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MSc Economics

Recourse in a Collateral Equilibrium Model

A Master's Thesis submitted for the degree of
"Master of Science"

supervised by
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Vienna, 08.06.2016

MSc Economics

Affidavit

I, Maximilian Huber

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Recourse in a Collateral Equilibrium Model

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Contents

List of Tables

List of Figures

Abstract

Recourse is introduced to the binomial collateral equilibrium model described by Geanakoplos and Fostl in their Econometrica paper “Leverage and default in binomial economies: A complete characterization”. First, the paper’s main results, culminating in the binomial no-default theorem, are examined in this generalized setting. This theorem ensures that any collateral equilibrium can be translated to a default-free equilibrium with the same prices and identical allocations of goods, yet different debt contract and asset allocations. It is shown that the theorem is valid in specific circumstances only. Then, welfare considerations are introduced. Examples motivate the conjecture that in an equilibrium in which the collateral is scarce and borrowing is maximal, every investor’s utility can be improved upon by increasing the recovery rate. Even though, closed-form expressions for marginal utility changes are derived, attempts to prove that they are positive were not successful yet.

1 Introduction

The Arrow-Debreu model of time and uncertainty in a general equilibrium model allows consumers to trade contingent claims on goods. This exchange type economy can be enhanced by allowing for production by firms which are owned by consumers. Furthermore, abstracting from production, assets can be introduced which are in positive net supply, such that consumers are endowed with them and have contingent payoffs in goods. These assets can be thought of as the stocks of a dividend-paying company. The assets can be traded at a price that equalizes the marginal disutility of buying an extra asset and the marginal utility from future payoffs. Going even further, a different kind of asset that is in zero net supply can be introduced, namely the debt contract. Those contracts have a buyer (borrower), a seller (lender), a face value which has to be repaid and a price.

This model can incorporate numerous real world situations. For instance, the asset could be a residential house or shares of a company. Borrowing through a debt contract is commonly not possible without putting up collateral in the form of an asset. This setting was modeled by John Geanakoplos.

Together with William Zame he shows the existence of a collateral equilibrium and describes novelties in asset pricing (Geanakoplos and Zame, 2013). Under the assumptions of strictly positive endowments individually and in the aggregate, continuous and quasi-concave utility functions that are weakly increasing in all consumptions, and strictly increasing in consumptions in period 0 and period 1, the existence of a collateral equilibrium is guaranteed. They also introduce a decomposition of an asset's price into a fundamental and a collateral value.

In a paper with Ana Fostl he uses a collateral equilibrium model to explain contagion between different assets in a situation of increased uncertainty (Fostel and Geanakoplos, 2008). In a three period model they describe how bad news in the intermediate period can force investors to rebalance their portfolio in order to meet collateral requirements. By doing so, investors flee to safe investments and sell other risky assets that are not directly affected by the bad news. This results in contagion.

Together with Ana Fostl he extensively discusses a special case of uncertainty structure, namely a binomial model (Fostel and Geanakoplos, 2015). They prove that any collateral equilibrium can be translated to an equilibrium without default, while having the same prices for the asset and the debt contract and maintaining the goods allocations. Only the asset and debt allocations have to be adjusted. The debt contracts in the paper are of non-recourse type, such that a borrower can simply default by handing the collateral over to the lender without any further responsibility. The lender cannot recourse over the difference between face value and collateral value.

This setting is a good description of the U.S. residential mortgage market, where the lender can easily default on the debt contract by surrendering the underlying house to the bank and does not need to fear legal action. In fact, this practice was used heavily from the beginning of the 2008 financial crisis on (Financial Crisis Inquiry Commission, 2010). But for different types of debt, or in other countries, there exist possibilities of recourse if the borrower defaults. Government bonds, for example, rarely default. But if they do, the law under which they were issued matters. In the course of the European debt crisis from 2010 on, Greece agreed to a 70 percent haircut on its local law bonds, while it had to repay English law bonds in full (Forbes, 2012). Indeed, countries with more investor protection, like the United Kingdom or Hong Kong, will have a higher expected recovery rate of the difference between the amount owed and the value of the collateral.

The rest of the paper is organized as follows. In section 2 a debt contract specific recovery rate is introduced to the 2 period model (Fostel and Geanakoplos, 2015). In section 3 results building up to the generalized binomial no-default theorem are proved and pricing is discussed. Section 4 introduces examples with two investors where a single asset and a single debt contract exist. An equilibrium is described in which the borrower owns all assets, borrows maximally and would like to borrow more. The conjecture, that increasing the recovery rate leads to utility improvement for all investors, arises from these examples.

2 Model

The model considered in this section is a general equilibrium model. There are two time periods $t \in \{0, T\}$and there is uncertainty over which of two states $S_T = \{U, D\}$ will realize in the last period. Period $t = 0$ will be referred to as state $s = 0$ and all three states are collected to the set $S = \{0\} \cup S_T$. Each investor $h \in H$ has subjective probabilities $\gamma_s^h \; \forall s \in S_T$ attached to the possible outcomes and a subjective discount factor β^h. There is a single consumption good c present in all states. Preferences over consumption plans $(c_0, c_U, c_D) \in \mathbb{R}^3_+$can be represented by the following utility function:

$$U^h(c_0, c_U, c_D) = u(c_0) + \beta^h \sum_{s \in S_T} \gamma_s^h u(c_s)$$

where $u^h : \mathbb{R}_+ \to \mathbb{R}$ is a differentiable, concave and monotonic utility function.
Furthermore, there is an asset y that is in positive net supply. It can be traded in state $s = 0$ and pays off $d_s \in \mathbb{R}_+ \; \forall s \in S_T$ units of the consumption good in the respective state in the last period. It is assumed that $d_U > d_D$.
Consumers are endowed with the consumption good in each state $e_s^h \in \mathbb{R}_+ \; \forall s \in S$ and with the asset $y_0^h \in \mathbb{R}_+$ in $s = 0$. It is assumed that a positive amount of the consumption good is present in every state.
An investor can borrow or lend to another investor via special asset, called the debt contract. There is a finite menu B of possible such contract, all of which are in zero net supply.

2.1 Collateral and Debt Contracts

Anticipating differences to the baseline model a comprehensive definition of debt contracts is given (Fostel and Geanakoplos, 2015).
Besides the single financial asset the investor can buy or sell a menu of debt contracts. A contract promises repayment of an amount j which is constant over states, i.e. $(j \cdot \vec{1}) \in \mathbb{R}^{S_T}$. It is collateralized by k units of the financial asset. This amount of assets has to be held by the borrower in order to fulfill the collateral requirement. In case of default, the collateral is handed over to the lender, but depending on the contract there is also a recourse process. Each contract has a recovery rate η. In default, if the value of the collateral falls short of the promise, a percentage of this gap can be reclaimed by the lender.
Hence, a debt contract can be specified by the triple (j, k, η).

2.2 Default and Delivery

When a state in S_T realizes the borrower chooses whether to repay the loan or to default. Rationality implies following delivery function:

$$\delta_s(j, k, \eta) = \min(\underbrace{\;j\;}_{\text{repay}}, \underbrace{kd_s + \eta \max(j - kd_s, 0)}_{\text{default}})$$

If $\eta = 0$, so there is no recourse, δ_s is homogeneous of degree one in j and k. As observed in (Fostel and Geanakoplos, 2015), contracts of the form $(j,k,0)$ can be normalized by $k = 1$ and hence be identified by j only. Furthermore if $\eta > 0$, δ_s is again homogeneous of degree one in j and k. If $j - kd_s \geq 0$, $\delta_s(\lambda j, \lambda k, \eta) = \min(\lambda j, \lambda kd_s + \eta\lambda(j - kd_s)) = \lambda \min(j, kd_s + \eta(j-k)) = \lambda\delta_s(j,k,\eta)$. Conversely if $j - kd_s < 0$, $\delta_s(\lambda j, \lambda k, \eta) = \min(\lambda j, \lambda kd_s + 0) = \lambda \min(j, kd_s) = \lambda\delta_s(j,k,\eta)$. Therefore, $k = 1$ is set for all contracts without loss of generality, and

$$\delta_s(j,\eta) = \min\left(j, d_s + \eta \max\left(j - d_s, 0\right)\right) \qquad \forall (j,\eta) \in B$$

Given j and η and final state the delivery function looks like:

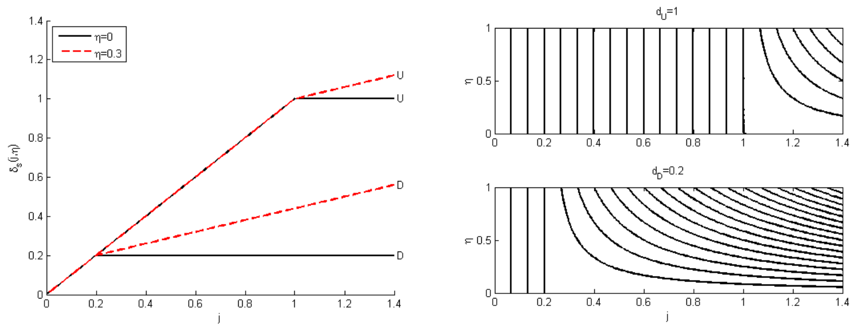

Figure 1: Delivery function

2.3 Budget Set

Each investor faces a budget set, constraining possible allocations. Given asset and debt contract prices $(p, (\pi_b)_{b \in B})$ and endowments of $(c_0^h, (c_s^h)_{s \in S_T})$ consumption goods and y_0^h financial assets, the budget set is:

$$B^h(p,\pi) = \{(c^h, y^h, \phi^h) \in \mathbb{R}_+^S \times \mathbb{R}_+ \times \mathbb{R}^B :$$

$$(c_0^h - e_0^h) + p(y^h - y_0^h) - \sum_{b \in B} \phi_b^h \pi_b \leq 0 \wedge$$

$$(c_s^h - e_s^h) - y^h d_s + \sum_{b \in B} \phi_b^h \min(j_b, d_s + \eta_b \max(j_b - d_s, 0)) \leq 0 \qquad \forall s \in S_T \wedge$$

$$\sum_{b \in B} \max(0, \phi_b^h) - y^h \leq 0 \,\}$$

The first constraint enforces affordability in the first period. The second kind of constraints concern final period affordability using the delivery function$\delta_s(j_b, \eta_b)$. The collateral requirement is ensured by the last constraint.

2.4 Collateral Equilibrium

Given a finite menu of debt contracts $B \subseteq \mathbb{R}^2_+$, utility functions U^h, consumption goods $(c_0^h, (c_s^h)_{s\in S_T})$ and financial asset endowments y_0^h, a collateral equilibrium with recourse is characterized just like the equilibrium without recourse. It is a vector of $((p,\pi),((c^h,y^h,\phi^h)_{h\in H}) \in (\mathbb{R}_+ \times \mathbb{R}^B_+) \times (\mathbb{R}^S_+ \times \mathbb{R}_+ \times \mathbb{R}^B)^H$ fulfilling:

$$\sum_{h\in H}(c_0^h - e_0^h) = 0 \tag{1}$$

$$\sum_{h\in H}(c_s^h - e_s^h) = \sum_{h\in H} y^h d_s \qquad \forall s \in S_T \tag{2}$$

$$\sum_{h\in H}(y^h - y_0^h) = 0 \tag{3}$$

$$\sum_{h\in H}\phi_b^h = 0 \qquad \forall b \in B \tag{4}$$

$$\begin{gathered}(c^h,y^h,\phi^h) \in B^h(p,\pi) \qquad \forall h \in H \\ (c',y',\phi') \in B^h(p,\pi) \Rightarrow U^h(c') \leq U^h(c^h) \qquad \forall h \in H\end{gathered} \tag{5}$$

Equations (1) and (2) ensure goods market clearing in every state. Equations (3) and (4) enforce that every asset must be held by someone and every debt contract must have a counterparty.

2.5 The Credit Surface

Given η, a contract's price π_b is strictly decreasing in j for $d_D \leq j$. If $\eta > 0$ and $d_D \leq j < j'$ but $\eta = \eta'$, $\delta_s(j',\eta) > \delta_s(j,\eta) \quad \forall s \in S_T$ and therefore $\pi_b > \pi_{b'}$, see figure 1. The case for $\eta = 0$ is discussed in (Fostel and Geanakoplos, 2015) with the same result for $d_D \leq j < j' \leq d_U$.
Given j such that $d_D < j$, a contract's price π_b is strictly increasing in η. Since $d_D < j$ there is a state s in which default occurs. If $\eta' > \eta$, the delivery is $\delta_s(j,\eta') > \delta_s(j,\eta)$ in this state, and for all states $\delta_s(j,\eta') \geq \delta_s(j,\eta)$. Therefore $\pi_b < \pi_{b'}$.
The gross interest rate r_b is defined by $(1+r_b) = \frac{j}{\pi_b}$, and is therefore strictly increasing in j for $d_D \leq j$ and strictly decreasing in η for $d < j$.

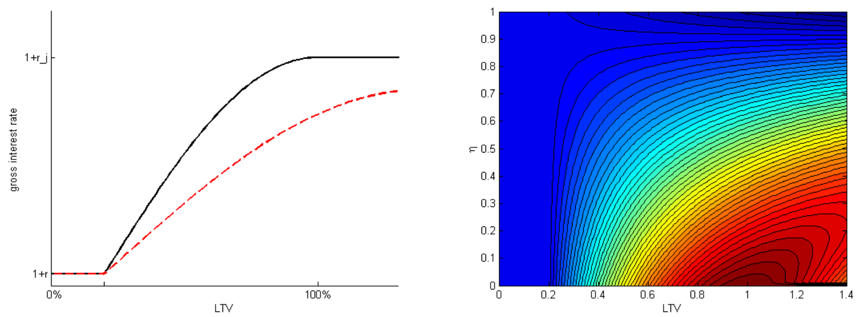

Figure 2: Credit Surface

3 A Generalized Binomial No-Default Theorem

Given a finite menu of debt contracts $B \subseteq \mathbb{R}^2_+$ and a two period model with $S_T = \{U, D\}$ and $d_U > d_D$, any collateral equilibrium in sense of chapter 2.4 with non-negative portfolio payoffs can be translated to a setting with only one, default-free debt contract $b^* = (d_D, 0) \in \mathbb{R}^2_+$. In order to proof this theorem, I give generalized lemmata on the geometric structure of the space of portfolio payoffs.

3.1 Payoff Cone

The portfolio of any investor consists of asset and debt holdings $\left(y^h, \phi^h\right)$ and produces a payoff

$$\left(w_U^h, w_D^h\right) = \left(y^h d_U - \sum_{b \in B} \phi_b^h \delta_U(j_b, \eta_b), \quad y^h d_D - \sum_{b \in B} \phi_b^h \delta_D(j_b, \eta_b)\right)$$

two respective states $s \in \{U, D\}$.
The first step in the translation to a no-default equilibrium is to find a portfolio that consists only of the asset and one debt contract and replicates arbitrary portfolio payoffs. However, this is only possible if the debt contract does not have the same payoffs as the asset.

Lemma 1. *Under the assumption that the debt contract b is not of the form $(j, 0)$ with $j \geq d_U$, any payoff vector $\left(w_U^h, w_D^h\right) \in \mathbb{R}^2$ lies in the space spanned by the payoffs of the financial asset and this debt contract.*

Proof. Suppose the vectors $\begin{pmatrix} d_U \\ d_D \end{pmatrix}$ and $\begin{pmatrix} \min(j_b, d_U + \eta_b \max(j_b - d_U, 0)) \\ \min(j_b, d_D + \eta_b \max(j_b - d_D, 0)) \end{pmatrix}$ were linearly dependent. Then $\exists \lambda \in \mathbb{R}$ such that

$$\begin{pmatrix} d_U \\ d_D \end{pmatrix} = \lambda \begin{pmatrix} \min(j_b, d_U + \eta_b \max(j_b - d_U, 0)) \\ \min(j_b, d_D + \eta_b \max(j_b - d_D, 0)) \end{pmatrix}$$

If now $\eta_b = 0$, it is by assumption true that $j < d_U$, but then $\begin{pmatrix} d_U \\ d_D \end{pmatrix} = \lambda \begin{pmatrix} j_b \\ \min(j_b, d_D) \end{pmatrix}$. If $j_b < d_D$, $d_U = d_D$ against the assumption that $d_U > d_D$, vice versa if $j_b \geq d_D$, $d_D = \lambda d_D$ hence $\lambda = 1$ and $d_U = j_b$ contradicting the same assumption.

Proof. If $\eta_b > 0$, there are more case distinctions needed. Let $j_b \geq d_U$ and therefore $j_b > d_D$, hence □

$$\begin{pmatrix} d_U \\ d_D \end{pmatrix} = \lambda \begin{pmatrix} \min(j_b, d_U + \eta_b (j_b - d_U)) \\ \min(j_b, d_D + \eta_b (j_b - d_D)) \end{pmatrix}$$

If now $j_b \geq d_U + \eta_b(j_b - d_U) > d_D + \eta_b(j_b - d_D)$, it follows that $d_U = d_D$. If $d_U + \eta_b(j_b - d_U) > j_b \geq d_D + \eta_b(j_b - d_D)$, it follows that $\begin{pmatrix} d_U \\ d_D \end{pmatrix} = \lambda \begin{pmatrix} j_b \\ d + \eta_b(j_b - d_D) \end{pmatrix}$ and hence $\lambda \leq 1$, but this and $j_b \geq d_U$ implies $d_U - \eta_b d_U \leq d_U - \eta_b j \leq d_D - \eta_b d_D$ and hence $d_U \leq d_D$. If $d_U + \eta_b(j_b - d_U) > d_D + \eta_b(j_b - d_D) > j_b$, $d_U = d_D$ follows immediately. All three possibilities contradict the $d_U > d_D$ assumption. Let $d_U > j_b \geq d_D$, hence $\begin{pmatrix} d_U \\ d_D \end{pmatrix} = \lambda \begin{pmatrix} j_b \\ \min(j_b, d_D + \eta_b(j_b - d_D)) \end{pmatrix}$ and $\lambda > 1$. If $j_b \geq d_D + \eta_b(j - d_D)$, $d_D < j$ follows. If $d_D + \eta_b(j - d_D) > j_b$, $d_U = d_D$ follows immediately. Both cases lead to the same contradiction. Let $d_U > d_D > j_b$, hence $\begin{pmatrix} d_U \\ d_D \end{pmatrix} = \lambda \begin{pmatrix} j_b \\ j_b \end{pmatrix}$ and $d_U = d_D$, which is the last needed contradiction. Linear dependence conflicts with an assumption in all possible cases, so the contrary must be true. □

Corollary 1. *Under the assumption that the debt contract b is not of the form $(j,0)$ with $j \geq d_U$, the artificial asset that pays off a positive amount only in state $s = U$, can be replicated with one unit of the asset and λ debt holdings, where $\lambda = \frac{d_D}{j_b}$ if $j_b \leq d_D$ and $\lambda = \frac{d_D}{d_D + \eta_b(j_b - d_D)}$* else.

Proof. Without loss of generality assume the financial asset holdings to be one. The resulting allocation can to be scaled up or down to achieve the payoff $(1,0)$ of the Arrow-U-security. The Arrow-U-asset demands zero payoff in the D state, hence $d_D - \lambda \min(j_b, d_D + \eta_b \max(j_b - d_D, 0)) = 0$ where λ is the number of debt contracts.
If now $j_b \leq d_D$, $\lambda = \frac{d_D}{j_b} \geq 1$. Conversely if $j_b > d_D$, $\lambda = \frac{d_D}{d_D + \eta_b(j_b - d_D)} \leq 1$. This debt holdings imply positive payoffs in the $s = U$ state. If $j_b \leq d_D$, $d_U - \frac{d_D}{j_b} j_b > 0$ by the assumption that $d_D < d_U$. If $d_D < j_b \leq d_U$, it follows that $d_U - \frac{d_D}{d_D + \eta_b(j_b - d_D)} j_b > 0$, because $\lambda \leq 1$ and if $j_b = d_U$, then $\eta_b > 0$ by assumption. Now let $d_D < d_U < j_b$ and suppose to the contrary $d_U - \frac{d_D}{d_D + \eta_b(j_b - d_D)}(d_U + \eta_b(j_b - d_U)) \leq 0$. The contradiction $d_U \leq d_D$ follows straight away. Hence, it follows that the choices of λ in the statement generate positive payoff in state $s = U$ and zero payoff in $s = D$. □

3.2 State and Asset Pricing

Corollary 2. *There exist unique state prices $e > 0$ and $f > 0$ pricing the financial asset and the max min debt contract b^* by solving $\pi_{b^*} = e\delta_U(j_{b^*}, \eta_{b^*}) + f\delta_D(j_{b^*}, \eta_{b^*})$ and $p = ed_U + fd_D$.*

Proof. Shown in (Fostel and Geanakoplos, 2015).
The max min contract clearly has payoffs linearly independent from the assets payoffs. But lemma 1 shows that also many other debt contracts have the same property. Using one of them leads to unique, positive state prices too. By market completeness any other debt contract can then be priced using the state prices. □

Lemma 2. *Given state prices, any debt contract can by priced by $\pi_b = e\delta_U(j_b, \eta_b) + f\delta_D(j_b, \eta_b)$.*

Proof. Suppose $\pi_b < e\delta_U(j_b, \eta_b) + f\delta_D(j_b, \eta_b)$. Then every consumer would buy b, while selling the replication portfolio consisting the Arrow-U-asset in corollary 1 and the debt contract b^*. Hence, debt market would not clear for b. Vice versa if $\pi_b > e\delta_U(j_b, \eta_b) + f\delta_D(j_b, \eta_b)$, every consumer would sell b, and buy the replication portfolio, which leads to excess supply of b. Either way, the equilibrium market clearing condition for the debt contract, equation (4), is not met. □

Corollary 3. *Given state prices, the price derivative with respect to the recovery rate is:*

$$\frac{\partial \pi_b}{\partial \eta_b} = \begin{cases} 0 & \text{if } j_b < d_D < d_U \\ f(j - d_D) & \text{if } d_D < j_b < d_U \\ e(j - d_U) + f(j - d_D) & \text{otherwise} \end{cases}$$

Proof. Partial derivatives of (δ_U, δ_D) are:

$$\frac{\partial \delta}{\partial \eta_b} = \begin{cases} (0,0)^T & \text{if } j_b < d_D < d_U \\ (0,(j - d_D))^T & \text{if } d_D < j_b < d_U \\ ((j - d_U),(j - d_D))^T & \text{otherwise} \end{cases}$$

and $\frac{\partial \pi_b}{\partial \eta_b} = (e, f) \cdot \frac{\partial \delta}{\partial \eta_b}$.U □

3.3 Default-free Equilibrium

Theorem 1. *Given an equilibrium $((p,\pi),((c^h,y^h,\phi^h)_{h\in H})$ in the sense of section 2.4 with portfolio payoffs $\left(w_U^h, w_D^h\right) \in \mathbb{R}_+^2\ \forall h \in H$, there is a default-free equilibrium with $b^* = (d_D, 0) \in \bar{B}$ having prices and allocation $((p,\pi),((c^h,\bar{y}^h,\bar{\phi}_{b^*}^h)_{h\in H})$.*

Proof. Altering the set of available debt contracts, either $\bar{B} = b^*$, or $\bar{B} = B \cup \{b^*\}$ and $\bar{\phi}_b^h = 0 \quad \forall b \in \bar{B} \setminus \{b^*\}$. Either way the payoff of investor h's entire portfolio is defined as $\left(w_U^h, w_D^h\right) = y^h\left(d_{U,}d_D\right) - \sum_{b\in B}\left(\delta_U\left(j_b,\eta_b\right),\delta_D(j_b,\eta_b)\right)$ and the allocations in the default-free environment are defined by $\bar{y}^h = \frac{w_U^h - w_D^h}{d_U - d_D}$ and $\bar{\phi}_{b^*}^h = \frac{\left(\bar{y}^h d_D - w_D^h\right)}{j_{b^*}} = \bar{y}^h - \frac{w_D^h}{j_{b^*}}$. For every investor h the portfolio of $\bar{y}^h$ asset and $\bar{\phi}_{b^*}^h$ debt contract holdings gives a payoff in $t = T$ of $\left(w_U^h, w_D^h\right)$ and, if b^* is priced according to lemma 2, it also costs the same. If and only if $w_D^h \geq 0\ \forall h \in H$, the collateral requirement condition is met too, because $\bar{\phi}_{b^*}^h \leq \bar{y}^h$. Now, if the investors optimality condition (5) was met before, it is also met in the new environment. Goods markets and financial markets clear (Fostel and Geanakoplos, 2015). □

Remark 1. As opposed to the construction of the default-free equilibrium without recourse, it has to be assumed that $\left(w_U^h, w_D^h\right) \in \mathbb{R}_+^2\ \forall h \in H$. This condition is not fulfilled, if $y^h = \sum_{b\in B} \phi_b^h$, that is, a investor borrows maximally according to his collateral constraint and $\exists b \in B : \phi_b^h > 0$ and $\eta_b > 0$ while all debt contracts have $j_b \geq d_D$. This is a quite regular situation, in which an investor who is wealthy in state D uses a recourse debt contract to sell some of his endowment. In this situation,$\exists h \in H :\ w_D^h < 0$, there is no possibility to synthesize the portfolio payoffs $\left(w_U^h, w_D^h\right)$ with a default-free debt contract, since the collateral requirement would not be fulfilled. Consider the portfolio payoffs in the following figure. In the no-recourse setting the menu of debt contracts lies on the cyan line. Therefore any payoff coming from a portfolio that conforms with the collateral requirement, has non-negative payoff in state D and can be replicated using the no-default debt contract only. But, for example, the payoff from holding one asset and one default debt contract with $\eta = 0.2$ can only be achieved using the no-default debt contract with strictly more than one of these debt contracts. This violates the collateral requirement.

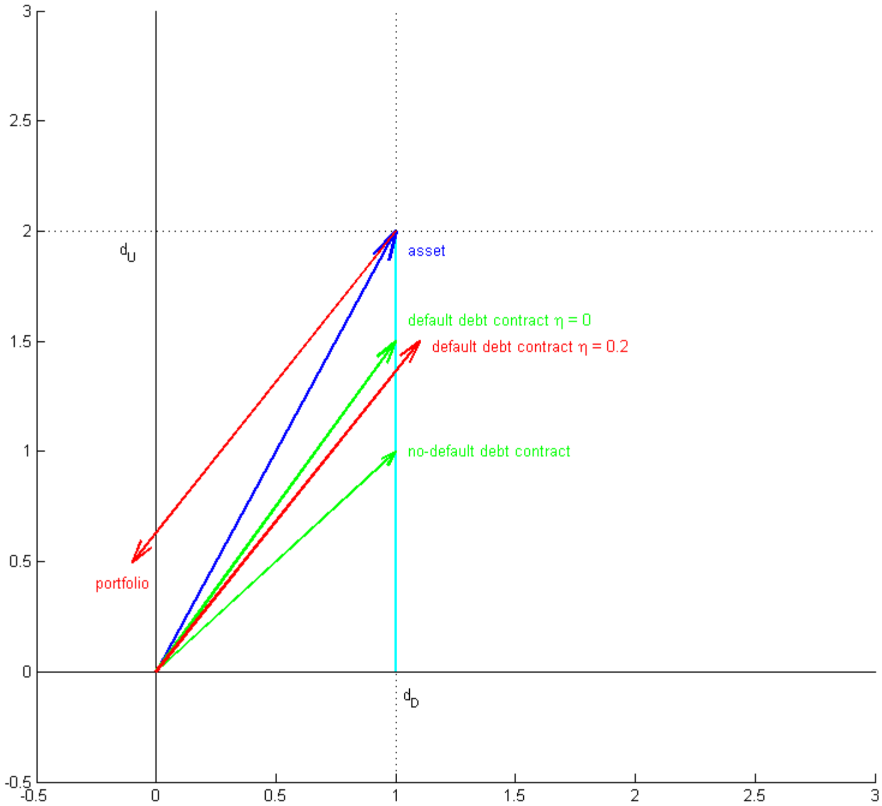

Figure 3: Portfolio Payoffs

4 Recourse and Welfare

In reality sizeable loans are rarely granted without collateral requirement. Claims on collateral are usually very easy to enforce, due to the fact that the collateral is often kept in the hands of a trustee, as in the case of stocks, or cannot be embezzled anyway, like a house.
Whether or not a remaining repayment gap after the seizure of collateral in default can be closed depends on regress laws. In the case of U.S. mortgage on residential houses before 2007, the debtor could stop repayment and surrender the house to the creditor without any further legal guilt. But in general the creditor has recourse possibilities.
In the case of a default, recourse enables the creditor to reclaim a fraction of the repayment gap. This is rationally expected by both the creditor and the debtor. The analysis in this chapter tries to explain in which circumstances a higher recovery rate leads to a welfare improvement for both parties.
In a binomial complete markets environment, as in chapter 3, a change of the recovery rate η does not change the linear subspace spanned by the asset's and debt contract's payoffs. Together they both still span the whole space. The generalized binomial no-default theorem allows for the replacement of a debt contract with or without recourse, only if there is not too much borrowing done via a recourse debt contract. An increase of η expands the space of possible payoffs taking the collateral requirement constraint into account. If it is binding in equilibrium, which is sufficient for the failure of the binomial no-default theorem if a recourse debt contract is used, the question whether a increase of η leads to an welfare improvement arises.
Also in a binomial environment with incomplete markets, e.g. with a third state at $t = 1$ but no further asset, equilibrium existence under very mild assumptions(Geanakoplos and Zame, 2013) showed. But the equilibrium allocations are generically inefficient in the sense of constrained Pareto efficiency (Geanakoplos and Polemarchakis, 1985). Furthermore, the equilibrium correspondence, which maps primitives, e.g. the debt contract menu B or endowments $\mathbf{e}^h$ and y_0^h, to equilibria, does not operate on Euclidean spaces but on Grassmannians, which makes the use of the implicit function theorem for local analysis very complicated. But also here the question whether an increase of η leads to a welfare improvement arises. Yet, it is harder to answer.

4.1 Complete Markets

4.1.1 An Example

In an environment as in section 3 with $U(c_0^h, c_U^h, c_M^h, c_D^h) = u(c_0^h) + \beta \sum_{s \in S_T} \gamma_s u(c_s^h)$, where $u(c) = c - \frac{1}{20}c^2$, let the asset payoffs be $d_U = 1$ and $d_D = 0.2$ and the available debt contract be $j = 0.3$, so that there is default in the D state. The quadratic utility function can be understood as Taylor-approximation of a true utility function that is sufficient for this example. Yet, endowments must be low enough to ensure monotonicity.
The following tables summarize consumption good and asset endowments and the (identical) discount factor and subjective probabilities:

A	0	U	D
e	0.5	1	8
β/γ	1	0.5	0.5
y	1		

B	0	U	D
e	3	5	5
β/γ	1	0.5	0.5
y	1		

Table 1: Example 1 Primitives

In this complete markets setup there are the following equilibria with varying recourse rates, where $\lambda_0, \lambda_U, \lambda_D$ are the Lagrange-multipliers for the budget constraints in each state and λ_{coll} is the collateral requirement multiplier.

	$\eta = 0$	$\eta = 0.3$	$\eta = 0.8$	$\eta = 1.0$	
U^A	3.834522	3.839590	3.847415	3.850329	$\uparrow$
U^B	6.003000	6.016710	6.039160	6.048000	$\uparrow$
p	0.439329	0.445175	0.453841	0.456932	$\uparrow$
π	0.162650	0.171379	0.185290	0.190632	$\uparrow$
c_0^A	0.385971	0.397583	0.416738	0.424332	$\uparrow$
c_U^A	2.400000	2.400000	2.400000	2.400000	$\rightarrow$
c_D^A	8.000000	7.940000	7.840000	7.800000	$\downarrow$
y^A	2.000000	2.000000	2.000000	2.000000	$\rightarrow$
ϕ^A	2.000000	2.000000	2.000000	2.000000	$\rightarrow$
c_0^B	3.114029	3.102417	3.083262	3.075668	$\downarrow$
c_U^B	5.600000	5.600000	5.600000	5.600000	$\rightarrow$
c_D^B	5.400000	5.460000	5.560000	5.600000	$\uparrow$
y^B	0.000000	0.000000	0.000000	0.000000	$\rightarrow$
ϕ^B	-2.000000	-2.000000	-2.000000	-2.000000	$\rightarrow$
λ_0^A	0.961403	0.960242	0.958326	0.957567	$\downarrow$
λ_U^A	0.380000	0.380000	0.380000	0.380000	$\rightarrow$
λ_D^A	0.100000	0.103000	0.108000	0.110000	$\uparrow$
λ_{coll}^A	0.022372	0.026875	0.033328	0.035543	$\uparrow$
λ_0^B	0.688597	0.689758	0.691674	0.692433	$\uparrow$
λ_U^B	0.220000	0.220000	0.220000	0.220000	$\rightarrow$
λ_D^B	0.230000	0.227000	0.222000	0.220000	$\downarrow$
λ_{coll}^B	0.036520	0.041663	0.049510	0.052395	$\uparrow$

Table 2: Example 1 Equilibrium Values

In this environments investor A would want to take out more debt than is feasible by the collateral constraint, as he already owns all assets and uses them as collateral. Increasing the recovery rate does lower his consumption in state D, where default occurs, since he has to pay back more. Yet, investor B is in equilibrium willing to pay more for a debt contract in state 0, which more than compensates investor A.

With a different endowment, $e_0^A = 2.5$, investor A does not want to borrow as much, because he is wealthier in state 0.

A	0	U	D
e	2.5	1	8
β/γ	1	0.5	0.5
y	1		

B	0	U	D
e	3	5	5
β/γ	1	0.5	0.5
y	1		

Table 3: Example 2 Primitives

This gives following equilibrium values:

	$\eta = 0$	$\eta = 0.3$	$\eta = 0.8$	$\eta = 1.0$	
U^A	5.516582	5.515209	5.515022	5.515558	$\downarrow\uparrow$
U^B	5.928000	5.971913	6.029899	6.048000	$\uparrow\uparrow$
p	0.507725	0.514558	0.523406	0.526170	$\uparrow\uparrow$
π	0.169655	0.176483	0.187862	0.192455	$\uparrow\uparrow$
c_0^A	2.261009	2.295815	2.343372	2.358741	$\uparrow\uparrow$
c_U^A	2.524800	2.472404	2.414287	2.400000	$\downarrow\downarrow$
c_D^A	8.083200	7.995509	7.853335	7.800000	$\downarrow\downarrow$
y^A	2.000000	2.000000	2.000000	2.000000	$\rightarrow\rightarrow$
ϕ^A	1.583999	1.758655	1.952376	2.000000	$\uparrow\uparrow$
c_0^B	3.238991	3.204185	3.156628	3.141259	$\downarrow\downarrow$
c_U^B	5.475200	5.527596	5.585713	5.600000	$\uparrow\uparrow$
c_D^B	5.316800	5.404491	5.546665	5.600000	$\uparrow\uparrow$
y^B	0.000000	0.000000	0.000000	0.000000	$\rightarrow\rightarrow$
ϕ^B	-1.583999	-1.758655	-1.952376	-2.000000	$\downarrow\downarrow$
λ_0^A	0.773899	0.770419	0.765663	0.764126	$\downarrow\downarrow$
λ_U^A	0.373760	0.376380	0.379286	0.380000	$\uparrow\uparrow$
λ_D^A	0.095840	0.100225	0.107333	0.110000	$\uparrow\uparrow$
λ_{coll}^A	0.000000	0.000000	0.000000	0.0000060	$\downarrow\downarrow$
λ_0^B	0.676101	0.679581	0.684337	0.685874	$\uparrow\uparrow$
λ_U^B	0.226240	0.223620	0.220714	0.220000	$\downarrow\downarrow$
λ_D^B	0.234160	0.229775	0.222667	0.220000	$\downarrow\downarrow$
λ_{coll}^B	0.070201	0.080109	0.092938	0.096886	$\uparrow\uparrow$

Table 4: Example 2 Equilibrium Values

For $\eta = \{0, 0.3, 0.8\}$ investor A borrows less than he could due to his asset holdings, hence $\lambda_0^A = 0$. Increasing η leads to an decrease in his utility.
Yet, from some $\eta^* \in (0.8, 1)$ on investor A borrows the maximal amount, and the corresponding $\lambda_0^A > 0$. At $\eta = 1$, he increase his utility in equilibrium.
These two examples lead to following conjecture:

Conjecture 1. *In an equilibrium, in which the borrower owns all assets, borrows the maximal amount and would like to borrow more than he can, increasing the recovery rate leads to a higher utility for borrower and lender.*

4.1.2 Attempt of a Proof

The equilibrium equation system f takes 20 equilibrium variables, that is 2 prices, 5 allocations per investor and 4 Lagrange-multipliers per investor, and consists of 23 equations. Those are 3 goods markets, an asset market, a debt market clearing equations and 9 optimality conditions per investor:

$$\nabla U^h(c_0^h, c_1^h, c_2^h) - \sum_{i=0}^{3} \lambda_i \nabla g_i(c_0^h, c_1^h, c_2^h, y^h, \phi^h) = 0$$

$$\lambda_i g_i(c_0^h, c_1^h, c_2^h, y^h, \phi^h) = 0 \quad \forall i \in \{0,1,2,3\}$$

where g_i are the budget constraints described in section 2.3.
The 20 variable solution to this 23 equations defines an equilibrium in a two investor environment. The 3 clearing condition on the consumption good markets are implied by the complementary slackness, asset clearing and debt clearing conditions in the following manner. Given that $\lambda_s^h > 0 \quad \forall h\, \forall s \in \{0, U, D\}$, adding up binding budget constraints (16) and (25) from the appendix and using asset and debt clearing (9) and (10) implies goods markets clearing in $t = 0$, that is equation (6). Adding up binding budget constraints (17) and (26) and using debt markets clearing implies goods markets clearing in $s = U$, that is equation (7). Similarly, equation (8) is implied too.
Besides the equilibrium variables z, the equation system depends on the primitives, that is utility functions, endowments and the recovery rate.
Therefore, an equilibrium can be describes by a root of the function $f: \mathbb{R}^{20} \times \mathbb{R} \to \mathbb{R}^{20}$ if all primitives except the recovery rate are substituted into the equations.
The goal is to proof:

$$\frac{\partial U^h}{\partial \eta} = \underbrace{u'(c_0^h)}_{\lambda_0^h} \cdot \frac{\partial c_0^h}{\partial \eta} + \underbrace{\frac{1}{2} u'(c_U^h)}_{\lambda_U^h} \cdot \frac{\partial c_U^h}{\partial \eta} + \underbrace{\frac{1}{2} u'(c_D^h)}_{\lambda_D^h} \cdot \frac{\partial c_D^h}{\partial \eta} \geq 0 \quad \forall h \in H$$

and

$$\exists h \in H: \quad \frac{\partial U^h}{\partial \eta} > 0$$

Using the implicit function theorem on $f(z, \eta) = 0$ gives

$$\frac{\partial z}{\partial \eta} = -\left(\frac{\partial f}{\partial z}(z^*; \eta^*)\right)^{-1} \frac{\partial f}{\partial \eta}(z^*; \eta^*)$$

if $\frac{\partial f}{\partial z}(z^*; \eta^*)$ is invertible. The latter condition is proved to hold almost everywhere by Wei Ma (Ma, 2015).

Imposing the assumption from conjecture 1, the Jacobian of the equilibrium equation system allows for a more intuitive representation of $\frac{\partial U^A}{\partial \eta}$. If I add up only the three respective rows concerning the complementary slackness conditions for the goods markets in the linear equation system $\frac{\partial f}{\partial z}(z^*;\eta^*)\frac{\partial z}{\partial \eta} = -\frac{\partial f}{\partial \eta}(z^*;\eta^*)$ yields:

$$\frac{\partial U^A}{\partial \eta} = \left(\phi^A\left(\frac{\partial \pi}{\partial \eta} - \frac{\partial p}{\partial \eta}\right) + \frac{\partial p}{\partial \eta} y_0^A\right)\lambda_0^A - (j - d_D)\,\phi^A \lambda_D^A$$

That is, the marginal utility change in equilibrium is equal to the marginal utility gained by the ability to borrow more and asset endowment which is worth more minus the utility lost due to a higher repayment in state D. A form for $\frac{\partial U^B}{\partial \eta}$ can be obtained by flipping the appropriate signs. Using the implicit derivatives of consumption and the assumptions that investor A owns all assets and borrows the maximal amount yields:

$$\begin{aligned}\frac{\partial U^A}{\partial \eta} \quad = \quad & \frac{-(j-d_D)}{\lambda_0^A\left(10\lambda_0^B + \pi y_0^A\right) - \lambda_0^B y_0^B(\pi - p)}\left(-y_0^A\left(10\lambda_D^B - \beta^B d_D \gamma_D^B\left(y_0^A + y_0^B\right)\right)\lambda_0^A\lambda_0^A\right. \\ & \left. + \lambda_D^A y_0^A\left(10\lambda_0^B + \pi\left(y_0^A + y_0^B\right)\right)\lambda_0^A - \lambda_0^B\lambda_D^A y_0^B\left(y_0^A + y_0^B\right)(\pi - p)\right)\end{aligned}$$

for investor A and for investor B:

$$\begin{aligned}\frac{\partial U^B}{\partial \eta} \quad = \quad & \frac{(j-d_D)}{\lambda_0^A\left(10\lambda_0^B + \pi y_0^A\right) - \lambda_0^B y_0^B(\pi - p)}\left(\beta^B d_D \gamma_D^B \lambda_0^A \lambda_0^B y_0^A\left(y_0^A + y_0^B\right)\right. \\ & \left. + \lambda_0^B y_0^B\left(-10\lambda_D^A\lambda_0^B - \lambda_D^B(\pi - p)\left(y_0^A + y_0^B\right)\right) + \lambda_0^A\lambda_D^B\left(10\lambda_0^B y_0^B + \pi y_0^A\left(y_0^A + y_0^B\right)\right)\right)\end{aligned}$$

A proof of the positivity of these expressions must entail properties of λ_{coll}^A and λ_{coll}^B, but attempts were not fruitful so far.

4.2 Incomplete Markets

Also in incomplete markets the pattern shown in the examples above can be observed.

4.3 An Example

In a model with two periods $t \in \{0,1\}$ and three possible states $S_T = \{U, M, D\}$ in period $t = 1$, two risk averse investors maximize their identical utility function $U(c_0^h, c_U^h, c_M^h, c_D^h) = u(c_0^h) + \beta \sum_{s \in S_T} \gamma_s u(c_s^h)$, where $u(c) = c - \frac{1}{20}c^2$. Let the asset payoffs be $d_U = 1$, $d_M = 0.2$, $d_D = 0.1$ and the available debt contract is $j = 0.2$, so that there is default in the D state. The following tables summarize consumption good and asset endowments and the (identical) discount factor and subjective probabilities:

A	0	U	M	D
e	1	1	5	5
β/γ	1	0.45	0.45	0.1
y	1			

B	0	U	M	D
e	3	5	5	5
β/γ	1	0.45	0.45	0.1
y	1			

Table 5: Example 3 Primitives

In this incomplete markets setup there are the following equilibria with varying recourse rates, where $\lambda_0, \lambda_U, \lambda_M, \lambda_D$ are the Lagrange-multipliers for the budget constraints in each state and λ_{coll} is the collateral requirement multiplier.

	$\eta = 0$	$\eta = 0.3$	$\eta = 0.8$	$\eta = 1.0$	
U^A	3.882713	3.882877	3.883040	3.883067	$\uparrow$
U^B	6.595116	6.595603	6.596352	6.596630	$\uparrow$
p	0.418866	0.419245	0.419664	0.419758	$\uparrow$
π	0.128119	0.130045	0.133135	0.134328	$\uparrow$
c_0^A	0.837373	0.840846	0.846605	0.848898	$\uparrow$
c_U^A	2.600000	2.600000	2.600000	2.600000	$\rightarrow$
c_M^A	5.000000	5.000000	5.000000	5.000000	$\rightarrow$
c_D^A	5.000000	4.940000	4.840000	4.800000	$\downarrow$
y^A	2.000000	2.000000	2.000000	2.000000	$\rightarrow$
ϕ^A	2.000000	2.000000	2.000000	2.000000	$\rightarrow$
c_0^B	3.162627	3.159154	3.153395	3.151102	$\downarrow$
c_U^B	5.400000	5.400000	5.400000	5.400000	$\rightarrow$
c_M^B	5.400000	5.400000	5.400000	5.400000	$\rightarrow$
c_D^B	5.200000	5.260000	5.360000	5.400000	$\uparrow$
y^B	0.000000	0.000000	0.000000	0.000000	$\rightarrow$
ϕ^B	−2.000000	−2.000000	−2.000000	−2.000000	$\rightarrow$
λ_0^A	0.916263	0.915915	0.915340	0.915110	$\downarrow$
λ_U^A	0.333000	0.333000	0.333000	0.333000	$\rightarrow$
λ_M^A	0.225000	0.225000	0.225000	0.225000	$\rightarrow$
λ_D^A	0.050000	0.050600	0.051600	0.052000	$\uparrow$
λ_{coll}^A	0.000791	0.000933	0.000975	0.000925	$\uparrow$
λ_0^B	0.683737	0.684085	0.684660	0.684890	$\uparrow$
λ_U^B	0.207000	0.207000	0.207000	0.207000	$\rightarrow$
λ_M^B	0.207000	0.207000	0.207000	0.207000	$\rightarrow$
λ_D^B	0.048000	0.047400	0.046400	0.046000	$\downarrow$
λ_{coll}^B	0.033194	0.033659	0.034288	0.034488	$\uparrow$

Table 6: Example 3 Equilibrium Values

With increasing η the utility of both investors increases, which constitutes a Pareto improvement. The driver of this improvement is the fact that the available collateral is scarce and that the marginal utility gain (loss) from the marginal dept price change times the Lagrange-multiplier of state 0, λ_0, is higher (lower) than the marginal utility loss (gain) from repaying (retrieving) more in state *D*, where the default actually matters.

5 Conclusion

In this paper, the limited applicability of the binomial no-default theorem is demonstrated. In equilibria where at least one investor has a negative portfolio payoff in state D the only portfolio which gives the same payoffs and at the same time uses only a no-default debt contract, does not meet the collateral requirement. These negative portfolio payoffs occur naturally in the presence of a recourse debt contract, if an investor borrows sufficiently much.

In equilibria, where an investor owns all assets and borrows maximally, I present examples where an increase of the recovery rate leads to a utility increase for all investors. The resulting conjecture, however, turned out to be very hard to prove, even in a setting with only two investors.

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A Equilibrium Equations for Examples

Market clearing:

$$c_0^A + c_0^B = e_0^A + e_0^B \tag{6}$$

$$c_U^A + c_U^B = e_U^A + y^A d_U + e_U^B + y^B d_U \tag{7}$$

$$c_D^A + c_D^B = e_D^A + y^A d_D + e_D^B + y^B d_D \tag{8}$$

$$y^A + y^B = y_0^A + y_0^B \tag{9}$$

$$\phi^A + \phi^B = 0 \tag{10}$$

For investor A:

$$u'(c_0^A) = 1 - \frac{1}{10}c_0^A = \lambda_0^A \tag{11}$$

$$\frac{1}{2}u'(c_U^A) = \frac{1}{2}\left(1 - \frac{1}{10}c_U^A\right) = \lambda_U^A \tag{12}$$

$$\frac{1}{2}u'(c_D^A) = \frac{1}{2}\left(1 - \frac{1}{10}c_D^A\right) = \lambda_D^A \tag{13}$$

$$d_U \lambda_U^A + d_D \lambda_D^A + \lambda_{coll}^A = p\lambda_0^A \tag{14}$$

$$\pi\lambda_0^A = j\lambda_U^A + (d_D + \eta(j - d_D))\,\lambda_D^A + \lambda_{coll}^A \tag{15}$$

$$\lambda_0^A\left(c_0^A - e_0^A + p\left(y^A - y_0^A\right) - \pi\phi^A\right) = 0 \tag{16}$$

$$\lambda_U^A\left(c_U^A - e_U^A - y^A d_U + j\phi^A\right) = 0 \tag{17}$$

$$\lambda_D^A\left(c_D^A - e_D^A - y^A d_D + (d_D + \eta\,(j - d_D))\,\phi^A\right) = 0 \tag{18}$$

$$\lambda_{coll}^A\left(\phi^A - y^A\right) = 0 \tag{19}$$

For investor B:

$$u'(c_0^B) = 1 - \frac{1}{10}c_0^B = \lambda_0^B \tag{20}$$

$$\frac{1}{2}u'(c_U^B) = \frac{1}{2}\left(1 - \frac{1}{10}c_U^B\right) = \lambda_U^B \tag{21}$$

$$\frac{1}{2}u'(c_D^B) = \frac{1}{2}\left(1 - \frac{1}{10}c_D^B\right) = \lambda_D^B \tag{22}$$

$$d_U\lambda_U^B + d_D\lambda_D^B + \lambda_{coll}^B = p\lambda_0^B \tag{23}$$

$$\pi\lambda_0^B = j\lambda_U^B + (d_D + \eta(j - d_D))\lambda_D^B \tag{24}$$

$$\lambda_0^B\left(c_0^B - e_0^B + p\left(y^B - y_0^B\right) - \pi\phi^B\right) = 0 \tag{25}$$

$$\lambda_U^B\left(c_U^B - e_U^B - y^B d_U + j\phi^B\right) = 0 \tag{26}$$

$$\lambda_D^B\left(c_D^B - e_D^B - y^B d_D + (d_D + \eta\,(j - d_D))\,\phi^B\right) = 0 \tag{27}$$

$$\lambda_{coll}^B\left(-y^B\right) = 0 \tag{28}$$

B Equilibrium Equations with Conjecture Assumptions

Market clearing:

$$c_0^A + c_0^B = e_0^A + e_0^B \tag{29}$$

$$c_U^A + c_U^B = e_U^A + \left(y_0^A + y_0^B\right)d_U + e_U^B \tag{30}$$

$$c_D^A + c_D^B = e_D^A + \left(y_0^A + y_0^B\right)d_D + e_D^B \tag{31}$$

$$y^A = y_0^A + y_0^B \tag{32}$$

$$\phi^A + \phi^B = 0 \tag{33}$$

For investor A:

$$u'(c_0^A) = 1 - \frac{1}{10}c_0^A = \lambda_0^A \tag{34}$$

$$\frac{1}{2}u'(c_U^A) = \beta^A\gamma_U^A\left(1 - \frac{1}{10}c_U^A\right) = \lambda_U^A \tag{35}$$

$$\frac{1}{2}u'(c_D^A) = \beta^A \gamma_D^A \left(1 - \frac{1}{10}c_D^A\right) = \lambda_D^A \tag{36}$$

$$d_U \lambda_U^A + d_D \lambda_D^A + \lambda_{coll}^A = p\lambda_0^A \tag{37}$$

$$\pi\lambda_0^A = j\lambda_U^A + d_D \lambda_D^A + \lambda_{coll}^A \tag{38}$$

$$c_0^A - e_0^A + py_0^B - \pi\left(y_0^A + y_0^B\right) = 0 \tag{39}$$

$$c_U^A - e_U^A = \left(y_0^A + y_0^B\right)(d_U - j) \tag{40}$$

$$c_D^A = e_D^A \tag{41}$$

$$\phi^A = y^A \tag{42}$$

For investor B:

$$u'(c_0^B) = 1 - \frac{1}{10}c_0^B = \lambda_0^B \tag{43}$$

$$\frac{1}{2}u'(c_U^B) = \beta^B \gamma_U^B \left(1 - \frac{1}{10}c_U^B\right) = \lambda_U^B \tag{44}$$

$$\frac{1}{2}u'(c_D^B) = \beta^B \gamma_D^B \left(1 - \frac{1}{10}c_D^B\right) = \lambda_D^B \tag{45}$$

$$d_U \lambda_U^B + d_D \lambda_D^B + \lambda_{coll}^B = p\lambda_0^B \tag{46}$$

$$\pi\lambda_0^B = j\lambda_U^B + d_D \lambda_D^B \tag{47}$$

$$c_0^B - e_0^B - py_0^B + \pi\left(y_0^A + y_0^B\right) = 0 \tag{48}$$

$$c_U^B - e_U^B - j\left(y_0^A + y_0^B\right) = 0 \tag{49}$$

$$c_D^B - e_D^B - d_D\left(y_0^A + y_0^B\right) = 0 \tag{50}$$

$$y^B = 0 \tag{51}$$

C Jacobi Matrix

Excluding the implied goods markets clearing condition, calculating partial derivatives and collecting them in the order
$\left(p, \pi, c_0^A, c_U^A, c_D^A, y^A, \phi^A, c_0^B, c_U^B, c_D^B, y^B, \phi^B, \lambda_0^A, \lambda_U^A, \lambda_D^A, \lambda_{coll}^A, \lambda_0^B, \lambda_U^B, \lambda_D^B, \lambda_{coll}^B\right)$ yields:

					1					1									
						1					1								
		$-\frac{1}{10}$										-1							
			$-\frac{\beta^A\gamma_U^A}{10}$										-1						
				$-\frac{\beta^A\gamma_D^A}{10}$										-1					
$-\lambda_0^A$												$-p$	d_U	d_D	1				
	λ_0^A											π	$-j$	$-\eta\left(j-d_D\right)-d_D$	-1				
$-\lambda_0^A \cdot \left(y_0^A-y_0^B\right)$	$-\lambda_0^A\phi^A$	λ_0^A			$\lambda_0^A p$	$-\lambda_0^A\pi$						$c_0^A-e_0^A -\pi\phi^A+p\left(y^A-y_0^A\right)$							
			λ_U^A		$-d_U\lambda_U^A$	$j\lambda_U^A$							$c_U^A-e_U^A +j\phi^A-d_U y^A$						
				λ_D^A	$-d_U\lambda_D^A$	$\left(d_D+\eta\left(j-d_D\right)\right) \cdot\lambda_D^A$								$c_D^A-e_D^A -d_D y^A+\phi^A\cdot \left(d_D+\eta\left(j-d_D\right)\right)$					
					$-\lambda_{coll}^A$	λ_{coll}^A									ϕ^A-y^A				
							$-\frac{1}{10}$									-1			
								$-\frac{\beta^B\gamma_U^B}{10}$									-1		
									$-\frac{\beta^B\gamma_D^B}{10}$									-1	
$-\lambda_0^B$																$-p$	d_U	d_D	1
	λ_0^B															π	$-j$	$-\eta\left(j-d_D\right)-d_D$	
$-\lambda_0^B \cdot \left(y_0^A-y_0^B\right)$	$-\lambda_0^B\phi^B$						λ_0^B			$\lambda_0^B p$	$-\lambda_0^B\pi$					$c_0^B-e_0^B -\pi\phi^B+p\left(y^B-y_0^B\right)$			
								λ_U^B		$-d_U\lambda_U^B$	$j\lambda_U^B$						$c_U^B-e_U^B +j\phi^B-d_U y^B$		
									λ_D^B	$-d_U\lambda_D^B$	$\left(d_D+\eta\left(j-d_D\right)\right) \cdot\lambda_D^B$							$c_D^B-e_D^B -d_D y^B+\phi^B\cdot \left(d_D+\eta\left(j-d_D\right)\right)$	
										$-\lambda_{coll}^B$									$-y_B$