed Robust Indirect Adaptive Fuzzy Control

This thesis presents a new method to design a Robust Indirect Adaptive Fuzzy Control for tracking control of a class of uncertain nonlinear MIMO systems with on-line tuning of linear fuzzy parameters based on H_∞ as well as VSS control techniques and the Strict Positive Real Lyapunov (SPR-Lyapunov) design approach. In particular, the Robust Indirect Adaptive Fuzzy Control does neither require that all system states are available for measurement nor the measured output signals or the reference trajectories to be smooth, necessarily. Robust Indirect Adaptive Fuzzy Control guarantees that all signals involved are bounded and provides the fuzzy modelling error cancellation by a VSS control term and the external bounded disturbances as well as measurement noise attenuation with H_∞ performance, obtained by a Riccati-like equation. Applying a dynamic fuzzy rule activation method the phenomenon which is called “the curse of dimensionality” can be significantly weakened. Simulation results of a MIMO magnetic levitation system demonstrate the effectiveness and robustness of the Robust Indirect Adaptive Fuzzy Control, considering several hardware specifications and constraints of an approved real-time experiment

7.2 Limitations of the proposed Robust Indirect Adaptive Fuzzy Control

The main limitation of the proposed RIAFC and ORIAFC is certainly the canonical normal form which the plant has to fulfil. Clearly, there are a lot of practical nonlinear systems which are already given in a feedback linearizable normal form or can be transformed into such a form by a local or global diffeomorphism, respectively. Nevertheless, not all plants fulfil this requirement. Also the difficulty of stable zero dynamics restricts the number of successful applications.

Adaptive Fuzzy Control, including the RIAFC and ORIAFC approaches, deals with “grey” plant models. The more information of the plant is available the faster the learning phase can be completed. In case of the ORIAFC the period is elongated by the state vector estimation. Hence, the initial performance of the closed- loop control circuit depends on the degree of pre-knowledge about the plant. The integration of VSS and H_∞ components in the controller design weakens the problem of missing information on one hand, but causes hard controller actions on the other hand.

The RIAFC and ORIAFC approaches provide systematic methods to design a tracking control for a class of plants with large uncertainty. Irrespective of this fact, there is some degree of freedom for a manual optimization of the design parameters (learning rates, control- and observer gains, etc.). Due to practical constraints like hard actuator limits, geometric limits of the hardware setup, limited sample rate, measurement noise etc. this optimization task is time consuming and has to be accomplished for each plant individually.

The adaptive fuzzy control developed in this thesis is designed to be extreme flexible concerning the domination of plant uncertainties in form of static plant nonlinearities. The price which has to be paid for this flexibility is the lack of freedom regarding fuzzy rule base optimization. It is well known, that without rule base modification high plant orders lead to a dramatic increase in fuzzy system complexity. The proposed dynamic fuzzy rule activation method weakens this phenomenon which is called the “curse of dimensionality”. However, this solution represents a compromise and is not able to fully compensate the higher demand on hardware resources in comparison to direct adaptive schemes with rule base optimization.

Applying the ORIAFC the impact of measurement noise can be suppressed by the H_∞ approach. Unfortunely, the usually high inherent feedback gains of the VSS and H_∞ components revoke this benifit partially. If the plant under control shows a low pass characteristic, like the introduced simulation examples, the measurement noise affects mainly the controller output signals and as a consequence the actuators.

7.3 Outlook

An interesting future research topic would be the theoretical extension of the in this thesis developed algorithms for the application in discrete-time control systems. A simple discretization of the continuous-time algorithms in combination with empirically determined sampling rate and measurement noise limits can easily be accomplished. But a theoretical analysis and prediction of stability and performance limits is a non-trivial task when complex uncertain nonlinear systems are under consideration.

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