DISSERTATION

Modelling Niklas Luhmann's Economic Theorems

ausgeführt zum Zwecke der Erlangung des akademischen Grades eines Doktors der technischen Wissenschaften unter der Leitung von

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1 Summary (German)

Dieser Arbeit liegt die Zielsetzung zugrunde, Ökonomische Thesen, die aus mehreren Publikationen des Deutschen Soziologen Niklas Luhmann ausgewählt wurden zu formalisieren und zu validieren. Die Arbeit enthält drei Teile:

- *Der erste Teil hat eine formalere Beschreibung der relevanten Arbeiten von Niklas Luhmann im Kontext von dynamischen Systemen zum Ziel.*
- *Im zweiten Teil wird ein Agenten basierendes Modell betreffend einer einfachen „Luhmann Ökonomie" entwickelt und spezifiziert.*
- *Ausgehend von einer Implementierung auf einem üblichen Personal Computer wird im dritten Teil das Verhalten einzelner Lösungen interaktiv untersucht, wobei Fuzzy-c-Means Clustering als Hilfe zur Visualisierung dient. Klassen von Lösungen werden mithilfe von Simulationen beleuchtet.*

Die These von Niklas Luhmann, dass die Wirtschaft von Ungleichheit ausgeht und weitere Ungleichheit produziert um fortzufahren, konnte in Computer Simulationen reproduziert werden.

Die wesentliche Charakteristik des gewählten Lösungsansatzes ist die Berücksichtigung der zusammenhängenden Struktur von Kommunikation (d.h. einer setzt einen kommunikativen Akt, mehrere Beobachter verstehen), die aus der Definition von Niklas Luhmann folgt. Das Modell gibt klare Anhaltspunkte, um weitergehende Aspekte aus der Gedankenwelt von Niklas Luhmann abzubilden.

2 Summary

This work tries to formalise and validate economic theorems that were collected from various publications of the German sociologist Niklas Luhmann. It contains three parts:

- The first part aims at a more formal description of relevant works of Niklas Luhmann in the context of dynamical systems.
- In the second part an agent based model for a simple Luhmann economy is developed and specified.
- Based on an implementation on a default personal computer the behaviour of individual solutions is studied interactively using fuzzy-c-means clustering as visualisation aid in the third part. Classes of solutions are explored by simulation runs.

Niklas Luhmann's proposition, that the economy starts from and produces further inequality in order to continue could be reproduced by computer simulation.

The main characteristic of the approach is the consideration of the cohesive structure of communication (i.e. one communicative act - many understanding observers) that follows from the definition of Niklas Luhmann. The model gives definite points for enhancement to model further aspects of Niklas Luhmann's thinking.

3 Steps towards a Formal Description of Niklas Luhmann's Theory

Among the motivation to start this work was the wish to gain further insight into Niklas Luhmann's thinking. When I came across a remark in "Die Wirtschaft der Gesellschaft" [1], I guessed that an economic perspective might be promising.

In the course of his theory Niklas Luhmann has developed his own terminology that frequently uses mathematical terms. Niklas Luhmann does not give definitions in a formal language and sometimes uses mathematically familiar terminology (e.g. system, differentiation, dimension, expectation) in a mathematically unfamiliar sense. This makes understanding Luhmann's writings more difficult for the mathematically literate reader. Glossaries and lexica have been published in German to serve as an aid for the reader of Niklas Luhmann's works[2], the English edition of "Ökologische Kommunikation"[3] contains a glossary of the most frequent terms in English.

I personally understand Niklas Luhmann's theory as an advanced description rather than a model. According to Niklas Luhmann *elements of social systems are events of communications*[4], *when they are understood*[5]. In this sense Niklas Luhmanns's theory of society contains (or has to explain) itself as a special type of communications.

Designing an agent based model that can be both mathematically formalised and implemented to run on a digital computer starts off with different design goals. An agent based model will be understood as a tool to aid the explanation of phenomena in defined (idealised) settings. It does not intend to be part of itself or explain itself.

[1] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 232) *One can oppose the theorem of a certain structural isomorphism of meaning and money – if one means something different by meaning. But insights, that are worth to be preserved are possibly lost in this case.*

[2] See Baraldi, Corsi, Esposito, Glossar zu Niklas Luhmanns Theorie sozialer Systeme,1999; Further Krause D., Luhmann-Lexikon, 2001

[3] See Niklas Luhmann, Ecological Communication, translated by John Bednarz, Jr., 1989

[4] see Niklas Luhmann, Soziale Systeme, p. 43: *An element is, what for a system functions as a not further dissoluble unity*; p. 240: *To the Question, what are the parts of social systems, we give a double answer: Communications and their attribution as action.*

[5] See Luhmann, Soziale Systeme, p. 226: *Put it differently, the level of constitution of communications cannot be lowered, ..., it cannot give up the melting of information, utterance and understanding without ending its operation.*

Given these design goals, the following approach towards a formal description of Niklas Luhmann's theory of social systems with focus on his economic theorems[6] uses simplifications and leaves many aspects of Niklas Luhmann's works uncovered.

3.1 Communications

I have chosen to start the presentation of Niklas Luhmann's theory by introducing his concept of communications. By progressing in a rather bottom-up manner, it seems easier to make assumptions for modelling.

Niklas Luhmann himself starts to explain his theory of social systems by explaining first systems and functions and moves later to the more detailed parts[7].

Niklas Luhmann's concept of communications tries to accomplish the following aspects:

- The definition is general, so that many types of communications are subsumed. The definition allows for written, oral and non-verbal communication. Many of Niklas Luhmann's writings treat the societal impact of writing and the printing press at great length and detail. So do European religious wars become unleashed because there is more than one holy bible. The impact of the evolution of communications media and technology, that allow communications across time (writing) and space (telecommunications) is closely linked to the evolution of society.
- Even more subtle distinctions – as that of direct and indirect communications[8] - can be made by means of the definition.

[6] Niklas Luhmann has neither developed nor as far as can be concluded from his writings has intended to develop economic theorems. The term economic theorem refers to topics in Niklas Luhmann's writings that are related to questions which are also considered in economic theories. They were collected from the following publications by Niklas Luhmann: Zweckbegriff und Systemrationalität, 1968; Soziale Systeme: Grundriss einer allgemeinen Theorie 1984/English: Social Systems, 1995; Ökologische Kommunikation, 1986/English: Ecological Communication, 1989; Die Wirtschaft der Gesellschaft, 1988, Soziologie des Risikos 1991/English: Risk, A Sociological Theory 1993

[7] See Niklas Luhmann, The general structure of "Soziale Systeme" is given a s follows: *1 – system an function, 2 – Meaning, 3 – double contingency, 4 – communications and action, 5 – system and environment, 6 – interpenetration, 7 – the individuality of psychic systems, 8 – structure and time, 9 – contradiction and conflict, 10 – society and interaction, 11 – self-reference and rationality, 12 consequences for epistemology*

[8] See André Kieserling, Kommunikation unter Anwesenden, Studien über Interaktionssysteme, 1999 (p. 147 ff)

Whatever an instance of communication is – it must be acceptable or reject-able[9]. So the mere usage of brute force (coercion) is not communication – from the perspective of the coerced. But it can be communication for observers of the coercion.

Niklas Luhmann puts the system, which has the capability and possibility to accept or reject the to-be understood communication into the centre of his definition. Accepting or rejecting a communication is defined as taking or not taking it as a premise for further action.

The system which is in the position to understand an element of communication is called Ego. Understanding (by Ego) involves the following steps:

- Selection (1) of the *information* – out of observed (empirical, physical) facts or traces. In Niklas Luhmann's terminology this step is called *information.*
- Selection (2) of an *utterance* by an author[10] called Alter. In Niklas Luhmanns's terminology this step is referred to as *utterance.*

  Niklas Luhmann stresses the fact that these two selections imply the detection/evaluation of a difference between them.

  In everyday language information and utterance are described as the answers to the different questions:

  - What has been said? and
  - Has it been said?

  If one thinks of watching a conversation in a language not understood – the information gets lost, where as the utterance is present.

  It is worth noting, that the author (Alter) is defined as an entity – a system, which is (by Ego) regarded as capable of communication. In terms of modelling this means two things:

  1. The observed behaviour (of Alter) is *contingent*, meaning it could have been different from Ego's perspective. This is the case, when the dynamics of the system Alter is not determinable by Ego and Ego nevertheless makes expectations about Alter's behaviour.
  2. Alter's behaviour was selected in consideration (i.e. *contingent* on) of Alter's expectations about Ego – according to Ego.

[9] See Niklas Luhmann, Soziale Systeme (p. 212) *Communication requires that refusals are possible*
[10] Instead of author (lat. auctor) on can think of an actor (in the sense of action) as well as an agent. See the etymological similarity of these notions.

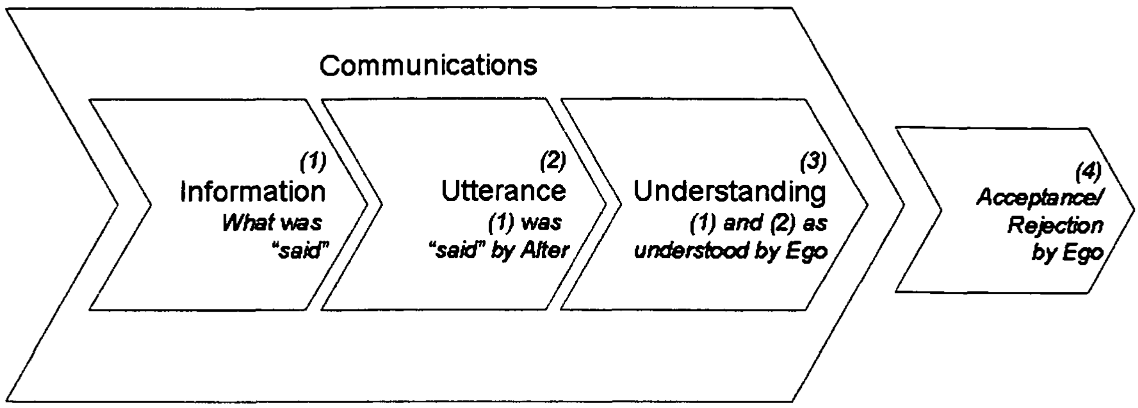


Figure 1: Elements of Communications

Note further, that this "communicative intent" of an information/utterance pair is a key concept for qualifying it as communications. It implies two contingencies. In Niklas Luhmann's terminology this state of affairs is referred to as *double contingency.* Formally it implies the application to two templates referred to as *expectations.* The setting is similar to a game-theoretic model of a two person game.

Niklas Luhmann is not precise, whether Alter is a social construction, so that his definition of communications allows for communication with unobservable entities (e.g. the dead or supernatural beings) or observable entities whose communicative abilities are not generally accepted (e.g. communication with and/or among animals) or for communications where the empirical author is uncertain or unobservable (e.g. finding a letter with no sender). From a modelling perspective this seems irrelevant, especially as the whole concept relies on the *individual construction* of the addressee Ego[11]. The *social construction* of a participant of communications is usually referred to as *person* in Niklas Luhmann's terminology.

- Selection (3) of a *Meaning* by Ego.

  Niklas Luhmann gives a description of important properties of *Meaning*[12]. In every-day language, I understand – *selection of a Meaning by Ego* - as fixing a point within an object, that contains everything that Ego can express by a sentence in (any) language Ego is capable of expressing him or herself. This idea is similar to that of a subjective category as given by F.W. Lawvere and S.H. Schanuel[13].

[11] See Niklas Luhmann, Soziale Systeme (p. 204): *Communication fixes a state of the addressee, that would no be defined without it – but this state is determined by the addressee him/herself.*

[12] See Niklas Luhmann, Soziale Systeme (p. 93) *The phenomenon of meaning appears in form of an abundance of references to further potentialities of Experiencing und Acting.*

[13] See F.W. Lawvere, S.H. Schanuel, Conceptual Mathematics, 1991, (p. 307) *In the subjective categories which are derived from formulas and rules of proof as alluded above, this is due to the fact that truth values 1 to Ω are themselves formulas. They might for example arise by composing*

The definition of a category of Meaning containing objects that fit to the above definition (an object corresponds to the meaning as understood by a *system*) is near at hand. The objects will have a further structure, yet undefined. Niklas Luhmann names three general aspects – he calls them factual, time and social *dimensions* – that are given with every element of meaning.

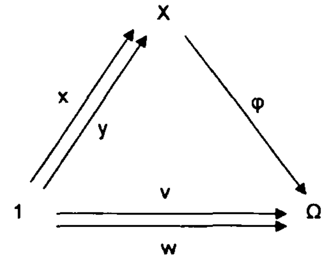
Based on the explanations given above Niklas Luhmann defines communication as follows:

Communication is the unity of information, utterance and understanding[14]. (Definition 1)

Niklas Luhmanns uses the notion of selection indicating that he thinks of operations that take time[15]. From a modelling perspective further assumptions about this operation seem necessary. One idea is to interpret selection as choosing an element of a set – in terms of a suitable definition using categories further definitions are required:

- The category of empirically observable facts or traces, with one object per observer. Following the terminology introduced by Heinz von Förster[16] it is called the <u>category of 1st order observations</u>.
- The 2nd selection involves systems, which are observable by Ego, to whom Ego attributes a communicative ability.

  Favouring empirically traceable systems (i.e. ruling out empirically unobservable authors of communication) this selection amounts two the following:
  - Selection of a cluster (involving more than one element) within the object of 1st order observations by Ego. This corresponds to the detection of the uttering.



where x, y are formulas naming elements of type X and where φ is a formula naming a property of elements of type X; in linguistics x and y might be noun phrases, φ a predicate, and v and w the two resulting sentences

[14] See Niklas Luhmann, Soziale Systeme, (p. 203): *If communication is defined as synthesis of three selections (the unity of information, utterance and understanding), it is realised when and to what extent understanding happens. Everything further happens outside the unity of an elementary communication. This is not to be confused with the acceptance of refusal of the (communicated, i.e. understood) selection as a basis for own behaviour. But it cannot be ignored.*

[15] See Niklas Luhmann, Soziale Systeme (p. 70): *Selection is a notion implying a notion of time ("Zeitbegriff"). A selection is pending, becomes necessary, is operated and has happened.*

[16] Heinz von Förster, Wissen und Gewissen, 1993

- o Identifying the source of this uttering with the predicate of being "able to communicate". This implies a reference to other objects within the category.

- The 3rd selection now links the prior two selections with an element in the object of Meaning (as understood by Ego).

  In terms of categories it is reasonable to assume that the *unity of information, utterance and understanding* involves maps between objects of the categories of 1st order observations and the category of meaning[17].

It is worth mentioning, that the notion of 2nd order observation (observers observing observers)[18] and communications as defined by Niklas Luhmann are very close from a modelling perspective. One could think of modelling 2nd order observations in the same way as selections (1) and (2) by replacing the "communicative intent" by the mere qualification as "another observer" or "another system with the ability to observe".

I would like to add some further comments:

- Niklas Luhmann describes communications are momentary elements without duration[19].
- Any observation multiplies (in a sense to be defined[20]) by the number of different observers observing the same. In terms of modelling this may give rise to some sort of cohesive (topological) structure within the category of 1st order observations. To observers, who are close in physical space the same events in physical space are observable – to a certain degree[21].
- There is research that has investigated to what extent advances in the cohesive structure of communication have been critical to the evolution of human language[22] (only two can groom, but more can simultaneously engage in communication).

[17] Niklas Luhmann describes meaning as a "category without differences", there is no such thing like "a meaning with not meaning". The fact that structure-preserving maps keep only positive properties, negative ones get lost, suggest the modelling approach. See Niklas Luhmann, Soziale Systeme (p. 96) *Meaning is a non negate-able, a difference-less category*

[18] See Heinz von Förster, Wissen und Gewissen, 1993

[19] See Niklas Luhmann, Soziale Systeme (p. 78) *The system builds itself from instable elements, that last only a short amount of time – or as in the case of action have no duration by themselves. ... A stable system consists of unstable elements. The stability are a consequence of itself being a system, not of the elements. Still the system is constituted by its elements – in this case events. It has no basis for duration outside of these events (this is why we experience the present necessarily as short)*

[20] It is at least suggestive to define this multiplicity of observation as product in terms of the approach followed so far.

[21] The formulation "to a certain degree" was used voluntarily to point to a fuzzy approach. See Bart Kosko, Fuzzy Thinking, 1993 (p. 3) *The fuzzy principle: Everything is a matter of degree*

[22] See Robin I.M. Dunbar, Grooming, Gossip and the Evolution of Human Language, 1996

3.2 Social and other Systems

To link this chapter to the preceding one, I start with Niklas Luhmann's definition of social systems:

Social systems consist of communications and their attribution as action[23]. (Definition 2)

Niklas Luhmann describes social systems as meaning-processing systems. If we conceive of social systems as a collection of communications in the sense of the preceding definition, the proposition that meaning is processed is obvious.

Niklas Luhmann explains why he gives no description of social systems in terms of the notions of statics and dynamics[24]. Nevertheless the mathematical description of a dynamic system requires specification of its dynamics and states. The study of dynamical systems may then lead to the question of how to model *structures* and *processes* in the sense of Niklas Luhmann's definitions.

The identification of states and dynamics that do not violate the propositions of Niklas Luhmann – or at least violate them only to an acceptable degree – can be seen as an objective of this work.

In order to model the dynamics the following propositions are summarised:

- Social systems show an autonomous dynamics, in the sense that the next operation in the system – and therefore the system's next state - is dependent on the system's own *structures* and *processes.*
  Social systems share this property of an autonomous dynamics with other systems - namely organic and psychic systems. They are described as *autopoietic* systems.
- This kind of autonomous dynamics implies that the states of the system cannot be determined from its environment. The system can only be irritated. The system is not autonomous in the sense, that it cannot control its environment but does react to its environment.
- The notion of *autopoiesis* is defined by the property that an *autopoietic* system creates the elements of which it consists by its own operations. These are cells for biological systems, (conscious) thoughts for psychic systems and (elements of) communications for social systems.

[23] See Niklas Luhmann, Soziale Systeme (p. 240)
[24] See Niklas Luhmann, Soziale Systeme (p. 73) *It would be wrong to understand structures as solely timeless and processes as solely time-full* (as they have structures). *Neither fits the dichotomy of statics and dynamics nor the dichotomy of constance and change*

- Communications refer to each other, meaning that elements of communications connect to prior elements of communications. Niklas Luhmann calls this property of social systems *self-referential*[25].

  With this focus on the connection of communications to prior communications rather than a focus on the connection of communications to its content, Niklas Luhmann departs from conventional thinking. Niklas Luhmann uses a similar line of thought when he connects the flow of money (solvability) to an opposed flow of insolvability – rather than connecting the flow of money with the opposed flow of goods and/or services.
- The conditioning of communications is achieved by the dynamics of social systems[26]. A key notion in this respect is *connectability* ("Anschlussfähigkeit")[27]. This is described as a tendency that the communications element that ranks highest in *connectability* gets chosen as the next one.
- According to Niklas Luhmann the emergence of social systems (i.e. the emergence of communications referring to each other in a "meaningful" way) is highly unlikely. Therefore certain mechanisms have to be available along with communications, so that the meaning of communication is rather accepted than rejected. Small social systems (called *interactions*) achieve this by direct (corporeal) observability (presence) of participants. Interactions typically do not last long.

  Bigger social systems (*organisations*) compensate participants who are referred to as members. Members are rewarded food, shelter, protection – or in modern society just money - in exchange for the burdening by meaning ("Sinnzumutung"). Organisations typically "live" longer than the psychic systems that constitute their (member-) environment.

  Societies achieve likelihood of acceptance (connectability) through the development of common expectations (languages, semantics).

  Modern society developed specialised codes (as part of so called *symbolically generalised media of communications*) that serve the same purpose.

[25] See Niklas Luhmann, Soziale Systeme (p. 31) *Self-referential systems exist. This means in a very general sense: Systems exist, such that a system has the ability to maintain relationships with itself and further is able to differentiate these relationships against relationships to its environment.*
[26] See Niklas Luhmann, Soziale Systeme (p. 236) *The answer is: By conditioning of communications, i.e. the building up of social systems*
[27] See Niklas Luhmann, Soziale Systeme (p. 258) .. *Re-production is constrained by Connectability*

According to the mathematical theory of discrete dynamical systems a discrete dynamical system is given by[28]

$$x_{n+1} = T_n(x_n) \tag{0.1}$$

$x_0 \in (X,d)$ a metric space, (T_n) is a sequence of endomaps $T: X \to X$, $n \in N_0$

The special case, when $\forall n: T_n = T$ is called autonomous. Otherwise the system is called non-autonomous. Given these notions of autonomy, I suggest to qualify meaning-processing systems in the context of Niklas Luhmann's theory as semi-autonomous.

In order to model states the following propositions are summarised:

- According to Niklas Luhmann, *a minimal system is just the set of relations between elements. This set is conditioned by an inclusion/exclusion rule plus conditions of countability (e.g. keep order constant during counting). Typically there are further conditionings*[29].
- The structures of social systems are *structures of expectations*[30].
- The set of all present *expectations* made by a system can be described as the present, internal construction of reality[31] by the same system. *Expectations* as described by Niklas Luhmann can come true (can be fulfilled) or not (when a deviation from the expectation is observed)[32].
- Niklas Luhmann uses this notion of *expectation* to further distinguish between cognitive and normative expectations. Systems are eager to modify *cognitive expectations* when expectations are not met. When systems prefer to keep their expectations unchanged, these expectations are described as *normative expectations.*
- Systems manage to maintain their boundary to their environment[33] and keep their autonomous dynamics intact[34].

[28] See Tim Nesemann, Stability Behaviour of Positive Nonlinear Systems with Applications to Economics, 1999 (p. 17)
[29] See Niklas Luhmann, Soziale Systeme (p. 44)
[30] See Niklas Luhmann, Soziale Systeme (p. 398) *The theory of events and structures plus the theory of expectations are put together by the proposition that structures of social systems are made of expectations, that they are structures of expectations, and that for social systems – because they temporalise their elements as events of action – there are no other options to build structures. That means: structures do only exist as present structures.*
[31] In order to avoid the ambiguity that according to Ernst von Glasersfeld was caused by the translation of "Vorstellung" in Immanuel Kant's works into "representation" I use "construction of reality" when translating Niklas Luhmann. See Ernst von Glasersfeld, Radikaler Konstruktivismus, Ideen, Ergebnisse, Probleme (p. 159)
[32] See Niklas Luhmann, Soziale Systeme (p. 363) *Expectations are a primitve technique. This technique can be used nearly without prerequisites. It is not necessary to know who you are or what you know about your environment. One can have an expectation, without knowing the world - for pure luck. It is solely important, that the expectation can be used by the autopoiesis, so that the access to connecting reality constructions ("Vorstellungen") is sufficiently pre-structured.*
[33] See Niklas Luhmann, Soziale Systeme (p. 35) *the starting point of a system-theoretic analysis is the difference between system and environment; boundary maintenance is system maintenance.*

- Niklas Luhmann gives several functionally equivalent strategies that systems pursue in order to maintain their boundary[35]. A basic strategy is the subjective construction of reality.

Among the systems that share the property of a semi-autonomous dynamics and the maintenance of expectations only social systems and psychic systems are described as meaning-processing systems. The idea that the same description applies for both kinds of system serves as a design goal for the further development of Niklas Luhmann's theory.

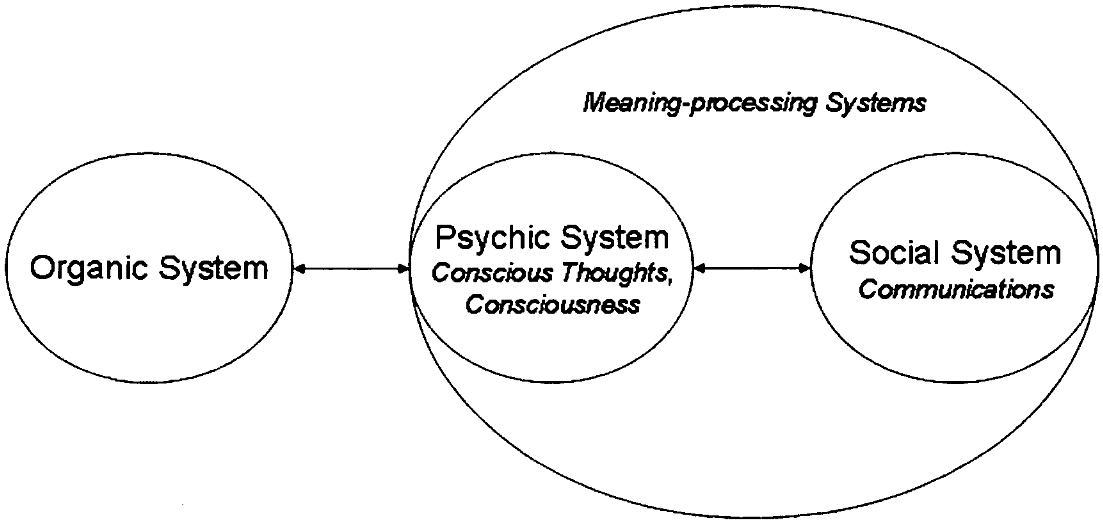


Figure 2: Meaning-processing systems

The interplay (interdependence) between psychic and social systems is referred to as *interpenetration* in Niklas Luhmann's terminology. The coupling between organic systems and psychic systems is not a primary focus in Niklas Luhmann's theory.

Neither social systems nor psychic systems are material systems, so they have no hardware or memory in terms of a computer analogy. Psychic systems may use their coupling with organic systems. Following the figure above social systems have no direct contact with organic systems (i.e. hardware).

The whole setting constitutes a burden regarding the analysis of social systems, as Niklas Luhmann comments[36], and he reasons that this is one plausible cause for a preference for theories of action (instead of theories of communication) in the field of sociology.

[34] See Niklas Luhmann, Soziale Systeme (p. 73) *A system has to do without a complete synchronization with its environment, and must therefore be able to absorb the risk of momentarily non-compliance*
[35] See Niklas Luhmann, Zweckbegriff und Systemrationalität, 1973 (p. 181 ff)
[36] See Niklas Luhmann, Soziale Systeme (p. 226) *communications cannot be directly observed, they can only be inferred*

Note that an important consequence of this approach is that human beings are understood as a sort of symbiosis of psychic and organic system and are no longer part of society which is understood as the social system made up of all communications.

Both types of systems (psychic and social) serve as a necessary environment to each other – and have evolved (co-evolved) together.[37] I regard this explicit formulation of a duality relation (Co-evolution) as one of the most important propositions of Niklas Luhmann's theory.

Taken literally it is hard to accept that an entity called social system is operating without hardware. It must then either use or take possession of the hardware of psychic systems. Given the autonomy of psychic systems this cannot be the case. Besides that, we have just banned the psychic systems into the environment of social systems. So the argument would lead to the conclusion that social systems do not exist.

But there are counterexamples:

- There are many situations in everyday life that can be understood and managed far better in terms of the social situation (e.g. traffic lights) than in terms of knowing the individual.
- It may be fairly practical to model generally accepted expectations, i.e. expectations that cannot easily be rejected by any respectable person - and how they change.

In view of the preceding definitions I suggest to use – in terms of steps towards a formal description - the notion of duality as common to categories[38]. The basic idea is to replace domain with co-domain of maps and to look in the reverse direction. Limits (terminal objects) relate to co-limits (initial objects, co-products) in this way.

I propose to think of evolution (of psychic systems) and co-evolution (of social systems) in the same way. When we further follow Niklas Luhmann's proposition that psychic systems and social systems belong to the same category of meaning-processing systems, the following ideas are near at hand:

- Psychic systems use their identity as *persons* to connect to a social system (one may think of an interaction, an organisation or a society) – in terms of universal mapping properties this points into the direction that a *person* might be understood as terminal object. From society there is one and only one map (of a certain kind) to each person.

[37] See Niklas Luhmann, Soziale Systeme (p. 92) *Psychic and social systems evolved by means of Co-evolution*
[38] See W.F. Lawvere, S.H. Schanuel, Conceptual Mathematics, (p. 215, 284)

- Social systems assign identity to persons – in this sense *the social system society* could be understood as an initial object because there can be only one map (of a certain kind) to any other object, i.e. system[39].

Niklas Luhmann's proposition of co-evolution of psychic and social systems, may finally lead to an interesting design guideline in the field of Artificial Intelligence: In order to make computers more intelligent – make them more social!

Autopoietic systems were originally described in the field of biology[40]. Niklas Luhmann gives – as an overview - the following hierarchy of systems:

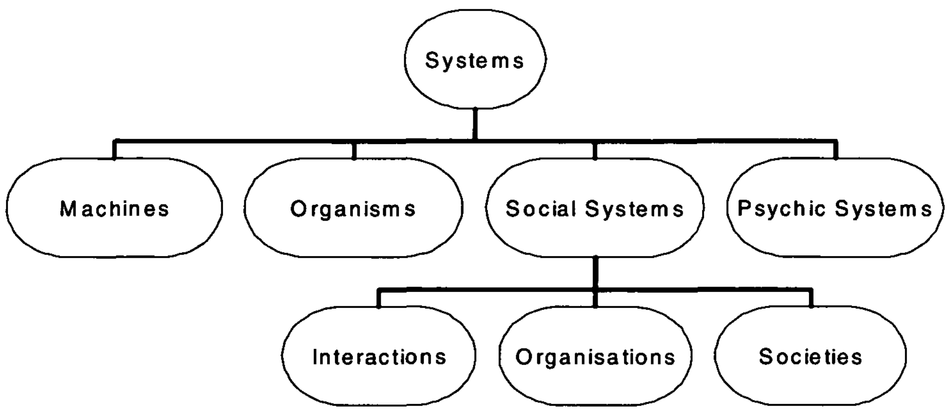


Figure 3: Systems[41]

I summarise the definitions for the types of social systems as shown in the figure above:

- Interactions are the smallest types of social systems. They are to be understood as communications among participants, who are present in the sense that there is real-time mutual observation (synchrony). A telephone conversation is an interaction.
- Organisations are social systems that are constituted by communications that are referred to as decisions (of the organisation by bodies/members of the organisation). It needs a decision to become a member of an organisation. Organisations have two environments – an environment of members and an environment of non-members.
- Societies are the biggest social systems containing all communications.

Having prepared some ground, a first very general formal definition of meaning-processing systems (social, psychic) that follows the ideas of Niklas Luhmann may be given:

[39] In certain game-theoretic models an initial move of a player "nature" is needed. See Alexander Mehlmann, Wer gewinnt das Spiel (p. 112), *The 3-(centi)pede game with information deficit.* In the setting of auction theory the "mechanism designer" appears as player with number 0.
[40] See Maturana H., Varela F.J., Der Baum der Erkenntnis, 1997
[41] See Niklas Luhmann, Soziale Systeme (p. 16)

Let M be the space of expectations in which all expectations of our system are made. We may further assume that this space is of finite (but large) dimension. If we further assume that single expectations (representing e.g. a single neuron) get imprinted in a finite way, e.g. $\in \{0,1\}$ we may conclude that that M is finite.

Given our earlier definitions we could think of M as related to a category of meaning as mentioned above and call it the space of all expectations of our system. I use the term space instead of set to indicate that meaningful expectations imply a further (cohesive) structure to be defined on it.

So M is the candidate to serve as state space for our dynamic system.

With regard to the semi-autonomous character of the dynamics as assumed above two endomaps have to be defined. Let us call them

- $S: M \to M$ and
- $E: M \to M$,

to denote basic system operation and expectation building. E does not stand for the system's environment as seen from an outside observer, but for the system's construction of reality i.e. the dynamics of changing expectations according to sensory sensations. The autonomous dynamics of our semi-autonomous Luhmann system is then given by:

$$x_{n+1} = E \circ S(x_n),\ x_0 \in M,\ n \in N_0 \qquad (0.2)$$

This formulation does not yet account for the non-autonomous part.

The time index n in definition (0.2) reflects the system's own point of view. $n \in N_0$ marks the development of the internal states of the system, whenever the system changes these states irrespectively of any irritations of the *larger environment*[42]. The systems basic operation S filters outside irritations.

[42] To distinguish between the system's environment as perceived by the system and the environment observed by an outside observer (with superior observation capacity) the latter is denoted as *larger environment*

If we model the dynamics of the system stressing our position as an outside observer, the dynamics should account for these irritations, irrespective if they are passed to the system as sensory signals or not. Let therefore U be the space of outside irritations as observed by an outside observer.

Focussing again on our system and assuming that the internal dynamics as well as the outside irritation is transparent to our outside observer with superior observation capacity we get the following description:

$$x_{n+1} = E \circ S(u_{n,0}, u_{n,1}, \ldots, u_{n,m}), x_0 \in M \subseteq U, \ u_{n,m} \in U, \ n, m \in N_0 \qquad (0.3)$$

Now $u_{n,m}$ refers to the observations that are made by our outside observer between n and $n+1$. m denotes the instances when the outside observer's states change, i.e. when the outside observer observes changes in the *larger environment*, where as n refers to the instances, when the system's states change.

Assuming - as constructors of the system and its dynamics – that the time instances, when something happens (both internally and externally) can be observed by our outside observer and that this observer is in the privileged position to be able to observe everything that is observable by our system, we can introduce time instances t as the instances when something happens either in the system or in the enlarged environment[43]. The enlarged environment fully contains the environment of our system.

In order to simplify the model we may further replace $S(x_n, u_{n,0}, u_{n,1}, \ldots, u_{n,m})$ $n, m \in N_0$ by a sequence $S_t(x)$, $t \in N_0$. This finally yields the following description:

$$x_{t+1} = E \circ S_t(x_t), x_0 \in M, \ t \in N_0, \qquad (0.4)$$

which is in accordance with definition (0.1) except that we do not claim M to be a metric space, but hold some other structure to be determined reflecting the idea that *understanding* maps expectations into a category of meaning.

[43] See Niklas Luhmann, Soziale Systeme, Niklas Luhmann is very precise in that respect: (p. 36) *No environment without a system. The environment receives its unity (firstly, only) by the system – and only relative to that system*; Further (p. 36-38) *There is a difference between the environment of a system – and systems within the environment of a system*

Niklas Luhmann gives some interesting perspectives regarding these different instances of time denoted by n and m. They are the so called strategies for systems to win (or gain) internal time[44] – as they are not reflected in his writings regarding the economy they are omitted.

The choosing of the order of composition $E \circ S$ rather than $S \circ E$ was made on the assumption that the system first operates its basic *autopoietic* operations and then maintains its expectations.

Following the propositions above, separate dynamics regarding 1st order observations and observations with communicative intent could be considered. This leads to a division of the maps E into pairs of retractions and sections E^{1st} indicating the change of expectations following 1st order observation and E^{2nd} indicating the change of expectations following 2nd order observation, or according to Niklas Luhmann a change of expectations triggered by observation with communicative intent. The resulting identity is

$$x_{t+1} = E^{2nd} \circ E^{1st} \circ S_t(x_t) \tag{0.5}$$

A similar approach may be followed taking into account the coupling of psychic systems and organic systems. Only organic systems have the ability of voluntary and involuntary movement. This would lead to division of the maps E^{1st} and E^{2nd} into maps $E^{1st} = A^{1st} \circ \bar{E}^{1st}$, and $E^{2nd} = A^{2nd} \circ \bar{E}^{2nd}$. The maps A refer to the action part, meaning that certain expectations set by the psychic system are followed by movements of the organic systems, A^{2nd} indicates conscious action, A^{1st} refers to the realms of reflexes. $\bar{E}$ denotes experience, subject to the dichotomy experiencing/acting ("Erleben"/"Handeln").

These definitions do not give any descriptions about the endomaps S_n, E^{1st} and E^{2nd}.

3.3 The Economy, a System Emerging within the System

Niklas Luhmann uses his basic assumptions about communications and social systems to further develop his theory which he applies to different aspects of society.

[44] See Niklas Luhmann, Soziale Systeme (p. 75) *In particular, there are very different ways to solve the problem of gaining time, … whenever,.. speed,.. aggregation and integration of temporal relations*

The Economic system has served as one of the first examples together with law, art and passionate love in the development of Niklas Luhmann's theory after the publication of "Soziale Systeme" in 1984. His publications about other function systems (e.g. politics, religion, science) have followed later[45].

The idea that leads the further way is that of systems within systems or *systems differentiation*[46].

Let us follow Niklas Luhmann's path and start with the definition of world society as the ultimate social system that contains all communications.

As mentioned earlier a key property of systems to maintain a boundary to its environment is their construction of reality. Given the proposition that a system is defined as being able to make a difference between itself and its environment[47], the system reintroduces this difference into its construction of reality. Following this argument Niklas Luhmann distinguishes two types of constructions of reality, namely *environmental differentation* and *systems differentiation.*

From the perspective of the system *systems differentiation* is self observation, where as *environmental differentiation* means observation of the environment. The same argument leads Niklas Luhmann to the statement that every observation starts with a distinction and his reference to George Spencer-Brown[48].

In terms of modern society Niklas Luhmann gives examples about *systems differentiation* - so called function systems - like the economy, law and politics. The primary difference is its function within society. Thereby he assumes that the primary form of differentiation of modern society is its differentiation according to its functions.

The birth of modern society, i.e. the timeframe when functional differentiation becomes the primary form of differentiation of society occurs around the age of enlightenment. One type of explanation given by Niklas Luhmann as to why *functional differentiation* becomes the primary form of differentiation is that functional differentiation allows for a higher capacity of self-observation of

[45] See Baraldi C., Corsi G., Esposito E., Glossar zu Niklas Luhmanns Theorie sozialer Systeme, 1999 for a list of Niklas Luhmann's publications
[46] See Niklas Luhmann, Soziale Systeme (p. 41) *The repetition of the system/environment difference within the system leads to a theory of systems differentiation. The repetition (analysis) of the element/relation difference within the system/environment leads to theory of system complexity*
[47] See Niklas Luhmann, Soziale Systeme (p. 31) *A self referential system has the ability to create/maintain relationships with itself. Further it is able to differentiate these relationships against relationships with its environment*
[48] See George Spencer-Brown, Laws of Form, 1969, 1994, (p. 3) *Forms taken out of the form – Construction – Draw a distinction. Content – Call it the first distinction, ...*

society. In Niklas Luhmann's thinking (given the interplay of psychic and social systems) this is equivalent to an increased capacity to make expectations in a world of increasing communications that need to be coped with.

Other (older) types of primary systems differentiation[49] are:

- Differentiation into hierarchies, e.g. stratification into ranks or classes (justified by birth and/or family membership) or castes (same with additional religious foundation).
- Differentiation into unequal parts, e.g. the distinctions of centre/periphery, city/countryside[50], conform/deviating -like speakers of the same language/strangers:
    - Greeks and barbarians or
    - Humans with the ability of the word слово/Slavs and the deaf немецкий/Germans to name just a few.

  Further examples iclude official/inofficial, formal/informal or on stage/backstage.
- Differentiation into equal parts, so called segmentary differentiation, e.g. differentiation into tribes and/or families.

Note that every form of differentiation is based on a leading (guiding) distinction.

Niklas Luhmann's focus is not the distribution of wealth or the differences of mortality or technological achievements, etc. in these different types of societies. Niklas Luhmann's aim is to build a theory about society[51], the social system with all communications as its *elements* that cannot be directly observed but only inferred as mentioned earlier.

So given the notion of world society (which has no other social systems in its environment) and the notions of *systems differentiation* and *environmental differentiation* Niklas Luhmann concludes that the economy is one among other function systems of world society. Its identity is defined by means of *systems differentiation* of world society.

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[49] See Niklas Luhmann, Soziale Systeme (p. 261 ff)

[50] The etymological origins of "politics" and "economy" - the ancient Greek words for city – "Polis", and the ancient Greek word for domestic households (based on property, in the countryside) – "Oikos", show the close similarity of the development of semantics and societal structure. A whole series of publications by Niklas Luhmann is devoted to a research program that addresses this issue (Gesellschaftsstruktur und Semantik 1-4)

[51] See Niklas Luhmann, Die Gesellschaft der Gesellschaft, 1997 (p.11) *Entering 1969 founded faculty of Sociology at the University of Bielefeld I was confronted with the approach to name research projects, with which I work. My project was called then and is to this day: A Theory of society; Duration 30 years; costs; none.*

At this point I cannot give a precise statement about Niklas Luhmann's assumptions about the guiding difference. In other words: What is the mechanism that society uses in order to make the distinction, whether an element of communications belongs to the function system economy or not. From his writings the following lines of thought are near at hand:

- The function of the economic system[52] – opposed to functions of other function systems.
- The specific coding – or the specific type of *symbolically generalised medium of communications.* A symbolically generalised medium of communications is defined as a template, to which elements of communication have to fit[53], in order to belong to the respective function system. In terms of the economy *money* serves as *symbolically generalised medium of communications.*

Niklas Luhmann uses the word function when referring to the method of comparative analysis. So the reasoning about functions or functional equivalents of societal subsystems is a task undertaken by advanced observers of society. It is therefore unlikely that Niklas Luhmann thought it to be a basic operation of the social system society. In other words one may be able to buy a pair of shoes (even in a foreign language[54]) without ever reflecting the theory of functional differentiation.

From a modelling perspective the coding mentioned above is the first candidate. In this case we have to take the existence of at least one symbolically generalised medium of communications as an axiom. This approach implies the following problems:

- Taking the existence of symbolically generalised media of communications as an axiom – we have to make a separate model to explain the evolution of *symbolically generalised media of communications.*
- According to Niklas Luhmann *symbolically generalised media of communications* do only appear within a society primarily differentiated into function systems. This requires taking the primacy of functional differentiation as another axiom. Given the everyday experience that communicative success (in terms of understanding) fairly often depends on other things than

[52] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 64) *With the increase of time-lasting, store-able goods, scarcity increases. A social mechanism has to be created that allows a future (and stable) provision to be connected with present distributions (of goods);* Further (p. 132)
The function of the economy – is roughly speaking – provision for the fulfilment of future needs – by payments that pass solvability

[53] This is very close to the ancient Greek meaning of "Symbolon"; see Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 257) *symbols bring divided things/entities together to unity; In a way such that unity is detectable on both sides, but without melting or destroying the difference.* The similarity with DNA molecules being templates to each other is evident (Metabolism). In conversations I sometimes tried to describe Niklas Luhmann's thinking as a shift from the dialectical trias thesis-antithesis-synthesis to a system-theoretic trias of symbolon-diabolon-metabolon

[54] The given example, that one is able to buy a pair of shoes without the need of mutual language comprehension shows both the validity and the limits of the concept of an economy understood as function system

codes of function systems – e.g. on the speaking the same language - it is hard to accept the primacy of functional differentiation in all situations of life[55].

- In my experience so-called borderline cases prove to be useful to test the quality of a model. Determining the function system by the usage of a specific code as a criterion may cause further difficulties. An element of communication, e.g. the buying of a wedding ring, may have a meaning in more than one function system.

Niklas Luhmann follows an evolutionary perspective assuming that the evolution of world society is leading to an increasing distance between interactions and the all-encompassing social system world society. The primacy[56] of functional differentiation may then be better understood as the most recent step in a process of emerging levels (discontinuities[57]) of systems differentiation.

I want to stop at this point with the top-down analysis from world society to its function systems with the suggestion to understand systems differentiation (from the viewpoint of the all-encompassing system) in terms of fuzzy memberships or fuzzy clustering.

Allowing fuzzy memberships an element of communication, say x (let X denote the set of communication elements under consideration) may belong to more than one group (or cluster C) with a varying degree. $\mu_C(x) \in [0,1]$, $x \in X$ is called the membership function.

Fuzzy clusters are linked to fuzzy similarity relations[58]. A fuzzy similarity relation explains a distance function.

[55] See Niklas Luhmann, Ecological Communication, 1989 (German: Ökologische Kommunikation: Kann die moderne Gesellschaft sich auf ökologische Gefährdungen einstellen?, 1986) (p. 36) Niklas Luhmann gives an interseting interpretation tp primacy: *...the communication of the streets, so to say, in a somewhat more high-sounding jargon: „life-world"-communication. Communication that affects society, however, depends on the possibilities of function systems.*

[56] The idea about primacy reminds me about the widespread belief that the human race equals a similar level in biological evolution. These People tend to forget, that although dinosaurs may have died out, reptiles are well among us. Not to forget about insects and other to this day evolutionary successful forms of life which are far older than the human race

[57] Niklas Luhmann describes these events as bifurcations, although he gives a mathematically unfamiliar explanation, See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 181) *A very general mechanism, that solves these problems (i.e. that transforms these problems to other problems) can be noted as bifurcation – and connecting to it as coding, A bifurcation prevents, that the constitutive paradox gains impact as unity. The unity is substituted by a difference, with the consequence that operations can orientate themselves within this difference – and the question regarding the unity of this difference does no longer occur.*

[58] See Olaf Wolkenhauer, Data Engineering, 2001, (p. 145-152)

As *systems differentiation* serves the purpose for systems to gain difference (distance) from themselves[59] – there is one more argument pointing to the usage of fuzzy methods as part of the modelling approach.

3.4 Economic Theorems

At the end of the introduction of "Die Wirtschaft der Gesellschaft" Niklas Luhmann clarifies that he was able to *reduce the number of unintentional coincidences with economic thinking*[60] in his work. He explains, that he has had no intention to describe economic aspects as opposed to social aspects of human behaviour. All economic action is societal action, or action within society – and this is why he chose the title "Die Wirtschaft der Gesellschaft" - the economy of society - for his publication. He undertakes an analysis of the economy using his theoretical framework[61].

Using his theoretical framework and his comparative approach Niklas Luhmann is able to come up with astonishing insights and new formulations of economic problems. I refer to the latter as Niklas Luhmann's economic theorems.

Among these I will try to describe the following

- The interdependence of the economy and the political system
- The emergence of money and the double cycle
- Prices and Markets

Because the economic system as described by Niklas Luhmann gives rise to complex organisations (such as the industry and banking sector) and does also give the means (money) to build organisations (remember that members need to be compensated) that operate in other function system (e.g. hospitals, government bureaucracies, universities) he adds a chapter about organisations in "Die Wirtschaft der Gesellschaft".

As the later modelling will focus on the macro level these topics are not covered.

[59] See Niklas Luhmann, Soziale Systeme (p. 38) *Differentiation provides the system with systemacity; besides its mere identity (difference from something else), it also gains a second version of unity (difference from itself).*
[60] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 12)
[61] See Niklas Luhmann, Ecological Communication, (p. 51) *Among society's many function systems the economy deserves first consideration*

3.4.1 The Economy and the Political System

Niklas Luhmann's theorems regarding the interdependence and/or irritability of the economy are based on the assumption that functional differentiation has been achieved. In this sense they are not valid for earlier types of economies.

Function systems of society – such as the economy – process their communications by a binary code. Communications that do not match the template of the function system's binary code are in this sense externalised by the function system, they belong to the function systems' environment[62].

For the economy, this binary code allows for the values having/not-having (property rights). For the political system the code of power/no power applies. Let us add the legal system using legal/illegal and the scientific system using true/false. The ongoing evolution of society led to a secondary code of solvability/insolvability (in terms of money) on top of the primary code having/not-having for the economy.

According to Niklas Luhmann the evolution of this secondary code leads to the so called double cycle of the economy, its closure and to the emergence of the economy as a self-referential (autopoietic) system.

As a direct consequence the economy cannot process communications of another function system. In terms of political interventions this means that these interventions can only be processed (can only be observed) by the economic system if they can be expressed in terms of the code solvability/insolvability (typically as costs). The comparable notion of an intervention of the political system into the legal system has to happen in the language/formalism of the legal system (e.g. a law).

Yet the function systems depend on each other, by the simple fact that they can relieve themselves of communications that are processed by other systems. In this sense there is no "externalisation" of costs, when externalisation is understood as a transfer to a function system other than the economy.

As *Codes* do not determine how the positive or the negative value has to be assigned (i.e. the code does not give directions if one should decide to buy a special good or not) nor does the code imply that one side is better than the other (i.e. the code does not aid in the decision if saving is better than

[62] See Niklas Luhmann, Ecological Communication, (pp.36-50) for a brief description

spending) – *Programs* (also called decision rules, criteria) are needed to guide the assignment of either the positive or negative value.

The concepts of *Coding* and *Programming* in the specific context of function systems do not appear in "Soziale Systeme". In "Die Wirtschaft der Gesellschaft" the notions of *generalised media of communications* and *Codes* are described as equivalent[63]. The notion of *Code* as described above is introduced with a description of its usage within the economic system[64]. Niklas Luhmann uses the notion of forms that get imprinted into a medium following a certain code.

The notion of programs or programming appears in earlier works of Niklas Luhmann. There are two kinds of programs, namely

- Conditional programs, based on if-then-else rules, and
- Anticipatory (goal) programs ("Zweckprogramme") based on the selection of a purpose (an ends as with means and ends, or a goal), that imply ordering of intrinsic values[65].

It is important to add that function systems can only learn within their programs. Codes are - as evolutionary achievements irreversible and invariant.

Regarding the interdependence of function systems Niklas Luhmann gives the following examples:

- Given functional differentiation, the autonomy of function systems is enhanced. None of the following: political power, scientific truth, passionate love, justice, salvation can be bought by money. At least the opposite is generally regarded as a state of corruption[66].
- Problems caused by the economy, e.g. the exploitation of resources can be treated by the political system only as a political problem[67]

[63] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 240) *symbolically generallised communications media are evolutionary emerged and proved answers to the problem of double contingency. This gives rise to and limits their design. They are special codes that can gain universal relevance*
[64] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 188) *Condensation (see George Spencer-Brown) requires repetition – Codes are condensed opposites – As with all bifurcations, they give rise for time and history.*
[65] See Niklas Luhmann, Zweckbegriff und Systemrationalität, Über die Funktion von Zwecken in Sozialen Systemen, 1968. The literal translation of „Zweckprogramm" is purpose-program. They appear frequently nowadays e.g. as so-called management-objectives. I chose anticipatory programs to reflect the notion of anticipatory systems according to R. Rosen.
[66] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 111)
[67] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 346) *Political programs can only be executed within the political system – economic programs within the economic system; no direct cybernetic mechanism across system boundaries. Subject to the condition that system-boundaries stay intact, politics can see its purpose, to influence programs of the economy ...* with all its difficulties and limitations

- There is a soothing impact of money. If we think of an economic transaction, say person A buys something from person B, the main function of money is to sooth third parties, say person C. The idea is that person C (or even more observers) refrains from violent access, because they understand that B has paid. Money prevents violence within the realm of its ordering capacity[68]
- Given the interdependence of function systems there is a tendency, that failure or underperformance in one function system leads to an overstrain in another[69]

In "Ecological Communications" Niklas Luhmann uses the idea of resonance to analyse the extent to which function systems can react to each other and to the environment.

In terms of modelling one could think of implementing an agent based model, where agents communicate following the rules of different codes, e.g. power (for politics/law) and payments (for economy). According to Niklas Luhmann the medium of power implies a setting that is usually modelled by asymmetric (non-zero sum) games such as the prisoners-dilemma[70].

3.4.2 The Emergence of Money and the Double Cycle

Niklas Luhmann explains the evolution of today's modern monetary economy (as a function system) in the following way:

- The first notion is scarcity. Niklas Luhmann distinguishes scarce from finite sets. Scarcity is a phenomenon that is generated by access (that limits further access). Assuming that scarcity motivates access, scarcity is its own cause. Niklas Luhmann calls this the scarcity paradox.[71].
- As a consequence scarcity is addressed in the social system. This is achieved by the establishment of ownership rights. In Niklas Luhmann's thinking this is the basic binary code[72]

[68] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 253) *Money ist he triumph of scarcity over violence*
[69] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 32) *This implies that the failure of one factor, e.g. the full political/governmental control of the banking sector, must lead to a an additional burden for the political system, in the sense that democratically led policies become more difficult*
[70] See Niklas Luhmann, Macht, 1975 (p. 22) *The use of power happens in the case, when the relationship between the alternatives, that are wished to be avoided by the involved parties, is structured in a way, that the person subdued to power is more likely to avoid his/her avoid-alternative (in our case physical fight) than the person in power; And that this situation is evident for all persons involved*
[71] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 179) *scarcity is a paradox problem. It creates what it wants to solve*
[72] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 189) *The ownership code says: with regard to all own-able goods – everyone can be owner or not-owner; there are no third possibilities. The inclusion is created by the difference – not by the positive code value (being the owner)*

using a symbolically generalised medium of communications. Due to an uneven distribution of wealth, this step does not eliminate scarcity.

- The next step is monetarisation. Niklas Luhmann argues that more than one condition must be present, in order to achieve monetarisation. One of which is the availability of goods, the second is that certain types of money (coins) are available[73]. He does not propose any alternative explanations regarding the evolution of money, but additional ones. The result is that the secondary code of solvability/insolvability replaces the basic code of having/not-having.
- Given monetarisation (and the intrinsic properties of the money code[74]) the economy gains full autonomy, as two cycles of information flow (one in the direction of money/solvability, the other in the direction of insolvability) get established.

  Niklas Luhmann distinguishes three types of participants in the economic systems – according to the means by which they have to regain solvability:
  - Families gaining solvability by labour[75]. Niklas Luhmann refrains from calling this sector consumers, as there is also consumption by other types of participants
  - Governments or the state gaining solvability by taxes
  - The proftability sector: Only in this sector money is spent in order to gain (more) money (value-add) at a later time (implying risk)[76].

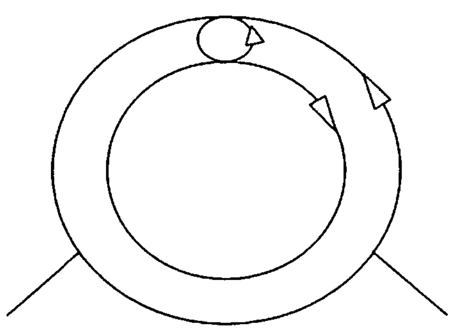

Figure 4: Double Cycle of the Economy[77]

[73] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 189) *People accept the exclusion of certain property/goods – and gain the inclusion into the economy. Economy can only evolve – when there are sufficient motives for this inclusion*

[74] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 198) *Although money can only be used for payments (the operation) – scarcity gives meaning to not paying. The keeping of money represents the wholeness of other possible money uses. It makes sense to consider – non-payment – although money can only be used for payments. The scarcity of money leads to a further bifurcation*

[75] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 215) *Everyone is the (sole) proprietor of one's own bodily and mental capabilities – and therefore one's own slave. Slavery becomes general – and gets abolished. Slavery now appears as freedom*

[76] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 55) *Profits occur, when payments benefit the payer*

The location of the profitability sector in the diagram, determines the self-conditioning of the economic system[78], because it is the location, where the price of money is determined. Niklas Luhmann gives this as reason why in everyday-language the profitability sector is often understood as "the economy".

Niklas Luhmann further notes that different tools (balance sheets/income statements) are used in the profitability sector (versus budgets) and other sectors.

- The development is finalised by the evolution of the banking sector[79] and capital markets. By comparison with the legal and political systems Niklas Luhmann explains, that the banking system (similar to courts) has a hierarchical differentiation in to central banks, merchant banks and banking customers.

3.4.3 Prices and Markets

An evolutionary achievement of the economic system – when differentiated as an *autopoietic* function system - is its ability to condition its operations. As its operations are payments this conditioning is achieved by prices[80]. In this context Niklas Luhmann stresses, that such a system is never in equilibrium (or equilibrium is not a desirable state)[81].

The usage of prices is explained by two conditions – one referencing the economy (self-reference) the other the environment (outside-refrence).

- *Needs must be distributed unevenly, so that goods appear more or less attractive at a given price, which is equivalent to assume that the environment of the economy requires a certain minimum of complexity*[82]

[77] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 137)

[78] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 224) *The code ownership/non-ownership is asymmetrical (ownership is preferred). The code pay/not-pay has nearly completely regained symmetry – both sides have their pros and cons. Decisions* (and therefore Organisations) *become necessary … – leading to programs*

[79] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 200) *Banks give incentives for saving and lending … The scarcity of money becomes autonomous, and dependent on the monetary policies (of the banking system) that is specially addressed to it. In its own way money is abundant and scarce for others. The scarcity paradox reappears again and unfolds into organisational differentiation*

[80] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 248) *One cannot transfer money without specifying an amount*

[81] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 17) *An adequate reference point for the observation and analysis of the system is therefore not the return to a state of rest, as theories of equilibrium suggest, but the continuing reproduction of moment-like activities, i.e. payments, which the systems consists of*

[82] Niklas Luhmann defines complexity as a property of cohesive sets: *we will call an interconnected collection of elements "complex" when, because of immanent constraints in the element's connective capacity, it is no longer possible at any moment to connect every element with every other element.*

- *Money must also be distributed unevenly, so that prices are too high for one but not for the other.*

Niklas Luhmann summarises that *inequality is at the same time the starting condition for the economy as well as its product*[83].

In terms of reducing external variety and at the same time increasing internal variety of the economic system - a state which serves as a motor for the economy – Niklas Luhmann comments that payments are characterised by a high loss of information[84].

The usage of money has two major advantages

- Because payments do not obligate reciprocity, one works for money and not to achieve salvation, the system gains independence of social structures outside the economy
- The motivation to work and to make decisions regarding consumption or investment (referred to as selections) is enhanced by the system itself[85].

It is in this sense that Niklas Luhmann describes money operating as a symbol (in terms of template) by connecting motivation and selectivity.

Prices serve as an (endogenous) means to make expectations. Prices are used at the core of self-descriptions of the economy[86]. Niklas Luhmann concludes that in order to achieve sufficient self-control, the economic system requires variable prices[87].

Using the framework of communications, social systems and system differentiation Niklas Luhmann analyses the typology of markets. Led by the observation that markets do not differentiate into

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"immanent constraint" refers to the internal complexity of the elements, which is not at the system's disposal, yet which males possible their capacity for unity ("Einheitsfähigkeit").In this respect complexity is a self-conditioning state of affairs; See Soziale Systeme (p. 46/English p. 24)

[83] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 112)

[84] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 18) *Neither needs nor wishes that can be fulfilled by monetary payments, need special explanation. Nor does the payer give any information regarding the origin of the money.*

[85] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 241) *One tries hard to sell one's products – and therefore uses a high amount of (re)investment and highly complex production-organisations – just for the purpose of getting rid of things. At the same time one tries hard to find more or less welcome jobs – All that, because of the magic formula: because it is paid for.*

[86] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 34) *If self-descriptions emerge, that the system creates and uses by its own communication processes, than there is no other (at the same efficiency level) way, than using prices*

[87] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 28) *In any case the goal is not the approximation to a perfect nature, but to find possibilities to control instabilities of the environment by instabilities of the system. And therefore variable prices are needed by the economy*

interactions and/or organisations but into specialised markets like good's markets or capital markets he defines markets as inner-economic environments but not as systems. This assumption rests on the following observations:

- Competition is (with the exception of price agreements in situations where we have few suppliers knowing each other well and assuming that they will meet again in a similar situation[88]) interaction free[89]. The participation in a market is therefore an efficient means to make expectations. This applies equally well to the supply and to the purchasing side.
- By reducing the need for interactions the economic system gains sensibility and reactivitiy[90].
- This increase in speed leads to the feature that the economy reacts primarily to events that are perceived as changes (e.g. changes in interest rates, changes in regulatory practices, changes in government policies)[91] and not to an intended determination of structure.

The classification (or the differentiation of markets) into customers, competitors and cooperation partners leads to a further level of abstraction. One can observe one's own interest regarding a specific scarce good in terms of this classification, i.e. without knowing each other individually. Niklas Luhmann argues that an adequate quality of markets is therefore the multiplicity of contexts for each participant (*poly-contextuality*) rather than the multiplicity of centres (*poly-centrality*).

Following Harrison C. White Niklas Luhmann describes markets as *tangible cliques of producers observing each other. Pressure from the buyer side creates a mirror in which producers see themselves*[92], leading – in order to fulfil their function as effective observation horizons – to the differentiation of markets into markets for special goods. This includes public goods. The impact of competition between politicians rather than demand regarding the supply of pobluc goods is mentioned as an example.

[88] See Robert Axelrod, The Evolution of Cooperation, 1984
[89] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 102) *Competition is interaction free – and therefore saves conflicts.* It has to be added, that Niklas Luhmann defines a conflict as any communication that opposes or contradicts a prior communication. By this conflicts serve as the immune-system of social systems in the sense that they do not protect a system's structures, but its autopoiesis.
[90] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 103) *The sensitivity and reactivity (speed) of the economic system depends heavily on the saving of interactions. The simultaneous reaction of many – to what many assume as reaction of the others – is a basic principle of a monetary economic system*
[91] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 103) *... and the system immediately reacts to the event by making new expectations, that might stand the test in the market given competition and the new circumstances*
[92] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 109), See further Harrison C. White, Where do Markets Come From? American Journal of Sociology 87, 1981 (p. 517-547)

Among markets money markets or capital markets have the distinctive feature, that they contain no outside-references (in terms of outside of the economic system)[93]. This leads to the specific risks of holding monetary assets and giving credit. Measures for internal stability have to be established, due to this lack of external orientation. This is gained by the hierarchical structure of the banking system into issuing banks, merchant banks and non-banks (customers of banks).

Assuming that markets imply risks and risks pose limits to rationality Niklas Luhmann concludes that structuring risks along markets is a sort of best-in-class strategy. Public organisations are therefore much more prone to failure regarding its interventions, especially when they believe that abstract calculations can lead to a description of the desired state without risk. Not without humour he adds, that it is therefore not surprising, when public organisations survive the ignorance of risks and/or their own failures more often than private participants – due to internally constituted securities[94].

[93] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 116) *The operations of capital markets are – to a high degree – determined by self-reference. ... regarding the market for beverages one can know what it means to be thirsty, how this depends on whether conditions*
[94] See Niklas Luhmann, Die Wirtschaft der Gesellschaft (p. 124)

4 A Model for a Simple Luhmann Economy

In order to study some of the theorems outlined above the technique of agent-based modelling has been applied. The design of such a model is oriented towards the following guiding questions:

- Assuming the economy starts off with inequality. To what degree does it produce further inequality in order to continue?
- What type or patterns of internal differentiation can be observed?

In order to study the questions mentioned above, an agent-based model of a simple Luhmann economy has been defined as follows[95].

4.1 Agents and Neighbourhoods

Each agent inhabits exactly one of $n \times m$ squares. The squares are positioned in a 2-dimensional array (world) with n rows and m columns. Both n and m are natural numbers ≥ 1.

The agents have the ability to observe other agents that are within reach of the observation horizon o. o is a natural number and denotes how many rows or columns apart an agent can "see". So in the straight forward case when $o = 1$ an observing agent can observe all it's eight (north, south, west, east and in between directions) neighbours and of course itself.

Edges are eliminated by closing the array like a torus, so that row 1 is also neighbouring row n, as well as column 1 is also neighbouring column m. By this the number of observable squares is the same for all agents with the same observation horizon o.

It is further assumed that all agents have the same observation horizon and behave according to the same rules. For simplicity reasons the model does not consider

- Movements of agents to "better" places
- Evolutionary settings, in the sense that agents copy or inherit behaviour of successful agents[96]

[95] Agent-based modelling refers to the term as used by Robert Axelrod, The Complexity of Cooperation, 1997 (p. 3) *Agent-based modelling is a third way of doing science. Like deduction, it starts with a set of explicit assumptions. But unlike deduction, it does not prove theorems. Instead, an agents-based model generates simulated data that can be analysed inductively*

[96] I expect the significance of Niklas Luhmann's theory of Social Systems and/or Society as a framework to explain social rather than biological evolution – so inheritance may not be the mechanism of primary interest

4.2 Portfolios

There are g different goods that can be owned and traded by agents. g is a natural number ≥ 2.

Each agent has a portfolio of g different goods. The $n \times m \times g$ stock matrix $\tilde{A}_t$ describes the portfolios (stocks) of all agents at discrete time $t \geq 0$. Because goods are modelled to be undividable all $\tilde{a}_{i,j,k}$ are natural numbers. There is no short selling (or credit), so

$$\forall i \forall j \forall k \forall t : \tilde{a}_{i,j,k} \in N_0 \tag{0.6}$$

In order to simplify the mathematical description of the system the $n \times m \times g$ matrix $\tilde{A}_t$ is reduced to a 2-dimensional, i.e. an $n \cdot m \times g$ matrix A_t, where the row identifies the agent and the column identifies the good.

4.3 Trades and Trade Runs

Agents trade with other agents within their observation horizon. A single trade involves two trading agents and two goods. One agent proposes to exchange an amount of good A in exchange for an amount of good B.

Using the same simplified notation as above an $n \cdot m \times n \cdot m \times g$ matrix P_t describes all the proposals from agent i to agent j regarding good k at $t \geq 0$.

A single trade (between agent i and agent j) comprises exactly one combination of two different goods A and B among the g goods. Therefore the following condition holds in the model:

$\forall i \forall j \forall t$: exactly one of the following 2 cases holds:

$$\begin{aligned} &\exists k_1 : p_{i,j,k_1} > 0 \Leftrightarrow \exists k_2 \neq k_1 : p_{i,j,k_2} < 0 \wedge \ \forall k \neq k_1 \neq k_2 : \ p_{i,j,k} = 0 \\ &\forall k : \ p_{i,j,k} = 0 \end{aligned} \tag{0.7}$$

This condition specifies, that at a given time instant $t \geq 0$ agent i can propose to agent j to trade only one combination of goods. Without loss of generality the quantities are to be understood as intended flows from agent i to agent j. $p_{i,j,k_1} > 0$ denotes a proposed flow of good k_1 from agent i

to agent j. $p_{i,j,k_2} < 0$ denotes a proposed flow of good k_2 in the reverse direction, i.e. from agent j to agent i.

The second (accepting) agent can only accept or refuse a proposal. There are no further options. The matrix $n \cdot m \times n \cdot m \times g$ (deal) matrix D_t has the same dimension as the proposal matrix P_t and describes the accepted proposals.

Trades are always completed by both agents in the way they are agreed. No defection/cooperation type of behaviour (e.g. agents take proposed good A without exchanging good B) is considered in the model. In a limited sense a trade is always "beneficial" for both partners, because there are no presents or giveaways.

A trade run – involves the following steps:

- Agents direct proposals to other agents in their respective observation horizon. The matrix P_t is calculated following the proposal rule.
- Agents accept the (economically) best proposals among the received proposals. The matrix D_t is calculated according to the acceptance rule
- The accepted proposals (deals) are cleared. The new stock matrix A_{t+1} is calculated from the old stock matrix A_t and the accepted trades D_t.

Given the definitions above, a trade run affects the stock matrix A_t without changing the total quantities of each good:

$$\forall k \forall t: \sum_{\forall i} a_{i,k,t} = \sum_{\forall i} a_{i,k,t+1} \tag{0.8}$$

Defining the g-vector $\bar{w}$, denoting the total wealth of the system in each good, the above condition can be rewritten in matrix notation ($1_{n \cdot m}$ is an $n \cdot m$-vector of 1s) as

$$\forall t: A^T_t \cdot 1_{n \cdot m} = \bar{w} \tag{0.9}$$

4.4 Observation

The rules that define the making of proposals and the acceptance of deals depend on mutual observation of the agents. Agents are modelled, so that they distinguish between themselves and their environment. An agent's (constructed) environment is constituted by the observable behaviour of the agents within the observation horizon of the observing agent.

Regarding itself an agent observes:

- Its own portfolio
- Its committed proposals during a trade run, to ensure that nothing is promised that cannot be fulfilled before deals are cleared

Regarding its environment (the behaviour of other agents within the observation horizon) an agent observes:

- "Showoffs" of all agents in its observation horizon. Showoffs are displays of an agent's wealth.
- Among the proposals only these, which are directed towards the agent can be observed. Proposals that other agents direct to other agents cannot be observed (even within the observation horizon). One can understand the directing of proposals (and their later acceptance) as a sealed bid auction.
- Accepted deals of all agents in its observation horizon.

The proposal and acceptance rules (that are the same for all agents, see 4.1) use no further variables, so the "decisions" of the agents are solely based on their "individual" observations (*cognitive expectations*) and on the functional form of the respective decision rule (*normative expectations*).

The model implements the definitions of *communications* and *double contingency* in the following way:

- Agents are not fully transparent to each other[97], because observation is limited to showoffs and deals
- *Understanding* – i.e. the event that makes a communication element a unity is the observation of another agent by an agent.
  The selection of information and uttering is obvious both in the showoff and proposal case. An identification of the utterer as an "observer with communicative ability" is due to the rule that this other agent is considered a possible addressee for a future proposal; the behaviour of this other observer is contingent.
  Meaning is generated by the two differences
  - Goods shown off versus owned goods. The difference indicates scarcity.

[97] See Thomas Kron, Peter Dittrich Doppelte Kontingenz nach Luhmann – ein Simulationsexperiment in Thomas Kron (Hg.), Luhmann modelliert, 2002, (p. 213); See further Papendick Sigmar, Wellner Jörg, Symbolergebnis und Strukturdifferenzierung, ibid (p. 189)

  - Sales versus purchase "prices". The difference indicates an opportunity for a profitable trade

The models presented in Thomas Kron (Hg.), "Luhmann modelliert" implement communication as a one-to-one interaction between agents. The model presented here makes use of the spatial structure and its restrictions to mutual observation and includes the excluded middle.

4.5 Displays of Wealth (Show Off)

Agents can perform showoffs or displays of wealth. Cheating, lying or misunderstandings (errors in information transfer) are not considered in the model.

As the model does not include prices or utilities the following simple showoff rule is implemented:

- With a probability p show off quantity and identity of the most numerous good. If the good is not unique the lexicographic order is used to select good and wealth to be displayed.

To study the pure behaviour of the system (i.e. the behaviour not influenced by other probabilities than the initial wealth) the showoff probability p is needs to be set to 1.

There are reasonable other rules, e.g.

- Given prior exchanges of goods at observed exchange rates (memory is needed) the most valuable instead of the most numerous could be selected
- More than one good is shown off

In terms of the mathematical description the showoff rule is a one-to-one map $\tilde{x}: A \to A$, of the set of $n \cdot m \times g$ (stock) matrices A into itself. Let the matrix $\tilde{X}_t = \tilde{x}(A_t)$ bet the observable (shown off) wealth. Note, that $\forall t: \tilde{X}_t \leq A_t$

4.6 Rule for Making Proposals

Every agent can direct proposals to all its neighbours. Depending on an agent's observations different rules are applied:

- The arbitrage rule is applied, when an agent observed (at least) two further agents exchanging the same goods in different quantity ratios ("prices")

- The proposal rule following showoff is applied, when no proposals can be made according to the arbitrage rule, e.g. when no deals were observed. By definition this is the case at time $t = 1$. One can understand the showoff rule as an activity rule in the sense that it triggers further activities[98].

The rule for making proposals comprises two sub rules that are slightly more complicated than the general setting. They are described in the following chapters. In mathematical notation the overall proposal rule can be described as a one-to-one map $\tilde{p} : \mathrm{A} \times \Delta \times \mathrm{A} \to \Delta$. A denotes the set of $n \cdot m \times g$ (stock) matrices as before. Δ denotes the set of $n \cdot m \times n \cdot m \times g$ (proposal, deal or flow) matrices.

As proposals depend on stock, prior deals and displayed wealth the following identity holds $\forall t \geq 1$: $P_t = \tilde{p}(A_t, D_{t-1}, \tilde{x}(A_t))$.

4.6.1 Arbitrage Rule

The main idea behind the arbitrage rule is the concept of "naïve" prognosis: Tomorrow will be the same as today. In the context of this model this means the following: If an agent observes another agent exchanging goods A and B in certain quantities, the agent expects the observed agent to do the same in the next run.

The situation, in which agent i makes proposals according to the arbitrage rule is characterised as follows:

- More than one exchange between two goods A and B have been observed at different quantity ratios, i.e. "prices".
- Agent i assumes by "naïve" prognosis, that the observed agents will repeat their behaviour and are therefore likely to accept a proposal, that is at least as good as the observed deal.
- Agent i owns a sufficient disposable quantity of goods A and B respectively, so a proposal, when accepted can be fulfilled.

Without loss of generality agent i selects good A as the primary object of the intended trade and selects good B as payment. Given this distinction, the set of observed deals (exchanges between A

[98] The term of activity rule corresponds to the term in auction theory, See Paul Milgrom, Putting Auction Theory to Work, 2003

and B) can be split up in observed purchases (of good A) and observed sales (of good A) according to the sign of the observed deal.

In this setting a purchase (of good A, say by agent $j \neq i$) constitutes an opportunity to sell the same good A (at the same price) to this agent j. Following the same reasoning a sale by agent $j \neq i$ constitutes an opportunity to buy.

Agent i now selects the cheapest price, i.e. the quantity of good B for one piece of good A among the opportunities to buy and the highest (dearest) price among the opportunities to sell.

If this cheapest purchase price is lower than the highest sales price, then a proposal is addressed to each of the agents, where the respective prices have been identified. If the disposable stock admits only one proposal, only one proposal is made.

If the cheapest purchase price or the highest sales price is not unique, the agent with the lexicographic smallest index is selected respectively.

Reasonable variations of this arbitrage rule could be the following:

- A higher purchase price or a lower sales price could be offered as an incentive to price out competition
- In case where prices are not unique, all “best price” agents could be addressed, as long as there is enough disposable stock.

The Arbitrage rule as outlined above was chosen as the most simple and straightforward one.

4.6.2 Proposal Rule following Showoff

Proposals are made according to the proposal rule following showoff, when no arbitrage rule proposals could be made. The showoff rule is less complex than the arbitrage rule. Before an agent i makes a proposal according to the showoff rule, it faces the following situation:

- other agents (within the observation horizon) gave displays of their wealth.
- From the above observations agent i computes, who the wealthiest neighbour regarding every good is.

For every good k_l the following procedure is followed:

- If agent i owns at least as much as (or 1 piece less than) the wealthiest neighbour, agent i "experiences" no scarcity, is confident and does nothing.
- If agent i owns less than 2 pieces than its wealthiest neighbour, scarcity is "experienced". Agent i now selects good $k_2 \neq k_1$ as the good where it owns the highest disposable quantity.
- If such a good k_2 can be identified, agent i proposes to give 1 piece of k_2 to its wealthiest neighbour in exchange for receiving half (truncated to integer) of the difference between the neighbour's and its own wealth (of good k_1).

When the wealthiest neighbour is not unique, the neighbour with the lexicographic smallest index is selected.

Thinkable variations of the proposal rule following showoff include

- Less greedy behaviour by offering more than one piece of good k_2, especially if a considerable quantity of k_2 (more than the observed minimum) is owned.
- Propose to all wealthiest neighbours, if there is more than one and sufficient "payment" is available

For simplicity reasons the show off rule as outlined above was selected in the model.

4.7 Rules for Accepting Proposals

Deals are accepted following a rule that is similar to the arbitrage rule. Before accepting deals between goods A and B agent j is in the following situation

- Zero, one or more proposals to exchange good A and B were addressed to agent j

For all suitable combinations of goods A and B agent j follows the following rule

- If no proposals are present, do nothing
- If one proposal is present, accept it, subject to fulfilment is possible given the no short selling condition.
- If more than one proposal is present - without loss of generality - agent j selects good A as the primary object of the intended trade and selects good B as payment. Proposals are split into two distinct sets, a set of proposals to buy good A and a set of proposals to sell good A.
- Among the proposals to buy good A the minimum purchase price is selected. Among the proposals to sell good A the highest sales price is selected.

- The proposal with the lowest purchase price and the proposal with the highest sales price are accepted if the lowest purchase price is lower than the highest sales price. Each proposal is made subject to the no short selling condition.
- If one of the sets is empty (i.e. when there are either only proposals to buy or only proposals to sell) the proposals with the lowest purchase price (or in the other case the proposal with the highest sales price) are accepted, subject to the no short selling condition.

If lowest purchase prices or highest sales prices are not unique, the lexicographic order is used to determine a unique proposing agent.

It is valid to say that the rule according to which a proposal to exchange good A with good B is accepted, if there is only one of its kind available (singleton proposal) is reasonable. If the (singleton) proposal was addressed to the agent according to the arbitrage rule then a proposal with the same price was selected in an earlier run. If – in the other case - the proposal was addressed to the agent according to the proposal rule following showoffs there is no competitive argument. Still the following holds:

- Let without loss of generality A be the good that is to be purchased by agent j and good B the payment.
- In this case B must have been the good that agent j showed off in the prior round. Given the chosen showoff rule - only these goods are shown off, that are owned in largest quantities - receiving quantities of the (scarcer) good A in compensation for the (less scarce) good B cannot be ruled out as uneconomic.

Thinkable variations of the deal acceptance rule include

- A consideration of transitivity (only needed with more than 2 goods). An "exchange rate" of good A with good B plus an "exchange rate" of good B with good C may influence the evaluation of exchanges between A and C. For the sake of simplicity (and in the absence of money) transitivity was not considered in the model.
- Accept all best proposals, if there is more than one and the no short-selling condition can be met.
- A more elaborate strategy with singleton proposals

For the sake of simplicity the acceptance rule as outlined above was selected.

In mathematical notation the deal acceptance rule can be described as a one-to-one map $\tilde{d}: A \times \Delta \rightarrow \Delta$. A as before denotes the set of $n \cdot m \times g$ (stock) matrices. Δ denotes the set of $n \cdot m \times n \cdot m \times g$ (proposal, deal, flow) matrices. The acceptance of deals depends on stock and (pending) proposals and reads in functional form as follows $\forall t \geq 1: D_t = \tilde{d}(A_t, P_t)$.

As all maps $\tilde{x}$, $\tilde{p}$ and $\tilde{d}$ are one-to-one they can be combined into a map $d: A \times \Delta \rightarrow \Delta$, so that the (simplified) identity holds $\forall t \geq 1: D_t = d(A_t, D_{t-1})$.

4.8 Clearing Deals

The last step of a trade run is the somewhat technical clearing of deals, as it does not involve any action of the agents. It is only needed for bookkeeping. It adds up the flows (deals) to the stock according to the straight forward rule:

$$\forall i \forall k: a_{i,k,t+1} = a_{i,k,t} + \sum_{\forall j} d_{i,j,k,t} - \sum_{\forall j} d_{j,i,k,t} \qquad (0.10)$$

Given the definitions above, the clearing of deals can be expressed in matrix notation as a one-to-one map $c: A \times \Delta \rightarrow \Delta$ giving the identity $A_{t+1} = c(A_t, D_t)$.

4.9 Dynamics

The dynamics of the system can be summarised by the following equalities:

$$\begin{aligned} &\text{Deals:} \quad & D_t = d(A_t, D_{t-1}) = \tilde{d}(A_t, \tilde{p}(A_t, D_{t-1}, \tilde{x}(A_t))) \\ &\text{Stocks:} \quad & A_{t+1} = c(A_t, D_t) \end{aligned} \qquad (0.11)$$

Subject to the following initial and transversality conditions:

The initial wealth is fixed at $t = 0$: A_0 fixed

There are no prior trades to $t = 1$: $D_0 = [0]$, so $A_1 = A_0$

(No short selling $\forall t$: $A_t \geq 0$ and integer

The total wealth remains constant $\forall t: A^T{}_t \cdot 1_{n \cdot m} = \bar{w}$

where

$A_t \in \mathrm{A}$, A denotes the set of $n \cdot m \times g$ finite integer matrices

$D_t \in \Delta$, Δ denotes the set of $n \cdot m \times n \cdot m \times g$ finite integer matrices

$n, m, g, t \in N_0$, $t \geq 0$, $n, m \geq 1$, $g \geq 2$

The specifics of the model are:

- There is no utility function – the only assumption is that more (of any scarce good) is better than less. None of the goods has a special role (like money). Which good is exchanged most is determined by the model's simulation runs.
- As there are no markets, there is no market clearing in the sense that supply equals demand. Bookkeeping rules make sure, that only available goods are exchanged. There is no short selling and no credit.
- The reality construction of agents contains *cognitive* expectations (variables) and *normative expectations* (functional form). The memory of agents is very limited in the sense that only the preceding transactions are memorized. Allowing for a bigger memory size does not require a redesign of the model, but only changing some parameters.
- The observation horizon can be chosen to study influences of the cohesive structure of the larger environment[99]
- Demand creation is endogenous to the model. Demand results from observed differences in ownership of scarce goods or by observed differences in purchase and sales prices.
- The focus is to study instability (not stability) – in terms of conditions that support the continuation of economic activity – not its decline to equilibrium

Potential Enhancements include:

- A relief of the no short selling condition (credit). In terms of modelling this implies an enlargement of the state space (agents need more memory-capacity as expectations regarding a longer timeframe into the future are needed).
  The granting of credit needs ways to implement creditworthiness or trust. In terms of Niklas Luhmann's thinking that implies a second medium (trustful/not trustful) of communication and/or power.
- Agents with longer memory (increased rationality over time)
- "Disturbances" in terms of either a declining stock (depreciation, consumption) and/or an increasing stock (production, labour endowments per period)

[99] As indicated earlier the term larger environment refers to the outside environment observable to the constructer of the model, who has – compared to the agents – superior observation capacity

- Individualisation (different strategies for different agents) by an evolutionary setting (successful behaviour is copied) makes it possible to look for evolutionary successful strategies and to compare these strategies. Investigating strategies understood as elements of function spaces (exponentiation maps) may show aspects where strategies converge and where they differ. I presume that this setting is a requirement to identify social systems in the sense of Niklas Luhmann's definition. So – in terms of modelling - individualisation is a requirement for the emergence of social systems. This corresponds to Niklas Luhmann's observations of modern society.
- A combination with other games by implementing the codes of other *symbolically generalised media of communications* may allow to study (in terms of specifying and analysing) the phenomenon of *resonance*[100] among function systems or the phenomenon of overstrain and relief between social systems and their meaning-processing environment.
- Agents are free to seek new places (move). Interesting questions are how player recognize each other individually (and remember earlier interactions) and how the make new acquaintances.

[100] See Niklas Luhmann, Ecological Communication (p. 15 ff)

5 Simulation and Analysis

The model was implemented in EXCEL and VBA (Visual Basic for Applications) to allow for simulation and exploration on a default personal computer.

As an aid for the detection and visualisation of the behaviour of solutions a fuzzy clustering algorithm[101] was additionally implemented and used.

Although in terms of programming I would have preferred to use a more advanced programming language that allows the use of pointers and recursions or alternatively to use dedicated systems for simulation (e.g. StarLogo) I opted for VBA to enhance the possibility of participation and exchange of software code with minimum requirements.

5.1 Interactive Model

The general idea was to implement an interactive model. The main design guideline was to use user-defined-functions. By this the full functionality of a spread-sheet program can be used, as changes in variables are immediately reflected in results (as far as computing time allows for immediacy).

This approach allows the construction of initial distributions of wealth and interactive observation of the behaviour of the model. The following figures give an overview, how the model works. The VBA Code can be found in the appendix A.

Only one parameter – the number of goods - has to be specified (in this case 3). The initial distribution then needs to be put into a matrix (a range in terms of EXCEL). The only requirement is that the number of rows must be a multiple of the number of goods specified initially.

The colours result from a classification by fuzzy clustering into 2 clusters (rich/poor). The clustering is achieved by the fuzzy-c-means clustering algorithm. The fields with blue background colour indicate that their membership in cluster with rank 1 is above 60%. The fields with a red background have a cluster membership in cluster with rank 2 above 60%. Uncoloured fields do not fulfil either of the two conditions. The clusters get ranked according to Euclidean norm of their cluster centres. The parameters for cluster weight can be chosen (in this setting 1.5 was used).

[101] See Olaf Wolkenhauer, Data Engineering (p. 94 ff)

n goods		3					
stock		column 1	column 2	column 3	column 4	column 5	column 6
A	row 1	1	3	2	3	7	0
	row 2	2	2	0	6	0	4
	row 3	3	1	0	1	0	7
	row 4	0	2	4	0	0	1
	row 5	0	1	0	6	2	1
	row 6	2	0	0	5	4	3
B	row 1	25	24	26	26	26	0
	row 2	18	34	27	8	24	12
	row 3	4	25	1	26	27	15
	row 4	32	30	16	25	27	4
	row 5	27	17	30	26	23	8
	row 6	20	32	6	30	7	13
C	row 1	1	4	1	4	2	3
	row 2	1	4	4	4	3	3
	row 3	4	1	4	4	3	4
	row 4	3	4	4	3	2	3
	row 5	3	2	3	3	4	4
	row 6	4	2	4	3	4	4

Figure 5: Initial wealth with 3 Goods in a 6×6 *world, Spreadsheet*

The example shows an initial distribution where goods A and C are owned fairly low in number. Good B is owned in larger quantities. In contrast to good A and good B, good C is distributed fairly equal.

This initial distribution was generated by using random numbers from beta distributions, each good with a different parameter setting and truncating the random numbers to integer values. Details regarding the generation of initial distributions of wealth are part of chapter 5.3.

After 500 trade runs with an observation horizon 1 (agents can observe other agents that are no more than 1 row or column apart) and a show off probability of 100% - these are the 3 input parameters - the following result is calculated:

n iter		500					
o horizon		1					
p showoff		1					
stock		column 1	column 2	column 3	column 4	column 5	column 6
A'	row 1	2	2	4	4	0	4
	row 2	0	4	0	4	4	0
	row 3	4	0	6	7	1	0
	row 4	1	8	0	3	2	0
	row 5	1	2	3	0	0	2
	row 6	1	1	0	0	3	0
B'	row 1	24	3	9	18	31	31
	row 2	17	11	20	2	22	20
	row 3	21	23	21	37	37	26
	row 4	12	9	24	21	21	21
	row 5	26	24	22	29	29	9
	row 6	24	1	4	11	30	31
C'	row 1	7	5	2	2	1	3
	row 2	0	6	6	2	4	6
	row 3	4	0	2	1	10	1
	row 4	5	7	4	4	0	1
	row 5	0	0	0	1	5	6
	row 6	0	5	5	8	0	0

Figure 6: Stock after 500 trade runs, observation horizon 1, show off probability 100%

Note that the evenness of distribution of wealth of good C has been lost.

The flows show the following structure:

fstat		1					
flow		column 1	column 2	column 3	column 4	column 5	column 6
A'	row 1	99	69	104	109	93	76
	row 2	[redacted]	80	76	78	[redacted]	64
	row 3	47	59	[redacted]	52	[redacted]	57
	row 4	81	84	[redacted]	[redacted]	[redacted]	79
	row 5	65	77	97	[redacted]	66	51
	row 6	[redacted]	81	66	159	83	[redacted]
B'	row 1	1313	871	993	848	1245	925
	row 2	747	993	793	880	670	948
	row 3	843	852	684	909	660	923
	row 4	1150	1051	292	698	590	973
	row 5	993	899	934	677	864	913
	row 6	718	965	1094	1305	797	520
C'	row 1	196	151	117	106	185	164
	row 2	137	152	140	148	109	163
	row 3	168	139	120	157	163	183
	row 4	192	161	68	143	116	160
	row 5	167	146	157	104	149	146
	row 6	148	133	151	139	120	90

Figure 7: Flow in 500 trade runs, observation horizon 1, show off probability 100%

The figure is obtained, by adding the the absolute value of goods flows throughout the trade runs. The colours now refer to 3 clusters. Blue is again the cluster which ranks highest in Euclidean norm of the cluster centre, green ranks lowest. So blue agents (one cell is occupied by one agent) trade most (in quantities – green ones trade lowest.

Not surprisingly, the given case clusters show a cohesive pattern, as deals are only made with direct neighbours (because the observation horizon is set to one).

Let us compare the result with an increased observation horizon:

n iter		500					
o horizon		2					
p showoff		1					
stock		column 1	column 2	column 3	column 4	column 5	column 6
A'	row 1	1	1	1	4	2	1
	row 2	1	4	0	1	2	3
	row 3	1	3	3	1	1	8
	row 4	0	4	2	2	1	0
	row 5	0	0	0	8	1	0
	row 6	1	1	1	6	3	5
B'	row 1	20	20	20	19	20	19
	row 2	20	20	20	21	21	21
	row 3	20	19	20	19	20	20
	row 4	20	19	20	21	21	20
	row 5	18	19	21	20	20	20
	row 6	21	21	21	20	19	21
C'	row 1	3	2	5	1	8	0
	row 2	3	8	5	2	3	4
	row 3	3	1	1	1	1	7
	row 4	1	4	2	5	7	4
	row 5	0	0	4	8	2	1
	row 6	1	1	2	7	2	4

Figure 8: Stock after 500 (137) trade runs, observation horizon 2, show off probability 100%

As an immediate consequence the system reaches equilibrium (a fixed point) after 137 trade runs. Equilibrium is defined as a situation in which no more deals occur.

From the results presented one can see, that equilibrium is primarily characterised by an even distribution in good B. Other goods are not distributed as evenly as good B. This is due to fact that individual showoffs only occur with the good that an agent owns most in number.

This leads to the question how many iterations are needed to reach equilibrium with observation horizon 1. The result is 159.219 trade runs, which is a big difference.

The clustering of the flows does no longer show the same pattern:

fstat		1					
flow		column 1	column 2	column 3	column 4	column 5	column 6
A'	row 1	22	12	13	11		9
	row 2	13					
	row 3	10	18	17	20	11	11
	row 4	8	10	10			1
	row 5			4	16	11	11
	row 6	11	7	7		17	14
B'	row 1	139	110	130	95	46	63
	row 2	94	30	47	41	27	49
	row 3	62	120	75	125	61	57
	row 4	102	85	116	56	30	70
	row 5	43	30	67	62	109	104
	row 6	85	69	81	36	122	84
C'	row 1	18	8	26	17	8	3
	row 2	18	12	19	8	6	3
	row 3	9	36	23	11	6	7
	row 4	16	8	16	10	17	13
	row 5	7	8	17	9	14	7
	row 6	11	7	8	6	12	10

Figure 9: Flow in 500 (137) trade runs, observation horizon 2, show off probability 100%

It is worth mentioning that a further increase of the observation horizon to 3 does not give a different result to observation horizon 2. In a 6×6 world observing other agents that are no more than 2 rows or columns apart (in each direction) is a situation that nearly amounts to "every agent observes every other agent". This may also explain the difference as to the number of iterations needed to reach an equilibrium state.

I would like to add the following comments regarding usage and limits of the model

- Given the rules of the model, an agent that owns no more than 0 (or 1) of every tradeable good will never participate in any deal. This allows for modelling of sparse populations.
- Neither exclusion nor inclusion can happen as a result of the dynamics of the system. Agents can become wealthier or poorer but they never drop out of business. There are two ideas to model exclusion/ inclusion: credit and goods that depreciate or are produced/consumed or get lost/endowed due to events in the environment

5.2 Characteristics of Solution Paths

In order to better observe individual solution paths the following statistics are collected:

- <u>Distance to mean</u>: As the total number of every good stays the same during all iterations it is straight forward to add up the absolute values of the differences of ownership of every agent with the mean. To make this measure comparable between goods, the total is divided by the number of agents.

Approaching the fixed point observed before this statistics must decrease in value. To be more exact equilibrium is better described as a region of fixed points, where distribution of wealth is almost even and no further proposals are made.

- Activities: The number of proposals following show off, the number of proposals following price differences and the number of deals is recorded and shown.
- Weighted average prices: In terms of prices (understood as exchange rates) the weighted average prices are calculated for every trade run and recorded. If no deals (exchanging good A and B) occur prices are set to zero.

Although these statistics amount to an outside observation of the larger environment it is nevertheless interesting for studying the behaviour of the system.

The following chapters show the statistics of the examples shown above.

5.2.1 Distance to Mean

The development of the (normed) distance to mean is shown in the following figures

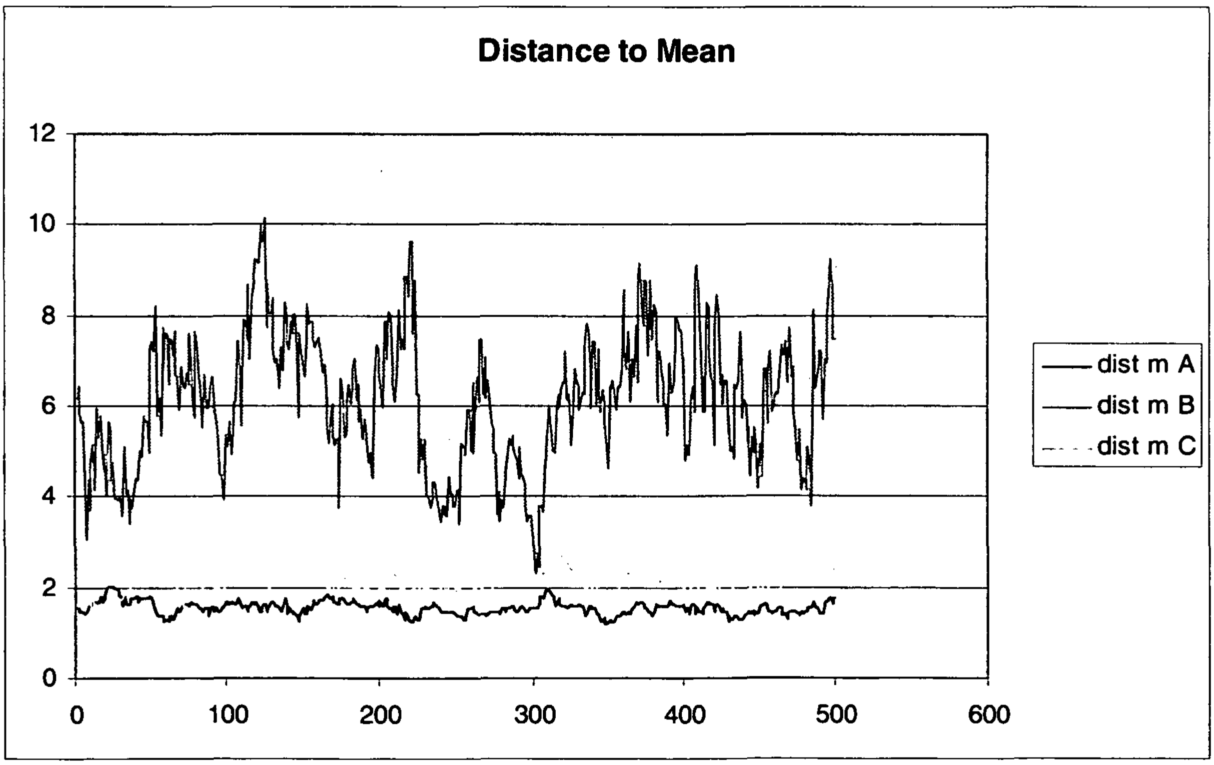

Figure 10: distance to mean (even distribution), observation horizon 1; only the first 500 trade runs are shown

The next figure will show the development of the distance to mean given the same initial distribution but an observation horizon 2. As indicated above, the system reaches equilibrium after 137 trade runs.

In both cases the distance to mean regarding good C is increased during the first trade runs, where as the distance to mean of good B lowers. At a certain point this development stops and the system shows a fairly irregular pattern. There are instances when the distance to mean is even bigger than the initial distribution for all goods.

This irregular pattern finally breaks down and the distance to mean of good B (the good that is most abundant) falls below 1, the system reaches equilibrium.

The difference between observation horizon 1 and 2 is, that the irregular phase lasts about 150.000 trade runs given observation horizon 1, and merely 80 trade runs given observation horizon 2.

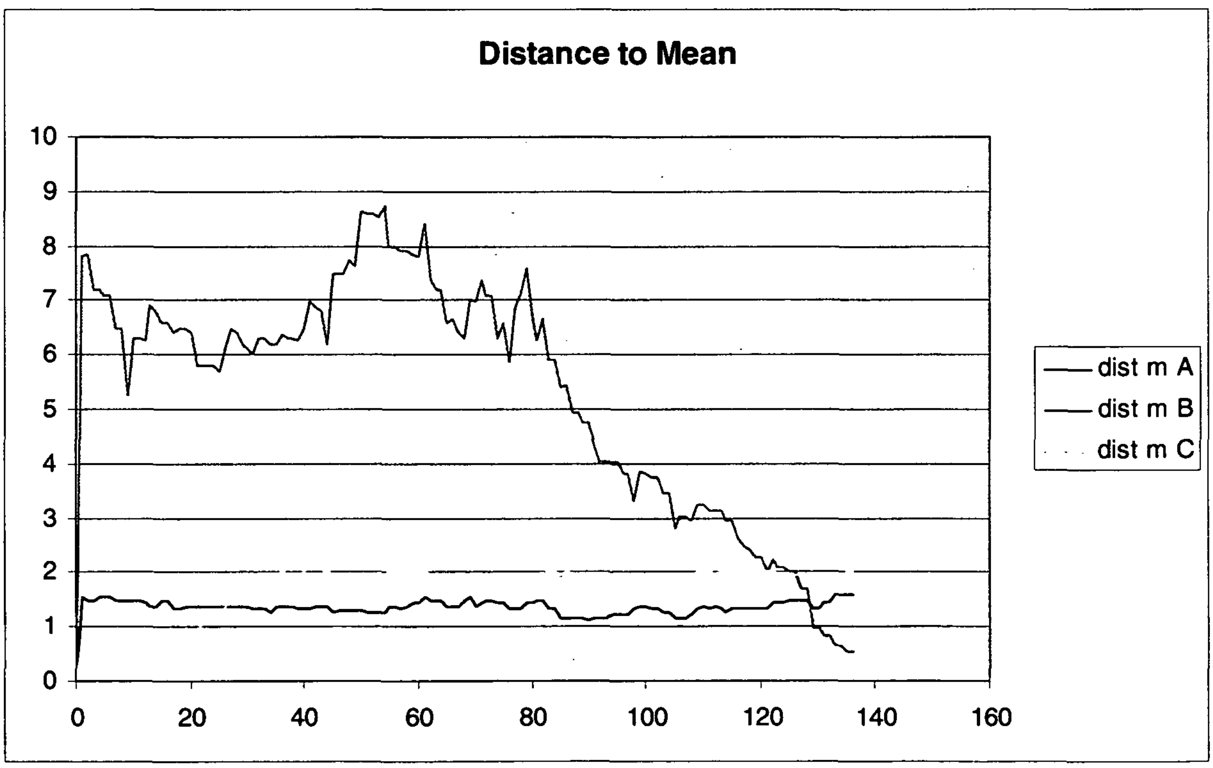

Figure 11: distance to even distribution, observation horizon 2

5.2.2 Activitites

The next statistics collected are the number of activities. As expected the increased observation horizon leads to more proposals but a smaller number of accepted deals, because only one best deal among all proposals (per pair of exchangable goods) is chosen.

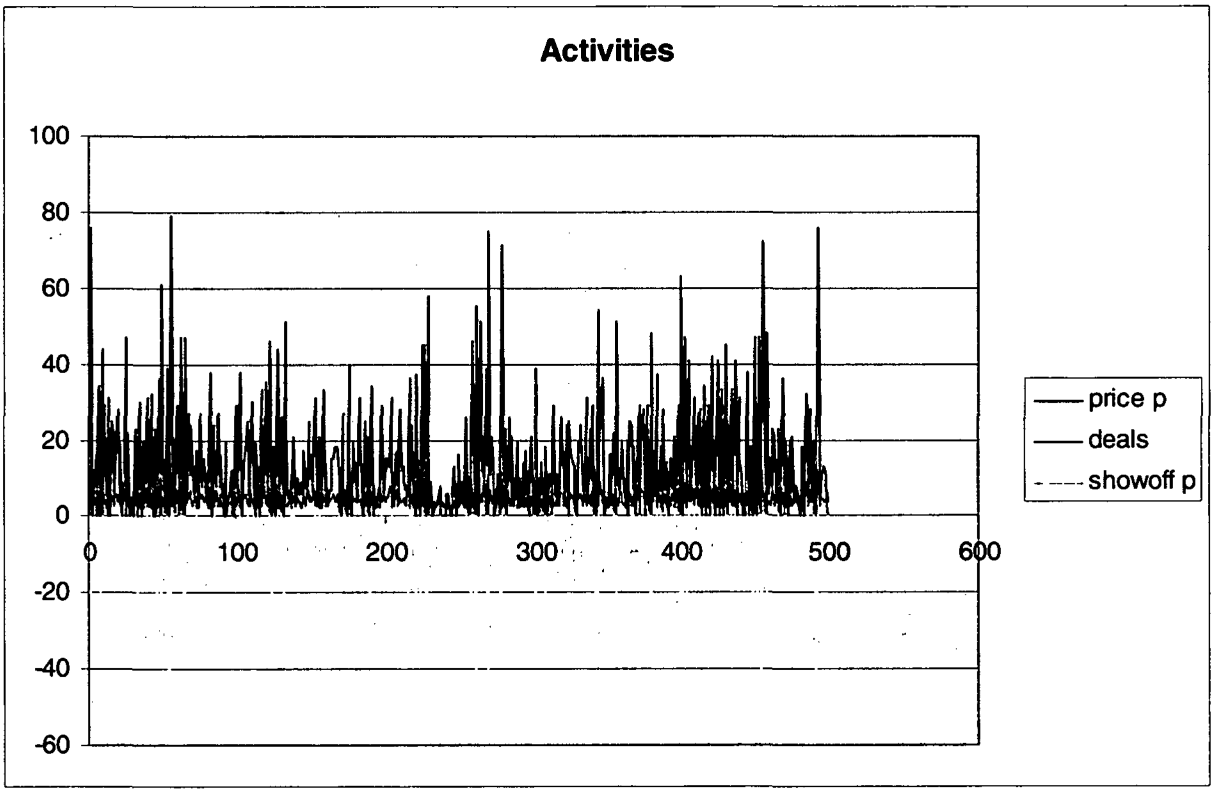

Figure 12: activities statistics, observation horizon 1; first 500 trade runs

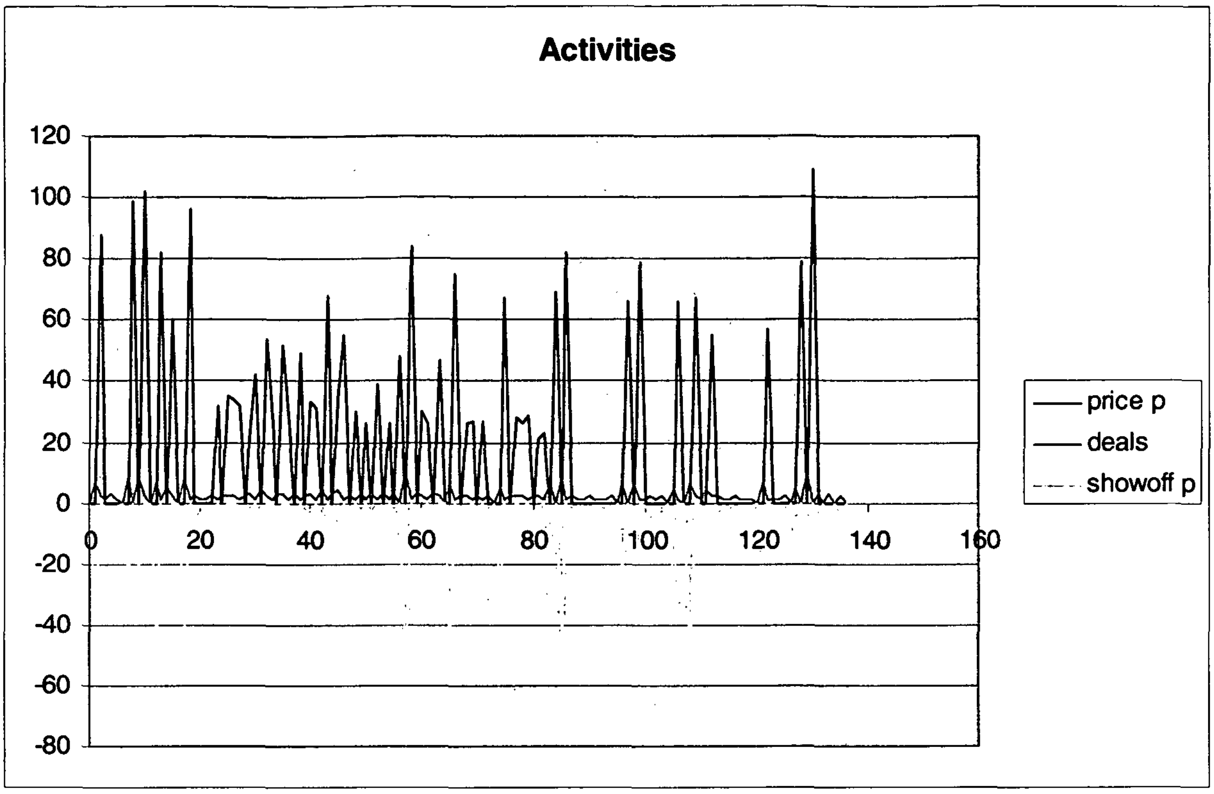

Figure 13: activities statistics, observation horizon 2

5.2.3 Prices

The last statistics collected are (weigthed) average prices. They show a similar pattern in both cases:

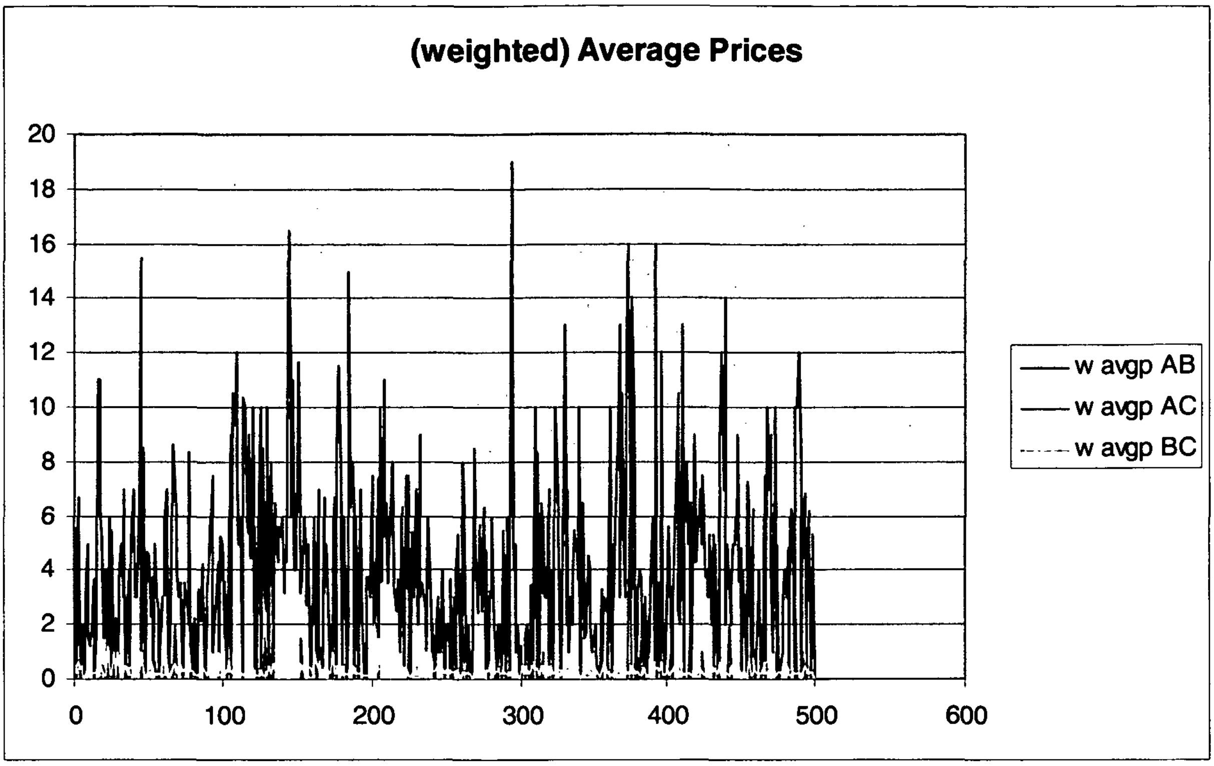

Figure 14: price statistics, observation horizon 1; first 500 trade runs

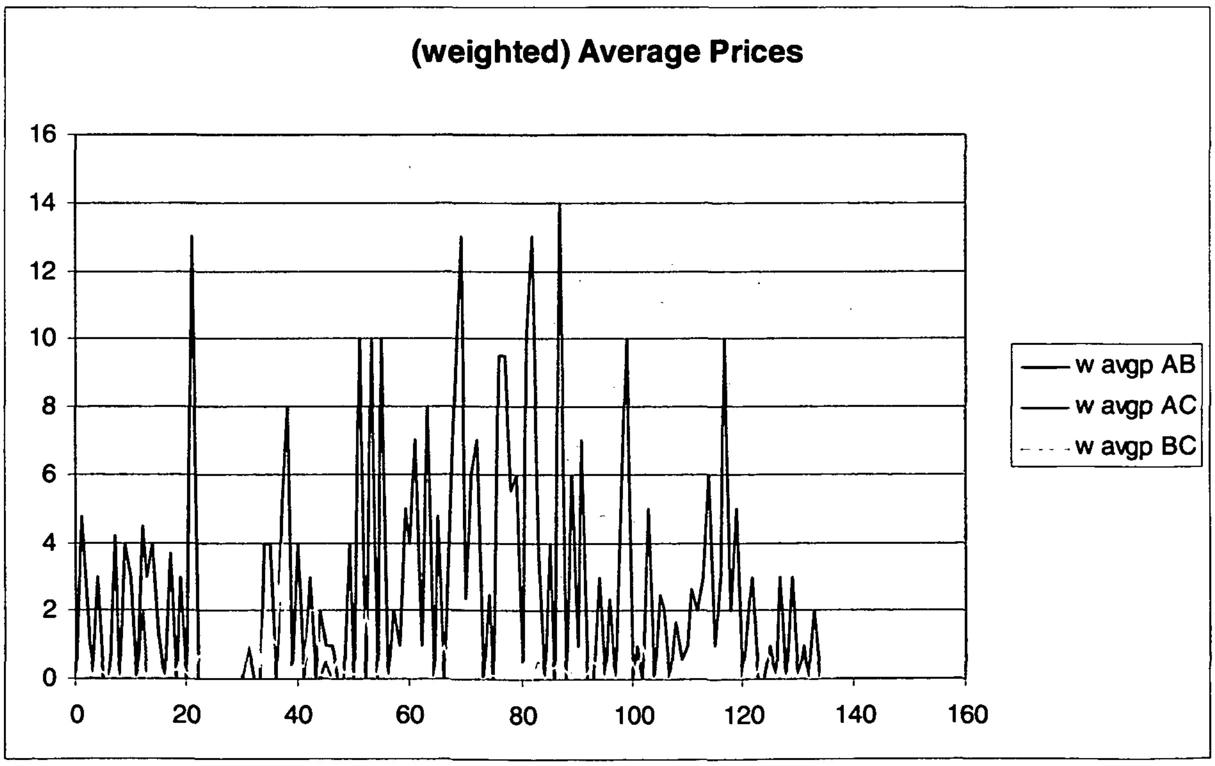

Figure 15: price statistics, observation horizon 2

5.3 Characteristics of Classes of Solution Paths

In order to analyse a wider range of solution paths scenarios were defined for simulation.

All of the scenarios (except scenario 1a) refer to a 6×6 world, 3 goods, an observation horizon 1 and a show off probability 1 (deterministic). Scenario 1a analyses the impact of different show off probabilities. As these sizes can be given as parameters to the software developed examples have been calculated also in a 4×4 and 12×12 world[102].

In the 4×4 world an observation horizon 1 is nearly equivalent to the situation of every agent observing every other agent. So this world was considered too small. In a 12×12 world the limits of a default personal computer regarding an acceptable performance for model exploration were reached.

The 6×6 world shows rich behaviour (cycles of different length, quick and late convergence) that a detailed analysis seems reasonable.

The following scenarios have been defined

- Scenario 1: All three goods are distributed according to a uniform distribution in the interval $[0,7]$[103] truncated to integer values.
- Scenario 1a: The showoff probability is varied.
- Scenario 2: All three goods are (still) distributed according to a uniform distribution – but the intervals vary. This is to reflect that goods vary in their abundance (scarcity). The intervals used are $[0,7]$, $[0,11]$ and $[0,5]$.
- Scenario 3: The intervals stay the same, but the distributions are changed. The distribution of the first good stays unchanged. For the second and third good Beta distributions are used[104] to compute the initial distribution of wealth. For good B the following parameters are used: $a=2$, $b=1$. For good C the values of the parameters are reversed.

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[102] A world with 12×12 agents is closer to the size of an early homo sapiens group. See Robin E. Dunbar, Grooming, Gossip and the Evolution in Human Language (p. 63)

[103] The probability density function of a uniform (or rectangular) distribution is given by $f(x)=\frac{1}{b-a}I_{[a,b]}(x)$, $-\infty<a<b<\infty$, See Mood A., Graybill F., Boes D., Introduction to the Theory of statistics (p. 540)

[104] The probability density function of a ($[0,1]$ standardised) Beta distribution is given by $f(x)=\frac{1}{\beta(a,b)}x^{a-1}(1-x)^{b-1}I_{[0,1]}(x)$, $a>0$, $b>0$, See Mood A., Graybill F., Boes D., Introduction to the Theory of statistics (p. 540)

- Scenario 4: intervals and distributions are the same as scenario 3. The parameters are varied to $a = 3$, $b = 1$ for good B, good C holds again the parameters of good B in reverse order.

5.3.1 Scenario 1

Each scenario is based on the selection of 100 initial distributions of wealth (goods A, B, C) according to the rules given above. For each of the initial distributions a maximum of 2000 trade runs is computed. With the exception of cycles within episodes that were not detected automatically all systems reach equilibrium within 2000 trade runs.

The following table gives information that was aggregated from the 100 cases of this scenario. The details can be found in appendix B.

In order to check for the robustness of the scenarios, the scenarios were repeated (with further 100 cases) and their results compared. There were no differences in qualitative terms (types of cycles, number of trade runs). In quantitative terms the differences of the aggregated results were considered small enough.

	last run no deals	number no deals	episode length	total A+B+C
mean	36,6	6,6	7,2	326,2
median	34,5	6,0	5,6	325,0
std deviation	11,9	3,7	5,3	21,8
std deviation in %	32%	56%	73%	7%

Table 1: aggregated results scenario 1

The table contains the following columns:

- Last run non deals: This refers to the number of the trade run, when no more deals are made in 2 consecutive steps and equilibrium is reached. The system dynamics (type 2nd order difference equation) demands that two steps need to be checked. In this case of a fairly even distribution of all goods the median number of trade runs needed to reach equilibrium is 34.5. The comparatively low standard deviation (in % of the mean) indicates that the behaviour is fairly homogenous.
- Number no deals: The rules of the model imply situations where no deal gets accepted. This is primarily due to repeated proposals following the arbitrage rule being not accepted given the no short selling condition. At these temporary rest points the system settles down – for one

moment – and the activity rule (proposals following show off) generates further business for the next rounds. The number of these temporary rest points is given. They mark the beginning and end of so called episodes.

- Episode length: The (average) length of episodes is given by the number of trade runs (i.e. the last trade run no deals) divided by the number of temporary rest points (number no deals).
- Total A+B+C: The total wealth was recorded because the possible number of states of the system (restricted to temporary restpoints) is given by $6 \cdot 6^{A+B+C-1}$, where A, B, C denote the total number of goods A,B,C in the system. The -1 in the exponent is due to the closedness of the torus. The total serves as a control variable, because a longer journey to equilibrium could also be due to the fact, that there are more states.

Although the quantitative results are not very surprising, this scenario shows rich qualitative behaviour that does not emerge in the other scenarios:

- Cycles: Nearly half of the cases obtained end with a cycle. The detected cycle lengths are 1, 2 and 5. As these lengths are calculated as distances between the last saved state that equals the actual state (considering symmetry), they refer to period lengths of 2, 3, and 6. A period 1 cycle is a fixed point.
- Cycle within episodes: As the exploration part of the software checks for cycles only at the temporary rest points (because they can be tested more easily) cycles that appear within an episode are not detected automatically. However 3 of these cases appeared.

All of the observed cycles do appear in states that are very close to the equilibrium.

If the mapping would be continuous, the observed period length 3 would point to chaotic behaviour by Sarkovskii's theorem[105]. As the system only has a finite number of states it cannot be chaotic in terms of the definition[106].

[105] See Devaney, Robert L., An Introduction to Chaotic Dynamical Systems (p. 60)

[106] See William F. Lawvere, Stephen H. Schanuel, Conceptual Mathematics (p. 317) *An Observable* $X \xrightarrow{f} Y$ *on a dynamical system* $X^{\supset\alpha}$ *is said to be chaotic if the induced* $S^{\supset}$ *-map* $X^{\supset\alpha} \xrightarrow{\overline{f}} (Y^{N})^{\supset\beta}$ *is 'onto for states', i.e. if for every possible sequence* $N \xrightarrow{y} Y$ *of future observations there is at least one state* x *of* X *for which* $\overline{f}(x) = y$.

5.3.2 Scenario 1a

Scenario 1a shows the influence of the show off probability on the characteristics of solutions to the model. The results are given in the following figures:

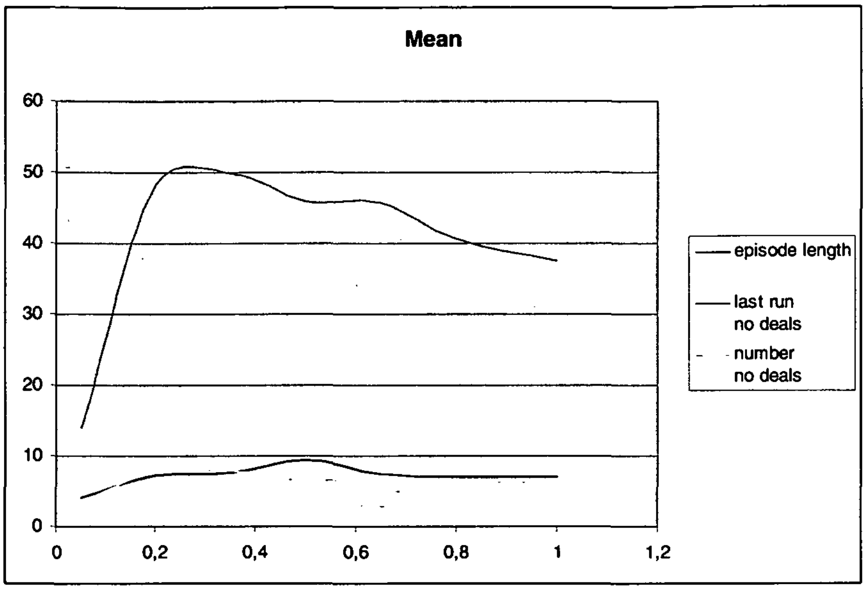

Figure 16: Means of characteristics varying with showoff probability

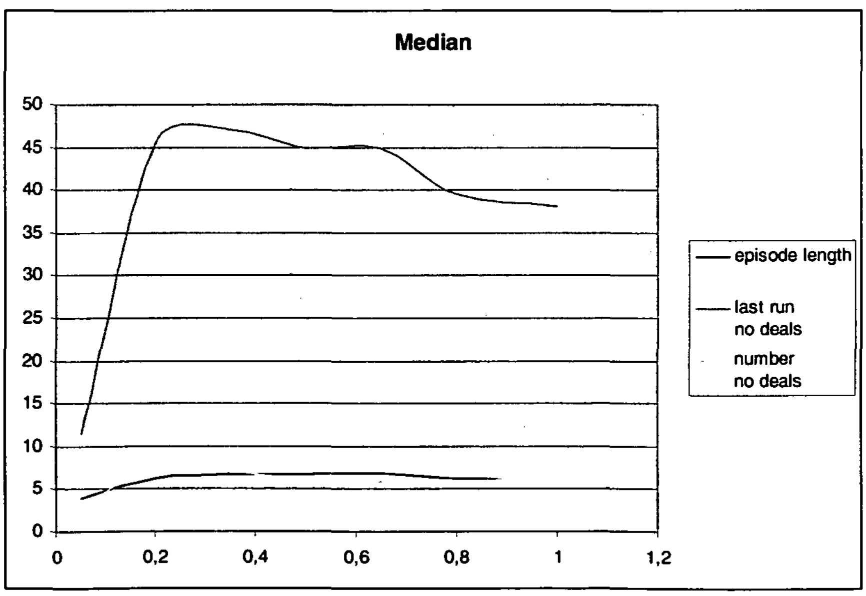

Figure 17: Median of characteristics varying with showoff probability

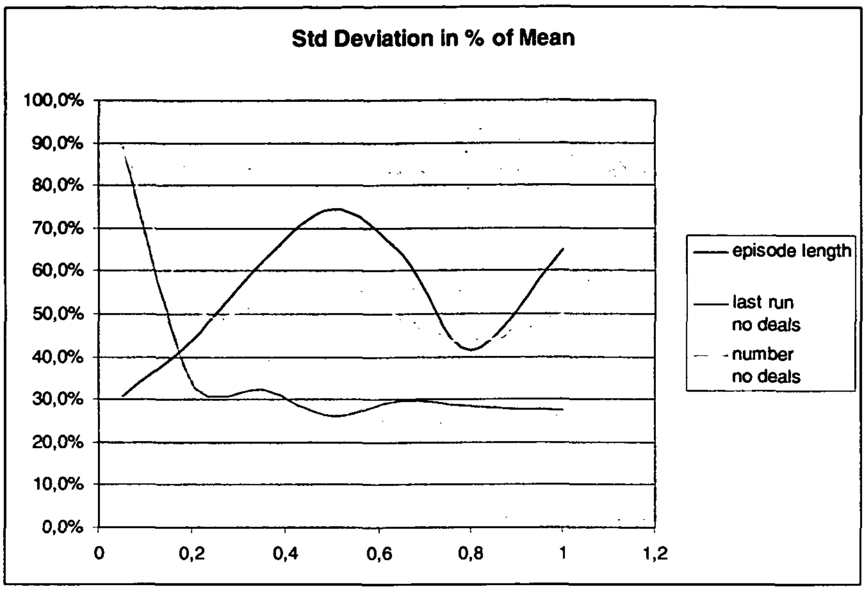

Figure 18: Relative Standard deviation of characteristics varying with showoff probability

The simulations indicate that episode length and the number of temporary rest points are fairly invariant to changes of the show off probability when the mean or median of the cases is considered. This is reasonable as episodes largely depend on the continuation of deals by applying the arbitrage rule. However the relative deviation shows an interesting pattern.

Regarding the number of trade runs required to reach equilibrium (last run no deals), a peak is achieved at a show off probability of 20% that is slowly decreasing to the value when show off is deterministic.

This can be interpreted as follows:

> The more agents are inclined to show off (as long as the show off probability is above 20%), the quicker the economy reaches equilibrium.

5.3.3 Scenario 2

Scenario 2 is very similar to scenario one, except that the intervals in which the goods A, B, C are distributed are no longer the same. Good B is must abundant. Good C is scarcest.

	last run no deals	number no deals	episode length	total A+B+C
mean	74,6	7,8	9,8	361,4
median	54,5	7,0	8,1	364,0
std deviation	81,6	4,8	5,3	25,3
std deviation in %	109%	62%	55%	7%

Table 2: aggregated results scenario 2

Compared to scenario 1 the mean number of trade runs required to reach equilibrium (last run no deals) more than doubles, the median moves from 34.5 to 54.5. The relative standard deviation at least doubles. The number and the length of episodes are increased only slightly (in all measures).

Cycling occurs far less, i.e. in about 14% compared to nearly half of the cases in scenario 1. A cycle of period length 4 is observed.

5.3.4 Scenario 3

In Scenario 3 the unevenness of distribution is further increased.

The initial wealth regarding good B (still most abundant) is now retrieved from a beta distribution with parameter values: $a = 2$, $b = 1$. The cumulative distribution function can be seen in the figure below. This parameter setting moves the mass of the distribution slightly to the right, i.e. more agents own a higher amount of good B.

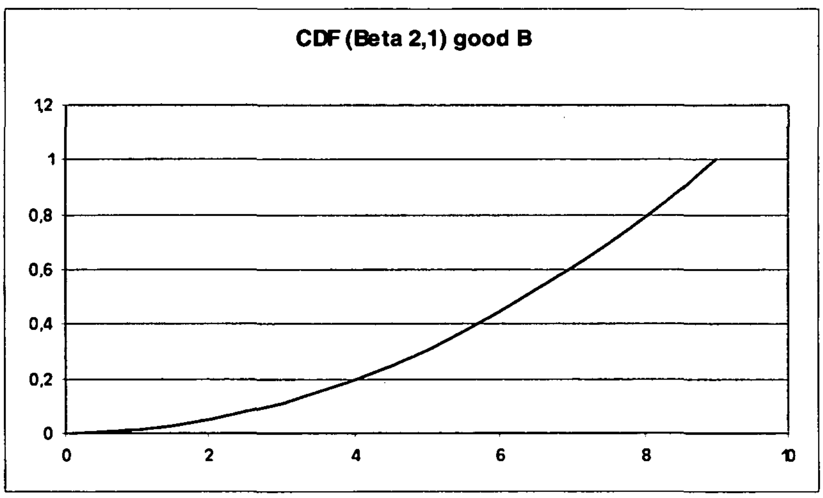

Figure 19: Cumulative distribution function of $\beta(2,1)$

For the scarcest good C, the situation is intensified by selecting parameter values: $a = 1$, $b = 2$ and moving the mass of the distribution slightly to the left. The cumulative distribution function is shown below:

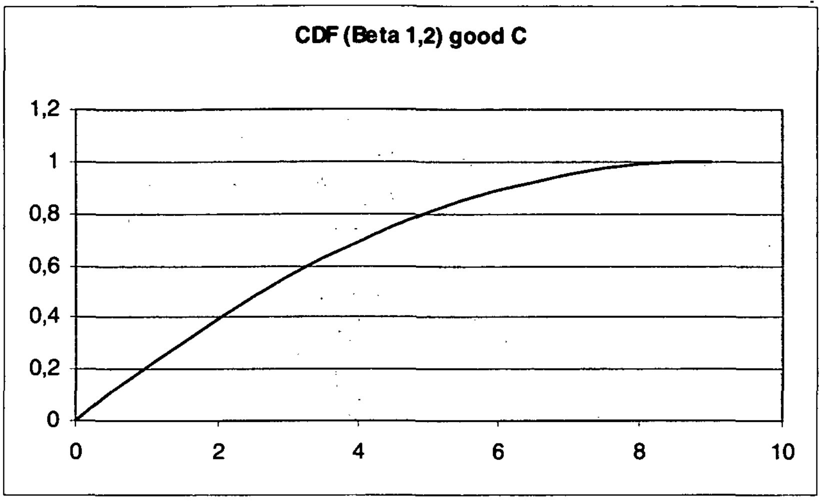

Figure 20: Cumulative distribution function of $\beta(1,2)$

The results are summarised in the following table:

	last run no deals	number no deals	episode length	total A+B+C
mean	150,8	11,4	12,0	396,6
median	96,5	9,0	11,0	396,5
std deviation	189,7	8,7	6,5	21,9
std deviation in %	126%	76%	54%	6%

Table 3: aggregated results scenario 3

It is interesting that the step from scenario 1 to 2 shows a similar pattern as the step from scenation2 to scenario 3. Mean and Median of the required number of trade runs to reach equilibrium nearly double again. The number of deals and episode length increase only slightly.

Cycling (with period length 2) occurs only once.

5.3.5 Scenario 4

The final scenario 4 is characterised by further intensifying abundance of good B and scarcity of good C. The unevenness of the total distribution of wealth is also increased.

The next two figures show the cumulative distribution functions from which the initial endowments regarding good B and good C were calculated.

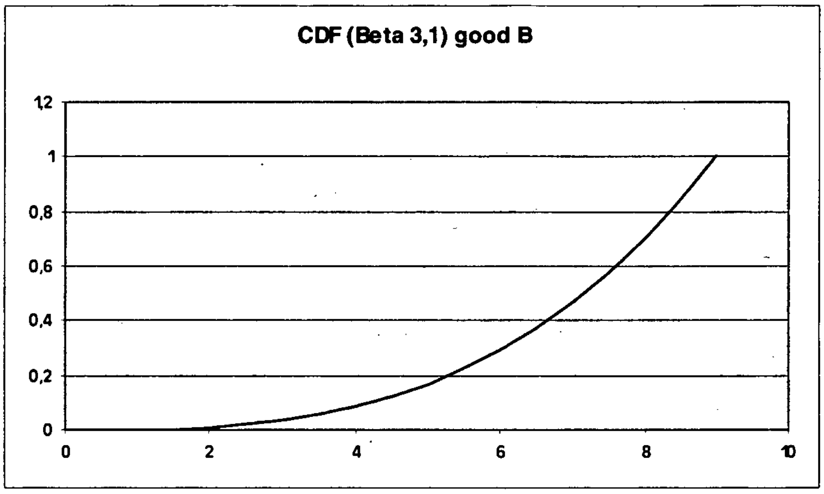

Figure 21: Cumulative distribution function of $\beta(3,1)$

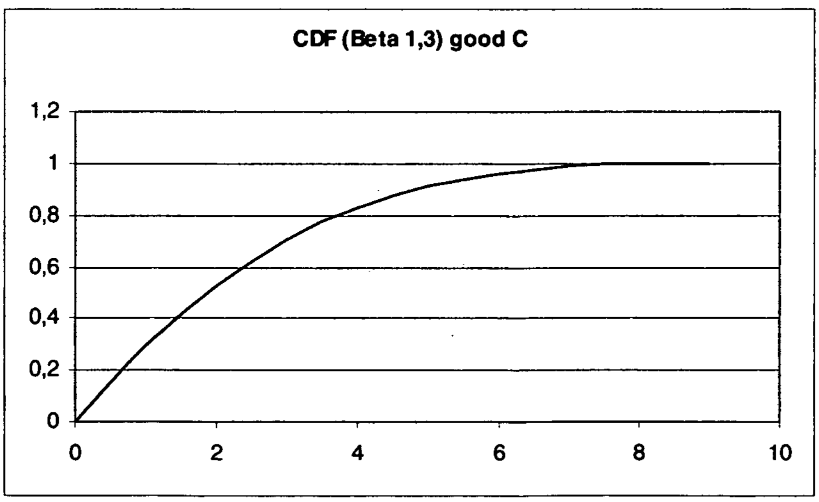

Figure 22: Cumulative distribution function of $\beta(1,3)$

The following results were obtained:

	last run no deals	number no deals	episode length	total A+B+C
mean	291,4	16,0	14,8	414,4
median	180,0	13,0	14,1	414,0
std deviation	331,6	12,4	9,1	17,5
std deviation in %	114%	78%	62%	4%

Table 4: aggregated results scenario 4

The pattern that was observed by moving from one scenario to the next is affirmed. Mean and median of the required trade runs to reach equilibrium nearly double again. The number of episodes and their length increases only slightly.

No more cycles are observed.

Combining the results of the scenarios with the solution that served as an example when the interactive model was shown leads to the following assumptions about the behaviour of the system:

- The number of trade runs needed by the system to reach equilibrium is significantly enhanced by the extent of unevenness of initial wealth. It reacts to it in a sensitive way.
- A comparatively long path to equilibrium can be observed, when the following conditions are met:
    - uneven initial distribution of wealth
    - sufficient large numbers of some, but not all goods (abundance/scarcity)
    - an observation horizon that does not imply every agent sees every other agent – this is equivalent to a requirement of a world sufficiently large
    - a showoff probability above 20%
- Given these conditions the system maintains a level of unevenness (subpaths seem to move away from equilibrium) and shows fairly irregular patterns for a considerable amount of trade runs (time) before it suddenly breaks down and reaches equilibrium.

  This timeframe may be long enough (thinking about settings with 100.000 agents and 10 goods; our example with only 36 agents, 3 goods altogether 900 in number needed more than 150.000 trade runs) to justify Niklas Luhmann's observation that the economy produces unevenness from unevenness.
- The observation indicates that the length of episodes is less sensitive to the parameters that guide unevenness. They seem to be more reactive to general settings like the number of agents and the size of the world.

- Although cycles (with various period lengths) have occurred in the simulations only close to equilibrium there are no indications that similar cycles (with presumably longer period lengths) could also occur at levels farther away from equilibrium.
- The emergence of systems within systems was not studied in this simple model.
  However the model describes how activities are caused by prior observations of activities. Our model maintains a certain distance from equilibrium by its own operations - not forever but for a considerable amount of time. In an evolutionary setting these are at least favourable conditions.

6 Conclusion

At the beginning of this work there has been the question, to what degree Niklas Luhmann's proposition, that the economy starts from and produces further inequality in order to continue could be verified.

Starting with Niklas Luhmann's definition about communications which emphasises understanding rather than exchange of information and referring to his theorems about closed societal function systems an agent based model for a simple Luhmann economy has been developed, specified and implemented.

The exploration of the model has shown, that under suitable conditions the proposition stated above can indeed be reproduced by computer simulation.

The steps undertaken to formalise some of Niklas Luhmann's theorems in the context of dynamic systems give ideas, how to understand Niklas Luhmann's central theorem of Co-evolution of social systems and psychic systems.

The second question is about types or patterns of internal differentiation. In terms of modelling internal differentiation is understood as fuzzy clustering of internal events. Fuzzy clustering – as part of the interactive model – has been used as an aid to identify certain types of system behaviour (primarily cycles) and has proved to be an efficient visualisation technique.

Nevertheless has the development and specification of an agent based model for a simple Luhmann economy given rise to further questions. But it also points to directions for future enhancements and gives ideas how these might be answered.

In terms of a formalisation or mathematisation of Niklas Luhmann's thinking this work can only be a beginning. I hope that it will enrich the work of others that have tried or will try to approach similar questions from other points of view.

7 Reference Matter

The following table gives authors and titles that were directly referenced:

Author(s)	Title	Publisher	Year
Axelrod, Robert M.	The Evolution of Cooperation	Harper	1984
Axelrod, Robert M.	The Complexity of Cooperation	Princeton	1997
Baraldi, Claudio, Corsi Giancarlo, Esposito, Elena	GLU, Glossar zu Niklas Luhmanns Theorie sozialer Systeme	Suhrkamp	1999
Devaney, Robert L.	An Introduction to Chaotic Dynamical Systems, 2nd Edition	Westview Press	2003
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Glasersfeld, Ernst von	Radikaler Konstruktivismus, Ideen, Ergebnisse, Probleme	Suhrkamp	1996, 1997
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Spencer-Brown, George	Laws of Form	Cognizer Co.	1969, 1994
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I wish to add the list of following authors and titles that guided the development of my ideas regarding the topics of this work but were not directly referenced:

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Weinberg, Gerald M.	An Introduction to General Systems Thinking, Silver Anniversary Edition	Dorset House	1975, 2001
Willke, Helmut	Systemtheorie I: Grundlagen, 6. Auflage	Lucius & Lucius	2000
Блауберг, И.В.	Проблема Целостности и системный Подход	Эдиториал УРСС	1997

8 Appendix A – VBA Code

The following pages contain the source code that was used to simulate the model as outlined above.

I have categorised the functions into the following subchapters:

- Functions in the section "Simple Luhmann Economy Model" are the core of the model. I have to apologise that some error messages are in German. The meaning should be obvious by the if-then-else clauses. *I tried to keep comments (although few) in English.*
- Functions in the section "Fuzzy clustering" refer to the implementation of the fuzzy-c-means clustering algorithm. Sepcial care regarding underflow (as a likely event) had to be taken in the modules.
- Functions of "Exploration" refer to functions that were used in the simulation and exploration part (e.g. the identification of cycles).
- Auxiliary functions like seeking minima, or taking care of the torus are contained in the last section.

Some debugging messages have not been erased but commented out because I regarded them as helpful in understanding the code.

Anyone who is interested to receive a spreadsheet with code and exemplary data included please contact me per email at anselm@fleischmann.at.

8.1 Simple Luhmann Economy Model

8.1.1 Displays of Wealth (Show Off)

```
Sub showoff(n, m, g, pshowoff, ab, xy)

Dim i, j, k, l, maxa As Integer

For i = 1 To n
    For j = 1 To m
        If Rnd() < pshowoff Then
            maxa = 0
            l = 1
            For k = 1 To g
                If ab(i, j, k) > maxa Then
                    maxa = ab(i, j, k)
                    l = k
                End If
            Next k
            xy(i, j, l) = maxa
        End If
    Next j
Next i

End Sub
```

8.1.2 Making Proposals

Making proposals includes the following functions:

- The function `propose` is the main loop throufh all agents
- The function `proposeij` takes care about the actions of an individual agent identified by row and column index i and j
- The functions `mmprice`, `pricerule` and `showoffrule` contain the respective rules to find price minima and maxima, create proposals following the price rule and for creating proposals according to the showoff rule
- The function `convertproposals` and `convertijproposals` are used to ease implementation and exchange (absolute and relative) addresses of proposer and proposee.

```
Sub propose(step, n, m, g, ni, mj, ab, ohorizon, o, xy, p, op, sp)

Dim i, j, k As Integer
ReDim own(1 To g) As Long

op = 0
sp = 0

For i = 1 To n
    For j = 1 To m
        For k = 1 To g
            own(k) = ab(i, j, k)
        Next k

        Call proposeij(step, n, m, g, ni, mj, i, j, own, ohorizon, o, xy, p, op, sp)
    Next j
Next i

End Sub

Sub proposeij(step, n, m, g, oi, oj, i, j, own, ohorizon, o, xy, p, op, sp)

'n,m rows,columns
'g goods
'i,j proposing entity
'own portfolio owned
'o observations of prior deals
'xy showoffs
'p result

Dim k, l, omade As Integer

Dim smax, bmin As Double
Dim smaxii, smaxjj As Integer
Dim bminii, bminjj As Integer


If ohorizon <= 0 Then GoTo pre_bye

omade = 0
For k = 1 To g
    For l = k + 1 To g

        Call mmprice(n, m, g, oi, oj, i, j, k, l, o, bmin, bminii, bminjj, smax, smaxii,
smaxjj, omade)
        'prices found for goods combination k,l

        Call pricerule(step, n, m, g, i, j, k, l, own, bmin, bminii, bminjj, smax, smaxii,
smaxjj, p, op, sp, omade)
        'proposals made
```

```
    Next l
Next k

pre_bye:

If omade = 0 Then
'    MsgBox ("proposeij> no observations made")
    Call showoffrule(n, m, g, oi, oj, i, j, own, xy, p, sp)
End If

bye:
End Sub


Sub mmprice(n, m, g, oi, oj, i, j, k, l, o, bmin, bminii, bminjj, smax, smaxii, smaxjj, omade)

Dim ii, jj As Integer
Dim oA, oB As Long
smax = 0
bmin = 9E+15

For ii = -oi To oi
    For jj = -oj To oj
'        exclude self, law of the first distinction
        If ii = 0 And jj = 0 Then GoTo jjloop

        For r = 1 To 2
'            r = 1 role accepter, 2 role proposer
'            s = 1, a sale of (k) - i.e. a chance for buying - was observed

            oA = o(i, j, ii, jj, r, 1, k)
            oB = o(i, j, ii, jj, r, 1, l)
            If oA = 0 Or oB = 0 Then GoTo rcontinue

            omade = omade + 1
            If Abs(oB / oA) < bmin Then
                bmin = Abs(oB / oA)
                bminii = ii
                bminjj = jj
            End If
rcontinue:
'            s = 2, a purchase of (k) - i.e. a chance for selling - was observed
            oA = o(i, j, ii, jj, r, 2, k)
            oB = o(i, j, ii, jj, r, 2, l)
            If oA = 0 Or oB = 0 Then GoTo rloop
            omade = omade + 1

            If Abs(oB / oA) > smax Then
                smax = Abs(oB / oA)
                smaxii = ii
                smaxjj = jj
            End If

rloop:
        Next r

jjloop:
    Next jj
Next ii

' smax and bmin identified
End Sub
```

```
Sub pricerule(step, n, m, g, i, j, k, l, own, bmin, bminii, bminjj, smax, smaxii, smaxjj, p,
op, sp, omade)

Dim buyA, sellA, B2buyA, B2sellA As Long


If smax > bmin And omade <> 0 Then

    B2buyA = own(l)
    buyA = Int(B2buyA / bmin)
    While B2buyA <> bmin * buyA And B2buyA > 0
        B2buyA = B2buyA - 1
        buyA = Int(B2buyA / bmin)
    Wend

'propose to buy A
    If B2buyA > 0 Then
        p(i, j, bminii, bminjj, k) = buyA
        p(i, j, bminii, bminjj, l) = -B2buyA
        op = op + 1
        own(l) = own(l) - B2buyA
    End If

    sellA = own(k)
    B2sellA = Int(smax * sellA)

    While B2sellA <> smax * sellA And sellA > 0
        sellA = sellA - 1
        B2sellA = Int(smax * sellA)
    Wend

'propose to sell A
    If sellA > 0 Then
        p(i, j, smaxii, smaxjj, k) = -sellA
        p(i, j, smaxii, smaxjj, l) = B2sellA
        op = op + 1
        own(k) = own(k) - sellA
    End If

End If
End Sub
```

```
Sub showoffrule(n, m, g, oi, oj, i, j, own, xy, p, sp)

Dim ii, jj, k, l As Integer
Dim a, pA, pB As Long

ReDim maxg(1 To g) As Long
ReDim maxwhor(1 To g) As Integer
ReDim maxwhoc(1 To g) As Integer

Dim maxM As Long
Dim maxMg As Integer

'ReDim minG(1 To g) As Long
'ReDim minwhor(1 To g) As Integer
'ReDim minwhoc(1 To g) As Integer

For k = 1 To g
'    minG(k) = 1000000000#
    For ii = -oi To oi
        For jj = -oj To oj
            If ii = 0 And jj = 0 Then GoTo jjloop

            a = xy(torus(i + ii, n), torus(j + jj, m), k)
            If a > maxg(k) Then
                maxg(k) = a
                maxwhor(k) = ii
                maxwhoc(k) = jj
            End If

'            If a <> 0 And a < minG(k) Then
'                minG(k) = a
'                minwhor(k) = ii
'                minwhoc(k) = jj
'            End If
jjloop:
        Next jj
    Next ii
Next k

For k = 1 To g
    If maxg(k) - own(k) > 2 Then

        maxM = 0
        For l = 1 To g
            If own(l) > maxM And l <> k Then
                maxM = own(l)
                maxMg = l
            End If
        Next l

        If maxM > 0 Then

            pA = Int((maxg(k) - own(k)) / 2)
            p(i, j, maxwhor(k), maxwhoc(k), k) = pA

'            pB = Int((minG(l) - own(l)) / 2)
            pB = -1
' greedy ... no more than 1 or other
            p(i, j, maxwhor(k), maxwhoc(k), maxMg) = pB

' do adapt ownership of maxMg not k
            own(maxMg) = own(maxMg) + pB

            sp = sp + 1
        End If
    End If
kloop:
Next k

End Sub
```

```
Sub convertproposals(n, m, g, ni, mj, z, d)

Dim i, j As Integer

For i = 1 To n
    For j = 1 To m
        Call convertijproposals(n, m, g, ni, mj, i, j, z, d)
    Next j
Next i

End Sub


Sub convertijproposals(n, m, g, ni, mj, i, j, z, ByRef d)

Dim ii, jj, k, l As Integer
Dim v1, v2 As Long

For ii = -ni To ni
    For jj = -mj To mj

        For k = 1 To g
            v1 = z(i, j, ii, jj, k)
            If v1 = 0 Then GoTo kloop

            For l = k + 1 To g
                v2 = z(i, j, ii, jj, l)
                If v2 = 0 Then GoTo lloop

                d(torus(i + ii, n), torus(j + jj, m), -ii, -jj, k) = -v1
                d(torus(i + ii, n), torus(j + jj, m), -ii, -jj, l) = -v2
's = "z( i: " + Str(i) + ", j: " + Str(j) + ", ii: " + Str(ii) + ", jj: " + Str(jj) + ", v1: "
+ Str(v1) + ", v2: " + Str(v2) + " )"
'MsgBox (s)
lloop:
            Next l
kloop:
        Next k
    Next jj
Next ii

End Sub
```

8.1.3 Accepting Deals

Accepting Deals includes the following functions:

- The function `acceptijdeals` is the main function. It loops through all combinations of tradeable goods.
- The functions `bestprice` and `acceptbestdeal` contain the identification of the best prices and the accepting of deals

```
Sub acceptijdeals(step, g, i, j, ni, mj, own, d, dd)

Dim ii, jj, k, l As Integer
Dim dA, dB As Long

Dim smax, bmin As Double
Dim smaxii, smaxjj As Integer
Dim bminii, bminjj As Integer

For k = 1 To g
    For l = k + 1 To g
        Call bestprice(i, j, ni, mj, k, l, own, d, smax, smaxii, smaxjj, bmin, bminii, bminjj)
         'best price selected
        Call acceptbestdeal(step, i, j, k, l, own, d, smax, smaxii, smaxjj, bmin, bminii,
bminjj, dd)
         'best deal accepted
    Next l
Next k
End Sub


Sub bestprice(i, j, ni, mj, k, l, own, d, smax, smaxii, smaxjj, bmin, bminii, bminjj)

Dim ii, jj As Integer
Dim dA, dB As Long

smax = 0
bmin = 9E+15

For ii = -ni To ni
    For jj = -mj To mj
        dA = d(i, j, ii, jj, k)
        dB = d(i, j, ii, jj, l)

        If dA = 0 Or dB = 0 Then GoTo jjloop

        If dA < 0 And dB > 0 Then
            If own(k) + dA > 0 And Abs(dB / dA) > smax Then
                smax = Abs(dB / dA)
                smaxii = ii
                smaxjj = jj
            End If
            GoTo jjloop
        End If

        If dA > 0 And dB < 0 Then
            If own(l) + dB > 0 And Abs(dB / dA) < bmin Then
                bmin = Abs(dB / dA)
                bminii = ii
                bminjj = jj
            End If
        End If

jjloop:
    Next jj
Next ii

End Sub
```

```
Sub acceptbestdeal(step, i, j, k, l, own, d, smax, smaxii, smaxjj, bmin, bminii, bminjj, dd)

Dim qK, qL As Long

If smax = 0 Then
    If bmin < 9E+15 Then

        qK = d(i, j, bminii, bminjj, k)
        qL = d(i, j, bminii, bminjj, l)

        dd(i, j, bminii, bminjj, k) = qK
        dd(i, j, bminii, bminjj, l) = qL
        own(k) = own(k) + qK
        own(l) = own(l) + qL

    End If
    GoTo bye
End If

If bmin >= 9E+15 Then
    If smax > 0 Then

        qK = d(i, j, smaxii, smaxjj, k)
        qL = d(i, j, smaxii, smaxjj, l)

        dd(i, j, smaxii, smaxjj, k) = qK
        dd(i, j, smaxii, smaxjj, l) = qL
        own(k) = own(k) + qK
        own(l) = own(l) + qL

    End If
    GoTo bye
End If

If smax > bmin Then
'If ni > 1 Then MsgBox ("double deal accepted")
    qK = d(i, j, bminii, bminjj, k)
    qL = d(i, j, bminii, bminjj, l)

    dd(i, j, bminii, bminjj, k) = qK
    dd(i, j, bminii, bminjj, l) = qL

    own(k) = own(k) + qK
    own(l) = own(l) + qL

    qK = d(i, j, smaxii, smaxjj, k)
    qL = d(i, j, smaxii, smaxjj, l)

    dd(i, j, smaxii, smaxjj, k) = qK
    dd(i, j, smaxii, smaxjj, l) = qL

    own(k) = own(k) + qK
    own(l) = own(l) + qL

End If

'accepted deals filled to dd
'ownership adaped

bye:
End Sub
```

8.1.4 Observing Deals

```
Sub observedeals(n, m, g, oii, ojj, dd, o)

' ReDim o(1 To n, 1 To m, -1 To 1, -1 To 1, 1 To 2, 1 To 2, 1 To 2) As Integer
' observations( observer(row,column), observee(relrow,relcol), _
              role(accepter,proposer), signA(sellA,buyA), scarce good) qty

Dim i, j, ii, jj, k, l, ni, nj, role, saleA As Integer
Dim ddA, ddB As Long

For i = 1 To n
    For j = 1 To m
'       dd(i,j,...) has accepted deal

        For ii = -oii To oii
            For jj = -ojj To ojj

                For k = 1 To g
                    ddA = dd(i, j, ii, jj, k)
                    If ddA = 0 Then GoTo kloop
                    For l = k + 1 To g
                        ddB = dd(i, j, ii, jj, l)
                        If ddB = 0 Then GoTo lloop

'                       inner loop

                        saleA = 1
                        If ddA > 0 Then saleA = 2
'                       saleA=1 it's a sale of A from the viewpoint of the accepter (and
proposer as *-1)
'                       saleA=2 it's a purchase of A from the viewpoint of the accepter (and
proposer as * -1)

'                       walk through all neighbours of accepter
                        For ni = -oii To oii
                            For nj = -ojj To ojj
                                o(torus(i + ni, n), torus(j + nj, m), -ni, -nj, 1, saleA, k) =
ddA
                                o(torus(i + ni, n), torus(j + nj, m), -ni, -nj, 1, saleA, l) =
ddB
                            Next nj
                        Next ni

'                       walk through all neighbours of proposer
                        For ni = -oii To oii
                            For nj = -ojj To ojj
                                o(torus(i + ii + ni, n), torus(j + jj + nj, m), -ni, -nj, 2,
torus(saleA + 1, 2), k) = -ddA
                                o(torus(i + ii + ni, n), torus(j + jj + nj, m), -ni, -nj, 2,
torus(saleA + 1, 2), l) = -ddB
                            Next nj
                        Next ni
lloop:
                    Next l
kloop:
                Next k
jjloop:
            Next jj
        Next ii
    Next j
Next i

End Sub
```

8.1.5 Clearing Deals

The function for clearing (additionally) verifies the bookkeeping (no short selling) rules.

```
Sub cleardeals(step, n, m, g, oii, ojj, ab, dd, ndeals, flow, fstat, delta)

Dim i, j, k, l, ii, jj As Integer
Dim oA, oB As Long
Dim fA, fB As Double

delta = 0
ndeals = 0

For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj
                If ii = 0 And jj = 0 Then GoTo jjloop

                For k = 1 To g
                    oA = dd(i, j, ii, jj, k)
                    If oA <> 0 Then GoTo kbreak
                Next k
kbreak:
                If oA = 0 Then GoTo jjloop

                For l = k + 1 To g
                    oB = dd(i, j, ii, jj, l)
                    If oB <> 0 Then GoTo lbreak
                Next l
lbreak:
                If oB = 0 Then GoTo jjloop
' Assertions
                If ab(i, j, k) + oA < 0 Then
                    GoTo jjloop
                End If
                If ab(torus(i + ii, n), torus(j + jj, m), k) - oA < 0 Then
                    GoTo jjloop
                End If
                If ab(i, j, l) + oB < 0 Then
                    GoTo jjloop
                End If
                If ab(torus(i + ii, n), torus(j + jj, m), l) - oB < 0 Then
                    GoTo jjloop
                End If
' Inner loop
                delta = delta + Abs(oA) + Abs(oB)
                ndeals = ndeals + 1

                fA = Abs(oA)
                fB = Abs(oB)

                ab(i, j, k) = ab(i, j, k) + oA
                If fstat = 1 Or (fstat = 2 And oA > 0) Or (fstat = 3 And oA < 0) Then
                    flow(i, j, k) = flow(i, j, k) + fA
                End If

                ab(torus(i + ii, n), torus(j + jj, m), k) = ab(torus(i + ii, n), torus(j + jj,
m), k) - oA
                If fstat = 1 Or (fstat = 2 And oA < 0) Or (fstat = 3 And oA > 0) Then
                    flow(torus(i + ii, n), torus(j + jj, m), k) = flow(torus(i + ii, n),
torus(j + jj, m), k) + fA
                End If

                ab(i, j, l) = ab(i, j, l) + oB
                If fstat = 1 Or (fstat = 2 And oB > 0) Or (fstat = 3 And oB < 0) Then
                    flow(i, j, l) = flow(i, j, l) + fB
                End If

                ab(torus(i + ii, n), torus(j + jj, m), l) = ab(torus(i + ii, n), torus(j + jj,
m), l) - oB
                If fstat = 1 Or (fstat = 2 And oB < 0) Or (fstat = 3 And oB > 0) Then
                    flow(torus(i + ii, n), torus(j + jj, m), l) = flow(torus(i + ii, n),
torus(j + jj, m), l) + fB
```

```
                End If
jjloop:
            Next jj
        Next ii
    Next j
Next i

'If Abs(delta) < 0.00001 Then MsgBox ("cleardeals>" + Str(step) + " no delta")

End Sub
```

8.1.6 Trade Runs

To compute trade runs, the following functions are used:

- The function `ngoodsrun` takes care of the interface to an .xls spreadsheet. The Output depends on the last two parameters. By setting them appropriately the function either delivers stocks, flows, or trade run statistics
- The function `dorun` is the main module that guides the calculation of traderuns. The main loop through all iterations given by an input parameter is located in that function. To be able to extract detailed information or statistics runs at a later stage (with a later call of the same function) the method of static variables is used.
- The function `iteration` takes care about one single trade run

```
Function ngoodsrun(g, numbiter, ohorizon, pshowoff, a As Range, Optional fstat = 0, Optional
showstat = 0)

If pshowoff < 0 Or pshowoff > 1 Then
    MsgBox ("wrong showoff probability")
    Exit Function
    End If

Dim i, j, k, l, ii, jj, n, m As Integer
n = a.Rows.Count
m = a.Columns.Count

If g < 2 Then
    MsgBox ("Anzahl Güter < 2 oder nicht ganzzahlig")
    Exit Function
End If
If Int(n / g) < 1 Then
    MsgBox ("Anzahl Zeilen zu klein")
    Exit Function
End If
If Int(n / g) * Int(g) <> n Then
    MsgBox ("Anzahl Güter " + Str(Int(n / g)) + " inkonsistent zu Zeilenzahl")
    Exit Function
End If
n = Int(n / g)

ReDim ab(1 To n, 1 To m, 1 To g) As Long                          'stock
ReDim flow(1 To n, 1 To m, 1 To g) As Double                      'flow

For k = 1 To g
    For i = 1 To n
        For j = 1 To m
            ab(i, j, k) = Int(a((k - 1) * n + i, j).Cells.Value)
        Next j
    Next i
Next k

ii = oh(ohorizon, n)
jj = oh(ohorizon, m)
```

```
ReDim z(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To g) As Long      'proposals
ReDim d(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To g) As Long      'proposed deals (converted)
ReDim dd(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To g) As Long     'deals
ReDim o(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To 2, 1 To 2, 1 To g) As Long     'observations

If showstat <> 0 Then GoTo statistics

ReDim xs(1 To 2, 1 To 18) As Double

Call dorun(1, numbiter, n, m, g, ii, jj, pshowoff, ohorizon, ab, z, d, dd, o, flow, fstat, xs)

' display

ReDim s(1 To g * n, 1 To m) As Double
If fstat = 0 Then
    For k = 1 To g
        For i = 1 To n
            For j = 1 To m
                s((k - 1) * n + i, j) = ab(i, j, k)
            Next j
        Next i
    Next k
Else
    For k = 1 To g
        For i = 1 To n
            For j = 1 To m
                s((k - 1) * n + i, j) = flow(i, j, k)
            Next j
        Next i
    Next k
End If
GoTo bye

statistics:

ReDim s(1 To minlong(numbiter, 5000), 1 To 18) As Double

Call dorun(0, numbiter, n, m, g, ii, jj, pshowoff, ohorizon, ab, z, d, dd, o, flow, fstat, s)

bye:
ngoodsrun = s

End Function

Sub dorun(do_show, numbiter, n, m, g, ii, jj, pshowoff, ohorizon, ab, zz, d, dd, o, flow,
fstat, xstat)

Dim ps As Double    'percentage showoff proposals
Dim op, sp, ndeals, lnod, nnod As Long

Dim delta, deltaold, mm, x, y, z As Double


Dim i, j, k, l As Long
Static s(1 To 5000, 1 To 18) As Double

If do_show = 1 Then

    lnod = 0
    mm = 0
    ReDim mab(1 To g) As Double
    For k = 1 To g
        x = gmean(n, m, k, ab)
        mab(k) = x
        mm = mm + x
    Next k

    Call inits(5000, 18, s)

    Dim c, cc As Integer     'cycle detection
    c = 5                    'depth of comparison
    ReDim abold(0 To c, 1 To n, 1 To m, 1 To g) As Double
    ReDim citer(0 To c) As Long

    cc = 0
    Call saveab(n, m, g, ab, abold, cc)
    citer(cc) = 0
```

```
'    y = dobserveablex(n, m, g, ohorizon, ab, mab)
'    z = dobserveablea(n, m, g, ohorizon, ab)
'    z = dfuzzycluster(n, m, g, 2, 1.5, ab)

    For i = 1 To numbiter

        Call iteration(i, n, m, g, ii, jj, pshowoff, ohorizon, ab, zz, d, dd, o, op, sp,
ndeals, flow, fstat, delta)

        If Abs(delta) < 0.00001 And Abs(deltaold) < 0.00001 Then
            MsgBox ("dorun> 2 phase no delta, step: " + Str(i) + " last no deals: " +
Str(lnod))
            GoTo ibreak
        End If
        deltaold = delta
        If ndeals = 0 Then lnod = i

' cycle check when no deals are made
        If ndeals = 0 Then
            nnod = nnod + 1
            j = cyclefound(n, m, g, ab, abold, citer, c)
            If j <> 0 Then
                MsgBox ("dorun> cycle found at " + Str(i + 1) + " same as " + Str(citer(j)) +
" last no deals: " + Str(lnod))
                GoTo ibreak
            End If

            cc = cc + 1
            If cc > c Then
                cc = 1
            End If

            Call saveab(n, m, g, ab, abold, cc)
            citer(cc) = i

        End If

'        j = i
'        If numbiter > 500 Then
'            If i <= numbiter - 500 Then
'                GoTo iloop
'            Else
'                j = i - numbiter + 500
'            End If
'        End If

' statistics collection
' only zero deals
'        If ndeals <> 0 Then GoTo iloop
' or

        j = torus(i, 5000)

        s(j, 1) = i

        s(j, 2) = dgmean(n, m, ab, mab(1), 1)
        s(j, 3) = dgmean(n, m, ab, mab(2), 2)
        s(j, 4) = dgmean(n, m, ab, mab(3), 3)

'        s(j, 5) = dall(n, m, g, ab, mab)
'        s(j, 6) = dweighted(n, m, g, ab, mab, mm)

        s(j, 5) = dobserveablex(n, m, g, ohorizon, ab, mab)
        s(j, 6) = dobserveablea(n, m, g, ohorizon, ab)

        s(j, 7) = op
        s(j, 8) = sp
        s(j, 9) = ndeals

        For k = 1 To minlong(3, g)
            For l = k + 1 To minlong(3, g)
                If k = l Then GoTo lloop
                    x = maxAprice(n, m, oh(ohorizon, n), oh(ohorizon, m), k, l, dd)
                    y = minAprice(n, m, oh(ohorizon, n), oh(ohorizon, m), k, l, dd)
                    z = avgAprice(n, m, oh(ohorizon, n), oh(ohorizon, m), k, l, dd)
' MsgBox ("dorun maxminstat> k " + Str(k) + " l " + Str(l) + " max(" + Str(9 + (k - 1) * 2 + l
- k) + " x " + Str(x))
                    s(j, 9 + (k - 1) * 2 + l - k) = x
```

```
                    s(j, 12 + (k - 1) * 2 + 1 - k) = y
                    s(j, 15 + (k - 1) * 2 + 1 - k) = z
lloop:
            Next l
        Next k

iloop:
    Next i

ibreak:
    MsgBox ("dorun> No deals encountered " + Str(nnod) + " times, last at " + Str(citer(cc)))

' if any inbetween step was reached call last one back
    If citer(cc) <> 0 Then Call abback(n, m, g, ab, abold, cc)

Else
    For k = 1 To minlong(5000, numbiter)
        If s(k, 1) = 0 Then GoTo kbreak
    Next k
kbreak:
    If k <= minlong(5000, numbiter) Then
        For i = 1 To k - 1
            For j = 1 To 18
                xstat(i, j) = s(k - i, j)
            Next j
        Next i
    Else
        For i = 1 To minlong(5000, numbiter)
            For j = 1 To 18
                xstat(i, j) = s(minlong(numbiter, 5000) - i + 1, j)
            Next j
        Next i
    End If
End If

End Sub
```

```
Sub iteration(i, n, m, g, ni, mj, pshowoff, ohorizon, ByRef ab, ByRef z, ByRef d, ByRef dd,
ByRef o, op, sp, ndeals, flow, fstat, delta)

ReDim xy(1 To n, 1 To m, 1 To g) As Integer
Call showoff(n, m, g, pshowoff, ab, xy)

Call initz(n, m, g, ni, mj, z)
Call propose(i, n, m, g, ni, mj, ab, ohorizon, o, xy, z, op, sp)

Call initz(n, m, g, ni, mj, d)
Call convertproposals(n, m, g, ni, mj, z, d)

Call initz(n, m, g, ni, mj, dd)
Call acceptdeals(i, n, m, g, ni, mj, ab, z, d, dd)

Call inito(n, m, g, ni, mj, o)
Call observedeals(n, m, g, ni, mj, dd, o)

Call cleardeals(i, n, m, g, ni, mj, ab, dd, ndeals, flow, fstat, delta)

'If i > 171 Then MsgBox ("iteration Z> " + Str(i))
End Sub
Sub acceptdeals(step, n, m, g, ni, mj, ab, z, d, dd)

Dim i, j, k, ii, jj As Integer
Dim oA As Long
ReDim own(1 To g) As Long

For i = 1 To n
    For j = 1 To m

        For k = 1 To g
            own(k) = ab(i, j, k)
If own(k) < 0 Then MsgBox ("acceptdeals A step" + Str(step) + "> i=" + Str(i) + " j=" + Str(j)
+ " own(" + Str(k) + ")= " + Str(own(k)))
        Next k

'       reduce for proposals made
        For ii = -ni To ni
            For jj = -mj To mj

                For k = 1 To g
                    oA = z(i, j, ii, jj, k)
                    If oA < 0 Then own(k) = own(k) + oA
                Next k

            Next jj
        Next ii

For k = 1 To g
    If own(k) < 0 Then MsgBox ("acceptdeals B step" + Str(step) + "> i=" + Str(i) + " j=" +
Str(j) + " own(" + Str(k) + ")= " + Str(own(k)))
Next k

        Call acceptijdeals(step, g, i, j, ni, mj, own, d, dd)

For k = 1 To g
    If own(k) < 0 Then MsgBox ("acceptdeals C step" + Str(step) + "> i=" + Str(i) + " j=" +
Str(j) + " own(" + Str(k) + ")= " + Str(own(k)))
Next k

    Next j
Next i

End Sub
```

8.2 Fuzzy Clustering

The following modules implement the fuzzy-c-means clustering algorithm[107].

8.2.1 Clustering

For clustering the following four functions are used:

- The function `showgfuzzycluster` takes care of the interface to an .xls spreadsheet
- The function `fuzzycluster` is the main module that guides the calculation
- The functions `nextu, nextv` calculate next approximation steps

```
Function showgfuzzycluster(g, ww, nnc, a As Range)

Dim n, m, i, j, k, nstocks As Integer

nstocks = Int(g)
n = a.Rows.Count
m = a.Columns.Count

If g < 2 Or nstocks <> g Then
    MsgBox ("Anzahl Güter < 2 oder nicht ganzzahlig")
    Exit Function
End If
If Int(n / g) < 1 Then
    MsgBox ("Anzahl Zeilen zu klein")
    Exit Function
End If
If Int(n / g) * Int(g) <> n Then
    MsgBox ("Anzahl Güter " + Str(Int(n / g)) + " inkonsistent zu Zeilenzahl")
    Exit Function
End If

n = Int(n / g)

Dim nc As Integer
nc = Int(nnc.Cells.Value)
If nnc.Cells.Value <> nc Then
    MsgBox ("Anzahl Cluster nicht ganzzahlig")
    Exit Function
End If
If nc < 1 Or nc > n * m Then
    MsgBox ("Anzahl Cluster < 1 oder >" + Str(n * m))
    Exit Function
End If


Dim w As Double
w = ww.Cells.Value
If w <= 1 Then
    MsgBox ("w muss > 1")
    Exit Function
End If


ReDim abc(1 To n, 1 To m, 1 To nstocks) As Double

For k = 1 To nstocks
    For i = 1 To n
```

[107] For a description of the fuzzy-c-means clustering algorithm see Olaf Wolkenhauer, Data Engineering (p. 94 ff)

```
        For j = 1 To m
            abc(i, j, k) = a((k - 1) * n + i, j).Cells.Value
        Next j
    Next i
Next k

ReDim c(1 To n, 1 To m, 1 To nc) As Double

'MsgBox ("g cluster> n" + Str(n) + " m" + Str(m) + " g" + Str(nstocks) + " nc" + Str(nc))

Call fuzzycluster(w, n, m, nstocks, nc, abc, c)

show:

ReDim cc(1 To n * nc, 1 To m) As Double
For k = 1 To nc
    For i = 1 To n
        For j = 1 To m
            cc((k - 1) * n + i, j) = c(i, j, k)
        Next j
    Next i
Next k

showgfuzzycluster = cc
End Function
```

```
Sub fuzzycluster(ByVal w, n, m, nstocks, nc, ByRef ab, ByRef c)

Dim i, j, k, l As Integer
Dim x, xvi As Double
Dim delta As Double
delta = 0.0001

'initialize u(ij)k=c(ij)k
For i = 1 To n
    For j = 1 To m

'no zero begin
        xvi = 0
        For l = 1 To nstocks
            xvi = xvi + Abs(ab(i, j, l))
        Next l
        If xvi = 0 Then GoTo jloop
'no zero end

        xvi = 0
        For k = 1 To nc - 1
            x = Rnd()
            If xvi + x < 1 Then
                c(i, j, k) = x
            Else
                c(i, j, k) = 1 - xvi
            End If
            xvi = xvi + c(i, j, k)
        Next k
        c(i, j, nc) = 1 - xvi
'smooth
        xvi = 0
        For k = 1 To nc
            If c(i, j, k) < 1 / (1.5 * nc) Then c(i, j, k) = 1 / (1.5 * nc)
            xvi = xvi + c(i, j, k)
        Next k
        For k = 1 To nc
            c(i, j, k) = c(i, j, k) / xvi
        Next k

jloop:
    Next j
Next i

'cluster centers
ReDim v(1 To nc, 1 To nstocks) As Double
Dim deltav As Double
deltav = 0
l = 0
Call nextv(w, n, m, nc, nstocks, ab, c, v, deltav)

While deltav > delta And l < 300
    l = l + 1
    Call nextu(w, n, m, nc, nstocks, ab, c, v)
    Call nextv(w, n, m, nc, nstocks, ab, c, v, deltav)
Wend

If deltav > delta Then
    MsgBox ("fuzzycluster> bad convergence, deltav: " + Str(deltav))
    GoTo bye
End If

For i = 1 To n
    For j = 1 To m
        xvi = 0
        For k = 1 To nc
            xvi = xvi + c(i, j, k)
        Next k
'if zeroes excluded
        If xvi <> 0 And Abs(xvi - 1) > delta Then
            MsgBox ("fuzzycluster> restriction violated xvi: " + Str(xvi) + " i: " + Str(i) +
" j: " + Str(j))
        End If

    Next j
Next i
bye:
End Sub
```

```
Sub nextv(ByVal w, n, m, nc, nstocks, ByRef ab, ByRef c, ByRef v, ByRef deltav)
On Error GoTo Sorry

Dim eps As Double
eps = 0.000000000000001

Dim k, l, ll As Integer
Dim vold, vnew, sux, su, xvi As Double
Dim i, j As Integer

deltav = 0
For k = 1 To nc
    For l = 1 To nstocks

'         MsgBox ("nextv> cluster" + Str(k) + " stock" + Str(l))

        sux = 0
        su = 0

'         MsgBox ("nextv> cluster" + Str(k) + " stock" + Str(l) + " sux,su init")
'         MsgBox ("nextv> again n" + Str(n) + " m" + Str(m))

        For i = 1 To n
'             MsgBox ("nextv cluster" * Str(k) + " stock" + Str(l) + " i" + Str(i) + " i pre
xvi")

            For j = 1 To m
'                 MsgBox ("nextv cluster" * Str(k) + " stock" + Str(l) + " i" + Str(i) + " j" +
Str(j) + " pre xvi")

                xvi = 0
                For ll = 1 To nstocks
                    xvi = xvi + ab(i, j, ll)
                Next ll
                If Abs(xvi) > eps Then

'                     MsgBox ("nextv cluster" * Str(k) + " stock" + Str(l) + " i" + Str(i) + "
j" + Str(j))
                    sux = sux + c(i, j, k) ^ w * ab(i, j, l)
                    su = su + c(i, j, k) ^ w
'                     MsgBox ("nextv cluster" * Str(k) + " stock" + Str(l) + " i" + Str(i) + "
j" + Str(j) + " completed")

                End If
            Next j
        Next i

'         MsgBox ("nextv> cluster" + Str(k) + " stock" + Str(l) + " pre completion")

        If Abs(su) > eps Then
            vnew = sux / su
            vold = v(k, l)
            deltav = deltav + Abs(vold - vnew)
            v(k, l) = vnew
        Else
'             MsgBox ("nextv> su=0")
'             GoTo bye
        End If

    Next l
Next k

bye:
Exit Sub
Sorry:
If Err.Number = 6 Then
    Resume Next
Else
    MsgBox "nextv> " & Err.Number & vbCrLf & vbCrLf & Err.Description
End If
End Sub
```

```
Sub nextu(ByVal w, n, m, nc, nstocks, ByRef ab, ByRef c, ByRef v)
On Error GoTo Sorry

Dim eps As Double
eps = 0.000000000000001

Dim i, j, k, l, ll As Integer
Dim xv, xvi, xvj, x, e As Double

ReDim singularity(1 To n, 1 To m) As Boolean
For i = 1 To n
    For j = 1 To m
        singularity(i, j) = False
    Next j
Next i
Dim nsing As Integer


For ll = 1 To nc

'   for all i<=>ll among clusters seek uik
    For i = 1 To n
        For j = 1 To m

'no zero begin
            xvi = 0
            For l = 1 To nstocks
                xvi = xvi + Abs(ab(i, j, l))
            Next l
            If xvi = 0 Then GoTo jloop
'no zero end
'           for all k<=>i,j among data seek uik

            xvi = 0
            For l = 1 To nstocks
                x = 0
                x = ab(i, j, l) - v(ll, l)
                xvi = xvi + x * x
'                xvi = xvi + ((ab(i, j, l) - v(ll, l)) ^ 2)
            Next l

            xv = 0
            For k = 1 To nc

                xvj = 0
                For l = 1 To nstocks
                    x = 0
                    x = ab(i, j, l) - v(k, l)
                    xvj = xvj + x * x
'                    xvj = xvj + ((ab(i, j, l) - v(k, l)) ^ 2)
                Next l

                If Abs(xvj) > eps Then
                    x = 0
                    x = xvi / xvj
                    e = 1 / (2 * (w - 1))
                    If Abs(x) > eps Then xv = xv + Exp(e * Log(x))
'                   xv = xv + (xvi / xvj) ^ (1 / (2 * (w - 1)))
                Else
'                    MsgBox ("nextu> xvj = 0")
'                    GoTo bye
                End If
            Next k

'           new uik computed
            If Abs(xv) > eps Then
                c(i, j, ll) = 1 / xv
'                MsgBox ("nextu> xv <> 0, ll" + Str(ll) + " i" + Str(i) + " j" + Str(j))
            Else
                c(i, j, ll) = 0
                singularity(i, j) = True
            End If

jloop:
        Next j
    Next i
Next ll
```

```
'repair singularities
For i = 1 To n
    For j = 1 To m
        If singularity(i, j) Then
'            MsgBox ("repair i" + Str(i) + " j" + Str(j))

            xvi = 0
            nsing = 0
            For k = 1 To nc
                If Abs(c(i, j, k)) <= eps Then
                    nsing = nsing + 1
                Else
                    xvi = xvi + c(i, j, k)
                End If
            Next k

            k = 1
            l = 1
            While k < nsing
                If c(i, j, l) = 0 Then
                    k = k + 1
'                    x = Rnd()
                    x = 2 * Rnd() / nc
                    If xvi + x < 1 Then
                        c(i, j, l) = x
                    Else
                        c(i, j, l) = 1 - xvi
                    End If
                    xvi = xvi + c(i, j, l)
                End If
                l = l + 1
            Wend
            While l <= nc
                If c(i, j, l) = 0 Then
                    c(i, j, l) = 1 - xvi
'                    GoTo jloop2
                End If
                l = l + 1
            Wend
        End If
jloop2:
    Next j
Next i

bye:
Exit Sub
Sorry:
If Err.Number = 6 Then
    Resume Next
Else
    MsgBox "nextu> " & Err.Number & vbCrLf & vbCrLf & Err.Description
End If
End Sub
```

8.2.2 Visualisation (Conditional Formatting)

To aid the visualisation of clusters the following functions are used:

- The function `showgcenter` takes care of the interface to an .xls spreadsheet
- The function `fuzzyclustercenter` computes the cluster centres

```
Function showgcenter(nstocks, ww, nnc, d As Range, a As Range)

Dim n, n2, m, i, j, k, l, g As Integer

Dim eps As Double
eps = 0.00001

g = Int(nstocks.Cells.Value)
n = a.Rows.Count
n2 = d.Rows.Count

m = a.Columns.Count
If m <> d.Columns.Count Then
    MsgBox ("showgcenter> Anzahl Spalten stimmen nicht überein")
    Exit Function
End If

If Int(n2 / g) < 1 Then
    MsgBox ("Anzahl (Daten) Zeilen zu klein")
    Exit Function
End If
If Int(n2 / g) * Int(g) <> n2 Then
    MsgBox ("Anzahl Güter " + Str(Int(n / g)) + " inkonsistent zu Zeilenzahl Daten")
    Exit Function
End If

n2 = Int(n2 / g)

If g < 2 Then
    MsgBox ("Anzahl Güter < 2 oder nicht ganzzahlig")
    Exit Function
End If

Dim nc As Integer
nc = Int(nnc.Cells.Value)
If nnc.Cells.Value <> nc Then
    MsgBox ("Anzahl Cluster nicht ganzzahlig")
    Exit Function
End If
If nc < 1 Or nc > n * m Then
    MsgBox ("Anzahl Cluster < 1 oder >" + Str(n * m))
    Exit Function
End If

If Int(n / nc) < 1 Then
    MsgBox ("Anzahl (Cluster) Zeilen zu klein")
    Exit Function
End If
If Int(n / nc) * Int(nc) <> n Then
    MsgBox ("Anzahl Cluster " + Str(Int(n / nc)) + " inkonsistent zu Zeilenzahl Cluster")
    Exit Function
End If

n = Int(n / nc)

Dim w As Double
w = ww.Cells.Value
If w <= 1 Then
    MsgBox ("w muss > 1")
    Exit Function
End If
```

```
If n <> n2 Then
    MsgBox ("showgcenter> Zeilen Daten und Zeilen Cluster inkonsistent")
    Exit Function
End If

'MsgBox ("showgcenter> n=" + Str(n) + " m=" + Str(m) + " nc=" + Str(nc) + " g=" + Str(g))

ReDim c(1 To n, 1 To m, 1 To nc) As Double

For k = 1 To nc
    For i = 1 To n
        For j = 1 To m
            c(i, j, k) = a((k - 1) * n + i, j).Cells.Value
        Next j
    Next i
Next k

ReDim abc(1 To n, 1 To m, 1 To g) As Double

For k = 1 To g
    For i = 1 To n
        For j = 1 To m
'MsgBox ("showgcenter> step A.(" + Str(i) + "," + Str(j) + ")")
            abc(i, j, k) = d((k - 1) * n + i, j).Cells.Value
        Next j
    Next i
Next k

ReDim cc(1 To nc, 1 To g) As Double

Call fuzzyclustercenter(w, n, m, g, nc, abc, c, cc)
showgcenter = cc

Exit Function
End Function


Sub fuzzyclustercenter(w, n, m, g, nc, abc, c, cc)

Dim i, j, k As Integer
Dim x, mu, mm, eps As Double
eps = 0.00001

For i = 1 To n
    For j = 1 To m
        x = 0
        For k = 1 To nc
            x = x + c(i, j, k)
        Next k
        If Abs(x - 1) > eps Then
'            MsgBox ("showgcenter> exclusion at x(" + Str(i) + "," + Str(j) + ")=" + Str(x))
'            Exit Function
        End If
    Next j
Next i

For k = 1 To nc
    For l = 1 To g

        mu = 0
        mm = 0

        For i = 1 To n
            For j = 1 To m
                x = c(i, j, k) ^ w
                mu = mu + x * abc(i, j, l)
                mm = mm + x
            Next j
        Next i

        If Abs(mm) > eps Then cc(k, l) = mu / mm
    Next l
Next k

End Sub
```

8.3 Exploration

The following modules were used for the exploration of the model

8.3.1 Simulation Runs

The following functions are used to generate initial distributions of wealth and to obtain statistics regarding the overall behaviour.

In the main function `nsimrun` the same technique (as in `dorun`) of using static variables to extract further details by a second call is applied.

```
Sub newab(g, n, m, a, b, c, a1, a2, b1, b2, c1, c2, ab)

Dim i, j, k, l As Integer

For i = 1 To n
    For j = 1 To m
        l = 1
        If g >= 3 Then
            ab(i, j, 1) = Int(Application.WorksheetFunction.BetaInv(Rnd(), a1, a2, 0, a))
            ab(i, j, 2) = Int(Application.WorksheetFunction.BetaInv(Rnd(), b1, b2, 0, b))
            ab(i, j, 3) = Int(Application.WorksheetFunction.BetaInv(Rnd(), c1, c2, 0, c))

            l = 4
        End If
        For k = l To g
            ab(i, j, k) = Int(Rnd() * a)
        Next k
    Next j
Next i
End Sub
```

```
Function nsimrun(Optional r = 10, Optional g = 3, Optional n = 6, Optional m = 6, Optional a =
7, Optional b = 11, Optional c = 5, Optional a1 = 1, Optional a2 = 1, Optional b1 = 2,
Optional b2 = 1, Optional c1 = 1, Optional c2 = 2, Optional save = 0)

Dim i, i2, j, k, l, ii, jj As Integer

Dim pshowoff As Double
pshowoff = 1

Dim ohorizon As Integer
ohorizon = 1

ii = oh(ohorizon, n)
jj = oh(ohorizon, m)
ReDim z(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To g) As Long                  'proposals
ReDim d(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To g) As Long                  'proposed
deals (converted)
ReDim dd(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To g) As Long                 'deals
ReDim o(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To 2, 1 To 2, 1 To g) As Long  'observations

ReDim ab(1 To n, 1 To m, 1 To g) As Long                                       'stock
Static absave(1 To 3, 1 To 6, 1 To 6, 1 To 3) As Long                          'stock save
ReDim flow(1 To n, 1 To m, 1 To g) As Double                                   'flow

Dim ns As Integer
ns = 10
ReDim s(1 To r, 1 To ns) As Double                                             'simulation
statistics
ReDim xs(1 To ns) As Double
Dim x As Double

If save <> 0 Then
    ReDim s(1 To g * n, 1 To m) As Double
    For k = 1 To g
        For i = 1 To n
            For j = 1 To m
                s((k - 1) * n + i, j) = absave(1, i, j, k)
                If save = 2 Then s((k - 1) * n + i, j) = absave(2, i, j, k)
            Next j
        Next i
    Next k
    GoTo bye
End If

For i = 1 To r
    Call newab(g, n, m, a, b, c, a1, a2, b1, b2, c1, c2, ab)
    Call saveab(n, m, g, ab, absave, 3)

    For k = 1 To g
        x = 0
        For i2 = 1 To n
            For j = 1 To m
                x = x + ab(i2, j, k)
            Next j
        Next i2
        If k <= ns - 7 Then s(i, 7 + k) = x
    Next k

    Call simrun(2000, n, m, g, ii, jj, pshowoff, ohorizon, ab, z, d, dd, o, flow, xs)

    s(i, 1) = i
    For j = 2 To 7
        s(i, j) = xs(j)
    Next j
    If s(i, 2) > 2000 Then
        Call moveab(n, m, g, absave, 3, 1)
        MsgBox ">2000 ..." + Str(absave(1, 6, 6, 3)) + "..2.." + Str(absave(3, 6, 6, 3))
    End If
    If s(i, 5) = 2 Then
        Call moveab(n, m, g, absave, 3, 2)
        MsgBox ">cycle 2..." + Str(absave(2, 6, 6, 3)) + "..2.." + Str(absave(3, 6, 6, 3))
    End If
Next i

bye:
nsimrun = s

End Function
```

```
Sub simrun(numbiter, n, m, g, ii, jj, pshowoff, ohorizon, ab, zz, d, dd, o, flow, s)

Dim ps As Double   'percentage showoff proposals
Dim op, sp, ndeals, lnod, nnod As Long
Dim delta, deltaold, mm, x, y, z As Double
Dim i, j, k, l As Long

lnod = 0
mm = 0
ReDim mab(1 To g) As Double
For k = 1 To g
    x = gmean(n, m, k, ab)
    mab(k) = x
    mm = mm + x
Next k

Dim c, cc As Integer     'cycle detection
c = 20                   'depth of comparison
ReDim abold(0 To c, 1 To n, 1 To m, 1 To g) As Double
ReDim citer(0 To c) As Long

cc = 0
Call saveab(n, m, g, ab, abold, cc)
citer(cc) = 0

For i = 1 To numbiter
    Call iteration(i, n, m, g, ii, jj, pshowoff, ohorizon, ab, zz, d, dd, o, op, sp, ndeals,
flow, fstat, delta)

    If Abs(delta) < 0.00001 And Abs(deltaold) < 0.00001 Then
        s(2) = i
        s(3) = lnod
        s(4) = 0
        s(5) = 0
        GoTo ibreak
    End If
    deltaold = delta
    If ndeals = 0 Then lnod = i

' cycle check when no deals are made
    If ndeals = 0 Then
        nnod = nnod + 1
        j = cyclefound(n, m, g, ab, abold, citer, c)
        If j <> 0 Then
            s(2) = i
            s(3) = lnod
            s(4) = citer(j)
            If j <= cc Then
                s(5) = cc - j
            Else
                s(5) = c + cc - j
            End If
            GoTo ibreak
        End If
        cc = cc + 1
        If cc > c Then
            cc = 1
        End If
        Call saveab(n, m, g, ab, abold, cc)
        citer(cc) = i
    End If

iloop:
Next i

ibreak:
s(6) = nnod
s(7) = citer(cc)

If i > numbiter Then
    s(2) = i
    s(3) = lnod
    s(4) = 0
    s(5) = 0
End If

End Sub
```

8.3.2 Model (Trade Run) Statistics

The following functions are used during trade runs, to compute and collect various statistics, e.g. average prices, number of deals, exchanged quantities etc.

```
Function qtyijAbuys(oii, ojj, i, j, dd)

Dim ii, jj As Integer
Dim noA As Double

For ii = -oii To oii
    For jj = -ojj To ojj
        If dd(i, j, ii, jj, 1) > 0 Then noA = noA + dd(i, j, ii, jj, 1)
    Next jj
Next ii

qtyijAbuys = noA
End Function


Function qtyijAsales(oii, ojj, i, j, dd)

Dim ii, jj As Integer
Dim noA As Double

'MsgBox ("qtyAijsales> i: " + Str(i) + " j: " + Str(j))

For ii = -oii To oii
    For jj = -ojj To ojj
        If dd(i, j, ii, jj, 1) < 0 Then

'MsgBox ("qtyAijsales> qty: " + Str(dd(i, j, ii, jj, 1)))
            noA = noA - dd(i, j, ii, jj, 1)
         End If
    Next jj
Next ii

qtyijAsales = noA
End Function


Function qtyAsales(n, m, oii, ojj, dd)

Dim i, j, ii, jj As Integer
Dim noA As Double

For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj
                If dd(i, j, ii, jj, 1) < 0 Then
                    noA = noA - dd(i, j, ii, jj, 1)
                End If
            Next jj
        Next ii
    Next j
Next i

qtyAsales = noA
End Function
```

```
Function qtyAbuys(n, m, oii, ojj, dd)

Dim i, j, ii, jj As Integer
Dim noA As Double

For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj
                If dd(i, j, ii, jj, 1) > 0 Then
                    noA = noA + dd(i, j, ii, jj, 1)
                End If
            Next jj
        Next ii
    Next j
Next i

qtyAbuys = noA
End Function


Function noAbuys(n, m, oii, ojj, dd)

Dim i, j, ii, jj, noA As Integer

For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj
                If dd(i, j, ii, jj, 1) > 0 Then noA = noA + 1
            Next jj
        Next ii
    Next j
Next i

noAbuys = noA
End Function


Function noAsales(n, m, oii, ojj, dd)

Dim i, j, ii, jj, k, noA As Integer

For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj
                If dd(i, j, ii, jj, 1) < 0 Then noA = noA + 1
            Next jj
        Next ii
    Next j
Next i

noAsales = noA
End Function
```

```
Function minAprice(n, m, oii, ojj, k, l, dd)

Dim i, j, ii, jj, a As Integer
Dim b, bmin As Double

bmin = 9E+15
For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj
                a = dd(i, j, ii, jj, k)
                If a <> 0 Then
                    b = Abs(dd(i, j, ii, jj, l) / a)
                    If b <> 0 And b < bmin Then bmin = b
                End If
            Next jj
        Next ii
    Next j
Next i

If bmin >= 9E+15 Then bmin = 0
minAprice = bmin
End Function


Function maxAprice(n, m, oii, ojj, k, l, dd)

Dim i, j, ii, jj, a As Integer
Dim b, bmax As Double

bmax = 0
For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj
                a = dd(i, j, ii, jj, k)
                If a <> 0 Then
                    b = Abs(dd(i, j, ii, jj, l) / a)
                    If b <> 0 And b > bmax Then bmax = b
                End If
            Next jj
        Next ii
    Next j
Next i

maxAprice = bmax
End Function


Function avgAprice(n, m, oii, ojj, k, l, dd)

Dim i, j, ii, jj As Integer
Dim a, ak, b, bl, aprice As Double

For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj
                ak = Abs(dd(i, j, ii, jj, k))
                bl = Abs(dd(i, j, ii, jj, l))
                If ak <> 0 And bl <> 0 Then
                    a = a + ak
                    b = b + bl
                End If
            Next jj
        Next ii
    Next j
Next i

If a <> 0 Then aprice = b / a
avgAprice = aprice

End Function
```

8.3.3 Finding Cycles

```
Function cyclefound(n, m, g, ab, abold, citer, c)

Dim cc, j As Integer

'MsgBox ("cyclefound> start")
j = 0
For cc = 0 To c
    If citer(cc) <> 0 Then
        If cyclefoundcc(n, m, g, ab, abold, citer, c, cc) Then
            j = cc
            GoTo ccbreak
        End If
    End If
Next cc
ccbreak:
cyclefound = j
'MsgBox ("cyclefound> passed")

End Function

Function cyclefoundcc(n, m, g, ab, abold, citer, c, cc)

Dim i, j, k As Integer
Dim starti, startj As Integer
Dim startok As Boolean
Dim a, maxg As Long

'MsgBox ("cyclefoundcc> start")

starti = 0
startj = 0
startok = False

For i = 1 To n
    For j = 1 To m

        startok = True
        maxg = 0

        For k = 1 To g
            a = ab(i, j, k)
            If a > maxg Then maxg = a
            If a <> abold(cc, i, j, k) Then startok = False
        Next k

        If startok Then
            If maxg = 0 Then GoTo jcontinue

            starti = i - 1
            startj = j - 1
            GoTo ibreak
        End If
jcontinue:
    Next j
Next i

'MsgBox ("cyclefoundcc> no startij identified")
GoTo bye

ibreak:
'assert.debug startok = True
'MsgBox ("cyclefoundcc> startij identified" + Str(starti) + ", " + Str(startj))

For i = 1 To n
    For j = 1 To m
        For k = 1 To g
'MsgBox ("cyclefoundcc> run" + Str(i) + ", " + Str(j) + ", " + Str(k))
            If ab(torus(starti + i, n), torus(startj + j, m), k) <> abold(cc, i, j, k) Then
'MsgBox ("cyclefoundcc> inequality identified")
                startok = False
                GoTo bye
            End If
```

```
        Next k
    Next j
Next i

bye:
cyclefoundcc = startok

'MsgBox ("cyclefoundcc> passed")
End Function
```

8.3.4 Computing Distances

```
Function dobserveablex(n, m, g, ohorizon, ab, mab)

Dim i, ii, j, jj, k As Integer
Dim x, y, z, mk, mm As Double

ReDim xy(1 To n, 1 To m, 1 To g) As Long
Call showoff(n, m, g, 1, ab, xy)

mm = 0
For k = 1 To g
    mm = mm + mab(k)
Next k

y = 0
For i = 1 To n
    For j = 1 To m

        For k = 1 To g
            z = ab(i, j, k)
            mk = mab(k)
            For ii = -oh(ohorizon, n) To oh(ohorizon, n)
                For jj = -oh(ohorizon, m) To oh(ohorizon, m)
                    If ii = 0 And jj = 0 Then GoTo jjloop
                    x = ab(torus(i + ii, n), torus(j + jj, m), k)
                    If x <> 0 Then y = y + Abs(z - x) * mk
jjloop:
                Next jj
            Next ii
        Next k

    Next j
Next i

x = (2 * oh(ohorizon, n) + 1) * (2 * oh(ohorizon, m) + 1) - 1
dobserveablex = y / (n * m * x * mm)

End Function
```

```
Function dfuzzycluster(n, m, g, nc, w, ab)

ReDim c(1 To n, 1 To m, 1 To nc) As Double
ReDim cc(1 To nc, 1 To g) As Double
ReDim x(1 To nc) As Double

Dim i, j As Integer
Dim y As Double

Call fuzzycluster(w, n, m, g, nc, ab, c)
Call fuzzyclustercenter(w, n, m, g, nc, ab, c, cc)

For i = 1 To nc
    x(i) = 0
    For j = 1 To g
        y = y + cc(i, j) ^ 2
    Next j
    x(i) = y ^ 0.5
Next i

y = 0
For i = 1 To nc - 1
    y = y + Abs(x(i) - x(i + 1))
Next i

dfuzzycluster = y
End Function

Function gmean(n, m, k, ab)

Dim i, j, a As Integer
Dim x, y As Double

x = 0
a = 0
For i = 1 To n
    For j = 1 To m
        y = ab(i, j, k)
'        If y <= 0 Then GoTo jloop
        x = x + y
        a = a + 1
jloop:
    Next j
Next i
If a > 0 Then gmean = x / a

End Function


Function dgmean(n, m, ab, mm, k)

Dim i, j, a As Integer
Dim x, y As Double

x = 0
y = 0
For i = 1 To n
    For j = 1 To m
        x = ab(i, j, k)
'        If x <= 0 Then GoTo jloop
        y = y + Abs(mm - x)
        a = a + 1
jloop:
    Next j
Next i
If a > 0 Then dgmean = y / a

End Function
```

```
Function dall(n, m, g, ab, mab)

Dim i, j, k As Integer
Dim x, y As Double

x = 0
For i = 1 To n
    For j = 1 To m
        y = 0
        For k = 1 To g
            y = y + Abs(ab(i, j, k) - mab(k))
        Next k
        x = x + y
    Next j
Next i
dall = x / (n * m)

End Function


Function dweighted(n, m, g, ab, mab, mm)

Dim i, j, k As Integer
Dim x, y As Double

x = 0
For i = 1 To n
    For j = 1 To m
        y = 0
        For k = 1 To g
            y = y + Abs(ab(i, j, k) - mab(k)) * mab(k)
        Next k
        x = x + y / mm
    Next j
Next i
dweighted = x / (n * m)

End Function


Function dobserveablea(n, m, g, ohorizon, ab)

Dim i, ii, j, jj, k As Integer
Dim x, y, z As Double

y = 0
For i = 1 To n
    For j = 1 To m

        For k = 1 To g
            z = ab(i, j, k)
            For ii = -oh(ohorizon, n) To oh(ohorizon, n)
                For jj = -oh(ohorizon, m) To oh(ohorizon, m)
                    If ii = 0 And jj = 0 Then GoTo jjloop
                    y = y + Abs(z - ab(torus(i + ii, n), torus(j + jj, m), k))
jjloop:
                Next jj
            Next ii
        Next k

    Next j
Next i

x = (2 * oh(ohorizon, n) + 1) * (2 * oh(ohorizon, m) + 1) - 1
dobserveablea = y / (n * m * g * x)

End Function
```

8.4 Auxiliary Functions

```
Function wealth(x, n)

Dim y As Double
y = x * n
wealth = Int(y + 0.49)

End Function


Sub saveab(n, m, g, ab, abold, cc)

Dim i, j, k As Integer

For i = 1 To n
    For j = 1 To m
        For k = 1 To g
            abold(cc, i, j, k) = ab(i, j, k)
        Next k
    Next j
Next i

End Sub


Sub abback(n, m, g, ab, abold, cc)

Dim i, j, k As Integer

For i = 1 To n
    For j = 1 To m
        For k = 1 To g
            ab(i, j, k) = abold(cc, i, j, k)
        Next k
    Next j
Next i

End Sub


Function ldistg(g, p)
Dim x As Double
Dim i, j As Integer

i = p
x = Log(g)
j = g
While j > 1 And i > 1
    j = j - 1
    i = i - 1
    x = x + Log(j)
Wend

ldistg = x
End Function


Function torus(ByVal x, n)

While x < 1
    x = x + n
Wend
While x > n
    x = x - n
Wend
torus = x

End Function
```

```
Function oh(ByVal x, n)

If x < 1 Then x = 1
While (2 * x) > (n - 1) And x > 1
    x = x - 1
Wend

oh = x
End Function


Sub initz(n, m, g, ni, mj, ByRef z)

Dim i, j, ii, jj, k As Integer
For i = 1 To n
    For j = 1 To m
        For ii = -ni To ni
            For jj = -mj To mj
                For k = 1 To g
                    z(i, j, ii, jj, k) = 0
                Next k
            Next jj
        Next ii
    Next j
Next i

End Sub


Sub inito(n, m, g, ni, mj, ByRef o)

Dim i, j, ii, jj, r, s, k As Integer
For i = 1 To n
    For j = 1 To m
        For ii = -ni To ni
            For jj = -mj To mj
                For r = 1 To 2
                    For s = 1 To 2
                        For k = 1 To g
                            o(i, j, ii, jj, r, s, k) = 0
                        Next k
                    Next s
                Next r
            Next jj
        Next ii
    Next j
Next i

End Sub


Sub inits(a, b, s)
Dim i, j As Integer

For i = 1 To a
    For j = 1 To b
        s(i, j) = 0
    Next j
Next i
End Sub


Function minlong(a, b)
Dim x As Long
x = a
If b < x Then x = b
minlong = x
End Function
```

```
Sub moveab(n, m, g, abold, f, t)

Dim i, j, k As Integer

For i = 1 To n
    For j = 1 To m
        For k = 1 To g
            abold(t, i, j, k) = abold(f, i, j, k)
        Next k
    Next j
Next i

End Sub
```

9 Appendix B

9.1 Scenario 1

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
1	44	44	40	1	4	42	96	99	100	295	11,00
2	32	31	0	0	7	31	98	116	108	322	4,43
3	33	32	0	0	5	32	129	107	87	323	6,40
4	29	28	0	0	3	28	110	108	128	346	9,33
5	47	47	43	1	17	45	105	105	106	316	2,76
6	2001	11	0	0	1	11	110	94	113	317	11,00
7	49	49	45	1	9	47	113	114	106	333	5,44
8	40	40	36	1	8	38	114	116	106	336	5,00
9	18	17	0	0	3	17	129	91	88	308	5,67
10	32	31	0	0	6	31	111	117	111	339	5,17
11	40	39	0	0	8	39	107	119	111	337	4,88
12	34	34	30	1	5	32	112	89	111	312	6,80
13	32	32	28	1	8	30	110	104	87	301	4,00
14	42	42	38	1	10	40	101	118	99	318	4,20
15	36	36	32	1	8	34	115	98	120	333	4,50
16	26	25	0	0	3	25	96	105	103	304	8,33
17	34	33	0	0	4	33	93	118	135	346	8,25
18	36	35	0	0	2	35	109	106	96	311	17,50
19	49	48	0	0	5	48	119	127	98	344	9,60
20	42	42	30	5	16	40	106	102	98	306	2,63
21	31	31	27	1	8	29	92	95	112	299	3,88
22	33	33	29	1	8	31	100	104	108	312	4,13
23	54	53	0	0	11	53	121	97	117	335	4,82

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
24	28	27	0	0	6	27	117	126	100	343	4,50
25	26	25	0	0	3	25	103	106	100	309	8,33
26	35	34	0	0	3	34	109	104	111	324	11,33
27	31	30	0	0	6	30	93	98	75	266	5,00
28	29	29	25	1	6	27	107	112	102	321	4,83
29	2001	2	0	0	1	2	113	103	96	312	2,00
30	33	32	0	0	5	32	114	104	110	328	6,40
31	30	29	0	0	3	29	121	81	135	337	9,67
32	60	60	56	1	14	58	124	98	96	318	4,29
33	53	52	0	0	9	52	124	122	130	376	5,78
34	47	46	0	0	8	46	97	111	88	296	5,75
35	25	24	0	0	3	24	92	109	83	284	8,00
36	36	36	32	1	9	34	119	125	96	340	4,00
37	64	64	58	2	14	62	115	122	114	351	4,57
38	48	47	0	0	6	47	101	109	113	323	7,83
39	56	56	44	5	11	54	106	109	115	330	5,09
40	30	29	0	0	6	29	103	120	105	328	4,83
41	52	52	48	1	9	50	121	110	94	325	5,78
42	40	39	0	0	7	39	91	98	119	308	5,57
43	2001	32	0	0	1	32	112	120	93	325	32,00
44	31	31	27	1	7	29	107	105	85	297	4,43
45	29	28	0	0	3	28	104	126	99	329	9,33
46	28	27	0	0	3	27	99	119	96	314	9,00
47	36	35	0	0	7	35	111	99	112	322	5,00
48	56	55	0	0	10	55	104	106	124	334	5,50
49	44	43	0	0	5	43	137	103	121	361	8,60
50	56	56	52	1	4	54	105	131	108	344	14,00
51	52	52	48	1	12	50	101	130	124	355	4,33
52	63	62	0	0	8	62	119	115	111	345	7,75

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
53	33	32	0	0	7	32	93	112	113	318	4,57
54	35	35	31	1	8	33	96	106	119	321	4,38
55	26	25	0	0	4	25	92	101	102	295	6,25
56	30	29	0	0	5	29	115	103	103	321	5,80
57	38	37	0	0	2	37	110	130	131	371	18,50
58	29	29	25	1	7	27	82	81	112	275	4,14
59	65	64	0	0	8	64	108	118	115	341	8,00
60	28	28	24	1	3	26	121	99	91	311	9,33
61	22	22	18	1	7	20	100	127	106	333	3,14
62	2001	2	0	0	1	2	118	110	86	314	2,00
63	41	40	0	0	4	40	100	105	124	329	10,00
64	41	41	37	1	10	39	108	117	104	329	4,10
65	36	35	0	0	3	35	103	120	124	347	11,67
66	63	63	51	5	15	61	118	128	108	354	4,20
67	32	32	28	1	6	30	104	119	119	342	5,33
68	41	40	0	0	4	40	114	124	123	361	10,00
69	68	68	64	1	19	66	112	134	128	374	3,58
70	30	29	0	0	5	29	127	112	104	343	5,80
71	35	34	0	0	11	34	113	96	106	315	3,09
72	34	33	0	0	5	33	127	96	96	319	6,60
73	37	36	0	0	3	36	131	82	83	296	12,00
74	46	45	0	0	10	45	103	122	111	336	4,50
75	38	38	34	1	8	36	102	124	112	338	4,75
76	34	33	0	0	8	33	106	128	103	337	4,13
77	35	34	0	0	6	34	103	115	107	325	5,67
78	30	29	0	0	8	29	88	104	134	326	3,63
79	41	41	37	1	8	39	105	126	115	346	5,13
80	35	34	0	0	8	34	111	107	104	322	4,25
81	24	23	0	0	3	23	107	129	129	365	7,67

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
82	33	32	0	0	6	32	100	115	103	318	5,33
83	27	26	0	0	6	26	101	134	98	333	4,33
84	45	44	0	0	4	44	110	137	97	344	11,00
85	44	44	40	1	13	42	109	125	129	363	3,38
86	25	24	0	0	1	24	92	107	106	305	24,00
87	25	24	0	0	3	24	100	100	106	306	8,00
88	52	51	0	0	8	51	112	133	115	360	6,38
89	35	35	31	1	10	33	100	103	112	315	3,50
90	34	33	0	0	6	33	103	124	110	337	5,50
91	39	38	0	0	4	38	94	124	106	324	9,50
92	36	36	32	1	6	34	109	117	119	345	6,00
93	38	37	0	0	1	37	84	118	116	318	37,00
94	37	36	0	0	6	36	95	88	85	268	6,00
95	47	47	43	1	13	45	128	121	116	365	3,62
96	2001	30	0	0	2	30	102	90	107	299	15,00
97	45	44	0	0	6	44	90	113	104	307	7,33
98	35	34	0	0	5	34	114	121	103	338	6,80
99	32	31	0	0	5	31	107	106	113	326	6,20
100	39	39	35	1	7	37	89	115	99	303	5,57

9.2 Scenario 2

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
1	28	27	0	0	3	27	113	170	67	350	9,00
2	49	49	41	3	13	47	118	176	70	364	3,77

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
3	51	50	0	0	6	50	117	233	74	424	8,33
4	44	43	0	0	7	43	133	183	58	374	6,14
5	147	146	0	0	12	146	109	178	67	354	12,17
6	52	51	0	0	6	51	103	191	72	366	8,50
7	39	38	0	0	5	38	124	158	82	364	7,60
8	50	49	0	0	9	49	103	210	72	385	5,44
9	48	47	0	0	6	47	107	172	81	360	7,83
10	58	57	0	0	5	57	97	208	70	375	11,40
11	59	58	0	0	10	58	124	161	79	364	5,80
12	40	39	0	0	2	39	109	162	73	344	19,50
13	57	56	0	0	9	56	115	175	72	362	6,22
14	39	39	35	1	5	37	98	169	91	358	7,80
15	18	17	0	0	6	17	107	173	68	348	2,83
16	57	56	0	0	6	56	104	175	67	346	9,33
17	62	62	58	1	10	60	112	177	68	357	6,20
18	59	58	0	0	11	58	93	166	86	345	5,27
19	55	54	0	0	8	54	125	198	77	400	6,75
20	83	82	0	0	8	82	125	190	75	390	10,25
21	41	40	0	0	4	40	89	191	80	360	10,00
22	53	52	0	0	10	52	91	226	80	397	5,20
23	67	66	0	0	3	66	98	187	82	367	22,00
24	63	63	59	1	10	61	105	190	81	376	6,30
25	103	102	0	0	13	102	110	197	73	380	7,85
26	71	70	0	0	7	70	82	193	59	334	10,00
27	45	44	0	0	2	44	102	188	78	368	22,00
28	39	39	35	1	6	37	124	163	68	355	6,50
29	15	14	0	0	1	14	107	201	62	370	14,00
30	32	31	0	0	4	31	87	168	81	336	7,75
31	132	131	0	0	4	131	126	195	71	392	32,75
32	35	34	0	0	7	34	99	194	79	372	4,86

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
33	463	462	0	0	24	462	123	201	74	398	19,25
34	113	112	0	0	6	112	120	170	75	365	18,67
35	41	40	0	0	9	40	119	194	69	382	4,44
36	27	26	0	0	2	26	70	178	79	327	13,00
37	45	44	0	0	6	44	108	195	77	380	7,33
38	148	148	144	1	9	146	130	165	78	373	16,44
39	57	56	0	0	5	56	92	156	76	324	11,20
40	61	60	0	0	7	60	100	159	71	330	8,57
41	393	392	0	0	21	392	126	188	53	367	18,67
42	114	113	0	0	8	113	118	200	94	412	14,13
43	103	102	0	0	14	102	106	192	67	365	7,29
44	40	39	0	0	6	39	93	154	57	304	6,50
45	16	15	0	0	3	15	74	186	74	334	5,00
46	317	316	0	0	15	316	130	180	76	386	21,07
47	21	20	0	0	2	20	82	168	70	320	10,00
48	560	559	0	0	33	559	117	219	66	402	16,94
49	81	80	0	0	8	80	96	182	72	350	10,00
50	48	47	0	0	4	47	102	174	61	337	11,75
51	95	94	0	0	6	94	126	185	83	394	15,67
52	57	56	0	0	8	56	114	163	71	348	7,00
53	62	61	0	0	12	61	100	169	68	337	5,08
54	76	75	0	0	3	75	101	186	74	361	25,00
55	44	44	40	1	7	42	99	167	88	354	6,29
56	57	57	53	1	10	55	90	161	66	317	5,70
57	121	120	0	0	8	120	105	205	55	365	15,00
58	54	53	0	0	5	53	124	204	68	396	10,60
59	183	182	0	0	23	182	102	217	63	382	7,91
60	33	32	0	0	4	32	115	171	88	374	8,00
61	41	41	37	1	8	39	105	153	70	328	5,13
62	39	38	0	0	8	38	115	198	53	366	4,75

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
63	44	44	40	1	8	42	132	197	76	405	5,50
64	66	65	0	0	9	65	130	191	80	401	7,22
65	115	114	0	0	13	114	111	173	81	365	8,77
66	76	75	0	0	7	75	95	189	83	367	10,71
67	49	48	0	0	4	48	100	171	63	334	12,00
68	111	110	0	0	8	110	119	217	85	421	13,75
69	31	30	0	0	2	30	93	180	78	351	15,00
70	51	50	0	0	6	50	118	176	79	373	8,33
71	32	32	28	1	4	30	81	140	70	291	8,00
72	117	117	113	1	9	115	122	174	68	364	13,00
73	34	33	0	0	5	33	96	188	69	353	6,60
74	56	56	52	1	12	54	115	159	77	351	4,67
75	57	56	0	0	4	56	102	164	75	341	14,00
76	32	31	0	0	8	31	105	169	60	334	3,88
77	33	32	0	0	9	32	96	190	66	352	3,56
78	39	38	0	0	3	38	101	180	75	356	12,67
79	66	66	62	1	14	64	116	157	61	334	4,71
80	102	101	0	0	13	101	102	153	56	311	7,77
81	34	33	0	0	5	33	95	162	71	328	6,60
82	74	73	0	0	9	73	92	202	72	366	8,11
83	78	77	0	0	6	77	113	168	75	356	12,83
84	54	53	0	0	9	53	119	185	87	391	5,89
85	72	71	0	0	5	71	113	185	77	375	14,20
86	39	38	0	0	3	38	115	193	63	371	12,67
87	54	53	0	0	6	53	107	202	80	389	8,83
88	33	32	0	0	5	32	115	216	67	398	6,40
89	61	60	0	0	10	60	108	163	72	343	6,00
90	22	21	0	0	5	21	118	172	68	358	4,20
91	38	37	0	0	8	37	108	200	71	379	4,63
92	122	121	0	0	6	121	120	199	61	380	20,17

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
93	74	73	0	0	8	73	101	205	63	369	9,13
94	39	38	0	0	8	38	97	157	72	326	4,75
95	57	56	0	0	9	56	115	168	69	352	6,22
96	30	29	0	0	3	29	87	180	78	345	9,67
97	77	76	0	0	9	76	115	174	81	370	8,44
98	65	64	0	0	6	64	105	173	57	335	10,67
99	56	55	0	0	9	55	118	146	68	332	6,11
100	54	53	0	0	12	53	107	172	80	359	4,42

9.3 *Scenario 3*

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
1	130	129	0	0	7	129	113	237	34	384	18,43
2	75	74	0	0	6	74	87	245	35	367	12,33
3	14	13	0	0	4	13	86	249	56	391	3,25
4	101	100	0	0	18	100	110	244	50	404	5,56
5	116	115	0	0	10	115	103	252	34	389	11,50
6	184	183	0	0	15	183	119	228	37	384	12,20
7	680	679	0	0	39	679	103	269	43	415	17,41
8	274	273	0	0	11	273	120	250	48	418	24,82
9	122	121	0	0	11	121	111	241	46	398	11,00
10	52	51	0	0	9	51	85	245	43	373	5,67
11	1480	1479	0	0	52	1479	117	266	38	421	28,44
12	70	69	0	0	6	69	116	259	61	436	11,50
13	99	98	0	0	7	98	104	227	49	380	14,00

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
14	51	50	0	0	9	50	117	272	45	434	5,56
15	63	62	0	0	6	62	95	265	47	407	10,33
16	130	129	0	0	13	129	110	262	46	418	9,92
17	411	410	0	0	28	410	112	249	41	402	14,64
18	175	174	0	0	14	174	84	248	38	370	12,43
19	26	25	0	0	6	25	94	238	45	377	4,17
20	70	69	0	0	12	69	77	230	50	357	5,75
21	52	51	0	0	3	51	92	246	34	372	17,00
22	130	129	0	0	10	129	106	239	44	389	12,90
23	231	230	0	0	13	230	118	250	35	403	17,69
24	78	77	0	0	10	77	86	238	40	364	7,70
25	84	83	0	0	8	83	98	248	39	385	10,38
26	40	39	0	0	5	39	106	272	46	424	7,80
27	118	117	0	0	8	117	125	266	29	420	14,63
28	196	195	0	0	8	195	119	251	43	413	24,38
29	637	636	0	0	24	636	114	260	45	419	26,50
30	60	59	0	0	6	59	108	237	49	394	9,83
31	99	98	0	0	9	98	111	242	59	412	10,89
32	500	499	0	0	32	499	108	244	44	396	15,59
33	48	47	0	0	7	47	100	253	54	407	6,71
34	145	144	0	0	9	144	101	195	35	331	16,00
35	19	18	0	0	5	18	131	257	69	457	3,60
36	125	124	0	0	10	124	105	251	46	402	12,40
37	16	15	0	0	2	15	119	259	43	421	7,50
38	36	35	0	0	3	35	88	280	48	416	11,67
39	35	34	0	0	4	34	110	235	44	389	8,50
40	155	154	0	0	14	154	116	240	42	398	11,00
41	469	468	0	0	26	468	113	262	51	426	18,00
42	413	412	0	0	30	412	118	256	52	426	13,73

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
43	65	64	0	0	9	64	99	243	64	406	7,11
44	44	43	0	0	8	43	102	262	32	396	5,38
45	191	190	0	0	13	190	109	257	44	410	14,62
46	358	357	0	0	38	357	113	234	42	389	9,39
47	70	69	0	0	5	69	120	235	50	405	13,80
48	130	129	0	0	8	129	104	228	49	381	16,13
49	101	100	0	0	4	100	84	251	53	388	25,00
50	66	65	0	0	4	65	114	239	44	397	16,25
51	399	398	0	0	18	398	124	254	38	416	22,11
52	314	313	0	0	20	313	95	238	54	387	15,65
53	91	90	0	0	14	90	101	226	38	365	6,43
54	152	151	0	0	6	151	95	263	45	403	25,17
55	117	116	0	0	7	116	104	254	33	391	16,57
56	36	35	0	0	6	35	105	233	53	391	5,83
57	217	216	0	0	24	216	99	253	40	392	9,00
58	547	546	0	0	34	546	124	255	42	421	16,06
59	64	63	0	0	4	63	102	236	40	378	15,75
60	285	284	0	0	17	284	129	252	46	427	16,71
61	134	133	0	0	12	133	94	248	41	383	11,08
62	123	122	0	0	10	122	108	241	38	387	12,20
63	49	48	0	0	8	48	114	280	41	435	6,00
64	94	94	90	1	9	92	117	238	40	395	10,44
65	135	134	0	0	11	134	124	228	49	401	12,18
66	126	125	0	0	11	125	96	237	60	393	11,36
67	185	184	0	0	11	184	119	238	49	406	16,73
68	72	71	0	0	14	71	89	249	37	375	5,07
69	106	105	0	0	9	105	112	273	61	446	11,67
70	42	41	0	0	4	41	99	254	44	397	10,25
71	27	26	0	0	3	26	93	256	41	390	8,67

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
72	81	80	0	0	9	80	112	235	34	381	8,89
73	42	41	0	0	6	41	99	228	42	369	6,83
74	88	87	0	0	2	87	99	250	41	390	43,50
75	287	286	0	0	17	286	114	244	38	396	16,82
76	96	95	0	0	11	95	119	245	49	413	8,64
77	39	38	0	0	8	38	97	248	41	386	4,75
78	93	92	0	0	12	92	103	250	44	397	7,67
79	44	43	0	0	5	43	95	239	43	377	8,60
80	289	288	0	0	15	288	93	276	32	401	19,20
81	116	115	0	0	5	115	102	264	34	400	23,00
82	65	64	0	0	8	64	107	230	48	385	8,00
83	101	100	0	0	11	100	128	221	48	397	9,09
84	111	110	0	0	15	110	108	240	40	388	7,33
85	95	94	0	0	15	94	117	234	46	397	6,27
86	85	84	0	0	11	84	109	228	27	364	7,64
87	38	37	0	0	6	37	96	270	44	410	6,17
88	50	49	0	0	4	49	115	236	49	400	12,25
89	185	184	0	0	21	184	92	235	37	364	8,76
90	67	66	0	0	8	66	106	248	44	398	8,25
91	86	85	0	0	7	85	111	227	33	371	12,14
92	279	278	0	0	14	278	113	270	47	430	19,86
93	42	41	0	0	8	41	81	220	35	336	5,13
94	22	21	0	0	4	21	86	279	42	407	5,25
95	7	6	0	0	3	6	85	242	31	358	2,00
96	234	233	0	0	17	233	112	247	33	392	13,71
97	35	34	0	0	7	34	116	260	50	426	4,86
98	112	111	0	0	8	111	126	251	48	425	13,88
99	23	22	0	0	6	22	79	257	43	379	3,67
100	21	20	0	0	3	20	90	236	56	382	6,67

9.4 Scenario 4

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
1	284	283	0	0	13	283	121	258	28	407	21,77
2	73	72	0	0	8	72	99	273	33	405	9,00
3	1161	1160	0	0	39	1160	125	297	33	455	29,74
4	132	131	0	0	14	131	127	285	27	439	9,36
5	450	449	0	0	24	449	120	295	24	439	18,71
6	417	416	0	0	12	416	116	280	21	417	34,67
7	455	454	0	0	22	454	107	269	40	416	20,64
8	961	960	0	0	40	960	106	283	29	418	24,00
9	25	24	0	0	2	24	121	257	27	405	12,00
10	19	18	0	0	5	18	119	268	24	411	3,60
11	65	64	0	0	8	64	103	287	31	421	8,00
12	542	541	0	0	31	541	131	287	35	453	17,45
13	186	185	0	0	11	185	96	276	26	398	16,82
14	263	262	0	0	22	262	115	275	17	407	11,91
15	176	175	0	0	5	175	87	254	20	361	35,00
16	916	915	0	0	42	915	131	264	28	423	21,79
17	372	371	0	0	25	371	112	275	21	408	14,84
18	214	213	0	0	12	213	108	264	22	394	17,75
19	91	90	0	0	13	90	93	266	29	388	6,92
20	225	224	0	0	13	224	108	288	24	420	17,23
21	675	674	0	0	30	674	119	267	17	403	22,47
22	305	304	0	0	9	304	119	285	24	428	33,78
23	38	37	0	0	5	37	91	281	23	395	7,40
24	477	476	0	0	18	476	109	277	27	413	26,44

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
25	44	43	0	0	5	43	96	254	31	381	8,60
26	147	146	0	0	5	146	119	295	33	447	29,20
27	58	57	0	0	16	57	92	282	27	401	3,56
28	91	90	0	0	7	90	108	271	27	406	12,86
29	7	6	0	0	3	6	107	292	28	427	2,00
30	38	37	0	0	6	37	106	256	32	394	6,17
31	348	347	0	0	19	347	105	266	35	406	18,26
32	261	260	0	0	18	260	92	287	35	414	14,44
33	350	349	0	0	17	349	112	269	37	418	20,53
34	428	427	0	0	21	427	104	282	28	414	20,33
35	92	91	0	0	3	91	111	274	29	414	30,33
36	15	14	0	0	3	14	103	266	31	400	4,67
37	167	166	0	0	14	166	126	264	30	420	11,86
38	148	147	0	0	9	147	114	285	19	418	16,33
39	68	67	0	0	7	67	93	272	20	385	9,57
40	402	401	0	0	23	401	112	286	43	441	17,43
41	76	75	0	0	9	75	106	279	36	421	8,33
42	80	79	0	0	14	79	110	270	32	412	5,64
43	94	93	0	0	20	93	88	290	33	411	4,65
44	233	232	0	0	11	232	123	267	30	420	21,09
45	270	269	0	0	13	269	105	299	22	426	20,69
46	5	4	0	0	2	4	101	282	26	409	2,00
47	31	30	0	0	4	30	112	271	23	406	7,50
48	1547	1546	0	0	53	1546	108	279	25	412	29,17
49	1068	1067	0	0	39	1067	108	281	23	412	27,36
50	229	228	0	0	7	228	105	287	34	426	32,57
51	486	485	0	0	31	485	109	287	33	429	15,65
52	5	4	0	0	2	4	80	267	29	376	2,00
53	294	293	0	0	10	293	108	281	24	413	29,30

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
54	650	649	0	0	30	649	113	280	20	413	21,63
55	106	105	0	0	11	105	95	281	25	401	9,55
56	244	243	0	0	24	243	107	302	40	449	10,13
57	353	352	0	0	17	352	117	297	34	448	20,71
58	914	913	0	0	35	913	117	280	24	421	26,09
59	567	566	0	0	25	566	127	294	23	444	22,64
60	252	251	0	0	16	251	110	272	24	406	15,69
61	14	13	0	0	4	13	102	288	22	412	3,25
62	84	83	0	0	10	83	118	265	37	420	8,30
63	40	39	0	0	8	39	102	287	29	418	4,88
64	32	31	0	0	5	31	105	275	38	418	6,20
65	145	144	0	0	15	144	120	266	38	424	9,60
66	199	198	0	0	16	198	94	264	29	387	12,38
67	259	258	0	0	18	258	101	278	26	405	14,33
68	114	113	0	0	6	113	100	267	25	392	18,83
69	464	463	0	0	24	463	115	275	29	419	19,29
70	574	573	0	0	22	573	129	279	28	436	26,05
71	763	762	0	0	32	762	125	267	26	418	23,81
72	385	384	0	0	24	384	123	278	24	425	16,00
73	16	15	0	0	5	15	101	278	34	413	3,00
74	14	13	0	0	4	13	106	289	28	423	3,25
75	21	20	0	0	3	20	114	262	38	414	6,67
76	361	360	0	0	13	360	131	264	32	427	27,69
77	231	230	0	0	22	230	129	264	18	411	10,45
78	626	625	0	0	24	625	109	285	31	425	26,04
79	1173	1172	0	0	32	1172	105	291	36	432	36,63
80	469	468	0	0	29	468	103	277	26	406	16,14
81	984	983	0	0	47	983	110	296	25	431	20,91
82	27	26	0	0	3	26	106	297	26	429	8,67

simulation	trade runs	last run no deals	cycle at run	cycle length	number no deals	last run no deals (compared)	total A	total B	total C	total A+B+C	episode length
83	17	16	0	0	6	16	103	309	26	438	2,67
84	18	17	0	0	4	17	97	290	33	420	4,25
85	39	38	0	0	8	38	96	285	24	405	4,75
86	1036	1035	0	0	55	1035	116	285	35	436	18,82
87	74	73	0	0	17	73	102	278	27	407	4,29
88	79	78	0	0	12	78	105	264	23	392	6,50
89	117	116	0	0	8	116	101	265	28	394	14,50
90	191	190	0	0	16	190	105	277	31	413	11,88
91	1320	1319	0	0	57	1319	113	286	32	431	23,14
92	12	11	0	0	4	11	103	276	35	414	2,75
93	49	48	0	0	11	48	107	259	12	378	4,36
94	15	14	0	0	3	14	92	259	28	379	4,67
95	222	221	0	0	16	221	103	288	23	414	13,81
96	95	94	0	0	10	94	107	264	20	391	9,40
97	9	8	0	0	4	8	124	296	20	440	2,00
98	71	70	0	0	9	70	94	274	32	400	7,78
99	118	117	0	0	13	117	94	280	39	413	9,00
100	69	68	0	0	5	68	107	286	34	427	13,60

10 Appendix C – Curriculum Vitae

I was born on March 27th, 1963 to Ewald Fleischmann and Ilse Fleischmann (Filip) in Vienna, Austria. My brothers Dominik and Oliver were born in 1964 and 1966 respectively. My sister Esther was born in 1967.

In 1963 my father worked as radiologist at the University of Vienna. He later held assignments in hospitals in Lower Austria and a practice in Tulln, to where the familiy eventually moved.

I spent my shool years, primary school 1969-1973, secondary school ("Neusprachliches Gymnasium") 1973-1981 in Tulln, where I graduated ("Matura") with honors.

After completing service in the Austrian Federal Army in 1982 I started to study computer science and economics ("Studienversuch Betriebsinformatik") at Technical University in Vienna. I graduated 1987 as Magister rerum socialum oeconomicarumque (Mag.rer.soc.oec.). The title of my master thesis was "Evaluation of Power Plants on the River Danube East of Greifenstein, ELECTRE (MCDM) Methods" which I completed at the institute for Operations Research.

Due to financial difficulties of my family I had to start my professional career as early as 1983, first as programmer with a software development firm and later as a trainer at the training institute of the Viennese chamber of commerce ("WIFI").

After completing my university education in 1987 I worked for 2 years at the Austrian institute for spatial planning and regional economics ("Österreichisches Institut für Raumplanung, ÖIR) in a research environment, but left in 1989 to pursue an opportunity in the computer industry.

Since then my professional experience includes 18 months at IKEA Eastern Europe, 5 years at Unisys (last position as branch manager for business process solutions, Austria in 1995), 4 years at KPMG Management Consulting (last position as senior manager, IT consulting, Austria in 1999) and 3 years at Siemens Business Services (last position as Principal and Director Management Consulting Austria in 2002).

From 2002 to today I work as independent management consultant. In 2003 I joined the newly founded firm act Managmenet Consulting as a part-time senior expert.

My areas of expertise and project references include

- Risk and crisis management for large projects
- IT strategy related consulting for banks, insurance, social insurance, healthcare, public sector and industry
- Efficiency measurement using mathematical methods (DEA) and decision support (MCDM)
- Business models for medium size organisations after major reorganisations (e.g. mergers)

In 1999 I qualified as Certified Management Consultant (CMC) with the Austrian chamber of commerce as approved by the ICMCI (International Council of Management Consulting Institutes). In 2003 I was listed as chartered Basel-II consultand by the same organisation. I further became a member of the Austrian society of actuaries in 2003.

I am married to Sabine Fleischmann-Preinreich since 1990. Our son Gregor Wenzel was born in 1996.