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Dynamic Stochastic General Equilibrium Model for Small Open Economy: Austria

A Master's Thesis submitted for the degree of
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supervised by
Michael Reiter

Gizem Akar
1425485

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MSc Economics

Affidavit

I, Gizem Akar

hereby declare

that I am the sole author of the present Master's Thesis,

Dynamic Stochastic General Equilibrium Model for Small Open Economy: Austria

20 pages, bound, and that I have not used any source or tool other than those referenced or any other illicit aid or tool, and that I have not prior to this date submitted this Master's Thesis as an examination paper in any form in Austria or abroad.

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Abstract

In this paper, I follow the model structure and the steps taken by the paper by Fenz et al. (2012). Model for small open economy is constructed. 14 shocks and many rigidities are included in it. Then, the main aim of the paper is to understand how model variables react to different shocks and which shocks in fact affect model variables more in the short-run and in the long-run.

1 Introduction

Today many of the central banks try to develop their own DSGE models. Many academicians are now working on DSGE models to understand the variations in the specific variables and try to predict the impact of changes in the policy. For example, Breuss and Rabitsch (2009) developed DSGE model for Austria and in the model EU seen as rest of the world. They investigated 2 cases. In the first one, they estimated the impact of foreign shocks on Austrian economy when Austria is not in the monetary union and in the second case they try to understand the impact of shocks when Austria is in the monetary union. At the end, they try to understand how being in the monetary union changed the impact of the shocks. Fenz et al. (2012) constructed DSGE model for Austrian economy to interpret the impact of different shocks during the 2008 crisis and again in the model they have Euro area as the foreign economy. Also, Curdia and Finocchiaro (2005) try to understand the impact of exchange rate regime change on the economy by building up Swedish DSGE model. They analyze how economy is affected from this change.

For my thesis, I focus on Fenz et al. (2012)'s paper. In the following sections, first I will explain the model that they constructed, then I will explain how the model is estimated and finally I will analyse the empirical performance of the model.

2 Households and Labor Market

In this economy, there are infinitely living identical households and they try to maximize their expected life time utility. They get positive utility from consumption and get negative utility from supplied working hours.

Households accumulate capital and in return they get capital income, they hold domestic and foreign bonds and they receive interest payments, they own the firms and they get dividend, finally to finance the unemployment benefits and government expenditures, they pay lump-sum taxes.

$$E_t \sum_{0}^{\infty} \beta^s e^b_{t+s}(ln(C_{h,t+s} - \kappa C_{t+s-1}) - (e^l_{t+s}/1 + \sigma_l) H^{(1+\sigma_l)}_{h,t+s}) \quad (1)$$

From Acemoglu (2008) book, we can interpret as $C_{h,t}$ is the consumption of household , $H_{h,t}$ are working hours supplied by household and C_{t-1} denotes the average consumption of the economy in the previous period. β is discount factor and κ denotes external habit formation.Preferences of the individuals are characterized by constant relative risk aversion and external habit formation. The parameter κ denotes the intensity of the habit formation and this results in non-separability of preferences over time. (As Fenz et al. (2012) explains in the model) Intuitively, this can be explained as more the individual eats today, the more hungrier she/he will wake up tomorrow.

Also, as it can be seen, there are 2 shocks; namely consumption shock and labor supply shock. e^b_{t+s} works as a demand shock and causes agents to consume more or less in the current period. e^l_{t+s} is a shock to labor supply and captures the movements in the observed wedge in the first order condition relating consumption and labor. Both shocks follow an autoregressive process of order 1: $e^l_t = (1-\rho_l) + \rho_l e^l_{t-1} + \epsilon_{l,t}$ is a labor supply shock and $e^b_t = (1-\rho_b) + \rho_b e^l_{t-1} + \epsilon_{b,t}$ is a consumption shock.

Now, what about the budget constraints of the households? Household budget constraint can be written in the following way:

$$C_{h,t} + I_{h,t} + T_t + B^f_{h,t}/(R^f_t \phi(nfa_t, e^{rp}_t) P_t) = B^f_{h,t-1}/P_t + W_{h,t} H_{h,t} + \\ + (R^k_t Z_{h,t} - \psi(Z_{h,t})) K_{h,t-1} + D_t + \tau_t + \int_0^1 \psi(Z_{h,t}) K_{h,t-1} d_i$$

where I_t is investment, T_t is lump-sum tax which is paid by the households to finance unemployment benefits and government expenditure, $B^f_{h,t}$ are foreign bonds held in period t, P_t is the price level, R^f_t is the gross foreign interest rate paid on bonds, $\phi(nfa_t, e^{rp}_t)$ denotes a risk premium on foreign bond holdings which can be seen as foreign bond holding's compensation for investors who tolerate risk rather than holding a risk free asset, R^k_t is the rate of return on physical capital that households hold, $W_{h,t}$ is the real wage rate in which fixed by each household according to their own work because households are monopolistic suppliers(subject to rigidities), Z_t is the utilization of capital, $\psi(Z_t)$ is the cost of utilization of capital, K_t is the stock of capital in which households receive capital income from it, D_t denote dividend payments from the firms that are owned by the households and τ_t is the net inflow from the state contingent securities(authors assume aka Arrow-Debreu market structure).

As "The Econometrics of DSGE Models" (2010) explains, this budget constraint is different than conventional budget constraints. Now, each household is a monopolistic supplier of his/her own type of work. So, households are fixing their wages by taking into account the wage rigidities and they work by how much is demanded by firms given this wage level.

Households own the capital stock. Evaluation of capital can be given by:

$$K_{h,t} = (1-\tau)K_{h,t-1} + (1 - S(e_t^i)I_{h,t}/(\mu^a I_{h,t-1}))I_{h,t} \tag{2}$$

where τ is the rate of depreciation. S(.) is the investment adjustment cost, μ_a denotes the trend growth rate of the economy. This construction of the investment adjustment cost captures the cost of moving away from the path of investment growth that that we have in the balanced growth path. e^i is a negative investment shock which actually shifts the relative price of capital and it follows an autoregressive process $e_t^i = (1-\rho_i) + \rho_i e_{t-1}^l + \epsilon_{i,t}$.

Households in the economy will want to maximize their objective function with respect to their budget constraint and law of motion of capital accumulation. Also, I want o emphasize the fact that $D_{h,t}$ and $\tau_{h,t}$ are taken as given by the authors in the model.

Each household $h\epsilon[0,1]$ provides a differentiated type of labor services. Perfectly competitive labor service firms aggregates differentiated labor services and turns them into homogeneous labor services. Then, in the paper it is assumed that intermediate good firms buy homogeneous labor services. This is a trick to get rid of heterogeneous agents in the model. Perfectly competitive labor service firms aggregates differentiated labor services according to CES aggregator.

$$H_t = (\int_0^1 (H_{h,t})^{(1/1+\lambda_w)} d_i)^{1+\lambda_w} \tag{3}$$

So, as Fenz et al. (2012) explains, this production function shows that $(1 + \lambda_w)/\lambda_w$ is the elasticity of substitution and H_t shows the aggregate labor demand. After solving the maximization problem, the demand for labor labor from household j can be written as:

$$H_{h,t} = (W_{h,t}/W_t)^{-(1+\lambda_w)/\lambda_w} H_t \tag{4}$$

Now, wage setting mechanism can be introduced. One way to introduce wage rigidity is time dependent rule called "Calvo pricing". In each period a fraction of $1 - \xi_w$ of the households receive a signal that permits them to re-optimize their nominal wage, all other households can only partially index their wages by past CPI inflation and fully to growth rate of the permanent technology process A_t. So, according to this rule, expected value of the real wage of household h in period t+s can be expressed in the following way:

$$E_t(W_{h,t+s}) = E_t(\xi_w (A_{t+s}/A_t) \Pi_{t+s-1}^{\gamma_w} W_{h,t+s-1} + (1 - \xi_w) W'_{h,t+s}) \tag{5}$$

where $W'_{h,t}$ stands for the solution to the optimization problem. Using the assumptions on nominal wage rigidity, the law of motion for wages can be formulated as CES aggregate over the prices of adjusters and non-adjusters:

$$W_t = [\xi_w (\mu_t^a W_{t-1} \Pi_{t-1} \Pi_t^{-1})^{-1/\lambda_w} + (1 - \xi_w)(W'_t)^{-1/\lambda_w}]^{-\lambda_w} \tag{6}$$

3 Production and Foreign Trade

There are 3 kinds of domestic firms in this economy. First, domestic intermediate firms rent capital from the households and buy homogeneous labor from labor service firms to produce differentiated domestic intermediate good. Then, domestic good assembling firms take differentiated intermediate goods and turns them into homogeneous good. Finally, final good producer firms combines domestically produced homogeneous good and imported good and turn it into one homogeneous final good. This final good is used for consumption, investment, government consumption and exports.

3.1 Domestic Intermediate Good Producers

As again mentioned above, there are continuum of domestic intermediate good producers in this economy and these firms rent capital from the households and buy homogeneous labor from labor service firms to produce differentiated domestic intermediate good. The production function of these firms can be expressed as follows:

Their production function is:

$$Y_{j,t} = A_t^{1-\alpha} e_t^a {K'_{j,t}}^{\alpha} H_{j,t}^{1-\alpha} - A_t \Phi \tag{7}$$

where A_t is a non-stationary global technology process, e_t^a is a stationary domestic technology process, $H_{j,t}$ and $K'_{j,t}$ denote labor and effective capital. $A_t \Phi$ is a fixed cost of production.

Non-stationary and stationary technology process evolves in the following way:

$$A_t / A_{t-1} =: \mu_t^a = (1 - \rho_{\mu^a}) \mu^a + \rho_{\mu^a} \mu_{t-1}^a + \mu^a \epsilon_t^{\mu^a} \tag{8}$$

$$e_t^a = (1 - \rho_a) + \rho_a e_{t-1}^a + \epsilon_t^a \tag{9}$$

Capital is rented by households to firms but the effective capital stock $K'_{j,t}$ is linked to the households' capital stock in this way:

$$\int_0^1 K'_{j,t} d_j = \int_0^1 Z_{h,t} K_{h,t-1} d_h \tag{10}$$

The problem of the intermediate good producer can be divided into 2 parts. First, given input prices(wage and rent), how much they should hire labor from labor servicing firms and how much capital they should rent from the households. They will minimize their cost subject to their production function. One can also get marginal cost equation. You can see the solution of the problem below:

$$K'_{j,t} / H_{j,t} = \alpha W_t / (1 - \alpha) R_t^k \tag{11}$$

The second problem of intermediate good producers is to set a price for the intermediate good. Again, like households, the intermediate good producer is subject to Calvo pricing. Now, probability of re-optimizing prices is $1-\xi_p$ and these firms set the price $P_t^{j\prime}$. As Fenz et al. (2012) explains, the remaining ξ_p firms follow a simple rule: $P_{j,t}^d = (\pi_{t-1}^d)^{\gamma_p} P_{j,t-1}^d$. Firms maximize $P_{j,t}^{d}{}'$ as $max \sum_0^\infty \xi_p^s \beta^s (\Lambda_{t+s}/\Lambda_t)(Profit_{j,t+s}/Profit_{t+s})$ subject to production function and the price of the domestic good can be written in the following way:

$$P_t^d = [\xi_p(P_{t-1}^d(\pi_{t-1}^d)^{\gamma_p})^{-1/\lambda_{p,t}} + (1-\xi_p)(P_t^{d\prime})^{-1/\lambda_{p,t}}]^{-\lambda_{p,t}} \qquad (12)$$

3.2 Competitive Domestic Good Assembling Firms

Competitive domestic good assembling firms buy differentiated goods from the domestic intermediate good producers and turns them into homogeneous domestic good which will be again used by the final good producing firms. These firms aggregate the intermediate good with the following production function:

$$Y_t = [\int_0^1 Y_{j,t}^{1/(1+\lambda_{p,t})} d_j]^{1+\lambda_{p,t}} \qquad (13)$$

where $(1+\lambda_{p,t})/\lambda_{p,t}$ shows the elasticity of substitution, Y_t denotes the domestic good, $Y_{j,t}$ the differentiated intermediate goods and $\lambda_{p,t}$ is a time varying mark-up subject to an i.i.d. cost push shock $(\lambda_{p,t} = \lambda_p + \epsilon_t^p)$. Competitive domestic good assembling firms maximize profits with respect to their production function taking as given all intermediate good prices. So, the input demand for differentiated intermediate good can be written as :

$$Y_{j,t} = (P_{j,t}^d/P_t^d)^{-(1+\lambda_{p,t})/\lambda_{p,t}} Y_t \qquad (14)$$

The aggregate price of domestic good is;

$$P_t^d = [\int_0^1 (P_{j,t}^d)^{-1/\lambda_{p,t}} d_j]^{-\lambda_{p,t}} \qquad (15)$$

3.3 Final Good Producers

In the model, it is assumed that there is only one good in the economy(F_t) as a final and this one final good is used for consumption, investment, exports and for government consumption. For production function, the final good producing firm uses domestically produced goods and imported goods as inputs. The production function of the final good assembling firm has the following form:

$$F(D_{i,t}, M_{i,t}) = [\mu^{\sigma_m/1+\sigma_m} D_{i,t}^{1/1+\sigma_m} + (1-\mu)^{\sigma_m/1+\sigma_m} (\phi_{i,t} M_{i,t})^{1/1+\sigma_m}]^{1+\sigma_m} \tag{16}$$

where $D_{i,t}$ refers to domestic good, $M_{i,t}$ refers to imported good, μ shows the weight to domestic good and $1+\sigma_m/\sigma_m$ shows the elasticity of substitution between imported and domestic good. Also, in the production function ϕ refers to adjustment of imported good. This adjustment cost has the following functional form:

$$phi_{i,t} = [1 - \phi_m(e_t^m - (M_{i,t}/D_{i,t})/(M_{i,t-1}/D_{i,t-1}))^2] \tag{17}$$

This adjustment cost exists when previous period's imported good over domestic good level changes this period. In this expression one can also see the shock to import adjustment cost. e_t^m evolves according to auto-regressive process with degree 1: $e_t^m = (1-\rho_m) + \rho_m e_{t-1}^m + \epsilon_{m,t}$

As in the previous cases, final good producing firm maximizes its profit by setting the price of the final good as P_t and finding the cost minimizing combination of domestic and foreign goods.

4 Rest of the World

In this model, rest of the world is constructed as rest of the European area. Since Austria is relatively small compare to European area, in the model it is assumed that rest of the world is not affected by the shocks coming from Austria and the rest of the world is exogenous to Austrian economy. Rest of the European area is modeled as a 3 equation system and these three equations refer to output, inflation and interest rates. Rest of the world variables will be described with subscript "f".

First, start with the households in the foreign economy.Again, like in the domestic economy, there are continuum of the households in the economy which is indexed by $h\epsilon[0,1]$. Like domestic households, they want to maximize their life time utility function:

$$E_0 \sum_0^{\infty} \beta^t e_t^{y_f} (ln(C_{h,t+s} - \kappa^f C_{t-1}^f) - (H_{h,t}^f)^{(1+\sigma_l^f)}/(1+\sigma_l) \tag{18}$$

where $e_t^{y_f}$ e_{t+s}^b works as a demand shock and causes agents to consume more or less in the current period in the foreign economy and follows an autoregressive process of order 1: $e_t^{y_f} = (1-\rho_{y_f}) + \rho_{y_f e_{t-1}^{y_f + \epsilon_{y_f,t}}}$. As I want to point out in here, there is no labor supply shock like in the domestic economy.

Now, what about the budget constraints of the households in the foreign economy? Household budget constraint can be written in the following way:

$$C_{h,t}^f + B_{h,t}^f / R_t^f P_t^f = B_{h,t-1}^f / P_t^f + W_t^f H_{h,t}^f + D_t^f \tag{19}$$

Also, now different than the domestic economy, wages are assumed to be flexible and therefore taken as given by the foreign households. Thanks to flexible wages, hours will be set optimally and wage will be equal to marginal rate of substitution between leisure and consumption.

In the production side, production function consists of aggregate working hours supplied by the households and technology (no capital). The functional form is the following: $Y_t^f = A_t H_t^f$ and $Y_t^f = C_t^f$.

Like intermediate good producing firms in the domestic economy, foreign economy producers produce differentiated good and again subject to Calvo pricing. With probability $(1-\xi_p^f)$ firms are allowed to change their prices and those firms re-optimize their wages but the rest of them adjust their price according to last period's inflation level. With this logic, the law of motion of foreign price level can be written in the following way:

$$P_t^f = [\xi_p^f (P_{t-1}^f (\pi_{t-1}^f)^{\gamma_p^f})^{-1/\lambda_{p,t}^f} + (1-\xi_p^f)(P_t^{f\prime})^{-1/\lambda_{p,t}^f}]^{-\lambda_{p,t}^f} \tag{20}$$

Also, in the foreign economy, monetary authority follows this rule to determine foreign interest rate:

$$R_t^{f\prime} = \rho_r R_{t-1}^f + (1-\rho_r)(\psi_\pi^f \pi_t^{f\prime} + \psi_\pi^f y_t^{f\prime}) + \epsilon_t^r \tag{21}$$

Foreign economy produces its own differentiated good and sells it domestic economy(Austria). In the foreign economy, the price of the good is P_t^f but foreign goods are sold to the domestic economy with price P_t^m and this is called mark-up process. this mark-up process is subject to a mark-up shock.

$$P_t^m = e_t^{\pi_m} P_t^f \tag{22}$$

where mark-up process evolves according to $e_t^{\pi_m} = \rho_{\pi m} e_{t-1}^{\pi_m} + (1 - \rho_{\pi m}) + e^{\pi_m} + e^{\pi_m} \epsilon_t^{\pi_m}$.

In the production of final foreign goods, domestically produced goods and imported goods from Austria are used.

$$F_{i,t}^f = \int_0^1 [\mu_f^{\sigma_{m_f}/(1+\sigma_{m_f})} (D_{i,t}^f)^{1/(1+\sigma_{m_f})} + (1-\mu_f)^{\sigma_{m_f}/(1+\sigma_{m_f})} (\phi_{i,t}^f M_{i,t}^f)^{1/(1+\sigma_{m_f})}]^{1+\sigma_{m_f}} d_i \tag{23}$$

In this final foreign good production function, μ_f shows the weight of the foreign good in the production, $(1+\sigma_{m_f})/\sigma_{m_f})$ shows the elasticity of substitution,shock process for adjustment cost: $e_t^{\pi_{m_f}} = (1-\rho_{\pi_{m_f}}) + \rho_{\pi_{m_f}} e_{t-1}^{m_f} + \epsilon_{m_f,t}$

The budget constraint of the government will be stated below in the market clearing section.

5 Clearing

Above, I mentioned that in the domestic economy, there is just one good in the economy as a final. So, when we aggregate, following equation can be written:

$$F_t = C_t + I_t + X_t + A_t e_t^g \tag{24}$$

For this equation, we can government expenditure consists of ,as authors also call, "productivity adjusted government consumption process" and it evolves according to $e_t^g = (1-\rho_g)g + \rho_g e_{t-1}^g + \epsilon_{g,t} g$.

Also, since Austria is a small interest rate coming from the rest of the world is taken as given. In the model, risk premium on foreign bond holdings are stated as ϕ' and risk adjusted interest rate is $R_t^f \phi'$. Next question is what is the functional form of this risk premium:

$$\phi'(nfa_t, e_t^{rp}) = exp(-\phi_a nfa_t + e_t^{rp}) \tag{25}$$

The budget constraint of the government:

$$A_t e_t^g + B_{g,t}^f / R_t^f \phi'(nfa_t, e_t^{rp}) P_t = T_t + B_{g,t-1}^f / P_t \tag{26}$$

and after getting this, we can show the law of motion of foreign bond holdings by combining government and household budget constraints:

$$B_t^f / R_t^f \phi'(nfa_t, e_t^{rp}) = B_{t-1}^f + P_t X_t - P_t^M M_t \tag{27}$$

6 Rigidities and Shocks

Now, after documenting the construction of the model, I want to summarize the shocks and the rigidities used in the model to make it more clear. In this model many nominal and real rigidities have been used. As I also described above, Calvo price mechanism is used for monopolistically competitive domestic intermediate good producing firms and foreign good producers. In this mechanism only with some probability, some firms are allowed to re-optimize their wages and rest of them are not allowed to re-optimize their prices but just they adjust their prices according to previous period's inflation level. Also, for the households, the same Calvo pricing mechanism used. Each household is a monopolistic supplier of his/her own type of work. So, household are fixing their wages by taking into account the wage rigidities. Each period, with some probability they are allowed to optimize their wages; otherwise rest of them just adjust their wages according to past period's inflation level.

Another rigidity in the economy is real adjustment costs which pop up in the investment and imported goods section. Investment adjustment cost captures the idea of costs which is the result of moving away from the investment growth that is observed in the balanced growth path. Also, in terms of real adjustment costs for the imported goods, changing the structure of the trade is costly because of adjustment cost and it happens only gradually.

In this economy, we also have external habit formation. Intuitively, this can be explained as more the individual eats today, when this individual wakes up tomorrow, he/she will want to eat even more.

Additionally, capital utilization rate is a variable in this economy. In their budget constraints, households want to increase their utilization rate from their capital but it is costly, so they have to decide on optimal capital utilization rate.

In this economy, there are 14 shocks. 3 of them belongs to shock which occur in the foreign economy and 11 of them belong to domestic economy. Since domestic economy is small compare to the rest of the world, foreign economy is not affected by the shocks coming from domestic economy but domestic economy is affected by the shocks coming from rest of the world and in the "result" section, I will try to understand which shocks especially affects the domestic economy in the short-run, medium-run and in the long-run. the list of shocks and rigidities can be seen below:

Table 1: Summary of shocks and rigidities

	Fenz&Reiss&Schneider (2012)
Technology Shocks	Permanent Tech.
	Stationary Tech.
Demand Shocks	Government Spending
	Investment Adjustment Cost
	Consumption Preference
Price Shocks	Price Mark-Up
	Labor Supply
	Foreign Inflation
	Relative Prices
Risk Premium Shock	Risk Premium
Foreign Monetary Shock	Foreign Monetary
Foreign Trade Shocks	Foreign IS Curve
	Domestic Import Preference
	Foreign Import Preference
Frictions	Consumption Habits
	Investment Adjustment Cost
	Import Adjustment Cost
	Price Rigidity
	Wage Rigidity

7 Solution and Estimation

Authors estimated the model by using Bayesian techniques.

First I will talk about the data that authors used and how I collect the data to replicate the paper. I collect the data for GDP, private consumption, investment, export, import, employment, real wages, domestic price level and import deflator data for Austria. Also, GDP, price level and short term interest rate data have been collected for rest of the European Area. The source of the data consists of Eurostat and Federal Reserve Bank of St. Louis data bank. The time interval for the data is between 1995Q1 to 2011Q1. Afterwards, the data has been seasonally adjusted, stationarized by taking the growth rates to the previous quarter and finally the growth rates have been de-trended.

Now, I will talk about how to link the data to the model. In order to construct the likelihood function, the model variables have to be linked to set observables (the data that I mentioned in the above paragraph).The measurement equations that have been used in Fenz et al. (2012) paper are the following ones:

$$y_t^{obs} = y_t' - y_{t-1}' + \mu_t^a \tag{28}$$

$$c_t^{obs} = c_t' - c_{t-1}' + \mu_t^a \tag{29}$$

$$i_t^{obs} = i_t' - i_{t-1}' + \mu_t^a \tag{30}$$

$$m_t^{obs} = m_t' - m_{t-1}' + \mu_t^a \tag{31}$$

$$w_t^{obs} = w_t' - w_{t-1}' + \mu_t^a \tag{32}$$

$$y_t^{f,obs} = y_t^{f\,\prime} - y_{t-1}^{f}{}' + \mu_t^a \tag{33}$$

$$N_t' = (1 - \beta\xi_e)(H_t' - N_t')/(1 - \xi_e) + \beta E_t N_{t+1}' \tag{34}$$

These measurement equations and the vector auto-regressive representation of the states(state transition equations for unique and stable solution) provide a state-space representation for the linearized(see appendix) DSGE model. Combining two of them will help us to construct the likelihood function of the model. For the estimation of the model, Bayesian method will be used but the problem is likelihood function for the Bayesian estimation should be constructed only from observables. Therefore, hidden states have to be integrated out to get the desired likelihood function and we need a filter for this purpose.

There are 3 ways to get the likelihood function. In the first one, algorithm uses Chapman-Kolmogorov and Bayes' theorem and by forecasting and updating, likelihood function can be obtained but in this approach practical implementation is hard and includes computational costs. Second, as also "The Econometrics of DSGE Models" (2010) mentiones, if we have linearity and normality, Kalman filter can be used. Third, if the transition and measurement equations are not linear and the shocks are not normally distributed "the particle filter" is a way to get likelihood function of the state space representation(for more information you can see "The Econometrics of DSGE Models" (2010) paper).

The next step is to determine which priors to use to get posterior distribution. Prior distributions are used to describe the initial knowledge about the our parameter at hand, before screening anything from the outside. For shock auto-correlations, authors assigned beta distribution, for variance of shock innovations, they assigned inverse gamma distribution and for structural parameters, it depends on parameter at hand.(in the appendix you can see the list of whole assigned prior distributions)

After obtaining prior and likelihood function, we can evaluate posterior for a given parameter but characterizing the whole posterior is nearly impossible. For this purpose, Metropolis Hastings algorithm will be used. The Metropolis-Hastings algorithm belongs to the class of Markov chain Monte Carlo(McMc) algorithms.The algorithm constructs a Markov chain such that the distribution associated with this chain equals the posterior distribution of interest. As Fernandez-Villaverde,Jesus (2010) explains, the initial draws obtained from an MH algorithm do not reflect the posterior distribution. There may be a large number of draws before the sampler has converged, that is, when a draw from the Markov chain approximately the same distribution as a direct draw from the posterior. In the paper, authors estimate the model by taking 1.000.000 draws from the Metropolis Hastings algorithm and they get rid of first 10% of the draws because they declare them as burn-in draws.

8 Results (Application)

8.1 Impulse Responses for Different Shocks

In this section, I will talk about how my variables in my model reacts to temporary technology shock, permanent technology shock, interest rate shock, world demand shock and price mark-up shock. In the appendix section,you can see the graphs for impulse responses in the following way.

I want to start with temporary technology shock. As it is expected, temporary technology shock increases production capacity of the domestic economy and therefore marginal cost of production decreases. What about the demand side? We have many nominal and real rigidities in this model like wage and price rigidities, investment and import adjustment costs and consumption habits. Therefore, demand does not increase as much as supply side. Therefore, firms need less input and capital utilization rate and hours worked decreases. Also, since marginal cost decreases thanks to temporary technology shock, inflation decreases. Additionally, since now domestic economy gains price competitiveness in the international environment, domestic economy exports increase.

The next one is permanent technology shock. The term "permanent" is used because variables do not return to their initial levels but they converge to their new levels after the shock. After the shock; consumption, export and investment increase because firms and households predict the increase the activity. I want to continue by comparing this permanent shock with temporary technology shock. In the temporary technology case, capital utilization rate and hours worked decrease because firms can produce more than what is needed. Yet, now with the permanent shock, capital utilization rate and hours worked increases because of the high activity and again by contrast to temporary shock, high input demand results in increase in real wages and rental rates of capital. Even though input prices are increased for the firms, inflation does not go up much because of the decrease in marginal cost.

The third one is monetary policy shock. This is a shock to the rest of the world interest rate and it is a contraction in the foreign economy since the interest rate is increased. What would happen? Since it is a contractionary movement in the economy, output and inflation will decrease in the foreign economy and since Austria is linked to foreign economy through financial flows and trade, output also decreases in the domestic economy. Yet, in the domestic economy, we have many nominal and real rigidities like I mentioned many times above, the movements in the domestic economy is much slower than the foreign economy since the foreign economy is just constructed according to 3 equation system. Increase in foreign interest rate results in decrease in inflation and thanks to this, foreign economy gains competitiveness in the international environment.

The fourth one is world demand shock. This shock refers to shock to IS curve in the foreign economy which results in increase in output and price level in the foreign economy. As the activity in the foreign economy increased, there will be more demand from the foreign economy to Austrian goods and therefore export of the domestic economy will increase. To cover the need for more domestic goods, consumption and investment decreases and the net foreign asset sign of the domestic economy turns into positive and after a while this positive sign is used to increase consumption and investment in the following periods.

The fifth one is price mark-up shock. This shock results in increase in Austrian mark-up and it manifests itself in increase in inflation rate. since now prices of the domestic goods become relatively expensive, domestic economy loses its competitiveness in the international environment. Also, since less is demanded from the outside and also since now prices of the goods in the domestic economy is more expensive, demand decreases and fall in output follows it. Since we have high inflation rate, real wages of the households decreases and also real rate of return of capital decreases as well which in return decreases demand again.

8.2 Variance Decompositions

After estimating the model, we want to see which shocks in fact has an huge impact on our model variables in the short-run, medium-run and in the long-run. For this purpose, forecast error variance decomposition has been used.

Let's start with domestic output. In the short run, domestic output is mostly affected by the shocks coming from foreign import preferences and domestic import preferences. This situation is something expected because Austria is a small open economy and shocks coming form the rest of the world would have more impact on domestic economy variables.

Let's take a look at domestic inflation. In the short run, domestic inflation is highly affected by the shocks coming from import prices, world inflation and domestic price mark-up. When we go from short-run to long-run, still import price shock remains strong and explains half of the fluctuations in the inflation.

In terms of employment, in the short run, most of the fluctuations are explained by transitory technology shock, domestic import shock and foreign import preferences. However, when we move from short-run to long-run, domestic price mark-up gains importance and domestic import and foreign import preferences still stay strong in explaining the fluctuations.

In terms of exports, in the short run, all most all the fluctuations in the domestic exports are explained by shocks to foreign import preferences. Yet, when we move to long-run, shocks to import preferences still explains half of the fluctuations but now permanent technology shock also has contribution.

In terms of investment, in the short run, shocks to investment explains all most everything and when we move from short-run to long-run, permanent technology shock also takes in explaining the fluctuations in investment.

Table 2: Variance Decomposition

	Output
Short-run	Foreign Import Preferences: 51%
	Domestic Import Preferences: 24%
Long-run	Permanent Tech. Shock: 45%
	Domestic Import Preferences: 15%
Short-run	**Inflation**
	Import Prices: 68%
	World Inflation:16%
Long-run	Import Prices: 50%
	Domestic Price Markup:16%
	Employment
Short-Run	Foreign Import Preferences: 37%
	Domestic Import Preferences:28%
Long-Run	Foreign Import Preferences:25%
	Domestic Price Markup:12%
	Export
Short-run	Foreign Import Preferences: 83%
	Foreign Output:12%
Long-run	Foreign Import Preferences: 53%
	Permanent Tech. Shock: 30%
	Investment
Short-run	Investment : 90%
Long-run	Investment:45%
	Permanent Tech. Shock: 23%

9 Conclusion

In this paper, I tried to replicate the paper of Fenz,Reiss,Schneider (2012); namely DSGE model for small open economy case for Austria. I first tried to understand how Fenz et al. (2012) set up the model and give detailed information about the setting. Then, I replicate the results by collecting the data, constructing the code and estimating the model.

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Curdia, Vasco, and Daria Finocchiaro. 2005. "An Estimated DSGE Model for Sweden with a Monetary Regime Change."

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A Appendix

In this section, I report the graphs of my model.

A.1 Impulse Response Graphs

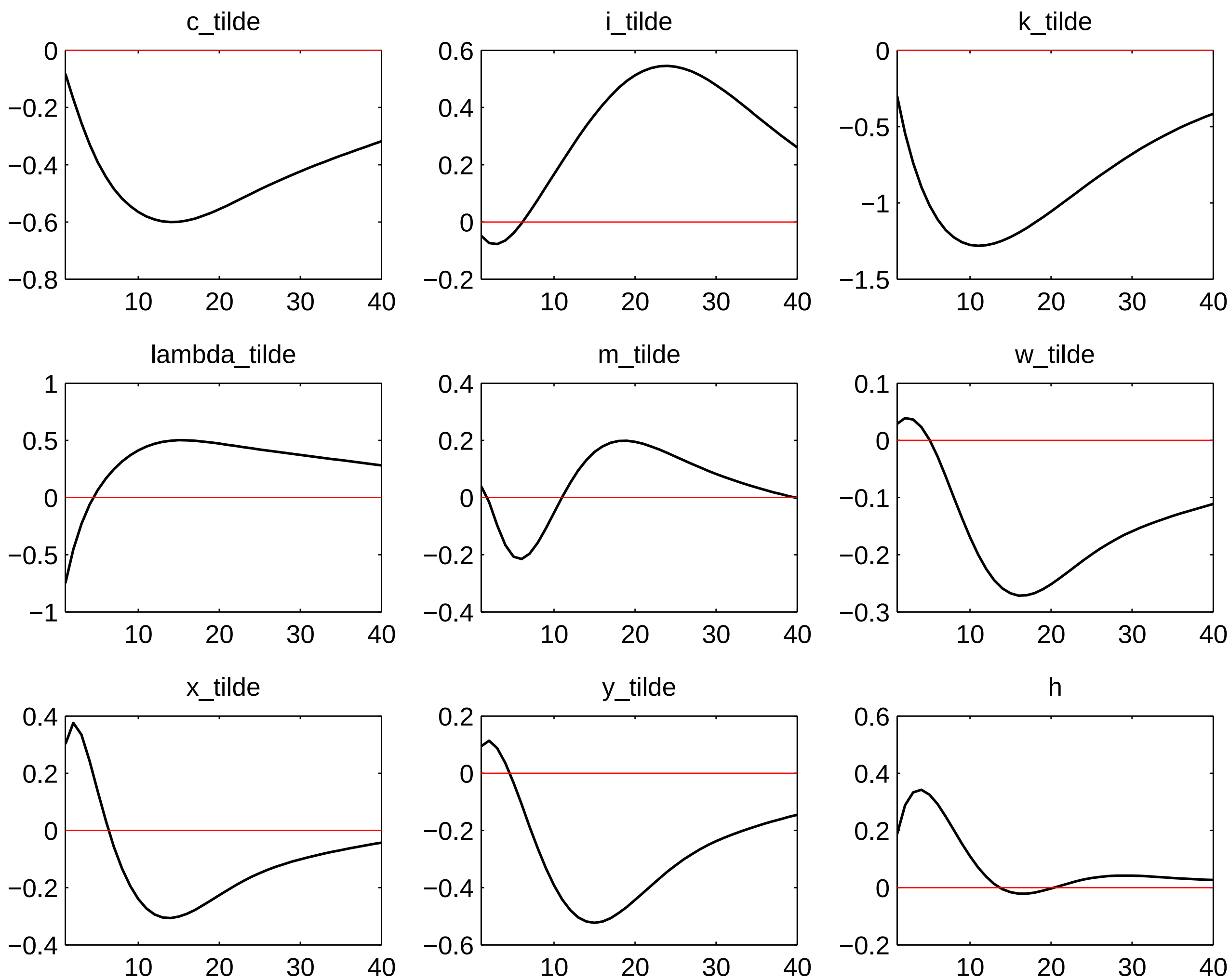
In this section, first 3 pages show temporary technology shock, next 4 pages show permanent technology shock, next 4 pages show monetary policy shock, next 4 pages show world demand shock, next 4 pages show price mark up shock.

A.2 Forecasting Graphs

In this section, tables with blue lines show the prediction of my model.

A.3 Log-Linearization

In the last 2 pages, you can find log-linearized version of my model.



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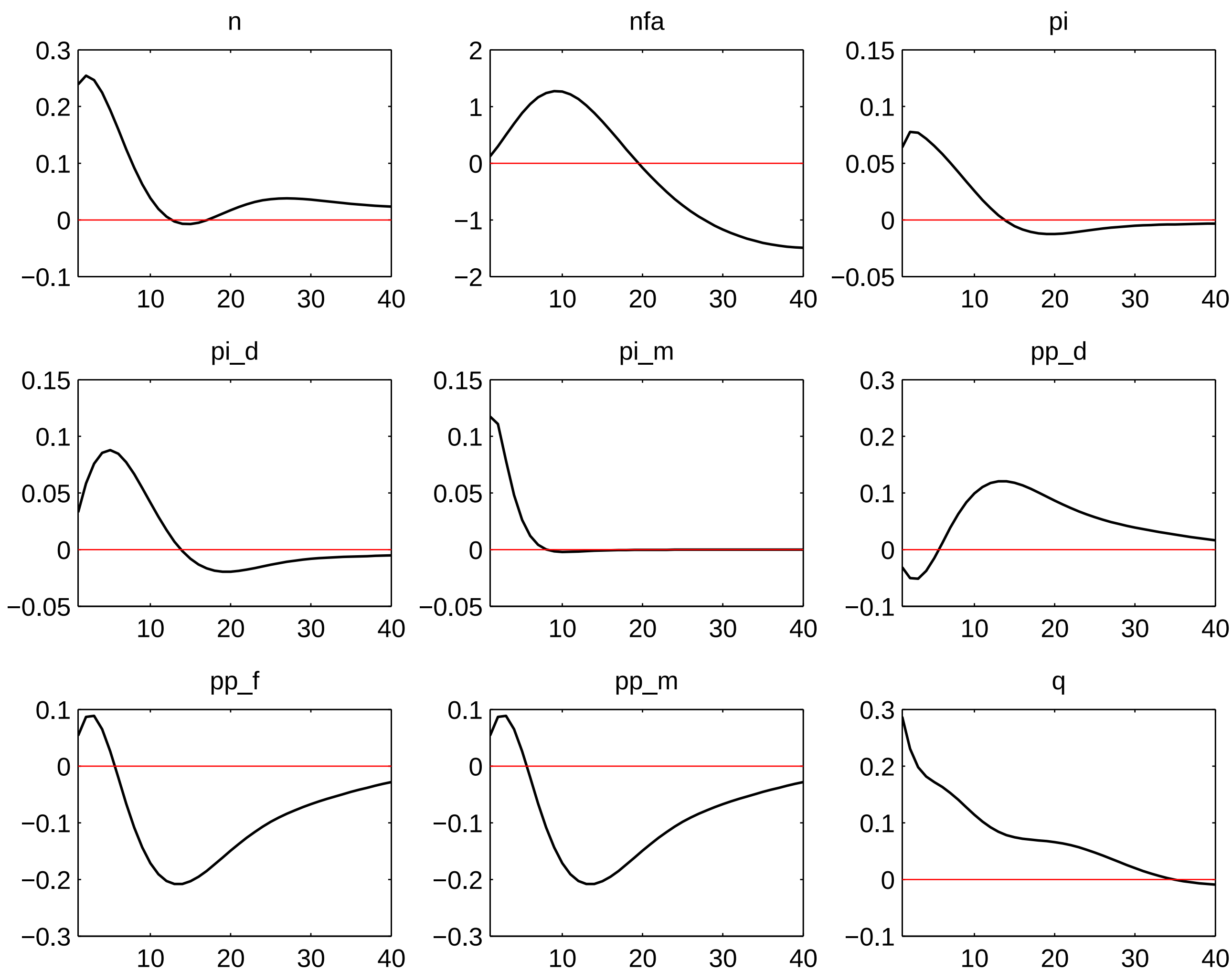

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pp_f
pp_m
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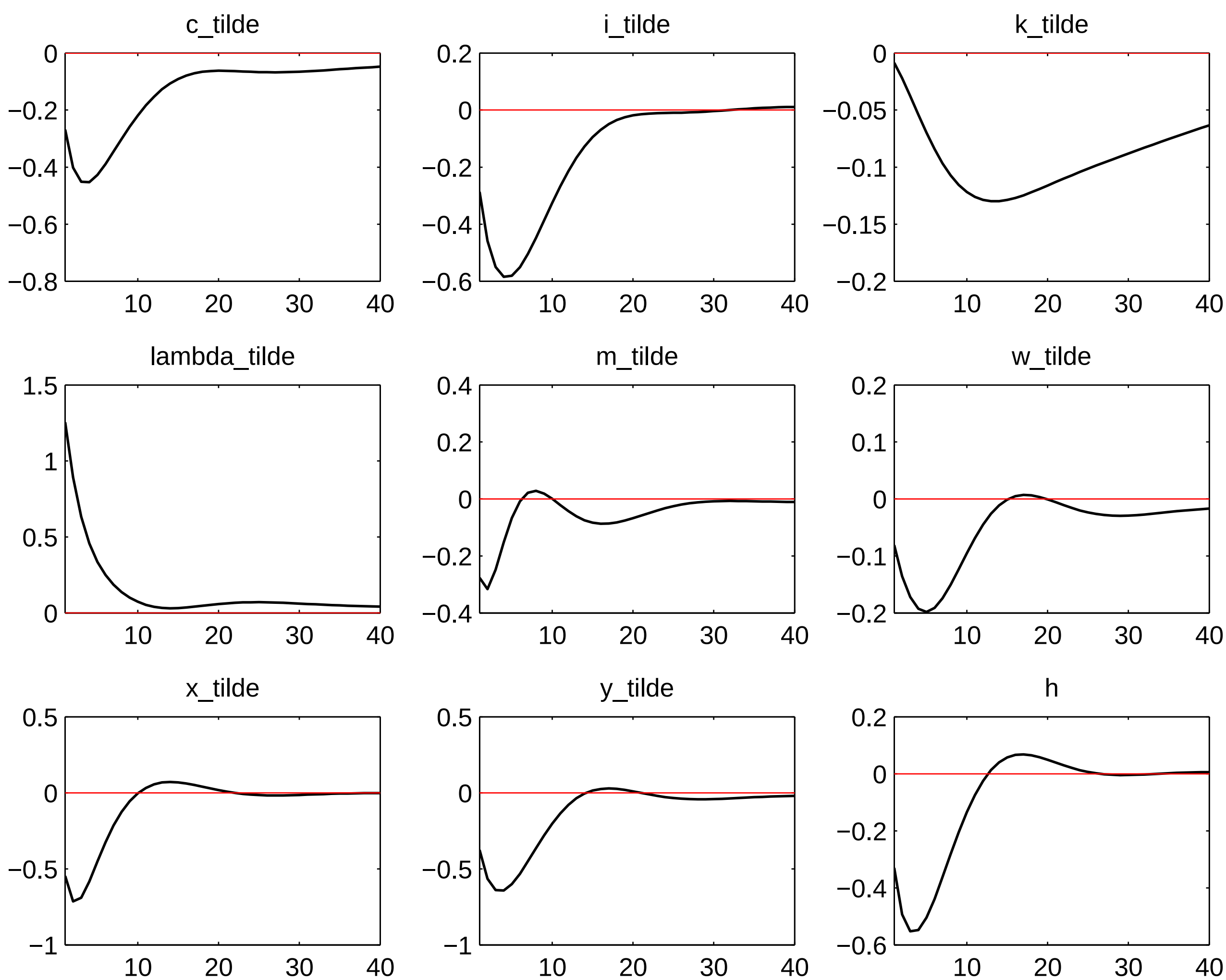
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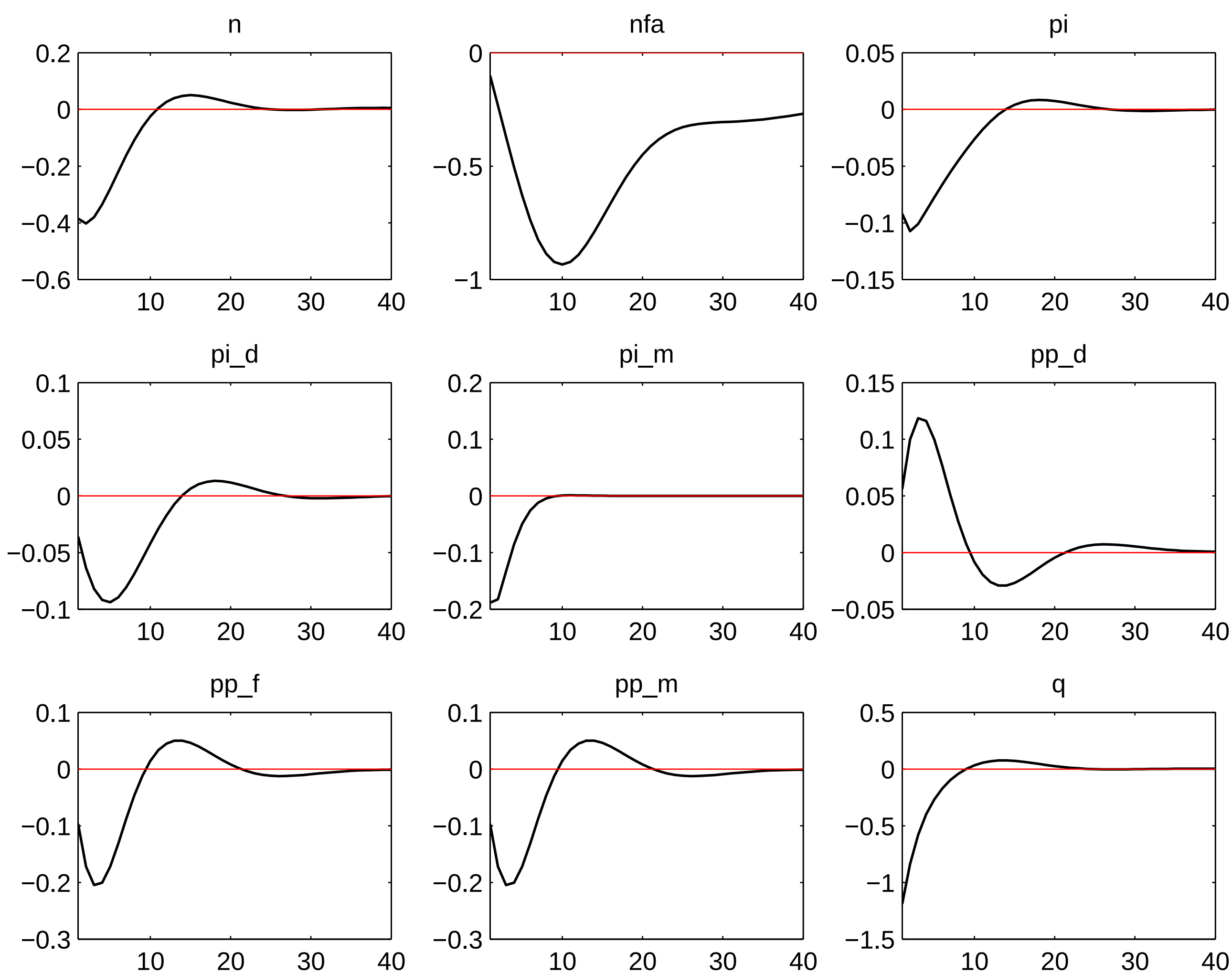
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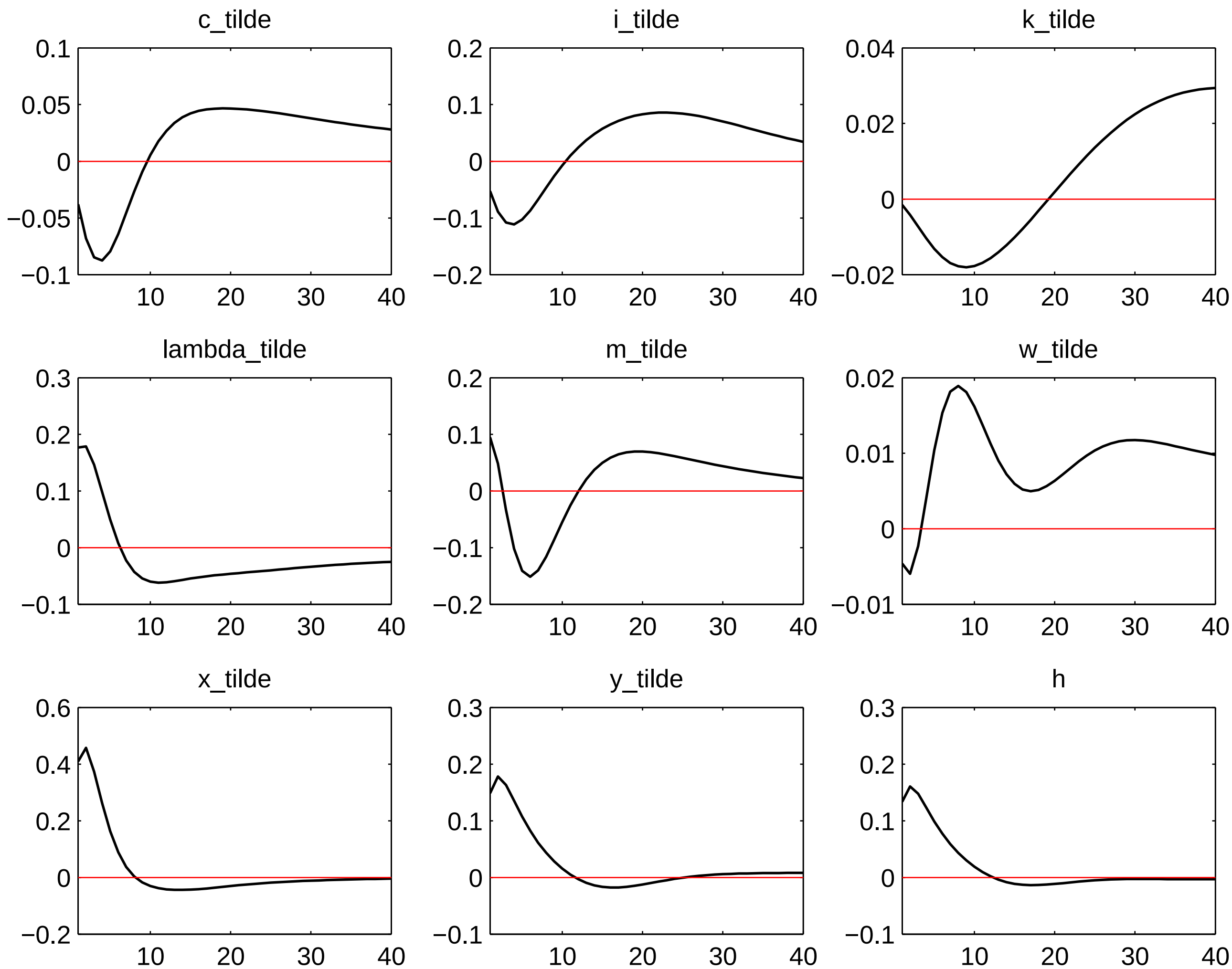
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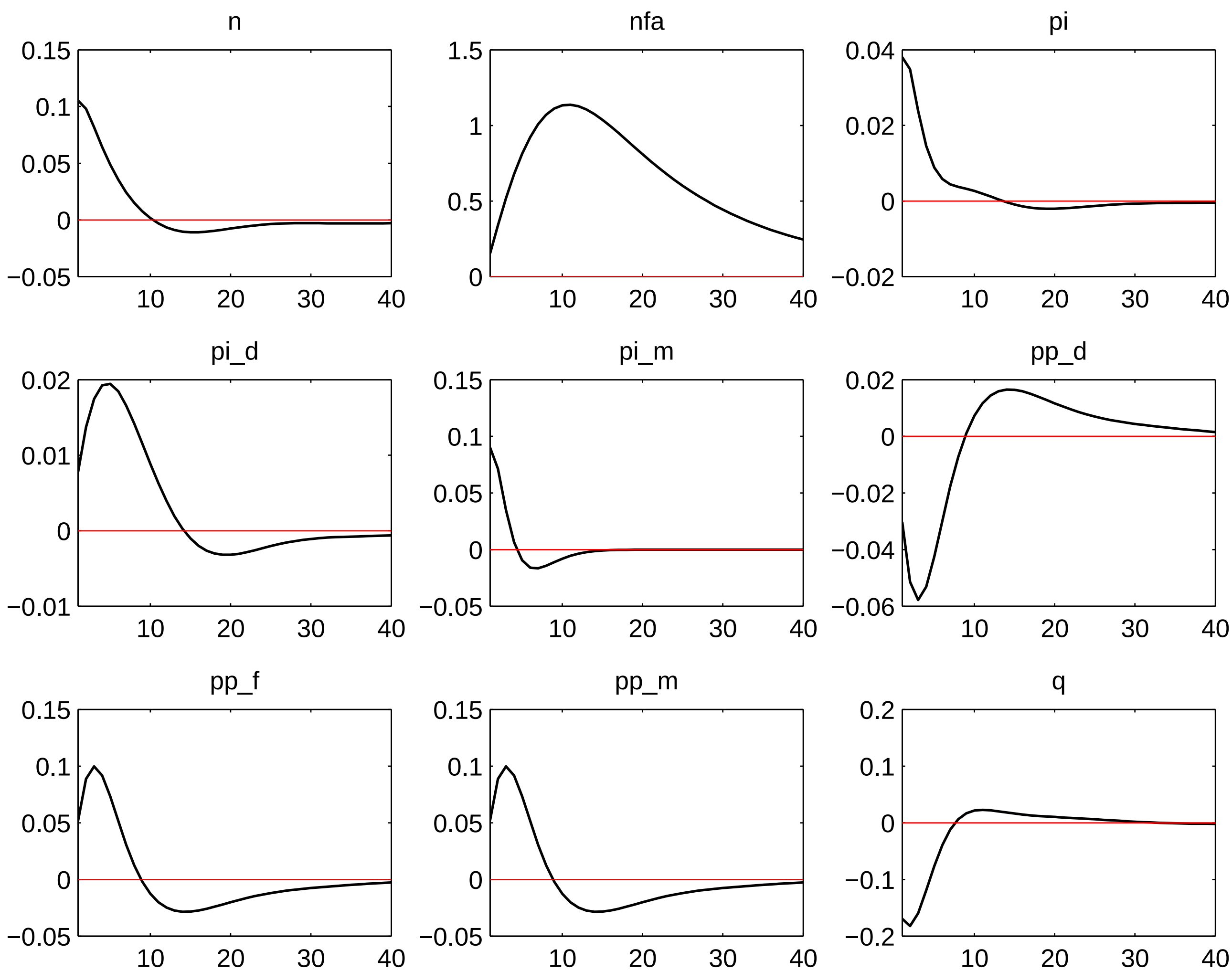

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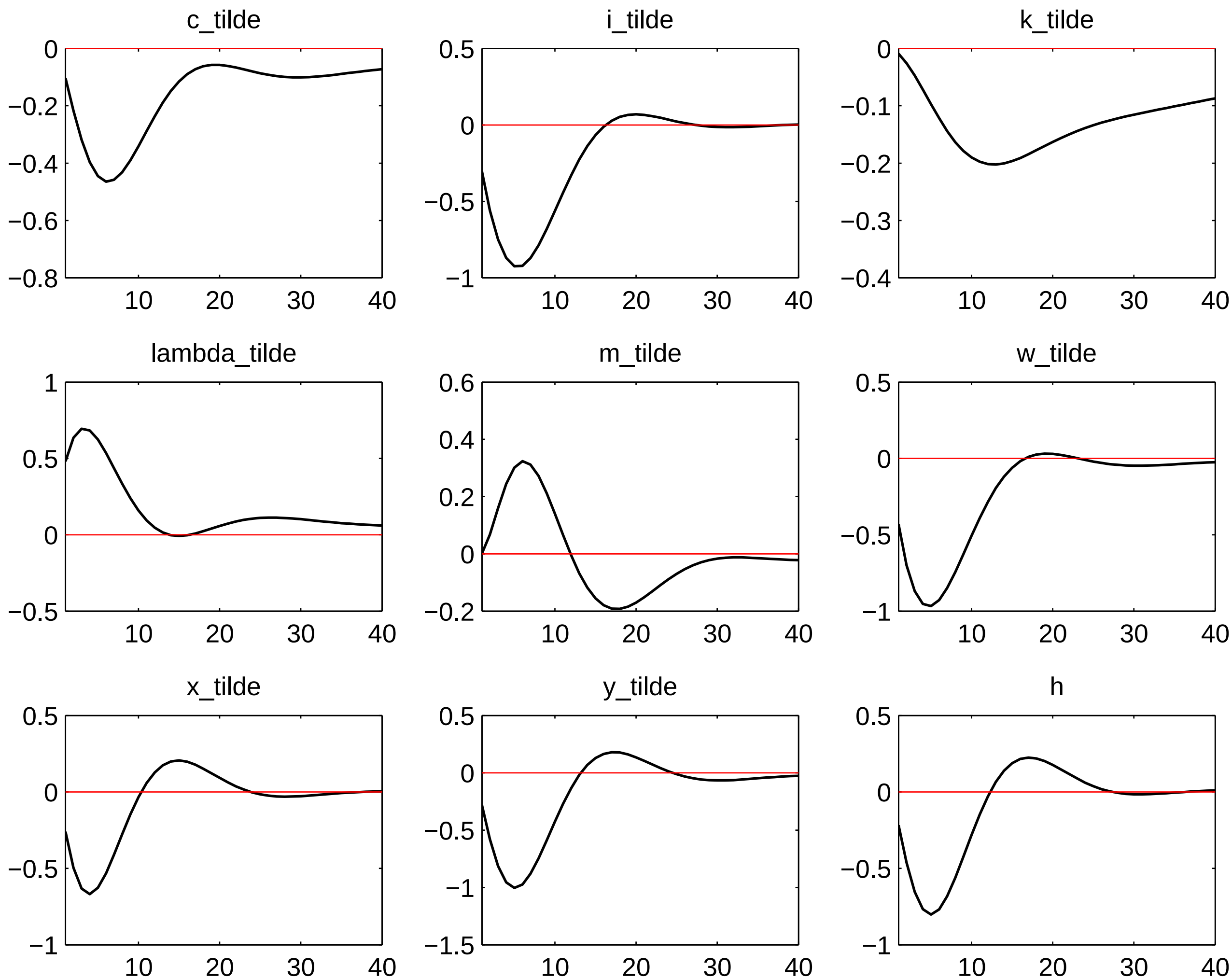

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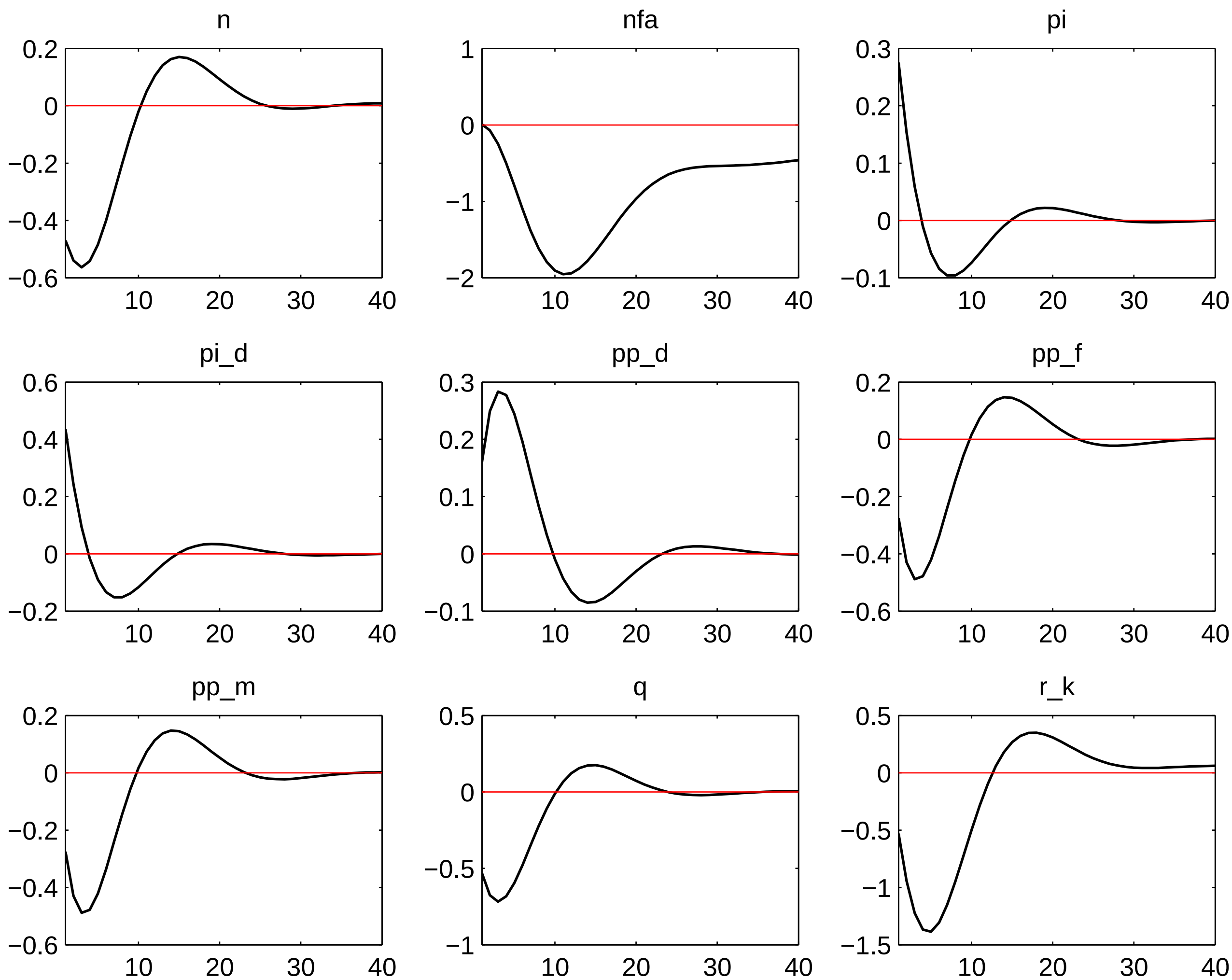
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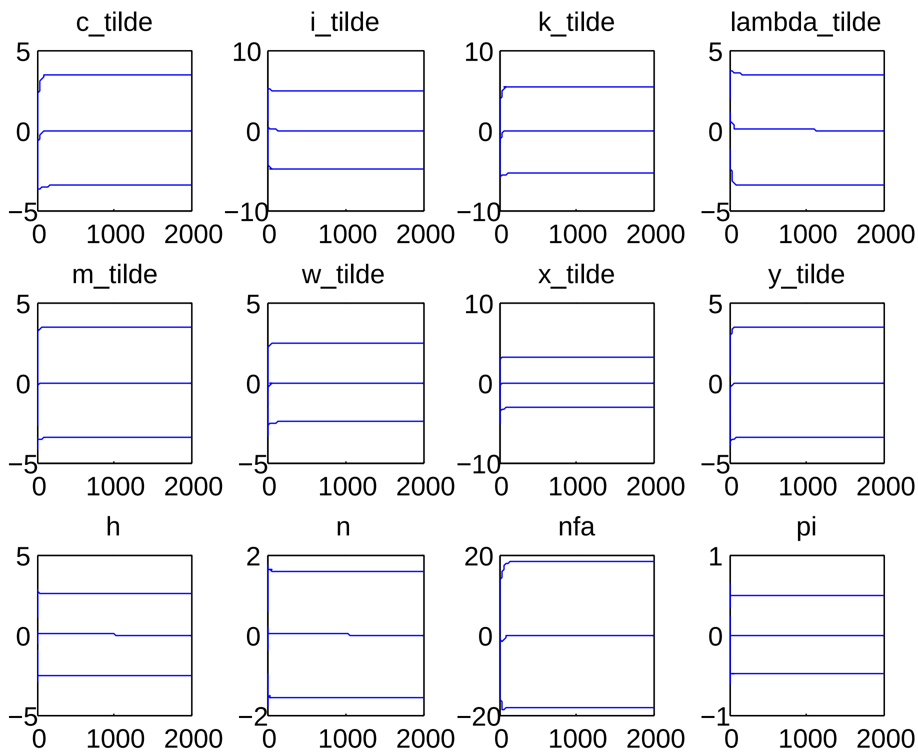
r_k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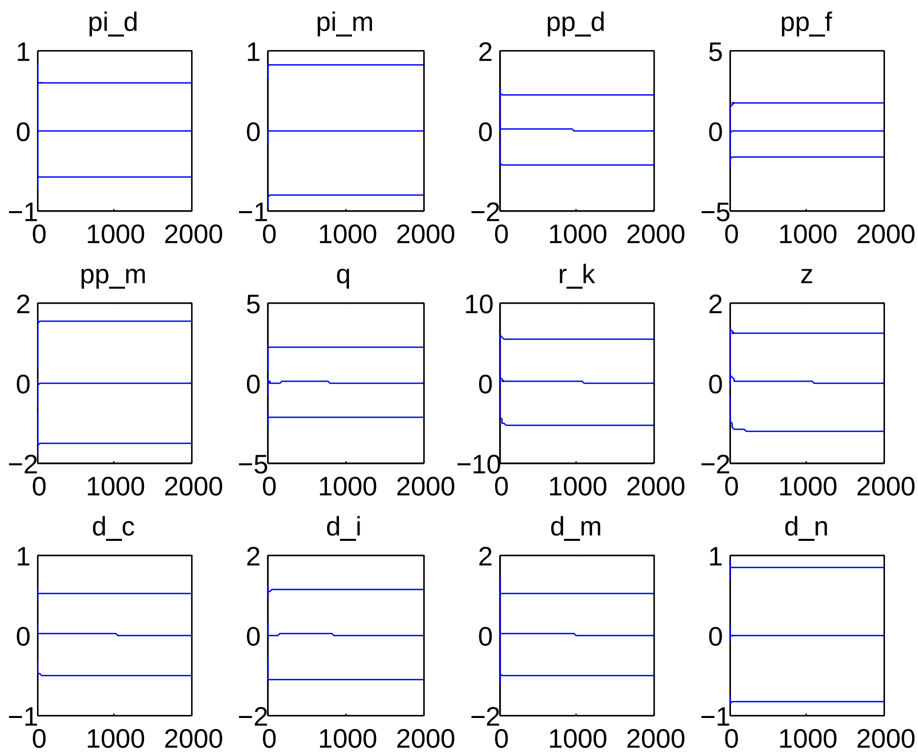
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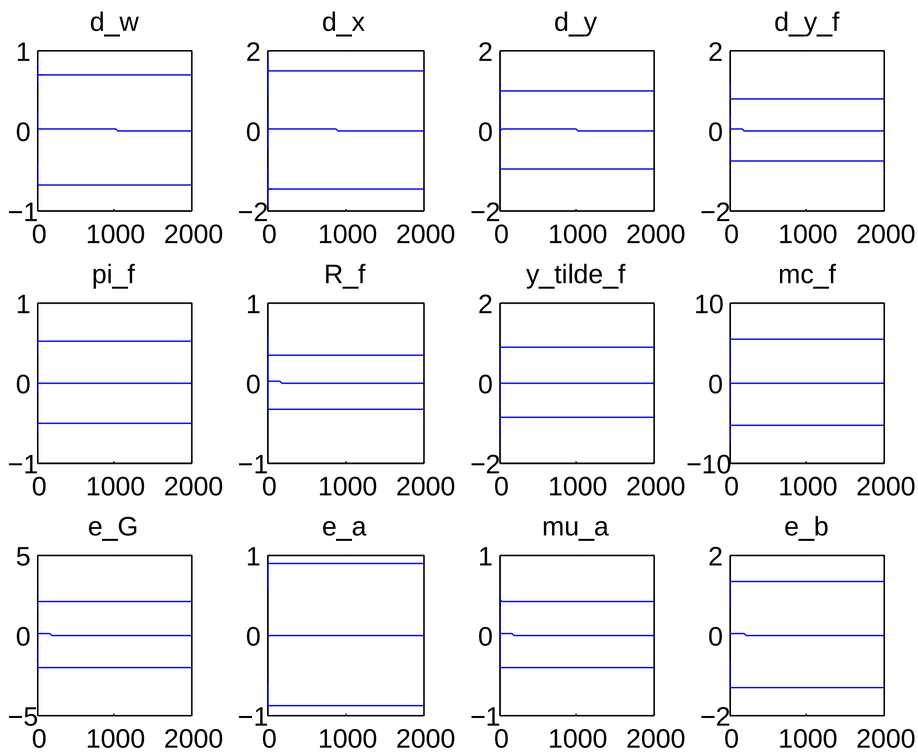

d_w
d_x
d_y
d_y_f
pi_f
R_f
y_tilde_f
mc_f
e_G
e_a
mu_a
e_b

$\mathrm{mu}_{ab}ar = 1 + mu\_a\_bar1 \;\; \mathrm{betta} = 0.99 \;\; \mathrm{mu\_a\_bar}_t \;\; \mathrm{zeta}_w = \frac{(1-betta_t\, xi\_w)\,(1-xi\_w)}{xi\_w\,(1+betta_t)\left(1+\frac{(1+lambda\_w\_bar)\,sigma\_L}{lambda\_w\_bar}\right)} \;\; \mathrm{r}_{kb}ar = \frac{mu\_a\_bar_t}{betta_t} - 1 + tau$

$$k\_tilde_t = \frac{1-tau}{mu\_a\_bar_t}\,(k\_tilde_{t-1} - mu\_a_t) + \left(1 - \frac{1-tau}{mu\_a\_bar_t}\right) i\_tilde_t \tag{1}$$

$$lambda\_tilde_t = e\_b_t - \frac{1}{1-\frac{kappa}{mu\_a\_bar_t}}\left(c\_tilde_t - \frac{kappa}{mu\_a\_bar_t}\,(c\_tilde_{t-1} - mu\_a_t)\right) \tag{2}$$

$$lambda\_tilde_t - lambda\_tilde_{t+1} = R\_f_t - phiatilde\, nfa_t + e\_RP_t - pi_{t+1} - mu\_a_{t+1} \tag{3}$$

$$i\_tilde_t = \frac{1}{1+betta_t}\,(i\_tilde_{t-1} - e\_i_t - mu\_a_t) + \frac{betta_t}{1+betta_t}\,(mu\_a_{t+1} + i\_tilde_{t+1} + e\_i_{t+1}) + \frac{chi}{1+betta_t}\,q_t \tag{4}$$

$$q_t = lambda\_tilde_{t+1} - lambda\_tilde_t - mu\_a_{t+1} + \frac{r\_k\_bar_t}{1+r\_k\_bar_t - tau}\,r\_k_{t+1} + \frac{1-tau}{1+r\_k\_bar_t - tau}\,q_{t+1} \tag{5}$$

$$z_t = psi\, r\_k_t \tag{6}$$

$$mu\_a_t + w\_tilde_t + h_t = k\_tilde_{t-1} + z_t + r\_k_t \tag{7}$$

$$w\_tilde_t\,(1+zeta\_w_t) = \frac{betta_t}{1+betta_t}\,w\_tilde_{t+1} + \frac{1}{1+betta_t}\,w\_tilde_{t-1} + pi_{t+1}\,\frac{betta_t}{1+betta_t} - \frac{1+betta_t\,gamma\_w}{1+betta_t}\,pi_t + \frac{gamma\_w}{1+betta_t}\,pi_{t-1} - zeta\_w_t\,(lambda\_tilde_t + h_t\,(-sigma\_L) - e\_L_t - e\_b_t) \tag{8}$$

$$0 = h_t\,\frac{(1-betta_t\,xi\_e)\,(1-xi\_e)}{xi\_e} + betta_t\,n_{t+1} - \left(1 + \frac{(1-betta_t\,xi\_e)\,(1-xi\_e)}{xi\_e}\right) n_t \tag{9}$$

$$pi\_d_t = \frac{betta_t}{1+betta_t\,gamma\_p}\,pi\_d_{t+1} + \frac{gamma\_p}{1+betta_t\,gamma\_p}\,pi\_d_{t-1} + \frac{(1-betta_t\,xi\_p)\,(1-xi\_p)}{(1+betta_t\,gamma\_p)\,xi\_p}\,(r\_k_t\,alfa + w\_tilde_t\,(1-alfa) - e\_a_t - pp\_d_t) + epsilon\_lambda\_p_t \tag{10}$$

$$pp\_m_t = pp\_f_t + e\_pio_t \tag{11}$$

$$pp\_d_t = pi\_d_t + pp\_d_{t-1} - pi_t \tag{12}$$

$$pp\_f_t = pp\_f_{t-1} + pi\_f_t - pi_t \tag{13}$$

$$pp\_m_t = pp\_m_{t-1} + pi\_m_t - pi_t \tag{14}$$

$$y\_tilde_t = (1+phi)\,(e\_a_t + alfa\,(k\_tilde_{t-1} + z_t - mu\_a_t) + h_t\,(1-alfa)) \tag{15}$$

$$pp\_d_t + y\_tilde_t = c\_tilde_t\,c\_y + i\_tilde_t\,inv\_y + g\_y\,e\_G_t + x\_bar\,(x\_tilde_t - (pp\_m_t + m\_tilde_t)) \tag{16}$$

$$betta_t\,nfa_t = \frac{1}{mu\_a\_bar_t}\,nfa_{t-1} + x\_bar\,(x\_tilde_t - m\_tilde_t - pp\_m_t) \tag{17}$$

$$pp\_m_t = \frac{sigma\_md}{1+sigma\_md}\left(y\_tilde_t\,\frac{1}{1+x\_bar} + m\_tilde_t\,\frac{x\_bar}{1+x\_bar} - m\_tilde_t\right) + 2\,chi\_md\,(y\_tilde_t - m\_tilde_t - y\_tilde_{t-1} + m\_tilde_{t-1} + e\_phi\_m_t) \tag{18}$$

$$pp\_d_t = \frac{sigma\_md}{1+sigma\_md}\left(y\_tilde_t\,\frac{1}{1+x\_bar} + m\_tilde_t\,\frac{x\_bar}{1+x\_bar} - y\_tilde_t\right) - (y\_tilde_t - m\_tilde_t - y\_tilde_{t-1} + m\_tilde_{t-1} + e\_phi\_m_t)\,chi\_md\,2\,x\_bar \tag{19}$$

$$y\_tilde\_f_t - x\_tilde_t = \frac{1+sigma\_f}{sigma\_f}\,(-pp\_f_t) - \frac{(1+sigma\_f)\,2\,chi\_f}{sigma\_f}\,(y\_tilde\_f_t - x\_tilde_t - y\_tilde\_f_{t-1} + x\_tilde_{t-1}) - \left(1 + \frac{(1+sigma\_f)\,2\,chi\_f}{sigma\_f}\right) e\_phi\_mf_t \tag{20}$$

$$y\_tilde\_f_t = \frac{\frac{kappa\_f}{mu\_a\_bar_t}}{1+\frac{kappa\_f}{mu\_a\_bar_t}}\,(y\_tilde\_f_{t-1} - mu\_a_t) + \frac{1}{1+\frac{kappa\_f}{mu\_a\_bar_t}}\,(mu\_a_{t+1} + y\_tilde\_f_{t+1}) - \frac{1-\frac{kappa\_f}{mu\_a\_bar_t}}{1+\frac{kappa\_f}{mu\_a\_bar_t}}\,(R\_f_t - pi\_f_{t+1} - e\_yf_t + e\_yf_{t+1}) \tag{21}$$

$$pi\_f_t = pi\_f_{t+1} \frac{betta_t}{1 + betta_t \, gamma\_pf} + \frac{gamma\_pf}{1 + betta_t \, gamma\_pf} pi\_f_{t-1} + \frac{\left(1 - betta_t \, xi\_pf\right) \left(1 - xi\_pf\right)}{\left(1 + betta_t \, gamma\_pf\right) xi\_pf} mc\_f_t + epsilon\_pif_t \tag{22}$$

$$mc\_f_t = y\_tilde\_f_t \, eta\_f + \frac{\frac{kappa\_f}{mu\_a\_bar_t}}{1 - \frac{kappa\_f}{mu\_a\_bar_t}} \left(mu\_a_t + y\_tilde\_f_t - y\_tilde\_f_{t-1}\right) \tag{23}$$

$$R\_f_t = rho\_r \, R\_f_{t-1} + \left(1 - rho\_r\right) \left(pi\_f_t \, psi\_f + y\_tilde\_f_t \, psi\_yf\right) + epsilon\_R_t \tag{24}$$

$$d\_y_t = mu\_a_t + y\_tilde_t - y\_tilde_{t-1} \tag{25}$$

$$d\_c_t = mu\_a_t + c\_tilde_t - c\_tilde_{t-1} \tag{26}$$

$$d\_i_t = mu\_a_t + i\_tilde_t - i\_tilde_{t-1} \tag{27}$$

$$d\_x_t = mu\_a_t + x\_tilde_t - x\_tilde_{t-1} \tag{28}$$

$$d\_m_t = mu\_a_t + m\_tilde_t - m\_tilde_{t-1} \tag{29}$$

$$d\_n_t = n_t - n_{t-1} \tag{30}$$

$$d\_w_t = mu\_a_t + w\_tilde_t - w\_tilde_{t-1} \tag{31}$$

$$d\_y\_f_t = mu\_a_t + y\_tilde\_f_t - y\_tilde\_f_{t-1} \tag{32}$$

$$mu\_a_t = rho\_mu\_a \, mu\_a_{t-1} + epsilon\_mu\_a_t \tag{33}$$

$$e\_L_t = rho\_L \, e\_L_{t-1} + epsilon\_L_t \tag{34}$$

$$e\_a_t = rho\_a \, e\_a_{t-1} + epsilon\_a_t \tag{35}$$

$$e\_b_t = rho\_b \, e\_b_{t-1} + epsilon\_b_t \tag{36}$$

$$e\_G_t = rho\_G \, e\_G_{t-1} + epsilon\_G_t \tag{37}$$

$$e\_i_t = rho\_i \, e\_i_{t-1} + epsilon\_i_t \tag{38}$$

$$e\_phi\_m_t = rho\_phi\_m \, e\_phi\_m_{t-1} + epsilon\_phi\_m_t \tag{39}$$

$$e\_phi\_mf_t = rho\_phi\_mf \, e\_phi\_mf_{t-1} + epsilon\_phi\_mf_t \tag{40}$$

$$e\_pio_t = rho\_pio \, e\_pio_{t-1} + epsilon\_pio_t \tag{41}$$

$$e\_RP_t = rho\_RP \, e\_RP_{t-1} + epsilon\_RP_t \tag{42}$$

$$e\_yf_t = rho\_yf \, e\_yf_{t-1} + epsilon\_yf_t \tag{43}$$