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Modelling Fertility and Human Capital - A Gender Specific Approach

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unter der Leitung von

Univ.Prof. Dipl.-Ing. Dr.techn. Alexia Fürnkranz-Prskawetz

durch

Andreas Hanisch, BSc

Lascygasse 24/4/5, 1170 Wien

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Abstract

Due to the changes in fertility and its familiar circumstances, like the timing of births and family structures, over the last century, fertility models had to be revised to reproduce and explain the observed empirical data. One very recent introduced feature in the framework of fertility models that can be key to analyze the current developments in the society is to assume gender specific agents with gender specific wages, educational choices and tasks in the household. In this master thesis I will present two papers that focus on these gender specific aspects in fertility models. The first, *"A model of voluntary childlessness"* by Gobbi (2013), concentrates on gender specific agents with specific fertility choices, the consequential effects on the matching process of spouses in the marriage market and as an outcome of this on the number of children. Furthermore, Gobbi introduces a fertility framework in which it can be an optimal solution for parents to stay childless, depending on their joint taste for children. Moreover, in her paper she shows the quite unintuitive result, that, under certain circumstances and assumptions, the correlation between childlessness and fertility can be positive.

The second paper I present, *"Gender Inequality, Endogenous Cultural Norms, and Economic Development"* by Hiller (2014), focuses on the quality instead of the quantity of children. Hiller introduces the gender specific aspect by assuming that girls and boys do receive a gender specific amount of education during their childhood. He chooses a framework in which parents always have exactly two children and decide the amount of education they provide to them dependent on an endogenous social norm of the society, without knowing that their decision will influence the future development of the norm. The social norm is driven by the female labour force participation, which itself is contingent on the wage and the human capital which is determined by the gender specific amount of education girls receive during their childhood. With his analysis Hiller identifies three different states of the society and gives policy advices, how a society can escape a low productive poverty regime and converge into a high productive steady state in which gender equality is accomplished.

CHAPTER
ONE

Motivation

1.1 Fertility Trends in Industrialized Countries

Over the last two centuries fertility dynamics changed quite markedly, especially in industrialized countries like the countries of Europe or the United States. These changes, concerning the timing of births, the total number of children born per women as well as the relation of fertility to other microeconomic variables, took place quite similar in the western countries after the second World War. *"The baby boom that peaked in 1963-1964 was followed by a steep fall in fertility rates in the late 1960s and early 1970s, which paved the way to the subsequent period of stable and persistently low fertility."*(SOBOTKA (2012)) To underline the significance of the drop in the fertility rate after the post World War II baby boom we take a look at the period total fertility rate of Austria. After peaking at around 2.8 in 1963 the total fertility rate declined drastically to a value of around 1.5 in 1974 and stays there for the recent years. These data are visualized in Figure 1.1, where one can also see the development of the period total fertility rate of Switzerland and Germany. Despite the common declining trend in all of the western countries these three countries are even more similar in their characteristics of fertility in the last century.

But not only the total number of children per woman changed during the last century, also the timing of births and the types of partnerships changed substantial. Regarding the former point empirical data clearly show that the mean age of a woman at her first birth has, after a slight decrease during the baby boom years in the 1960s, increased. Again taking Austria as an example for the western countries the available data shows that the mean age of a woman at her first birth increased from around twenty-three years during the baby boom to around twenty-eight years in the recent years. While in the early post World War II years mainly the number of children changes and then stays quite stable, in the last few decades it is the timing of births that changes primarily. This change can be explained by the development of social norms like the increasing female labour force participation and the increased education provided to women. Furthermore, also medical inventions in the field of contraception during the last decades contributed to this trend and delayed the average timing of the first birth. Especially in the age group of the 12-27 year-old women the decline in the fertility rate is quite drastic. Figure 1.2 shows the increase in the mean age at first birth in Austria, Germany and Switzerland.

Along with the timing of births also the family structures changed due to the social and biological developments. We experienced a change in family structures over the last century which influenced the trend of the fertility rate. On the one hand the amount of single mothers increased which does not have had a significant effect on the fertility, but on the other hand also

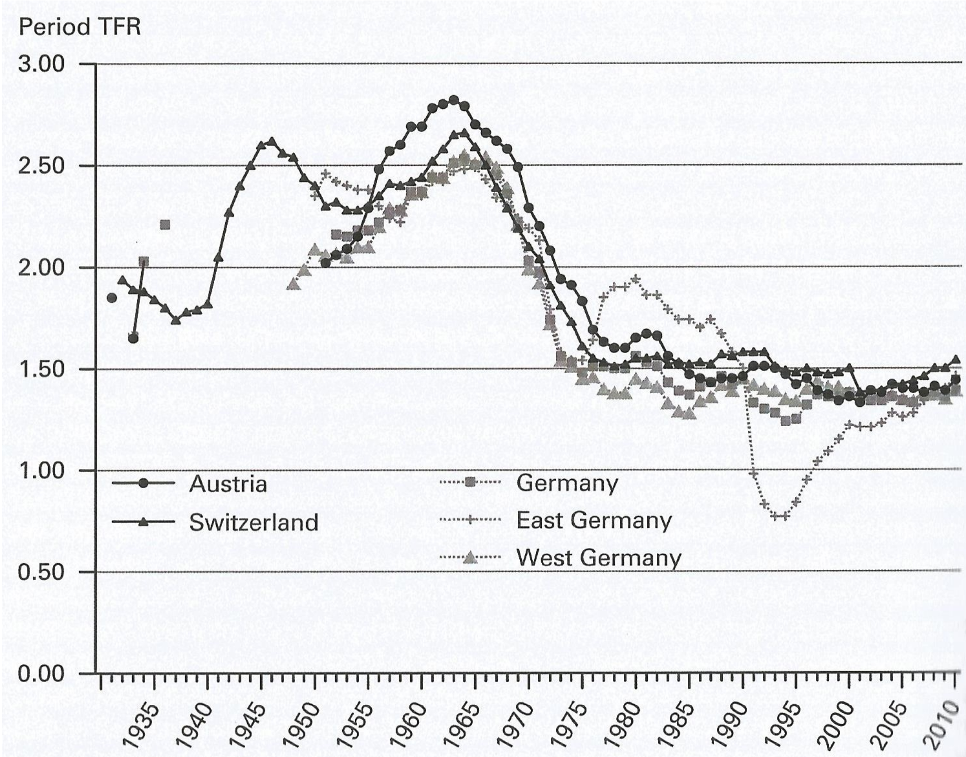

Figure 1.1: Period Total Fertility Rate in Austria, Switzerland Germany (Sobotka (2010), page 266, *Fig. 1*)

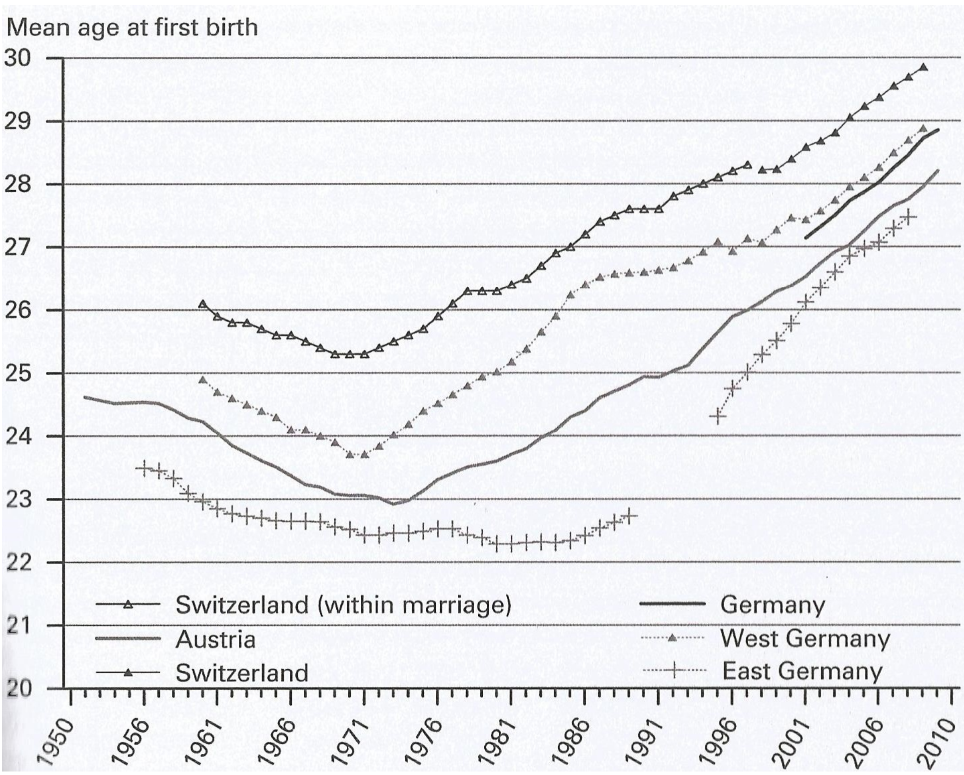

Figure 1.2: Mean age of a mother at her first birth in Austria, Switzerland Germany (Sobotka (2010), page 277, *Fig. 6*)

more and more women and couples decide to stay childless. The development of the share of women born between 1900 and 1968 who stayed childless can be observed in figure 1.3. Furthermore, also the institution of marriage became less important as a factor in having children in the last century which also contributes to the changes in fertility rates. This development can be seen in figure 1.4 and serves as an example of how social structures have changed over the last 60 years and therefore have had influenced the evolution of the fertility rate.

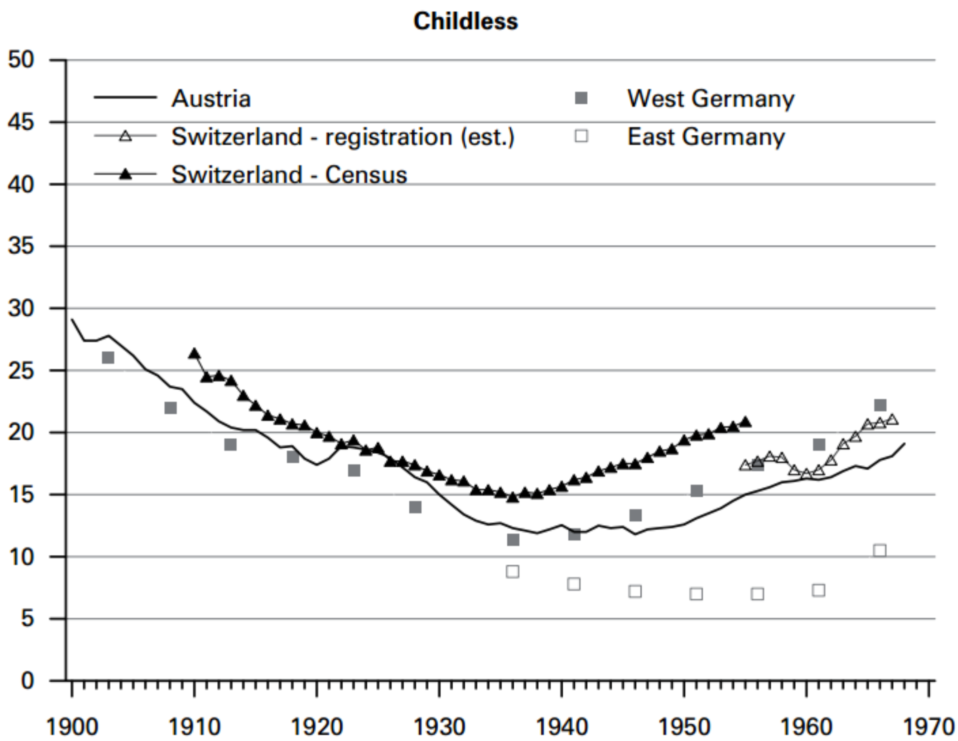

Figure 1.3: Share of childless women born between 1900-1968 in Austria, Switzerland Germany (Sobotka (2010), page 274, *Fig. 5*)

Another change in the social structures over the last century relates to the fact that more and more women started to enter the labour market. Figure 1.5 shows the employment rate of women of the age group from 20 to 64 between 1994 and 2014 in Austria. This data, which is provided and visualized by Eurostat, shows a distinct increase in the employment rate of women over the last years from about 60.5% to around 70%. This change clearly influences many different aspects of the society and economy, especially the fertility. Related to this topic, which is also important to mention for a later chapter of this thesis, is the development of the gender pay gap. The unadjusted gender pay gap is defined by EuroStat as the average gross hourly earnings of male paid employees minus the average gross hourly earnings of female paid employees in comparison to the average gross hourly earnings of male paid employees. The average earnings used for this definition of the gender pay gap were calculated as arithmetic means. *"The indicator has been defined as unadjusted (e.g. not adjusted according to individual characteristics that may explain part of the earnings difference) because it should give an overall picture of gender inequalities in terms of pay. The gender pay gap is the consequence of various inequalities (structural differences) in the labour market such as different working pattern, differences in institutional mechanisms and systems of wage setting. Consequently, the pay gap is linked to a number of legal, social and economic factors which go far beyond the single issue of equal pay for equal work."*(EuroStat (2016b)) It seems intuitive

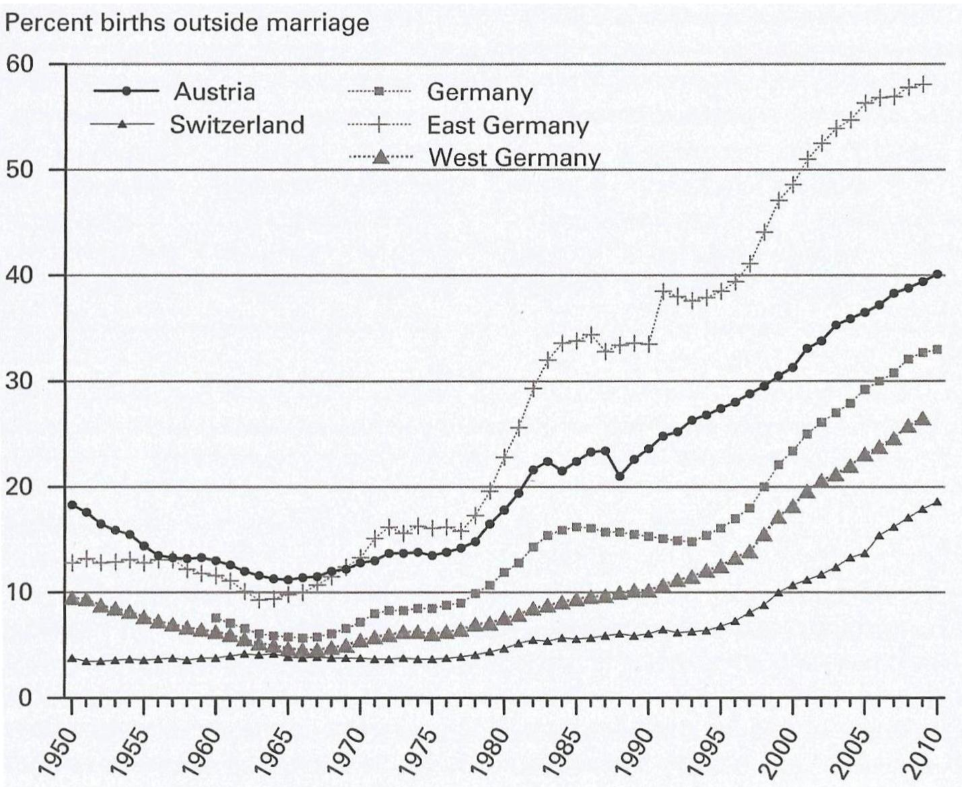

Figure 1.4: Share of births outside of marriage in Austria, Switzerland Germany (Sobotka (2010), page 280, *Fig. 8*)

that the gender wage pay gap also influences the fertility of a country by influencing the decision of women to participate in the labour market and therefore should be considered in the analysis of fertility development. Figure 1.6 shows the gender pay gap expressed as defined by Eurostat in Austria, Germany, Sweden and the average of the EU28 countries. As can be seen the gender wage gap between females and males decreased in Austria, Germany and Sweden, while it seems to stagnate if one considers all 28 countries of the European Union. Furthermore, in my opinion it is quite interesting that the gender pay gap is significantly higher in Austria and Germany compared to Sweden.

These quite drastic changes in the total fertility rate and the development of its correlates, make it difficult to develop fertility models that are able to describe and reproduce these characteristics. Essential for the development of such fertility models is the finding of empirical studies across time and for different countries all over the world, that the fertility is negatively correlated to income. This relation can be observed in figure 1.7, in which the correlation between fertility and the (husband's) income is plotted for different birth cohorts born from 1828 up to 1958. Although some literature criticizes that this correlation should indeed be positive and just shows this negative character because of some missing explanatory variables, the negative relationship has become the origin of most of the economic fertility models. Therefore, most fertility models try to explain the decline in fertility rates in Western countries by the increase in income over the last decades. Generally speaking, there are two different approaches to model the negative correlation between the fertility of a family and its income, the aspect of time costs of child-rearing and the quantity-quality trade-off thesis. The former is described in *"Fertility Theories: Can They Explain the Negative Fertility-Income Relationship?"* in the following way: *"The basic idea is that the price of children is largely time, and because of this, children are more*

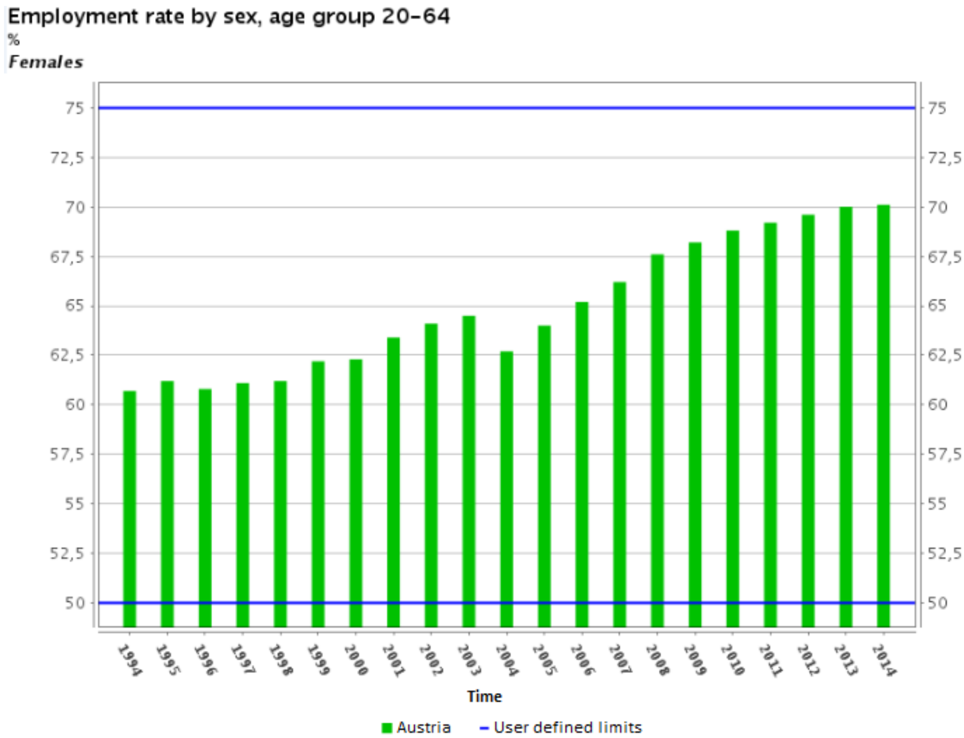

Figure 1.5: Female employment rate in Austria, redefined scale from 50% to 75%
(Data from EuroStat (8th of February 2016), last updated 3rd of February 2016)

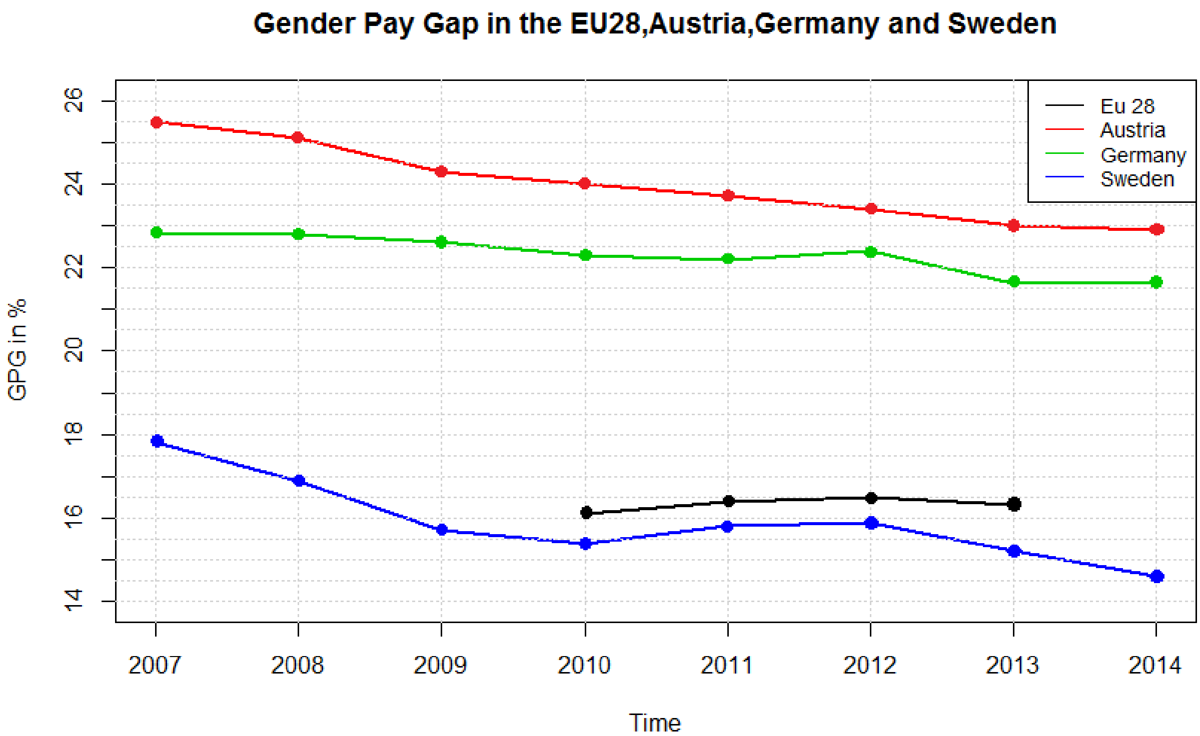

Figure 1.6: Gender Pay Gap in unadjusted form in %
(Data from EuroStat (9th of February 2016), last updated 4th of February 2016)

expensive for parents with higher wages." (Jones, Schoonbroodt und Tertilt (2010),page 44). The second approach, the quantity-quality trade-off, follows the argument that higher-wage parents will invest more time, respectively income, in the quality of their children which increases their opportunity costs, respectively the pure costs, per child and therefore they will have fewer children. However, it is worth mentioning that in the early development stages like in agrarian economies the relation between income and fertility was positive. In these societies income is often described by farm size and thereby it might be intuitive that income and fertility are positively correlated. But over the last century in almost every western country from Japan over Europe to the US, the fact that parents with higher wages have fewer children seems very robust.

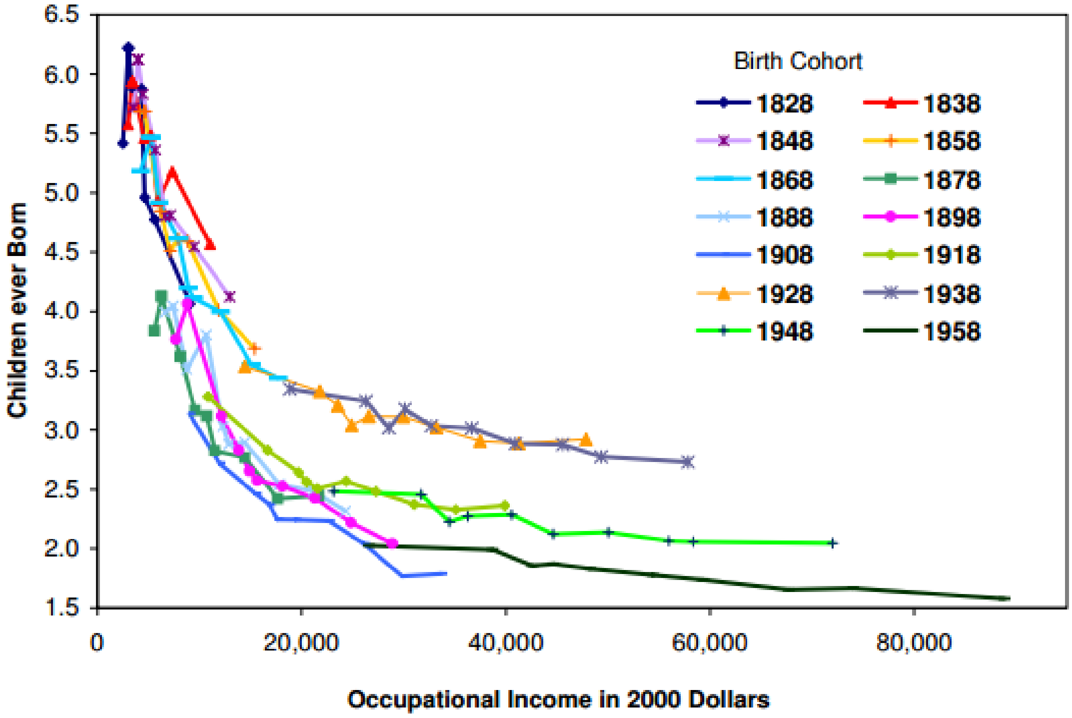

Figure 1.7: Fertility-Income Relationship
(Jones, Schoonbroodt und Tertilt (2008)), page 7, *Figure 1*)

1.2 Fertility Models

Based on these empirical results of the negative correlation between fertility and income I will review different approaches of constructing fertility models. Therefore I will follow the chapter *"Fertility Theories: Can They Explain the Negative Fertility-Income Relationship?"* in *"Demography and the Economy"* by Jones, Schoonbroodt und Tertilt (2010), which gives an excellent overview over the topic of fertility models in general. In their work they introduce different benchmark models and approaches and analyze if extensions of these models are needed to generate the negative correlation between fertility and income. They start with a basic model in which individuals just try to optimize their utility, given by a specific utility function (with focus on separable utilities in the choice of children, consumption and leisure) and subject to a budget constraint and a time constraint. People can gain utility from consumption goods, the number of children they have, the leisure time they enjoy and the average quality of their children, which is given by a particular quality function. The agents can earn wage by using part of their given amount of time, normalized to one unit, at the labour market. The amount of time they are not working they can spend on childrearing and leisure time. It is assumed that "producing" children incurs two different costs, units of goods costs and time costs, both of them per child. A basic model described in that way could for example look like

$$
\begin{aligned}
\max_{c,n,q,e,l_w} \quad & U(c,n,q,l) \\
s.t. \quad & l_w + b_1 n + l \leq 1 \\
& c + (b_0 + s)n \leq y + w l_w \\
& q = f(s)
\end{aligned}
\tag{1.1}
$$

where $U(c, n, q, l)$ is the utility function of the individuals, c denotes the consumption, n stands for the number of children and q for their average quality, l denotes leisure time and l_w the time spent working. The variable y stands for nonlabour income while w denotes the wage per unit of time spent working. The children's quality q results out of the quality function $f(s)$, which depends on the educational expenditures for the children, denoted by s. Furthermore b_0 describes the goods costs and b_1 the unit time costs of having children. The first constraint of the maximization problem states that the total time endowment of an individual, normalized to one, can either be used for working, leisure of child-rearing. The second constraint describes the budget constraint of the individuals. The sum of the labour income and the non-labour income can be used for consumption goods, which price is normalized to one, and for child-rearing and educational costs for children, $(b_o + s)n$. As the following illustrations will show the time cost factor, together with increasing opportunity costs of parent's time is crucial to obtain the negative correlation between income and fertility.

In a first approach the authors choose a logarithmic form for the utility function in which parents do not gain any utility from child quality or leisure, which means analytically that in the following maximization problem the weight of the child quality α_q and the weight of the leisure time $\alpha_l = 0$ in the utility function are set equal to zero. Thereby, individuals just gain utility through consumption and having children, $\alpha_c > 0$ and $\alpha_n > 0$. Furthermore, the individuals are restricted to the usual budget constraint: an individual receives one unit of time, which they can whether spend at the labour market and earn the wage w or use for child rearing, which costs them b_1 per child. The resulting income can then be used to buy the numeraire consumption good c or for child rearing, which good costs are equal to b_0 per child.

$$
\begin{aligned}
\max_{c,n} \quad & \alpha_c \log(c) + \alpha_n \log(n) \\
s.t. \quad & c + b_0 n \leq w(1 - b_1 n)
\end{aligned}
\tag{1.2}
$$

This first model (1.2) does not provide the empirical observed relationship between fertility and income even if the good costs of children are set to zero and even not if quality of children and leisure are considered. To obtain evidence, that the time cost of child rearing is the reason for the negative relationship between income and fertility, the authors assume general constant elasticity of substitution (CES) utility functions. Furthermore, they introduce non-labour income to the framework of the model. The time cost of raising children is still needed to receive the desired correlation between fertility and income, while the good costs of children are set to zero. Such a model takes the following form

$$
\begin{aligned}
\max_{c,n} \quad & \alpha_c \frac{c^{1-\sigma} - 1}{1-\sigma} + \alpha_n \frac{n^{1-\sigma} - 1}{1-\sigma} \\
s.t. \quad & c \leq y + w(1 - b_1 n)
\end{aligned}
$$

If the elasticity of substitution between children and consumption is high enough this framework leads to the stylized empirical facts. This is the case because the effect of an increase in wage on the number of children depends on the proportion between the substitution effect and the income effect. On the one hand with a higher wage families can afford more children but on the other hand they can also earn more wage for consumption goods if they spend less time for child rearing. Formulated in the notation of demand theory, the substitution effect must be stronger than the income effect and therefore the substitution elasticity between children and consumption greater than one. The result of the negative correlation between fertility and income holds for both cases, i.e. with the assumption of non-labour income and in the absence of it. Some literature questions the meaningfulness of non-labour income, because of its nonexistence in most households in the world. Other authors use the introduction of non-labour income in fertility models as a way to distinguish between the husband's earnings, which is then treated as non-labour income, and the wife's wage, assuming that only the wife's income is crucial for the relation between income and fertiltiy.

Another approach is to assume that people differ in their taste for children. It seems intuitive that some people want to have more children than others and therefore accept the fact that they do not have the same amount of time to increase their human capital than people who want to have fewer children. But this decision, to invest fewer time in the allocation of human capital, causes these people to have lower future market-based skills and therefore earn lower wages. A similar attempt would be to assume that the individuals differ in their fertility desires and accordingly the individuals choose their investments in human capital. Both approaches lead to the wanted negative fertility-income correlation the empirical data clearly shows. Furthermore, there is empirical evidence, that breaks in the working life due to childbearing have long lasting negative effects on the future income because of the missed working experience during this time. Working experience can be seen as a specific type of human capital accumulation, which supports these two approaches to explain the negative relationship between fertility and income. The same applies to a delay in childbirth. Empirical studies suggest *"that an exogenous delay in childbirth leads to a substantial increase in earnings, wage rates, and hours worked."*(JONES, SCHOONBROODT UND TERTILT (2010),page 61)

As mentioned above the second basic approach to explain the negative relation between income and fertility is the quality-quantity trade-off theory. The basic idea is that richer parents have a demand for more child quality and therefore would have fewer children but will invest more in their quality. But there are critics of this theory because: *"While richer parents do spend more on their children (better schools, better clothes, higher bequests, etc.), richer people spend more on everything. ... richer people would want more quality, but probably not less quantity, the same way they also would not want better but fewer cars."*(JONES, SCHOONBROODT UND TERTILT (2010),page 63) Hence, with this argumentation it follows that quantity should be slightly increasing in income and the relationship between quality and income should be clearly positively correlated. Under these assumptions the model can be formulated as follows:

$$\max_{c,n,q,s} \alpha_c \log(c) + \alpha_n \log(n) + \alpha_q \log(q)$$
$$s.t.\ c + sn \leq w(1 - b_1 n)$$
$$q = f(s)$$

Again it is assumed that it takes no unit costs to produce children, $b_0 = 0$. Adding a quality choice to the model does not generate the observed negative relationship between income and fertility the empirical data show, but rather some specifications in the design of the model are needed. One way to replicate the stylized facts is to assume again preference heterogeneity for children among the individuals. In terms of the above mentioned optimization model this means a different α_n for every individual in its utility funciton. The more children parents want to have, the less they will work at the labour market to earn income and therefore they will spend less income on the quality of their children. In this approach the income is negatively correlated to fertility and the quality is positively related to income. A second way to design the model to reproduce the stylized facts is to keep assuming wage heterogeneity and specify the characteristics of the quality function. An example would be to choose a quality function which consists of two additive parts and at least one of the two parts is perfectly positively correlated with the income, like

$$f(s) = d_0 + d_1 s = d_0 + (\kappa w)s$$

For example, the first part d_o could be interpreted as public schooling, which is independent of the income, while the second part stands for investments in the quality of the child s multiplied by a part d_1 that depends, to a certain degree κ, on the wage of the parents w . This specification would also lead to the result that while quality is increasing with the income, fertility is negatively correlated to it.

Another very important aspect of fertility models is the possibility to consider gender differences between the individuals. This distinction provides the possibility to distinguish for example between the wages and the available time of the husband and the wife. This is important because as an example, nowadays it is still the case that child care is mainly the responsibility of mothers, thereby it would be possible that just her time is crucial. Furthermore the gender specific considerations raise the question, if the negative correlation between income and fertility still holds even if just the man earns wages at the labour market, while the woman does not work at all and thereby can use her time to look after the children. The distinction between the individual's gender also allows to introduce a certain matching mechanism. For example, if it is assumed that all individuals differ in their taste for children it would make sense to model a mechanism which improves the possibility, that women with a high preference for children get together with a man with the same desire to have children. Such matching rules are quite important in fertility models and can affect the outcome quite drastically. Furthermore, the data suggests that such matching mechanics exist in our society: *"... assortative mating in education has long been documented in the data... ."* (JONES, SCHOONBROODT UND TERTILT (2010) ,page 89) Gender specific fertility models provide also the possibility to analyze the development of social variables like the female labour participation, the gender wage gap or other gender related aspects that give information about the state of gender equality in a society.

In my thesis I will present two papers that consider gender specific fertility models. The first, *"A model of voluntary childlessness"* by GOBBI (2013), considers the gender distinction in labour and in wages and in the matching process of the individuals at the marriage market. Gobbi's fertility model focuses on the quantity of children and introduces the possibility for couples to stay childless as an optimal solution. This is especially important due to the increasing ratio of childless couples and childless single women that can be observed over the last century. The different number of children among the couples in this framework results from the preference heterogeneity of the individuals that Gobbi assumes. Due to this heterogeneity in preferences and the distinction by gender, the model provides a special way of matching in which a woman and a man are matched randomly and their individual taste for children yields their desire to have children as a couple. *"Gender Inequality, Endogenous Cultural Norms, and Economic Development"* by HILLER (2014) is the second paper I will present in my master thesis. Compared to the former paper, Hiller does not focus on the quantity, but on the quality of the children. He assumes that parents do always have exactly two children, a girl and a boy. Depending on their gender and the income of the parents the children receive a gender specific amount of education that determines their future human capital and thereby their future wage. It is assumed that females receive less human capital and the difference between male and female human capital investment depends on an introduced endogenous social norm that measures the inequality between women and men. In his paper Hiller shows steady state solutions for the income and the social norm of his model and on the basis of them provides policy implications and possibilities to escape from gender inequality with respect to wage and education.

CHAPTER

TWO

A model of voluntary childlessness by Paula E. Gobbi

2.1 Introduction

One way to model childlessness as an important factor for the fertility of a society is provided by a model designed by Paula E. Gobbi in *"A model of voluntary childlessness"* Gobbi (2013). In her work she also investigates how childlessness is connected with fertility and which aspects affect the direction of this correlation. In my presentation of her paper I will follow Gobbi's work and start first with an introduction and an explanation of the framework of the model and later derive its results.

2.2 The Model

Gobbi chooses an overlapping-generation (OLG) model in which the agents live for two periods. The first period is the childhood in which they are raised up by their parents. In the second period the agents are in adulthood. In this period they decide about their level of consumption and the number of kids they want to have and finance their decisions by spending part of their given amount of time working and receive a certain wage. In her model Gobbi assumes, that the agents are all the same during their childhood, so it makes no difference in terms of costs if the child is female or male. Nonetheless during the adulthood period it does matter if the agent is male or female, because the wage differs related to gender. Furthermore only women are able to have children and are responsible for raising them. In addition there will be different agents with different desires to have children for both genders. When a woman and a man with different taste for having children form a couple an average taste will be derived which influences their common decisions about consumption and fertility. Following Gobbi's paper I will start on the household level by defining the household maximization problem and deriving its solution.

2.2.1 The Household Maximization Problem

Agents that reach adulthood form a couple with another agent from the opposite sex. These couples, indicated by a superscript j, decide on a common level of consumption c_t^j and the number of children n_t^j they want to have. These decisions lead to a joint utility for the couple, given for the period t by the following function:

$$U_t^j(c_t^j, n_t^j) = \ln(c_t^j) + \gamma^j n_t^j \tag{2.1}$$

This utility function consists of two additive parts. First the natural logarithm of the consumption of the couple j at the period t, $\ln(c_t^j)$, and second the couple's fertility n_t^j multiplied by $\gamma^j > 0$, which represents the couple's average willingness to have kids. If we take a look at the utility function (2.1) we notice that Gobbi assumes that the parents gain no utility from the future utility of their children, just from the number of children they have.
To be able to consume goods and to raise children the individuals need to earn money and therefore every adult individual is given one unit of time. As stated above Gobbi makes the assumption that only women are able to bear and raise children and therefore men use their whole unit of time to earn their gender specific wage w_t^m. With this assumption still in mind woman can just spend $(1 - \theta n_t^j)$ of their given time in the labour market, where $\theta \in [0, 1]$ indicates the amount of time it takes to raise each child. Therefore the gender specific income of the mother is given by $(1 - \theta n_t^j)w_t^f$. Furthermore having the first child raises fix costs k to the parents. Altogether the household's budget constraint has the form

$$c_t^j = w_t^m + (1 - \theta n_t^j)w_t^f - kI(n_t^j) \tag{2.2}$$

where $I(n_t^j)$ is a variable that indicates, if the couple j has at least on child or not and therefore is defined by:

$$I(n_t^j) = \begin{cases} 0 & \text{if } n_t^j = 0 \\ 1 & \text{if } n_t^j > 0. \end{cases}$$

Knowing the amount of time it takes to raise one child also provides the maximum number of children a mother can raise with her given time. This delivers the following constraint for the number of children a couple j can have:

$$0 \leq n_t^j \leq \frac{1}{\theta} \tag{2.3}$$

Every couple tries to maximize its utility (2.1) considering their budget constraint (2.2) and fertility constraint (2.3). This leads to three possible solutions for the consumption and the fertility, two corner solutions and one interior solution. The first corner solution (c_t^0, n_t^0) represents the case in which a couple decides not to have any children and therefore uses all its money for consumption:

$$\begin{aligned} c_t^0 &= w_t^m + w_t^f \\ n_t^0 &= 0 \end{aligned} \tag{2.4}$$

On the other hand the second corner solution would imply that the couple has as many children as possible. Because of that, the mother would not have any time left which she could spend at work and therefore just the man's wage can be used for consumption. This leads to the following solution (c_t^{max}, n_t^{max}):

$$\begin{aligned} c_t^{max} &= w_t^m - k \\ n_t^{max} &= \frac{1}{\theta} \end{aligned} \tag{2.5}$$

The last solution is the interior solution (c_t^*, n_t^*) for which both optimal solutions are greater than zero and which can be derived by solving the maximization problem via the Lagrange function

$$L = \ln(c_t^j) + \gamma^j n_t^j + \lambda_t(w_t^m + (1-\theta n_t^j)w_t^f - c_t^j - k) \tag{2.6}$$

by setting the derivatives with respect to the control variables c_t^j and n_t^j equal to zero.

$$\frac{\partial L}{\partial c_t^j} \stackrel{!}{=} 0 \Leftrightarrow \frac{1}{c}^j_t = \lambda_t \qquad \frac{\partial L}{\partial n_t^j} \stackrel{!}{=} 0 \Leftrightarrow \gamma^j = \lambda_t \theta w_t^f$$

$$\Rightarrow \gamma^j = \frac{1}{c_t^j}\theta w_t^f$$

$$\Rightarrow c_t^j = \frac{\theta w_t^f}{\gamma^j}$$

This leads to the following transformations via the budget constraint (2.2) and the solution for the maximization problem.

$$\begin{aligned}
\Rightarrow \qquad & c_t^j + kI(n_t^j) = w_t^m + (1-\theta n_t^j)w_t^f \\
\Longleftrightarrow\ & \frac{\theta w_t^f}{\gamma^j} + kI(n_t^j) = w_t^m + (1-\theta n_t^j)w_t^f \\
\Longleftrightarrow\ & \frac{\theta}{\gamma^j} + \frac{kI(n_t^j)}{w_t^f} = \frac{w_t^m}{w_t^f} + 1 - \theta n_t^j \\
\Longleftrightarrow\ & n_t^{j*} = \frac{w_t^m - kI(n_t^j)}{w_t^f \theta} + \frac{1}{\theta} - \frac{1}{\gamma^j} \\
\Longleftrightarrow\ & n_t^{j*} = \frac{w_t^m + w_t^f - kI(n_t^j)}{w_t^f \theta} - \frac{1}{\gamma^j}
\end{aligned}$$

Because of the fact that for the interior solution $n_t^j > 0$ holds it follows that $I(n_t^j) = 1$ and therefore the third possible solution (c_t^*, n_t^*) is given by

$$\begin{aligned}
c_t^* &= \frac{\theta w_t^f}{\gamma^j} \\
n_t^* &= \frac{w_t^m + w_t^f - k}{\theta w_t^f} - \frac{1}{\gamma^j}
\end{aligned} \tag{2.7}$$

Because of the composition of the solutions (2.4),(2.5) and (2.7) it seems natural to study, if there exists a specific value for the willingness to have children $\gamma^j = \gamma^*$ for which a couple would be indifferent between becoming parents and having no children at all. Gobbi investigates this case in her paper within the *"Proposition 1"* (Gobbi (2013), page 970). A couple is indifferent between having children and staying childless if and only if the values of the respective utility functions are the same. Therefore a couple decides to have children if

$$U(c_t^{j*}, n_t^{j*}) \geq U(c_t^{j0}, n_t^{j0})$$
$$\ln\Big(\frac{\theta w_t^f}{\gamma^j}\Big) + \gamma^j\Big(\frac{w_t^m + w_t^f - k}{w_t^f \theta} - \frac{1}{\gamma^j}\Big) \geq \ln(w_t^m + w_t^f) \tag{2.8}$$

is the case, because then the utility of having at least one child is higher than the one the couple would achieve staying childless. The equation above can be rewritten as

$$\ln\Big(\frac{\theta w_t^f}{\gamma^j}\Big) - \ln(w_t^m + w_t^f) \geq \Big(1 - \frac{\gamma^j(w_t^m + w_t^f - k)}{w_t^f \theta}\Big)$$
$$\iff \ln\Big(\frac{\theta w_t^f}{\gamma^j(w_t^m + w_t^f)}\Big) \geq \Big(1 - \frac{\gamma^j(w_t^m + w_t^f - k)}{w_t^f \theta}\Big)$$

To analyze the behavior of this equation we define the left-hand side and the right-hand side as

$$v(\gamma^j) := \ln\Big(\frac{\theta w_t^f}{\gamma^j(w_t^m + w_t^f)}\Big)$$
$$z(\gamma^j) := 1 - \frac{\gamma^j(w_t^m + w_t^f - k)}{w_t^f \theta}$$

Therefore the initial problem can be rewritten by formulating the equivalent problem using our above defined functions:

$$\Rightarrow U(c_t^{j*}, n_t^{j*}) \geq U(c_t^{j0}, n_t^{j0}) \iff v(\gamma^j) \geq z(\gamma^j)$$

Now first we have a look at the characteristics of these two functions to gain some first insights into the properties of the value γ^* we are looking for. It can easily be shown that the first derivative of $v(\gamma^j)$ is less than zero,

$$\begin{aligned}
\frac{dv(\gamma^j)}{d\gamma^j} &= \frac{1}{\frac{\theta w_t^f}{\gamma^j(w_t^m + w_t^f)}}(\theta w_t^f(-1)(\gamma^j(w_t^m + w_t^f))^{-2}(w_t^m + w_t^f)) = \\
&= \frac{\gamma^j(w_t^m + w_t^f)}{\theta w_t^f}\theta w_t^f \frac{-(w_t^m + w_t^f)}{(\gamma^j(w_t^m + w_t^f))^2} = \\
&= \frac{-(w_t^f + w_t^f)}{\gamma^j(w_t^m + w_t^f)} < 0
\end{aligned}$$

while the second derivative is positive for all γ^j.

$$\begin{aligned}
\frac{d^2 v(\gamma^j)}{d(\gamma^j)^2} &= -(w_t^m + w_t^f)(-1)(\gamma^j(w_t^m + w_t^f))^{-2}(w_t^m + w_t^f) = \\
&= \frac{(w_t^m + w_t^f)^2}{(\gamma^j(w_t^m + w_t^f))^2} \\
&= \frac{(w_t^m + w_t^f)^2}{\gamma^{j2}(w_t^m + w_t^f)^2} \\
&= \frac{1}{\gamma^{j2}} > 0
\end{aligned}$$

Furthermore if we analyze the behavior of the function $v(\gamma^j)$ for $\gamma^j \to 0$ and $\gamma^j \to +\infty$ it can be shown that

$$\lim_{\gamma^j \to 0} v(\gamma^j) = \lim_{\gamma^j \to 0} \ln(\frac{\theta w_t^f}{\gamma^j (w_t^m + w_t^f)}) = \ln(+\infty) = +\infty$$
$$\lim_{\gamma^j \to +\infty} v(\gamma^j) = \lim_{\gamma^j \to +\infty} \ln(\frac{\theta w_t^f}{\gamma^j (w_t^m + w_t^f)}) = \ln(0) = -\infty$$

Now we conduct the same analyses for the right-hand side function $z(\gamma^j)$ and see, that the first derivative is always negative and the second derivative is equal to zero for all γ^j. Furthermore the function starts at 1 and converges towards minus infinity in the long run.

$$\frac{dz(\gamma^j)}{d\gamma^j} = -\frac{(w_t^m + w_t^f - k)}{w_t^f \theta} < 0$$
$$\frac{d^2 z(\gamma^j)}{d(\gamma^j)^2} = 0$$
$$z(0) = 1$$
$$\lim_{\gamma^j \to +\infty} z(\gamma^j) = \lim_{\gamma^j \to +\infty} 1 - \frac{\gamma^j (w_t^m + w_t^f - k)}{w_t^f \theta} = -\infty$$

Moreover we inspect for which values the two functions $v(\gamma^j)$ and $z(\gamma^j)$ switch their algebraic sign.

$$v(\gamma^{j1}) \stackrel{!}{=} 0 \iff \gamma^{j1} = \frac{w_t^f \theta}{w_t^m + w_t^f}$$
$$z(\gamma^{j2}) \stackrel{!}{=} 0 \iff \gamma^{j2} = \frac{w_t^f \theta}{w_t^m + w_t^f - k}$$

It is obvious that $\gamma^{j1} \leq \gamma^{j2}$ holds. So from the results above it follows, that $v(\gamma^j)$ is decreasing and convex while $z(\gamma^j)$ is decreasing and linear. This implies, that in the long run

$$\Rightarrow \lim_{\gamma^j \to +\infty} \Big(v(\gamma^j) - z(\gamma^j) \Big) > 0 \tag{2.9}$$

must hold. Furthermore we know, that for $\gamma^j = \gamma^{j2}$ the optimal solution would be $n_t^* = 0$ and therefore that the two functions $v(\gamma^j)$ and $z(\gamma^j)$ have to intersect twice once before γ^{j2} because of their initial values for $\gamma^j = 0$ and once after γ^{j2} because of their long run behaviour (2.9). For all values of $\gamma^j < \gamma^{j2}$ it holds that the solution n_t^j would be less than zero, which means that the couple would choose the optimal solution n_t^0, which implies that they would stay childless. Therefore the second intersection which is greater than γ^{j2} is the value we are looking for, the level of the willingness to have children γ^*, for which the couple is indifferent between being childless ($n_t = n_t^0$) and having children ($n_t = n_t^*$). In figure 2.1 I visualize the above discussed characteristics of the two functions $v(\gamma^j)$ and $z(\gamma^j)$ by showing their behaviour for $\gamma^j \in [0, 0.4]$ and setting the other variables equal to $\theta = \frac{1}{3}, w_t^m = 0.565, w_t^f = 0.4407$ and $k = 0.0836$. These values correspond with the values Gobbi estimates in her paper for the

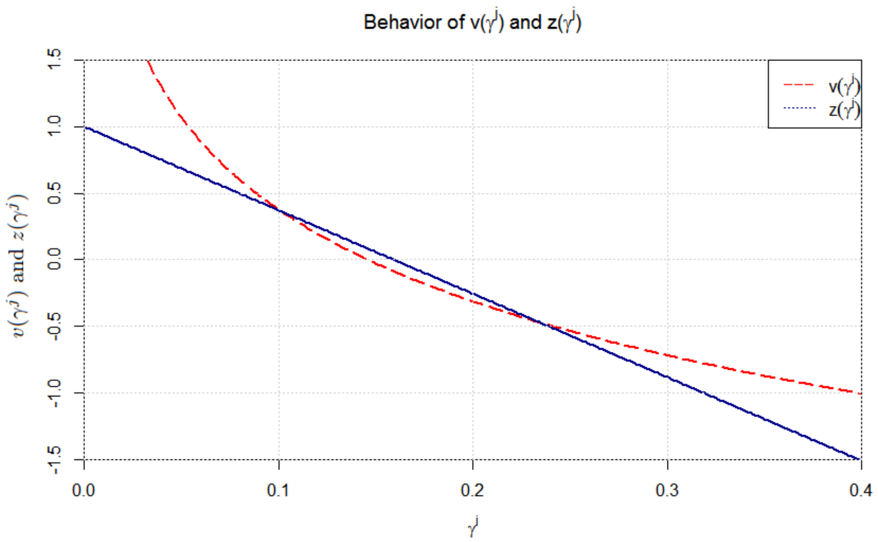

Figure 2.1: Own Calculation of $v(\gamma^j)$ and $z(\gamma^j)$ with RStudio

historic fertility development in the US, with the exception of k. I increased the value of k from 0.00236 to 0.0836 to recieve explicit intersections between the two functions.

With this result of γ^* it can be shown, that $\frac{\partial \gamma^*}{\partial w_t^m} < 0$, which means that an increase in the wage of the husbands leads to a reduction of γ^* which is why couples will decide to have children for even smaller values of γ^j. The contrary effect comes with an increase in the fix costs k which increases the critical level γ^* while the effect in an increase in the wage of women is ambiguous. These results can be seen in figure 2.2, which shows the effects of an increase in w_t^m to 0.765, which decreases the value of γ^* and in figure 2.3, which displays an increase in k to 0.136 and therefore an increase in γ^*. The ambiguous effect of a change in the wage of women w_t^f is vizualized in figure 2.4 and 2.5. The first figure shows that with the earlier chosen parameters an increase in w_t^f leads to an increase in γ^*. Nonetheless if the parameters are adjusted (for example $\theta = 0.133, w_t^m = 0.165, w_t^f = 0.4407$ and $k = 0.166$) as in the case of figure 2.5, an increase in the wage of women can also lead to a lower value of γ^*. The calculation to verify these propositions is done in the appendix part A.1 at the end of this paper.

2.2.2 Types of Marriages and their Willingness to have Children

To simplify the model Gobbi assumes that there are only two different levels for the value of the agent's taste for children. A high value, $\overline{\gamma}$, for those individuals with a great desire to become a parent and, $\underline{\gamma}$, for those with a low willingness to have children. As mentioned in the introduction to this model Gobbi assumes that the individuals in her model are randomly matched. Because of the assumption that a couple's taste for children results out of the average willingness to have children of both parents it leads to the fact, that there are just three different possible values γ^j for a couple:

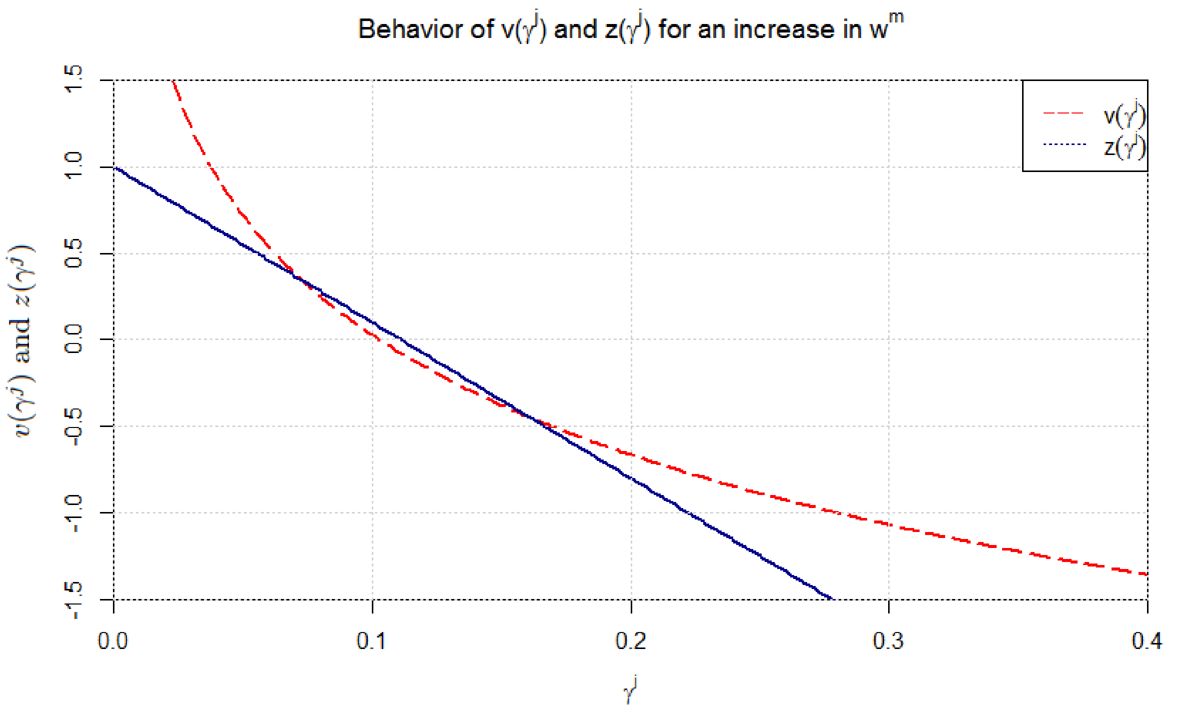

Figure 2.2: Own Calculation of $v(\gamma^j)$ and $z(\gamma^j)$ with RStudio

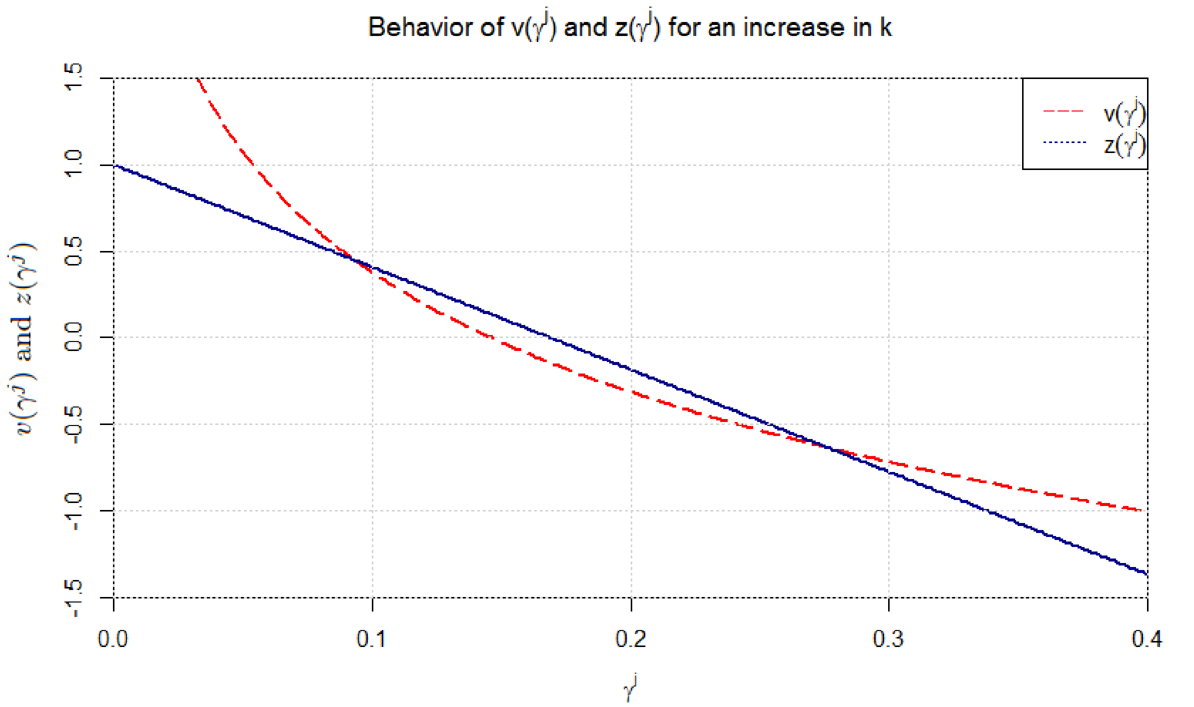

Figure 2.3: Own Calculation of $v(\gamma^j)$ and $z(\gamma^j)$ with RStudio

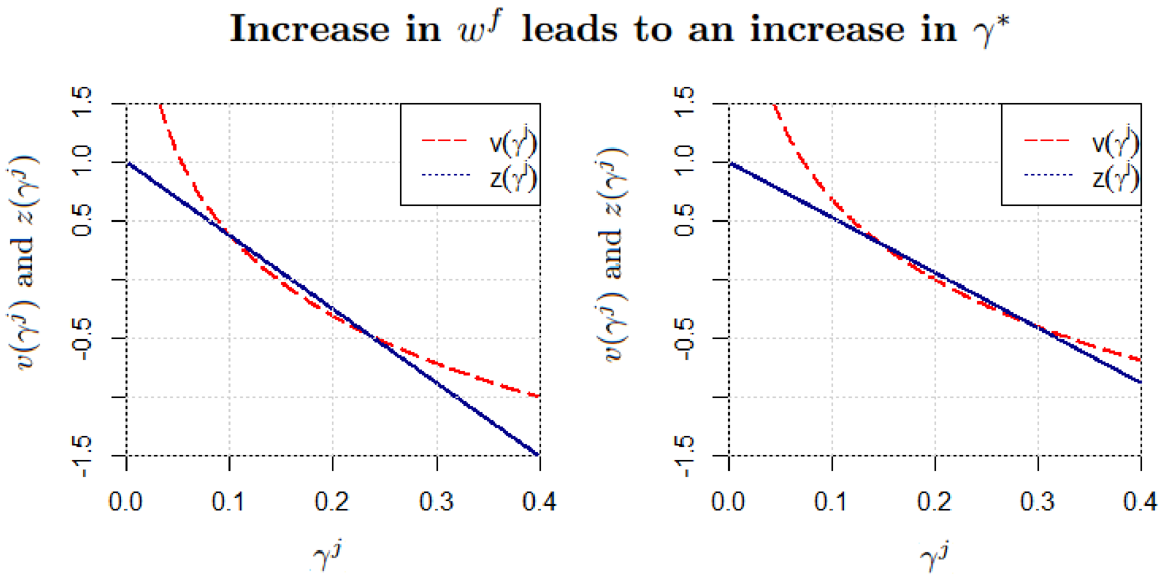

Figure 2.4: Left panel: initial position, right panel: effect of an increase in w_t^f
Own Calculation of $v(\gamma^j)$ and $z(\gamma^j)$ with RStudio

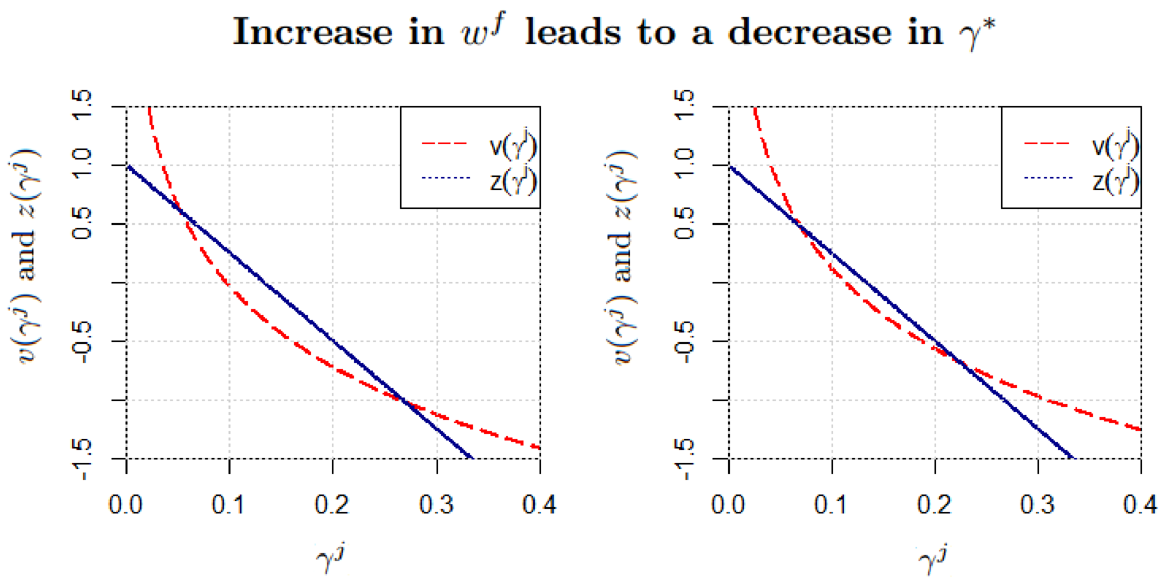

Figure 2.5: Left panel: initial position, right panel: effect of an increase in w_t^f
Own Calculation of $v(\gamma^j)$ and $z(\gamma^j)$ with RStudio

$$(\underline{\gamma},\underline{\gamma}) \rightarrow \gamma^1 = \underline{\gamma}$$
$$(\underline{\gamma},\overline{\gamma}) \rightarrow \gamma^2 = \frac{\overline{\gamma}+\underline{\gamma}}{2}$$
$$(\overline{\gamma},\overline{\gamma}) \rightarrow \gamma^3 = \overline{\gamma}$$

Furthermore Gobbi assumes, that $\underline{\gamma} = \gamma^1 < \gamma^*$, $\gamma^2 \geq \gamma^*$ and $\overline{\gamma} = \gamma^3 > \gamma^*$ holds. So couples with a desire to have children which is equal to γ^1 will remain childless because their value of γ is below the threshold level of γ^*. To derive the solution for the other two types of couples we use the optimal interior solution we obtained above (2.7). This leads for the number of children of the couples of type two and three to the following solutions, where $\overline{n}$ describes the solution for a couple with willingness γ^2 and $\overline{\overline{n}}$ for couples with taste γ^3.

$$\begin{aligned}\overline{n} &= \frac{w_t^m + w_t^f - k}{\theta w_t^f} - \frac{2}{\overline{\gamma}+\underline{\gamma}} \\ \overline{\overline{n}} &= \frac{w_t^m + w_t^f - k}{\theta w_t^f} - \frac{1}{\overline{\gamma}}\end{aligned} \tag{2.10}$$

In a final step Gobbi describes the share of each type of indiviuals in the whole population. Therefore the whole population at a time t is defined as P_t, containing both individuals with high and low willingness to have children. The part of the population with a great desire for kids is denoted as $\overline{P_t}$ and the portion with a low taste of children is denoted as $\underline{P_t}$. Because of the fact that couples are matched randomly the proportions of the three different types of couples are given by the probabilities that two individuals out of certain groups are matched. Therefore the shares of the three different mariages with the above discussed average taste for children γ^1,γ^2 and γ^3 in the population are given by:

$$\begin{aligned}&\left(\frac{\underline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2 \quad \text{for } \gamma = \gamma^1 \\ &\frac{2\underline{P_t}\,\overline{P_t}}{(\overline{P_t}+\underline{P_t})^2} \quad \text{for } \gamma = \gamma^2 \\ &\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2 \quad \text{for } \gamma = \gamma^3\end{aligned} \tag{2.11}$$

2.2.3 Production Function

The next step is to design the model's production sector. Therefore we consider a representative firm which produces the numeraire final good Y_t using the following production function.

$$F(L_t^m, L_t^f) = Y_t = (\alpha(L_t^m)^{-\rho} + (1-\alpha)(L_t^f)^{-\rho})^{-\frac{1}{\rho}} \tag{2.12}$$

L_t^f and L_t^m denote the amount of used labour time by women and men respectively. For the parameters α and ρ it holds that $\alpha \in (0,1)$ and $\rho \neq 0$ and $\rho \geq -1$. Because of the three different types of families defined in the previous subchapter the amount of women labour time can be split up in three parts. First the amount of women labour contributed by childless women denoted by L_t^{f1}. The next part is the workforce supplied by women out of couples of type two

L_t^{f2} who have as defined above $\overline{n}$ children. The last part of the women labour is denoted by L_t^{f3} and represents the amount of labour time used in production from women who belong to marriages of type three with a taste for children $\gamma = \gamma^3$ and $\overline{\overline{n}}$ children. Therefore the women's labour can be rewritten as:

$$L_t^f = L_t^{f1} + L_t^{f2} + L_t^{f3}$$

To receive equations for the wages of men and women we solve the representative firm's profit maximization problem given by

$$\max_{L_t^m, L_t^{f1}, L_t^{f2}, L_t^{f3}} \Pi_t = F(L_t^m, L_t^{f1}, L_t^{f2}, L_t^{f3}) - w_t^m L_t^m - w_t^f (L_t^{f1} + L_t^{f2} + L_t^{f3}) \tag{2.13}$$

By building the derivatives with respect to women's labout L_t^f and men's labour L_t^m and setting these derivatives equal to zero (marginal productivities of labour = marginal costs of labour), as shown in the appendix part A.1, the following equations for the two different wages are obtained.

$$\begin{aligned} \Rightarrow w_t^m &= \alpha\Big(\alpha + (1-\alpha)(\frac{L_t^f}{L_t^m})^{-\rho}\Big)^{-\frac{1+\rho}{\rho}} \\ w_t^f &= (1-\alpha)\Big(\alpha(\frac{L_t^m}{L_t^f})^{-\rho} + (1-\alpha)\Big)^{-\frac{1+\rho}{\rho}} \end{aligned} \tag{2.14}$$

These equations show that in the case where $\rho \neq -1$ the gender specific wage increases if the respectively other gender labour input increases. On the other hand the wage decreases if the amount of labour time supplied by the own gender increases.

$$\begin{aligned} \Rightarrow \rho \neq -1: \qquad & \frac{\partial w_t^m}{\partial L_t^m} < 0 \qquad & \frac{\partial w_t^m}{\partial L_t^f} > 0 \\ & \frac{\partial w_t^f}{\partial L_t^f} < 0 \qquad & \frac{\partial w_t^f}{\partial L_t^m} > 0 \end{aligned}$$

2.3 Dynamics

Until now we can summarise the framework of this model in the following way:
Given the size of the adult population $P_t = \overline{P_t} + \underline{P_t}$, which consists to equals parts of women P_t^f and men P_t^m, and their corresponding willingness to have children $\overline{\gamma}$ and $\underline{\gamma}$, an equilibrium composed of $\Big(c_t^j, n_t^j, \gamma_t, P_t^m, P_t^f, P_t, L_t^m, L_t^{f1}, L_t^{f2}, L_t^{f3}, Y_t, w_t^m, w_t^f\Big)$ has to meet several constraints. First this equilibrium vector has to satisfy the household constraints (2.2) and (2.3) and the value of the consumption c_t^j and the fertility n_t^j have to solve the households utility maximization problem (2.1). Second the labour inputs $L_t^m, L_t^{f1}, L_t^{f2}, L_t^{f3}$ and therefore the level of the output Y_t have to maximize the representative firm's profit (2.13) which leads to the equilibrium level of the wages of women and men, w_t^f and w_t^m. Furthermore the wages have to clear the labour market. With help of the share of the different kind of couples with respect to the whole population (2.11) the labour inputs of men and women can be written as:

$$L_t^m = \frac{P_t}{2} = \frac{\overline{P_t} + \underline{P_t}}{2}$$
$$L_t^{f1} = \frac{\underline{P_t}^2}{2(\underline{P_t} + \overline{P_t})}$$
$$L_t^{f2} = \frac{\overline{P_t}\underline{P_t}}{\underline{P_t} + \overline{P_t}}(1 - \theta\overline{n})$$
$$L_t^{f3} = \frac{\overline{P_t}^2}{2(\underline{P_t} + \overline{P_t})}(1 - \theta\overline{\overline{n}})$$

While the labour amount of men just equals half of the population, the female labour supply again can be obtained by looking at the other half ot the population, multiplied by the respective proportions of the different couples of the whole population and multiplied by the time this specific proportion has left after taking into account child rearing. For example the first type of female labour force L_t^{f1} results from the female half of the population, $\frac{\overline{P_t}+\underline{P_t}}{2}$, multiplied by the share of childless couples in the society, $\left(\frac{\underline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2$ and multiplied by one, because this share of women do not need any time for childrearing. The different amounts of labour supply $L_t^m, L_t^{f1}, L_t^{f2}, L_t^{f3}$ in an equilibrium have to also satisfy these constraints.

If we now take a look at the interior solution of the optimal fertility in the equilibrium n^* (2.7) we can analyse the effects of changes in the gender specific wage on the fertility. It is easy to see that an increase in the husband's wage increases the fertility. This is intuitive because in this model childrearing is done by the wife alone. Therefore an increase in man's wage gives the woman the possibility to reduce her time spent for working and increases her time at home without suffering a loss of consumption. While the effect of an increase in man's wage is clear the effect of an increase in the woman's wage while the man's wage stays constant is not clear. To study the effect of an increase in the wife's wage we take a look at the inner solution of the fertiltiy.

$$n_t^* = \frac{w_t^m + w_t^f - k}{\theta w_t^f} - \frac{1}{\gamma^j}$$

The derivative of n_t^* with respect to the woman's wage w_t^f yields:

$$\begin{aligned}\frac{\partial n_t^*}{\partial w_t^f} &= \frac{1\theta w_t^f - (w_t^m + w_t^f - k)\theta}{(\theta w_t^f)^2} \\ &= \frac{\theta w_t^f - \theta w_t^f + (k - w_t^m)\theta}{(\theta w_t^f)^2} \\ &= \frac{k - w_t^m}{\theta (w_t^f)^2}\end{aligned}$$

Therefore the increase of women's wage just leads to an increase in fertility if and only if the fix costs of having the first child are greater than the man's wage, so if $k > w_t^m$ holds. Otherwise if $k < w_t^m$ or even $k = 0$, which means that there are no fix costs for the first child, the relationship is directed in the opposite way and an increase in the women's wage leads to a decrease in fertility. In this case the income effect of an increase in wage, the additional amount of money which is available and could be used for children, is overcompensated by the substitution effect because the opportunity costs of staying at home and not earning money are higher. Because of that the woman would reduce her fertility to spend more time at the labour market. Gobbi also provides a graphic of this case in which one can see the relationship between the woman's wage and fertility. While the fertility stays constant at the corner solution $n_t^{max} = \frac{1}{\theta}$ for very low wages, it starts to decline at $w_t^f = \frac{\gamma^j(w_t^m - k)}{\theta}$. This 'breaking-point'-value for w_t^f can easily be obtain by setting the utility of the corner solution (c_t^{max}, n_t^{max}) equal to the utility of the interior solution (c_t^*, n_t^*)

$$\ln(w_t^m - k) + \gamma^j(\frac{1}{\theta}) \stackrel{!}{=} \ln(\frac{\theta w_t^f}{\gamma^j}) + \gamma^j(\frac{w_t^m + w_t^f - k}{\theta w_t^f} - \frac{1}{\gamma^j})$$

and checking for which w_t^f this equation is fulfilled. This value can be obtained trivially by

$$w_t^m - k \stackrel{!}{=} \frac{\theta w_t^f}{\gamma^j}$$

or

$$\frac{1}{\theta} \stackrel{!}{=} \frac{w_t^m + w_t^f - k}{\theta w_t^f}$$

which leads to $w_t^f = \frac{\gamma^j(w_t^m - k)}{\theta}$. Beyond this level of the women's wage fertility starts to decline until it reaches the point $w_t^f = \frac{\gamma^j}{\theta - \gamma^j}(w_m^t - k)$, which can be obtained in an analog way as above, where it drops to the second corner solution $n_t^0 = 0$. This abrupt decline is due to the fix costs of having at least one child which makes it not optimal to have just a small number of children.

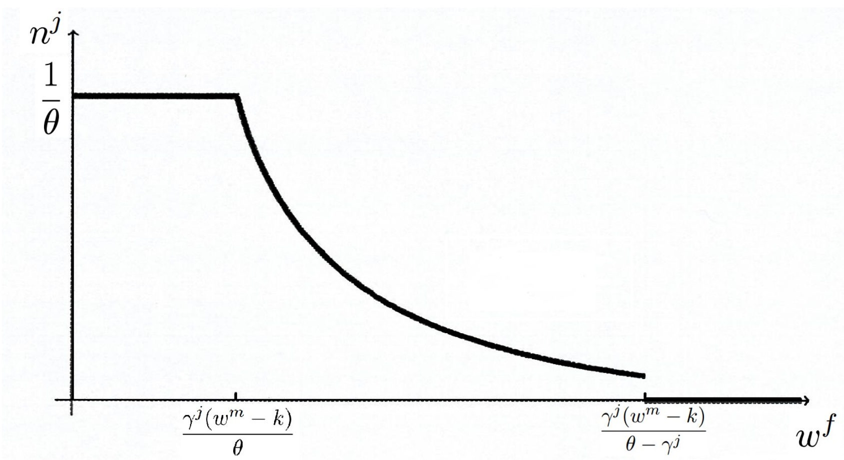

Figure 2.6: Gobbi (2012), page 973, We added the notation for the second jump discontinuity

Next Gobbi looks at the dynamics of the population, respectively the dynamics of the two different population groups $\overline{P_t}$ and $\underline{P_t}$. Therefore she assumes that $\rho = -1$, which means that female and male labour input are perfect substitutes. This assumption simplifies the equations for the wages (2.14) to

$$w_t^m = \alpha$$
$$w_t^f = 1 - \alpha$$

Furthermore she assumes that there are exogenous probabilities that a child has a certain willingness to have children oneself, depending on the desire to have children γ of the parents. So the probability that parents who both have a high willingness to have children $\overline{\gamma}$ have a child with a high taste for children is denoted by a. Of course the probability of such a couple to have a child with a low willingness to have children is given by $(1-a)$. In a similar way we denote the probabilty of a couple of type $(\overline{\gamma}, \underline{\gamma})$ with a medium taste for children to have a child with a high desire to have kids with b. If we now assume that $a > b$ the dynamics of the two different population groups are given by

$$\begin{aligned} \overline{P_{t+1}} &= a\overline{\overline{n}}\Big(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\Big)^2 \cdot (\overline{P_t}+\underline{P_t}) + b\overline{n}\frac{2\overline{P_t}\underline{P_t}}{(\overline{P_t}+\underline{P_t})^2} \cdot (\overline{P_t}+\underline{P_t}) \\ \underline{P_{t+1}} &= (1-a)\overline{\overline{n}}\Big(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\Big)^2 \cdot (\overline{P_t}+\underline{P_t}) + (1-b)\overline{n}\frac{2\overline{P_t}\underline{P_t}}{(\overline{P_t}+\underline{P_t})^2} \cdot (\overline{P_t}+\underline{P_t}) \end{aligned} \tag{2.15}$$

The right hand side in both equations is composed of two terms. The first term is the product of the randomly matched proportion of couples with a high desire to have children in the whole population, multiplied by the number of children they will have, $\overline{\overline{n}}$, and multiplied by the probability to have children with a certain desire to have children themselves, a respectively $(1-a)$. The second term is the product of the proportion of couples of type 2 with a mixed desire to have children, multiplied by the amount of children they will have, $\overline{n}$ and multiplied by the probability to have children with a high, respectively low taste for children, b respectively $(1-b)$. These two dynamics (2.15) can be combined to a single equation representing the ratio of agents with a high willingness to have children to those with a low taste for children, z_t. The detailed derivation is done in the appendix part A.2.

$$z_{t+1} = \frac{a\overline{\overline{n}}z_t + 2b\overline{n}}{(1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}} =: \phi(z_t) \qquad \text{with } z_t = \frac{\overline{P_t}}{\underline{P_t}} \tag{2.16}$$

Equation (2.16) is a difference equation with a lag of order one and for future discussions we define $z_{t+1} := \phi(z_t)$. With this definition of z_t the proportion of childless women, which is equal to the probability that two individuals with a low taste for children are matched randomly at the marriage market, can be rewritten as

$$\chi := \frac{1}{(1+z_t)^2} = \Big(\frac{\underline{P_t}}{\overline{P_t}+\underline{P_t}}\Big)^2$$

A more detailed derivation is done in part A.2 of the appendix at the end of this thesis. It can easily be shown, as done in the appendix A.2, that the average fertility of the society can be expressed as

$$n_t = \frac{z_t}{(1+z_t)^2}(z_t\overline{\overline{n}} + 2\overline{n})$$

Using the first order difference equation (2.16) the steady state value for z_t can be determined. A steady state is characterised by the fact that the ratio of individuals with high willingness to have children relative to the ones with low taste stays constant over time, or to formulate it analytically, that $z_t = z^* \; \forall t$. Therefore we set $z_t \overset{!}{=} z^*$ and $z_{t+1} \overset{!}{=} z^*$ in (2.16) and transform it,as it is shown in the appendix part A.2, to obtain the following solution for the steady state value z^*.

$$z^* = \frac{-\left((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\right) + \sqrt{((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}})^2 + 2(1-a)\overline{n}b\overline{\overline{n}}}}{(1-a)\overline{\overline{n}}} \tag{2.17}$$

Furthermore it is easy to show that the function $\phi(z_t) = z_{t+1}$ from (2.16) fulfills $\phi'(z_t) > 0$ and $\phi''(z_t) < 0$. Additionally it holds that $\lim_{z_t \to +\infty} \phi(z_t) = \frac{a}{1-a} > \frac{b}{1-b} = \phi(0)$ because of the assumption made above that $a > b$. All these characteristics are shown in part A.2 of the appendix. These characteristics cause that $z_t < z_{t+1} \; \forall t$ and $z(0) < z^*$ in particular. Therefore regardless if the initial value for the ration $z(0)$ is smaller or greater than the steady state value z^* it converges to the steady state level estimated above which means that the equilibrium is globally stable for all starting values $z(0)$. This means that depending on the initial value, the ratio of individuals with a high willingness to have children constantly increases respectively decreases, until it reaches the steady state level. This behaviour is shown well in Figure 2.7, by setting the values of the variables equal to $a = 0.670, b = 0.536$, $\overline{n} = 0.817$ and $\overline{\overline{n}} = 1,60949$. Again these values correspond with the ones Gobbi estimates in her paper for the United States. Furthermore it can be seen that the steady state value z^* is always positive and smaller than infinity, which implies that none of the two different population groups will disappear over time.

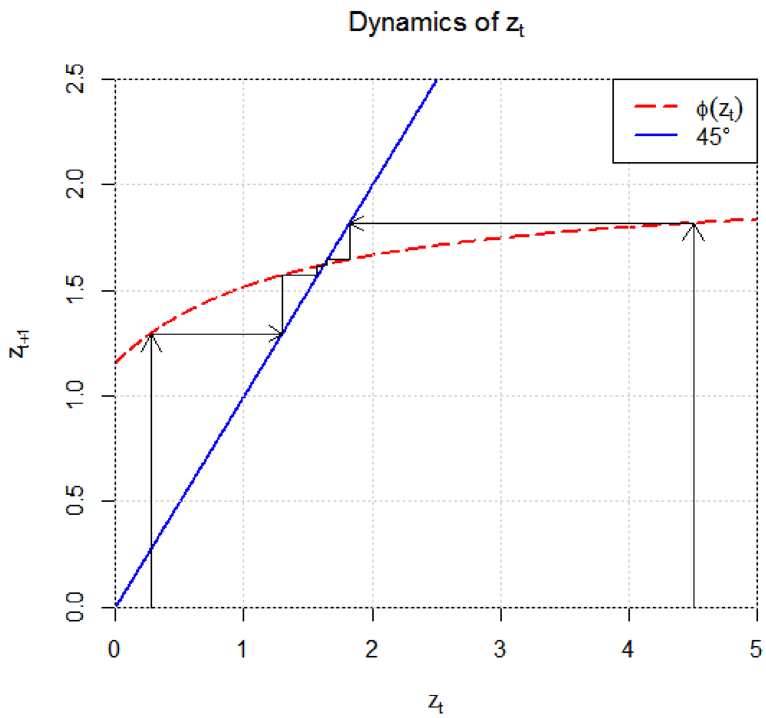

Figure 2.7: Dynamics of z_t and the steady state value z^*, own computation in RStudio

The reason for this dynamic can be explained if we consider the case in which the starting value is very low at the start and therefore $z_0 < z*$. If we take a look at the fractions of children coming from the two different groups of couples with high respectively low taste for children:

$$\frac{2\overline{n}}{\overline{\overline{n}}z_t + 2\overline{n}} \qquad \text{children from couples with a mixed willingness to have children } (\overline{\gamma}, \underline{\gamma})$$

$$\frac{\overline{\overline{n}}z_t}{\overline{\overline{n}}z_t + 2\overline{n}} \qquad \text{children from couples with a high willingness to have children } (\overline{\gamma}, \overline{\gamma})$$

For very low values of z_t the number of children of couples with a different willingness to have children is higher than the respectively one of parents with a high taste for children. Over time z_t improves as shown above and because of the fact that

$$\overline{n} = \frac{w_t^m + w_t^f - k}{\theta w_t^f} - \frac{2}{\overline{\gamma} + \underline{\gamma}} \qquad < \qquad \frac{w_t^m + w_t^f - k}{\theta w_t^f} - \frac{1}{\overline{\gamma}} = \overline{\overline{n}}$$

and that for the possibilities of having children of a certain kind of willingness of having children on its own $a > b$ holds, the share of children from parents with a high taste for kids in the overall fertility increases. Therefore also z_t increases even further until it reaches the steady state level z^* and stays there.

For better understanding of the characteristics of the steady state value z^* I numerically calculated various bifurcation diagrams for the different variables that influence the level of z^*. Figure 2.8 shows the effects of changes in $\overline{n}$ on z^*. The influence is quite intuitive, because for higher values of $\overline{n}$ the number of children from couples with a mixed desire for children are higher. These children have a lower chance to have a high desire for children themselves, compared to children from parents who both have a high desire for children $\overline{\overline{n}}$. Therefore there will be more adults with a low willingness to have children in the next period and therefore $\underline{P_t}$ increases and as a consequence z^* decreases.
The bifurcation analysis of the effects of changes in $\overline{\overline{n}}$ displays a similar picture and is shown in figure 2.9. It follows the same dynamics as the ones observed for changes in $\overline{n}$, just in the opposite direction. Therefore a rise in $\overline{\overline{n}}$ increases the number of children with a high desire to have children themselves and therefore raises $\overline{P_t}$ and the steady state value z^*.
Also the bifurcation analysis for the probabilities a and b, figure 2.10 and figure 2.11, display an expected result. An increase in a, respectively b, implies that it becomes more likely that couples have children with a high desire for children. This means that the proportion of adults with a high willingness to have children increases in the next period, which implies that $\overline{P_t}$ increases, which leads to an increase of the steady sate value of the proportion of people with a high taste for children relative to the ones with a low taste, z^*.

Another interesting dynamic of z^* comes from a change of the weight of women in the production function of the representative firm $(1-\alpha)$. If α decreases then $(1-\alpha)$ increases which means that labour input of women carries more weight on the level of the output Y_t. With the help of equation (2.14) it is easy to see that an increase in $(1 - \alpha)$ leads to a rise of the wage women receive w_t^f, while the men's wage w_t^m decreases. As a result both $\overline{n}$ and $\overline{\overline{n}}$ decrease by the same amount, but the number of children from couples with a mixed willingness to have children $\overline{n}$ is more effected by these changes, relative to the fewer number of children they already had before, compared to couples of type 3. Thereby, the fertility of couples who are more likely to have children with a low taste for children themselves is the most affected. This implies, that in

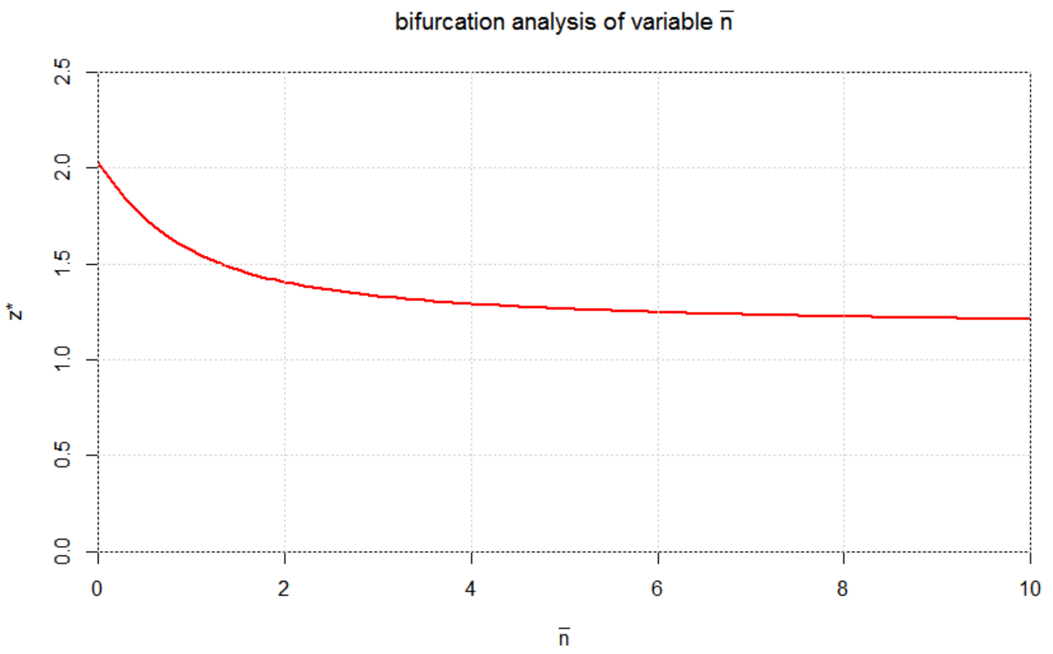

Figure 2.8: Own Calculation with RStudio

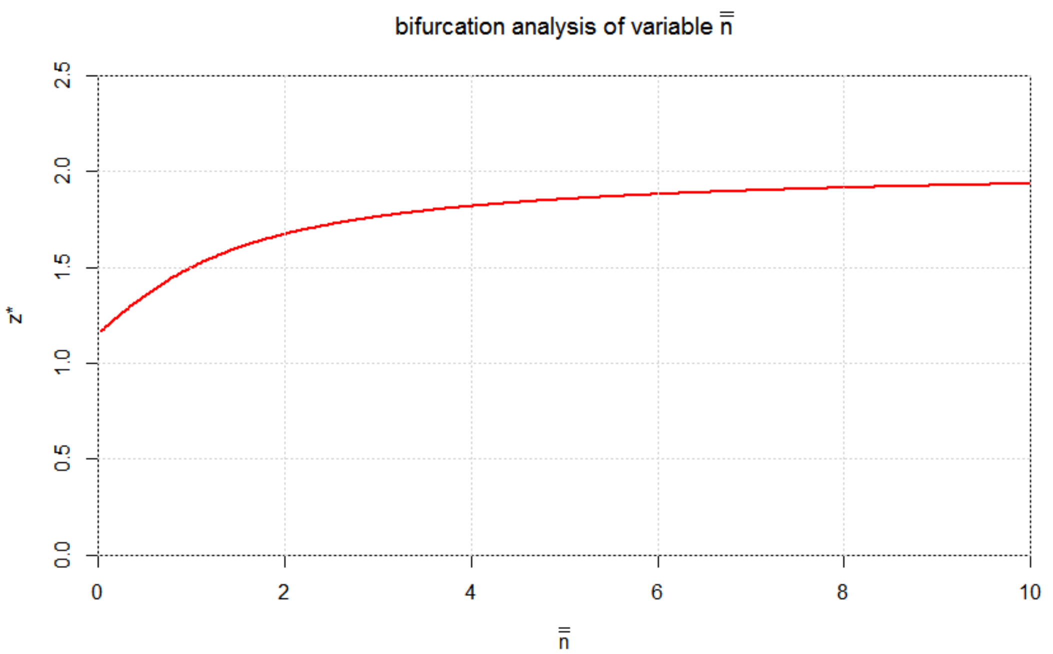

Figure 2.9: Own Calculation with RStudio

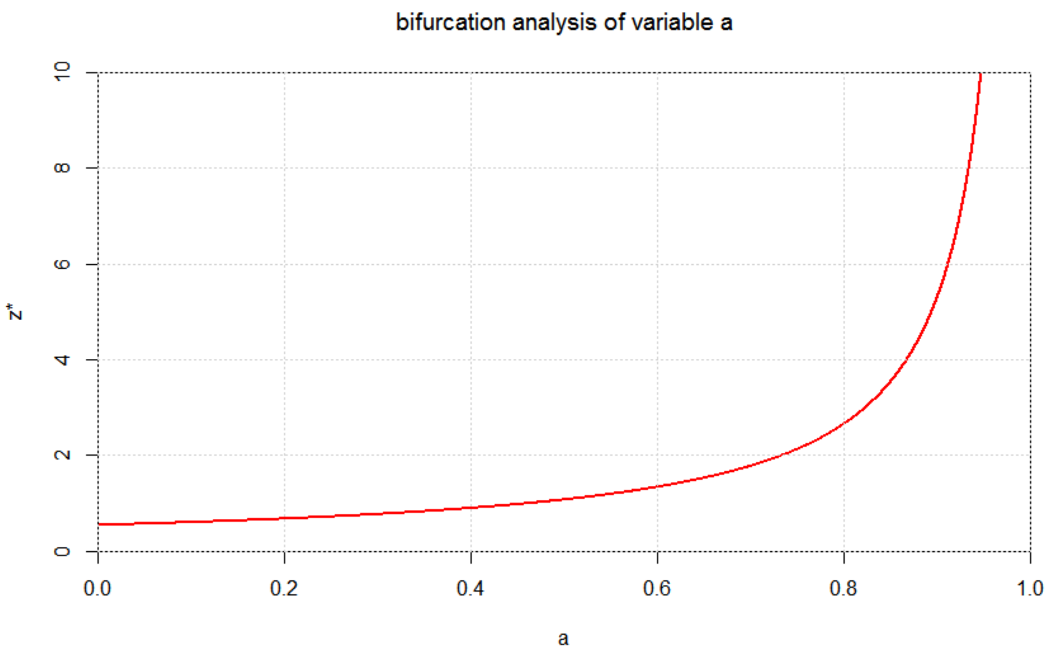

Figure 2.10: Own Calculation with RStudio

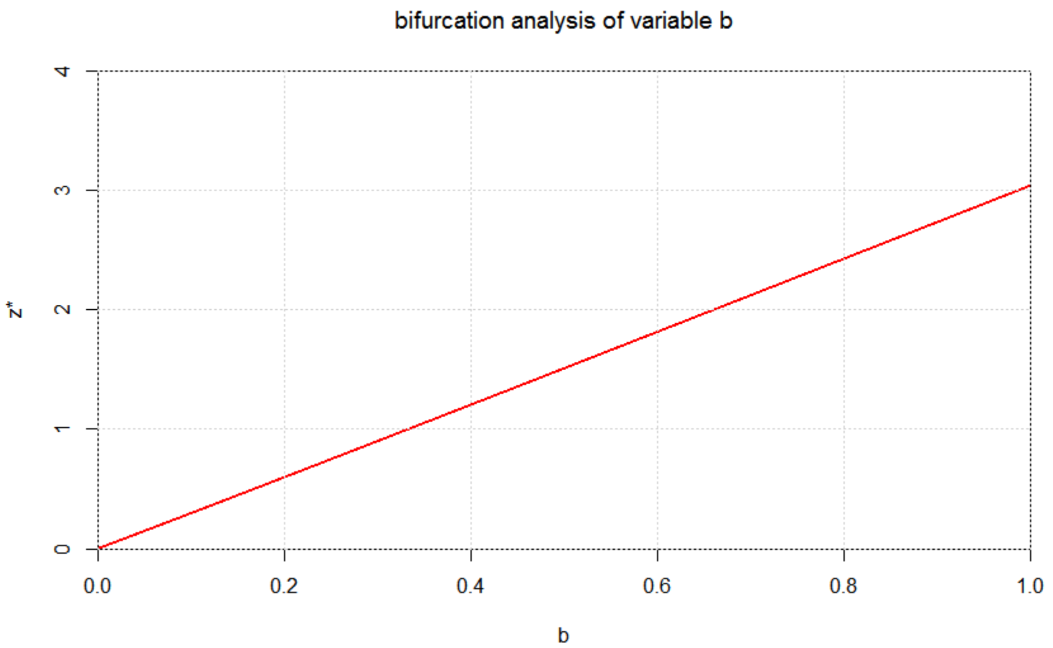

Figure 2.11: Own Calculation with RStudio

the next period, when the children of this affected generation have been grown up, the number of adults with a low taste for children has decreased relatively to the ones with a high desire to have children. Due to the random matching process on the marriage market these adults with a low taste for children are the most likely to end up childless, which means that a decrease in the ratio of adults with a low taste for children to the ones with a high taste for children also decreases childlessness. Therefore, these composition leads to an increase of the steady state value z^*.

However it can be shown that this increase of z^* decreases the steady state values of the average fertility $n^* = n_t(z^*)$ and the share of childless women $\chi^* = \chi_t(z^*)$. The decrease of n^* can be explained by the decrease of both $\overline{n}$ and $\overline{\overline{n}}$. Whereas the decrease in χ^* might not seem intuitive at first sight but gets clearer if one considers the particular impact on $\overline{n}$ and $\overline{\overline{n}}$. Because the reduction in fertility for couples with a mixed taste for children $\overline{n}$ is relatively higher than for couples with a high willingness to have children $\overline{\overline{n}}$ and the fact that these couples are less likely to have children with a high taste for children themselves implies, that in the next period the proportion of people with a high willingness to have children will increase and the share of childless couples χ^* will decrease. This implies that in this model there can exist a positive relationship between childlessness, χ^*, and fertility, n^*. So the overall effect of a decreasing α can be split up into a direct effect on the fertility due to the increase in female wages and into an indirect effect that causes a change in the population structures in the next period.

A similar effect on fertility and childlessness can be observed when the fixed costs of raising the first child k increases. This increase has a direct negative effect on the fertility of mothers of both types of couples which do not want to stay childless and decreases both, $\overline{n}$ and $\overline{\overline{n}}$, by the same amount. But again the fertility of couples with a mixed taste for children $\overline{n}$ is affected more relatively to the lower number of children these couples have compared to the couples with a high taste for children. Thereby, with the same argumentation as in the case of a decrease in α, the childlessness rate and the fertility rate will decrease.

2.4 Summary

With this paper Gobbi presents a fertility model in which it can be optimal for individuals to stay childless. She does this by implementing agents with different tastes for children which means that they do gain a different amount of utility by becoming parents. Therefore, there exists a unique level for the variable γ^j which defines the scope of staying childless and becoming parents for the randomly matched couples. Another factor that can cause childlessness in this model is the level of women's wage because it raises the opportunity costs for women to raise children. Furthermore, Gobbi shows that the correlation between childlessness and fertility can be positive in her model. This interesting fact can be observed by the two effects that occur if the weight of women in the production function is increased or if the fix costs for child rearing rises. First both shocks increase the opportunity costs of having children for mothers which decreases the fertility. The second effect is that couples with a mixed taste for children are relatively affected stronger by these shocks than couples with a high taste for children. These couples are the ones who are more likely to have children with a low taste for children themselves, who are therefore most likely to stay childless in the next period. Thereby, a relatively stronger decline in fertility rates for couples with a mixed taste for children will drop the future childlessness rate. In her paper Gobbi also tries to simulate the historic fertility development in the US for different cohorts. Her model is able to reproduce the evolution of fertility and childlessness for these cohorts and also to model the development under the above discussed shocks.

CHAPTER
THREE

Gender Inequality, Endogenous Cultural Norms, and Economic Development by Victor Hiller

3.1 Introduction

Gender Inequality, Endogenous Cultural Norms, and Economic Development is a paper by Victor Hiller that investigates the economic effects of gender inequality due to cultural norms. Therefore he assumes an interdependency between the cultural norms and the economic situation of the society, by modelling the amount of education girls and boys receive, depending on the joint income of husband and wife, which again depends on the participation of the wife in the working life and the wage gap between men and women. Thus this paper provides a framework in which the dynamics of the cultural norm is endogenous and drives the magnitude of the gender inequality and the economic development.

3.2 The Model

The framework of this model is a two-sex overlapping generation model. Parents always have two children, a daughter and a son, for which they can decide separately how much education they provide for them. The amount of education of the daughter is denominated as e^f_{t+1}, while the son's education is denoted as e^m_{t+1}. The decision of the parents about the education of their children is influenced by the cultural norm $\theta_t \in [0, 1]$, which is modelled as endogenous variable. The higher this cultural norm θ_t is, the higher is the standing of women in the society and the lower is gender inequality, respectively a lower wage gap and a higher participation of women in the labour force prevail. When parents decide about the education of their children they take θ_t as given, not taking into account that their decision will influence the future development of the norm. With their decision parents try to maximize their own utility function, which is given by

$$U_t = \mu \ln(C_t) + (1 - \mu) \ln(D_t) + \xi \Big(\theta_t \ln(h^f_t) + \ln(h^m_t)\Big) + \beta \Big(\theta_t \ln(h^f_{t+1}) + \ln(h^m_{t+1})\Big) \quad (3.1)$$

The variable C_t describes the joint consumption of market goods of the parents, D_t their joint consumption of household goods. The parameter $\mu \in (0, 1)$ determines the relative preference of market goods over household goods. The function h^j_t, with $j \in \{f; m\}$, denominates the human capital production function of the current generation and h^j_{t+1} the human capital production function of the next generation, respectively the children. The future utility of the children is taken into account through considering the human capital of children in the utility function of parents. The human capital of the current generation and their children is weighted by the parameters ξ and β. The human capital production function is given by

$$h_t^j = h_t^j(e_t^j) = \Big(c + ae_t^j\Big)^{\alpha} \tag{3.2}$$

This equation shows that also if parents do not invest in the education of their children, they will always have a basic level of education $c > 0$. The parameter $a > 0$ stands for the productivity of expenditures for education in the human capital production function. The elasticity of human capital with regard to the education is given by $\alpha \in (0, 1)$. The amount of market produced consumption goods the parents can afford is given by the following budget constraint

$$(1 - l_t^f)w_t^f + (1 - l_t^m)w_t^m = C_t + \tau(e_{t+1}^f + e_{t+1}^m) \tag{3.3}$$

Here the right-hand side denotes the expenditures, where $\tau > 0$ indicates the costs per unit of education. The income of the parents is given by their individual income w_t^j, which they earn if they spend part of their total time of one unit producing market goods. Because individuals can also spend time at home producing household goods, l_t^j, the wage of each parent is multiplied by the term $(1 - l_t^j)$, the time they actually spend at the labour market. The level of the wage is influenced by the amount of education an individual received during his/her childhood. However it is assumed that uneducated men earn more than uneducated women.

$$\begin{aligned} &w_t^m = h(e_t^m) \\ &w_f^f = h(e_t^f) - \delta(e_t^f)s \qquad \text{with} \qquad \delta(e_t^f) = \begin{cases} 1 & \text{if } e_t^f = 0 \\ 0 & \text{if } e_t^f > 0 \end{cases} \end{aligned} \tag{3.4}$$

The indicator function $\delta(e_t^f)$ takes the above discussed assumption into account that men are physically stronger than women and therefore are more productive even though they are uneducated, compared to non-educated women. The parameter s measures this difference in productivity between uneducated women and uneducated men. Thereby this assumption considers the assumption that in a low developed economy jobs in most instances require physical strength and not skill and therefore a gender productivity gap exists. For later derivations Hiller also defines the total possible income of a couple if both spend their whole given time at the labour market by

$$y_t = w_t^f + w_t^m = h(e_t^m) + h(e_t^f) - \delta(e_t^f)s \tag{3.5}$$

As mentioned above individuals can also spend their time at home producing household goods D_t, whose consumption contributes to their utility function.

$$D_t = (l_t^f)^{\gamma} + (l_t^m)^{\gamma} \tag{3.6}$$

$\gamma \in (0, 1)$ is an index of the elasticity of domestic good production with regard to time spent at the household to produce them l_t^j. The dynamic of the social norm is given by

$$\theta_t = \sigma\theta_{t-1} + (1 - \sigma)\Big(\frac{l_t^m}{l_t^f}\Big)^{\kappa} \tag{3.7}$$

The parameter $\kappa \in (0,1)$ describes the elasticity of the change in the norm from one period to the next, $\theta_{t+1} - \theta_t$, with regard to relative labour supply. The paramter $\sigma \in (0,1)$ describes the level of influence of the current social norm on its future development. As can be seen the evolution of the norm is driven by the gender inequality in the labour market, respectively by the difference in the education which the two genders receive. For example if the term $\frac{l_t^m}{l_t^f}$ increases it means, that either men spend more time at housework or women less. This increase induces an increase in the social norm θ_t, which means that the social opinion, that men should earn most of the money and women should stay at home, gets less popular. Thereby parents should be motivated to increase the amount of education they provide for their daughters, which allows them to earn more wage during their adulthood and thus close the gender wage gap. But also the existing social norm affects the development. This can be explained by the assumption, that children are influenced by their parents social views. Therefore they adopt their parents' social norm θ_{t-1}, but adapt it to the recent development of the society, respectively to the development of the amount of education provided to girls.
Under these constraints the parents aim to maximize their utility, given by equation (3.1), by choosing the amount of market goods C_t and household goods D_t they want to consume, the amount of time they both work at the household, respectively in the labour market $(1 - l_t^f)$ and $(1 - l_t^m)$ and the amount of education they provide for their children e_{t+1}^f and e_{t+1}^m. Altogether this leads to the following maximization problem and the corresponding constraints:

$$\max_{C_t, D_t, l_t^f, l_t^m, e_{t+1}^f, e_{t+1}^m} \mu \ln(C_t) + (1-\mu)\ln(D_t) + \xi\Big(\theta_t \ln(h_t^f) + \ln(h_t^m)\Big) + \beta\Big(\theta_t \ln(h_{t+1}^f) + \ln(h_{t+1}^m)\Big)$$

$$\begin{aligned} \text{subject to:} \quad & (1 - l_t^f)w_t^f + (1 - l_t^m)w_t^m = C_t + \tau(e_{t+1}^f + e_{t+1}^m) \\ & D_t = (l_t^f)^\gamma + (l_t^m)^\gamma \end{aligned} \tag{3.8}$$

As shown in the appendix part B.1, the maximization of this problem with respect to the amount of time both partners work at home l_t^f and l_t^m yields for an interior solution

$$\begin{aligned} & \frac{w_t^f \mu}{C_t \gamma (1-\mu)} = \frac{(l_t^f)^{\gamma-1}}{D_t} \\ & \Rightarrow \frac{(l_t^m)^{\gamma-1}}{(l_t^f)^{\gamma-1}} = \Big(\frac{l_t^m}{l_t^f}\Big)^{\gamma-1} = \frac{w_t^m}{w_t^f} \qquad \text{or} \qquad \Big(\frac{l_t^m}{l_t^f}\Big)^{1-\gamma} = \frac{w_t^f}{w_t^m} \end{aligned} \tag{3.9}$$

The equality sign holds because it is assumed that for an interior solution both partners work at home, respectively in the market, which means that $l_t^j \in (0,1)$. The second equation in (3.9) describes the optimal share of housework between women and men. On the right-hand side we see the relative opportunity costs of working at home and not at the market, which equals the ratio of the wages, while on the left-hand side shows the relative productivity of the time spent at home. The optimization of (3.8) with respect to the amount of education the children receive e_{t+1}^m and e_{t+1}^f, which is also calculated in the appendix part B.1, leads to

$$\Rightarrow \frac{h'(e_{t+1}^m)}{h(e_{t+1}^m)} = \frac{\tau\mu}{C_t\beta} \qquad \text{and} \qquad \frac{h'(e_{t+1}^f)}{h(e_{t+1}^f)} = \frac{\tau\mu}{C_t\beta\theta_t} \tag{3.10}$$

Again it is assumed that an interior solution exists and both values e^f_{t+1} and e^m_{t+1} are strictly positive and thereby the equality in both equations holds. These two first-order conditions combined with the budget constraint (3.3) and the definition of the household production function (3.6) can be used to obtain the optimal values of the education the daughter and the son receive. The exact calculations consist of quite a lot of rather easy transformations, which are not performed completely in part B.1 of the appendix, rather I chose to provide a guideline of the separate steps and just performed some of the transformations and the most important intermediate results. So, as can be seen in the appendix part B.1, after several transformations one obtains the following values for the optimal choices of education for daughters and sons in this model.

$$e^f_{t+1} = \begin{cases} 0 & \text{if } y_t < \tilde{y}_t(\theta_t) \\ \frac{a\alpha\beta\theta_t y_t - \tau c\Big(\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t)\Big)}{a\tau\Big(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)\Big)} & \text{if } y_t \geq \tilde{y}_t(\theta_t) \end{cases}$$

$$e^m_{t+1} = \begin{cases} 0 & \text{if } y_t < \tilde{y}_t(1) \\ \frac{a\alpha\beta y_t - \tau c\Big(\mu+\gamma(1-\mu)\Big)}{a\tau\Big(\mu+\gamma(1-\mu)+\alpha\beta\Big)} & \text{if } y_t \in [\tilde{y}_t(1), \tilde{y}_t(\theta_t)] \\ \frac{a\alpha\beta y_t - \tau c\Big(\mu+\gamma(1-\mu)-\alpha\beta(1-\theta_t)\Big)}{a\tau\Big(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)\Big)} & \text{if } y_t \geq \tilde{y}_t(\theta_t) \end{cases} \tag{3.11}$$

where $\tilde{y}_t(\theta_t)$ is given by

$$\tilde{y}_t(\theta_t) = \frac{\tau c}{a}\Big(\frac{\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t)}{\alpha\beta\theta_t}\Big) \tag{3.12}$$

As one can see there are three different types of regimes the society can be in, depending on the amount of education parents provide to their children, respectively on the income of the parents y_t. Due to the existing basic level of human capital that children have without any education (equation (3.2)) parents can choose not to provide any education to one or both of their children. Because of the assumption that women have a physical disadvantage compared to men parents will not educate just their daughters and leave their sons uneducated. Therefore we receive three different types of regimes for the society, depending on the income of the parents. The first case in which the income is too low and the amount of education the children receive would be negative, the parents decide not to educate any of their children. In the paper this case is called the poverty regime which occurs if $y_t < \tilde{y}(1)$. In the second case, the inequality regime the income of the parents is a little bit higher, $y_t \in [\tilde{y}_t(1), \tilde{y}_t(\theta_t)]$. In this regime only the sons receive education, because of the above mentioned physical advantage it is assumed they have over their sisters. As can be seen in the definition of $\tilde{y}_t(\theta_t)$, the upper boundary for this regime depends on the social norm θ_t. The higher θ_t, the smaller is the gender inequality gap and thereby the lower is the upper limit for the income y_t for the inequality regime. The final case describes the interior solution of the optimization problem, in which the income of the parents is high enough ($y_t \geq \tilde{y}_t(\theta_t)$) so that both children receive education. As explained in the former case of the inequality regime, the lower boundary in the interior regime depends on the social standing of women θ_t in a negative relationship. These three different regimes with respect to the education can be seen in figure 3.1. In the next section I will discuss the results of these conclusions for the dynamics of the endogenous variables, the income y_t and the social norm θ_t, and their steady state solutions.

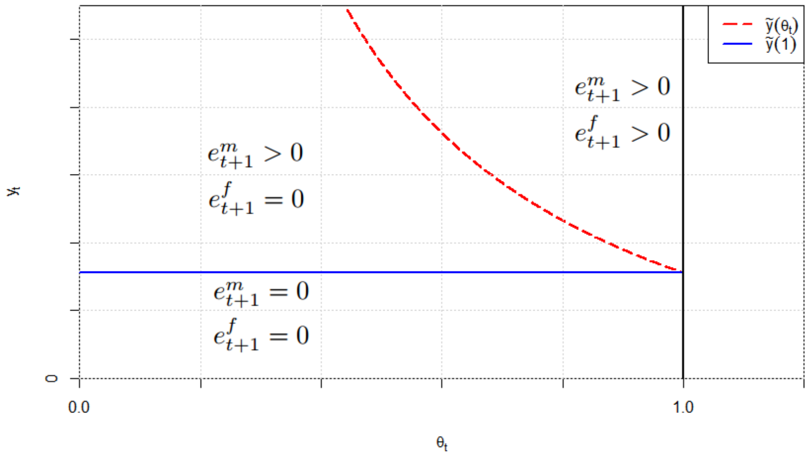

Figure 3.1: Three regimes with respect of education, own calculations with: $\tau = 3, \mu = 0.5, \gamma = 0.2, \beta = 2, \alpha = 0.3, c = 2, a = 5.8$

3.3 Dynamics of the Model

First I start with the dynamics of the total possible income y_t. Due to the definition of y_t given by equation (3.5) and the optimal values of the amount of education the children receive, which I derived in the last section, the optimal dynamic of the total possible income is given as

$$y_{t+1} = \begin{cases} 2c^\alpha - s & \text{if } y_t < \tilde{y}_t(1) \\ c^\alpha - s + \chi(0)(ay_t + tc)^\alpha & \text{if } y_t \in [\tilde{y}_t(1), \tilde{y}_t(\theta_t)] \\ \chi(\theta_t)(1 + \theta_t^\alpha)(ay_t + 2\tau c)^\alpha & \text{if } y_t \geq \tilde{y}_t(\theta_t) \end{cases} \tag{3.13}$$

For reasons of readability the term $\chi(\theta_t)$ is defined as

$$\chi(\theta_t) = \left[\frac{\alpha\beta}{\tau\Big(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t)\Big)}\right]^\alpha$$

The calculation to receive these equations can be found in the appendix B.2. In the dynamics of y_t one can again see the differences between the three possible regimes the society can be in. The first solution represents the poverty regime in which the parents do not spend any money for the education of their children. Therefore the total possible income stays constant over time. In the second case y_t experiences an increase over time, which however is independent of the social norm θ_t. This characteristic of the dynamic results because in the gender inequality regime just the boys receive education. In the last case, the interior regime, the education of boys and girls stimulates the growth of the income even more and because of the fact that girls receive education the term depends on the social norm θ_t.

Next I derive the dynamics of the social norm θ_t for the case that the optimal values for the education are chosen. Therefore we use the dynamics of the social norm (3.7) and the definition of the variables it contains. The exact derivation is provided in part B.2 of the appendix. After some transformations the optimal dynamic of the social norm θ_t is given by

$$\theta_{t+1} = \begin{cases} \sigma\theta_t + (1-\sigma)\left(\frac{c^\alpha - s}{c^\alpha}\right)^{\frac{\kappa}{(1-\gamma)}} & \text{if } y_t < \tilde{y}_t(1) \\ \sigma\theta_t + (1-\sigma)\left(\frac{c^\alpha - s}{\chi(0)(ay_t+\tau c)^\alpha}\right)^{\frac{\kappa}{(1-\gamma)}} & \text{if } y_t \in [\tilde{y}_t(1), \tilde{y}_t(\theta_t)] \\ \sigma\theta_t + (1-\sigma)\theta_t^{\frac{\alpha\kappa}{(1-\gamma)}} & \text{if } y_t \geq \tilde{y}_t(\theta_t) \end{cases} \tag{3.14}$$

Again the existence of three different regimes lead to the three different solutions obtained, depending again on the level of the total possible income y_t. Key to the development of the social norm is the female labour supply, which is given by the second term in all of the three cases. Again only in the third case, the interior regime, the second term depends on the current level of the social norm. This is due to the fact that girls as well as boys are educated, which again influences the wage gap and therefore the development of the social norm. In the first two cases, girls do not receive any education at all, thereby their weight on the female labour supply is the constant term $c^\alpha - s$. In the poverty regime, also men are not educated and therefore the whole female labour supply is given by a constant term as you can see in the first case. In the inequality regime just the boys receive education and therefore the development of the social norm is affected by the development of the income, which again is driven by the amount of education the boys receive. In fact the income is increasing and partly reinvested in the education of the children, however just in the education of men. To secure the convergence towards a steady state solution in all three cases, the author of the paper assumes that

$$\alpha\kappa < 1 - \gamma$$

This assumption is needed for the case of the interior regime. In the poverty regime and the gender inequality regime it can easily be seen that the social norm θ_t converges towards a steady state solution.
Now that one knows the dynamics of the total possible income y_t and the social norm θ_t we can derive the steady state solution of the whole model, because all other endogenous variables can be expressed as functions of y_t and θ_t. Therefore also the path, alongside which the economy is developing, is given by the path of the income and the social norm $\{y_t, \theta_t\}_{t=0}^{\infty}$. To solve for the steady state solutions one has to derive the isoclines of y_t and θ_t, alongside which the two variables stay constant. Hiller calls them in his paper the yy locus and the $\theta\theta$ locus. The loci are defined by

$$yy = \{(y_t, \theta_t) \ : \ y_{t+1} = y_t\}$$
$$\theta\theta = \{(y_t, \theta_t) \ : \ \theta_{t+1} = \theta_t\}$$

The exact derivations of the two loci are given in the appendix part B.2. There it can be seen that the value of the productivity of educational expenditure a is crucial for the shape of the yy locus. We receive two boundaries that divide the different cases

$$\bar{a} = \frac{\tau c(\mu + \gamma(1-\mu))}{2\alpha\beta c^{\alpha}}$$

$$\tilde{a} = \frac{\tau c(\mu + \gamma(1-\mu))}{\alpha\beta(2c^{\alpha} - s)}$$

with $\bar{a} < \tilde{a}$. This leads to three different cases: If $a < \bar{a}$ is fulfilled, the yy locus exists just in the poverty regime. The condition entails that $y_t < \tilde{y}(1)\ \forall t$, which means that the yy locus is given by an horizontal line at $y_t = 2c^{\alpha} - s$ in the (θ_t, y_t) space. If the variable for the productivity of educational expenditure satisfies $a \in [\bar{a}, \tilde{a})$ the yy locus in the (θ_t, y_t) space consits of a horizontal line at $y_t = 2c^{\alpha} - s$ which belongs to the poverty regime and a function $y_i^{yy}(\theta_t)$, exactly defined in the appendix B.2, which belongs to the interior regime. As shown in part B.2 of the appendix, this function $y_i^{yy}(\theta_t)$ is a concave function of θ, which means that for higher values of θ_t the stationary value of total possible income $y_i^{yy}(\theta_t)$ is higher. This can be explained by the fact that a higher value of the social norm θ_t decreases the education gap between boys and girls and thereby increases the total possible income, because the marginal productivity of human capital of women is relatively higher than the marginal productivity of men, as it can be seen in (3.10). This scenario can be seen in figure 3.2. In the last case $a \geq \tilde{a}$ the yy locus consists of a horizontal line at the value y_{gi}^{yy}, which is defined in part B.2 of the appendix, and again of the function $y_i^{yy}(\theta_t)$. All pairs of (θ_t, y_t) which belong to y_{gi}^{yy} are stationary states for the total possible income y_t that belong to the gender inequality regime. Like the yy locus that belongs to the poverty regime, y_{gi}^{yy} is a horizontal line in the (θ_t, y_t) space, because they are independent of the social norm θ_t. This again reflects the fact that in both regimes women do not receive any education and therefore the total possible income y_t develops without any impacts of θ_t. This last case is plotted in figure 3.3.

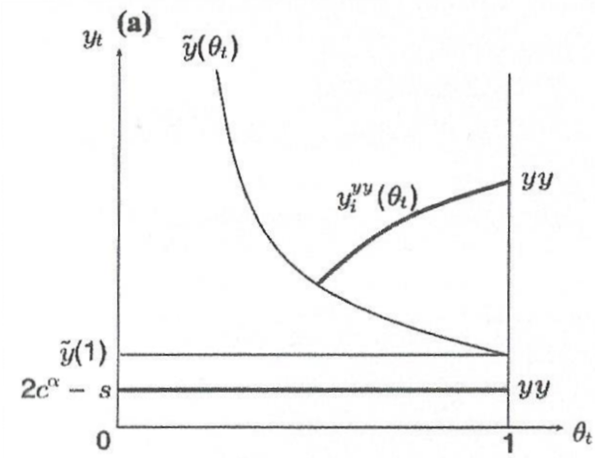

Figure 3.2: yy locus in the case of $a \in [\bar{a}, \tilde{a})$
(Hiller (2014), page 467, *Fig. 4 The yy locus*)

The $\theta\theta$ locus in the (θ_t, y_t) space, visualized in figure 3.4, can also be split up in three parts, every part belonging to one of the described regimes. In the poverty regime, the dynamic of θ_t is independent of y_t, because of the fact, that neither daughters nor sons receive any education. This means that even if the income increases, the relative labour supply of women stays the same and therby also the social norm, because there is no increase in the investments in education. Therefore the $\theta\theta$ locus in the poverty regime can be described as vertical line at

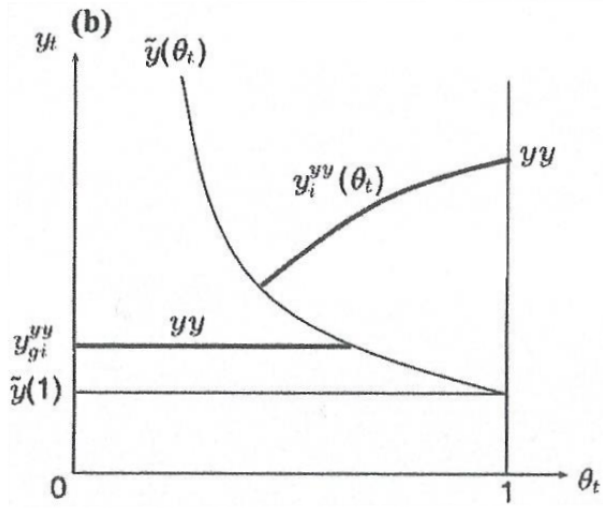

Figure 3.3: yy locus in the case $a \geq \tilde{a}$
(Hiller (2014), page 467, *Fig. 4 The yy locus*)

$\theta_t = \left(\frac{c^\alpha - s}{c^\alpha}\right)^{\frac{\kappa}{(1-\gamma)}}$ from $y_t = 0$ to $y_t = \tilde{y}(1)$. This value is smaller than 1 due to our assumption that unskilled men earn more wage than uneducated women, due to their physical advantage. Therefore boys are favored in this state of the economy, which means that the social norm is smaller than 1. In the gender inequality regime the $\theta\theta$ locus is described by a function denominated as $y_{gi}^{\theta\theta}(\theta_t)$. As shown in the appendix B.2, along with all its other characteristics, this function is decreasing in θ_t, which reflects the fact, that girls do not receive education in the gender inequality regime. Therefore a higher income y_t means that just boys receive more education, which increases their human capital, thereby decreases the female labour supply and as a consequence decreases the social norm θ_t. In the last case, the interior regime, the fact that boys and girls are both educated leads to the situation that the productivity gap vanishes over time. This means that women and men are socially equal which implicates that the steady state value of the social norm is given by $\theta_\infty = 1$. Therefore the $\theta\theta$ locus in the interior regime is given by a vertical line at $\theta_t = 1$ starting at $y_t = \tilde{y}(\theta_t) = \tilde{y}(1)$, as shown in figure 3.4.

3.4 Steady State Solutions of the Model

Now that we have described the yy locus and the $\theta\theta$ locus we can analyze the steady state behavior of this model. A steady state solution is defined as a state of the model, in which neither y_t nor θ_t change over time. Therefore these pairs of (y_t, θ_t) can be found by looking for intersections between the yy locus and the $\theta\theta$ locus. Because of the characteristics of the yy locus we analyzed above it is obvious that the number and characteristics of the steady state solution depends on the level of the paramter a. Therefore there are four different solutions for the steady state solution, which Hiller explains in his paper under Proposition 1. All of the following results are visualized as phase diagrams in figure 3.5, which is taken out of Hiller's paper. (Hiller (2014), page 470)

The first case occurs if the equation $a < \bar{a}$ holds, because under this assumption the yy locus is given by just a horizontal line which belongs to the poverty regime. In this case there is only one intersection of the two loci and therefore the unique globally stable steady state solution

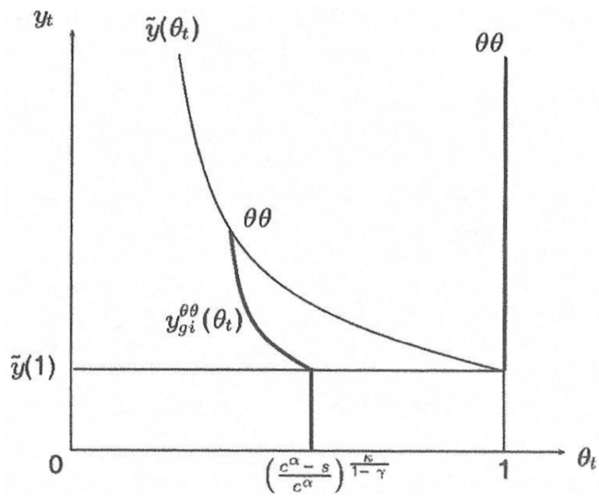

Figure 3.4: $\theta\theta$ locus
(Hiller (2014), page 468, *Fig. 5 The $\theta\theta$ locus*)

is located in the poverty regime, Hiller refers to this case as the poverty trap. Because of the low level of a and therefore the low level of productivity of investments in education a high level of income y_t would be needed to even start education spending. Therefore in this case "*norms about gender roles are fully shaped by differences in physical strength.*"(HILLER (2014), page 469) The fact that the steady state is globally stable is proved in the appendix part B.3 of this thesis. This case is shown in figure 3.5 (a) and the steady state equilibrium is visualized by a red dot.

In the second case the paramter for the productivity of expenditures for education a fulfills $a \in [\bar{a}, \tilde{a})$. For the yy locus this means that it consists as discussed above of a horizontal line belonging to the poverty regime, $y_t < \tilde{y}(1)$, and a monotonically increasing concave function located in the interior regime. The $\theta\theta$ locus stays, as in all four cases, unchanged. Therefore there now exists not only the steady state equilibrium we received in the previous case, the poverty trap solution, but also one situated in the interior regime. The second solution is located at the intersection of the concave part of the yy locus and the vertical part of the $\theta\theta$ locus at $\theta_t = 1$. Obviously different to the previous case these both equilibria are now only locally stable, which is again shown in the appendix part B.3 at the end of this thesis. In figure 3.5 this situation is described in the top right panel (b) of the figure. This means that the initial situation of the economy decides if the poverty trap solution will be reached or the high level equilibrium where the income is high and gender equality ($\theta = 1$) is achieved.

For the last two cases we assume that $a > \tilde{a}$ holds. But depending on the level of a, two different situations can occur. Therefore we define another boundary for a denoted by $\hat{a}$, which dinstinguishes between the two cases. The exact definiton and deviation of the boundary $\hat{a}$ can be found in the appendix part B.3. Depending on the value of a, the horizontal part of the yy locus that belongs to the gender inequality regime is located at a constant low or constant high level of y_t. So if $a \in [\tilde{a}, \hat{a})$ the horizontal part of the locus is still low enough to intersect with $\theta\theta$ locus in the gender inequality regime. The part of the yy locus in the interior regime that follows a concave function again intersects with the $\theta\theta$ locus at its vertical line situated $\theta_t = 1$. So this means that the poverty trap solution does not exist anymore, but additional to the interior steady state solution we received above, we get a new locally stable equilibrium, situated in the inequality regime, as can be seen in figure 3.5 (c). In this steady state equilibrium the steady state value of the social norm θ_∞ is lower than in the poverty trap solution, because the fact that only men receive education in this regime intensifies the initially assumed social inequality, that men have a physical advantage over women.

In the last case we assume that the paramter $a \geq \hat{a}$. Now the level of the part of the yy locus, which is given by a horizontal line in the gender inequality regime, is located too high to intersect with the $\theta\theta$ locus. Therefore the locally stable equilibrium belonging to the gender inequality regime disappears and there is just the globally stable equilibrium in the interior regime left. The steady state value of the total possible income y_∞ is higher than in the previous case, because the increase in a also increases the level of the concave part of the yy locus. The disappearance of the gender inequality solution can be explained by the high level of the productivity of education. Thereby it can no longer be optimal to exclude women from education. So the increase in the income might just be reinvested in the education of the sons at the start but over time it will also be invested in the education of the daughters, which pushes the level of the possible total income even further. Again all the exact proofs are derived in detail in the appendix B.3. This last case is visualized in figure 3.5 part (d).

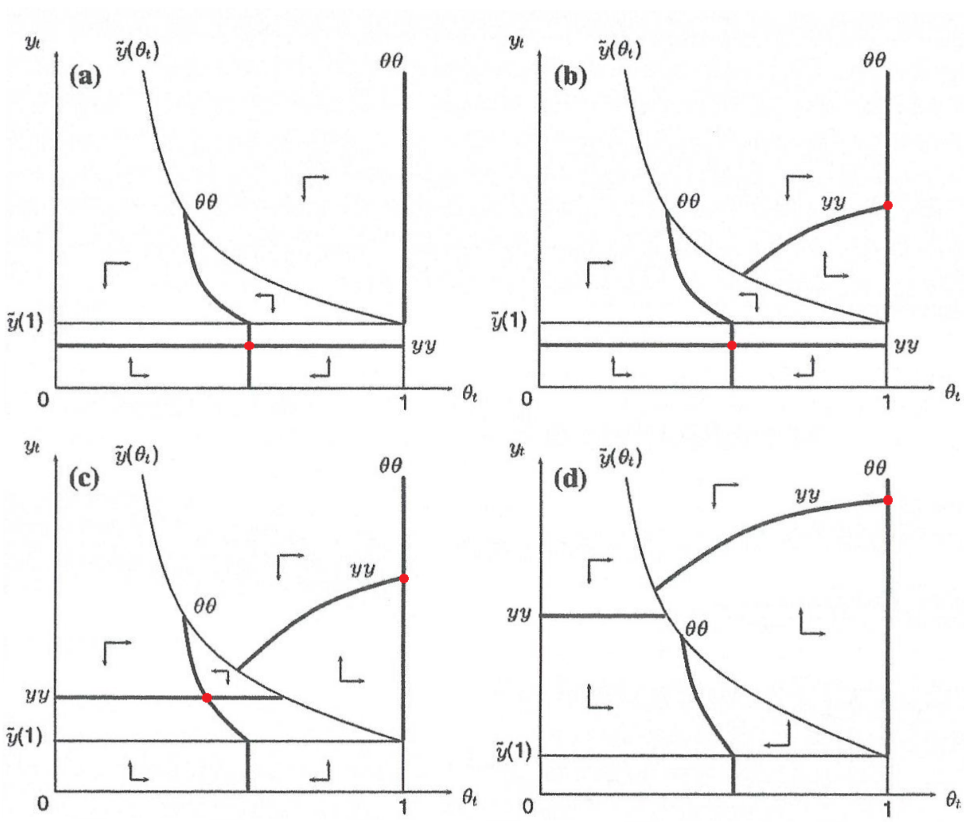

Figure 3.5: Four steady state equilibria
(Hiller (2014), page 470, *Fig. 6 Phase Diagrams*, red dots added)

3.5 Policy Implications

As one has seen through the analysis in the last section, the initial values of the total possible income y_t and of the social norm θ_t can be key for the development of the economy and the society. If the specific value of the productivity of educational expenditures a provides a case in which two equilibria exist then the initial values of the income and the norm decide in which of the two equilibria the economy develops. For example if $a \in [\tilde{a}, \hat{a})$ then a low social norm that favours men over women would drive the economy into the lower equilibrium that is situated in the gender inequality regime. Hiller talks in his paper in relation to this case about a gender inequality trap, because the society starts and ends in a state of gender inequality due to the low level of the social norm θ_t.
The existence of the poverty trap and the gender inequality trap raises the question if there are ways, respectively possible decisions from the policy makers that help to escape from them. Hiller responds to this question in his paper by presenting comparative statistics within *Proposition 2* (Hiller (2014), page 471). Due to the analysis in the last section we see that a rise in the productivity of educational spending a would provide an escape out of the poverty trap by increasing the stationary value of the possible total income y_t. But on the other hand it also decreases the social norm θ_t and therefore the increase in income would just lead to an increased investment in men's education and therefore more gender inequality. But if the shock is large enough, both traps can be overcome and the economy and the society can converge towards the high steady state equilibrium. In this case the total possible income takes the highest value of all three regimes and the social norm is equal to one which means that there is no more gender gap.
Another way to escape from the gender inequality trap is to reduce the educational costs τ. As shown in the appendix part B.4, a decrease of τ in the gender inequality regime would increase the steady state value of the total possible income y_t, but on the other hand would reduce the steady state equilibrium of θ_t. This means that although the income is increasing, the surplus is used only to invest in the education of boys. But again if the shock is large enough it might force the economy to escape from the gender inequality trap and reach the gender equity high output steady state equilibrium.

3.6 Full Transition and the U-shaped Female Labour Force Participation

The following proof of the U-shape of the female labour participation during the full transition from the poverty regime to the interior regime that I will present now follows the one in Hiller's original paper, which can be found in the *Appendix D* of his paper (Hiller (2014), page 478). At the start of the transition the economy is stuck in the poverty regime and therefore the steady state value of the social norm in this situation is given by $\theta_t = \left(\frac{(c^\alpha - s}{c^\alpha}\right)^{\frac{\kappa}{(1-\gamma)}} = (1 - \frac{s}{c^\alpha})^{\frac{\kappa}{(1-\gamma)}}$. Now we assume a huge productivity shock or a technologie shock that pushes the economy towards the interior solution, for example a permanent increase of a. This implies that the economy moves out of the poverty regime and through the gender inequality regime. In this phase as proven previously the possible total income y_t increases and the social norm θ_t decreases. After this stage the economy leaves the gender inequality regime, enters the interior regime and shifts towards the steady state equilibirum that is located in this regime. This steady state equilibirum is characterised by an increase in both, y_t and θ_t, compared to the gender inequality regime. To analyze the behaviour of the female labour force participation during this transition we take a look at the term

$$l_t^f = \Big(\frac{\gamma(1-\mu)}{\mu(1+(\frac{w_t^f}{w_t^m})^{\frac{\gamma}{(1-\gamma)}})}\Big)\frac{C_t}{w_t^f} \tag{3.15}$$

The transformations to receive this equation and the following result can be found in part B.5 of the appendix of this thesis. The last term of the equation above can be rewritten as

$$\frac{C_t}{w_t^f} = \begin{cases} \frac{\mu}{a}\Big(\frac{a\left(1+\frac{w_t^m}{w_t^f}\right)+\frac{\tau c}{w_t^f}}{\mu+\gamma(1-\mu)+\alpha\beta}\Big) & \text{if } y_t \in [\tilde{y}_t(1), \tilde{y}_t(\theta_t)) \\ \frac{\mu}{a}\Big(\frac{a\left(1+\frac{w_t^m}{w_t^f}\right)+\frac{2\tau c}{w_t^f}}{\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)}\Big) & \text{if } y_t \geq \tilde{y}_t(\theta_t) \end{cases} \tag{3.16}$$

The second case of this equation differs from the solution Hiller provides in his paper Hiller (2014).Instead of $\frac{2\tau c}{w_t^f}$ Hiller receives the term $\frac{\tau c}{w_t^f}$. But this difference has no further effects on the results and might just be a typing error. If we take these two equations into account the U-shaped evolution of the female labour force participation during the full transition of the economy from the poverty regime into the high output solution in the interior regime can be explained. When the economy moves from the poverty regime into the gender inequality regime parents start to provide education to their sons, while girls remain uneducated. This means that men are able to increase their human capital and therefore their wage w_t^m with the rise in y_t, while the women due to the lack of investment in their education cannot do this and w_t^f remains at the same level. Thereby the term $\frac{w_t^m}{w_t^f}$ increases which leads, as can be seen in (3.16), to an increase in $\frac{C_t}{w_t^f}$. Because of the fact that both of the terms of the right-hand side of equation (3.15) are increasing, also the time women spent at producing household goods l_t^f has to increase. This is equivalent to a decrease of the female labour force participation $1 - l_t^f$ during this first phase of the transition. After the economy has gone through the inequality regime it reaches the interior regime and therefore the last step of the transition. In the interior regime parents start also to provide education to their daughters, which allows them also to generate more human capital $h(e_t^f)$, which increases the income of females w_t^f and therefore increases the total possible income y_t even further. Moreover, as shown in the last section, the increase in w_t^f leads to the closing of the wage gap and therefore the social norm θ_t increases, which means that women and men are more emancipated. So the higher values of the steady state solutions of y_t and θ_t in the interior regime, compared to the ones in the gernder inequality regime, correspond to a higher w_t^f. As shown in the appendix part B.5, the equation $\frac{w_t^m}{w_t^f} = \left(\frac{1}{\theta_{t-1}}\right)^\alpha$ holds in the interior regime. So with these results and the equations (3.15) and (3.16) we see that $\frac{C_t}{w_t^f}$ is decreasing and therefore l_t^f is decreasing as well. This again means that the female labour force participation is increasing during the convergence to the steady state solution of the interior regime. Thus during the whole transition from the poverty regime to the interior regime the relation of the female labour force participation and the economic situation can be described as a U-shaped function in this model.

3.7 Summary

The model by Victor Hiller provides a framework to analyze the effects of inegalitarian social gender norms on the economic development of the society. Therefore the development of the social norm is endogenous and driven by the differences of the education expenditures for boys and girls. Parents take the social norm as given when they decide about the income they want to spend on each of their children, so they do not consider the impact they have on the dynamics of the norm. Furthermore it is important to mention that Hiller assumes a productivity gap between uneducated women and men, which is crucial for some of the results of this paper. This assumption accommodates the fact that in pre-industrial societies the physical advantage of men over women leads to a productivity gap between the two genders. With this assumption the paper provides optimal values for the variables of the parents utility-optimization-problem and steady state solutions of the total possible income and the social norm for the society. These assumptions imply that three different states exists in which the society can be: the poverty regime, the inequality regime and the interior regime. Hiller also provides implications for public policies which can force the economy out of the poverty trap and to converge into the egalitarian high output steady state equilibrium which is located in the interior regime. He also shows that his model provides a theory of the relationship between economic development and gender inequality. In his model this relationship expressed by the female labour force participation can be described by an U-shaped function which also fits, according to Hiller, to recent empirical findings.

CHAPTER
FOUR

Discussion

In this last chapter of my master thesis I will once more give a quick overview over the two papers I presented in this work and discuss how they could be extended or combined with models from other papers.

In the first paper I presented, *A model of voluntary childlessness* written by Paula E. Gobbi (2013), the author analyzes the fertility decision of a couple and how it is influenced by different shocks in some of the parameters. Therefore she uses an OLG model in which the agents are all the same during their childhood, but start to differ when they grow up. In adulthood the agents are characterized by their gender, because it is assumed that only women take care of the children. Furthermore Gobbi differentiates between people with a high taste for children and agents with a low willigness to have children. She assumes that couples are matched randomly and their joint taste for children is given by an average of the tastes of the two partners. To simplify the model, Gobbi further assumes that just three possible kinds of couples exist in the population. The spouses with a high desire for children, the couples with a low taste for children and the couples who will stay childless. Furthermore she derives steady state solutions for the ratio of agents with a high willingness to have children to those with a low taste and analyzes how a change in the weight of female labour supply in the production function or the fix costs of child rearing change this equlibrium. Additional I performed an extensive sensitivity analysis of the steady state to see how various parameters of the model effect the steady state equilibrium. Last Gobbi shows that in her model there exists a positive correlation between childlessness and fertility. This is quite interessting because although certain shocks can lead to a decrease of childless couples the negative effect on the fertility is even strong enough to reduce the overall part of the population that have a strong desire to have children. Aother interesting and I think quite new result is that Gobbi provides a fertility model in which it can be optimal for the agents to stay childless. Due to the recent development of fertility rates in developed countries, especially in Europe, this aspect of Gobbi's model is in my opinion especially innovative. A way in which this model could be extended would be to give up the assumption of random matching at the marriage market. Intuitive it would make sense that individuals with a certain taste for children will rather marry another individual with the same taste than marry someone with a complete different attitude to this important aspect of their common future. So therefore in my opinion it would make sense to either have higher, respectively lower, probabilities that agents with a common taste, respectively different view, of having children get together or introduce the chance, that the randomly matched agents do no get married and stay single until the end of their lifetime. I found a similar approach to this specific part of the model that was chosen by Baudin, De La Croix and Paula Gobbi herself in their paper *Fertility and Childlessness in the*

United States (2015), where they assumed that individuals with the same taste for children have a higher possibility to become a couple than agents that differ in their attitude to have children. Another idea how Gobbi's model could be extended is to not only allow agents to have children or stay childless, but also take the time of birth into account. This idea was developed into a mathematical framework by Alessandro Cigno and John Ermisch in *A Microeconomic Analysis of the Timing of Births* published in 1988, where they not only observe the total fertility of women, but also the timing of birth and how it shifts when income, the education or other variables change over time.

The second paper, *Gender Inequality, Endogenous Cultural Norms, and Economic Development*, by Victor Hiller (2014) tries to analyze the effects of social gender norms in the society on the economy. Therefore he chooses an OLG model in which two parents will always have a daughter and a son. The distinction between the genders just matters during childhood while in the adulthood parents make joint decisions and try to maximize a joint utility function. Hiller models the social norm as an endogenous variable that is driven by its past and by the development of the relative female labour force supply. The relative female labour supply itself depends on the education women receive during their childhood, because education increases their human capital and thereby the wage they could earn on the labour market during their adulthood. Just with a high enough wage and therefore high enough opportunity costs of staying at home and producing household goods, women will choose to increase their time at the labour market and thereby change their social norm to a more egalitarian one. The education expenditures of the parents depends solely on the social norm, not considering that their decision will be crucial for the development of the social norm and therefore for their children's and their own utility function. With this definition of the model Hiller is able to define three different situations for the steady state equilibrium of the economy, depending, among other variables, on the productivity of the educational expenditures regarding the human capital. The worst case is referred by Hiller as the *poverty regime* in which neither girls nor boys receive any education and just live with their innate basic level of human capital. The total income of the parents and the social norm take the lowest values in this case because of the lack of education. If the income increases the economy converges into another steady state equilibrium. In this situation the parents decide to provide education to their children but because of the assumed physical advantage that men have over women, just the boys receive education. This case for the steady state is denoted as the *inequality regime*, because in this state women do not receive any education. This situation can result because the value for the social norm is not high enough to make it for a given income optimal for parents to invest in the education of their daughters too. The last and best possible steady state equilibrium is defined as the *interior solution* in which both, boys and girls, receive education. In this regime the total possible income of the parents and the social norm take the highest values. With these results Hiller tries to give policy advices how poor countries could escape out of poverty traps, respectively leave the poverty regime and converge towards the prosperous interior solution. Furthermore with his model Hiller is able to reproduce the U-shaped process of the female labour force participation during the transition from a low level income economy to a wealthy developed country. For me personal the introduction of the endogenous social norm is the most interesting aspect of this model. On the one hand it has a crucial influence on all variables of the model and the characteristics of the steady state equilibrium. But on the other hand at the same time its dynamic is influenced by the decisions of the individuals in this model, although they do not consider the influence of their choices. Therefore I think the introduction of the endogenous social norm is an elegant way for gender specific frameworks to model gender inequalities like the wage gap or the education gap.

CHAPTER
FIVE

Appendices

A A model of voluntary childlessness by Paula E. Gobbi

A.1 The Model

Effects on γ^*
To show the impact of w^m, w^f and k on the unique level of γ^j for which parents are indifferent between having children and staying childless, γ^*, I apply the implicit function theorem, as suggested by Gobbi. Therefore we look at γ^* as a function of the different variables of equation (2.8), which are needed to define its exact value, $\gamma^* = \gamma^*(w^m, w^f, k, \theta)$. But for better readability I keep the short form γ^*. Now I assumes that one knows the exact value of γ^* and again have a look at (2.8), which after some transformations can be rewritten as

$$\ln\Big(\frac{\theta w^f}{\gamma^*}\Big) + \gamma^*\Big(\frac{w^m + w^f - k}{\theta w^f}\Big) - 1 \overset{!}{=} \ln(w^m + w^f) \tag{5.1}$$

To analyze the effects from changes in w^m, w^f or k on $\gamma^*(w^m, w^f, k, \theta)$ I build the derivate of this equation with respect to w^m, w^f and k and solve for the partial derivative of γ^* with respect to the specific variable. I start with the partial derivative of γ^* with respect to the male's wage w^m, so building the derivative of equation (5.1) leads to:

$$\begin{aligned}
&\Big(\frac{1}{\frac{\theta w^f}{\gamma^*}}\Big)\Big(\frac{0\cdot\gamma^* - \theta w^f \frac{\partial\gamma^*}{\partial w^m}}{(\gamma^*)^2}\Big) + \frac{\partial\gamma^*}{\partial w^m}\Big(\frac{w^m + w^f - k}{\theta w^f}\Big) + \gamma^*\Big(\frac{1}{\theta w^f}\Big) = \frac{1}{w^m + w^f}\\
\iff &\frac{\gamma^*}{\theta w^f}\Big(-\frac{\theta w^f \frac{\partial\gamma^*}{\partial w^m}}{(\gamma^*)^2}\Big) + \frac{\partial\gamma^*}{\partial w^m}\Big(\frac{w^m + w^f - k}{\theta w^f}\Big) + \Big(\frac{\gamma^*}{\theta w^f}\Big) = \frac{1}{w^m + w^f}\\
\iff &-\frac{\frac{\partial\gamma^*}{\partial w^m}}{\gamma^*} + \frac{\partial\gamma^*}{\partial w^m}\Big(\frac{w^m + w^f - k}{\theta w^f}\Big) + \Big(\frac{\gamma^*}{\theta w^f}\Big) = \frac{1}{w^m + w^f}\\
\iff &-\frac{\partial\gamma^*}{\partial w^m} + \frac{\partial\gamma^*}{\partial w^m}\Big(\frac{w^m + w^f - k}{\theta w^f}\Big)\gamma^* + \Big(\frac{\gamma^{*2}}{\theta w^f}\Big) = \frac{\gamma^*}{w^m + w^f}\\
\iff &\frac{\partial\gamma^*}{\partial w^m}\Big[\Big(\frac{w^m + w^f - k}{\theta w^f}\Big)\gamma^* - 1\Big] = \frac{\gamma^*}{w^m + w^f} - \Big(\frac{\gamma^{*2}}{\theta w^f}\Big)\\
\iff &\frac{\partial\gamma^*}{\partial w^m} = \Big[\frac{\gamma^*}{w^m + w^f} - \Big(\frac{\gamma^{*2}}{\theta w^f}\Big)\Big]\Big[\Big(\frac{w^m + w^f - k}{\theta w^f}\Big)\gamma^* - 1\Big]^{-1} < 0
\end{aligned}$$

The validity of the last inequality can be shown if one takes a look at the two terms which are equal to $\frac{\partial\gamma^*}{\partial w^m}$. As shown in the chapter 2.2.1 $\gamma^* > \frac{\theta w^f}{w^m + w^f - k}$ must hold and therefore the first term is strictly negative. The second term $\Big[\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)\gamma^*-1\Big]^{-1}$ is equal to $(n_t^*)^{-1}$ and therefore greater than zero. So altogether these two facts lead to the stated proposition

$$\frac{\partial\gamma^*}{\partial w^m}<0$$

Next I analyze the behaviour of γ^* in the case of a change in women's wage. Therefore again I build the derivative of the equation (2.8), but this time with respect to w^f. This leads to

$$\begin{aligned}
&\frac{1}{\frac{\theta w^f}{\gamma^*}}\Big(\frac{\theta\gamma^*-\theta w^f\frac{\partial\gamma^*}{\partial w^f}}{(\gamma^*)^2}\Big)+\frac{\partial\gamma^*}{\partial w^f}\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)+\gamma^*\Big(\frac{\theta w^f-(w^m+w^f-k)\theta}{(\theta w^f)^2}\Big)=\frac{1}{w^m+w^f}\\
\Longleftrightarrow\ &\frac{\gamma^*}{\theta w^f}\Big(\frac{\theta\gamma^*-\theta w^f\frac{\partial\gamma^*}{\partial w^f}}{(\gamma^*)^2}\Big)+\frac{\partial\gamma^*}{\partial w^f}\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)+\gamma^*\Big(\frac{\theta w^f-(w^m+w^f-k)\theta}{\theta^2(w^f)^2}\Big)=\frac{1}{w^m+w^f}\\
\Longleftrightarrow\ &\frac{\gamma^*-w^f\frac{\partial\gamma^*}{\partial w^f}}{\gamma^*w^f}+\frac{\partial\gamma^*}{\partial w^f}\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)+\gamma^*\Big(\frac{w^f-(w^m+w^f-k)}{\theta(w^f)^2}\Big)=\frac{1}{w^m+w^f}\\
\Longleftrightarrow\ &\frac{1}{w^f}-\frac{\frac{\partial\gamma^*}{\partial w^f}}{\gamma^*}+\frac{\partial\gamma^*}{\partial w^f}\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)+\gamma^*\Big(\frac{k-w^m}{\theta(w^f)^2}\Big)=\frac{1}{w^m+w^f}\\
\Longleftrightarrow\ &\frac{\gamma^*}{w^f}-\frac{\partial\gamma^*}{\partial w^f}+\frac{\partial\gamma^*}{\partial w^f}\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)\gamma^*+(\gamma^*)^2\Big(\frac{k-w^m}{\theta(w^f)^2}\Big)=\frac{\gamma^*}{w^m+w^f}\\
\Longleftrightarrow\ &\frac{\partial\gamma^*}{\partial w^f}\Big[\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)\gamma^*-1\Big]=\frac{\gamma^*}{w^m+w^f}-\frac{\gamma^*}{w^f}-(\gamma^*)^2\Big(\frac{k-w^m}{\theta(w^f)^2}\Big)\\
\Longleftrightarrow\ &\frac{\partial\gamma^*}{\partial w^f}=\Big[\gamma^*\Big(\frac{1}{w^m+w^f}-\frac{1}{w^f}-\gamma^*\Big(\frac{k-w^m}{\theta(w^f)^2}\Big)\Big)\Big]\Big[\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)\gamma^*-1\Big]^{-1}\\
\Longleftrightarrow\ &\frac{\partial\gamma^*}{\partial w^f}=\Big[\gamma^*\Big(-\frac{w^m}{(w^m+w^f)w^f}-\gamma^*\Big(\frac{k-w^m}{\theta(w^f)^2}\Big)\Big)\Big]\Big[\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)\gamma^*-1\Big]^{-1}
\end{aligned}$$

Although we know again that the second term on the right-hand side is positive we can't draw a conclusion about the sign of $\frac{\partial\gamma^*}{\partial w^f}$ because of the indeterminacy of the first term. Therefore the impact of a change in the women's wage w^f on the level of γ^* is unassigned.
The last effect of a parameter change I analyze is the impact of a change in the fix costs of having a child k on γ^* and hence I again build the derivative of (2.8) with respect to k:

$$\begin{aligned}
&\Big(\frac{1}{\frac{\theta w^f}{\gamma^*}}\Big)\Big(\frac{0\cdot\gamma^*-\theta w^f\frac{\partial\gamma^*}{\partial k}}{(\gamma^*)^2}\Big)+\frac{\partial\gamma^*}{\partial k}\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)+\gamma^*\Big(-\frac{1}{\theta w^f}\Big)=0\\
\Longleftrightarrow\ &\frac{\gamma^*}{\theta w^f}\Big(-\frac{\theta w^f\frac{\partial\gamma^*}{\partial k}}{(\gamma^*)^2}\Big)+\frac{\partial\gamma^*}{\partial k}\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)=\Big(\frac{\gamma^*}{\theta w^f}\Big)\\
\Longleftrightarrow\ &-\frac{\frac{\partial\gamma^*}{\partial k}}{\gamma^*}+\frac{\partial\gamma^*}{\partial k}\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)=\Big(\frac{\gamma^*}{\theta w^f}\Big)\\
\Longleftrightarrow\ &-\frac{\partial\gamma^*}{\partial k}+\frac{\partial\gamma^*}{\partial k}\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)\gamma^*=\Big(\frac{(\gamma^*)^2}{\theta w^f}\Big)\\
\Longleftrightarrow\ &\frac{\partial\gamma^*}{\partial k}\Big[\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)-1\Big]=\Big(\frac{(\gamma^*)^2}{\theta w^f}\Big)\\
\Longleftrightarrow\ &\frac{\partial\gamma^*}{\partial k}=\Big[\Big(\frac{(\gamma^*)^2}{\theta w^f}\Big)\Big]\Big[\Big(\frac{w^m+w^f-k}{\theta w^f}\Big)-1\Big]^{-1}>0
\end{aligned}$$

Using the same argument as in the other cases, one knows, that the second term on the right-hand side is equal to $(n_t^*)^{-1}$ and therefore positive. The first term must also be positive because $(\gamma^*)^2, \theta$ and w^f are positive. Therefore we know that $\frac{\partial \gamma^*}{\partial k} > 0$ and an increase in the fix costs of having children increases the unique level of γ^j for which parents are indifferent between having children and staying childless.These conclusions are quite intuitive, even the indeterminacy of the effect in changes in the women's wage. On the one hand an increase in the women's wage means that a couple would be able to afford more children and therefore γ^* should decrease. But on the other hand an increase in the wage also increases the opportunity costs for women to stay at home and care for the children. So depending on whether the income effect (more children) or the substitution effect (more work, less children) is bigger, the overall effect on γ^* is positive respectively negative.

Woman's and Man's Wage
The level of women's and men's wage the firm is willing to pay results from the firm's profit optimization process. Therefore the firm maximizes the profit function (2.13) by choosing the amount of male and female labour input they want to use. To find the profit maximizing value for the labour inputs one has to build the derivative of the profit function with respect to women's labour input L_t^f and men's labour input L_t^m and set the respective derivatives equal to zero, which is equal to set the marginal productivity of the respective labour input equal to its marginal costs. So doing this we receive for the men's wage

$$
\begin{aligned}
&\frac{\partial \Pi_t}{\partial L_t^m} = F_{L_t^m} - w_t^m \overset{!}{=} 0 \\
&\Longleftrightarrow\ -\frac{1}{\rho}\Big(\alpha (L_t^m)^{-\rho} + (1-\alpha)(L_t^f)^{-\rho}\Big)^{-\frac{1+\rho}{\rho}} \alpha(-\rho)(L_t^m)^{-\rho-1} - w_t^m = 0 \\
&\Longleftrightarrow \alpha (L_t^m)^{-\rho-1}\Big(\alpha (L_t^m)^{-\rho} + (1-\alpha)(L_t^f)^{-\rho}\Big)^{-\frac{1+\rho}{\rho}} = w_t^m \\
&\Longleftrightarrow w_t^m = \alpha\Big(\alpha + (1-\alpha)(L_t^f)^{-\rho}(L_t^m)^{\rho}\Big)^{-\frac{1+\rho}{\rho}}
\end{aligned}
$$

and in the same way the women's wage

$$
\begin{aligned}
&\frac{\partial \Pi_t}{\partial L_t^f} = F_{L_t^f} - w_t^f \overset{!}{=} 0 \\
&\Longleftrightarrow\ -\frac{1}{\rho}\Big(\alpha (L_t^m)^{-\rho} + (1-\alpha)(L_t^f)^{-\rho}\Big)^{-\frac{1+\rho}{\rho}} (1-\alpha)(-\rho)(L_t^f)^{-\rho-1} - w_t^f = 0 \\
&\Longleftrightarrow (1-\alpha)(L_t^f)^{-\rho-1}\Big(\alpha (L_t^m)^{-\rho} + (1-\alpha)(L_t^f)^{-\rho}\Big)^{-\frac{1+\rho}{\rho}} = w_t^f \\
&\Longleftrightarrow w_t^f = (1-\alpha)\Big((1-\alpha) + \alpha (L_t^m)^{-\rho}(L_t^f)^{\rho}\Big)^{-\frac{1+\rho}{\rho}}
\end{aligned}
$$

A.2 Dynamics

Using the newly defined variable z_t, the ratio of agents with a high willingsness to have children to those with a low taste for children, the two equations from (2.15) can be combined to obtain the dynamics of $z_t + 1$:

$$
\begin{aligned}
z_{t+1} = \frac{\overline{P_{t+1}}}{\underline{P_{t+1}}} &= \frac{a\overline{\overline{n}}\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2 \cdot (\overline{P_t}+\underline{P_t}) + b\overline{n}\frac{2\overline{P_t}\underline{P_t}}{(\overline{P_t}+\underline{P_t})^2}\cdot(\overline{P_t}+\underline{P_t})}{(1-a)\overline{\overline{n}}\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2 \cdot (\overline{P_t}+\underline{P_t}) + (1-b)\overline{n}\frac{2\overline{P_t}\underline{P_t}}{(\overline{P_t}+\underline{P_t})^2}\cdot(\overline{P_t}+\underline{P_t})} = \\
&= \frac{\left(\overline{P_t}+\underline{P_t}\right)\left(a\overline{\overline{n}}\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2 + b\overline{n}\frac{2\overline{P_t}\underline{P_t}}{(\overline{P_t}+\underline{P_t})^2}\right)}{\left(\overline{P_t}+\underline{P_t}\right)\left((1-a)\overline{\overline{n}}\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2) + (1-b)\overline{n}\frac{2\overline{P_t}\underline{P_t}}{(\overline{P_t}+\underline{P_t})^2}\right)} = \\
&= \frac{\left(\frac{\overline{P_t}}{(\overline{P_t}+\underline{P_t})^2}\right)\left(a\overline{\overline{n}}\overline{P_t}+2b\overline{n}\underline{P_t}\right)}{\left(\frac{\overline{P_t}}{(\overline{P_t}+\underline{P_t})^2}\right)\left((1-a)\overline{\overline{n}}\overline{P_t}+2b\overline{n}\underline{P_t}\right)} = \frac{a\overline{\overline{n}}\overline{P_t}+2b\overline{n}\underline{P_t}}{(1-a)\overline{\overline{n}}\overline{P_t}+2b\overline{n}\underline{P_t}} = \\
&= \frac{\frac{1}{\underline{P_t}}\left(a\overline{\overline{n}}z_t+2b\overline{n}\right)}{\frac{1}{\underline{P_t}}\left((1-a)\overline{\overline{n}}z_t+2b\overline{n}\right)} = \frac{a\overline{\overline{n}}z_t+2b\overline{n}}{(1-a)\overline{\overline{n}}z_t+2b\overline{n}}
\end{aligned}
$$

To show that the proportion of childless women can be rewritten as stated in chapter 2.3, I just use the definition of z_t from (2.16) to describe the probability that two people with a low taste for children are randomly matched together as a couple. This leads to the following transformations

$$
\chi_t = \frac{1}{(1+z_{t+1})^2} = \frac{1}{(1+\frac{\overline{P_t}}{\underline{P_t}})^2} = \left(\frac{\underline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2
$$

Proving that the new form of the average fertility of the society n_t is equal to the proportions of types of marriages multiplied by the number of children they have, I substitute z_t with its definition:

$$
\begin{aligned}
n_t &= \frac{z_t}{(1+z_t)^2}(z_t\overline{\overline{n}}+2\overline{n}) = \frac{\frac{\overline{P_t}}{\underline{P_t}}}{\left(\frac{\overline{P_t}}{\underline{P_t}}+1\right)^2}\left(\frac{\overline{P_t}}{\underline{P_t}}\overline{\overline{n}}+2\overline{n}\right) = \frac{\frac{\overline{P_t}}{\underline{P_t}}}{\frac{(\overline{P_t}+\underline{P_t})^2}{\underline{P_t}^2}}\left(\frac{\overline{P_t}}{\underline{P_t}}\overline{\overline{n}}+2\overline{n}\right) = \\
&= \frac{\overline{P_t}\underline{P_t}^2}{(\overline{P_t}+\underline{P_t})^2\underline{P_t}}\left(\frac{\overline{P_t}}{\underline{P_t}}\overline{\overline{n}}+2\overline{n}\right) = \frac{\overline{P_t}\underline{P_t}}{(\overline{P_t}+\underline{P_t})^2}\left(\frac{\overline{P_t}}{\underline{P_t}}\overline{\overline{n}}+2\overline{n}\right) = \\
&= \frac{\overline{P_t}\underline{P_t}}{(\overline{P_t}+\underline{P_t})^2}\frac{\overline{P_t}}{\underline{P_t}}\overline{\overline{n}} + \frac{\overline{P_t}\underline{P_t}}{(\overline{P_t}+\underline{P_t})^2}2\overline{n} = \frac{\overline{P_t}^2}{(\overline{P_t}+\underline{P_t})^2}\overline{\overline{n}} + 2\frac{\overline{P_t}\underline{P_t}}{(\overline{P_t}+\underline{P_t})^2}\overline{n}
\end{aligned}
$$

For receiving the steady state value of z_t I use the definition of the steady state and set $z_t \overset{!}{=} z^*$ and $z_{t+1} \overset{!}{=} z^*$ in 2.16

$$
\begin{aligned}
z_{t+1} &= \frac{a\overline{\overline{n}}z_t+2b\overline{n}}{(1-a)\overline{\overline{n}}z_t+2(1-b)\overline{n}} \\
&\iff z^* = \frac{a\overline{\overline{n}}z^*+2b\overline{n}}{(1-a)\overline{\overline{n}}z^*+2(1-b)\overline{n}} \\
&\iff z^{*2}(1-a)\overline{\overline{n}} + z^*(2(1-b)\overline{n}-a\overline{\overline{n}}) - 2b\overline{n} = 0
\end{aligned}
$$

This equation can now be solved by using the solution formula for quadratic equations

$$\Rightarrow z^* = \frac{-\Big(2(1-b)\overline{n} - a\overline{\overline{n}}\Big) \pm \sqrt{\Big(2(1-b)\overline{n} - a\overline{\overline{n}}\Big)^2 + 8(1-a)\overline{\overline{n}}b\overline{n}}}{2(1-a)\overline{\overline{n}}}$$

$$= \frac{-2\Big((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\Big) + \sqrt{4\Big[\Big((1-b)\overline{n} - a\overline{\overline{n}}\Big)^2 + 2(1-a)\overline{\overline{n}}b\overline{n}\Big]}}{2(1-a)\overline{\overline{n}}}$$

$$= \frac{-\Big((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\Big) + \sqrt{\Big((1-b)\overline{n} - a\overline{\overline{n}}\Big)^2 + 2(1-a)\overline{\overline{n}}b\overline{n}}}{(1-a)\overline{\overline{n}}}$$

The minus sign was dropped in the second line due to the fact that z^* must positive by definition.

Next I analyze the characteristics of the function $\phi(z_t) = z_{t+1}$ which was defined in 2.16

$$\phi(z_t) = \frac{a\overline{\overline{n}}z_t + 2b\overline{n}}{(1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}}$$

First I show the positive sign of the first derivative of $\phi(z_t)$ with respect to z_t:

$$\frac{\partial\phi(z_t)}{\partial z_t} = \frac{a\overline{\overline{n}}\Big((1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}\Big) - (1-a)\overline{\overline{n}}(a\overline{\overline{n}}z_t + 2b\overline{n})}{\Big((1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}\Big)^2} =$$

$$= \frac{a\overline{\overline{n}}(1-a)\overline{\overline{n}}z_t - (1-a)\overline{\overline{n}}a\overline{\overline{n}}z_t + 2(1-b)\overline{n}a\overline{\overline{n}} - 2b\overline{n}(1-a)\overline{\overline{n}}}{\Big((1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}\Big)^2} =$$

$$= \frac{2\overline{n}\overline{\overline{n}}\Big((1-b)a - b(1-a)\Big)}{\Big((1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}\Big)^2} =$$

$$= \frac{2\overline{n}\overline{\overline{n}}(a - ab - b + ab)}{\Big((1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}\Big)^2} =$$

$$= \frac{2\overline{n}\overline{\overline{n}}(a-b)}{\Big((1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}\Big)^2} > 0$$

The direction of the inequality holds because the denominator is always positive and the numerator is also positive due to the assumption that $a > b$. The second derivative of $\phi(z_t)$ with respect to z_t is strictly negative as can be seen below.

$$\frac{\partial^2\phi(z_t)}{\partial(z_t)^2} = -2 \cdot 2\overline{n}\overline{\overline{n}}(a-b)\frac{1}{\Big((1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}\Big)^3}(1-a)\overline{n} =$$

$$= \frac{4\overline{n}\overline{\overline{n}}(b-a)(1-a)\overline{n}}{\Big((1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}\Big)^3} < 0$$

While the denominator again is positive because $a < 1$, $b < 1$ and the positive sign of $\overline{n}$, $\overline{\overline{n}}$ and z_t, the numerator is now negative because $(b - a) < 0$.
Next we analyze the long term behaviour of the function $\phi(z_t)$:

$$\lim_{z_t \to +\infty} \phi(z_t) = \lim_{z_t \to +\infty} \frac{a\overline{\overline{n}} + \frac{2b\overline{n}}{z_t}}{(1-a)\overline{\overline{n}} + \frac{2(1-b)\overline{n}}{z_t}} = \frac{a\overline{\overline{n}}}{(1-a)\overline{\overline{n}}} = \frac{a}{(1-a)} > \frac{b}{(1-b)} = \phi(0)$$

The inequality at the end holds again because of the assumption that $a > b$.
Showing the impact of an increase of z^* on the steady state values of the average fertility $n^* = n_t(z^*)$ can be done by building the derivative of n_t with respect to z_t:

$$\begin{aligned}
\frac{\partial n_t}{\partial z_t} &= \frac{(1 + z_t^2 - z_t 2(1+z_t)1}{(1+z_t)^4}(z_t\overline{\overline{n}} + 2\overline{n}) + \frac{z_t}{(1+z_t)^2}\overline{\overline{n}} = \\
&= \frac{(1 + 2z_t + z_t^2 - 2z_t - 2z_t^2}{(1+z_t)^4}(z_t\overline{\overline{n}} + 2\overline{n}) + \frac{z_t}{(1+z_t)^2}\overline{\overline{n}} = \\
&= \frac{1 - z_t^2}{(1+z_t)^4}(z_t\overline{\overline{n}} + 2\overline{n}) + \frac{z_t}{(1+z_t)^2}\overline{\overline{n}} = \\
&= \frac{z_t\overline{\overline{n}} + 2\overline{n} - z_t^3\overline{\overline{n}} - 2\overline{n}z_t^2}{(1+z_t^4)} + \frac{z_t\overline{\overline{n}} + 2z_t^2\overline{\overline{n}} + z_t^3\overline{\overline{n}}}{(1+z_t)^4} = \\
&= \frac{2z_t\overline{\overline{n}} + 2\overline{n} - 2z_t^2\overline{n} + 2z_t^2\overline{\overline{n}}}{(1+z_t)^4} = \\
&= \frac{2z_t^2(\overline{\overline{n}} - \overline{n}) + 2(z_t\overline{\overline{n}} + \overline{n})}{(1+z_t)^4} > 0
\end{aligned}$$

It is easy to see that the denominator and the numerator are always positive because $z_t > 0$ and $(\overline{\overline{n}} > \overline{n})$. Therefore an increase in z_t, or the steady state value z^*, increases the average number of children n_t, respectively its steady state level $n^* = n_t(z^*)$.

B Gender Inequality, Endogenous Cultural Norms, and Economic Development by Victor Hiller

B.1 The Model

In this section I will derive the optimal values or rather the relations of the optimal values to the other variables and parameters which are given by equations (3.9) and (3.10). Because it is assumed that there exists an interior solution all the inequality signs in the equations can be changed to equality signs, which I will do in this derivation. Therefore I substitute the values C_t and D_t given by the constraints and by equations (3.6) and (3.3) into the maximization problem (3.8), which leads to:

$$\begin{aligned}
U_t =& \mu \ln\Big((1-l_t^f)w_t^f + (1-l_t^m)w_t^m - \tau(e_{t+1}^f + e_{t+1}^m)\Big) + (1-\mu)\ln\Big((l_t^f)^\gamma + (l_t^m)^\gamma\Big) + \dots \\
&\dots + \xi\Big(\theta_t \ln\Big((c + ae_t^f)^\alpha\Big) + \ln\Big((c + ae_t^m)^\alpha\Big)\Big) + \beta\Big(\theta_t \ln\Big((c + ae_{t+1}^f)^\alpha\Big) + \ln\Big((c + ae_{t+1}^m)^\alpha\Big)\Big)
\end{aligned} \tag{5.2}$$

First I will derive equation (3.9) by building the first derivative of the obtained utility function (5.2) with respect to the amount of women's and men's household labour time, l_t^f respectively

l_t^m, and set them equal to zero, to find the maximum. (It is easy to see that the second derivatives are negative so it is indeed a maximum) I will start with the derivative with respect to l_t^m:

$$\begin{aligned}\frac{\partial U_t}{\partial l_t^m} =& \mu \frac{1}{C_t}(-w_t^m) + (1-\mu)\frac{1}{D_t}\gamma (l_t^m)^{\gamma-1} \overset{!}{=} 0\\ \iff \quad \frac{w_t^m \mu}{C_t \gamma (1-\mu)} &= \frac{(l_t^m)^{\gamma-1}}{D_t}\end{aligned}$$

Similarly the first derivative of the utility function w.r.t. the amount of female's time spent in household l_t^f leads to:

$$\begin{aligned}\frac{\partial U_t}{\partial l_t^f} =& \mu \frac{1}{C_t}(-w_t^f) + (1-\mu)\frac{1}{D_t}\gamma (l_t^f)^{\gamma-1} \overset{!}{=} 0\\ \iff \quad \frac{w_t^f \mu}{C_t \gamma (1-\mu)} &= \frac{(l_t^f)^{\gamma-1}}{D_t}\end{aligned}$$

Dividing the first result by the second result yields:

$$\frac{\frac{(l_t^m)^{\gamma-1}}{D_t}}{\frac{(l_t^f)^{\gamma-1}}{D_t}} = \frac{(l_t^m)^{\gamma-1}}{(l_t^f)^{\gamma-1}} = \left(\frac{l_t^m}{l_t^f}\right)^{\gamma-1} = \frac{w_t^m}{w_t^f} \qquad \iff \qquad \left(\frac{l_t^m}{l_t^f}\right)^{1-\gamma} = \frac{w_t^f}{w_t^m}$$

To derive the equations (3.10) I will proceed the same way and build the derivative of the utility function with respect to e_{t+1}^m and e_{t+1}^f, the amount of education the parents provide to their daughter and their son. This leads after a simple transformation to the two equations in (3.10) we are looking for:

$$\begin{aligned}\frac{\partial U_t}{e_{t+1}^m} =& \mu \frac{1}{C_t}(-\tau) + \beta \frac{1}{h_{t+1}^m}\Big(\alpha (c + a e_{t+1}^m)^{\alpha-1} a\Big) \overset{!}{=} 0\\ \iff \quad \frac{(h_{t+1}^m)'}{h_{t+1}^m} &= \frac{\mu\tau}{C_t \beta}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial U_t}{e_{t+1}^f} =& \mu \frac{1}{C_t}(-\tau) + \beta\theta_t \frac{1}{h_{t+1}^f}\Big(\alpha (c + a e_{t+1}^f)^{\alpha-1} a\Big) \overset{!}{=} 0\\ \iff \quad \frac{(h_{t+1}^f)'}{h_{t+1}^f} &= \frac{\mu\tau}{C_t \beta\theta_t}\end{aligned}$$

where $h_{t+1}^j = (c + a e_{t+1}^j)^\alpha$ as given in (3.2) and therefore $\alpha(c + a e_{t+1}^j)^{\alpha-1} a = (h_{t+1}^j)'$.

Deriving the optimal values that the parents provide to their children e_{t+1}^m and e_{t+1}^f is quite a long calculation consisting mostly of simple transformations. Therefore, for reasons of simplicity I will not perform every single step, but the most important intermediate results. To receive the result for the optimal vales of e_{t+1}^f and e_{t+1}^m one has just to use the budget constraint (3.3), the relation for the optimal amount of time spent for housework (3.9) and the relation for the optimal amount of education (3.10). We start with equation (3.10) which can be rewritten as

$$(c + ae^f_{t+1})\tau\mu = a\alpha\beta\theta_t C_t \qquad \text{and} \qquad (c + ae^m_{t+1})\tau\mu = a\alpha\beta C_t \tag{5.3}$$

The next step is to express C_t explicitly from the budget constraint (3.3) as a function of e^f_{t+1} and e^m_{t+1} and then substitute it in the equations (5.3) and transform them to

$$e^f_{t+1} = \frac{a\alpha\beta\theta_t\Big((1-l^f_t)w^f_t + (1-l^m_t)w^m_t - \tau e^m_{t+1}\Big) - c\tau\mu}{a\tau(\mu + \alpha\beta\theta_t)}$$
$$e^m_{t+1} = \frac{a\alpha\beta\Big((1-l^f_t)w^f_t + (1-l^m_t)w^m_t - \tau e^f_{t+1}\Big) - c\tau\mu}{a\tau(\mu + \alpha\beta)}$$

Now one can solve these equations for e^m_{t+1} respectively e^f_{t+1} and substitute the solutions of the explicitly expressed terms, e^m_{t+1} and e^f_{t+1}, in the respectively other equation.

$$\begin{aligned} e^f_{t+1} &= \frac{a\alpha\beta\theta_t\Big((1-l^f_t)w^f_t + (1-l^m_t)w^m_t\Big) - c\tau\Big(\mu + \alpha\beta(1-\theta_t)\Big)}{a\tau\Big(\mu + \alpha\beta(1+\theta_t)\Big)} \\ e^m_{t+1} &= \frac{a\alpha\beta\Big((1-l^f_t)w^f_t + (1-l^m_t)w^m_t\Big) - c\tau\Big(\mu - \alpha\beta(1-\theta_t)\Big)}{a\tau\Big(\mu + \alpha\beta(1+\theta_t)\Big)} \end{aligned} \tag{5.4}$$

For the next transformation we first have to show, how the term $\Big((1-l^f_t)w^f_t + (1-l^m_t)w^m_t\Big)$ can be rewritten. Therefore one can use equation (3.9) and the definition $D_t = (l^f_t)^\gamma + (l^m_t)^\gamma$. With these two equations the time spent in the household for each gender can be written as

$$l^f_t = \frac{\gamma(1-\mu)}{\mu\Big(1 + (\frac{w^f_t}{w^m_t})^{\frac{\gamma}{1-\gamma}}\Big)} \cdot \frac{C_t}{w^f_t} \qquad \text{and} \qquad l^m_t = \frac{\gamma(1-\mu)}{\mu\Big(1 + (\frac{w^m_t}{w^f_t})^{\frac{\gamma}{1-\gamma}}\Big)} \cdot \frac{C_t}{w^m_t}$$

respectively

$$l^f_t w^f_t = \frac{\gamma(1-\mu)C_t}{\mu\Big(1 + (\frac{w^f_t}{w^m_t})^{\frac{\gamma}{1-\gamma}}\Big)} \qquad \text{and} \qquad l^f_t w^m_t = \frac{\gamma(1-\mu)C_t}{\mu\Big(1 + (\frac{w^m_t}{w^f_t})^{\frac{\gamma}{1-\gamma}}\Big)}$$

and therefore

$$l^f_t w^f_t + l^m_t w^m_t = \frac{\gamma(1-\mu)C_t}{\mu}$$

Now again the term C_t can be replaced by the one obtained from the budget constraint (3.3) and after some transformations one receives

$$l_t^f w_t^f + l_t^m w_t^m = \frac{\gamma(1-\mu)}{\mu+\gamma(1-\mu)}\Big(y_t - \tau(e_{t+1}^f + e_{t+1}^m)\Big)$$

where $y_t = w_t^f + w_t^m$. Thereby the term $\Big((1-l_t^f)w_t^f + (1-l_t^m)w_t^m\Big)$ can be rewritten as

$$\begin{aligned}\Big((1-l_t^f)w_t^f + (1-l_t^m)w_t^m\Big) &= (w_t^f + w_t^m) + l_t^f w_t^f + l_t^m w_t^m = \\ &= y_t + \frac{\gamma(1-\mu)}{\mu+\gamma(1-\mu)}\Big(y_t - \tau(e_{t+1}^f + e_{t+1}^m)\Big)\end{aligned}$$

Now one can continue the transformations of e_{t+1}^f and e_{t+1}^m by substituting this term in the equations (5.4) and after some simple transformations one obtains

$$e_{t+1}^f = \frac{a\alpha\beta\theta_t\mu y_t + a\alpha\beta\theta_t\gamma(1-\mu)\tau e_{t+1}^m - c\tau\Big(\mu+\alpha\beta(1-\theta_t)\Big)\Big(\mu+\gamma(1-\mu)\Big)}{a\tau\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)) + \alpha\beta\gamma(1-\mu)\Big)}$$

$$e_{t+1}^m = \frac{a\alpha\beta\mu y_t + a\alpha\beta\gamma(1-\mu)\tau e_{t+1}^f - c\tau\Big(\mu-\alpha\beta(1-\theta_t)\Big)\Big(\mu+\gamma(1-\mu)\Big)}{a\tau\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)) + \alpha\beta\gamma(1-\mu)\Big)}$$

The next step is to substitute the e_{t+1}^m term in the e_{t+1}^f equation, respectively the e_{t+1}^f term in the e_{t+1}^m equation, and express e_{t+1}^f explicitly, respectively express e_{t+1}^m explicitly. After some transformations one obtains the final form of the optimal values for the amount of education parents provide for their daughter and their son, e_{t+1}^f and e_{t+1}^m:

$$\begin{aligned}e_{t+1}^f &= \frac{a\alpha\beta\theta_t\mu y_t\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)+\alpha\beta\gamma(1-\mu)(1+\theta_t))\Big)}{a\tau\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t))\Big)\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)+\alpha\beta\gamma(1-\mu)(1+\theta_t))\Big)} - \dots \\ &\dots - \frac{c\tau\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)+\alpha\beta\gamma(1-\mu)(1+\theta_t))\Big)\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t))\Big)}{a\tau\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t))\Big)\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)+\alpha\beta\gamma(1-\mu)(1+\theta_t))\Big)} \\ e_{t+1}^m &= \frac{a\alpha\beta y_t\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)+\alpha\beta\gamma(1-\mu)\theta_t)\Big)}{a\tau\Big(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)\Big)\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)+\alpha\beta\gamma(1-\mu)\theta_t)\Big)} - \dots \\ &\dots - \frac{c\tau\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)+\alpha\beta\gamma(1-\mu)\theta_t)\Big)\Big(\mu+\gamma(1-\mu)-\alpha\beta(1-\theta_t)\Big)}{a\tau\Big(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)\Big)\Big(\mu(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)+\alpha\beta\gamma(1-\mu)\theta_t)\Big)}\end{aligned}$$

$$\iff$$

$$e_{t+1}^f = \frac{a\alpha\beta\theta_t y_t - c\tau\Big(\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t))\Big)}{a\tau\Big(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)\Big)}$$

$$e_{t+1}^m = \frac{a\alpha\beta y_t - c\tau\Big(\mu+\gamma(1-\mu)-\alpha\beta(1-\theta_t)\Big)}{a\tau\Big(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)\Big)}$$

These values must always be positive, therefore we define a lower boundary for the income y_t, so that for all values of the income which are lower than this lower limit $\tilde{y}_t(\theta_t)$, the amount of education provided to the children is equal to zero. This value can easily be found:

$$
\begin{aligned}
e^f_{t+1} &= \frac{a\alpha\beta\theta_t y_t - c\tau\Big(\mu + \gamma(1-\mu) + \alpha\beta(1-\theta_t))\Big)}{a\tau\Big(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t)\Big)} \geq 0 \\
\iff y_t &\geq \frac{\tau c\Big(\mu + \gamma(1-\mu) + \alpha\beta(1-\theta_t)\Big)}{a\alpha\beta\theta_t} = \frac{\tau c}{a}\Big(\frac{\mu + \gamma(1-mu) + \alpha\beta(1-\theta_t)}{\alpha\beta\theta_t}\Big) =: \tilde{y}_t(\theta_t)
\end{aligned}
$$

For the level of education sons receive there is another even lower boundary $\tilde{y}_t(1)$, for which the amount of education e^m_{t+1} is already positive, while girls still do not receive any education.

$$
\begin{aligned}
e^m_{t+1} &= \frac{a\alpha\beta y_t - c\tau\Big(\mu + \gamma(1-\mu) - \alpha\beta(1-\theta_t)\Big)}{a\tau\Big(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t)\Big)} \geq 0 \\
\iff y_t &\geq \frac{\tau c\Big(\mu + \gamma(1-\mu) - \alpha\beta(1-\theta_t)\Big)}{a\alpha\beta} = \frac{\tau c}{a}\Big(\frac{\mu + \gamma(1-\mu) - \alpha\beta(1-\theta_t)}{\alpha\beta}\Big)
\end{aligned}
$$

B.2 Dynamics

For deriving the dynamics of the income y_{t+1} one just has to put the different values of e^m_{t+1} and e^f_{t+1}, depending on the income y_t, into the equation which defines the future income (3.5). Thereby there are three different cases depending on the level of the income of the last period t. If $y_t < \tilde{y}_t(1)$ the equation for the income in the next period takes the form

$$y_{t+1} = h(0) + h(0) - \delta(0)s = (c + a0)^\alpha + (c + a0)^\alpha - 1s = 2c^\alpha - s$$

If $y_t \in [\tilde{y}_t(1), \tilde{y}_t(\theta_t)]$ just the level of educations the sons receive is positive and therefore the income in the period $t+1$ is given as

$$
\begin{aligned}
y_{t+1} &= h(e^m_{t+1}) + h(0) + \delta(0)s = \Big(c + a\frac{a\alpha\beta y_t - \tau c(\mu + \gamma(1-\mu))}{a\tau(\mu + \gamma(1-\mu) + \alpha\beta)}\Big)^\alpha + c^\alpha - s = \\
&= c^\alpha - s + (a y_t + \tau c)^\alpha \Big(\frac{\alpha\beta}{\tau(\mu + \gamma(1-\mu) + \alpha\beta)}\Big)^\alpha
\end{aligned}
$$

Finally in the third case where the income y_t is high enough so that boys and girls receive education both terms e^m_{t+1} and e^f_{t+1} are greater than zero and therefore the future income is defined as

$$
\begin{aligned}
y_{t+1} &= h(e^m_{t+1}) + h(e^f_{t+1}) + \delta(e^f_{t+1})s \\
&= \Big(c + a\frac{a\alpha\beta y_t - \tau c(\mu + \gamma(1-\mu) - \alpha\beta(1-\theta_t))}{a\tau(\mu + \gamma(1-\mu) + \alpha\beta)}\Big)^\alpha + \dots \\
&\dots + \Big(c + a\frac{a\alpha\beta\theta_t y_t - \tau c(\mu + \gamma(1-\mu) + \alpha\beta(1-\theta_t))}{a\tau(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t))}\Big)^\alpha = \\
&= \Big(\frac{a\alpha\beta y_t - \tau c(2\alpha\beta)}{\tau(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t)}\Big)^\alpha + \Big(\frac{a\alpha\beta\theta_t y_t - \tau c(2\alpha\beta\theta_t)}{\tau(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t)}\Big)^\alpha = \\
&= \Big(\frac{\alpha\beta}{\tau(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t)}\Big)^\alpha \Big((ay_t - 2\tau c)^\alpha + (a\theta_t y_t - 2\tau c\theta_t)^\alpha\Big) = \\
&= \chi(\theta_t)(ay_t - 2\tau c)^\alpha(1+\theta_t^\alpha) \qquad \text{whith} \qquad \chi(\theta_t) := \Big(\frac{\alpha\beta}{\tau(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t)}\Big)^\alpha
\end{aligned}
$$

With this definition of $\chi(\theta_t)$ the second case where $y_t \in [\tilde{y}_t(1), \tilde{y}_t(\theta_t)]$ can be rewritten as

$$
y_{t+1} = c^\alpha - s + (ay_t + \tau c)^\alpha \chi(0) \tag{5.5}
$$

The derivation of the dynamics of θ_{t+1} works in a similar way. The equation (3.7) defines the evolution of θ_t, which depends on the amount of time spent for housework l_t^m and l_t^f. The dynamic of these to variables is given by (3.9), which is driven by the wage, which again depends on the amount of the education girls and boys receive (3.4). Therefore again one has to check the three different cases that can occur, depending on the level of income y_t. The first case is again the one in which the income is lower than $\tilde{y}_t(1)$ and therefore neither girls nor boys receive any education:

$$
\begin{aligned}
\theta_{t+1} &= \sigma\theta_t + (1-\sigma)\Big(\frac{l_t^m}{l_t^f}\Big)^\kappa = \sigma\theta_t + (1-\sigma)\Big(\frac{w^f_{t+1}}{w^m_{t+1}}\Big)^{\frac{\kappa}{(1-\gamma)}} = \\
&= \sigma\theta_t + (1-\sigma)\Big(\frac{c^\alpha - s}{c^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}}
\end{aligned}
$$

In the second case $y_t \in [\tilde{y}_t(1), \tilde{y}_t(\theta_t))$, which means that just boys receive education. Therefore the evolution of the social norm θ_t is given by

$$
\begin{aligned}
\theta_{t+1} &= \sigma\theta_t + (1-\sigma)\Big(\frac{l_t^m}{l_t^f}\Big)^\kappa = \sigma\theta_t + (1-\sigma)\Big(\frac{w^f_{t+1}}{w^m_{t+1}}\Big)^{\frac{\kappa}{(1-\gamma)}} = \\
&= \sigma\theta_t + (1-\sigma)\Big(\frac{c^\alpha - s}{(c + a\frac{a\alpha\beta y_t - \tau c(\mu+\gamma(1-\mu))}{a\tau(\mu+\gamma(1-\mu)+\alpha\beta)})^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} = \\
&= \sigma\theta_t + (1-\sigma)\Big(\frac{c^\alpha - s}{(\frac{a\alpha\beta y_t + \tau c\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta)})^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} = \sigma\theta_t + (1-\sigma)\Big(\frac{c^\alpha - s}{\chi(0)(ay_t + \tau c)^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}}
\end{aligned}
$$

In the final case $y_t \geq \tilde{y}_t(\theta_t)$ and thus girls as well as boys receive a positive amount of education which leads to the following dynamic:

$$
\begin{aligned}
\theta_{t+1} &= \sigma\theta_t + (1-\sigma)\Big(\frac{l_t^m}{l_t^f}\Big)^{\kappa} = \sigma\theta_t + (1-\sigma)\Big(\frac{w_{t+1}^f}{w_{t+1}^m}\Big)^{\frac{\kappa}{(1-\gamma)}} = \\
&= \sigma\theta_t + (1-\sigma)\Big(\frac{(c + a\frac{a\alpha\beta\theta y_t - \tau c(\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t))}{a\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)})^{\alpha}}{(c + a\frac{a\alpha\beta y_t - \tau c(\mu+\gamma(1-\mu))}{a\tau(\mu+\gamma(1-\mu)+\alpha\beta)})^{\alpha}}\Big)^{\frac{\kappa}{(1-\gamma)}} = \\
&= \sigma\theta_t + (1-\sigma)\Big((\frac{aa\alpha\beta\theta_t y_t + a\tau c\alpha\beta 2\theta_t}{aa\alpha\beta y_t + a\tau c 2\alpha\beta})^{\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} = \\
&= \sigma\theta_t + (1-\sigma)\theta_t^{\frac{\alpha\kappa}{(1-\gamma)}}
\end{aligned}
$$

The next derivation of the characteristics of the yy and $\theta\theta$ loci and the consequential restrictions for the parameter a are following the derivation of Hiller (2014) in the appendix of his paper. First I will explain the derivation of the yy locus and to do that one again has to consider the different regimes regarding the income one could be in. The first case is the poverty regime in which $y_t < \tilde{y}_t(1)$. In this case the dynamic of y_t is given by $y_{t+1} = y_t = 2c^{\alpha} - s$. Therefore a condition for the steady state value of the income y_{∞} must be that $y_{\infty} < \tilde{y}_t(1)$ holds. This leads to the following restriction for the parameter a

$$
\begin{aligned}
y_{\infty} < \tilde{y}_t(1) &\iff 2c^{\alpha} - s < \frac{\tau c}{a}\Big(\frac{\mu+\gamma(1-\mu)}{\alpha\beta}\Big) \\
&\iff a < \frac{\tau c(\mu+\gamma(1-\mu))}{\alpha\beta(2c^{\alpha}-s)} := \tilde{a}
\end{aligned}
$$

If $a < \tilde{a}$ than the yy locus is a horizontal line and belongs to the poverty regime where $y_t < \tilde{y}_t(1)$ for all time. The second case is described by the fact, that $y_t \in [\tilde{y}_t(1), \tilde{y}_t(\theta_t))$ which leads with the dynamic (5.5) to

$$
y_{t+1} = c^{\alpha} - s + (ay_t + \tau c)^{\alpha}\Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta)}\Big)^{\alpha} := g(y_t)
$$

with

$$
\begin{aligned}
\frac{dg(y_t)}{dy_t} &= \underbrace{a\alpha}_{>0}\underbrace{\Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta)}\Big)^{\alpha}}_{>0}\underbrace{\Big(ay_t+\tau c\Big)^{\alpha-1}}_{>0} > 0 \\
\frac{d^2g(y_t)}{dy_t^2} &= \underbrace{aa\alpha}_{>0}\underbrace{(\alpha-1)}_{<0}\underbrace{\Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta)}\Big)^{\alpha}}_{>0}\underbrace{\Big(ay_t+\tau c\Big)^{\alpha-2}}_{>0} < 0 \\
g(0) &= \underbrace{c^{\alpha}-s}_{\text{assumed to be}>0} + \underbrace{\Big(\frac{c\alpha\beta}{\mu+\gamma(1-\mu)+\alpha\beta}\Big)^{\alpha}}_{>0} > 0
\end{aligned}
$$

This implies that the function $g(y_t)$ is concave and always greater than zero which implies that the equation $y_{t+1} = g(y_t)$ has a unique solution y_{gi}^{yy} for which $y_{gi}^{yy} = g(y_{gi}^{yy})$ holds. This stationary solution belongs to the gender inequality regime if and only if $y_{gi}^{yy} \geq \tilde{y}_t(1)$.

$$\begin{aligned}
y_{gi}^{yy} = \tilde{y}_t(1) &\iff g(\tilde{y}_t(1)) = \tilde{y}_t(1) \\
&\iff \frac{\tau c}{a}\Big(\frac{\mu+\gamma(1-\mu)}{\alpha\beta}\Big) = c^\alpha - s + \Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta)}\Big)^\alpha \Big(a(\frac{\tau c}{a}(\frac{\mu+\gamma(1-\mu)}{\alpha\beta})) + \tau c\Big)^\alpha \\
&\iff \frac{\tau c}{a}\Big(\frac{\mu+\gamma(1-\mu)}{\alpha\beta}\Big) = c^\alpha - s + c^\alpha = 2c^\alpha - s \\
&\iff a = \frac{\tau c(\mu+\gamma(1-\mu))}{\alpha\beta(2c^\alpha - s)} = \tilde{a}
\end{aligned}$$

It can be proofed that $\tilde{y}_t(1)$ is decreasing in a, while y_{gi}^{yy} is increasing in a. This means that the stationary locus, which is again a horizontal line, belongs to the gender inequality regime ($y_\infty \geq \tilde{y}_t(1)$) as long as $a > \tilde{a}$. The last case is described by the interior solution where $y_t \geq \tilde{y}_t(\theta_t)$ which means that $y_{t+1} \overset{!}{=} y_t$ can be written as

$$y_t = \Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t))}\Big)^\alpha (ay_t + 2\tau c)^\alpha (1+\theta_t^\alpha) =: \rho^i(y_t, \theta_t)$$

It is easy to see that $\rho^i(y_t,\theta_t)$ is concave in y_t and that $\rho^i(0,\theta_t) > 0$ which again means that there exists a unique solution $y_i^{yy}(\theta_t)$ that fulfills $y_{t+1} = y_t$. Furthermore, the equation can be rewritten as:

$$f(y_t) := \frac{y_t}{(ay_t + 2\tau c)^\alpha} = \Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t))}\Big)^\alpha (1+\theta_t^\alpha) := h(\theta_t)$$

and therefore it holds that

$$\frac{dy_t}{d\theta_t} = \frac{h'(\theta_t)}{f'(y_t)} \iff \operatorname{sign}\Big(\frac{dy_t}{d\theta_t}\Big) = \operatorname{sign}\Big(\frac{h'(\theta_t)}{f'(y_t)}\Big)$$

The derivative of $f(y_t)$ is given by

$$\frac{df(y_t)}{dy_t} = \frac{(ay_t + 2\tau c)^{\alpha-1}(2\tau c + ay_t(1-\alpha))}{(ay_t+2\tau c)^{2\alpha}} > 0$$

and the derivative of $h(\theta_t)$ by

$$\begin{aligned}
\frac{dh(\theta_t)}{d\theta_t} &= \alpha\Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t))}\Big)^{\alpha-1}\Big(-\frac{\alpha\beta\tau\alpha\beta}{(\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)))^2}\Big)(1+\theta^\alpha) + \dots \\
&\dots + \Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t))}\Big)^\alpha \alpha\theta_t^{\alpha-1} = \\
&= \underbrace{\alpha\Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t))}\Big)^\alpha}_{>0} \underbrace{\Big(\frac{(\mu+\gamma(1-\mu))\theta_t^{\alpha-1} + \alpha\beta(\theta_t^{\alpha-1}-1)}{\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)}\Big)}_{>0} > 0
\end{aligned}$$

because $\theta_t \in [0;1]$ as well as $\alpha \in [0;1]$, which means that $\theta_t^{\alpha-1} > 1$. This means that the unique solution, respectively the yy locus $y_i^{yy}(\theta_t)$, is increasing in θ_t. Furthermore it holds that

$$\begin{aligned}\frac{d^2h(\theta_t)}{d\theta_t^2} &= \alpha\alpha\Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t))}\Big)^{\alpha-1}\Big(-\Big(\frac{\alpha\beta\tau\alpha\beta}{(\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)))^2}\Big)^{\alpha}\Big)\cdot\ldots\\ &\ldots\cdot\Big(\frac{(\mu+\gamma(1-\mu))\theta_t^{\alpha-1}+\alpha\beta(\theta_t^{\alpha-1})}{\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)}\Big)+\alpha\Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t))}\Big)^{\alpha}\cdot\ldots\\ &\ldots\cdot\Big(\frac{((\alpha-1)(\mu+\gamma(1-\mu))\theta_t^{\alpha-2}+\alpha\beta(\alpha-1)\theta_t^{\alpha-2})(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t))}{(\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)))^2}-\ldots\\ &\ldots-\frac{((\mu+\gamma(1-\mu))\theta_t^{\alpha-1}+\alpha\beta(\theta_t^{\alpha-1}-1))\alpha\beta}{(\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t)))^2}\Big)<0\end{aligned}$$

which means that the yy locus is a concave curve $y_i^{yy}(\theta_t)$. Last we have to find a restriction for the value a that grants the existence of the stationary locus in the interior regime, a value that guarantees that $y_i^{yy}(1) > \tilde{y}(1)$. The equality between these two values holds if

$$\begin{aligned}&\tilde{y}(1) = \rho^i(\tilde{y}(1),1)\\ \iff\ &\frac{\tau c}{a}\Big(\frac{\mu+\gamma(1-\mu)}{\alpha\beta}\Big) = \Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+2\alpha\beta)}\Big)^{\alpha}(1+1^{\alpha})\Big(a\frac{\tau c}{a}(\frac{\mu+\gamma(1-\mu)}{\alpha\beta})+2\tau c\Big)^{\alpha}\\ \iff\ &\frac{\tau c}{a}\Big(\frac{\mu+\gamma(1-\mu)}{\alpha\beta}\Big) = 2c^{\alpha}\\ \iff\ &a = \frac{\tau c(\mu+\gamma(1-\mu))}{2c^{\alpha}\alpha\beta} =: \bar{\alpha}\end{aligned}$$

Again we see that $y_i^{yy}(\theta_t)$ is increasing in a, while $\tilde{y}(1)$ is decreasing in a, which means that the yy locus exists in the interior regime for all $a \geq \bar{a}$. Furthermore it is easy to see that $\tilde{a} > \bar{a}$, so the stationary locus in the interior regime exists for all values of $a \in [\bar{a}; \tilde{a})$.

The $\theta\theta$ locus can be obtained in the same way as the yy locus. Again one has to check the characteristic of the locus for the three different regimes the society can be in, respectively the three different cases for the level of income y_t. I start with the poverty regime in which the income y_t is strictly lower than $\tilde{y}(1)$. The $\theta\theta$ locus is again defined as all pairs of (y_t, θ_t) for which $\theta_t = \theta_{t+1}$ holds. According to the equation of the dynamic of the norm (3.14) this is equivalent to

$$\begin{aligned}\theta_t = \theta_{t+1} \iff\ &\theta_t = \sigma\theta_t + (1-\sigma)\Big(\frac{c^{\alpha}-s}{c^{\alpha}}\Big)^{\frac{\kappa}{(1-\gamma)}}\\ \iff\ &\theta_t = \Big(\frac{c^{\alpha}-s}{c^{\alpha}}\Big)\end{aligned}$$

Thereby the $\theta\theta$ locus in the povery regime is described as a vertical line at $\left(\frac{c^{\alpha}-s}{c^{\alpha}}\right)^{\frac{\kappa}{(1-\gamma)}}$, from $y_t = 0$ to $y_t = \tilde{y}(1)$ in the (y_t, θ_t) space. For the second case, the gender inequality regime, the constraint $y_t \in [\tilde{y}_t(1), \tilde{y}_t(\theta_t))$ holds. Therefore the locus restriction $\theta_t = \theta_{t+1}$ and the dynamic of θ_t yield

$$
\begin{aligned}
\theta_t &= \sigma\theta_t + (1-\sigma)\Big(\frac{c^\alpha - s}{\chi(0)(ay_t+\tau c)^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} = \sigma\theta_t + (1-\sigma)\Big(\frac{c^\alpha - s}{(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta)})^\alpha(ay_t+\tau c)^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} \\
\iff \theta_t &= \Big(\frac{c^\alpha - s}{(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta)})^\alpha(ay_t+\tau c)^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} \\
\iff \theta_t^{\frac{(1-\gamma)}{\alpha\kappa}} &= \frac{\tau(\mu+\gamma(1-\mu)+\alpha\beta)(c^\alpha - s)^{\frac{1}{\alpha}}}{\alpha\beta a y_t + \alpha\beta\tau c} \\
\iff \alpha\beta a y_t &= \frac{\tau(\mu+\gamma(1-\mu)+\alpha\beta)(c^\alpha - s)^{\frac{1}{\alpha}}}{\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}}} - \alpha\beta\tau c \\
\iff y_{gi}^{\theta\theta}(\theta_t) := y_t &= \frac{\tau(\mu+\gamma(1-\mu)+\alpha\beta)(c^\alpha - s)^{\frac{1}{\alpha}}}{a\alpha\beta\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}}} - \frac{\tau c}{a}
\end{aligned}
$$

$y_{gi}^{\theta\theta}(\theta_t)$ describes the graph of the $\theta\theta$ locus in the (y_t, θ_t) plane between $y_t = \tilde{y}_t(1)$ and $y_t = \tilde{y}_t(\theta_t)$. To check the characteristics of this graph, I build the first and second derivative of $y_{gi}^{\theta\theta}(\theta_t)$ with respect to θ_t.

$$
\frac{dy_{gi}^{\theta\theta}(\theta_t)}{d\theta_t} = \frac{0 - \frac{(1-\gamma)}{\alpha\kappa}a\alpha\beta\theta_t^{\frac{(1-\gamma)-\alpha\kappa}{\alpha\kappa}}}{(a\alpha\beta\theta_t^{\frac{(1-\gamma)}{\alpha\beta}})^2} < 0
$$

The first derivative is less than zero because $\gamma \in (0;1)$. The second derivative is given by

$$
\begin{aligned}
\frac{d^2y_{gi}^{\theta\theta}(\theta_t)}{d\theta_t^2} &= \frac{\frac{(1-\gamma)}{\alpha\kappa}a\alpha\beta(a\alpha\beta\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}})(-\frac{(1-\gamma)-\alpha\kappa}{\alpha\kappa}\theta_t^{\frac{(1-\gamma)-2\alpha\kappa}{\alpha\kappa}}(a\alpha\beta\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}}) + \theta_t^{\frac{(1-\gamma)-\alpha\kappa}{\alpha\kappa}}2a\alpha\beta\theta_t^{\frac{(1-\gamma)-\alpha\kappa}{\alpha\kappa}}\frac{(1-\gamma)}{\alpha\kappa})}{(a\alpha\beta\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}})^4} = \\
&= \frac{\frac{(1-\gamma)}{\alpha\kappa}a\alpha\beta(-\frac{(1-\gamma)-\alpha\kappa}{\alpha\kappa}a\alpha\beta\theta_t^{\frac{2(1-\gamma)-2\alpha\kappa}{\alpha\kappa}} + 2a\alpha\beta\frac{(1-\gamma)}{\alpha\kappa}\theta_t^{\frac{2(1-\gamma)-2\alpha\kappa}{\alpha\kappa}})}{(a\alpha\beta\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}})^3} = \\
&= \frac{\frac{1}{\kappa}(1-\gamma)a^2\alpha\beta^2\theta_t^{\frac{2(1-\gamma)-2\alpha\kappa}{\alpha\kappa}}(\frac{(1-\gamma)-\alpha\kappa}{\alpha\kappa})}{(a\alpha\beta\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}})^3} = \\
&= \frac{(1-\gamma)\theta_t^{\frac{-(1-\gamma)-2\alpha\kappa}{\alpha\kappa}}(\frac{(1-\gamma)-\alpha\kappa}{\alpha\kappa})}{a\kappa\alpha^2\beta} > 0
\end{aligned}
$$

The last inequality holds because we assume that $\alpha\kappa < (1-\gamma)$. Together this means that $y_{gi}^{\theta\theta}(\theta_t)$ is a convex function of θ_t. Furthermore the graph intersects with $\tilde{y}(1)$ at $\Big(\frac{c^\alpha - s}{c^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}}$, which can easily be shown by solving

$$
\begin{aligned}
y_{gi}^{\theta\theta}(\theta_t) &= \frac{\tau(\mu+\gamma(1-\mu)+\alpha\beta)(c^\alpha - s)^\alpha}{a\alpha\beta\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}}} - \frac{\tau c}{a} \overset{!}{=} \frac{\tau c}{a}\Big(\frac{\mu+\gamma(1-\mu)}{\alpha\beta}\Big) = \tilde{y}(1) \\
\iff \theta_t &= \Big(\frac{c^\alpha - s}{c^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}}
\end{aligned}
$$

The same way one can check where the $y_{gi}^{\theta\theta}(\theta_t)$ intersects with the $\tilde{y}(\theta_t)$ curve:

$$
\begin{aligned}
y_{gi}^{\theta\theta}(\theta_t) &= \frac{\tau(\mu+\gamma(1-\mu)+\alpha\beta)(c^\alpha - s)^\alpha}{a\alpha\beta\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}}} - \frac{\tau c}{a} \overset{!}{=} \frac{\tau c}{a}\Big(\frac{\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t)}{\alpha\beta\theta_t}\Big) = \tilde{y}(\theta_t) \\
\iff \; & \theta_t(c^\alpha - s)^{\frac{1}{\alpha}} = c\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}} \\
\iff \; & \theta_t = \Big(\frac{c^\alpha - s}{c^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)-\alpha\kappa}}
\end{aligned}
$$

All of these characteristics of the $\theta\theta$ locus in the inequality regime, the $y_{gi}^{\theta\theta}(\theta_t)$ curve, together describe the form of the graph shown in figure 3.4. In the last case, in which the system reaches the interior solution and $y_t \geq \tilde{y}(\theta_t)$ the $\theta\theta$ locus is given by a vertical line at $\theta = 1$. This can easily be seen if one takes a look at the dynamics in this case under the restriction of the locus that $\theta_{t+1} = \theta_t$ holds.

$$
\begin{aligned}
&\theta_t = \sigma\theta_t + (1-\sigma)\theta_t^{\frac{\alpha\kappa}{(1-\gamma)}} \\
&\iff \theta_t = \theta_t^{\frac{\alpha\kappa}{(1-\gamma)}} \\
&\iff \theta = 1
\end{aligned}
$$

B.3 Steady State Solutions of the Model

The first case is given by the assumption that $a < \bar{a}$. In this case the yy locus is given by a horizontal line in the (θ_t, y_t) space and belongs to the poverty regime, which means that $y_t < \tilde{y}(1)$ holds. Because the $\theta\theta$ locus in the poverty regime is given by a vertical line in the same space it is obvious that there can just be one intersection between these two loci. To prove that this steady state equilibrium is globally stable, we take a look at the dynamics of the two variables, under the assumption that, because of the value of a, $y_t < \tilde{y}(1)$ holds. Considering the dynamic of the possible total income y_t derived in the previous sections, y_t stays constant over time at its steady state value for every given value of θ_t. On the other hand if one takes the income $y_t < \tilde{y}(1)$ as given and looks at the dynamic of the social norm θ_t we have to consider three different cases to analyze its development, depending if it is greater, lower or equal to its steady state value.

$$
\begin{aligned}
\text{if } \; \theta_t < \Big(\frac{c^\alpha - s}{c^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} &\Rightarrow \theta_{t+1} - \theta_t = \sigma\theta_t + (1-\sigma)\Big(\frac{c^\alpha - s}{c^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} - \theta_t = \\
&= (1-\sigma)\Big(\frac{c^\alpha - s}{c^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} - (1-\sigma)\theta_t > 0 \\
\text{if } \; \theta_t > \Big(\frac{c^\alpha - s}{c^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} &\Rightarrow \theta_{t+1} - \theta_t < 0 \\
\text{if } \; \theta_t = \Big(\frac{c^\alpha - s}{c^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} &\Rightarrow \theta_{t+1} - \theta_t = 0
\end{aligned}
$$

This means that if the social norm is initially lower than the steady state value it increases over time, which means that the economy travels along horizontal yy locus towards the steady state equilibrium. Then again, if the value of the social norm gets higher as its equilibrium value it starts to decline. This continues until the social norm reaches its steady state value and stays there. Altogether this indicates the global stability of the unique steady state solution in the poverty regime one receives in this first case.

In the second case it is assumed that $a \in [\bar{a}, \tilde{a})$ holds, in which case the yy locus in the (θ_t, y_t) space consists of an horizontal line in the poverty regime and a concave function in the interior regime. The same arguments as in the first case guarantee the local stability of the equilibrium located in the poverty regime. Proving the local stability of the steady state solution situated in the interior regime one assumes that for a given θ_t the income fulfills $y_t \geq \tilde{y}(\theta_t)$. From the previous section in the appendix, when the yy locus was derived one knows that

$$y_{t+1} = \rho^i(y_t, \theta_t) = \Big(\frac{\alpha\beta}{\tau(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t))}\Big)^\alpha (ay_t + 2\tau c)^\alpha (1 + \theta_t^\alpha)$$

where $y_i^{yy}(\theta_t)$ describes the level of the possible total income which grants $y_t = \rho^i(y_t, \theta_t)$. As I have already mentioned it is easy to see that the function $\rho^i(y_t, \theta_t)$ is concave and that $\rho^i(0, \theta_t) > 0$ holds. Therefore the function guarantees that for every $y_t < y_i^{yy}(\theta_t)$ it follows that $y_{t+1} > y_t$, respectively for every $y_t > y_i^{yy}(\theta_t)$ it follows that $y_{t+1} < y_t$. I illustrated this beaviour in figure 5.1 for a better understanding and visibility of this conclusion.This means that the dynamic of y_t leads to the steady state solution $y_i^{yy}(\theta_t)$ over time. For the social norm one knows that for a given $y_t \geq \tilde{y}(\theta_t)$ the dynamic of θ_t is given by

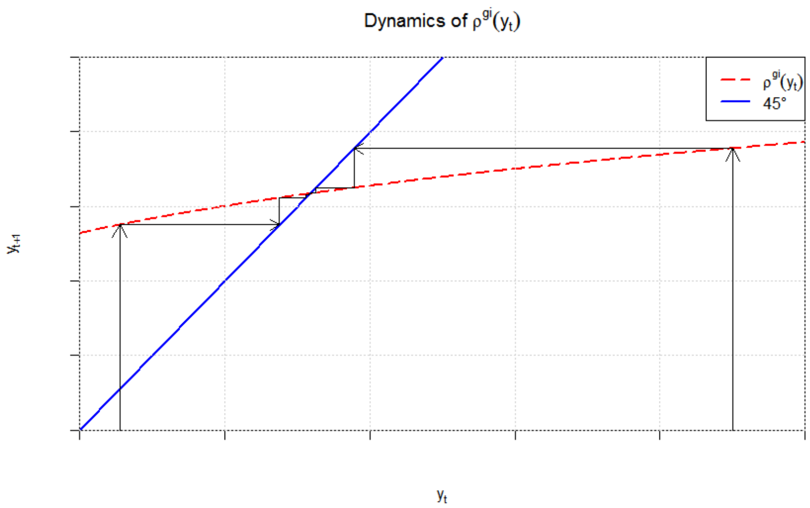

Figure 5.1: Own Calculations with RStudio

$$\theta_{t+1} = \sigma\theta_t + (1-\sigma)\theta_t^{\frac{\alpha\kappa}{(1-\gamma)}}$$
$$\Rightarrow \theta_{t+1} - \theta_t = \sigma\theta_t + (1-\sigma)\theta_t^{\frac{\alpha\kappa}{(1-\gamma)}} - \theta_t = (1-\sigma)\theta_t\Big(\theta^{\frac{\alpha\kappa}{(1-\gamma)}-1} - 1\Big)$$

As assumed in the paper $\frac{\alpha\kappa}{1-\gamma} < 1$ holds and therefore if follows that $\theta^{\frac{\alpha\kappa}{(1-\gamma)}-1} \geq 1$, because $\theta \in [0, 1]$. This means that $\theta_{t+1} - \theta_t > 0$ until the social norm reaches the value 1 and stays there. This behaviour, together with the characteristics of the dynamic of y_t, proves the local stability for the steady state solution located in the interior regime.

For the last two cases I start with showing the local stability of the steady state equilibrium of the gender inequality regime, starting again with the dynamic of the income y_t. Because the solution is situated in the gender inequality regime the fact $y_t \in [\tilde{y}(1), \tilde{y}(\theta_t))$ holds. For a given θ_t the dynamic of the possible total income is given by

$$y_{t+1} = c^\alpha - s + \chi(0)(ay_t + \tau c)^\alpha =: g^{gi}(y_t)$$

The level of income that fulfills $y_t = g^{gi}(y_t)$ is, as defined, given by y_{gi}^{yy}. Again it is easy to see, that under the definition of $\alpha \in (0, 1)$, the function $g^{gi}(y_t)$ is concave and that $g^{gi}(0) > 0$ holds. These characteristics ensure the fact that every $y_t < y_{gi}^{yy}$ leads to $y_{t+1} > y_t$, respectively that every $y_t > y_{gi}^{yy}$ leads to $y_{t+1} < y_t$. Therefore the income converges against the steady state solution. The dynamic of the social norm by a given $y_t \in [\tilde{y}(1), \tilde{y}(\theta_t))$ is given by the following equation.

$$\begin{aligned}
&\theta_{t+1} = \sigma\theta_t + (1-\sigma)\Big(\frac{c^\alpha - s}{\chi(0)(ay_t + \tau c)^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} \\
&\Rightarrow \theta_{t+1} - \theta_t = \sigma\theta_t + (1-\sigma)\Big(\frac{c^\alpha - s}{\chi(0)(ay_t + \tau c)^\alpha}\Big) - \theta_t = (1-\sigma)\Big(\Big(\frac{c^\alpha - s}{\chi(0)(ay_t + \tau c)^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} - \theta_t\Big)
\end{aligned}$$

The level of income that guarantees $\theta_{t+1} - \theta_t = 0$ is as shown in the previous section given by $y_{gi}^{\theta\theta}$. Furthermore it can be shown that $\theta_{t+1} - \theta_t$ is decreasing in y_t:

$$\frac{d(\theta_{t+1} - \theta_t)}{dy_t} = \underbrace{(1-\sigma)}_{>0}\underbrace{\frac{\kappa}{(1-\gamma)}}_{>0}\underbrace{\Big(\frac{c^\alpha - s}{\chi(0)(ay_t + \tau c)^\alpha}\Big)^{\frac{\kappa-1+\gamma}{(1-\gamma)}}}_{>0}\underbrace{\frac{c^\alpha - s}{\chi(0)}}_{>0}\underbrace{(-1)}_{<0}\underbrace{(ay_t + \tau c)^{-2}a}_{>0} < 0$$

This inequality holds because in the paper it is assumed that $c^\alpha - s > 0$ holds. Therefore $\theta_{t+1} - \theta_t$ is decreasing in y_t and equal to zero if $y_t = y_{gi}^{\theta\theta}$. Therefore for every $y_t < y_{gi}^{\theta\theta}$, respectively $y_t > y_{gi}^{\theta\theta}$, for the social norm it holds that $\theta_{t+1} > \theta_t$, respectively $\theta_{t+1} < \theta_t$. Altogether these characteristics ensures the local stability of the steady state solution located in the gender inequality regime.

Last I have to explain the definition of the extra boundary for the parameter a, $\hat{a}$. Therefore one has to find the value of a that shifts the yy locus in the gender inequality regime high enough so that the yy locus, the $\theta\theta$ locus and the $\tilde{y}(\theta_t)$ curve meet at the same point. To find this value I need the definition of these three curves:

$$\begin{aligned}
\tilde{y}(\theta_t) &= \frac{\tau c}{a}\Big(\frac{\mu + \gamma(1-\mu) + \alpha\beta(1-\theta_t)}{\alpha\beta\theta_t}\Big) \\
y_{gi}^{\theta\theta}(\theta_t) &= \frac{\tau(\mu + \gamma(1-\mu) + \alpha\beta)(c^\alpha - s)^{\frac{1}{\alpha}}}{a\alpha\beta\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}}} - \frac{\tau c}{a}
\end{aligned}$$

and y_{gi}^{yy} is given as the solution of $y_t = g(y_t)$. First I find the value of θ where $\tilde{y}(\theta_t)$ and $y_{gi}^{\theta\theta}(\theta_t)$ intersect. Therefore I equalize these both terms:

$$
\begin{aligned}
\tilde{y}(\theta_t) = y_{gi}^{\theta\theta}(\theta_t) \iff & \frac{\tau c}{a}\Big(\frac{\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t)}{\alpha\beta\theta_t}\Big) = \frac{\tau(\mu+\gamma(1-\mu)+\alpha\beta)(c^\alpha - s)^{\frac{1}{\alpha}}}{a\alpha\beta\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}}} - \frac{\tau c}{a} \\
\iff & \frac{\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t)}{\alpha\beta\theta_t} = \frac{(\mu+\gamma(1-\mu)+\alpha\beta)(c^\alpha - s)^{\frac{1}{\alpha}}}{c\alpha\beta\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}}} - 1 \\
\iff & \frac{\theta_t^{\frac{(1-\gamma)}{\alpha\kappa}}}{\theta_t} = (c^\alpha - s)^{\frac{1}{\alpha}} \\
\iff & \hat{\theta}_t = (c^\alpha - s)^{\frac{\kappa}{(1-\gamma-\alpha\kappa)}}
\end{aligned}
$$

where $\hat{\theta}_t$ is the term we are looking for. As one can see this value of θ_t is independent of the level of the productivity of educational expenditures a. Next I will analyze the intersection between y_{gi}^{yy} and $\tilde{y}(\theta_t)$. Because y_{gi}^{yy} is the solution of $y_t = g(y_t)$, the intersection between these two curves is the solution of the equation

$$
\begin{aligned}
& \tilde{y}(\theta_t) = g(\tilde{y}(\theta_t)) \\
\iff & \frac{\tau c}{a}\Big(\frac{\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t)}{\alpha\beta\theta_t}\Big) = \dots \\
& \dots = c^\alpha - s + \Big(a[\frac{\tau c}{a}\Big(\frac{\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t)}{\alpha\beta\theta_t}\Big)] + \tau c\Big)^\alpha \Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta)}\Big)^\alpha \\
\iff & \frac{\tau c}{a}\Big(\frac{\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t)}{\alpha\beta\theta_t}\Big) = c^\alpha - s + \Big(\frac{c^\alpha}{\theta_t}\Big)^\alpha \\
\iff & \frac{\tau c}{a} = \frac{\alpha\beta(\theta_t(c^\alpha - s) + c^\alpha\theta^{1-\alpha})}{\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t)}
\end{aligned}
$$

The value of the social norm θ one is looking for is the solution of this equation and will be denoted as $\tilde{\theta}(a)$. Now I take a look at the right-hand side term of the equation and how it changes, if θ_t changes, to analyze how $\tilde{\theta}(a)$ reacts, if a changes. Therefore I build the derivative of the right-hand side with respect to θ_t.

$$
\begin{aligned}
& \frac{d}{d\theta_t}\Big(\frac{\alpha\beta(\theta_t(c^\alpha - s) + c^\alpha\theta^{1-\alpha})}{\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t)}\Big) = \\
& = \frac{(\alpha\beta(c^\alpha - s) + (1-\alpha)c^\alpha\theta_t^{-\alpha})(\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t)) - (\alpha\beta(\theta_t(c^\alpha - s) + c^\alpha\theta^{1-\alpha}))(-\alpha\beta)}{(\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t))^2} = \\
& = \frac{\overbrace{(\alpha\beta(c^\alpha - s)}^{>0} + \overbrace{(1-\alpha)c^\alpha\theta_t^{-\alpha})}^{>0}\overbrace{(\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t))}^{>0} + \overbrace{\alpha^2\beta^2}^{>0}\overbrace{(\theta_t(c^\alpha - s) + c^\alpha\theta^{1-\alpha}))}^{>0}}{\underbrace{(\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_t))^2}_{>0}} > 0
\end{aligned}
$$

This implies, that if a increases, the left-hand side of the upper equation will decrease. To compensate this decrease the previous derivative shows that the value of θ has to decrease so that the whole right-hand side decreases as well. This means that the defined $\tilde{\theta}(a)$ is decreasing in a. Furthermore it shows that $\lim\limits_{a\to 0}\tilde{\theta}_t = +\infty$ and $\lim\limits_{a\to+\infty}\tilde{\theta}_t = 0$ holds, which means that there exists a unique value of a so that $\tilde{\theta}(a) = \hat{\theta}$. This unique value of a is exactly the extra boundary we were looking for which Hiller denotes by $\hat{a}$.

With this newly defined extra boundary we can define our last two cases that can appear in the analysis of the steady state equilibria. If the parameter of the productivity of education fulfils $a \in [\tilde{a}, \hat{a})$ it guarantees that there exists a steady state solution in the gender inequality regime. In this case y_{gi}^{yy} and $y_{gi}^{\theta\theta}$, the parts of the two loci that belong to the gender inequality regime, intersects exactly once. This is because the value of the social norm where the part of the yy locus intersects with $\tilde{y}(\theta)$, $\tilde{\theta}(a)$, is higher than the value of θ_t where the part of the $\theta\theta$ locus intersects with $\tilde{y}(\theta)$. Therefore in this case there exists, as shown, a locally stable equilibrium in the gender inequality regime and one locally stable solution in the interior regime.

In the last case of our analysis it holds that $a \geq \hat{a}$. In this case the value of the parameter is high enough so that the intersection between y_{gi}^{yy} and $y_{gi}^{\theta\theta}$ disappears, because $\tilde{\theta}(a) \leq \hat{\theta}$. This means that the horizontal line y_{gi}^{yy} intersects with $\tilde{y}(\theta)$ before it can intersect with the vertical line $y_{gi}^{\theta\theta}$. This means that in this case just the equilibrium located in the interior regime is left, which in this case then is globally stable.

B.4 Policy Implications

First I show the effects of paramter changes of a and τ on $\tilde{y}(\theta_t)$. From its definition

$$\tilde{y}(\theta_t) = \frac{\tau c}{a}\Big(\frac{\mu + \gamma(1-\mu) + \alpha\beta(1-\theta_t)}{\alpha\beta\theta_t}\Big)$$

it follows, that a decrease in τ as well as an increase in a leads to an left shift of $\tilde{y}(\theta_t)$ in the (θ_t, y_t) space. To analyze the changes on the steady state value of the income in the gender inequality regime, y_{gi}^{yy}, one considers its implicit definition, given by

$$\begin{aligned} y_{t+1} &= c^\alpha - s + \chi(0)(ay_t + \tau c)^\alpha =: g^{gi}(y_t) \\ y_t &= g^{gi}(y_t) =: y_{gi}^{yy} \end{aligned}$$

For a parameter change in a it is easy to see, that an increase in a leads to an increase of the right-hand side and therefore to an increase of the steady state value y_{gi}^{yy}. The effects of changes in τ are not that obvious and therefore I build the derivative of the right-hand side with respect to τ.

$$\begin{aligned} &\frac{\partial}{\partial\tau}\Big(c^\alpha - s + \chi(0)(ay_t + \tau c)^\alpha\Big) = \frac{\partial}{\partial\tau}\Big(c^\alpha - s + \Big(\frac{\alpha\beta}{\tau(\mu + \gamma(1-\mu) + \alpha\beta)}\Big)^\alpha (ay_t + \tau c)^\alpha\Big) = \\ &\quad= \frac{\partial}{\partial\tau}\Big(c^\alpha - s + \frac{\alpha^\alpha\beta^\alpha(ay_t + \tau c)^\alpha}{\tau^\alpha(\mu + \gamma(1-\mu) + \alpha\beta)^\alpha}\Big) = \\ &\quad= \frac{\alpha^\alpha\beta^\alpha\alpha(ay_t + \tau c)^{\alpha-1}\tau^{\alpha-1}(\mu + \gamma(1-\mu) + \alpha\beta)^\alpha(\tau c - (ay_t + \tau c))}{\tau^{2\alpha}(\mu + \gamma(1-\mu) + \alpha\beta)^{2\alpha}} = \\ &\quad= \frac{\alpha^\alpha\beta^\alpha\alpha(ay_t + \tau c)^{\alpha-1}(-ay_t)}{\tau^{1+\alpha}(\mu + \gamma(1-\mu) + \alpha\beta)^\alpha} = -\frac{\alpha^{\alpha+1}\beta^\alpha(ay_t + \tau c)^{\alpha-1}ay_t}{\tau^{1+\alpha}(\mu + \gamma(1-\mu) + \alpha\beta)^\alpha} < 0 \end{aligned}$$

This implies that an increase in τ leads to a decrease of the steady state value of the total possible income in the gender inequality regime y_{gi}^{yy}. To receive results about the effect of paramter changes on the steady state value of the social norm θ_t I will pursue in a similar way and analyze the effect of the specific paramters on the impicit definition of $y_{gi}^{\theta\theta}$.

$$\begin{aligned}
\theta_{t+1} - \theta_t &= (1-\sigma)\Big(\Big(\frac{c^\alpha - s}{\chi(0)(ay_t + \tau c)^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} - \theta_t\Big) = \\
&= (1-\sigma)\Big(\Big(\frac{c^\alpha - s}{\Big(\frac{\alpha\beta}{\tau(\mu+\gamma(1-\mu)+\alpha\beta)}\Big)^\alpha (ay_t + \tau c)^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} - \theta_t\Big) = \\
&= (1-\sigma)\Big(\Big(\frac{c^\alpha - s}{\Big(\frac{(\alpha\beta)^\alpha}{\tau^\alpha(\mu+\gamma(1-\mu)+\alpha\beta)^\alpha}\Big)(ay_t + \tau c)^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} - \theta_t\Big) = \\
&= (1-\sigma)\Big(\Big(\frac{c^\alpha - s}{\Big(\frac{(\alpha\beta)}{(\mu+\gamma(1-\mu)+\alpha\beta)}\Big)^\alpha (\frac{ay_t+\tau c}{\tau})^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} - \theta_t\Big) = \\
&= (1-\sigma)\Big(\Big(\frac{c^\alpha - s}{\Big(\frac{(\alpha\beta)^\alpha}{(\mu+\gamma(1-\mu)+\alpha\beta)^\alpha}\Big)(\frac{ay_t}{\tau} + c)^\alpha}\Big)^{\frac{\kappa}{(1-\gamma)}} - \theta_t\Big)
\end{aligned}$$

It follows that an increase in a leads to a decrease in the steady state value of the social norm and a decrease in τ induces an increase in it.

B.5 Full Transition and the U-shaped Female Labour Force Participation

To show the validity of equation (3.15) and (3.16) I need the budget constraint (3.3), the definition of the household good production function (3.6), the first order condition for the time spend for the production of household goods (3.9) and the optimal values of the education expenditures (3.11). We start with equation (3.9), which can be transformed with help of equation (3.6) the following way:

$$\begin{aligned}
&\frac{w_t^f \mu}{C_t\gamma(1-\mu)} = \frac{(l_t^f)^{\gamma-1}}{D_t} \iff \frac{w_t^f \mu}{C_t\gamma(1-\mu)} = \frac{(l_t^f)^{\gamma-1}}{(l_t^f)^\gamma + (l_t^m)^\gamma} \\
&\iff \frac{w_t^f \mu}{C_t\gamma(1-\mu)} = \frac{\frac{1}{l_t^f}}{1 + (\frac{w_t^f}{w_t^m})^{\frac{\gamma}{(1-\gamma)}}} \iff l_t^f = \Big(\frac{\gamma(1-\mu)}{(1 + (\frac{w_t^f}{w_t^m})^{\frac{\gamma}{(1-\gamma)}})}\Big)\frac{C_t}{w_t^f}
\end{aligned}$$

The last equation is exactly the one given by (3.15). To receive the second equation (3.16), one takes the budget constraint (3.3) and transforms it to

$$\begin{aligned}
&(1-l_t^f)w_t^f + (1-l_t^m)w_t^m = C_t + \tau(e_{t+1}^f + e_{t+1}^m) \\
&\iff w_t^f + w_t^m - (w_t^f l_t^f + w_t^m l_t^m) = C_t + \tau(e_{t+1}^f + e_{t+1}^m) \\
&\iff y_t - \Big(\frac{\gamma(1-\mu)C_t}{\mu}\Big) = C_t + \tau(e_{t+1}^f + e_{t+1}^m) \\
&\iff C_t + \Big(\frac{\gamma(1-\mu)C_t}{\mu}\Big) = y_t - \tau(e_{t+1}^f + e_{t+1}^m) \\
&\iff C_t = \frac{\mu(y_t - \tau(e_{t+1}^f + e_{t+1}^m))}{\mu + \gamma(1-\mu)}
\end{aligned} \tag{5.6}$$

To receive the result I am looking for one has to insert the regime specific optimal values for e_{t+1}^f and e_{t+1}^m from equation (3.11) into the equation (5.6). I start with the gender inequality regime:

$$e^f_{t+1} + e^m_{t+1} = 0 + \frac{a\alpha\beta y_t - \tau c(\mu + \gamma(1-\mu))}{a\tau(\mu + \gamma(1-\mu) + \alpha\beta)}$$

$$\Rightarrow C_t = \frac{\mu(y_t - \tau(\frac{a\alpha\beta y_t - \tau c(\mu+\gamma(1-\mu))}{a\tau(\mu+\gamma(1-\mu)+\alpha\beta)}))}{\mu + \gamma(1-\mu)} = \frac{\mu(\mu + \gamma(1-\mu))(ay_t + \tau c)}{(\mu + \gamma(1-\mu))(a(\mu + \gamma(1-\mu) + \alpha\beta))} =$$

$$= \frac{\mu(ay_t + \tau c)}{a(\mu + \gamma(1-\mu) + \alpha\beta)}$$

For the interior regime one receives:

$$e^f_{t+1} + e^m_{t+1} = \frac{a\alpha\beta\theta_t y_t - \tau c(\mu + \gamma(1-\mu) + \alpha\beta(1-\theta_t))}{a\tau(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t))} + \frac{a\alpha\beta y_t - \tau c(\mu + \gamma(1-\mu) - \alpha\beta(1-\theta_t))}{a\tau(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t))} =$$

$$= \frac{a\alpha\beta y_t(1+\theta_t) - \tau c(2\mu + 2\gamma(1-\mu))}{a\tau(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t))}$$

$$\Rightarrow C_t = \frac{\mu(y_t - \tau(\frac{a\alpha\beta y_t(1+\theta_t) - \tau c(2\mu+2\gamma(1-\mu))}{a\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t))}))}{\mu + \gamma(1-\mu)} = \frac{\mu(\frac{ay_t(\mu+\gamma(1-\mu)) + 2\tau c(\mu+\gamma(1-\mu))}{a(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_t))})}{(\mu + \gamma(1-\mu))} =$$

$$= \frac{\mu(ay_t + 2\tau c)}{a(\mu + \gamma(1-\mu) + \alpha\beta(1+\theta_t))}$$

Lastly to receive equation (3.16) one has to divide both equations for C_t by w^f_t.

To proof the equation for the relative wage $\frac{w^m_t}{w^f_t}$ one just needs the defined formula of the gender specific wage (3.4) and the human capital function (3.2) and the optimal values of the education expenditures in the interior regime given by equation (3.11).

$$\frac{w^m_t}{w^f_t} = \frac{(c + ae^m_t)^\alpha}{(c + ae^f_t)^\alpha} = \Big(\frac{c + ae^m_t}{c + ae^f_t}\Big)^\alpha =$$

$$= \Big(\frac{c + a(\frac{a\alpha\beta y_{t-1} - \tau c(\mu+\gamma(1-\mu)-\alpha\beta(1-\theta_{t-1}))}{a\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_{t-1}))})}{c + a(\frac{a\alpha\beta\theta_{t-1} y_{t-1} - \tau c(\mu+\gamma(1-\mu)+\alpha\beta(1-\theta_{t-1}))}{a\tau(\mu+\gamma(1-\mu)+\alpha\beta(1+\theta_{t-1}))})}\Big)^\alpha =$$

$$= \Big(\frac{a\alpha\beta y_{t-1} + \tau c\alpha\beta(1+\theta_{t-1}) + \tau c\alpha\beta(1-\theta_{t-1})}{a\alpha\beta\theta_{t-1} y_{t-1} + \tau c\alpha\beta(1+\theta_{t-1}) - \tau c\alpha\beta(1-\theta_{t-1})}\Big)^\alpha = \Big(\frac{a\alpha\beta y_{t-1} + \tau c(2\alpha\beta)}{a\alpha\beta\theta_{t-1} y_{t-1} + \tau c(2\alpha\beta\theta_{t-1})}\Big)^\alpha =$$

$$= \Big(\frac{1}{\theta_{t-1}}\frac{a\alpha\beta y_{t-1} + \tau c(2\alpha\beta)}{a\alpha\beta y_{t-1} + \tau c(2\alpha\beta)}\Big)^\alpha = \Big(\frac{1}{\theta_{t-1}}\Big)^\alpha =$$

$$= \frac{1}{\theta^\alpha_{t-1}}$$

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