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Pricing of and Hedging with Traffic Light Options

Ausgeführt am

Institut für Stochastik und Wirtschaftsmathematik

unter der Leitung von

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durch

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Abstract

Nowadays structured products have become an important component on capital markets worldwide. This thesis is about a new designed structured product which is called traffic light option. First, there will be a short introduction about structured products and some historical developments. Then, in accordance with previous works of Thomas Kokholm and Peter Løchte Jørgensen, it is a more detailed approach to pricing of and hedging with traffic light options in the LIBOR market model. At the end, a simulation regarding the pricing and an example for hedging will be run.

Contents

Chapter 1

Introduction

Derivatives, options and structured products are financial vocabularies and there are no simple definitions. Economists, accountants, lawyers, and government regulators have all struggled to develop a precise definition for derivatives[1]. Imprecision in the use of the term, moreover, is more than just a semantic problem. It is also a real problem for firms that must operate in a regulatory environment where the meaning of the term often depends on which regulator is using it.

Although there are several competing definitions, we define a derivative as a contract that derives most of its value from some underlying asset, reference rate or index. As our definition implies, a derivative must be based on at least one underlying. An underlying is the asset, reference rate or index from which a derivative inherits its principal source of value. Falling within our definition, there are several different types of derivatives, including commodity derivatives and financial derivatives.

A commodity derivative is a derivative contract specifying a commodity or commodity index as the underlying. For example, a crude oil forward contract specifies the price, quantity, and date of a future exchange of the grade of crude oil that underlies the forward contract. Because crude oil is a commodity, a crude oil forward contract would be a commodity derivative.

A financial derivative is a derivative contract specifying a financial instrument, interest rate, foreign exchange rate, or financial index as the underlying. For example, a call option on IBM stock gives its owner the right to buy the IBM shares that underlie the option at a predetermined price. In this sense, an IBM call option derives its value from the value of the underlying shares of IBM stock. Because IBM stock is a financial instrument, the IBM call option is a financial derivative.

In practice, financial derivatives cover a diverse spectrum of underlyings, including stocks, bonds, exchange rates, interest rates, credit characteristics, or stock market indexes. Practically nothing limits the financial instruments, reference rates, or indices that can serve as the underlying for a financial derivative contract. Some derivatives, moreover, can be based on more than one underlying, a definition for structured products. For example, the value of a financial derivative may depend on the difference between a domestic interest rate and a foreign interest rate (i.e. two separate reference rates) or as seen in the master thesis on an interest rate and a stock portfolio.

[1] see [KO14]

Chapter 2

Structured products

2.1 Definition

Structured products are designed to facilitate highly customized risk-return objectives.[1] This is accomplished by taking a traditional security, such as a conventional investment-grade bond and replacing the usual payment features e.g. periodic coupons and final principal with non-traditional payoffs derived not from the issuer's own cash flow but from the performance of one or more underlying assets.
The payoffs from these performance outcomes are contingent in the sense that if the underlying assets return value "x", then the structured product pays out "y". Hence, structured products closely relate to traditional models of option pricing; Though they may also contain other derivative types such as swaps, forwards and futures as well as embedded features such as leveraged upside participation or downside buffers.
Structured products originally became popular in Europe and have gained currency in the U.S., where they are frequently offered as SEC-registered products, which means they are accessible to retail investors in the same way as stocks, bonds, ETFs (Exchange Traded Funds) and mutual funds. Their ability to offer customized exposure, including hard-to-reach asset classes and subclasses, make structured products useful as a complement to these other traditional components of diversified portfolios.

There are three main types of structured products[2]:

1. the privately placed and individually negotiated transactions that are done for a single investor or a very small number of investors.

2. those that are sold to the public through retail networks, such as bank branches of financial advisers.

3. products listed and traded on public exchanges or otherwise widely available to retail investors and institutional clients alike.

For further information, I highly recommend the book "Structured Products-Evolution and Analysis" by Clarke Pitts, who gives a deeper insight to the evolution of structured products.

[1] see [Lam16]
[2] see [Pit13]

2.2 History of Options

The very first options and futures were traded in ancient Greece, when olives were sold before they had reached ripeness. Thereafter, the market evolved in the following way.

16th century[3]: Ever since the 15th century tulips, which were liked for their exotic appearance, were grown in Turkey. The head of the royal medical gardens in Vienna, Austria, was the first to cultivate those Turkish tulips successfully in Europe. When he fled to Holland because of religious persecution, he took the bulbs along. As the new head of the botanical gardens of Leiden, Netherlands, he cultivated several new strains. It was from these gardens that avaricious traders stole the bulbs to commercialize them because tulips were a great status symbol.

17th century: The first futures on tulips were traded in 1630. As of 1634, people could buy special tulip strains by the weight of their bulbs; For the bulbs, the same value was chosen as for gold. Along with the regular trading, speculators entered the market and the prices skyrocketed. A bulb of the strain "Semper Octavian" was worth two wagonloads of wheat, four loads of rye, four fat oxen, eight fat swine, twelve fat sheep, two hogsheads of wine, four barrels of beer, two barrels of butter, 1,000 pounds of cheese, one marriage bed with linen and one sizable wagon. People left their families, sold all their belongings, and even borrowed money to become tulip traders. When in 1637, this supposedly risk-free market crashed, traders as well as private individuals went bankrupt. The government prohibited speculative trading; the period became famous as Tulipmania.

18th century: In 1728, the Royal West-Indian and Guinea Company, the monopolist in trading with the Caribbean Islands and the African coast issued the first stock options. Those were options on the purchase of the French Island of Ste. Croix where sugar plantings were planned. The project was realized in 1733, and paper stocks were issued in 1734. Along with the stock, people purchased a relative share of the island and the valuables as well as the privileges and the rights of the company.

19th century: In 1848, 82 businessmen founded the Chicago Board of Trade (CBOT). Today it is the biggest and oldest futures market in the entire world. Most written documents were lost in the great fire of 1871, however, it is commonly believed that the first standardized futures were traded as of 1860. CBOT now trades several futures and forwards. Not only T-bonds and treasury bonds are traded there but also options and gold. In 1870, the New York Cotton Exchange was founded. In 1880, the gold standard was introduced.

20th century: In 1914, the gold standard was abandoned because of the war.
In 1919, the Chicago Produce Exchange which was in charge of trading agricultural products was renamed to Chicago Mercantile Exchange. Today, it is the most important futures market for Eurodollar, foreign exchange and livestock.
Most developments in terms of option markets and products were done from the 1970s to 2000.

[3]see [Wys07, Ch.1]

21th century: Now, structured products are frequently used in financial markets. There are hardly boundaries in variety and combinations. As we can see in the following Fig. 2.1[4], the peak of sold structured products was in 2007, right before the housing bubble and credit crisis in 2008. Driven by cost pressure and new regulations, the amount of structured products, which were bought, descended very dramatically from almost \$250*bn* in 2007 to about \$100*bn* nowadays. Despite that fact, the number of structured products issued is almost constantly increasing.

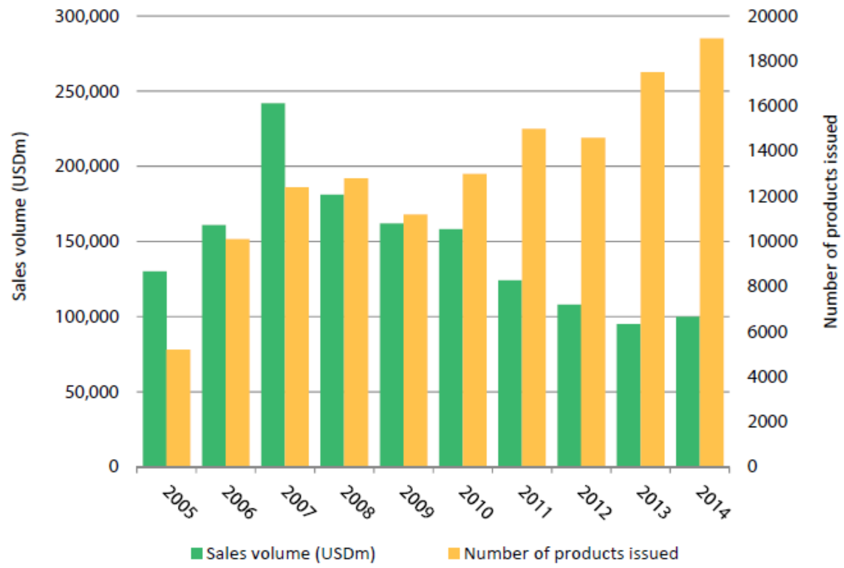


Figure 2.1: Structured products sales and issuance 2005-2014

[4]Source:Research Report for OIC (The Options Industry Council)–Analysis on Structured Products and Listed Equity Options in Europe 2015. For more details: `http://www.optionseducation.org/content/dam/oic/documents/literature/files/srp-part1-2015.pdf`

2.3 Goals and purposes

The main goals and purposes of structured products are[5]:

1. Arbitrage: Both investors and issuers can carry out arbitrage trades with derivatives and underlying assets by means of structured products.

2. Investment restrictions: Such groups of investors as pension and mutual funds and insurance companies can access derivatives transaction via structured products.

3. Taxation and accounting: Structured products are easy from the perspective of accounting and taxation as they are considered as a separate security and the value of derivatives is already included in the product price.

4. Creation of products "à la carte": The freedom of products' creation is pretty unbounded. They are customised to fit the unique requirements of investors.

5. Hedging: They can be used not only for investments but also as hedge of positions against market risks.

6. Access to new markets: Investors can access exotic instruments and new markets with the help of structured products. For instance the assets and instrument of developing markets that would otherwise be difficult for investors to access directly.

7. Cheap funding source: Part of funds, intended for fixed income investment, can be used by the issuer for its own financing more cheaply than the market rates.

2.4 Classification

In this section, I determine the classification of structured products which I have to consider while talking in terms of structured products:

1. **By levels of principal protection**:
   On degree of protection of the capital, the following products can be divided:

   - Pricipal-protected products: Those products provide full protection of the initial capital, not depending on the underlying asset's price move.
   - Partially protected products: In this case the return of initial capital is guaranteed only at certain level in the form of percent against originally invested sum.

2. **By quantity (periodicity) of payments**:

   - Coupon products: Throughout the whole period of a product's life, those instruments provide more than one payment like usual bonds.
   - Non-coupon products: Those products provide only one payment at the maturity date, which includes both the return of initial capital and profit-and-loss amount.

[5]for a more detailed insight see [Ome09]

3. **By type of underlying asset**:
   Among underlying assets to which the product can be linked to, the following assets can be mentioned:

   - Security, interest rate, currency, index, commodity, basket of assets (currencies, securities, commodities), credit quality, volatility, spread, consumer price index and other macroeconomic indicators, property price index.

4. **By the form of a structured product**:
   Structured products can be issued in the following forms:
   Security, Deposit and Fund.

5. **By the type of investor**:
   Each structured product is prepared for its own predetermined group of investors and customers. It is possible to outline three basic groups of investors:

   - Retail group: group of mass consumer.
   - Group of institutional investors: among them are large investment banks, mutual and pension funds and state funds.
   - Individual investors: group of wealthy consumers.

6. **By behaviour of the underlying asset**:
   Structured products' payoffs depend on dynamics of the underlying asset they are linked to. The following behaviour models can be defined:

   - Growth/falling
   - Lateral movements
   - Occurrence/non-occurrence of an event
   - High/low volatility

7. **By degree the payoff depends on the price path of the underlying asset**:
   Payoffs of structured products can be either defined by the value of a variable at the maturity date or the value of a variable throughout all the life time of a product. Thus, the payoff can be independent and dependent upon the price path of the underlying.

8. **By the payoff functions**:
   The basic peculiarity of structured products is their core element: derivative financial instruments. Almost all derivatives can be used for creation of structured products. The type of derivatives and their combinations do certainly define the payoffs' functions that differ one product from another. The given criterion is the most complex for definition. Having investigated the products offered on the market, the following types or payoff functions can be detached:

   - Tracking functions: Their payoffs are fully defined by the movement of the underlying asset and its change of 1 percent provides 1 percent change in price of the product. Example: Protected Tracker

- Leveraged functions: Financial leverage is used. Those products bear a risk of a partial loss of the initial capital. Example: Leverage long with stop loss note.
- Basket functions: Payoffs are defined here by dynamics of one asset versus a basket of underlying assets. Example: Altiplano note
- Functions with floating parameters: Here the main parameters can be changed, for example, a strike, when the underlying asset has overcome a certain level. Example: Cliquet note.
- Fixed payoff functions: Payments in this case are fixed. Example: Reverse convertible.
- Swap functions: Within those functions the payoffs are defined by spreads between prices (values) of certain underlying assets or by their volatility. Example: Dispersion note.

The disclosure of mentioned indicators and their detailed description will allow all market participants to outline the borders and possibilities of market's functioning and further development more accurately. It is worth mentioning at least three fields where the disclosure of information mentioned above is highly necessary:

- Creation of investment memorandum at the stage of product launching.
- Placement of restrictions and limits on structured products by the regulating authorities. This thresholding can be used in relation to institutional investors, mutual and pension funds. Market members need similar thresholding criteria as well.
- Ranking of structured products by independent associations and organisations including rating of products.

In Fig. 2.2 all different parameters are depicted in terms of structured products.

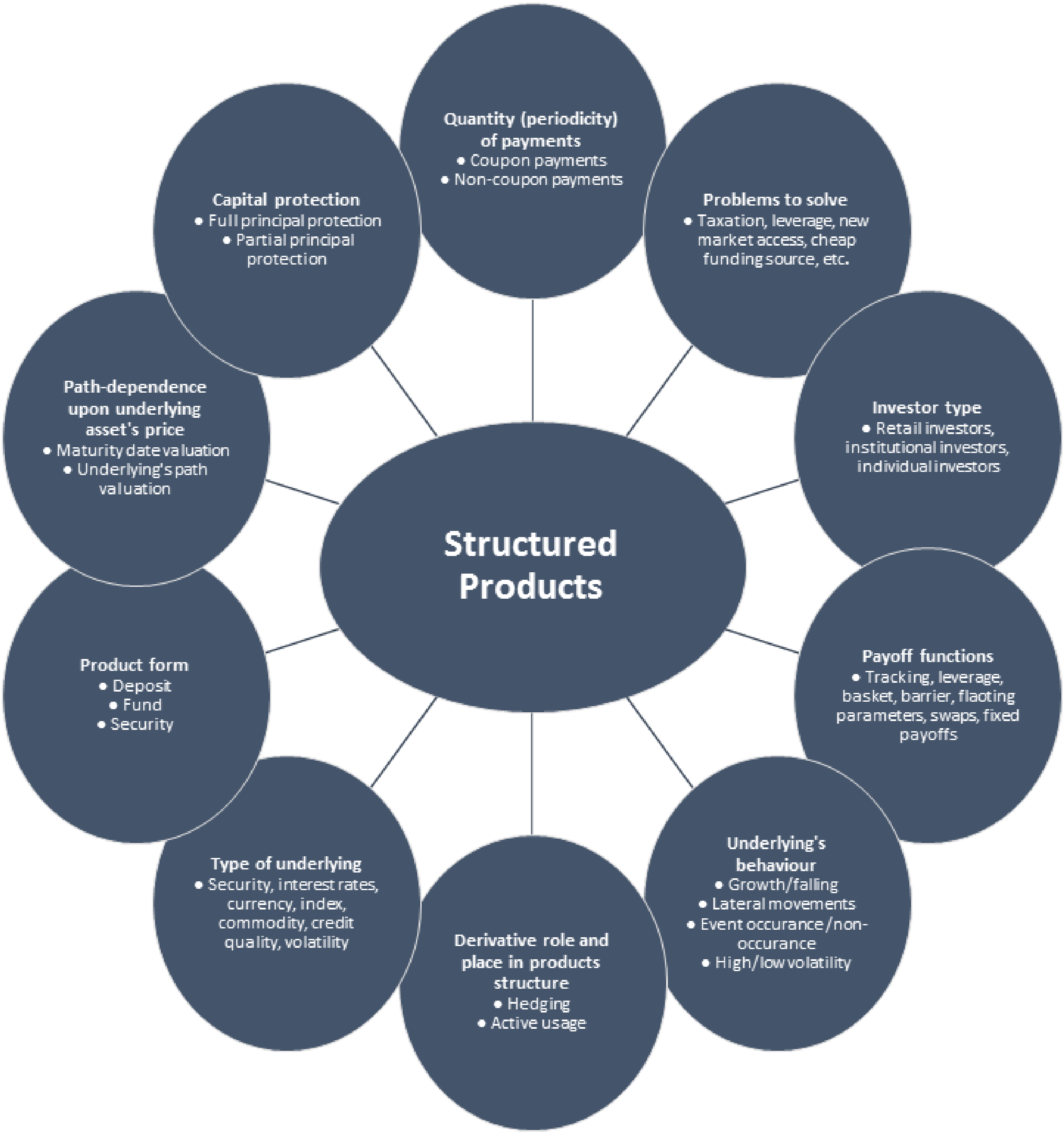


Figure 2.2: Classification of structured products according to [Ome09]

Chapter 3

Pricing of Traffic Light Options

3.1 Introduction of Correlation Options

The main part of this master thesis is about an innovative structured product which was independently developed by several London-based investment banks, such as Dresdner Kleinwort Wasserstein and Goldman Sachs International.[1]
The sharp decline in the stock market that occurred in early 2000 and the subsequent drop in interest rates weakened companies' financial strength. Many companies saw a substantial decrease in their margin for risk taking when their risk-bearing capital was eroded.
In 2001 the Danish Supervisory Authorities developed a new supervising tool which includes a traffic light scenario in order to measure companies' state of solvency. This tool consists of different scenarios where both the interest rate level and stock prices fall simultaneously. Shocks of real estate values are usually ignored since it is not practical, and real estate investments constitute an insignificant part of total portfolios. Especially, the Danish Life and Pension (L&P) sector is exposed in these scenarios.
There are two main reasons for L&P companies:

1. The duration is typically much longer on the liability side than on the asset side making the company exposed to negative shocks to interest rate levels.
2. Many L&P companies have issued guarantees on policy holder contributions which, with the low interest rate levels today, forces the companies to invest in the stock market in order to capture the higher return here.
   This investment behaviour exposes the companies to negative shocks of the stock market.

Companies which ignored to take adjustments in their risk exposure in accordance with the new rules introduced in mid-2001 had to report the red light status[2] after the equity markets collapse after "9/11".

[1] see [Kok09]
[2] see Def. 3.1.

After more than a decade of falling interest rates, L&P companies finally initiated hedging strategies involving the purchase of protection against further interest rate drops in the form of derivatives. The reported market value of Danish L&P companies holdings of financial derivatives increased from 0 in the first half of 2000 to USD 14.5*bn* in the late of 2005.[3]
The fundamental idea behind these instruments has been to construct derivatives which pay off in the traffic light scenarios in such a way, that however over-hedging is avoided. Over-hedging may result if the L&P company buys protection against downside interest rate and stock market risk separately. Thus the challenge is to structure products which pay off more when interest rates and stock prices fall simultaneously and less when only one of the variables moves adversely.

[3]Source: Danmarks Nationalbank, `https://www.nationalbanken.dk`. For comparison the 2005-position in derivatives corresponds to about 5% of the total market value of Danish $L\&P$ companies' liabilities which were estimated at DKK 1842*bn* in the same quarter.

Definition 1 (Traffic light scenarios).
There are 2 stress-test scenarios on the base capital of companies to point out the solvency state:

1. *Red light scenario involves:*
   - 70*bps*[4] *decrease in interest rates,*
   - 12% *decline in general stock prices and*
   - 8% *decrease in real estate investment values.*

   If an L&P company's base capital falls below a given critical level in this scenario, then the company is categorised with red light status.

   Consequences: In practical, this implies strict monitoring by the DFSA, and the company will be required to submit more frequent (monthly) solvency reports.

2. *Yellow light scenario involves:*
   - 100*bps* *decrease in interest rates,*
   - 30% *decline in general stock prices and*
   - 12% *decrease in real estate investment values.*

   If an L&P company's base capital falls below a given critical level in this scenario, then the company is categorised with yellow light status.

   Consequences: The company will be required to submit quarterly solvency reports.

3. *Green light scenario:*
   A company which can withstand the yellow light scenario without experiencing solvency problems will operate in the green light status.

   There are no additional consequences on the reporting side for green light companies.

Definition 2 (Correlation options).
In general, correlation options are represented as the following payoff:

$$(S_T - \bar{S})^+ \mathbb{1}_{R_T > \bar{R}} \quad or \quad (\bar{S} - S_T)^+ \mathbb{1}_{\bar{R} > R_T},$$

with given strike levels $\bar{S}$ *and* $\bar{R}$. S_T *and* R_T *are the values of the assets at maturity* T.

In this framework we model European style correlation options.

[4] One basis point is equivalent to 0.01% (1/100th of a percent) or 0.0001 in decimal form.

Another variation leads us to:

Definition 3 (Payoff structure of correlation options)**.**

$$C(S_T, R_T) = \begin{cases} (\bar{S} - S_T)^+ \cdot (\bar{R} - R_T)^+, \\ (\bar{S} - S_T)^+ \cdot (R_T - \bar{R})^+, \\ (S_T - \bar{S})^+ \cdot (\bar{R} - R_T)^+, \\ (S_T - \bar{S})^+ \cdot (R_T - \bar{R})^+, \end{cases} \tag{3.1}$$

with given strike levels $\bar{S}$ and $\bar{R}$.

Remark. *The question of alternative definitions for the traffic light option payoff arises. In [Jør07, Ch.2] is a short discussion of other possibilities, i.e. Jøergensen stated, that*

$$aS(t) - bL(t),$$

with suitable chosen constants a and b and a put option on that variable would be an alternative. Another piecewise linear payoff function could be obtained by specifying

$$C(S_T, R_T) = a\big[\bar{S} - S_T\big]^+ \mathbb{1}_{\bar{R} > R_T} + b\big[\bar{R} - R_T\big]^+ \mathbb{1}_{\bar{S} > S_T},$$

but to the best of Jøergensen's knowledge, none of these linear structures are seen in practice and therefore not further analysed.
This discussion came up through comments from referees and by discussions with members of the Structured Products group at Goldman Sachs International.
Investment bankers offering these structured products stated that the multiplicative payoff, given by (3.1)*, fits the needs for clients best since over-hedging is avoided.*

3.2 Model Framework

In this section, a framework of the basic traffic light option will be introduced with dependence on both an underlying stock portfolio and an underlying benchmark interest rate. Due to the fact that the most common and important benchmark interest rates in the financial industry are the London Inter-Bank Offered Rates (with different maturities) or LIBOR, we will use these rates for pricing.
Basic assumptions:

- The existence of a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ with the physical probability measure $\mathbb{P}$.
- Efficient and perfect market conditions are assumed.
- The settlement dates are given by $0 \leq T_0 < T_1 < \cdots < T_n$ which is called the tenor structure.
- Length between two tenor dates: $\tau_i = T_i - T_{i-1}$.

Definition 4 (Zero-coupon bond).

A T-maturity zero-coupon bond (ZCB) is a contract that guarantees its holder the payment of one unit of currency at time T with no intermediate payments. $B(t,T)$ is defined as the contract value at time $t < T$ and $B(T,T) = 1$ for all $T \in \mathbb{R}_+$ is also known as the face value of the ZCB.

For all tenor dates T_j from $0 \le j \le n$ we denote $B(0, T_j)$ as the ZCB maturing at time T_j.

Definition 5 (Forward LIBOR rates).

$$L_i(t) := \frac{1}{\tau_{i+1}} \left(\frac{B(t,T_i)}{B(t,T_{i+1})} - 1 \right) \quad \forall i = 1, \cdots, n, \tag{3.2}$$

is the simply compounded forward interest rate from T_i to T_{i+1}, as seen at time $t < T_i$.

Proposition 1 (Forward measure $\mathbb{Q}^{T_{i+1}} =: \mathbb{Q}^{i+1}$).

If the market is arbitrage-free, then for every $i = 1, \cdots, n$, there exists an equivalent martingale measure denoted by $\mathbb{Q}^{T_{i+1}}$ with given numeraire $B(t, T_{i+1})$, under which the LIBOR rate process $L_i(t)$ is a martingale.

Definition 6 (Terminal measure $\mathbb{Q}^{T_{n+1}} =: \mathbb{Q}^{n+1}$).

For $i = n$ follows that $\mathbb{Q}^{n+1}$ is the last equivalent martingale measure and it is called the Terminal measure. As we obtained, under this measure, the last forward LIBOR rate process $L_n(t)$ is a martingale.

Definition 7 (Numeraire). [5]

A numeraire is a price process or asset $N(t)_{0 \le t \le T}$, which is strictly positive for all $t \in [0, T]$. Numeraires are used to express all prices in a market. In this work we consider mostly T_i-bonds or discrete bank accounts as numeraires.

Definition 8 (Equivalent martingale measure (EMM)).

Let $(\Omega, \mathcal{F}, \mathbb{P})$ denote the probability space as before. The set of EMM is the set of probability measures $\mathbb{Q}^{i+1}$ with the following properties:

1. *$\mathbb{Q}^{i+1}$ is equivalent to $\mathbb{P}$, i.e. both measures have the same nullsets, for all $i = 1, \cdots, n$.*

2. *the forward LIBOR rates $L_i(t)$ are martingales under $\mathbb{Q}^{i+1}$ for all $i = 1, \cdots, n$, i.e. $\mathbb{E}^{Q^{i+1}} \left[\frac{L_i(t)}{B(t,T_{i+1})} \middle| \mathcal{F}_s \right] = \frac{L_i(s)}{B(s,T_{i+1})}$ for all $s \le t$.*

The definition of the EMM implies the need for a theorem that connects non-existence of arbitrage opportunities and completeness with equivalent martingale measures.

[5] see [Kaj04]

Theorem 1 (Unique EMM).
A market is free of arbitrage opportunities and every claim is attainable if for every choice of numeraire there exists a unique EMM.

Other representation of the forward LIBOR rates leads to[6]

$$B(t, T_{i+1})L_i(t) = \Big(B(t, T_i) - B(t, T_{i+1})\Big)\frac{1}{\tau_{i+1}},$$

this represents the price of a tradeable asset (difference between two discount bonds with notional amounts $\frac{1}{\tau_{i+1}}$). As such, when its price is expressed with respect to the numeraire $B(t, T_{i+1})$, it has to be a martingale under the corresponding measure
$\mathbb{Q}^{T_{i+1}} =: \mathbb{Q}^{i+1}$(Forward measure).
Hence, L_i is modeled according to a diffusion process under the forward measure $\mathbb{Q}^{i+1}$:

$$dL_i(t) = L_i(t)\lambda_i(t)dW^{i+1}(t), \quad \text{for all } i = 1, \cdots, n, \tag{3.3}$$

where W^{i+1} is a Brownian motion under $\mathbb{Q}^{i+1}$ and since we are in the log-normal LIBOR market model, the diffusion term $L_i(t)\lambda_i(t)$ is given by some deterministic function $\lambda_i(t)$.[7]
Each of these stochastic differential equations is called the LIBOR Market Model for the forward LIBOR rate process $L_i(t)$ under the equivalent martingale measure $\mathbb{Q}^{i+1}$.

Remark. *The solution of these stochastic differential equations(SDE) is given by*

$$L_i(t) = L_i(0)\exp\Big(-\frac{1}{2}\int_0^t \lambda_i(s)^2 ds + \int_0^t \lambda_i(s)dW_s^{i+1}\Big) \quad , i = 1, \cdots, n.$$

We get this explicit solution of each SDE by applying Itô's Formula to the process $L_i(t)$ using the function $f(t,x) = \log(x)$ or $f(t, L_i(t)) = \log(L_i(t))$.
Apply Itô:

$$f(t,x) = \log(x) \Rightarrow \frac{\partial f}{\partial t}(t,x) = 0, \quad \frac{\partial f}{\partial x}(t,x) = \frac{1}{x}, \quad \frac{\partial^2 f}{\partial x^2}(t,x) = -\frac{1}{x^2}.$$

$$df(t,x) = \frac{\partial f}{\partial t}(t,x)dt + \frac{\partial f}{\partial x}(t,x)dx + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(t,x)d[x,x]$$

Now inserting $L_i(t)$ leads to:

$$\begin{aligned} d\log(L_i(t)) =& \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial L_i}dL_i(t) + \frac{1}{2}\frac{\partial^2 f}{\partial L_i^2}d[L_i, L_i] \\ =& 0dt + \frac{1}{L_i}(L_i(t)\lambda_i(t)dW^{i+1}(t)) + \frac{1}{2}\Big(-\frac{1}{L_i(t)^2}\Big)(L_i(t)^2\lambda_i(t)^2dt) \\ =& \lambda_i(t)dW^{i+1}(t) - \frac{1}{2}\lambda_i(t)dt. \quad \square \end{aligned}$$

[6]see [BM06], p. 208
[7]will be discussed later in Section 3.3

Theorem 2 (Martingale pricing).

Suppose the equivalent martingale measure $\mathbb{Q}^N$ connected with the numeraire $N(t)$ is chosen. The price process $\pi(t)$ of any attainable claim $C(.)$ is given by the martingale pricing formula:

$$\pi(t) = N(t)\mathbb{E}^{\mathbb{Q}^N}\left[\frac{C(.)}{N(T)}\bigg|\mathcal{F}_t\right] \tag{3.4}$$

In our setup, the pricing of a T_i-claim $C\big(S(T_1), \ldots, S(T_{i+1}), L_0(T_0), \ldots, L_n(T_i)\big)$ is considered where S denotes some stock portfolio with $i \leq n$. Now we can use the martingale pricing theorem formula (3.4) to get:

$$\pi(t) = N(t)\mathbb{E}^{\mathbb{Q}^N}\left[\frac{1}{N(T_i)}C(S(T_0), \ldots, S(T_i), L_0(T_0), \ldots, L_n(T_i))|\mathcal{F}_t\right], \tag{3.5}$$

where $N = N(t)_{0\leq t\leq T_i}$ is a strictly positive price process, for the change of measure as a numeraire.
Now the question of what numeraire N (or equally what EMM $\mathbb{Q}^N$) naturally arises.
The fact, that L_n is a martingale under $\mathbb{Q}^{T_n+1} =: \mathbb{Q}^{n+1}$ with given dynamics (3.3) leads us to the obvious choice of the ZCB maturing at time T_{n+1} as the intuitive numeraire.
The remaining problem is to find the dynamics for S and all other LIBOR rates under this measure, which will be discussed in section 3.2.1.

3.2.1 Pricing of the traffic light option under the forward measure

Let us consider the valuation of the traffic light option with T_{n+1}- payoff given by the following theorem:

Theorem 3 (Traffic light option T_{n+1}-payoff)**.**

$$C\big(S(T_{n+1}), L_n(T_n)\big) = \big[\bar{S} - S(T_{n+1})\big]^+ \cdot \big[\bar{L} - L_n(T_n)\big]^+, \tag{3.6}$$

where $\bar{S}$ and $\bar{L}$ are given strike levels and $S(T_{n+1})$ is the stock portfolio price at time T_{n+1} and $L_n(T_n)$ is the value of the LIBOR rate at time T_n for the next period T_n to T_{n+1}.

Remark. *The payoff function is a product of the payoffs as seen in* (3.1) *of a standard interest rate floorlet and a plain vanilla equity put option with a European-style payoff structure.*

Due to the previous section, we will now choose the ZCB maturing at time T_{n+1} as numeraire. Now inserting the T_{n+1}-claim from (3.6) in (3.4) leads us to:

$$\pi(t) = B(t, T_{n+1})\mathbb{E}^{\mathbb{Q}^{n+1}}\left[\frac{[\bar{S} - S(T_{n+1})]^+ \cdot [\bar{L} - L_n(T_n)]^+}{B(T_{n+1}, T_{n+1})}\bigg|\mathcal{F}_t\right] \tag{3.7}$$

$$= B(t, T_{n+1})\mathbb{E}^{\mathbb{Q}^{n+1}}\left[\left[\bar{S} - \frac{S(T_{n+1})}{B(T_{n+1}, T_{n+1})}\right]^+ \cdot [\bar{L} - L_n(T_n)]^+\bigg|\mathcal{F}_t\right]. \tag{3.8}$$

Note that in the framework of the log-normal forward model, the LIBOR rate $L_n(t)$ is log-normal under its own measure. Due to the dependence of the instantaneous development in the ZCB maturing at T_{n+1} for the stock portfolio price dynamics, we use the fact from the FTAP[8] that the discounted stock portfolio process $\frac{S(t)}{B(t,T_{n+1})}$ is a martingale. Hence, it is actually the forward stock price by the no-arbitrage assumption in this framework. Assuming lognormality of the forward stock price process[9] $\frac{S(t)}{B(t,T_{n+1})}$ leads us to the following two stochastic differential equations:

$$d\left(\frac{S(t)}{B(t, T_{n+1})}\right) = \left(\frac{S(t)}{B(t, T_{n+1})}\right)\sigma_t dW_{\frac{S}{B}}^{n+1}, \tag{3.9}$$

$$dL_n(t) = L_n(t)\lambda_n(t)dW_L^{n+1}(t), \tag{3.10}$$

with

$$d\big\langle W_{\frac{S}{B}}^{n+1}, W_L^{n+1}\big\rangle(t) = \rho_t dt, \tag{3.11}$$

where ρ_t and σ_t are deterministic functions of time. W_L^{n+1} and $W_{\frac{S}{B}}^{n+1}$ are defined as the Brownian motion generated from the LIBOR rates respectively the discounted asset price process with respect to the terminal measure $\mathbb{Q}^{n+1}$.

[8]see Appendix A

[9]is the future stock price, which is discounted by a ZCB

From our assumptions above we can derive the volatility σ_t of the discounted asset price process from market prices on plain vanilla European call options since a closed form solution is given by a Black 1976 formula[10].

$$\begin{aligned}\Pi^{Call}(t) =& B(t,T)\mathbb{E}^{\mathbb{Q}^T}\left[\frac{[S(T)-K]^+}{B(T,T)}\bigg|\mathcal{F}_t\right]\\ =& S(t)N(d_1) - B(t,T)N(d_2),\end{aligned} \tag{3.12}$$

where

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S_t}{B(t,T)K}\right) + \frac{1}{2}\sigma_S^2}{\sigma_S},\\ d_2 &= \frac{\ln\left(\frac{S_t}{B(t,T)K}\right) - \frac{1}{2}\sigma_S^2}{\sigma_S},\end{aligned}$$

and

$$\sigma_S^2 = \int_t^T \sigma_u^2 du.$$

Due to the fact of lognormality of the forward stock price process, the volatility of this process can be derived from quoted prices by inversion of (3.12). [11]

The following theorem represents the main result in this paper:

Theorem 4 (Analytical formula for the value of a traffic light option at time t)**.**
The T_{n+1}-payoff at time t is given by:

$$\begin{aligned}\pi(S(t),L_n(t),t;\rho_{SL}) =& S(t)L_n(t)\Bigg[\tilde{S}\cdot\tilde{L}\cdot M\left(\frac{\ln\tilde{S}-\mu_x}{\sigma_x},\frac{\ln\tilde{L}-\mu_y}{\sigma_y};\rho_{SL}\right)\\ &- \tilde{L}\cdot M\left(\frac{\ln\tilde{S}-\mu_x}{\sigma_x}-\sigma_x,\frac{\ln\tilde{L}-\mu_y}{\sigma_y}-\rho_{SL}\sigma_x;\rho_{SL}\right)\\ &- \tilde{S}\cdot M\left(\frac{\ln\tilde{S}-\mu_x}{\sigma_x}-\rho_{SL}\sigma_y,\frac{\ln\tilde{L}-\mu_y}{\sigma_y}-\sigma_y;\rho_{SL}\right)\\ &+ e^{\sigma_{xy}}\cdot M\left(\frac{\ln\tilde{S}-\mu_x}{\sigma_x}-\rho_{SL}\sigma_y,\frac{\ln\tilde{L}-\mu_y}{\sigma_y}-\rho_{SL}\sigma_x;\rho_{SL}\right)\Bigg],\end{aligned} \tag{3.13}$$

where

$$\begin{aligned} \tilde{S} &= \frac{\bar{S}B(t,T_{n+1})}{S(t)}, & \sigma_x^2 &= \int_t^{T_{n+1}}\sigma_s^2 ds,\\ \tilde{L} &= \frac{\bar{L}}{L_n(t)}, & \sigma_y^2 &= \int_t^{T_n}\lambda_n^2(s)ds,\\ \mu_x &= -\frac{1}{2}\int_t^{T_{n+1}}\sigma_s^2 ds, & \sigma_{xy} &= \int_t^{Tn}\sigma_s\lambda_n(s)\rho_s ds,\\ \mu_y &= -\frac{1}{2}\int_t^{T_n}\lambda_n(s)^2 ds, & \rho_{SL} &= \frac{\sigma_{xy}}{\sigma_x\sigma_y},\end{aligned}$$

[10]for more detailed information see [BM06].

[11]since the price function is monoton and increasing => existence of a inverse function => implicit volatility.

and $M(.,.,;\rho)$ *is the cumulative probability of the standardized bivariate normal distribution with correlation coefficient* ρ.

Remark. *Before we can start with the proof, we need some essential properties of the standardised bivariate normal distribution:*
Suppose:

$$\vec{v} := \begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}\right), \tag{3.14}$$

with $\mu_x, \mu_y, \sigma_x^2, \sigma_y^2$ and σ_{xy} are the constant mean, variance and covariance coefficients. The coefficient of correlation is given as:

$$\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y}.$$

Density of a bivariate normal distribution is given by:

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left(-\frac{z}{2(1-\rho^2)}\right),$$

with $z = \left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2$.
The standardised bivariate density can be factorized as:

$$f(x,y) = f(x)f(x|y), \tag{3.15}$$

where

$$f(x) = \int_{-\infty}^{+\infty} f(x,y)dy = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}, \tag{3.16}$$

and

$$f(y) = \int_{-\infty}^{+\infty} f(x,y)dx = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-\frac{(y-\mu_y)^2}{2\sigma_y^2}},$$

are the marginal density of x and y.
The conditional density $f(y|x)$ is the density of y conditional on x:

$$f(y|x) = \frac{1}{\sqrt{2\pi}\sigma_y\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2\sigma_y^2(1-\rho^2)}\left(y - \mu_y - \frac{\rho\sigma_y}{\sigma_x}(x-\mu_x)\right)^2\right).$$

Now we have all properties for the proof:

Proof. It is essential that

$$L_n(t)e^Y = \mathbb{E}^{\mathbb{Q}^{n+1}}[L_n(T_n)|\mathcal{F}_t],$$
$$\frac{S(t)}{B(t,T_{n+1})}e^X = \mathbb{E}^{\mathbb{Q}^{n+1}}\left[\frac{S(T_{n+1})}{B(T_{n+1},T_{n+1})}\middle|\mathcal{F}_t\right],$$

holds with $\begin{pmatrix} X \\ Y \end{pmatrix}$ are bivariate normally distributed, independent of $\mathcal{F}_t$ and

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}\right),$$

with

$$\mu_x = -\frac{1}{2}\int_t^{T_{n+1}} \sigma_s^2 ds, \quad \mu_y = -\frac{1}{2}\int_t^{T_n} \lambda_n(s)^2 ds, \quad \sigma_x^2 = \int_t^{T_{n+1}} \sigma_s^2 ds,$$
$$\sigma_y = \int_t^{T_n} \lambda_n(s)^2 ds \text{ and } \sigma_{xy} = \int_t^{T_n} \sigma_s \lambda_n(s)\rho_s ds.$$

Now we can use the formula (3.8) and calculate this expression more in detail:

$$\begin{aligned}
\pi(t) &= B(t,T_{n+1})\mathbb{E}^{\mathbb{Q}^{n+1}}\left[\left[\bar{S} - \frac{S(T_{n+1})}{B(T_{n+1},T_{n+1})}\right]^+ \cdot [\bar{L} - L_n(T_n)]^+ \middle| \mathcal{F}_t\right] \\
&= B(t,T_{n+1})\mathbb{E}^{\mathbb{Q}^{n+1}}\left[\left[\bar{S} - \frac{S(t)}{B(t,T_{n+1})}e^X\right]^+ \cdot [\bar{L} - L_n(t)e^Y]^+ \middle| \mathcal{F}_t\right] \\
&= S(t)L_n(t)\mathbb{E}^{\mathbb{Q}^{n+1}}\left[\left[\tilde{S} - e^X\right]^+ \cdot [\tilde{L} - e^Y]^+ \middle| \mathcal{F}_t\right] \\
&= S(t)L_n(t) \cdot \Big[\tilde{S}\tilde{L}\mathbb{E}^{\mathbb{Q}^{n+1}}[\mathbb{1}_{\{e^X<\tilde{S}\}}\mathbb{1}_{\{e^Y<\tilde{L}\}}] \\
&\quad - \tilde{L} \cdot \mathbb{E}^{\mathbb{Q}^{n+1}}[e^X \mathbb{1}_{\{e^X<\tilde{S}\}}\mathbb{1}_{\{e^Y<\tilde{L}\}}] \\
&\quad - \tilde{S} \cdot \mathbb{E}^{\mathbb{Q}^{n+1}}[e^Y \mathbb{1}_{\{e^X<\tilde{S}\}}\mathbb{1}_{\{e^Y<\tilde{L}\}}] \\
&\quad + \mathbb{E}^{\mathbb{Q}^{n+1}}[e^X e^Y \mathbb{1}_{\{e^X<\tilde{S}\}}\mathbb{1}_{\{e^Y<\tilde{L}\}}]\Big].
\end{aligned}$$

Inserting the argument of lognormality shows the second equality. Redefining $\tilde{S} := \frac{\bar{S}B(t,T_{n+1})}{S(t)}$, $\tilde{L} = \frac{\bar{L}}{L_n(t)}$ and the independence between the variables and the σ-algebra lead to the last equation.
Now, we evaluate the four expectations in order to find a price of the traffic light option. Since we are in the framework of standard bivariate normal distribution, we can now use the basics from the remark above.
First, we compute the first expectation:

$$\begin{aligned}
\mathbb{E}^{\mathbb{Q}^{n+1}}[\mathbb{1}_{\{e^X<\tilde{S}\}}\mathbb{1}_{\{e^Y<\tilde{L}\}}] &= \mathbb{Q}^{n+1}(x < \ln\tilde{S}, y < \ln\tilde{L}) \\
&= \int_{-\infty}^{\ln\tilde{S}}\int_{-\infty}^{\ln\tilde{L}} f(x,y)dydx \\
&= \int_{-\infty}^{\ln\tilde{S}}\int_{-\infty}^{\ln\tilde{L}} \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}\exp\Big(-\frac{1}{2(1-\rho^2)}\cdot z\Big)dydx,
\end{aligned}$$

with

$$z = \Big(\frac{x-\mu_x}{\sigma_x}\Big)^2 - 2\rho\Big(\frac{x-\mu_x}{\sigma_x}\Big)\Big(\frac{y-\mu_y}{\sigma_y}\Big) + \Big(\frac{y-\mu_y}{\sigma_y}\Big)^2,$$

leads with an appropriate substitution $u = \frac{x-\mu_x}{\sigma_x}$ and $v = \frac{y-\mu_y}{\sigma_y}$ to the following result:

$$M\Big(\frac{\ln\tilde{S}-\mu_x}{\sigma_x}, \frac{\ln\tilde{L}-\mu_y}{\sigma_y}; \rho_{SL}\Big).$$

Second expectation term leads to:

$$\mathbb{E}^{\mathbb{Q}^{n+1}}[e^X \mathbb{1}_{\{e^X<\tilde{S}\}} \mathbb{1}_{\{e^Y<\tilde{L}\}}] = \int_{-\infty}^{\ln \tilde{S}} \int_{-\infty}^{\ln \tilde{L}} e^x f(y) f(x|y) dx dy.$$

Now, the exponent is given by:

$$x - \frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2 - \frac{1}{2\sigma_x^2(1-\rho^2)}\left(x-\mu_x-\frac{\rho\sigma_x}{\sigma_y}(y-\mu_y)\right)^2 = \mu_x + \frac{1}{2}\sigma_x^2 - \frac{1}{2(1-\rho^2)}(u^2-2\rho uv+v^2)$$

with the substitution: $u = \frac{x-\mu_x}{\sigma_x} - \sigma_x$ and $v = \frac{y-\mu_y}{\sigma_y} - \rho\sigma_x$.
Then, we get:

$$\mathbb{E}^{\mathbb{Q}^{n+1}}[e^X \mathbb{1}_{\{e^X<\tilde{S}\}} \mathbb{1}_{\{e^Y<\tilde{L}\}}] = e^{\mu_x+\frac{1}{2}\sigma_x^2} \cdot M\left(\frac{\ln \tilde{S}-\mu_x}{\sigma_x} - \sigma_x, \frac{\ln \tilde{L}-\mu_y}{\sigma_y} - \rho_{SL}\sigma_x; \rho_{SL}\right).$$

With the argument of the symmetry of the bivariate normal distribution, we can write the third expectation:

$$\mathbb{E}^{\mathbb{Q}^{n+1}}[e^Y \mathbb{1}_{\{e^X<\tilde{S}\}} \mathbb{1}_{\{e^Y<\tilde{L}\}}] = e^{\mu_y+\frac{1}{2}\sigma_y^2} \cdot M\left(\frac{\ln \tilde{S}-\mu_x}{\sigma_x} - \rho_{SL}\sigma_y, \frac{\ln \tilde{L}-\mu_y}{\sigma_y} - \sigma_y; \rho_{SL}\right).$$

The last expectation term can be calculated:

$$\mathbb{E}^{\mathbb{Q}^{n+1}}[e^X e^Y \mathbb{1}_{\{e^X<\tilde{S}\}} \mathbb{1}_{\{e^Y<\tilde{L}\}}] = \int_{-\infty}^{\ln \tilde{S}} \int_{-\infty}^{\ln \tilde{L}} e^x e^y f(y) f(y|x) dy dx.$$

Then we get again for the exponent:

$$\begin{aligned} x + y - \frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - \frac{1}{2\sigma_y^2(1-\rho^2)}\left(y-\mu_y-\frac{\rho\sigma_y}{\sigma_x}(x-\mu_x)\right)^2 = \\ = \mu_x + \mu_y + \frac{1}{2}\sigma_x^2 + \frac{1}{2}\sigma_y^2 + \rho\sigma_x\sigma_y - \frac{1}{2(1-\rho^2)}(u^2-2\rho uv+v^2) \end{aligned}$$

with the substitution $u = \frac{x-\mu_x}{\sigma_x} - \rho\sigma_y - \sigma_x$ and $v = \frac{y-\mu_y}{\sigma_y} - \rho\sigma_x - \sigma_y$.
Putting everything together leads to:

$$\begin{aligned} &\mathbb{E}^{\mathbb{Q}^{n+1}}[e^X e^Y \mathbb{1}_{\{e^X<\tilde{S}\}} \mathbb{1}_{\{e^Y<\tilde{L}\}}] \\ &= e^{\mu_x+\mu_y+\frac{1}{2}\sigma_x^2+\frac{1}{2}\sigma_y^2+\rho\sigma_x\sigma_y} \cdot M\left(\frac{\ln \tilde{S}-\mu_x}{\sigma_x} - \rho_{SL}\sigma_y - \sigma_x, \frac{\ln \tilde{L}-\mu_y}{\sigma_y} - \rho_{SL}\sigma_x - \sigma_y; \rho_{SL}\right). \end{aligned}$$

Inserting the four expectations gives us the following result:

$$\begin{aligned} \pi(t) = &S(t)L_n(t)\Big[\tilde{S}\tilde{L} \cdot M(.,.;.) \\ &- \tilde{L}e^{\mu_x+\frac{1}{2}\sigma_x^2} \cdot M(.,.;.) \\ &- \tilde{S}e^{\mu_y+\frac{1}{2}\sigma_y^2} \cdot M(.,.;.) \\ &+ e^{\mu_x+\mu_y+\frac{1}{2}\sigma_x^2+\frac{1}{2}\sigma_y^2+\rho\sigma_x\sigma_y} \cdot M(.,.;.)\Big]. \end{aligned}$$

Now using the fact that $\mu_x = -\frac{1}{2}\sigma_x$ and $\mu_y = -\frac{1}{2}\sigma_y$ leads us to the final result:

$$\begin{aligned}\pi(t) =& S(t)L_n(t)\Big[\tilde{S}\tilde{L}\cdot M(.,.;.)\\ &- \tilde{L}\cdot M(.,.;.)\\ &- \tilde{S}\cdot M(.,.;.)\\ &+ e^{\sigma_{xy}}\cdot M(.,.;.)\Big]\end{aligned}$$

$\square$

Remark. *We want to compute the integrals used in Theorem* (4)*:*

$$\sigma_y^2 = \int_t^{T_n} \lambda_n^2(s)ds \text{ with } \lambda_n(t) = (a + (T_n - t)b)\cdot e^{-c(T_n-t)} + d.$$

First we want to compute all single integrals and then put everything together.

$$\begin{aligned}\sigma_y^2 = \int_t^{T_n} \Big(& a^2\cdot e^{-2c(T_n-s)} + b^2\cdot T_n^2\cdot e^{-2c(T_n-s)} + b^2\cdot s^2\cdot e^{-2c(T_n-s)} + d^2\\ &+ 2\cdot a\cdot b\cdot T_n\cdot e^{-2c(T_n-s)} - 2\cdot a\cdot b\cdot s\cdot e^{-2c(T_n-s)}\\ &+ 2\cdot a\cdot d\cdot e^{-c(T_n-s)} - 2\cdot b^2\cdot s\cdot T_n\cdot e^{-2c(T_n-s)}\\ &+ 2\cdot b\cdot d\cdot T_n\cdot e^{-c(T_n-s)} - 2\cdot b\cdot d\cdot s\cdot e^{-c(T_n-s)}\Big)ds.\end{aligned}$$

The first terms, containing only the exponential function, will be now calculated:

$$\int_t^{T_n} a^2\cdot e^{-2c(T_n-s)}ds = a^2\cdot \frac{e^{-2c(T_n-s)}}{2c}\Big|_{s=t}^{T_n} = a^2\cdot\frac{e^{-2c(T_n-T_n)}}{2c} - a^2\cdot\frac{e^{-2c(T_n-t)}}{2c} = \frac{a^2}{2c}\cdot\big(1 - e^{-2c(T_n-t)}\big).$$

$$\int_t^{T_n} 2\cdot a\cdot b\cdot T_n\cdot e^{-2c(T_n-s)}ds = \frac{a\cdot b\cdot T_n}{c}\cdot\big(1 - e^{-2c(T_n-t)}\big).$$

$$\int_t^{T_n} b^2\cdot T_n^2\cdot e^{-2c(T_n-s)}ds = \frac{b^2\cdot T_n^2}{2c}\cdot\big(1 - e^{-2cT_n+2ct}\big).$$

$$\int_t^{T_n} 2\cdot a\cdot d\cdot e^{-c(T_n-s)}ds = \frac{2\cdot a\cdot d}{c}\cdot\big(1 - e^{-c(T_n-t)}\big).$$

$$\int_t^{T_n} 2\cdot b\cdot d\cdot T_n\cdot e^{-c(T_n-s)}ds = \frac{2\cdot b\cdot d\cdot T_n}{c}\cdot\big(1 - e^{-c(T_n-t)}\big).$$

$$\int_t^{T_n} d^2 ds = (T_n - t)\cdot d^2.$$

Next, we calculate the terms given by $\int_t^{T_n} s\cdot e^{-2c(T_n-s)}ds$ *with integration by parts:*

$$\begin{aligned}\int_t^{T_n} 2\cdot a\cdot b\cdot s\cdot e^{-2c(T_n-s)}ds =& 2\cdot a\cdot b\cdot \int_t^{T_n} s\cdot e^{-2c(T_n-s)}ds\\ =& 2\cdot a\cdot b\cdot\left(s\cdot\frac{e^{-2c(T_n-s)}}{2c}\Big|_{s=t}^{T_n} - \int_t^{T_n}\frac{e^{-2c(T_n-s)}}{2c}ds\right)\\ =& 2\cdot a\cdot b\cdot\left(\frac{1}{2c}\big(T_n - t\cdot e^{-2c(T_n-t)}\big) - \int_t^{T_n}\frac{e^{-2c(T_n-s)}}{2c}ds\right).\end{aligned}$$

Now, we calculate the intermediate result:

$$\int_t^{T_n} \frac{e^{-2c(T_n-s)}}{2c} ds = \frac{e^{-2c(T_n-s)}}{4 \cdot c^2}\Big|_{s=t}^{T_n} = \frac{1}{4c^2}\Big(1 - e^{-2c(T_n-t)}\Big).$$

The results are given by:

$$\int_t^{T_n} 2 \cdot a \cdot b \cdot s \cdot e^{-2c(T_n-s)} ds = 2 \cdot a \cdot b \cdot \left(\frac{1}{2c}\big(T_n - t \cdot e^{-2c(T_n-t)}\big) - \frac{1}{4c^2}\big(1 - e^{-2c(T_n-t)}\big)\right).$$

$$\int_t^{T_n} 2 \cdot b^2 \cdot T_n \cdot s \cdot e^{-2c(T_n-s)} ds = 2 \cdot b^2 \cdot T_n \cdot \left(\frac{1}{2c}\big(T_n - t \cdot e^{-2c(T_n-t)}\big) - \frac{1}{4c^2}\big(1 - e^{-2c(T_n-t)}\big)\right).$$

$$\int_t^{T_n} 2 \cdot b \cdot d \cdot s \cdot e^{-c(T_n-s)} ds = 2 \cdot b \cdot d \cdot \left(\frac{1}{c}\big(T_n - t \cdot e^{-c(T_n-t)}\big) - \frac{1}{c^2}\big(1 - e^{-c(T_n-t)}\big)\right).$$

The last term has to be evaluated with two times integration by parts:

$$\int_t^{T_n} b^2 \cdot s^2 \cdot e^{-2c(T_n-s)} ds = b^2\left(s^2 \cdot \frac{e^{-2c(T_n-s)}}{2 \cdot c}\Big|_{s=t}^{T_n} - [\int_t^{T_n} 2 \cdot s \cdot \frac{e^{-2c(T_n-s)}}{2 \cdot c} ds]\right) =$$

$$= b^2\left(\frac{1}{2c}\big(T_n^2 - t^2 \cdot e^{-2c(T_n-t)}\big) - \left[2 \cdot s \cdot \frac{e^{-2c(T_n-s)}}{4 \cdot c^2}\Big|_{s=t}^{T_n} - \left[\int_t^{T_n} 2 \cdot \frac{e^{-2c(T_n-s)}}{4 \cdot c^2} ds\right]\right]\right)$$

With the intermediate results:

$$2 \cdot s \cdot \frac{e^{-2c(T_n-s)}}{4 \cdot c^2}\Big|_{s=t}^{T_n} = \frac{1}{2c^2} \cdot \big(T_n - t \cdot e^{-2c(T_n-t)}\big),$$

and

$$\int_t^{T_n} 2 \cdot \frac{e^{-2c(T_n-s)}}{4 \cdot c^2} ds = 2 \cdot \frac{e^{-2c(T_n-s)}}{8c^3}\Big|_{s=t}^{T_n} = \frac{1}{4c^3} \cdot \big(1 - e^{-2c(T_n-t)}\big),$$

follows:

$$\begin{aligned}&\int_t^{T_n} b^2 \cdot s^2 \cdot e^{-2c(T_n-s)} ds \\ &= b^2\left(\frac{1}{2c}\big(T_n^2 - t^2 \cdot e^{-2c(T_n-t)}\big) - \frac{1}{2c^2} \cdot \big(T_n - t \cdot e^{-2c(T_n-t)}\big) + \frac{1}{4c^3} \cdot \big(1 - e^{-2c(T_n-t)}\big)\right).\end{aligned}$$

Overall, we get the following result:

$$\begin{aligned}\sigma_y^2 =& \left(1 - e^{-c(T_n-t)}\right) \cdot \left(\frac{2 \cdot a \cdot d}{c} + \frac{2 \cdot b \cdot d \cdot T_n}{c}\right) \\ &+ \big(1 - e^{-2c(T_n-t)}\big)\left(\frac{a^2}{2c} + \frac{a \cdot b \cdot T_n}{c} + \frac{b^2 \cdot T_n^2}{2c}\right) \\ &+ 2 \cdot b \cdot \left(\frac{1}{2c}\big(T_n \cdot -t \cdot e^{-2c(T_n-t)}\big) - \frac{1}{4c^2}\big(1 - e^{-2c(T_n-t)}\big)\right)(-a - b \cdot T_n) \\ &- 2 \cdot b \cdot d \cdot \left(\frac{1}{c}\big(T_n - t \cdot e^{-c(T_n-t)}\big) - \frac{1}{c^2}\big(1 - e^{-c(T_n-t)}\big)\right) \\ &+ b^2\left(\frac{1}{2c}\big(T_n^2 - t^2 \cdot e^{-2c(T_n-t)}\big) - \frac{1}{2c^2} \cdot \big(T_n - t \cdot e^{-2c(T_n-t)}\big) + \frac{1}{4c^3} \cdot \big(1 - e^{-2c(T_n-t)}\big)\right) \\ &+ \big(T_n - t\big) \cdot d^2.\end{aligned}$$

$$\sigma_x^2 = \int_t^{T_{n+1}} \sigma_s^2 ds = \big(T_{n+1} - t\big) \cdot \sigma_s^2, \quad \textit{with } \sigma_s \textit{ deterministic.}$$
$$\mu_x = -\frac{1}{2} \int_t^{T_{n+1}} \sigma_s^2 ds = -\frac{1}{2}\sigma_x^2.$$
$$\mu_y = -\frac{1}{2} \int_t^{T_n} \lambda_n(s)^2 ds = -\frac{1}{2}\sigma_y^2.$$

The last equation for σ_{xy} will be evaluated:

$$\sigma_{xy} = \int_t^{T_n} \sigma_s \lambda_n(s) \rho_s ds, \quad \textit{with } \sigma_s \textit{ and } \rho_s \textit{ deterministic.}$$
$$\sigma_{xy} = \rho_s \cdot \sigma_s \int_t^{T_n} \Big((a + (T_n - s)b) \cdot e^{-c(T_n - s)} + d \Big) ds$$
$$= \rho_s \cdot \sigma_s \int_t^{T_n} \Big(a \cdot e^{-c(T_n - s)} + T_n \cdot b \cdot e^{-c(T_n - s)} - s \cdot b \cdot e^{-c(T_n - s)} + d \Big) ds$$

$$\int_t^{T_n} a \cdot e^{-c(T_n - s)} ds = a \cdot \frac{e^{-c(T_n - s)}}{c} \Big|_{s=t}^{T_n} = \frac{a}{c} \Big(1 - e^{-c(T_n - t)} \Big).$$
$$\int_t^{T_n} T_n \cdot b \cdot e^{-c(T_n - s)} ds = \frac{T_n \cdot b}{c} \Big(1 - e^{-c(T_n - t)} \Big).$$

$$\int_t^{T_n} b \cdot s \cdot e^{-c(T_n - s)} ds = b \cdot \left[s \cdot \frac{e^{-c(T_n - s)}}{c} \Big|_{s=t}^{T_n} - \int_t^{T_n} \frac{e^{-c(T_n - s)}}{c} ds \right]$$
$$= b \cdot \left[\frac{1}{c} \big(T_n - t \cdot e^{-c(T_n - t)} \big) - \int_t^{T_n} \frac{e^{-c(T_n - s)}}{c} ds \right]$$
$$= b \cdot \left[\frac{1}{c} \big(T_n - t \cdot e^{-(T_n - t)c} \big) - \frac{e^{-c(T_n - s)}}{c^2} \Big|_{s=t}^{T_n} \right]$$
$$= b \cdot \left[\frac{1}{c} \big(T_n - t \cdot e^{-c(T_n - t)} \big) - \frac{1}{c^2} \Big(1 - e^{-c(T_n - t)} \Big) \right].$$

All together we get for σ_{xy}:

$$\sigma_{xy} = \rho_s \cdot \sigma_s \Bigg[\big(1 - e^{-c(T_n - t)} \big) \cdot \left(\frac{a}{c} + \frac{T_n \cdot b}{c} \right)$$
$$- b \cdot \left(\frac{1}{c} \Big(T_n - t \cdot e^{-c(T_n - t)} \Big) - \frac{1}{c^2} \Big(1 - e^{-c(T_n - t)} \Big) + (T_n - t) \cdot d \right) \Bigg].$$

3.2.2 Valuation under the spot measure

In theory, it is often common to use the stochastic differential equation of any asset (S) under the risk neutral measure($\mathbb{Q}$):

$$dS_t = S_t(r_t dt + \sigma_t dW_t^{\mathbb{Q}}), \tag{3.17}$$

By applying $It\hat{o}'s$-Formula we get:

$$S_t = S_0 \exp \int_0^t (r_s - \frac{1}{2}\sigma_s^2)ds + \int_0^t \sigma_s dW_s^{\mathbb{Q}}. \tag{3.18}$$

Since the involvement of instantaneous rates, it is more practical to stick to the LIBOR rates. The aim is to use a discrete rate version of the dynamics (3.17). We will now introduce the discretely compounded bank account as the numeraire with common used strategy.

3.2.3 Discretisation for the spot measure

The spot LIBOR portfolio invests in the ZCB using the following strategy:

Definition 9 (Rolling strategy in ZCB)**.**
The self-financing strategy follows:

1. *At time* 0*, start with 1 euro, buy* $\frac{1}{B(0,T_0)}$ T_0*-bonds.*
2. *At time* T_0*, receive* $\frac{1}{B(0,T_0)}$ *euro, buy* $\frac{1}{B(0,T_0)}/B(T_0,T_1)$ T_1*-bonds.*
3. *At time* T_1*, receive* $\frac{1}{B(0,T_0)}/B(T_0,T_1)$ *euro, buy* $\frac{1}{B(0,T_0)}/B(T_0,T_1)/B(T_1,T_2)$ T_2*-bonds.*

The value of this self-financing strategy at any time t is given by:

$$B^d(t) := \frac{B(t,T_{i(t)})}{B(0,T_0)} \prod_{j=0}^{i(t)-1} \frac{B(T_j,T_j)}{B(T_j,T_{j+1})} = \frac{B(t,T_{i(t)})}{B(0,T_0)} \prod_{j=0}^{i(t)-1} (1+\tau_{j+1}L_j(T_j)), \tag{3.19}$$

with index function: $i(t) = \inf\{k | T_{k-1} \leq t < T_k\}$.

Remark. *For* $T_0 = 0$ *and* $t = T_k$ *for some* k*, the discrete bank account reduces to:*

$$B^d(T_k) = \prod_{j=0}^{k-1} \big(1+\tau_{j+1}L_j(T_j)\big). \tag{3.20}$$

Assuming that the bond prices have dynamics:

$$dB(t,T_j) = B(t,T_j)(\alpha(t,T_j)dt + \beta(t,T_j)dW(t)),$$

under some underlying probability measure, then using $It\hat{o}'s$ lemma on $B^d(t)$ gives:

$$dB^d(t) = B^d(t)(\alpha(t,T_{i(t)})dt + \beta(t,T_{i(t)})dW(t)).$$

Due to the fact that the dynamics involve instantaneous drift and diffusion terms from the bond dynamics, it is easier to let the future claims to be priced tied to the settlement dates of the LIBOR rates[12] while modeling.
Determining the discretely compounded analog of the asset price process directly reveals

$$S_t = S_0 \frac{B(t, T_{i(t)})}{B(0, T_0)} \prod_{j=0}^{i(t)-1} (1 + \tau_{j+1} L_j(T_j)) \exp\left(\int_0^t -\frac{1}{2}\sigma_s^2 ds + \int_0^t \sigma_s dW_S^d(s) \right). \tag{3.21}$$

The equivalent martingale measure $\mathbb{Q}^{B^d} =: \mathbb{Q}^d$ corresponding to the discrete bank account B^d as numeraire is denoted as the spot LIBOR measure. The discounted asset price process $\frac{S(t)}{B^d(t)}$ is then a martingale under $\mathbb{Q}^d$:

$$\frac{S(t)}{B^d(t)} = \mathbb{E}^d \left[\frac{S(T)}{B^d(T)} \middle| \mathcal{F}_t \right], \quad \text{for all } t \leq T. \tag{3.22}$$

The discounted asset price process $\frac{S(t)}{B^d(t)}$ obviously satisfies the martingale property.
Pricing under the spot LIBOR measure requires that the dynamics of the LIBOR rates L_i for $i = 1, ..., n$ have to be found in order to calculate the expectation:

$$\pi(t) = B^d(t) \mathbb{E}^d \left[\frac{1}{B^d(T_i)} C(S(T_0), \dots, S(T_i), L_0(T_0), \dots, L_n(T_i)) \right]. \tag{3.23}$$

These dynamics are derived in the lognormal LIBOR market model in [Jam97] and given by:

$$dL_i = L_i(t) \sum_{j=i(t)}^{i} \frac{\tau_{j+1} L_j(t) \rho_{i,j} \lambda_j(t)}{1 + \tau_{j+1} L_j(t)} \lambda_i(t) dt + L_i(t) \lambda_i(t) dW_i^d(t), \tag{3.24}$$

where W_i^d is a Wiener process under the spot LIBOR measure $\mathbb{Q}^d$ and $\rho_{i,j}$ is the correlation coefficient between the Wiener processes W_i^d and W_j^d.
It is important that the model is only completely determined at the tenor dates of the LIBOR rates. This can be deduced from the equation (3.23) where the time t price depends on the discrete bank account at time t. As also noted in [Jam97], a simple linear interpolation between the two nearest tenor dates is suggested to get $B^d(t)$ if the time t price of the derivative is needed.

3.3 Instantaneous volatilities and correlation

3.3.1 For simulation

In general, it is not possible to find the simultaneous distribution of the various stochastic variables under the expectation (3.23) which is required analytically. Hence, the evolution in the corresponding processes has to be implemented by simulation.

[12] that is not a strict assumption

The main issue here, is to get appropriate instantaneous correlations between the different LIBOR rates and the stock portfolio.

$$\begin{bmatrix} dW_1^d(t) \\ dW_2^d(t) \\ \vdots \\ dW_n^d(t) \\ dW_S^d(t) \end{bmatrix} \cdot [dW_1^d(t), dW_2^d(t), \cdots, dW_n^d(t), dW_S^d(t)] = \begin{bmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,n} & \rho_{1,S} \\ \rho_{2,1} & 1 & \cdot & \cdot & \rho_{2,S} \\ \vdots & \cdot & 1 & \cdot & \vdots \\ \rho_{n,1} & \cdot & \cdot & 1 & \rho_{n,S} \\ \rho_{S,1} & \cdot & \cdot & \rho_{S,n} & 1 \end{bmatrix} dt.$$

In this implementation, correlation between the LIBOR rates are described by deterministic functions depending on the length between the corresponding tenor dates, T_i and T_j. With the correlations involving the stock portfolio, the dependence is on the length between the time t and the corresponding tenor date of the LIBOR rate T_i. For our purpose, it is sufficient to specify the instantaneous volatility of the LIBOR rates following as:

Definition 10 (Instantaneous volatility of LIBOR rates)**.**
The structure with a hump shaped functional equation is defined as

$$\lambda_i(t) = g(T)f(T_i - t), \tag{3.25}$$

in [Kok09], it argued that the functional form of f

$$f(T_i - t) = (a + (T_i - t)b) \cdot e^{-c(T_i - t)} + d, \tag{3.26}$$

is flexible enough to capture desirable criteria such as being hump shaped. And $g(T)$ is set to 1.

In the next chapter, we will analyse the hump shaped form.
Now, we want to model the correlations between the LIBOR rates:
The instantaneous correlation matrix between the LIBOR rates should fulfil four criteria:

1. Symmetry: $\rho_{i,j} = \rho_{j,i}$ for all i, j.
2. Positive semidefinite: $x^T \rho x \geq 0$ for all $x \in \mathbb{R}^{\mathbb{N}}$.
3. Only 1 on the diagonal: $\rho_{i,i} = 1$ for all i.
4. All entries are in the interval $[-1, 1]$.

Further we will model the matrix as a time homogeneous function for $T_i, T_j > t$ and $i \neq j$.

Definition 11 (Instantaneous correlation between the LIBOR rates)**.**
For our purpose, we introduce a simple correlation function that satisfies the requirements as mentioned above:

$$\rho_{i,j}(t) = \exp(-\beta|T_i - T_j|), \tag{3.27}$$

with $\beta > 0$ and $i, j, t \geq 0$.

Last question is how to specify the volatility of the stock portfolio, and how it correlates with the LIBOR rates.
For simplicity, we will let the volatility be constant $\sigma_S = \sigma$. It is reasonable to let the Wiener process for the stock portfolio $W_S^d(t)$ be correlated most with the LIBOR rates with the shortest distance to maturity $T_i - t$.

Definition 12 (Instantaneous correlation between LIBOR rates and stock portfolio)**.** *A convenient form for the instantaneous correlation is given by:*

$$\rho_{S,i}(t) = \frac{1 - \exp(-\frac{\alpha}{(t-T_i-\gamma)})}{1 + \exp(-\frac{\alpha}{(t-T_i-\gamma)})} = \tanh\left(\frac{\alpha}{2(t - T_i - \gamma)}\right), \tag{3.28}$$

where $\gamma > 0$.

Remark. *Positive values of α give rise to negative correlations and vice versa. This specific choice of function ensures correlation between -1 and 1, and if more flexibility is needed, additional parameters can be included inside the brackets in* (3.28). *Of course, for $T_i < t$ the LIBOR rate has matured and the correlation is set to zero.*

3.4 Numerical Implementation

In this section, different approaches for the valuation of traffic light options will be done. First, we want to analyse several assumptions from the previous sections:

Payoff function of $T_{n+1}-$ claim

The graphic characterisation of the payoff function (3.6) is given by:

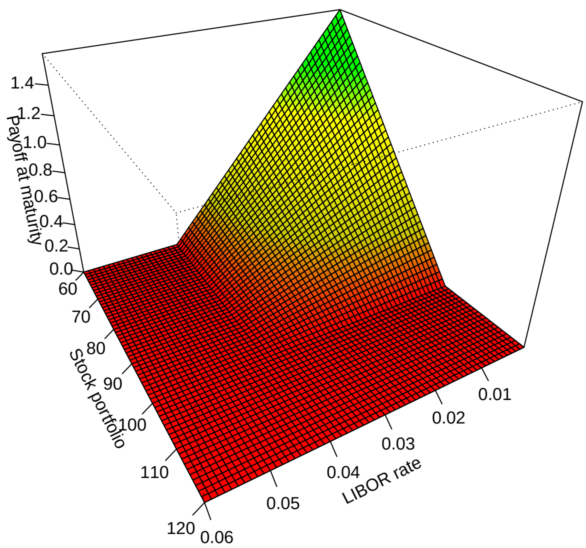


Figure 3.1: The payoff profile of the traffic light option with $\bar{S} = 100$ and $\bar{L} = 0.04$.

The illustration in Fig.3.1 represents the payoff profile of a traffic light option with a benchmark stock portfolio and the LIBOR rate. This can also be plotted with some benchmark interest rate[13].

[13]see [Jør07]

3.4.1 Volatility structure of the LIBOR rates

According to the previous section, the LIBOR rates are assumed to have the form (3.26) with parameters taken from ([BM06],p.320):

$$a = 0, \quad b = 0.29342753, \quad c = 1.25080230, \quad d = 0.13145869,$$

which is illustrated in the following plot:

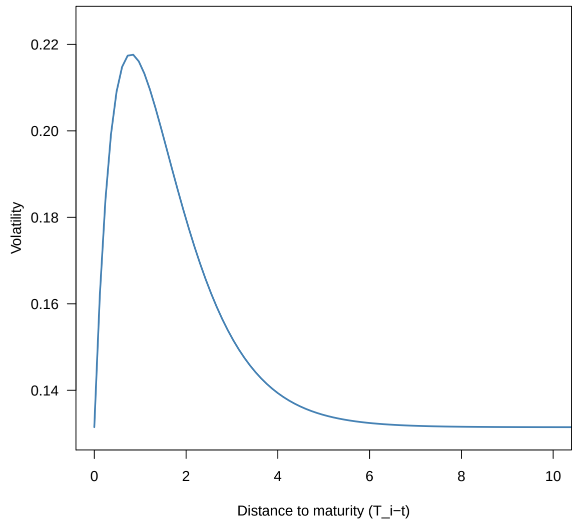


Figure 3.2: The instantaneous LIBOR rate volatility as a function of distance to maturity $T_i - t$.

In Fig.3.2 it is seen that the instantaneous LIBOR rate volatility as a function of distance to maturity $T_i - t$ is decreasing with the particular choice of the correlation function. This is also a reasonable property. It is the rate, maturing nearest from now, that reacts the most to the market information and also drives the stock market.

Interpretation

This form is clearly time-homogenous and displays, for suitable choices of the parameter set, a nicely humped term structure of volatility. However, $g(T)$ allows a possibility for a perfect calibration in some cases and is therefore very useful. In order to preserve time-homogenousity it is, however, important to assure that $g(T)$ are as close as possible to 1. In order to preserve the short and long time behavior and the humped form of the term structure of volatilities one may not choose the parameters a, b, c and d completely free. For the interpretation of the function as a well behaved instantaneous volatility, the following conditions must be satisfied:

- $a + d > 0$
- $c, d > 0$

Furthermore, when $\hat{\delta} := T_i - t$ tends to zero, instantaneous and average volatilities tend to coincide and therefore the quantity $a + d$ should at least approximately assume values given by the shortest maturities implied volatilities. On the other hand, when $\hat{\delta}$ tends to large values d has to be connected with the very-long-maturity volatilities.

- $a + d \longrightarrow$ short maturities implied volatilities
- $d \longrightarrow$ very long maturities implied volatilities

Considering the first derivative of the time-homogeneous part of equation for the instantaneous volatility function with respect to $\hat{\delta}$: $f'(\hat{\delta}) = e^{-c\hat{\delta}}\left(b - ca - cb\hat{\delta}\right)$ gives some final information:

- $\frac{b-ca}{cb}$: The location of the extremum (the top of the hump) should be greater 0 and not too large.
- $b > 0$: Constraint for the extremum to be a maximum.

Interpretation about the characteristics is in BRIgo Mercurio, Brigo and Mercurio (2001), Rebonato (2002, 2005) and White and Rebonato (2009) for a justification of the choice and description of the properties of this function.

3.4.2 Correlation structure of the LIBOR rates

Let the coefficient in (3.27) be given by $\beta = 0.1$ as illustrated in Fig.3.3:

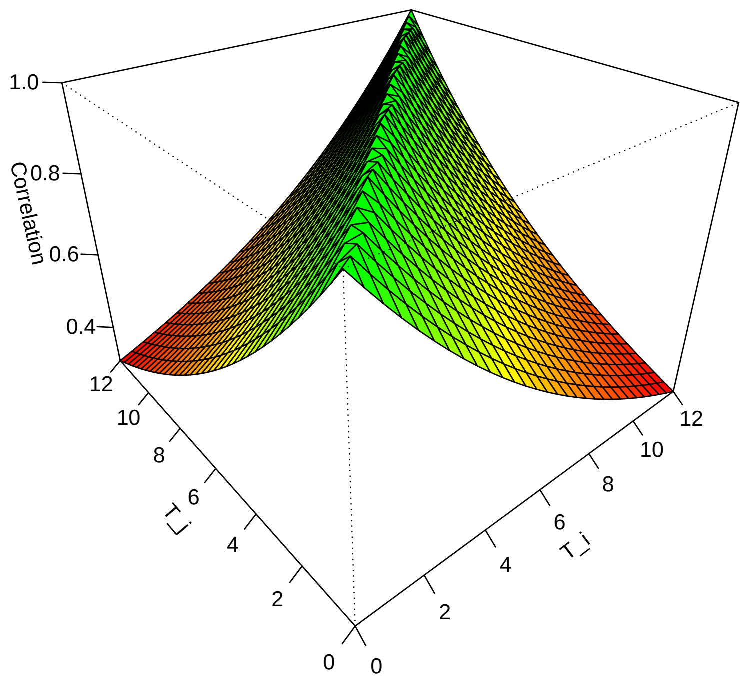


Figure 3.3: Correlation between LIBOR rates with $\beta = 0.1$

Interpretation

This one-parameter parametrisation always produces a valid correlation matrix in the sense that it produces a real, symmetric, positive-definite matrix. However, correlation is only dependent on the distance between maturities and is constant with regard to t. Under the assumption of constant volatilities, instantaneous and terminal correlation are equal. β is called the de-correlation factor or rate of de-correlation as it controls the decrease in correlation with increasing maturity interval. Setting $\beta := 0$ results in a model with perfect instantaneous correlation, thereby reducing the number of driving factors of the model to 1. [14] In general, this correlation function can be assigned a functional dependence on calendar times and on the maturities of the two forward LIBOR rates:

$$\rho_{ij} = \rho(t, T_i, T_j)$$

[14] see [Pac05]

For simplicity of modelling, we assume that the correlation function is time-homogenous and only depends on the relative distance between the two forward LIBOR rates with different tenor dates.
The following form of the correlation function is:

$$\rho_{ij}(t) = \rho(|T_i - T_j|)$$

with the characteristics:

$$\rho(|T_3 - T_1|) = \rho(|T_3 - T_2|) \cdot \rho(|T_2 - T_1|).$$

In other terms, the logarithm of ρ must be a linear function. Hence, in general, there must exist some $\beta \geq 0$ such that:

$$\rho_{ij}(t) = \rho(|T_i - T_j|) = e^{-\beta|T_i - T_j|}$$

For our purpose we restrict the condition from $\beta \geq 0$ to $\beta > 0$.

3.4.3 Correlation between the stock portfolio and the LIBOR rates

The instantaneous correlation is represented by (3.28). Now we want to choose appropriate parameters α and γ to measure the correlation. It is not clear to find correct values for the parameters. Therefore a more detailed table of the instantaneous correlation values α, γ and the distance to maturity $T_i - t$ of the corresponding LIBOR rate is illustrated in [Kok09].

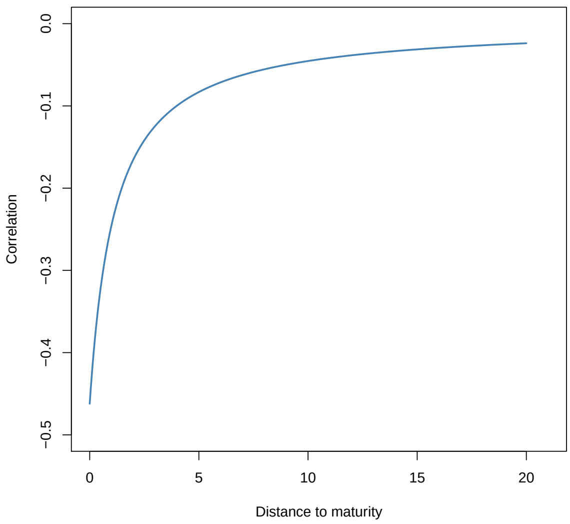


Figure 3.4: Correlation between LIBOR rates and stock portfolio with $\alpha = \gamma = 1$.

The α parameter controls the level of the correlation and higher absolute values of this parameter increases the absolute correlation across maturities, though not in a parallel way.
The γ parameter controls the curvature of the function. This is also clear from looking at the table, where it is seen that the absolute decrease in correlation as distance to maturity increases is highest for small values of γ. Or loosely stated, the staring point of the function in Fig 4. shifts closer to zero for higher γ values.

3.4.4 Pricing of the TLO with Theorem 4

In comparison to the payoff function the price of the traffic light option is found with Theorem (4) and depicted as a function of the stock portfolio price and the LIBOR rate at time $t = 0$:

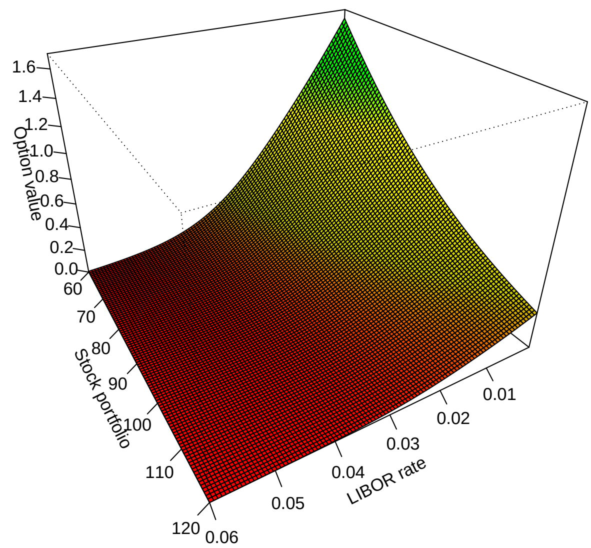


Figure 3.5: Here the parameter values are $\bar{S} = 100$, $\bar{L} = 0.04$, $T_{n+1} = 3$, $\rho = -0.5$, $\sigma_s = \sigma = 0.2$ and the term structure is assumed flat equal to the initial LIBOR rate.

In Fig. 3.5 we can see that in the critical areas, which are $(100, 0.00) \times (120, 0.04)$ and $(60, 0.04) \times (100, 0.06)$, the values are above zero in comparison to the payoff function in 3.6. The price for a traffic light option slightly converges to zero, if we take the values to $(120, 0.06)$. The x-and y-axis are symmetric, if we take the 45 degree line between stock portfolio and LIBOR rate for reflection.

3.4.5 TLO price in dependence of correlation

Interpretation

With increasing correlation between LIBOR rate and stock portfolio the price of the traffic light option rises as well, with fixed tenor date, bond value and the initial values of the stock portfolio resp. the LIBOR are set at their strike levels.

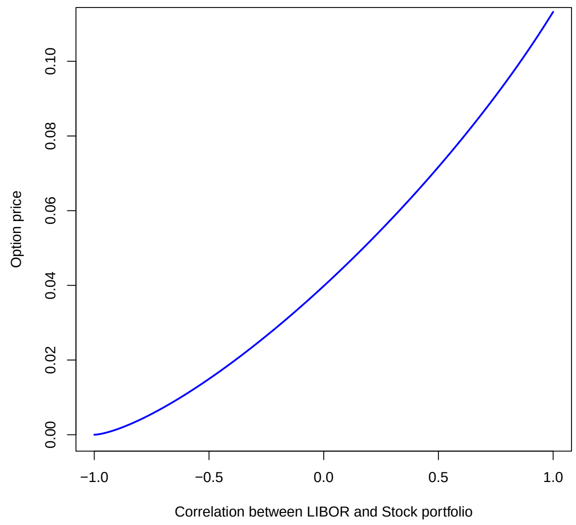


Figure 3.6: The traffic light option as a function of correlation with $S_0 = 100$, $L_n(0) = 0.04$, $\bar{S} = 100$, $\bar{L} = 0.04$, $T_{n+1} = 3$ and $B(0, T_{n+1}) = 0.8890$.

3.4.6 Pricing with Monte Carlo simulation

In practice it is possible to price any European type T_i-payoff given by $C(S(T_0), \ldots, S(T_i), \ldots, L_0(T_0), \ldots, L_n(T_i))$ via simulation techniques. In this section the payoff function of a traffic light option will be implemented and run with a Monte Carlo simulation.

Payments similar to this form is being increasingly used in the construction of structured products. The pricing is performed under the spot LIBOR measure.

In a simulation of the LIBOR rates, the first choice is to fix the time grid of the future time points over which to simulate $0 = t_0 < t_1 < \cdots < t_m < t_{m+1}$. In this time grid, it is convenient to let the tenor dates $T_0 < T_1 < \cdots < T_n$ be a subset.

Further by letting the time difference between two simulation points be constant ($t_{j+1} - t_j = \delta$), the notation is reduced.

LIBOR Simulation with Euler-scheme

Simulating the LIBOR rates with an Euler-scheme on $\log\left(\widehat{L_i}\right)$ results in(see [Gla04]):

$$\widehat{L}_i(t_{j+1}) = \widehat{L}_i(t_j) \cdot \exp\left\{\left(\mu_i(t_j) - \frac{1}{2}\lambda_i(t_j)^2\right)\delta + \sqrt{\delta}\lambda_i(t_j)Z_{j+1}\right\},$$

with

$$\mu_i(t_j) = \sum_{l=i(t)}^{i} \frac{\tau_{l+1}\widehat{L}_i(t_j)\rho_{i,j}\lambda_l(t_j)}{1 + \tau_{l+1}\widehat{L}_i(t_j)}\lambda_i(t_j), \tag{3.29}$$

and $Z_1, \cdots, Z_{m+1}$ are independent $N(0,1)$ random variables. In the equation above, the hats have been added to clarify that the continuous LIBOR rates have been discretized.

The simulation is initialized with (3.2) by setting[15]:

$$\widehat{L}_i(0) = \frac{1}{\tau_{i+1}}\left(\frac{B(0,T_i)}{B(0,T_{i+1})} - 1\right), \quad i = 1, \cdots, n.$$

For instance, the simulated path of the LIBOR rate $\hat{L}_6(t)$ with maturity date $T_6 = 3$ years is seen in Fig.3.7. After simulating 1000 possible paths for $L_6(t)$ via Monte Carlo simulation the mean can be observed in Fig.3.8.

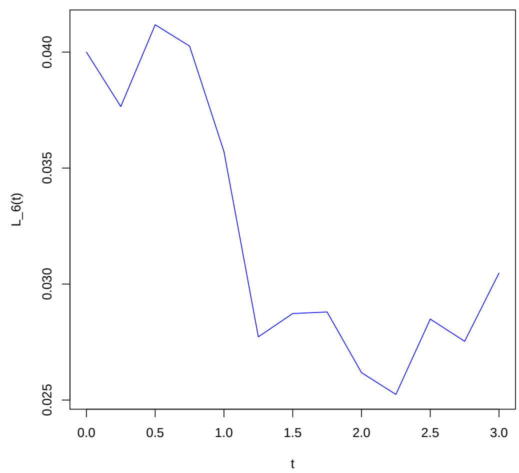


Figure 3.7: Simulated LIBOR rate $L_6(t)$

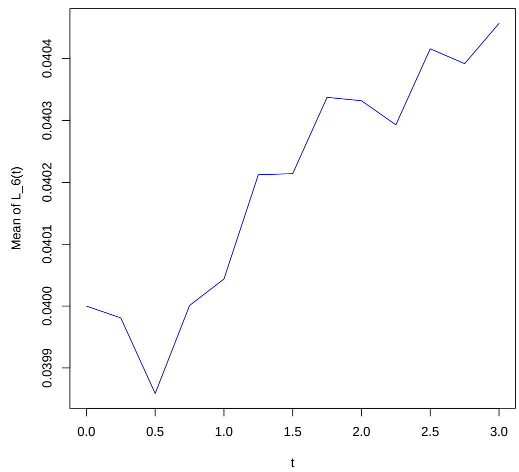


Figure 3.8: Mean of 1000 simulated LIBOR rates $L_6(t)$

[15]in the implementation the initial forward LIBOR rates are flat with 3%.

The distribution of the LIBOR rate L_6 at time $t = 1$ year is noticed in Fig.3.9.

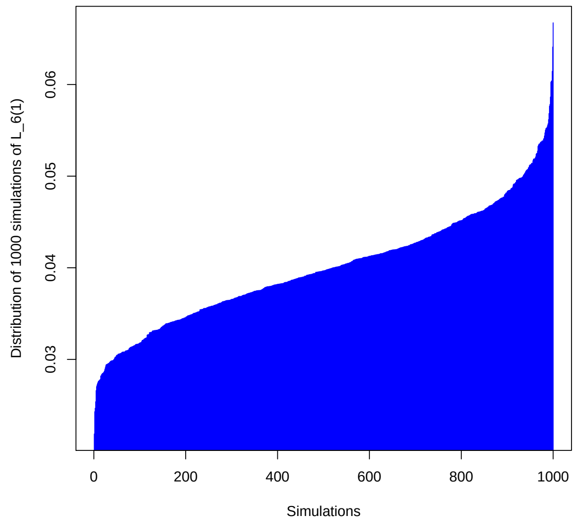


Figure 3.9: Distribution of 1000 simulated LIBOR rates L_6 evaluated at time $t = 1$ year

The value of the bank account at maturity T_n can be derived from the simulation of the LIBOR rates using (3.20):

$$\widehat{B}^d(T_k) = \prod_{j=0}^{k-1}(1 + \tau_{j+1}\widehat{L}_j(T_j)) \quad \text{for all } k = 2, \cdots, n,$$

with $\widehat{B}^d(T_k) = 1$ for $k = 0, 1$.
In order to simulate the stock price which is given by:

$$S(T_n) = B^d(T_n)S_0 \exp\left(\int_0^{T_n} \sigma_s dW_S^d(s) - \frac{1}{2}\int_0^{T_n} \sigma_s^2 ds\right).$$

We will split the simulation in two discretisation schemes, where we define:

$$X(T_n) := S_0 \exp\left(\int_0^{T_n} \sigma_s dW_S^d(s) - \frac{1}{2}\int_0^{T_n} \sigma_s^2 ds\right),$$

which can be simulated with the discretisation scheme:

$$\widehat{X}(T_{j+1}) = \widehat{X}(T_j) \cdot \exp\left\{-\frac{1}{2}\sigma_s^2\tau + \sqrt{\tau}\sigma_s Z_{j+1}\right\}.$$

Now we can get the simulated stock portfolio with:

$$\widehat{S}(T_{j+1}) = \widehat{X}(T_{j+1}) \cdot \widehat{B}^d(T_{j+1}). \tag{3.30}$$

After running the simulation with the Euler-scheme for 1000 possible paths of stock portfolio prices with initial value 100, we get the significant plot in Fig.3.10:

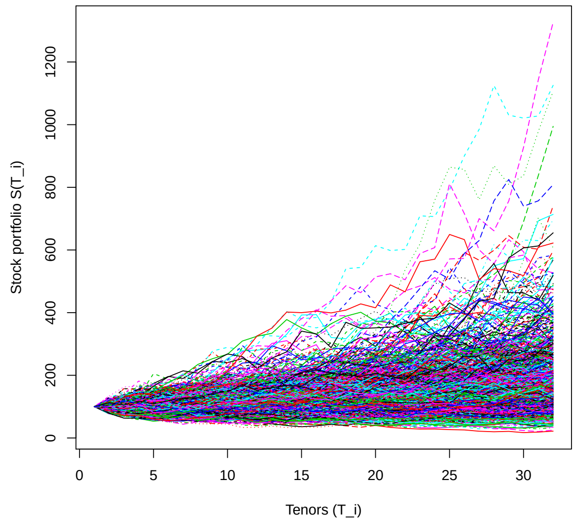


Figure 3.10: Simulation of 1000 possible paths of the discounted stock portfolio prices $\hat{S}(T)$

We can see through building the mean over this 1000 simulations, that the stock portfolio is strictly increasing:

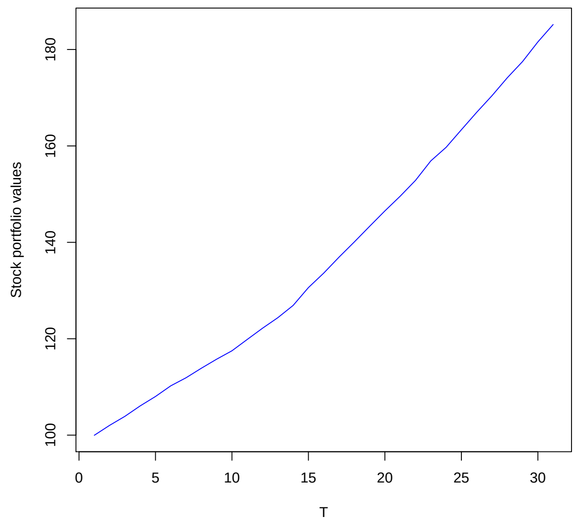


Figure 3.11: Mean over 1000 simulations of possible paths of the discounted stock portfolio prices $\hat{S}(T)$

In the following example our payoff is described by:

$$\begin{aligned} C\big(S(T_{n+1}), L_n(T_n)\big) &= \big[\bar{S} - S(T_{n+1})\big]^+ \cdot \big[\bar{L} - L_n(T_n)\big]^+, \\ &= \big[\bar{S} - \widehat{S}(T_{n+1})\big]^+ \cdot \big[\bar{L} - \widehat{L}_n(T_n)\big]^+. \end{aligned}$$

For each linear combination of the strike levels $\bar{S}$ and $\bar{L}$, the simulation will be evaluated and then averaged. By use of the discretisations above, the time zero price of the derivative:

$$\pi(0) = \mathbb{E}^d\left[\frac{1}{B^d(T_i)} C\big(S(T_{n+1}), L_n(T_n)\big)\right],$$

can then be approximated with the Monte Carlo simulation. Now implementing the Monte Carlo simulation for the analytical formula of the traffic light option in Theorem (4) leads to the option prices in the following plot:

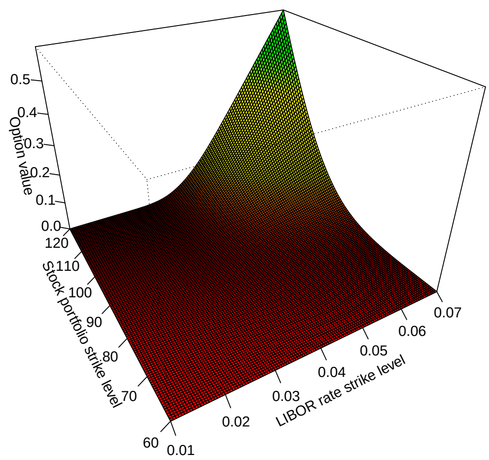


Figure 3.12: Monte Carlo simulation of the analytical traffic light option formula

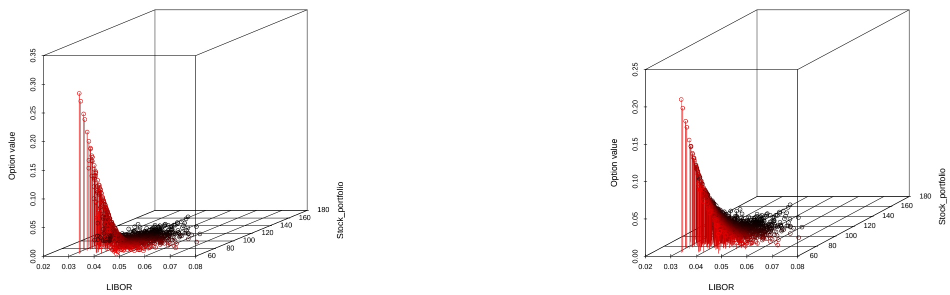
We can see the characteristics as mention before. The plot is symmetric, if we take the 45 degree line for reflection. Moreover the unique shape, as in Theorem (4), is preserved.

(a) Theoretical formula (b) Simulation

Figure 3.13: Theoretical formula versus simulation of a TLO

If we compare the simulation to the analytical formula, we can easily state, that our implemented simulation needs more optimisation. We evaluated the standard deviation and mean of 1000 simulated payoff scenarios. After that we built a confidence interval with mean $+/-$ 3 times the standard deviation. The lower bound is negative and therefore the lower bound is set to 0 and the upper bound of the confidence interval is always 350% above the mean. This is a large simulation error. Here an optimised implementation would be wise.[16]

Back to the previous example with 1000 simulations of the stock portfolio and the LIBOR rate, the conditional distribution of the option price scenarios have the following plots:



(a) Computed claim (b) Analytical formula

Figure 3.14: Conditional distribution of 1000 simulations compared between claim versus analytical formula

Now we have finished the simulation chapter. In the next chapter there will be more insights regarding traffic light options in terms of their most practical use: Hedging the balance sheet of a typical Danish L&P company to stay solvent in the yellow light scenario.

[16] not part of this thesis

Chapter 4

Hedging with Traffic Light Options

4.1 The traffic light option as a hedging instrument

The goal of this section is, with the help of the traffic light option, to hedge a typical balance sheet of an L&P company.
First, we will do this from the theoretical point of view, and then by way of an illustrative numerical example.

Definition 13 (Solvency ratio). [1]

Solvency ratio (=:SR) is a key metric used to measure an enterprise's ability to meet its debt and other obligations. The solvency ratio indicates whether a company's cash flow is sufficient to meet its liabilities. The lower a company's solvency ratio, the higher the probability that it will default on its debt obligations. The exact definition is given by:

$$SR = \frac{\textit{Net income} + \textit{Deprecation}}{\textit{Liabilities}}. \tag{4.1}$$

For simplification, our balance sheet, which will be explained in the next section, we only need the following definition for the solvency ratio:

$$SR = \frac{\textit{Net income}}{\textit{Liabilities}}. \tag{4.2}$$

Who needs the solvency ratio?
Solvency ratio is of interest to long-term creditors and shareholders. These groups are interested in the long-term health and survival of business firms. In other words, solvency ratio has to prove that business firms can service their debt or pay the interest on their debt as well as pay the principal, when the debt matures. It also helps business owner keep an eye on downtrends that could eventuate in a possible bankruptcy.

[1] see `http://www.investopedia.com/terms/s/solvencyratio.asp`

4.1.1 Theoretical Approach

The simplified balance sheet[2] of an L&P company at time t:

Assets	Liabilities & Free equity
$S(t)$	$\Theta(L_j, t, T_j)$
$B(L_i, t, T_i)$	$FE(t)$

The asset side of the balance sheet consists of the market value of the well-diversified stock portfolio at time t, represented by $S(t)$, and the bond investments at time t with LIBOR rate L_i and maturity T_i, represented by $B(L_i, t, T_i)$.
The liability side of the balance sheet consists of $\Theta(L_j, t, T_j)$, which denotes the market value of the company's fixed pension obligations at time t with LIBOR rate L_j and maturity T_j. For simplicity, we consider that the fixed pension obligations behave like bonds, with a longer duration. $FE(t)$ is the market value of the free equity at time t.
In order to ensure a balanced sheet, we fix:

$$FE = S + B - \Theta, \tag{4.3}$$

residually.
Using $It\hat{o}'s$ lemma on the above expression, we can now deduce the following dynamics under the EMM $\mathbb{Q}$ for the free equity:

$$\begin{aligned} dFE(L, S, t) =& L_i(t) FE(t) dt + \sigma_S S(t) dW_S^{\mathbb{Q}}(t) \\ &+ \sigma_{L_i} \left(\frac{\partial B(L_i, t, T_i)}{\partial L_i} \right) dW_{L_i}^{\mathbb{Q}}(t) \\ &- \sigma_{L_j} \frac{\partial \Theta(L_j, t, T_j)}{\partial L_j} dW_{L_j}^{\mathbb{Q}}(t) \end{aligned}$$

For our purpose it is more simple to set all LIBOR rates flat ($L_i = L_j$) for $i, j \in \{1, \ldots, n\}$, resulting in the following dynamics:

$$\begin{aligned} dFE(L, S, t) =& L_i(t) FE(t) dt + \sigma_S S(t) dW_S^{\mathbb{Q}}(t) \\ &+ \sigma_L \left(\frac{\partial B(L, t, T_i)}{\partial L} - \frac{\partial \Theta(L, t, T_j)}{\partial L} \right) dW_L^{\mathbb{Q}}(t). \end{aligned}$$

In theory the asset liability mismatch can easily be repaired by selling all stocks and investing in bonds such that $\frac{\partial B}{\partial L} = \frac{\partial \Theta}{\partial L}$. For various reasons, however, this is rarely done in practice[3]. Typically the pension obligations have a much longer duration (between 15 to 25 years (seen in [Jør07])), in comparison to the bonds duration with a typical duration of 6 years. Hence, in practice a typical L&P portfolio manager more often attempts to control the risk to the free equity via rearrangement of the asset side. They include an appropriate amount of structured products such as traffic light options. Such an asset reallocation from a portfolio manager could include the following new arrangements:

[2]unhedged
[3]according to [Jør07]

Hedged Portfolio with Traffic Light Options

Assets	Liabilities & Free equity
$S^{new}(t)$	$\Theta(L_k, t, T_k)$
$B^{new}(L_j, t, T_j)$	$FE(t)$
$H(L_i, S, t)$	

The liability and free equity side is unaffected by the reallocation in the new composition. On the asset side, to keep the example simple, we only sell bonds and buy instead traffic light option for hedging the balance sheet. Hence, we have the new allocation on the asset side: $S^{new} := S$ and $B^{new} := B - H$. Through the reallocation our $\mathbb{Q}$-dynamics for free equity changed as well:

$$\begin{aligned} dFE(t) =& L(t)FE(t)dt + \sigma_S S^{new}(t)\left(1 + \frac{\partial H(L_i, S, t)}{\partial S}\right)dW_S^{\mathbb{Q}}(t) \\ &+ \sigma_{L_i}\left(\frac{\partial H(L_i, S, t)}{\partial L_i}\right)dW_{L_i}^{\mathbb{Q}}(t) + \sigma_{L_j}\left(\frac{\partial B^{new}(L_j, t, T_j)}{\partial L_j}\right)dW_{L_j}^{\mathbb{Q}}(t) \\ &- \sigma_{L_k}\left(\frac{\partial \Theta(L_k, t, T_k)}{\partial L_k}\right)dW_{L_k}^{\mathbb{Q}}(t). \end{aligned}$$

It is necessary for calculations that $T_i < T_j < T_k$ for all $i, j, k \in \mathbb{N}$. In case of hedging with traffic light options H, we have to consider the strike levels $\bar{L}$ and $\bar{S}$ as well as maturities T and the time of evaluation t. From the dynamics above we can see the perfect instantaneous hedge of the free equity fulfils the following conditions:

$$\begin{aligned} &1 + \frac{\partial H(L, S, t)}{\partial S} = 0, \\ &\frac{\partial H(L_i, S, t)}{\partial L_i} = 0, \\ &\frac{\partial B^{new}(L_j, t, T_j)}{\partial L_j} = 0, \\ &\frac{\partial \Theta(L_k, t, T_k)}{\partial L_k} = 0, \end{aligned}$$

and

$$H(L_i, S, t) = \big(S(t) - S^{new}(t)\big) + \big(B(L_j, t, T_j) - B^{new}(L_j, t, T_j)\big).$$

As we can see from the conditions above, we have an under-determined system of equations. Hence it is not possible to find one true solution. Despite that fact, we can find values to hedge the yellow light scenario.

Remark. *For simplicity, as mentioned in the non-hedged case, we consider all LIBOR rates as flat, resulting in the following dynamics:*

$$\begin{aligned} dFE(t) =& L(t)FE(t)dt + \sigma_S S^{new}(t)\left(1 + \frac{\partial H(L, S, t)}{\partial S}\right)dW_{S^{new}}^{\mathbb{Q}}(t) \\ &+ \sigma_L\left(\frac{\partial H(L, S, t)}{\partial L} + \frac{\partial B^{new}(L, t, T_j)}{\partial L} - \frac{\partial \Theta(L, t, T_k)}{\partial L}\right)dW_L^{\mathbb{Q}}(t). \end{aligned}$$

Now it is possible to solve the following equations for a perfect instantaneous hedge:

$$
\begin{aligned}
&1 + \frac{\partial H(L,S,t)}{\partial S} = 0,\\
&\frac{\partial H(L,S,t)}{\partial L} + \frac{\partial B^{new}(L,t,T_j)}{\partial L} - \frac{\partial \Theta(L,t,T_k)}{\partial L} = 0
\end{aligned}
$$

and

$$H(L,S,t) = \big(S(t) - S^{new}(t)\big) + \big(B(L,t,T_j) - B^{new}(L,t,T_j)\big).$$

4.2 Numerical Example of a publicly-listed Insurance Company

In this section a typical balance sheet of a publicly-listed insurance company will be shocked in two scenarios: firstly without traffic light options and then with TLOs included as a hedging instrument.
The asset side includes a well-diversified stock portfolio to the extent of 30 units and 70 units of zero-coupon bonds with duration of 6 years with the LIBOR rate as the benchmark interest rate. On the liability & equity side, we find pension obligations, which will be handled for simplicity like zero-coupon bonds with a longer duration of 15 years[4] and the resulting free equity[5] of 8 units.

Unhedged balance sheet at time $t = 0$

Assets		Liabilities & Free equity	
Stocks	30.00	92.00	Pension obligations (D= 15 years)
Bonds (D=6 years)	70.00	8.00	Free equity (SR: 8.70%)
Total	100.00	100.00	Total

Now the unhedged balance sheet will be shocked by the yellow light scenario in Def. 1. The LIBOR rate drops from 4% to 3% and the stock portfolio loses 30% of its initial value at $t = 0$. The shock will be modelled in the Vasicek-model and BMG-model.

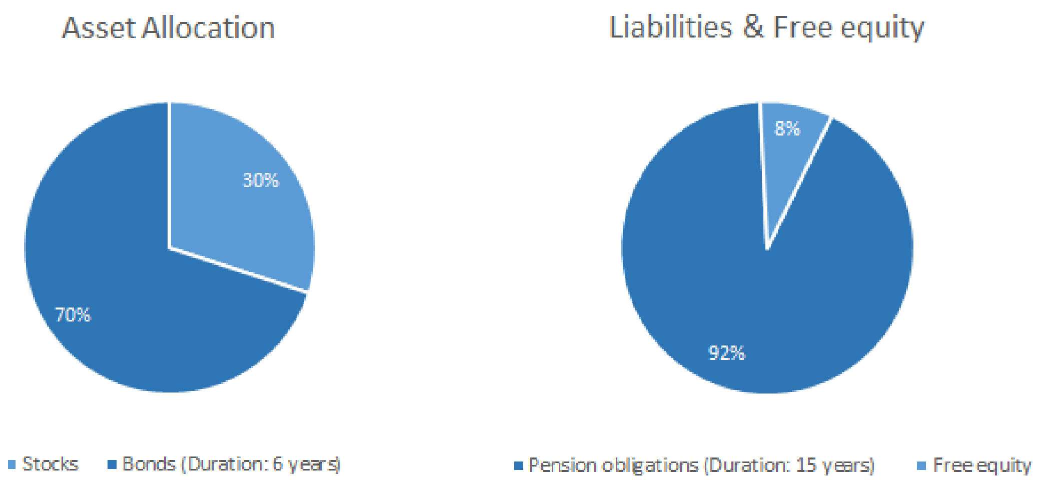


Figure 4.1: Unhedged balance sheeet at time $t = 0$

[4]the actual pension fund liability durations vary between 15 and 25 years depending on the exact age distribution of the policy holders. See [Jør07]

[5]Free equity = Total of the asset side - Pension obligations

Unhedged balance sheet shocked in the yellow light scenario right after $t = 0$ in the Vasicek-model

Assets		Liabilities & Free equity	
Stocks	21.00	95.66	Pension obligations (D= 15 years)
Bonds (D=6 years)	72.21	-2.45	Free equity (SR: -2.56%)
Total	93.21	93.21	Total

After the yellow light scenario in the Vasicek-model, the insurance company is technically insolvent with a solvency ratio of -2.56%. In comparison to the unprotected balance sheet at time $t = 0$ we can see that the long-term bonds on the liabilities and free equity side do not react significant in the short-term framework.

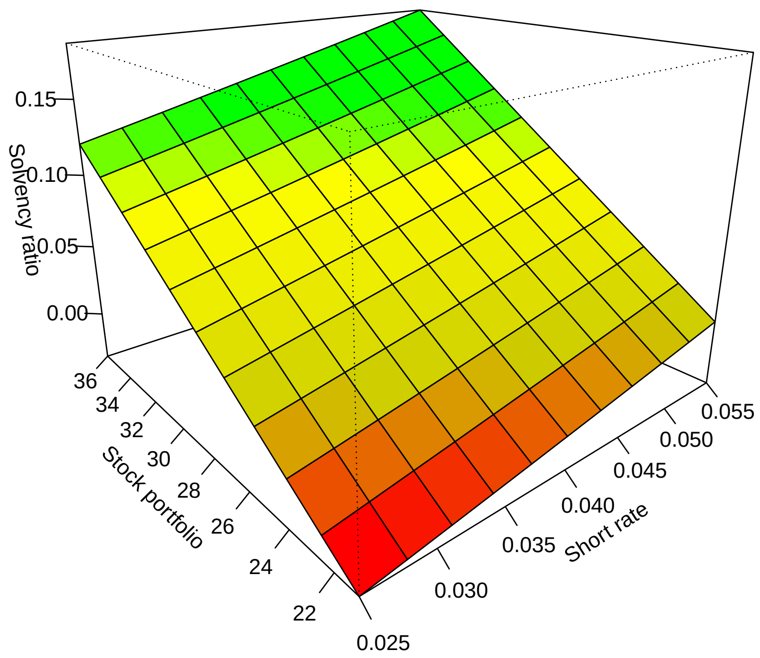


Figure 4.2: Unhedged balance sheet shocked in the Vasicek-model

Unhedged balance sheet shocked in the yellow light scenario right after $t = 0$ in the BMG-model

Assets		Liabilities & Free equity	
Stocks	21.00	106.61	Pension obligations (D= 15 years)
Bonds (D=6 years)	74.25	-11.36	Free equity (SR: -11.36%)
Total	95.25	95.25	Total

After the yellow light scenario in the BMG-model, the insurance company is technically insolvent with an solvency ratio of -11.36%. This dramatic difference in solvency's state, in the Vasicek-model "only" -3% and in the BMG-model -12%, arises because of the different calculations in the bond formula. In the Vasicek-model long-term bonds do not react that sensitive as in the BMG-model, as we can see in the plots and tables.
In order to avoid insolvency, we will now sell some bonds and buy instead traffic light options. This will act as an protection against insolvency.

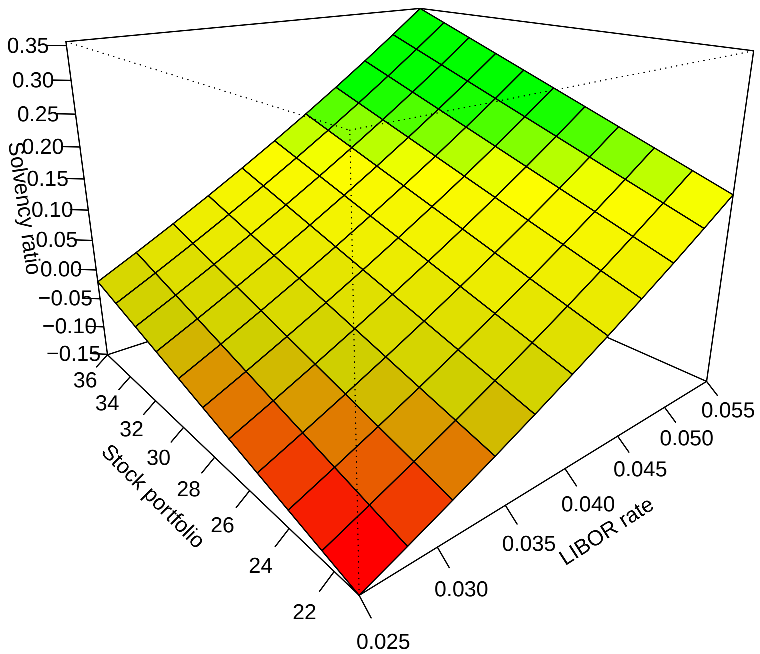


Figure 4.3: Unhedged balance sheet shocked in the BMG-model

4.3 Hedged balance sheet

Now we will buy 250 units of traffic light options to protect against the yellow light scenario. The parameters for the traffic light options are:
$T_{n+1} = 5, t = 0, \rho = 0.0, \sigma_s = 0.2, \bar{S} = 30$ and $\bar{L} = 0.04$.

Hedged balance sheet at time $t = 0$

Assets		Liabilities & Free equity	
Stocks	30.00	92.00	Pension obligations (D= 15 years)
Bonds (D=6 years)	66.19		
Traffic Light Options	3.81	8.00	Free equity (SR: 8.70%)
Total	100.00	100.00	Total

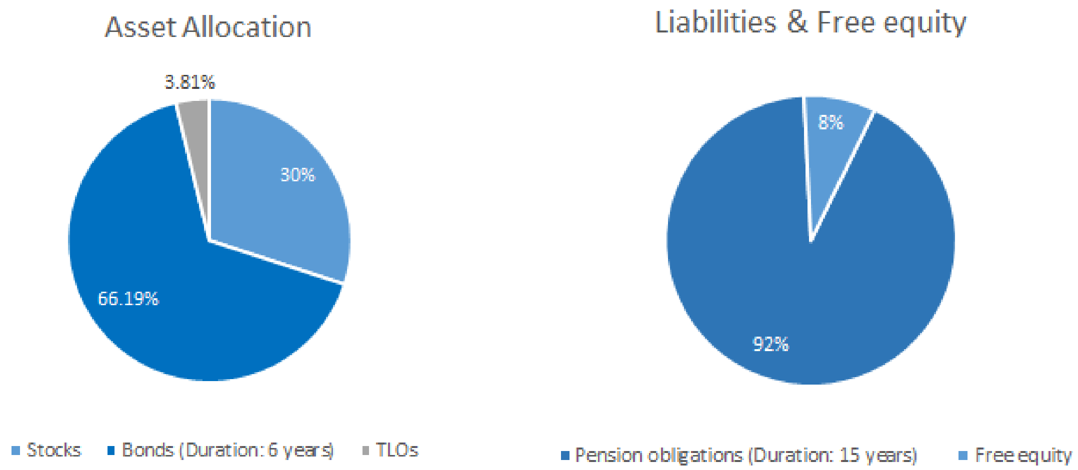


Figure 4.4: Unhedged balance sheet at time $t = 0$

For the hedging part, we only sell bonds and buy TLOs instead. In Fig.4.4 the asset allocation has changed to the new split.

Hedged balance sheet shocked in the yellow light scenario right after $t = 0$ in the BMG-model

Assets		Liabilities & Free equity	
Stocks	21.00	106.61	Pension obligations (D= 15 years)
Bonds (D=6 years)	70.21		
Traffic Light Options	20.28	4.88	Free equity (SR: 4.58%)
Total	111.49	111.49	Total

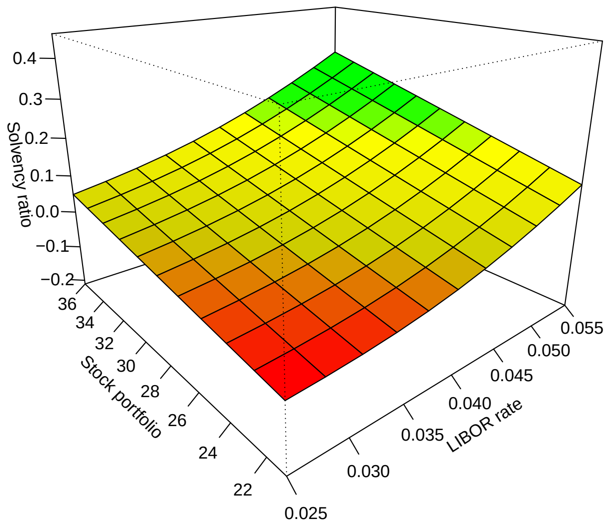


Figure 4.5: Hedged balance sheet shocked in the BMG-model

In comparison to [Jør07, Ch.4], where the Vasicek-model is used, the BMG-model is also a good framework for stress-testing. As mentioned above, long durations in the BMG-model react more sensitive, than in the Vasicek-model, since in the short rate model, long durations in pension obligations do not react significantly.
The aim in this thesis, was to find an appropriate hedge with TLOs in the framework of the BMG-model, to stay solvent in the yellow-light scenario, which we successfully achieved.

4.3.1 Conclusion

The thesis has introduced an exotic financial derivative as a potential hedging instrument to meet regulatory requirements. In the 21^{st} century requirements regarding financial institutions, for instance in terms of base capital, have become very strict. And there will be even more requirements for companies to cope with. As mentioned in [Jør07, Ch.5], there is a potential risk that companies will focus too narrowly on passing just the regulator's stress tests. Companies could easily hedge away the equity part and/or the interest rate of the regulatory risk regarding the yellow light scenario by purchasing a digital-type option paying a suitable fixed amount if the market goes down between 29% and 31%, and zero otherwise. This scenario would cost the companies very little and enable them to preserve a green light status.

Summarising all facts and simulations, we can state, that this innovative structured product is easy to simulate, but the problem is to find an optimisation, which can fit the theoretical approach best. The goal of this thesis, is to give a snapshot about structured products with a deeper insight on pricing traffic light options in order to hedge the introduced stress-tests from the Danish Supervisory Authorities for Danish Life & Pension companies. The results in the first part of the work are a more detailed version of Thomas Kokholm's paper [Kok09] and the second part is based on the work of Peter Løchte Jørgensen [Jør07] with an introduction to the LIBOR Market Model.

Appendix A

Basics

This chapter will provide some basic knowledge regarding financial mathematics.

Definition 14 (Martingale measure and equivalent martingale measure).

A probability measure $\mathbb{Q}$ absolutely continuous with respect to $\mathbb{P}$ is a martingale measure for S if and only if S is a $\mathbb{Q}-$martingale. It is called an equivalent martingale measure if it is equivalent to $\mathbb{P}$, i.e. $\mathbb{Q} \in \mathcal{M}^e$.

Definition 15 (Strategy).

A strategy ν is an S-integrable process. The value process V associated to an initial capital c and a strategy ν in the risky asset S is given as a stochastic integral process

$$V = c + \int \nu \, dS.$$

Definition 16 (Admissible strategy).

A strategy ν is called an admissible strategy if the gain process $\int vdS$ is a $\mathbb{Q}-$martingale for every martingale measure $\mathbb{Q} \in \mathcal{M}^e$.

Definition 17 (Arbitrage opportunity).

A strategy ν is called an arbitrage opportunity if we have for the associated value process V that

- $V_0 \leq 0$
- $V_T \geq 0 \quad \mathbb{P} - a.s.$
- $\mathbb{P}(V_T > 0) > 0.$

Definition 18 (1.Fundamental Theorem of Asset Pricing (FTAP)).

If there exists an equivalent martingale measure $\mathbb{Q}$ for S then there are no arbitrage opportunities with admissible strategies.[1]

[1] see [RS11]. "Easy direction"

Definition 19 (Claim).

A Claim C is an $\mathcal{F}_T$-measurable random variable. The claim C is attainable if there exists a constant c and an admissible strategy ν such that

$$C = c + \int_0^T \nu_t \, dS_t.$$

The quintuple $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P}, S)$ is called a market.
A market is complete if all bounded claims are attainable.

Definition 20 (Predictable representation property).

The process $M \in \mathcal{M}^2_{loc}$ has the (PRP) if $\mathcal{S}(\mathcal{M}) = \mathcal{M}^2$. That is, every $N \in \mathcal{M}^2$ can be written as $N = N_0 + \int \nu \, dM$ where $\nu \in L^2(M)$

Definition 21 (2.FTAP).

The following assertions are equivalent:

- *The market is complete.*
- $|\mathcal{M}^e| = 1. \quad (\exists! \mathbb{Q} \in \mathcal{M})^e$
- *There exists $\mathbb{Q} \in \mathcal{M}^e$ such that S has the PRP with respect to $(\mathbb{Q}, \mathcal{F})$.*

Definition 22 (Numeraire). [2]

A numeraire is an asset B with strictly positive $B(t)$ at any time t in $[0, T]$.
The role of a numeraire is to discount other asset prices processes $B_1, \cdots, B_n$ by expressing the relative price process $B_i^{'} := \frac{B_i}{B}$, $i = 1, \cdots, n$. In this work, the numeraires that we consider will mostly be $T-$bonds or bank accounts.
An equivalent martingale measure (associated to a numeraire B) is a probability measure $\mathbb{Q}$ on the same filtered probabilisable space $(\Omega, \mathcal{F}, \mathbb{F})$ such that

- *$\mathbb{Q}$ and $\mathbb{P}$ have the same null sets,*
- *the discounted price processes $B_i^{'}$, $i = 1, \cdots, n$, are martingales under $\mathbb{Q}$.*

[2] see Swap Market Models for pricing Interest rate derivates Monte Carlo Simulations by Mbele Bidima Martin Le Doux 2004

Appendix B

R-codes

For the numerical implementation, we use two additional R-packages:

- ”pbivnorm”, which will be used for the computing of the standardized bivariate normal distribution.
- ”scatterplot3d”, which is required for graphic illustrations.

B.1 Payoff–profile of a traffic light option

The following code includes: Fig.3.1

```
# Payout-profile of a traffic light option

Payout <- function (x, Sstrike, Lstrike){
  if (Sstrike>=x[1] & Lstrike>=x[2]){
    (Sstrike-x[1])*(Lstrike-x[2])
  }
  else 0
}

# x-axis stock portfolio prices

S <- seq(60,120,length.out = 61)


# y-axis LIBOR rates from 0.1 % to 6%
# with 61 data points

L <- seq(0.0001,0.06,length.out = 61)

# Strike for the stock portfolio

Sstrike <- 100

# Strike for LIBOR

Lstrike <- 0.04

# Create all linear combinations between
```

```
# stock portfolio prices and LIBOR rates

grid <- expand.grid(S, L)

# Create a vector z with all payoff-profile calculations

z <- apply(grid, 1, Payout, Sstrike = Sstrike, Lstrike = Lstrike)

# In order for the numerical illstrations we need
# z to be a 61x61-matrix

z <- matrix(z, ncol = 61, nrow = 61)

# Colour Plot

# Colour surface parameters

par(bg = "white")
x <- L
y <- S
z <- z
nrz <- nrow(z)
ncz <- ncol(z)

# Create a function interpolating colors in the range of specified colors

jet.colors <- colorRampPalette( c("red","yellow3","yellow2","yellow1","
    green1", "green") )

# Generate the desired number of colors from this palette

nbcol <- 1000000
color <- jet.colors(nbcol)

# Compute the z-value at the facet centres

zfacet <- z[-1, -1] + z[-1, -ncz] + z[-nrz, -1] + z[-nrz, -ncz]

# Recode facet z-values into color indices

facetcol <- cut(zfacet, nbcol)

# Colour Plot
persp(x, y, z, col = color[facetcol],ylab="Stock portfolio", xlab = "LIBOR
    rate",
        zlab = "Payoff at maturity",#main="Traffic light option payoff at
    maturity",
        expand = 0.75,ticktype = "detailed",
        nticks =8, phi = 30, theta = 150)

# Saving the graphic

pdf(file= "Payout_of_TLO.pdf")

# Plot
```

```

persp(x, y, z, col = color[facetcol],
      ylab="Stock portfolio",
      xlab = "LIBOR rate",
      zlab = "Payoff at maturity",
      expand = 0.75,ticktype = "detailed",
      nticks =8, phi = 30, theta = 150)

dev.off()
```

B.2 Volatility structure of LIBOR rates

The following code includes: Fig.3.2

```
# Volatiliy Structure of the LIBOR rates

# Formula for the lambda-function

# Be careful in the paper we have T_i-t,
# but for the plotting it is only dependend
# on the lag between T_i-t

lambda <- function(t) {

  # Parameters are set as in [BM06]

  a <- 0
  b <- 0.29342753
  c <- 1.25080230
  d <- 0.13145969

  # Note that t is the lag of T_i-t, hence Time to maturity

  return((a+(t)*b)*exp(-(t)*c)+d)
}

# x-axis as time to maturity

x_axis <- seq(0,12,length.out = 100)

# Volatility values

w <- lambda(x_axis)

# Plot area for x-axis and y-axis

xlim <- c(0,10)
ylim <- c(0.13, 0.225)


# Plot function of the volatility structure

plot(x = x_axis, y =w , type = "l", xlab = "Distance to maturity (T_i-t)",
     ylab = "Volatility",
     #main = "Volatility structure of LIBOR rates",
     col = "steelblue",
     lwd = 2, xlim = xlim, ylim = ylim, las = 1)

# Saving the graphic

pdf(file= "Volatility_structure_of_the_LIBOR_rates.pdf")

# Plot

plot(x = x_axis, y =w , type = "l",
     xlab = "Distance to maturity (T_i-t)",
```

```r
      ylab = "Volatility",
      col = "steelblue",
      lwd = 2, xlim = xlim, ylim = ylim, las = 1)

dev.off()
```

B.3 Correlation between the LIBOR rates

The following code includes: Fig.3.3

```
# Correlation between the LIBOR rates (between the different maturities)

# Correlation function formula

corrfunc <- function(b = 0.1 ,T1,T2){

    exp(-b*abs(T1-T2))

  }

# For the plot
# Create 20 maturity dates = T_1 to T_20

T_i <- T_j <- 0:20

a <- corrfunc(0.1,0,T_j)
b <- corrfunc(0.1,T_i,0)

# Surface colours corresponding to z-values

par(bg = "white")
x <- seq(0, 12, length = 30)
y <- seq(0, 12, length = 35)
z <- outer(x, y, function(a, b) corrfunc(0.1,a,b))
nrz <- nrow(z)
ncz <- ncol(z)

# Create a function interpolating colors in the range of specified colors

jet.colors <- colorRampPalette( c("red","yellow", "green") )

# Generate the desired number of colors from this palette

nbcol <- 100000
color <- jet.colors(nbcol)

# Compute the z-value at the facet centres

zfacet <- z[-1, -1] + z[-1, -ncz] + z[-nrz, -1] + z[-nrz, -ncz]

# Recode facet z-values into color indices

facetcol <- cut(zfacet, nbcol)

# 3D Plot of Correlation between LIBOR rates with different maturities

persp(x, y, z, col = color[facetcol],expand=0.75,
      #main = "Correlation between LIBOR rates with different maturities",
      ylab="T_j",xlab = "T_i",zlab = "Correlation",ticktype = "detailed",
      phi = 30, theta = -40)

# Saving the graphic
```

```r

pdf(file= "Correlation_between_the_LIBOR_rates.pdf")

# Plot

persp(x, y, z, col = color[facetcol],expand=0.75,
      #main = "Correlation between LIBOR rates with different maturities",
      ylab="T_j",xlab = "T_i",zlab = "Correlation",ticktype = "detailed",
      phi = 30, theta = -40)

dev.off()
```

B.4 Correlation between LIBOR rates and the stock portfolio

The following code includes: Fig.3.4

```r
# Correlation between Stock and Interest rate

# The parameters a and g stand for alpha and gamma

# Correlation function

corr_SL <- function(a,g,t,Ti){

  return ((1-exp(-a/(t-Ti-g)))/ (1+exp(-a/(t-Ti-g))))

}

# For the plotting it is the same "problem"! We need to switch the order
# of the distance to maturity

a<-rev(seq(0,20,by=0.05))

# Fix alpha and gamm with value = 1

y<-corr_SL(a=1,g=1, a,20)

# Plot area for x-axis and y-axis

xlim <- c(0,21)
ylim <- c(-0.5,0)

b<-seq(0,20,by=0.05)

# Plot-Funktion

plot(x = b, y =y , type = "l", xlab = "Distance to maturity",
     ylab = "Correlation",
     #main = "Correlation structure between LIBOR rates and the stock index",
     col = "steelblue",
     lwd = 2, xlim = xlim, ylim = ylim, las = 0.01)

# Saving the graphic

pdf(file= "Correlation_between_Stock_index_and_LIBOR_rates.pdf")

# Plot

plot(x = b, y =y , type = "l", xlab = "Distance to maturity",
     ylab = "Correlation",
     #main = "Correlation structure between LIBOR rates and the stock index",
     col = "steelblue",
     lwd = 2, xlim = xlim, ylim = ylim, las = 0.01)

```

```
dev.off()
```

B.5 Analytical formula for pricing TLOs

The following code includes: Fig.3.5

```
# Pricing Traffic Light Option
# with analytical formula

# Remark: r is equally flat to LIBOR rates

# Need the package pbivnorm for the standardized
# bivariate normal distribution

require(pbivnorm)

# Formula for TLO

priceTLO<-function(L,S,t=0,Tn1=3,SStrike=100,LStrike=0.04,sigma_s=0.2,rho
    =-0.5) {

  # Difference between two tenor dates

  tau <- 0.5

  # Tn

  Tn <- Tn1 - tau

  # Need tenor for the semi-annual calculations
  # Starting with T_0=0, T_1=0.5, ...

  tenor <- seq( 0 , Tn1 , by = tau )

  # Bond with maturity at T_(n+1)
  # Due to the fact of semi-annual tenor dates
  # we need all tenor evaluation points = length(tenor)

  bTn1 <- 1/(1+tau*L)^(length(tenor)-t-1) # Bond with r = LIBOR r = x

  SSchlange <-(SStrike*bTn1)/S

  LSchlange <- LStrike/L

  sigma_xq <-(Tn1-t)*sigma_s^2

  sigma_x <- sqrt(sigma_xq)

  # We need lambda for the sigma_y

  lambda <- function(t,Tn) {

    a <- 0
    b <- 0.29342753
    c <- 1.25080230
    d <- 0.13145969

    result <- (a+(Tn-t)*b)*exp(-(Tn-t)*c)+d
```

```
  return (result)
}

integrand1 <- function(x) {lambda(t = x, Tn = Tn)^2}
sigma_yq1 <-integrate(integrand1, lower = t, upper = Tn )

# Returns only the value without abs error

sigma_yq<-sigma_yq1[[1]]

# sigma_y^2=sigma_yq

sigma_y <- sqrt(sigma_yq)

# sigma_xy

integrand2 <- function(x) {sigma_s * rho * lambda(t=x,Tn = Tn)}
sigma_xy1 <- integrate(integrand2,lower = t, upper= Tn)
sigma_xy <- sigma_xy1[[1]]

# mu_x

mu_x <- sigma_xq*(-0.5)

# mu_y

mu_y <- sigma_yq*(-0.5)


# rho_SL

rho_SL <- sigma_xy/(sigma_x*sigma_y)

# For a better reading of the formula

a1 <- as.numeric((log(SSchlange)-mu_x)/sigma_x)
b1 <- as.numeric((log(LSchlange)-mu_y)/sigma_y)
a2 <- as.numeric(a1-sigma_x)
b2 <- as.numeric(b1-rho_SL*sigma_x)
a3 <- as.numeric(a1-rho_SL*sigma_y)
b3 <- as.numeric(b1-sigma_y)
a4 <- as.numeric(a1-rho_SL*sigma_y-sigma_x)
b4 <- as.numeric(b1-rho_SL*sigma_x-sigma_y)

p1 <- pbivnorm(x = a1, y = b1, rho = rho_SL)
p2 <- pbivnorm(x = a2, y = b2, rho = rho_SL)
p3 <- pbivnorm(x = a3, y = b3, rho = rho_SL)
p4 <- pbivnorm(x = a4, y = b4, rho = rho_SL)


# Print section
# Comment for the 3D plot
# For a test run

```

```
  print(c("bTn1",bTn1))
  print(c("SSchlange",SSchlange))
  print(c("LSchlange",LSchlange))
  print(c("sigma_xq",sigma_xq))
  print(c("sigma_x",sigma_x))
  print(c("sigma_yq",sigma_yq))
  print(c("sigma_y",sigma_y))
  print(c("sigma_xy",sigma_xy))
  print(c("mu_x", mu_x))
  print(c("mu_y",mu_y))
  print(c("rho_SL",rho_SL))
  print(c("a1,b1,p1",a1,b1,p1))
  print(c("a2,b2,p2",a2,b2,p2))
  print(c("a3,b3,p3",a3,b3,p3))
  print(c("a4,b4,p4",a4,b4,p4))

  result <- as.numeric((L*S*(SSchlange*LSchlange*p1
                        - LSchlange*p2
                        - SSchlange*p3
                        + exp(sigma_xy)*p4)))

  print(c("Result",result))

  return (result)

}

################################################

# TLO price plot

# x-axis = stock portfolio prices

S <- seq(60,120,length.out=100)

# y-axis = LIBOR rates from 0.01% to 6%
# with 100 evaluation points

L <- seq(0,0.06,length.out = 100)
L[1] <- 0.00001 # To avoid L = 0.00000

# Strike for stock portfolio prices

SStrike <- 100

# Strike for LIBOR rates

LStrike <- 0.04

# Volatility of the stock prices

sigma_s <- 0.2

# Correlation between stocks and LIBOR rates

```

```r
rho_SL <-(-0.5)

# Create all linear combinations between stocks
# and LIBOR rates in a list

grid <- expand.grid(L, S)

###########################################

# For the example just uncomment the print
# area in the formula above
# Test run

priceTLO(grid[6965,1],grid[6965,2], t=0,Tn1=3)

bTn1<-1/(1+0.5*grid[6965,1])^7
bTn1

SSchlange<-100/grid[6965,2]*bTn1

LSchlange <- 0.04/grid[6965,1]
LSchlange

sigma_xq <- 0.2^2*(3-0)
sigma_xq
sigma_x <- sqrt(sigma_xq)
sigma_x


lambda <- function(t,Tn) {

  a<-0
  b<- 0.29342753
  c <- 1.25080230
  d <- 0.13145969

  (a+(Tn-t)*b)*exp(-(Tn-t)*c)+d

}

integrand1 <- function(x) {lambda(t = x, Tn = 2.5)^2}
sigma_yq1 <-integrate(integrand1,lower = 0, upper= 3 - 0.5)
sigma_yq <-sigma_yq1[[1]]
sigma_yq
sigma_y <-sqrt(sigma_yq)
sigma_y
mu_y<--1/2*sigma_yq
mu_y
mu_x <- -1/2*sigma_xq
mu_x
a1 <- (log(SSchlange)-mu_x)/sigma_x
b1 <- (log(LSchlange)-mu_y)/sigma_y
a1
b1
p1 <- pbivnorm(x = a1, y = b1, rho = -0.5)
```

```r
p1

#############################################

# TLO price plot (colour surface)

# LIBOR rates = x-axis

x <- L

# Stock portfolio = y-axis

y <- S

# Outer creats all linear combinations
# between L (=x) and S (=y) and then
# computes the price formula for the TLO

# TLO-prices = z-axis

z <- outer(x,y,priceTLO)

# Check if all values are greater than 0

length(which(z<0))

# Some pars

par(bg = "white")
nrz <- nrow(z)
ncz <- ncol(z)

# Create a function interpolating colors in the range of specified colors

jet.colors <- colorRampPalette( c("red1","yellow3","yellow2","yellow1","green1", "green") )

# Generate the desired number of colors from this palette

nbcol <- 1000000
color <- jet.colors(nbcol)

# Compute the z-value at the facet centres

zfacet <- z[-1, -1] + z[-1, -ncz] + z[-nrz, -1] + z[-nrz, -ncz]

# Recode facet z-values into color indices

facetcol <- cut(zfacet, nbcol)

# 3D-Plot

persp(x, y, z, col = color[facetcol],xlab="LIBOR rate", ylab = "Stock portfolio",
      zlab = "Option value",
```

```
      #main="Traffic light option price at maturity",
      ticktype = "detailed",zlim = c(0,1.7) ,nticks = 8,
      expand = 0.75,
      phi = 30, theta = 150)


# Saving the graphic

pdf(file= "TLO_price_with_analytical_formula.pdf")

# Plot

persp(x, y, z, col = color[facetcol],xlab="LIBOR rate", ylab = "Stock
    portfolio",
      zlab = "Option value",
      #main="Traffic light option price at maturity",
      ticktype = "detailed",zlim = c(0,1.7) ,nticks = 8,
      expand = 0.75,
      phi = 30, theta = 150)

dev.off()
```

B.6 Pricing TLO in dependence of the correlation

The following code includes: Fig.3.6

```
# Correlation function of the TLO price
# with dependence of correlation

# Need the package pbivnorm for the standardized
# bivariate normal distribution

require(pbivnorm)

# Formula for the TLO price with dependence
# of correlation

priceTLO_rho_SL<-function(rhos_SL,S0=100,L0=0.04,t=0,SStrike=100,LStrike
    =0.04,Tn1=3,sigma_s=0.2,bTn1=0.889) {

  # Time at evaluation = t = 0

  t <- 0

  # Difference between two tenor dates

  tau <- 0.5

  # T_n

  Tn <- Tn1 - tau

  # Need tenor for the semi-annual calculations
  # Starting with T_0=0, T_1=0.5, ...

  tenor <- seq( 0 , Tn1 , by = tau )

  # Bond with maturity at T_(n+1)
  # Due to the fact of semi-annual tenor dates
  # we need all tenor evaluation points = length(tenor)

  SSchlange <-(SStrike*bTn1)/S0

  LSchlange <- LStrike/L0

  sigma_xq <-(Tn1-t)*sigma_s^2

  sigma_x <- sqrt(sigma_xq)

  # We need lambda for the sigma_y

  lambda <- function(t,Tn) {

    a <- 0
    b <- 0.29342753
    c <- 1.25080230
    d <- 0.13145969

```

```
  result <- (a+(Tn-t)*b)*exp(-(Tn-t)*c)+d

  return (result)
}

integrand1 <- function(x) {lambda(t = x, Tn = Tn)^2}
sigma_yq1 <-integrate(integrand1, lower = t, upper = Tn )

# Returns only the value without abs error

sigma_yq<-sigma_yq1[[1]]

# sigma_y^2=sigma_yq

sigma_y <- sqrt(sigma_yq)

# sigma_xy

integrand2 <- function(x) {lambda(t=x,Tn = Tn)}
sigma_xy1 <- integrate(integrand2,lower = t, upper= Tn)
sigma_xy <- sigma_xy1[[1]] * sigma_s * rho_SL

# mu_x

mu_x <- sigma_xq*(-0.5)

# mu_y

mu_y <- sigma_yq*(-0.5)

# sigma_xy

sigma_xy  <- rho_SL*(sigma_x*sigma_y)

# For a better reading of the formula

a1 <- as.numeric((log(SSchlange)-mu_x)/sigma_x)
b1 <- as.numeric((log(LSchlange)-mu_y)/sigma_y)
a2 <- as.numeric(a1-sigma_x)
b2 <- as.numeric(b1-rho_SL*sigma_x)
a3 <- as.numeric(a1-rho_SL*sigma_y)
b3 <- as.numeric(b1-sigma_y)
a4 <- as.numeric(a1-rho_SL*sigma_y-sigma_x)
b4 <- as.numeric(b1-rho_SL*sigma_x-sigma_y)

p1 <- pbivnorm(x = a1, y = b1, rho = rho_SL)
p2 <- pbivnorm(x = a2, y = b2, rho = rho_SL)
p3 <- pbivnorm(x = a3, y = b3, rho = rho_SL)
p4 <- pbivnorm(x = a4, y = b4, rho = rho_SL)

result <- as.numeric((L0*S0*(SSchlange*LSchlange*p1
                            - LSchlange*p2
                            - SSchlange*p3
                            + exp(sigma_xy)*p4)))

```

```r
  return (result)

}

# Values for rho_SL are in [-1,1]
# Declaration for the x-axis and y-axis

rho_SL <- seq(-1,1,by=0.01)

y<-priceTLO_rho_SL(rho_SL)

# Plot

plot(rho_SL, y ,type = "l", ylim = c(0,0.11), xlim = c(-1,1),
     xlab = "Correlation between LIBOR and Stock portfolio",
     ylab = "Option price",
     col="blue",
     #main="Traffic light option price as function of correlation",
     lwd=2)

# Saving the graphic

pdf(file= "TLO_price_in_dependence_of_rho_SL.pdf")

# Plot

plot(rho_SL, y ,type = "l", ylim = c(0,0.11), xlim = c(-1,1),
     xlab = "Correlation between LIBOR and Stock portfolio",
     ylab = "Option price",
     col="blue",
     lwd=2)

dev.off()
```

B.7 Monte Carlo Simulation of the LIBOR rates and Stock portfolio

The following code includes: Fig.3.7, Fig.3.8, Fig.3.9, Fig.3.10, Fig.3.11, Fig.3.12, Fig.3.13, Fig.3.14

```
# LIBOR rates and Stock portfolio Monte Carlo Simulation


# Discretitzation points (Evaluation points for Euler Scheme)
# from t_0=0 to t_61=15 years by 0.25 steps
# quaterly evaluated

t <- seq(0, 15, by = 0.25)

# Tenor dates with T_1=0 to T_30=15 years by 0.5 steps
# semi-annually evaluated

tenor <- seq(0.5, 15, 0.5)

# Generate a matrix for all discrete LIBOR rates for the EULER scheme
# Matrix with coloumn length t and row length tenor

m.L <- matrix(0, ncol = length(t), nrow = length(tenor))

# Write all initial LIBOR rates L_i(0) for all
# i from 1 to length of tenor in the first coloumn

# Initial Term structure is flat due to the definition of the BOND
# Due to the assumption that the initial term structure is flat
# We will write for all initial LIBOR rates L_i(0) = 4 %

m.L[, 1] <- 0.04

# Function for correlations of LIBOR rates between different tenors
# It is a tenor*tenor matrix with 1 as diagonal entries
# Compare to the definition in the paper

rhoil <- function(tenor){

  beta <-0.1
  m.rho <- matrix(0, ncol = length(tenor) , nrow = length(tenor))
  p <- length(tenor)

  for (i in 1:p){
    for (j in 1:p){
      m.rho[i, j] <- exp(-beta*abs(tenor[i]-tenor[j]))
    }
  }

  return(m.rho)
}

# Valuation of the correlation matrix

```

```
m.rho <- rhoil(tenor)

# Function for the volatility-structure of the LIBORs
# as matrix

lambda <- function(t, tenor) {

  # Parameters are chosen as discussed in the paper

  a<-0
  b<- 0.29342753
  c <- 1.25080230
  d <- 0.13145969

  m.lambda <- matrix(0, ncol = length(t), nrow = length(tenor))

  for (j in 1:length(t)){
    for (i in 1:(length(tenor))){
      if (t[j] >= tenor[i]){
        m.lambda[i, j] <- 0
      } else {
        m.lambda[i, j] <- (a+(tenor[i]-t[j])*b)*exp(-(tenor[i]-t[j])*c)+d
      }
    }
  }
  return(m.lambda)
}

# Write the volatility structure in a matrix

m.lambda <- lambda(t, tenor)

##################################################

# Now we have all data for the Euler- scheme of the Libor rates


# Euler scheme for LIBOR rate

euler_LIBOR <- function(t, tenor, tau = 0.5, delta = 0.25){

  # Generate the random variables

  # set.seed generate always the same random variables

  # set.seed(2)

  Z <- rnorm(length(t)+1)

  # Calculate the entries for the discrete LIBOR rates
  # in a matrix

  for (j in 1:(length(t)-1)){
    for (i in 1:(length(tenor))){
      mu <- 0
```

```r
      for (l in 1:i){
        mu <- mu + (tau*m.L[l, j]*m.rho[i, l]*m.lambda[l, j])/(1+tau*m.L[l, j])*m.lambda[i, j]
      }
      m.L[i, j+1] <- m.L[i, j] *
                     exp((mu - 0.5*m.lambda[i, j]^2)*delta + sqrt(delta)*m.lambda[i, j]*Z[j+1])
    }
  }

  # In order to circumvent the complex index sum for mu

  for (i in 1:nrow(m.L)){
    i.tmp <- which(diff(m.L[i, ], lag = 1) == 0)[2]
    if (!is.na(i.tmp)){
      m.L[i, i.tmp:ncol(m.L)] <- 0
    }
  }

  # Declaration for the matrix

  m.L <- as.data.frame(m.L)
  colnames(m.L) <- paste0("t_", t)
  rownames(m.L) <- paste0("T_", tenor)

return(m.L)
}


# Euler scheme on a Matrix to get all simulated LIBOR rate entries

m.Libor <- euler_LIBOR(t, tenor)

# Now we have our discrete LIBOR rates via Euler Scheme in a matrix
# T_15 means T for 15 years and in our defintion we actually have
# T_30 for T=15 years (appears due to semi-annually tenors)


##########################################################################

# We want to evaluate the TLO (Libor with maturity at 3 years)
# at time 1 year (that means tenor[2]=T_2=1 or t[5])
# is important for S and L

# LIBOR rates with maturity T_6= 3 years(in the matrix declared as T_6)
# and 13 time steps for the discretisation
# L_6(t) with t from 0.00 to 3 years with 0.25 interval steps

# Evaluationpoint: t=1 years = t[5]

t[5]

# Last calculation point for T_6= 3 years and t[13]=3.00

m.Libor[6,13]
```

```r

# m.Libor is data.frame, hence set as numeric

m.Libor6 <- as.numeric(m.Libor[6,1:13])

# Plot for L_6(t)....T_6= tenor[6]=3 years

 plot(y=m.Libor6,x=t[1:13],xlab = "t",
      #main ="LIBOR rate (L_6(t)) with Maturity 3 years",
      ylab = "L_6(t)", type = "l", col="blue")

# Saving the graphic

 pdf(file= "LIBOR_rate_simulation_with_maturity_3y.pdf")

# Plot

# Figure ?.?

 plot(y=m.Libor6,x=t[1:13],xlab = "t",
      #main ="LIBOR rate (L_6(t)) with Maturity 3 years",
      ylab = "L_6(t)", type = "l", col="blue")

 dev.off()

###########################################################

# Simulation of 1000 paths of LIBOR rates L_6

m.sim <- NULL

 for (j in 1:1000){

   test <- euler_LIBOR(t, tenor)
  m.sim <- rbind(m.sim, test[6, 1:13])
   print(j)

   }

# Declaration of the entries of m.sim
# First m.sim is data.frame
# hence we declare it as a matrix

m.sim <-as.matrix(m.sim)

# Dimension of m.sim: 1000x13-matrix

# Declaration of m.sim

colnames(m.sim) <- paste("L6(t)", 1:ncol(m.sim), sep = "_")
rownames(m.sim) <- paste("Simulation", 1:nrow(m.sim), sep = "_")

# For evaluation at time 1 year we need t[5]=1
# hence m.sim[,5]

```

```
# Compute the mean of all 1000 simulations
# of L_6(t) for t=0.00 to 3.00 in 0.25 steps
# Need m.sim[,5] for the evaluation date 1 year

mean_sim <- colMeans(m.sim)

# Plot of the mean of 1000 simulations of L_6(t)

plot(mean_sim, x=t[1:13],xlab = "t",col="blue",
     #main  = "Mean of 1000 Simulations of L_6(t)",
     ylab = "Mean of L_6(t)", type = "l")

# Saving the graphic

pdf(file= "Mean_of_LIBOR_3y_of_1000_simulations.pdf")

# Plot

plot(mean_sim, x=t[1:13],xlab = "t",col="blue",
     #main  = "Mean from 1000 Simulations of L_6(t)",
     ylab = "Mean of L_6(t)", type = "l")


dev.off()


# All 1000 Simulations of L_6(t) with evaluation point at t[5]=1 year


matplot(m.sim[order(m.sim[,5]),5],xlab="Simulations",
        #main = "Distribution of 1000 Simulations of L_6(t_5=1 year)",
        ylab="Distribution of 1000 simulations of L_6(1)",type = "h",col =
    "blue")

# Mean of the simulation

mean(m.sim[,5])

# Median of the simulation

median(m.sim[,5])

# Saving the graphic

pdf(file= "Distribution_of_1000_sim_of_L_6_at_1y.pdf")

# Plot

matplot(m.sim[order(m.sim[,5]),5],xlab="Simulations",
        #main = "Distribution of 1000 Simulations of L_6(t_5=1 year)",
        ylab="Distribution of 1000 simulations of L_6(1)",type = "h",col =
    "blue")

dev.off()

```

```
################################################################################

# Euler for Stock-Simulation

# Euler for L_N(T_N)

# Function for the last entries of the LIBOR matrix m.sim

lastentries <- function(m.Libor){

  # Generate a vector with length of tenor

  B <- rep(0, length(tenor))

  # Generate a vector v.Libor with length of the row of m.Libor

  v.Libor <- rep(0, nrow(m.Libor))

  # Want the last entries L_n(t_n)
  # which is the last entry before 0

  for (i in 1:nrow(m.Libor)){

    i.tmp <- which(m.Libor[i, ] == 0)[1] - 1

    if (is.na(i.tmp)){

      i.tmp <- ncol(m.Libor)

    }

    v.Libor[i] <- m.Libor[i, i.tmp]
  }

  return(v.Libor)

}

v.Libor<-lastentries(m.Libor)

# v.Libor is a vector with L_N(T_N) for N=1,..,30
# where T_1=0.5 year,..., T_30= 15.00 years

# 1000 simulations of L_N(T_N) in order to
# discount the stock portfolio simulations
# Create v.sim as a 1000 x 30 - matrix

v.sim <- NULL

for (j in 1:1000){

  test <- euler_LIBOR(t,tenor)

  test1 <- lastentries(test)

```

```
  v.sim <- rbind(v.sim, test1)

  print(j)
}

# Declaration of the entries of v.sim

colnames(v.sim) <- paste("L_T(T)", 1:ncol(v.sim), sep = "_")
rownames(v.sim) <- paste("Simulation", 1:nrow(v.sim), sep = "_")

# The finished simulation of v.sim contains 1000 simulations
# of L_N(T_N). (dim(v.sim)=1000 x 30)

##################################################################

# Generate a matrix Bdis for discounting with LIBOR rates

# We need a (ncol(v.sim)+1) x (nrow(v.sim))-matrix for Bdis
# since we need B.d(T_31) for discounting S(T_31)
# for L_30(T_30)

Bdis <- matrix(0, ncol = (ncol(v.sim)+1), nrow = nrow(v.sim))

# Set L_0(T_0)=1

Bdis[,1]<-1

# Formula for the Bdis matrix

Bd <- function(v.sim,tau=0.5){

  for (i in 1:nrow(v.sim)){

    for (j in 2:(ncol(v.sim)+1)){

      Bdis[i,j] <-(1+tau*v.sim[i,j-1])*Bdis[i,j-1]

    }

  }

  colnames(Bdis) <- paste("BdT", 1:(ncol(v.sim)+1), sep = "_")
  rownames(Bdis) <- paste("Sim", 1:(nrow(v.sim)), sep = "_")

  return(Bdis)

}


# Now we have all Bdis for 1000 Simulations

Bdis <-Bd(v.sim)

#########################################################################

```

```
# Stock Simulation with Euler
# start with S(T_0) and need a length of tenor + 2
# for the last calculation S(T_31)

S <- numeric(length(tenor)+2)
S[1]<-100

# Euler-scheme for Stockprices 1 dimensional

euler_stock <- function(tenor, delta = 0.25,sigmas=0.2){

  #Generate the random variables

  Z <- rnorm(length(tenor)+2)

  # Calculate the entries for the discrete Stockprices
  # in a vector

  for (i in 1:(length(tenor)+1)){

    S[i+1] <- S[i] * exp((- 0.5*sigmas^2)*delta + sqrt(delta)*sigmas*Z[i
   +1])
  }

  # Declaration for the vector entries

  S <- as.vector(S)

  names(S)<- paste("S_T", 0:(length(tenor)+1), sep = "_")

  return (S)
}

# 1 Simulation of a stock portfolio with initial value
# of 100 and from 0 to 31 Tenor dates

Stockprice<-euler_stock(tenor)

# Create v.stock as the matrix with 1000 rows and in each coloumn S(T_i)
# from T_0=0 until T_31= 15.50 with different random variables

v.stock <- NULL

for (j in 1:1000){

  test12 <- euler_stock(tenor)

  v.stock <- rbind(v.stock, test12)

  print(j)
}

# v.stock is a 1000 x 32- matrix

# Finally to get the simulated discounted stock portfolio
```

```
# prices with respect to the simulated LIBOR rates

Stockpricediscounted <- function(Bdis, Stockprice){

  # Generate a matrix

  a <- matrix(0, nrow = nrow(Bdis), ncol = ncol(Bdis))

  for (i in 1:(nrow(Bdis))){

    for (j in 1:(ncol(Bdis))){

      a[i,j]<- Bdis[i,j]*Stockprice[i,j+1]
    }

  }

  colnames(a) <- paste("S_T", 1:ncol(Bdis), sep = "_")

  return (a)
}

# The final simulated stock portfolio prices

Stocknew<-Stockpricediscounted(Bdis,v.stock)

# need Stocknew[,2] for evaluation at 1 year=T_2

# Simulation of 1000 simulated discounted stock portfolio values

matplot(t(Stocknew[1:1000,]),
        xlab="Tenors (T_i)",ylab = "Stock portfolio S(T_i)" ,
        #main= "1000 Simulated discounted stock portfolio"
        type = "l")

# Saving the graphic

pdf(file= "Discounted_stock_portfolio_1000_sim_start_at_ST1.pdf")

# Plot

matplot(t(Stocknew[1:1000,]),
        xlab="Tenors (T_i)",ylab = "Stock portfolio S(T_i)" ,
        #main= "Discounted_stock_portfolio_1000_sim_start_at_ST1"
        type = "l")

dev.off()

# Short insertion

# Add in first entry 100 the initial Stockprice
# Just for S Simulation plot with initial value 100

Stocknew1 <- cbind(S_T_0 = rep(100, nrow(Stocknew)), Stocknew)

```

```
# Simulation of 1000 simulated discounted stock portfolio values
# with added initial value S_(T_0)=100

matplot(t(Stocknew1),xlab="Tenors (T_i)",ylab = "Stock portfolio S(T_i)" ,
        #main= "Discounted_stock_portfolio_1000_sim_start_at_S_T_0",
        type = "l")

# Saving the graphic

pdf(file= "Discounted_stock_portfolio_1000_sim_start_at_S_T_0.pdf")

# Plot

matplot(t(Stocknew1),xlab="Tenors (T_i)",ylab = "Stock portfolio S(T_i)" ,
        #main= "Discounted_stock_portfolio_1000_sim_start_at_S_T_0",
        type = "l")

dev.off()


# Mean of 1000 simulations of the discounted stock portfolio

Stockmean <- colMeans(Stocknew)

plot(Stockmean, type = "l", ylab = "Stock portfolio values",
     xlab = "T",
     #main="Stock portfolio mean value",
     col="blue")

# Saving the graphic

pdf(file= "Stock_portfolio_mean_of_1000_simulations.pdf")

# Plot

plot(Stockmean, type = "l", ylab = "Stock portfolio values",
     xlab = "T",
     #main="Stock portfolio mean value",
     col="blue")

dev.off()

# Now the simulation is finished

##################################################

# Create a data.frame with 2 coloumns and 30 rows
# first coloumn is defined as the LIBOR rates (m.sim)
# and second coloumn is defined as the corresponding
# stock portfolio (Stocknew)
# at time 1 year with maturity 3 years

SL <- data.frame(LIBOR = m.sim[,5], Stock_portfolio = Stocknew[,2])

# hence we have a data.frame we need as.matrix
```

```r

SL1 <- as.matrix(SL)

rownames(SL1)<- paste("Sim", 1:nrow(SL1), sep = "_")

###########################################################################

# Monte Carlo Simulation
# With 1000 simulations of LIBOR rates and stock prices
# Here we  insert the simulations into the payoff function

# Function for the payoff profile

# Pricing scenarios for payoff function of a traffic light option
# Calculates all combination in this special scenario
# with strike for Stock portfolio and LIBOR rate


PayoffSL <- function (x, Sstrike=100, Lstrike=0.04){

  z <- rep(0, nrow(x))

  for (i in 1:nrow(x)){

    if (Lstrike >= x[i,1] & Sstrike >= x[i,2]){

      z[i] <- (Lstrike-x[i, 1])*(Sstrike-x[i, 2])

    }
    else
      0
  }


  return(z)

}

# Valuation points for the monte carlo simulation

liborseq <-seq(0.01,0.07, by=0.0005)

stockseq <- seq(60,120, length.out = length(liborseq))

# Function for the computation of all strike
# combinations between LIBOR and Stock portfolio

pay <- function (SL1, liborseq, stockseq){

  g <- matrix(0, ncol = length(liborseq), nrow = length(stockseq))

  z <- NULL

  for (i in 1:length(liborseq)){

```

```r
      for (j in 1:length(stockseq)){

      z<- PayoffSL(SL1, stockseq[j] , liborseq[i])
      g[i,j] <- mean(z)

      }
  print(i)

  }

 rownames(g)<- paste("Stockvalues", 1:nrow(g), sep = "_")
 colnames(g) <-paste("LIBORvalues", 1:ncol(g), sep = "_")

return (g)
}

# Evaluation of the Monte Carlo simulation
# for each strike level combination, the mean will be computed
# and then be plotted

g <-pay(SL1,liborseq,stockseq)

# g contains payout values for all linear combinations
# of all strikes from LIBOR rates and stock indices

# Colour plot (colour surface)

# Some pars

par(bg = "white")
x <- liborseq
y <- stockseq
z <- g
nrz <- nrow(z)
ncz <- ncol(z)

# Create a function interpolating colors in the range of specified colors

jet.colors <- colorRampPalette( c("red","yellow3","yellow2","yellow1","
    green1", "green") )

# Generate the desired number of colors from this palette

nbcol <- 10000
color <- jet.colors(nbcol)

# Compute the z-value at the facet centres

zfacet <- z[-1, -1] + z[-1, -ncz] + z[-nrz, -1] + z[-nrz, -ncz]

# Recode facet z-values into color indices

facetcol <- cut(zfacet, nbcol)

# Plot
```

```
persp(x, y, z, col = color[facetcol],
      xlab="LIBOR rate strike level",
      ylab = "Stock portfolio strike level",
      zlab = "Option value",
      #main="Traffic light option Monte Carlo Simulation",
      ticktype = "detailed",
      #zlim = c(0,1.7),
      nticks = 8,
      expand = 0.75,
      phi = 30,
      theta = -30)

# Saving the graphic

pdf(file= "Traffic light option Monte Carlo Simulation.pdf")

# Plot

persp(x, y, z, col = color[facetcol],
      xlab="LIBOR rate strike level",
      ylab = "Stock portfolio strike level",
      zlab = "Option value",
      #main="Traffic light option Monte Carlo Simulation",
      ticktype = "detailed",
      #zlim = c(0,1.7),
      nticks = 8,
      expand = 0.75,
      phi = 30,
      theta = -30)
dev.off()

# Plot in relation to the analytical formula

persp(x, y, z, col = color[facetcol],
      xlab="LIBOR rate strike level", ylab = "Stock portfolio strike level"
  ,
      zlab = "Option value",
      #main="Traffic light option Monte Carlo Simulation in-rel to
  analytical formula",
      ticktype = "detailed",
      zlim = c(0,1.7),
      nticks = 8,
      expand = 0.75,
      phi = 30,
      theta = -30)

# Saving the graphic

pdf(file= "Traffic light option Monte Carlo Simulation in rel to analytical
  formula.pdf")

# Plot

persp(x, y, z, col = color[facetcol],
```

```
      xlab="LIBOR rate strike level", ylab = "Stock portfolio strike level"
  ,
      zlab = "Option value",
      #main="Traffic light option Monte Carlo Simulation in rel to
  analytical formula",
      ticktype = "detailed",
      zlim = c(0,1.7),
      nticks = 8,
      expand = 0.75,
      phi = 30,
      theta = -30)

dev.off()

#############################################################

# Detailed describtion with one example

# 1000 simulations with Sstrike=120 and Lstrike=0.05

p<-PayoffSL(SL1, Sstrike = 120,Lstrike = 0.05)

# Evaluate the standard deviation of p

sd_p <- sd(p)

# Generate the mean over all values

p_mean <- mean(p)

# build a confidence interval with
# mean +/- 3 times the standard deviation

p_low <- p_mean-3*sd_p
p_high <- p_mean+3*sd_p
p_high/p_mean

# p_low is negative and therfore the lower bound
# is set to 0
# p_high is more than 370 percent above the mean of
# p
# this results in a large simulation error
# --> more optimisation is needed
# not part of this thesis

######################################################

# Distribution of the 1000 simulations with the
# payout function

w<-PayoffSL(SL1)
order(w)

wnew <- w[order(w)]

```

```
# Plot for the option price in this special scenario

plot(wnew, col="blue",
     #main = "Distribution of the option price with 1000 sim",
     xlab = "Simulations", ylab = "Option price",
     type = "h")

# Saving the graphic

pdf(file= "Distribution_of_the_option_price_with_1000_sim.pdf")

# Plot

plot(wnew, col="blue",
     #main = "Distribution of the option price with 1000 sim",
     xlab = "Simulations", ylab = "Option value",
     type = "h")

dev.off()

# Pricing Traffic Light Option with Prop 2.1

# 1000 Simulations TLO values
# priceTLO formula from TLO_Price_with_analytical_formula.R

Optionvalue <- function (SL1){

  TLO <- rep(0,1000)

  for (i in 1:1000){
    TLO[i]<-priceTLO(SL1[i,1],SL1[i,2],t=1)
  }
  return (TLO)


}

# TLO contains all option values

TLO<- Optionvalue(SL1)

# Check if all TLO options are greater than 0

length(which(TLO>0))

#Create a data.frame for TLO

dfTLO <- data.frame(LIBOR = SL[,1] , Stock_portfolio = SL[,2], Option_
    Values = TLO)

# hence we have a data.frame we need as.matrix

SL1 <- as.matrix(SL)

rownames(SL1)<- paste("Sim", 1:nrow(SL1), sep = "_")
```

```
# Plot  Payoff  simulation  of  TLO  with  formula

library(scatterplot3d)

# Simulation  of  the  conditional  payout  scenario

with(dfTLO, {
  scatterplot3d(LIBOR,    # x  axis
                Stock_portfolio ,      # y  axis
                Option_Values ,     # z  axis
                angle =24,
                zlab="Option  value",
                #box = FALSE,
                type = "h",
                highlight.3d=TRUE,
                #main="Simulation_of_TLO_conditional_payout_distribution_at
    _1y_with_maturity_3y")
                col.grid = "black")})

# Saving  the  graphic

pdf(file= "Simulation_of_TLO_conditional_payoff_distribution_at_1y_with_
    maturity_3y.pdf")

# Plot

with(dfTLO, {
  scatterplot3d(LIBOR,    # x  axis
                Stock_portfolio ,      # y  axis
                Option_Values ,     # z  axis
                angle =24,
                zlab="Option  value",
                #box = FALSE,
                type = "h",
                highlight.3d=TRUE,
                #main="Simulation_of_TLO_conditional_payoff_distribution_at
    _1y_with_maturity_3y")
                col.grid = "black")})

dev.off()

# Plot  of  Payout  profile

with(dfTLO, {
  scatterplot3d(LIBOR,    # x  axis
                Stock_portfolio ,      # y  axis
                w,     # z  axis
                angle =24,
                #box = FALSE,
                type = "h",
                zlab="Option  value",
                highlight.3d=TRUE,
                #main="Simulation_of_TLO_conditional_payoff_distribution_at
    _1y_with_maturity_3y_theoretical",
```

```
                  col.grid = "black")})


# Saving the graphic

pdf(file= "Simulation_of_TLO_conditional_payoff_distribution_at_1y_with_
    maturity_3y_theoretical.pdf")

# Plot

with(dfTLO, {
  scatterplot3d(LIBOR,    # x axis
                  Stock_portfolio,      # y axis
                  w,     # z axis
                  angle =24,
                  #box = FALSE,
                  type = "h",
                  zlab="Option value",
                  highlight.3d=TRUE,
                  #main="Simulation_of_TLO_conditional_payoff_distribution_at
    _1y_with_maturity_3y_theoretical",
                  col.grid = "black")})

dev.off()
```

B.8 Unhedged balance sheet in the Vasicek-model

The following code includes: Fig.4.2

```
# Hedging with TLO

# Unhedged balance sheet
# Yellow Light Scenario

# Bonds are priced in the
# Vasicek model

# Stock prices
# generate vector of length 11
# Stockprices from -30% to 20%

# Names in percentage

Sperchar1 <- seq(-0.3,0.2,length.out = 11)
Sperchar1

# Function for percentage

percent <- function(x, digits = 2, format = "f", ...) {
  paste0(formatC(100 * x, format = format, digits = digits, ...), "%")}

Sperchar<-percent(Sperchar1)

Sperchar

Spercentage<-seq(0.7,1.2, length.out = 11)

names(Spercentage)<-Sperchar
Spercentage

# Initial value

S <- 30

# Stockprice Vector

Stockprices<- S*Spercentage
Stockprices


# LIBOR Rates vector of length 11
# ranges from -1.5 % to +1.5 %
# compare to [Jor07] with -3% to +3%

liborchange  <- seq(-0.015,0.015,length.out = 11)

Liborchar <- percent(liborchange)

Liborvalues<- seq(0.025,0.055, length.out =11)

names(Liborvalues)<-Liborchar
```

```
Liborvalues

# 100%  =[6]....unchanged scenario

Liborvalues[6]

# Bonds with duration tnn = 6 years
# Here bonds are priced in the vasicek
# model

PSI <- function (x,k=0.25){

  e<-NULL

  e <- (1-exp(-k*x))/k
  return (e)
}


Bond <- function(L =0.04, k=0.25, theta=0.012, sigma_l=0.02, t=0, tnn=3){

  b <- rep(0,1)

  integrand <-function(s) theta*PSI(tnn-s,k)
  a <-integrate(integrand,lower = t, upper=tnn)

  anew <- a[[1]]

  gamm <- -anew+sigma_l^2/(2*k^2)*(tnn-t)-sigma_l^2/(2*k^2)*PSI(tnn-t,k)-
    sigma_l^2/(4*k)*PSI(tnn-t,k)^2

  b <- exp(gamm-PSI(tnn-t)*L)

  return (b)
}

# Bond face value = 70*1/Bond(L=0.04,tnn=6)=90.58077
# Duration (y) = tnn
# LIBOR rate = 0.04

temp<-70*1/Bond(L=0.04,tnn=6)
Bondvalues<-temp*Bond(L= Liborvalues,tnn=6)

# Pension obligations like bonds
# Face value = 177.88 = 92/Bond(L=0.04,tnn=15)

temp1 <- 92/Bond(L=0.04,tnn=15)
temp1
PO <-temp1*Bond(L=Liborvalues,tnn=15)

names(PO) <- Liborchar

# All changes in
# stockprices, Bonds, Pensionobligation(PO)

```

```
Stockprices
Bondvalues
PO

################################################
# generate data.frame for Bonds and PO

Table1 <- data.frame(Bonds = Bondvalues, Pension = PO)

# hence we have a data.frame we need as.matrix

m.Table1 <- as.matrix(Table1)

# Function for the linearcombination between the
# stockprices and m.Table1

lincom <- function(Stockprices,m.Table1){

  z <- matrix(0,nrow = 11*11,ncol = 3)

  for ( i in 1:11){

    for (j in 1:11){

      k <-11*(i-1)+j
      z[k,1] <- Stockprices[i]
      z[k,2] <- m.Table1[j,1]
      z[k,3] <- m.Table1[j,2]
    }
  }
  return (z)
}


f<-lincom(Stockprices,m.Table1)

m.Table <- data.frame(Stockprices=f[,1],Bonds=f[,2],PO=f[,3])

m.Table_new <- as.matrix(m.Table)

# f contains all linearcombinations between stockprices
# and BONDS with PO

freeequity <- function (m.Table_new){

  u <- rep(0,nrow(m.Table_new))

  for (i in 1:nrow(m.Table_new)){

    u[i]<- m.Table_new[i,1]+m.Table_new[i,2]-m.Table_new[i,3]

  }
  return (u)

}
```

```r

fequity <- freeequity(m.Table_new)

fequity

# Function for the solvency ratio
# which is defined as:
# Solv Ratio = Free Equity / Pension Obligations

solvencyratio <- function(fequity,f){

  o <- rep(0,length(fequity))

  for (i in 1:length(fequity)){

    o[i]<-fequity[i]/f[i,3]
  }
  return (o)
}

solvperc <- solvencyratio(fequity,m.Table_new)

sp <- matrix(solvperc,ncol=11)

# Full table of the balance sheet

spnew <- as.vector(solvperc)

m.Table2 <- data.frame(m.Table_new, Solv_perc=spnew)

# Initial balance sheet at time t=0

m.Table2[72,]

# Worst case solvency ratio

sp[which.min(sp)]
m.Table_new[which.min(sp),]

# Best case solvency ratio

sp[which.max(sp)]
m.Table_new[which.max(sp),]

# persp PLOT

par(bg = "white")
x <- Liborvalues
y <- Stockprices
z <-sp
nrz <- nrow(z)
ncz <- ncol(z)
# Create a function interpolating colors in the range of specified colors
jet.colors <- colorRampPalette( c("red","yellow3","yellow2","yellow1","green1", "green") )
```

```r
# Generate the desired number of colors from this palette
nbcol <- 1000
color <- jet.colors(nbcol)
# Compute the z-value at the facet centres
zfacet <- z[-1, -1] + z[-1, -ncz] + z[-nrz, -1] + z[-nrz, -ncz]
# Recode facet z-values into color indices
facetcol <- cut(zfacet, nbcol)

# Plot

persp(x,
      y,
      z,
      col = color[facetcol],
      xlab="Short rate",
      ylab = "Stock portfolio",
      zlab = "Solvency ratio",
      #main="Unhedged balance sheet in the Vasicek model",
      ticktype = "detailed",nticks = 8,
      expand = 0.75,
      phi = 20, theta =-40)

# Saving the graphic

pdf(file= "Unhedged_balance_sheet_in_the_Vasicek_model.pdf")

# Plot

# Figure ?.?

persp(x,
      y,
      z,
      col = color[facetcol],
      xlab="Short rate",
      ylab = "Stock portfolio",
      zlab = "Solvency ratio",
      #main="Unhedged balance sheet in the Vasicek model",
      ticktype = "detailed",nticks = 8,
      expand = 0.75,
      phi = 20, theta =-40)

dev.off()

# Yellow light scenario

# Stock portfolio drops 30%
# LIBOR drops 100bps=1%

# S=30 -> S=21

S_shocked <- 21

# Bonds before shocked

```

```
Bonds <- 70

# PO before shocked

PO <- 92


# Bond face value = 70*1/Bond(L=0.04,tnn=6)=90.58077
# Duration (y) = tnn
# LIBOR rate = 0.04


temp<-Bonds*1/Bond(L=0.04,tnn=6)
temp
Bonds_shocked<-temp*Bond(L= 0.03,tnn=6)

Bonds_shocked

# Pension obligations like bonds
# Face value = 177.88 = 92/Bond(L=0.04,tnn=15)

temp1 <- PO/Bond(L=0.04,tnn=15)
PO_shocked <-temp1*Bond(L=0.03,tnn=15)

PO_shocked

totalassetside<-S_shocked+Bonds_shocked
freeequityshocked<-totalassetside - PO_shocked

freeequityshocked

# Solvency percentage in the yellow light scenario

freeequityshocked/PO_shocked

# Initial balance sheet before getting shocked

Initial_balance_sheet <-data.frame(Stock_portfolio=S,
                                   Bonds=Bonds,
                                   PO=PO,
                                   Free_Equity=S+Bonds-PO,
                                   Solvency_ratio=
                                     (S+Bonds-PO)/PO)

Initial_balance_sheet

# Yellow light scenario with all entries

Yellowlightscenario_balance_sheet<-data.frame(Stock_portfolio=S_shocked,
                                              Bonds=Bonds_shocked,
                                              PO=PO_shocked,
                                              Free_Equity=freeequityshocked
    ,
                                              Solvency_ratio=
```

```
                                                    freeequityshocked/PO_
    shocked)

Yellowlightscenario_balance_sheet


# Double check if the balance sheet is complete
# total asset side = total liabilities and free equity side

Yellowlightscenario_balance_sheet[1,1]+Yellowlightscenario_balance_sheet
    [1,2]
Yellowlightscenario_balance_sheet[1,3]+Yellowlightscenario_balance_sheet
    [1,4]

2.45/95.66
```

B.9 Unhedged balance sheet in the BMG-model

The following code includes: Fig.4.3

```
# Hedging with TLO

# Unhedged balance sheet
# Yellow Light Scenario

# BMG- Model framework!
# Bonds are priced with the product
# formula


# Stock prices
# generate vector of length 11
# Stockprices from -30% to 20%

# Names in percentage

Sperchar1 <- seq(-0.3,0.2,length.out = 11)
Sperchar1

# Function for percentage

percent <- function(x, digits = 2, format = "f", ...) {
  paste0(formatC(100 * x, format = format, digits = digits, ...), "%")}

Sperchar<-percent(Sperchar1)

Sperchar

Spercentage<-seq(0.7,1.2, length.out = 11)

names(Spercentage)<-Sperchar
Spercentage

# initial value

S <- 30

# Stockprice Vector

Stockprices<- S*Spercentage
Stockprices

# LIBOR Rates vector of length 11
# ranges from -1.5 % to +1.5 %

liborchange  <- seq(-0.015,0.015,length.out = 11)
liborchange
Liborchar <- percent(liborchange)

Liborvalues<- seq(0.025,0.055, length.out =11)

names(Liborvalues)<-Liborchar
```

```r
Liborvalues

# 100%  =[6]....unchanged scenario

Liborvalues[6]

# BOND Value in BMG-model

BondBMG <- function(L=0.04, t = 0, tnn = 3)
  {
  delta <- 0.5 # Difference between the tenor dates

  tenor <- seq(0,tnn, by =0.5)

  btnn <- 1/(1+delta*L)^(length(tenor)-t-1)

  return (btnn)
}

BondBMG(c(0.01,0.02,0.03,0.04),tnn=6)

# Bond face value = 70*1/BondBMG(L=0.04,tnn=6)=90.55246 for 6 years

temp<-70*1/BondBMG(L=0.04,tnn=6)

Bondvalues<-temp*BondBMG(L= Liborvalues,tnn=6)

# Pension obligations like bonds
# Face value = 166.65 = 92/BondBMG(L=0.04,tnn=15)

temp1 <- 92/BondBMG(L=0.04,tnn=15)
temp1
PO <-temp1*BondBMG(L=Liborvalues,tnn=15)

names(PO) <- Liborchar

# All changes in
# stockprices, Bonds, Pensionobligation(PO)

Stockprices
Bondvalues
PO

###############################################
# generate data.frame for Bonds and PO

Table1 <- data.frame(Bonds = Bondvalues, Pension = PO)

# hence we have a data.frame we need as.matrix

m.Table1 <- as.matrix(Table1)

# Function for the linearcombination between the
# stockprices and m.Table1

```

```
lincom <- function(Stockprices,m.Table1){

  z <- matrix(0,nrow = 11*11,ncol = 3)

  for ( i in 1:11){

    for (j in 1:11){

      k <-11*(i-1)+j
      z[k,1] <- Stockprices[i]
      z[k,2] <- m.Table1[j,1]
      z[k,3] <- m.Table1[j,2]
    }
  }
  return (z)
}

f<-lincom(Stockprices,m.Table1)

m.Table <- data.frame(Stockprices=f[,1],Bonds=f[,2],PO=f[,3])

m.Table_new <- as.matrix(m.Table)

# f contains all linearcombinations between stockprices
# and BONDS with PO

freeequity <- function (m.Table_new){

  u <- rep(0,nrow(m.Table_new))

  for (i in 1:nrow(m.Table_new)){

    u[i]<- m.Table_new[i,1]+m.Table_new[i,2]-m.Table_new[i,3]

  }
  return (u)

}

fequity <- freeequity(m.Table_new)

fequity

# Function for the solvency ratio
# which is defined as:
# Solv Ratio = Free Equity / Pension Obligations

solvencyratio <- function(fequity,f){

  o <- rep(0,length(fequity))

  for (i in 1:length(fequity)){

    o[i]<-fequity[i]/f[i,3]
  }
```

```r
  return (o)
}

solvperc <- solvencyratio(fequity,m.Table_new)

sp <- matrix(solvperc,ncol=11)

# Full table of the balance sheet

spnew <- as.vector(solvperc)

m.Table2 <- data.frame(m.Table_new, Solv_perc=spnew)

# Initial balance sheet at time t=0

m.Table2[72,]

# Worst case solvency ratio

sp[which.min(sp)]
m.Table_new[which.min(sp),]

# Best case solvency ratio

sp[which.max(sp)]
m.Table_new[which.max(sp),]

# Some pars

par(bg = "white")
x <- Liborvalues
y <- Stockprices
z <-sp
nrz <- nrow(z)
ncz <- ncol(z)

# Create a function interpolating colors in the range of specified colors

jet.colors <- colorRampPalette( c("red","yellow3","yellow2","yellow1","
    green1", "green") )

# Generate the desired number of colors from this palette

nbcol <- 1000
color <- jet.colors(nbcol)

# Compute the z-value at the facet centres

zfacet <- z[-1, -1] + z[-1, -ncz] + z[-nrz, -1] + z[-nrz, -ncz]

# Recode facet z-values into color indices

facetcol <- cut(zfacet, nbcol)

# 3D Plot
```

```
persp(x,
      y,
      z,
      col = color[facetcol],
      xlab="LIBOR rate",
      ylab = "Stock portfolio",
      zlab = "Solvency ratio",
      #main="Unhedged Balance Sheet shocked in the BGM Model",
      ticktype = "detailed",nticks = 8,
      expand = 0.75,
      phi = 20, theta =-40)

# Saving the graphic

pdf(file= "Unhedged_balance_sheet_shocked_in_the_BGM_Model.pdf")

# Plot

persp(x,
      y,
      z,
      col = color[facetcol],
      xlab="LIBOR rate",
      ylab = "Stock portfolio",
      zlab = "Solvency ratio",
      #main="Unhedged Balance Sheet shocked in the BGM Model",
      ticktype = "detailed",nticks = 8,
      expand = 0.75,
      phi = 20, theta =-40)

dev.off()

# Yellow light scenario

# Stock portfolio drops 30%
# LIBOR drops 100bps=1%

# S=30 -> S=21

S_shocked <- 21

# Bonds before shocked

Bonds <- 70

# PO before shocked

PO <- 92


# Bond face value = 70*1/Bond(L=0.04,tnn=6)=90.58077
# Duration (y) = tnn
# LIBOR rate = 0.04

```

```r
temp<-Bonds*1/BondBMG(L=0.04,tnn=6)
temp
Bonds_shocked<-temp*BondBMG(L= 0.03,tnn=6)

Bonds_shocked

# Pension obligations like bonds
# Face value = 166.65 = 92/BondBMG(L=0.04,tnn=15)

temp1 <- PO/BondBMG(L=0.04,tnn=15)
PO_shocked <-temp1*BondBMG(L=0.03,tnn=15)

PO_shocked

totalassetside<-S_shocked+Bonds_shocked
freeequityshocked<-totalassetside- PO_shocked

freeequityshocked

# Solvency percentage in the yellow light scenario

freeequityshocked/PO_shocked

# Initial balance sheet before getting shocked

Initial_balance_sheet <-data.frame(Stock_portfolio=S,
                                   Bonds=Bonds,
                                   PO=PO,
                                   Free_Equity=S+Bonds-PO,
                                   Solvency_ratio=
                                     (S+Bonds-PO)/PO)

Initial_balance_sheet

# Yellow light scenario with all entries

Yellowlightscenario_balance_sheet<-data.frame(Stock_portfolio=S_shocked,
                                              Bonds=Bonds_shocked,
                                              PO=PO_shocked,
                                              Free_Equity=freeequityshocked
    ,
                                              Solvency_ratio=
                                                freeequityshocked/PO_
    shocked)

Yellowlightscenario_balance_sheet

# Double check if the balance sheet is complete
# total asset side = total liabilities and free equity side

Yellowlightscenario_balance_sheet[1,1]+Yellowlightscenario_balance_sheet
    [1,2]
Yellowlightscenario_balance_sheet[1,3]+Yellowlightscenario_balance_sheet
    [1,4]
```

B.10 Hedged balance sheet in the BMG-model

The following code includes: Fig.4.5

```
# Pricing Traffic Light Option
# with analytical formula

# Remark: r is equally flat to LIBOR rates

# Need the package pbivnorm for the standardized
# bivariate normal distribution

require(pbivnorm)

# Formula for TLO

priceTLO<-function(L,S,t=0,Tn1=3,SStrike=100,LStrike=0.04,sigma_s =0.2,rho
    =-0.5) {

  # Difference between two tenor dates

  tau <- 0.5

  # T_n

  Tn <- Tn1 - tau

  # Need tenor for the semi-annual calculations
  # Starting with T_0=0, T_1=0.5, ...

  tenor <- seq( 0 , Tn1 , by = tau )

  # Bond with maturity at T_(n+1)
  # Due to the fact of semi-annual tenor dates
  # we need all tenor evaluation points = length(tenor)

  bTn1 <- 1/(1+tau*L)^(length(tenor)-t-1) # Bond with r = LIBOR r = x

  SSchlange <-(SStrike*bTn1)/S

  LSchlange <- LStrike/L

  sigma_xq <-(Tn1-t)*sigma_s^2

  sigma_x <- sqrt(sigma_xq)

  # We need lambda for the sigma_y

  lambda <- function(t,Tn) {

    a <- 0
    b <- 0.29342753
    c <- 1.25080230
    d <- 0.13145969

    result <- (a+(Tn-t)*b)*exp(-(Tn-t)*c)+d
```

```
  return (result)
}

integrand1 <- function(x) {lambda(t = x, Tn = Tn)^2}
sigma_yq1 <-integrate(integrand1, lower = t, upper = Tn )

# Returns only the value without abs error

sigma_yq<-sigma_yq1[[1]]

# sigma_y^2=sigma_yq

sigma_y <- sqrt(sigma_yq)

# sigma_xy

integrand2 <- function(x) {sigma_s * rho * lambda(t=x,Tn = Tn)}
sigma_xy1 <- integrate(integrand2,lower = t, upper= Tn)
sigma_xy <- sigma_xy1[[1]]

# mu_x

mu_x <- sigma_xq*(-0.5)

# mu_y

mu_y <- sigma_yq*(-0.5)

# rho_SL

rho_SL <- sigma_xy/(sigma_x*sigma_y)


# For a better reading of the formula

a1 <- as.numeric((log(SSchlange)-mu_x)/sigma_x)
b1 <- as.numeric((log(LSchlange)-mu_y)/sigma_y)
a2 <- as.numeric(a1-sigma_x)
b2 <- as.numeric(b1-rho_SL*sigma_x)
a3 <- as.numeric(a1-rho_SL*sigma_y)
b3 <- as.numeric(b1-sigma_y)
a4 <- as.numeric(a1-rho_SL*sigma_y-sigma_x)
b4 <- as.numeric(b1-rho_SL*sigma_x-sigma_y)

p1 <- pbivnorm(x = a1, y = b1, rho = rho_SL)
p2 <- pbivnorm(x = a2, y = b2, rho = rho_SL)
p3 <- pbivnorm(x = a3, y = b3, rho = rho_SL)
p4 <- pbivnorm(x = a4, y = b4, rho = rho_SL)


result <- as.numeric((L*S*(SSchlange*LSchlange*p1
                         - LSchlange*p2
                         - SSchlange*p3
                         + exp(sigma_xy)*p4)))
```

```
  print(c("Result",result))

  return (result)

}

# Hedging with TLO

# Hedged balance sheet
# Yellow Light Scenario

# BMG- Model framework!
# Bonds are priced within the BGM-Model

# Stock prices
# generate vector of length 11
# Stockprices from -30% to 20%

# Names in percentage

Sperchar1 <- seq(-0.3,0.2,length.out = 11)
Sperchar1

# Function for percentage

percent <- function(x, digits = 2, format = "f", ...) {
  paste0(formatC(100 * x, format = format, digits = digits, ...), "%")}

Sperchar<-percent(Sperchar1)

Sperchar

Spercentage<-seq(0.7,1.2, length.out = 11)

names(Spercentage)<-Sperchar

# initial value for the stock portfolio

S <- 30

# Stockprice Vector

Stockprices<- S*Spercentage
Stockprices

# In the BGM-model the problem arises
# with extrem LIBOR rate changes quite to
# the contrary as seen in the short rate
# model (Vasicek), where the long term rates
# resp. the long term bonds do not react as much
# as in the BGM-model
# Hence we put the range from -150bps
# to 150bps instead as in the Joergensen paper
# with the short rate from -300bps to 300bps
```

```
# LIBOR Rates vector of length 11
# ranges from -1.5 % to +1.5 %

liborchange  <- seq(-0.015,0.015,length.out = 11)

liborchange

Liborchar <- percent(liborchange)

Liborvalues<- seq(0.025,0.055, length.out =11)

names(Liborvalues)<-Liborchar
Liborvalues

# 100%  =[6]....unchanged scenario

Liborvalues[6]

# BOND Value in BMG-model

BondBMG <- function(L=0.04, t = 0, tnn = 3){

  delta <- 0.5 # Difference between the tenor dates

  tenor <- seq(0,tnn, by =0.5)

  btnn <- 1/(1+delta*L)^(length(tenor)-t-1)

  return (btnn)
}

BondBMG(c(0.01,0.02,0.03,0.04),tnn=6)

# Price for one TLO with
# maturity T_n+1=5 at time t=0 and LStrike=0.04
# SStrike=30, sigma_s=0.2, and rho=0.0

TLO1<-priceTLO(0.04,30,t=0,Tn1 = 5,SStrike = 30,LStrike = 0.04, sigma_s =
    0.2,rho= 0.0)

# Assumption sell bonds and buy 250 units of TLOs
# We need more units of TLOs, since in the BGM
# the longterm rates react more with the bonds
# as in the short rate model.
# But in the end the worst case solvency ratio will
# be %!!!!!!

amountofTLOs <- 250

TLO<-amountofTLOs*TLO1

# 70- TLO = new amount of Bonds

newbonds<-70-TLO
```

```
newbonds

# On the asset side we have bonds with a 6 year duration
# Bond face value = 62.3887/BondBMG(L=0.04,tnn=6)=79.12391

newbonds/BondBMG(L=0.04,tnn=6)

temp<-newbonds*1/BondBMG(L=0.04,tnn=6)
temp
Bondvalues<-temp*BondBMG(L= Liborvalues,tnn=6)

Bondvalues

# Due to the fact that longterm rates in the BGM
# react more than in the short rate model
# We take the Pension obligations on the
# lower end with a duration of 15 years

# Pension obligations like bonds
# Face value = 166.6453 = 92/BondBMG(L=0.04,tnn=15)

temp1 <- 92/BondBMG(L=0.04,tnn=15)
temp1
PO <-temp1*BondBMG(L=Liborvalues,tnn=15)

names(PO) <- Liborchar
PO

# All changes in
# stockprices, Bonds, Pensionobligation(PO)

Stockprices
Bondvalues
PO

##############################################
# generate data.frame for Bonds and PO

Table <- data.frame(Bonds = Bondvalues, Pension = PO)

# hence we have a data.frame we need as.matrix

m.Table <- as.matrix(Table)
m.Table

nrow(m.Table)

# Function for the linearcombination between the
# stockprices and m.Table1
# due to the fact that if the LIBOR rates drops
# Bonds and PO (like bonds) drop equally and
# therefore a "fixed" pair

lincom <- function(Stockprices,m.Table){

```

```
  z <- matrix(0,nrow = 11*11,ncol = 3)

  for ( i in 1:11){

    for (j in 1:11){

      k <-11*(i-1)+j
      z[k,1] <- Stockprices[i]
      z[k,2] <- m.Table[j,1]
      z[k,3] <- m.Table[j,2]
    }
  }
  return (z)
}

# Contains all linearcombinations

f<-lincom(Stockprices,m.Table)

m.Table1 <- data.frame(Stockprices=f[,1],Bonds=f[,2],PO=f[,3])

m.Table1 <- as.matrix(m.Table1)
head(m.Table1)

# z contains all TLO values

z <- outer(Liborvalues,Stockprices,priceTLO)

z
# Initial point

z[72]

m.Table1[72,]

nrow(m.Table1)

# Now we have to normalize
# the TLO values to the inital
# value of 7.611335=TLO

znew <- z/z[72]

which.max(znew)
znew1 <- znew*(TLO)
znew1

length(znew1)
which.max(znew1)
head(znew1)
znew2 <- as.vector(znew1)

# Table with TLO, Stockprices, Bonds and PO

m.Table1
```

```
m.Table2 <- data.frame( TLO=znew2, m.Table1)

m.Table2 <- as.matrix(m.Table2, ncol=4,ncol=121)

m.Table2

# Initial valuation point

m.Table2[72,]

# m.Table_comp2 contains all linearcombinations between stockprices
# and BONDS with PO

freeequity <- function (m.Table2){

  u <- rep(0,nrow(m.Table2))

  for (i in 1:nrow(m.Table2)){

    u[i]<- m.Table2[i,1]+m.Table2[i,2]+m.Table2[i,3]-m.Table2[i,4]

  }
  return (u)

}

# fequity contains all entries
# of Free equity

fequity <- freeequity(m.Table2)

m.Table2
Table3 <- data.frame(m.Table2, Free_equity=fequity)

m.Table3 <- as.matrix(Table3, ncol=5)
is.matrix(m.Table3)
head(m.Table3)
dim(m.Table3)
nrow(m.Table3)
m.Table3[72,]

# Test if all values in the balance sheet fit

m.Table3[50,1]+m.Table3[50,2]+m.Table3[50,3]
m.Table3[50,4]+m.Table3[50,5]

# Function for the solvency ratio
# which is defined as:
# Solv Ratio = Free Equity / Pension Obligations

solvencyratio <- function (m.Table3){

  o <- rep(0,nrow(m.Table3))

```

```
  for (i in 1:nrow(m.Table3)){
    o[i]<- m.Table3[i,5]/m.Table3[i,4]

  }
  return (o)
}

solvperc <- solvencyratio(m.Table3)

solvperc

m.Table3
m.Table4 <- data.frame(m.Table3,Solvency_Ratio=solvperc)

# Complete Table with all datas

head(m.Table4)

length(m.Table4)

# The full table with all data

m.Table4[72,]

# Corresponding solvency ratios

z_values<- matrix(m.Table4[,6], ncol=11)

############

## 3D PLOT

par(bg = "white")
y <- Stockprices
x <- Liborvalues
z <- z_values
nrz <- nrow(z)
ncz <- ncol(z)
# Create a function interpolating colors in the range of specified colors
jet.colors <- colorRampPalette( c("red","yellow3","yellow2","yellow1","
    green1", "green") )
# Generate the desired number of colors from this palette
nbcol <- 1000
color <- jet.colors(nbcol)
# Compute the z-value at the facet centres
zfacet <- z[-1, -1] + z[-1, -ncz] + z[-nrz, -1] + z[-nrz, -ncz]
# Recode facet z-values into color indices
facetcol <- cut(zfacet, nbcol)

# Plot

persp(x,
      y,
      z,
      col = color[facetcol],
```

```
      xlab="LIBOR rate",
      ylab = "Stock portfolio",
      zlab = "Solvency ratio",
      zlim = c(-0.2112645,0.45865), # Scaling as in the unhedged scenario
      #main="Hedged_balance_sheet_shocked_in_the_BGM_model",
      ticktype = "detailed",nticks = 8,
      expand = 0.75,
      phi = 20,
      theta = -40)

# In the worst case scenario
# with 250 TLOs we have a
# solvency ratio of

z_values[which.min(z)]
which.min(z)

# Saving the graphic

pdf(file= "Hedged_balance_sheet_shocked_in_the_BGM_model.pdf")

# Plot

# Figure ?.?

persp(x,
      y,
      z,
      col = color[facetcol],
      xlab="LIBOR rate",
      ylab = "Stock portfolio",
      zlab = "Solvency ratio",
      zlim = c(-0.2112645,0.45865),
      #main="Hedged_balance_sheet_shocked_in_the_BGM_model",
      ticktype = "detailed",nticks = 8,
      expand = 0.75,
      phi = 20,
      theta = -40)

dev.off()

######################################

# Yellow light scenario

# Stock portfolio drops 30%
# LIBOR drops 100bps=1%

# S=30 -> S=21

# Price of 1 TLO before shocking

TLO1

# Price of 250 TLOs
```

```
TLO

# Bonds before shocked

newbonds

# temp = for the 6y bonds for discounting

newbondshocked <- temp*BondBMG(L=0.03,tnn=6)

# temp1 = for the 15y PO bonds for discounting

newPOshocked <- temp1*BondBMG(L=0.03,tnn=15)

# Price for one TLO in the yellow light scenario

TLOshockedtemp<-priceTLO(0.03,21,t=0,Tn1 = 5,SStrike = 30,LStrike = 0.04,
                          sigma_s = 0.2,rho= 0.0)

TLOshockedtemp
TLO1
TLOshocked <- TLOshockedtemp*amountofTLOs
TLOshocked

newPOshocked
newbondshocked
newstockshocked <-21
TLOshocked
TLO

totalassetside<-TLOshocked+newbondshocked+newstockshocked

freeequityshocked<-totalassetside- newPOshocked

freeequityshocked

# Solvency percentage in the yellow light scenario

percent(freeequityshocked/newPOshocked)

# Initial balance sheet before getting shocked

Initial_balance_sheet <-data.frame(Stock_portfolio=S,
                                   Bonds=Bondvalues[6],
                                   TLO=TLO, PO=PO[6],
                                   Free_Equity=S+Bondvalues[6]+TLO-PO[6],
                                   Solvency_ratio=
                                   percent((S+Bondvalues[6]+TLO-PO[6])/PO
    [6]))

Initial_balance_sheet

# Yellow light scenario with all entries

```

```
Yellowlightscenario_balance_sheet<-data.frame(Stock_portfolio=
    newstockshocked,
                                             Bonds=newbondshocked,
                                             TLO=TLOshocked, PO=newPOshocked,
                                             Free_Equity=freeequityshocked,
                                             Solvency_ratio=
                                             percent(freeequityshocked/newPOshocked))

Yellowlightscenario_balance_sheet

# Double check if the balance sheet is complete
# total asset side = total liabilities and free equity side

Yellowlightscenario_balance_sheet[1,1]+Yellowlightscenario_balance_sheet
    [1,2]+Yellowlightscenario_balance_sheet[1,3]
Yellowlightscenario_balance_sheet[1,4]+Yellowlightscenario_balance_sheet
    [1,5]
```

List of Figures

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